Coordination and Crisis in Monetary Unions

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Abstract

We characterize fiscal and monetary policy in a monetary union with the potential for rollover crises in sovereign debt markets. Member-country fiscal authorities lack commitment to repay their debt and choose fiscal policy independently. A common monetary authority chooses inflation for the union, also without commitment. We first describe the existence of a fiscal externality that arises in the presence of limited commitment and leads countries to over borrow; this externality rationalizes the imposition of debt ceilings in a monetary union. We then investigate the impact of the composition of debt in a monetary union, that is the fraction of high-debt versus low-debt members, on the occurrence of self-fulfilling debt crises. We demonstrate that a high-debt country may be less vulnerable to crises and have higher welfare when it belongs to a union with an intermediate mix of high- and low-debt members, than one where all other members are low-debt. This contrasts with the conventional wisdom that all countries should prefer a union with low-debt members, as such a union can credibly deliver low inflation. These findings shed new light on the criteria for an optimal currency area in the presence of rollover crises.

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1 Introduction

Monetary unions like the euro zone are characterized by centralized monetary policy and decentralized fiscal policy. The on going crisis in the euro zone highlights the well known problems associated with stabilizing asymmetric shocks with a common monetary policy. These problems have been studied in depth starting with the seminal work of Mundell (1961) on optimal currency areas. The crisis however has brought to the forefront a much less understood phenomena of the consequences of heterogenous sovereign debt positions in a monetary union on monetary policy and the conflicts that can arise. For instance there is significant disagreement among euro zone members on how to confront the sovereign debt crisis in high-debt countries. Countries like Germany are concerned about the fiscal and inflationary consequences of the ECB's promise to purchase sovereign debt of periphery economies in the event of a crisis. On the other hand, crisis economies like Spain, Portugal, Ireland and Greece argue that a lender of last resort is required to remove or mitigate the threat of a self-fulfilling rollover crisis.[1] In this paper we shed light on these under studied issues. We introduce a framework that allows us to address the role of uncoordinated fiscal policy but centralized monetary policy in nominal debt dynamics and exposure to self-fulfilling debt crises.[2] Our findings shed new light on the criteria for an optimal currency area in the presence of debt crises.[3]

The environment consists of individual fiscal authorities that choose how much to consume and borrow by issuing nominal bonds. A common monetary authority chooses inflation for the union, taking as given the fiscal policy of its member countries. Both fiscal and monetary policy is implemented without commitment. The lack of commitment on fiscal policy raises the possibility of default. The lack of commitment on monetary policy makes the central bank vulnerable to the temptation to inflate away the real value of its members' nominal debt. In choosing the optimal policy ex post, the monetary authority trades off the distortionary costs of inflation against the fiscal benefits of debt reduction. Lenders recognize this temptation and charge a higher nominal interest rate ex ante, making ex post inflation

[1] De Grauwe (2011) emphasizes the importance of the lender of last resort role for the ECB.

[2] Araujo et al. (2012) consider some implications of currency denomination of debt in the presence of self-fulfilling crisis. Dixit and Lambertini (2001) and Dixit and Lambertini (2003) examine the implications for output and inflation in a monetary union where fiscal policy is decentralized and monetary policy is centralized, allowing for the authorities to have conflicting goals for output and inflation. Cooper et al. (2009) and Cooper et al. (2010) examine the interaction between fiscal and monetary policy in a monetary union including exploring the incentives for a monetary bailout in the presence of regional debt. There exists an important literature jointly analyzing fiscal and monetary policies in a monetary union in the presence of New Keynesian frictions such as for example Beetsma and Jensen (2005), Gali and Monacelli (2008), Ferrero (2009) and Farhi and Werning (2013). The focus of our paper differs from this literature as it is on debt, inflation and crises.

[3] For a survey on optimal currency areas see Silva and Tenreyro (2010).

self defeating.[4]

The joint lack of commitment and coordination gives rise to a fiscal externality in a monetary union. The monetary authority's incentive to inflate depends on the aggregate value of debt in the union. Each country in the union ignores the impact of its borrowing decisions on the evolution of aggregate debt and hence on inflation. We compare this to the case of a small open economy where the fiscal and monetary authority coordinate on decisions while maintaining the assumption of limited commitment. We show that a monetary union leads to higher debt, higher long-run inflation and lower welfare. While coordination eliminates the fiscal externality, it does not replicate the full-commitment outcome. We show that full commitment in monetary policy gives rise to the first best level of welfare, with or without coordination on fiscal policy. These two cases allow us to decompose the welfare losses in the monetary union due to lack of coordination versus lack of commitment.[5] The presence of this fiscal externality rationalizes the imposition of debt ceilings in a monetary union.[6]

In this context of debt overhang onto monetary policy, we explore the composition of the monetary union. In particular, we consider a union comprised of high- and low-debt economies, where the groups differ by the level of debt at the start of the monetary union. Consider first the case without rollover crises, that is there is no coordination failure among lenders in rolling over maturing debt. While inflation is designed to alleviate the real debt burden of the members, all members, regardless of debt levels, would like to be part of a low-debt monetary union. This is because in a high-debt monetary union the common monetary authority is tempted to inflate to provide debt relief ex post but the lenders anticipate this and the higher inflation is priced into interest rates ex ante. Consequently, the members in a union obtain no debt relief and only incur the dead weight cost of inflation. A low-debt monetary union therefore better approximates the full-commitment allocation of low inflation and correspondingly low nominal interest rates. High-debt members recognize they will roll over their nominal bonds at a lower interest rate in such a union, thereby benefiting from joining a low-debt monetary union. This agreement on membership criteria however does not survive the possibility of rollover crises.

In particular, we consider equilibria in which lenders fail to coordinate on rolling over

[4]Barro and Gordon (1983) in a seminal paper demonstrate the time inconsistency of monetary policy and the resulting inflationary bias.

[5]Chari and Kehoe (2007) describe the roll of commitment in eliminating the fiscal externality in a monetary union. We demonstrate the separate role of coordination and of commitment in affecting inflation, debt dynamics and welfare in a monetary union.

[6]Debt ceilings on member countries are a feature of the Stability and Growth pact in the eurozone. Similarly debt ceilings exist on individual states in the U.S. Von Hagen and Eichengreen (1996) provide evidence of debt constraints on sub-national governments in a large number of countries, each of which works like a monetary union. Beetsma and Uhlig (1999) provide an argument for debt ceilings in a monetary union that arise from political economy constraints, namely short-sighted governments.

maturing debt. This opens the door to self-fulfilling debt crises for members with high enough debt levels. In this environment, there is a trade off regarding membership criteria. As in the no-crisis benchmark, a low-debt union can credibly promise low inflation, which leads to low nominal interest rates and low distortions. However, in the presence of rollover crises monetary policy not only should deliver low inflation in tranquil times but also serve as a lender of last resort to address (and potentially eliminate) coordination failures among lenders. The monetary authority of a union comprised mainly of low-debtors may be unwilling to inflate in the event of a crisis, as such inflation benefits only the highly indebted members at the expense of higher inflation in all members. That is, while low-debt membership provides commitment to deliver low inflation in good times, it undermines the central banks credibility to act as lender of last resort. Therefore, highly indebted economies prefer a monetary union in which a sizeable fraction of members also have high debt, balancing commitment to low inflation against commitment to act as a lender of last resort.

Importantly, the credibility to inflate in response to a crisis (an off-equilibrium promise) may eliminate a self-fulfilling crisis without the need to inflate in equilibrium. This is reminiscent of the events in the summer of 2012 when the announcement by the ECB president Mario Draghi to defend the euro at all costs sharply reduced the borrowing costs for Spain, Italy, Portugal, Greece and Ireland. This put the brakes on what arguably looked like a self-fulfilling debt crises in the euro zone, without the ECB having to buy any distress country debt.[7]

One way to interpret these findings is to consider the decision of an indebted country to join a monetary union or to have independent control over its monetary policy. In the absence of rollover crises the country is best served by joining a monetary union with low aggregate debt, as in such a union the monetary authority will deliver low inflation. This is the classic argument for joining a union with a monetary authority that has greater credibility to keep inflation low.[8] By contrast, in the presence of self-fulfilling roll-over crises, the country can be better off by joining a monetary union with intermediate level of aggregate debt, as this reduces its vulnerability to self-fulfilling crises compared to a union with low aggregate debt.

Importantly, inflation credibility can be influenced endogenously through the debt composition of the monetary union. These findings shed new light on the criteria for an optimal currency area and relates to the literature on institutional design for monetary policy. Rogoff

[7] An alternative strategy would be for the core countries to promise fiscal transfers to the periphery in the event of the crisis. The political economy constraints on engineering such transfers and the weak credibility of such promises make the ECB intervention more practical and credible, which is why we focus on the latter.

[8] Alesina and Barro (2002) highlight the benefits of joining a currency union whose monetary authority has greater commitment to keeping inflation low in an environment where Keynesian price stickiness provides an incentive for monetary authorities to inflate ex post.

(1985) highlighted the virtues of delegating monetary authority to a central banker whose objective function can differ from society's, so as to gain inflation credibility. Implementing such delegation however may be difficulty if society disagrees with the central banker's objectives. Here we demonstrate how debt characteristics of monetary union members endogenously impacts the inflation credibility of the monetary authority.

The rest of the paper is structured as follows. Section 2 presents the model in an environment without roll-over crises. It characterizes the fiscal externality in a monetary union. Section 3 analyzes the case with roll-over crises. Section 5 discusses the implications for the optimal composition of a union an indebted country is considering joining and Section 6 concludes.

2 Model

2.1 Environment

There is a measure-one continuum of small open economies, indexed by $i \in [0, 1]$, that form a monetary union. Fiscal policy is determined independently at the country-level, while monetary policy is chosen by a single monetary authority. In this section we consider the case where economies are not subject to roll-over risk, that is lenders can commit to roll-over debt. We introduce rollover risk in section 3.

Time is continuous and there is a single traded consumption good with a world price normalized to one. Each economy is endowed with $y_i = y$ units of the good each period that is assumed to be constant. The local currency price at time t is denoted $P_t = P(t) = P(0)e^{\int_0^t \pi(t)dt}$, where $\pi(t)$ denotes the rate of inflation at time t.[9] The domestic-currency price level is the same across member countries and its evolution is controlled by the central monetary authority.[10]

Preferences Each fiscal authority has preferences over paths for consumption and inflation given by:

$$U^f = \int_0^\infty e^{-\rho t} \left(u\left(c_i(t)\right) - \psi(\pi(t)) \right) dt. \tag{U$^\text{f}$}$$

[9] As we shall see, we assume that the monetary authority's policy selects $\pi(t) \leq \bar{\pi} < \infty$, and so the domestic price level is a continuous function of time. Moreover, we treat the initial price level $P(0)$ as a primitive of the environment, which avoids complications arising from a large devaluation in the initial period.

[10] For evidence of convergence in euro area inflation rates and price levels see Lopez and Papell (2012) and Rogers (2001).

Utility over consumption satisfies the usual conditions, $u' > 0, u'' < 0$, $\lim_{c\downarrow 0} u'(c) = \infty$. As the fiscal authority controls $c_i(t)$, $u(c)$ is the relevant portion of the objective function in terms of fiscal choices. The second term, $\psi(\pi(t))$ reflects the preferences of the fiscal authority in each country over the inflation choices made by the central monetary authority. This term captures in reduced form the distortionary costs of inflation borne by the individual countries. For tractability purposes we assume $\psi(\pi(t)) \equiv \psi_0 \pi(t)$ and we restrict the choice of inflation to the interval $\pi \in [0, \bar{\pi}]$.

The monetary authority preferences are an equally-weighted aggregate:

$$U^m = \int_0^\infty e^{-\rho t} \left(\int_i u\left(c_i(t)\right) di - \psi(\pi(t)) \right) dt. \tag{U$^{\mathrm{m}}$}$$

Bond Markets Each country i can issue a non-contingent nominal bond that must be continuously rolled over. Denote $B_i(t)$ the outstanding stock of country i's debt, the real value of which is denoted $b_i(t) \equiv \frac{B_i(t)}{P(t)}$. We normalize the price of a bond to one in local currency and clear the market by allowing the equilibrium nominal interest rate $r_i(t)$ to adjust. Denoting country i's consumption by $c_i(t)$, the evolution of nominal and real debt is given by:

$$\begin{aligned} \dot{B}_i(t) &= P(t)\left(c_i(t) - y\right) + r_i(t) B_i(t) \\ \dot{b}_i(t) &= c_i(t) - y + \left(r_i(t) - \pi(t)\right) b_i(t), \end{aligned}$$

where the second line uses the identity $\dot{b}(t)/b(t) = \dot{B}(t)/B(t) - \pi(t)$.

Fiscal authorities cannot commit to repay loans. At any moment, a fiscal authority can default and pay zero. If it defaults, it is punished by permanent loss of access to international debt markets plus a loss to output given by the parameter χ. We assume that when an individual country makes the decision to default it is not excluded from the union. We let $\underline{V}$ represent the continuation value after a default.

$$\underline{V} = \frac{u((1-\chi)y)}{\rho} - \int_0^\infty e^{-\rho t} \psi(\pi(t)) dt. \tag{1}$$

Note that the default payoff depends on currency-wide inflation, but does not depend on the amount of debt prior to default.

Bonds are purchased by risk-neutral lenders who behave competitively and have an opportunity cost of funds $r^\star = \rho$. We ignore the resource constraint of lenders as a group by assuming that the monetary union is small in world financial markets (although each country is a large player in terms of its own debt). In particular, we assume that country i's bond market clears as long as the expected real return is $r^\star$.

2.2 Symmetric Markov Perfect Equilibrium

We are interested in the equilibrium of the game between competitive lenders, individual fiscal authorities, and a centralized monetary authority. In particular, we construct a Markov perfect equilibrium in which each member country behaves symmetrically in terms of policy functions. The payoff-relevant state variables are the outstanding amounts of nominal debt issued by member countries. We can substitute the real value of debt under the assumption that $P(0)$ is given; that is, the monetary authority cannot erase all nominal liabilities at the start of time with a discrete devaluation of the price level. This is similar to bounding the initial capital levy in a canonical Ramsey taxation program.[11]

In general, the aggregate state is the distribution of bonds across all members of the monetary union. We are interested in environments in which members differ in their debt stocks, allowing us to explore potential disagreement among members regarding policy and the optimal composition of the monetary union. On the other hand, tractability requires limiting the dimension of the state variable. To this end, we consider a union comprised of high and low debt countries in the initial period. Let $\eta \in (0, 1]$ denote the measure of high-debt economies, and denote this group H and the low-debt group L. For tractability, we assume that there is no within-group heterogeneity; that is, $b_i(0) = b_H(0)$ for all $i \in H$ and $b_j(0) = b_L(0)$ for all $j \in L$, with $b_H(0) > b_L(0)$.

We focus on equilibria with symmetric policy functions, and so the initial within-group symmetry is preserved in equilibrium. It is useful to introduce the following notation. Let $\boldsymbol{b}(t)_H = \frac{1}{\eta}\int_{i\in H} b_i(t)di$ denote the mean debt stock of the high-debt group, and similarly $\boldsymbol{b}(t)_L = \frac{1}{1-\eta}\int_{i\in L} b_i(t)di$ denote the debt stock of the low-debt group. Let $\boldsymbol{b} = (\boldsymbol{b}_H, \boldsymbol{b}_L)$ denote the vector of mean debt stocks in the two subgroups of members.

Using this notation, the relevant state variable for an individual fiscal authority is the triplet $(b, \boldsymbol{b}_H, \boldsymbol{b}_L) = (b, \boldsymbol{b})$, where the first argument is the country's own debt level and the latter characterizes the aggregate state. Let $C(b, \boldsymbol{b})$ denote the optimal policy function for the representative fiscal authority in the symmetric equilibrium. The monetary authority's policy function is denoted $\Pi(\boldsymbol{b})$, where we incorporate in the notation that monetary policy is driven by aggregate states alone and does not respond to idiosyncratic deviations from the symmetric equilibrium.

[11] This also speaks to the differences between our environment and the "fiscal theory of the price level." In that literature, the initial price level adjusts to ensure that real liabilities equal a given discounted stream of fiscal surpluses. In our environment, we take the initial price level as given and solve for the equilibrium path of fiscal surpluses and inflation.

The individual fiscal authority faces an equilibrium interest rate schedule denoted $r(b, \boldsymbol{b})$. The interest rate depends on the first argument via the risk of default and the latter two arguments via anticipated inflation. In the current environment we abstract from rollover crises and focus on perfect-foresight equilibria. Lenders will not purchase bonds if default is perfectly anticipated, and thus fiscal authorities will have debt correspondingly rationed. From the lender's perspective, the real return on government bonds absent default is $r(b, \boldsymbol{b}) - \Pi(\boldsymbol{b})$, which must equal $r^\star$ in equilibrium.[12] In the deterministic case, there is no interest rate that supports bond purchases if the government will default. Let $\bar{\Omega} \subset [0, \infty)$ denote the endogenous domain of debt stocks that can be issued in equilibrium.[13]

Each fiscal authority takes the inflation policy function of the monetary authority $\Pi(\boldsymbol{b})$ as given, as well as the consumption policy functions of the other members of the union, which we distinguish using a tilde, $\tilde{C}(b, \boldsymbol{b})$. Given an initial state $(b, \boldsymbol{b}) \in \bar{\Omega}^3$ and facing an interest rate schedule $r(\boldsymbol{b})$ and domain $\overline{\Omega}$, the fiscal authority with initial debt $b \in \overline{\Omega}$ solves the problem:

$$V(b, \boldsymbol{b}) = \max_{c(t)} \int_0^\infty e^{-\rho t} \left(u\left(c(t)\right) - \psi_0 \pi(\boldsymbol{b}(t))\right) dt, \tag{P1}$$

$$\begin{aligned}
&\text{subject to} \\
&\dot{b}(t) = c(t) - y + (r(\boldsymbol{b}(t)) - \pi(\boldsymbol{b}(t)))b(t) \text{ with } b(0) = b \\
&\dot{\boldsymbol{b}}_j(t) = \tilde{C}(\boldsymbol{b}_j(t), \boldsymbol{b}(t)) - y + (r(\boldsymbol{b}(t)) - \Pi(\boldsymbol{b}(t)))\boldsymbol{b}_j(t), \text{ for } j = H, L \\
&b(t) \in \bar{\Omega}, t \geq 0.
\end{aligned}$$

Note that this problem is written under the premise the government does not default. This will be the case for any domain $\bar{\Omega}$ that is sustainable in equilibrium.

The monetary authority sets inflation $\pi(t)$ in every period without commitment. The decision of the monetary authority can be represented by a sequence problem where the

[12] To expand on this break-even condition, consider a bond purchased in period t that matures in period $t+m$ and carries a fixed interest rate $r_t = r(b(t), \boldsymbol{b}(t))$. The nominal return of this bond is $e^{r_t m}$. Equilibrium requires that the real return per unit time is $r^\star$:

$$\left(\frac{P^e(t+m)}{P(t)}\right) e^{r_t m} = e^{r^\star m},$$

where superscript e denotes equilibrium expectations. Taking logs of both sides, dividing by m, letting $m \to 0$, and using the definition that $\pi^e(t) = \lim_{m \downarrow 0} \frac{\ln P^e(t+m) - \ln P(t)}{m}$, gives the condition $r_t = r^\star - \pi^e(t)$. In equilibrium, $\pi^e(t) = \Pi(\boldsymbol{b}_H(t), \boldsymbol{b}_L(t))$, which gives the expression in the text.

[13] More specifically, let $D(b, \boldsymbol{b})$ denote the default policy function, with $D(b, \boldsymbol{b}) = 1$ if the fiscal authority defaults and zero otherwise. The additive separability in U implies that the equilibrium default decision of an idiosyncratic fiscal authority is independent of inflation, and hence aggregate debt. Therefore, $\bar{\Omega} = \{b | D(b, \boldsymbol{b}) = 0\}$ does not depend on the aggregate states. The restriction that $b \geq 0$ is not restrictive in our environment, as no fiscal authority has an incentive to accumulate net foreign assets.

monetary authority takes the interest rate function $r(\boldsymbol{b}_H, \boldsymbol{b}_L)$ and the representative fiscal authority's consumption function $C(b, \boldsymbol{b})$ as a primitive of the environment. For any debt level $(\boldsymbol{b}_H, \boldsymbol{b}_L) \in \overline{\Omega}^2$ the monetary authority solves the following problem:

$$
\begin{aligned}
& J(\boldsymbol{b}) = \\
& \max_{\pi(t)\in[0,\bar{\pi}]} \int_0^\infty e^{-\rho t} \left[\eta u(C(\boldsymbol{b}_H(t), \boldsymbol{b}_H(t), \boldsymbol{b}_L(t))) + (1-\eta) u(C(\boldsymbol{b}_L(t), \boldsymbol{b}_H(t), \boldsymbol{b}_L(t))) \right. \\
& \left. -\psi_0 \pi(t)) \right] dt, \\
& \text{subject to} \\
& \dot{\boldsymbol{b}}_j(t) = C(\boldsymbol{b}_j(t), \boldsymbol{b}(t)) + (r(\boldsymbol{b}(t)) - \pi(t)) \boldsymbol{b}_j(t) - y \\
& \quad \text{with } \boldsymbol{b}_j(0) = \boldsymbol{b}_j \text{ for } j = H, L.
\end{aligned} \tag{P2}
$$

Note that the monetary authority takes the equilibrium interest rate schedule $r(\boldsymbol{b})$ as given. From the lenders' break-even constraint, we have that $r(\boldsymbol{b}) = r^\star + \pi^e$, where π^e is the lenders' expectation of inflation. In this sense the monetary authority is solving its problem taking inflationary expectations as a given. This is why the solution to the sequence problem P2 is time consistent; the monetary authority is not directly manipulating inflationary expectations with its choice of inflation. In equilibrium, $\pi^e = \Pi(\boldsymbol{b})$, but this equivalence is not incorporated into the monetary authority's problem as the central bank cannot credibly manipulate market expectations. This contrasts with the full-commitment Ramsey problem in which the monetary authority commits to a path of inflation at time zero and thereby selects market expectations. The solution to that problem is to set $\pi = 0$ every period.

Before discussing the solution to the problem of the fiscal and monetary authorities, we define our equilibrium concept as follows.

Definition 1. *A symmetric* Recursive Competitive Equilibrium (RCE) *is an interest rate schedule r; a fiscal authority value function V and associated policy function C; and a monetary authority value function J and associated policy function Π, such that:*

(i) V is the value function for the solution to the fiscal authority's problem (P1) and C is the associated policy function when Problem (P1) satisfies the consistency condition $\tilde{C} = C$;

(ii) J is the value function for the solution to the monetary authority's problem and Π is the associated policy function for inflation;

(iii) Bond holders break even: $r(\boldsymbol{b}) = r^\star + \Pi(\boldsymbol{b})$;

(iv) $V(b, \boldsymbol{b}) \geq \underline{V}$ *for all* $(b, \boldsymbol{b}) \in \overline{\Omega}^3$.

The last condition imposes that default is never optimal in equilibrium. In the absence of rollover risk, there is no uncertainty and any default would be inconsistent with the lender's break-even requirement. As we shall see, this condition imposes a restriction on the domain of equilibrium debt levels. It also ensures that problem (P1), which presumes no default, is consistent with equilibrium. That is, by construction the constraint $b(t) \in \bar{\Omega}$ in (P1) ensures that the government would never exercise its option to default in any equilibrium.

Equilibrium Allocations As $\rho = r^{\star}$, a natural starting point for characterizing the equilibrium is that fiscal authorities would like to maintain a constant level of consumption and stationary debt. Of course, this conjecture must be verified given that fiscal policy is implemented with nominal bonds rather than real bonds. To this end, we conjecture and verify that stationary debt is an equilibrium.

Consider the problem of the fiscal authority when r satisfies the equilibrium condition $r(\boldsymbol{b}) - \Pi(\boldsymbol{b}) = r^{\star} = \rho$. As $\boldsymbol{b}$ is beyond the control of an individual fiscal authority, we can substitute this condition into Problem (P1), and, focusing on the part of the objective function that is relevant for the fiscal authority, consider the simple consumption-savings problem:

$$\max_{c(t)} \int_0^{\infty} e^{-\rho t} u(c(t)) dt,$$

subject to $\dot{b}(t) = c(t) - y + \rho b(t)$. For $b(0) \in \overline{\Omega}$, the solution to this problem is constant consumption; that is, $C(b, \boldsymbol{b}) = y - \rho b$ for all $(b, \boldsymbol{b}) \in \overline{\Omega}^3$. As this policy is followed for all level of debt, $\dot{b}(t) = \dot{\boldsymbol{b}}_H(t) = \dot{\boldsymbol{b}}_L(t) = 0$. The associated value function for $(b, \boldsymbol{b}) \in \overline{\Omega}^3$ is therefore:

$$\begin{aligned} V(b, \boldsymbol{b}) &= \frac{u(y - \rho b)}{\rho} - \psi_0 \int_0^{\infty} e^{-\rho t} \Pi(\boldsymbol{b}(t)) dt \\ &= \frac{u(y - \rho b) - \psi_0 \Pi(\boldsymbol{b})}{\rho}, \end{aligned}$$

which is conditional on the inflation policy function of the monetary authority.

The equilibrium domain $\bar{\Omega}$ can be determined from the condition:

$$V(b, \boldsymbol{b}) \geq \underline{V},$$

which can be rewritten as

$$\frac{u(y-\rho b)-\psi_0\Pi(\boldsymbol{b})}{\rho} \geq \frac{u((1-\chi)y)-\psi_0\Pi(\boldsymbol{b})}{\rho},$$

or

$$b \leq \frac{\chi y}{\rho}.$$

Therefore, $\overline{\Omega} = \left[0, \frac{\chi y}{\rho}\right]$. Note that this outcome verifies the conjecture that $\overline{\Omega}$ is independent of aggregate states.

Turning to the monetary authority, faced with the above fiscal policy functions its problem becomes:

$$J(\boldsymbol{b}) = \max_{\pi(t)\in[0,\bar{\pi}]} \int_0^\infty e^{-\rho t}\left[\eta u(y-\rho \boldsymbol{b}_H(t)) + (1-\eta)u(y-\rho\boldsymbol{b}_L(t)) - \psi_0\pi(t))\right] dt,$$

subject to

$$\begin{aligned}\dot{\boldsymbol{b}}_j(t) &= C(\boldsymbol{b}_j(t), \boldsymbol{b}_H(t), \boldsymbol{b}_L(t)) + (r(\boldsymbol{b}(t)) - \pi(t))\boldsymbol{b}_j(t) - y,\\ &= \left[r(\boldsymbol{b}(t)) - \pi(t) - \rho\right]\boldsymbol{b}_j(t), \quad j = H, L,\end{aligned}$$

where the second line of the constraint substitutes $C(b,\cdot) = y - \rho b$.

The solution to this problem satisfies the recursive Bellman equation:

$$\rho J(\boldsymbol{b}) = \max_{\pi\in[0,\bar{\pi}]} u(y-\rho\boldsymbol{b}) - \psi_0\pi + (r(\boldsymbol{b}) - \pi - \rho)\nabla J(\boldsymbol{b}_H, \boldsymbol{b}_L)\cdot\boldsymbol{b}',$$

wherever $\nabla J(\boldsymbol{b}) = (J_H, J_L) = \left(\frac{\partial J}{\partial \boldsymbol{b}_H}, \frac{\partial J}{\partial \boldsymbol{b}_L}\right)$ exists. The first order condition with respect to π yields:

$$\Pi(\boldsymbol{b}) = \begin{cases} 0 & \text{if} \quad \psi_0 > -\nabla J(\boldsymbol{b})\cdot\boldsymbol{b}', \\ \in[0,\bar{\pi}] & \text{if} \quad \psi_0 = \nabla J(\boldsymbol{b})\cdot\boldsymbol{b}', \\ \bar{\pi} & \text{if} \quad \psi_0 < -\nabla J(\boldsymbol{b})\cdot\boldsymbol{b}'. \end{cases} \tag{2}$$

The inequalities that determine whether inflation is zero, maximal, or intermediate, have a natural interpretation. The marginal disutility of inflation is ψ_0. The gain from inflation is a reduction in real debt levels conditional on consumption. This reduction is proportional to the level of debt, and is translated into utility units via the terms $\nabla J = (J_H, J_L)$.

Conditional on the optimal inflation policy, as well as the equilibrium behavior of lenders and the fiscal authorities, the monetary authority's value function is:

$$J(\boldsymbol{b}) = \frac{\eta u(y - \rho \boldsymbol{b}_H) + (1-\eta)u(y - \rho \boldsymbol{b}_L) - \psi_0 \Pi(\boldsymbol{b})}{\rho}. \tag{3}$$

For $\boldsymbol{b}$ such that $\nabla \Pi(\boldsymbol{b}) = \left(\frac{\partial \Pi}{\partial \boldsymbol{b}_H}, \frac{\partial \Pi}{\partial \boldsymbol{b}_L}\right)$ exists, this implies

$$-\nabla J(\boldsymbol{b}) = \begin{bmatrix} \eta u'(y - \rho \boldsymbol{b}_H) \\ (1-\eta)u'(y - \rho \boldsymbol{b}_L) \end{bmatrix} + \frac{\psi_0}{\rho} \nabla \Pi(\boldsymbol{b}). \tag{4}$$

We can construct an equilibrium by finding a pair $(J(\boldsymbol{b}, \Pi(\boldsymbol{b}))$ that satisfies (2) and (3). There are many such pairs. The multiplicity arises because the monetary authority takes the nominal interest rate function $r(\boldsymbol{b})$ as given and chooses $\Pi(\boldsymbol{b})$ as its best response. Correspondingly, lenders set $r(\boldsymbol{b})$ based on the monetary authority's policy function. There may be many pairs of functions that are best-response pairs.

One natural property is for the equilibrium to be monotonic, i.e. that $\Pi(\boldsymbol{b})$ (and equivalently $r(\boldsymbol{b})$) be weakly increasing in each argument. From (4), monotonicity implies that

$$-\nabla J(\boldsymbol{b}) \cdot \boldsymbol{b} \geq \eta u'(y - \rho \boldsymbol{b}_H)\boldsymbol{b}_H + (1-\eta)u'(y - \rho \boldsymbol{b}_L)\boldsymbol{b}_L.$$

From (2), if the right hand side is strictly greater than ψ_0, then optimal inflation is $\bar{\pi}$ in any monotone equilibrium. It is useful to define the locus of points that defines this region. In particular, for each $\boldsymbol{b}_L \in \overline{\Omega}$, let the cutoff $\boldsymbol{b}_\pi(\boldsymbol{b}_L)$ be defined by:

$$\eta u'(y - \rho \boldsymbol{b}_\pi)\boldsymbol{b}_\pi + (1-\eta)u'(y - \rho \boldsymbol{b}_L)\boldsymbol{b}_L = \psi_0. \tag{5}$$

Note that the concavity of u implies that $\boldsymbol{b}_\pi$ is a well defined function and strictly decreasing in $\boldsymbol{b}_L$. We thus have:

Lemma 1. *In any monotone equilibrium,* $\Pi(\boldsymbol{b}) = \bar{\pi}$ *for* $\boldsymbol{b} \in \overline{\Omega}^2$ *such that* $\boldsymbol{b}_H > \boldsymbol{b}_\pi(\boldsymbol{b}_L)$.

As inflation is a deadweight loss in a perfect-foresight equilibrium, the best case scenario in a monotone equilibrium is for $\pi = 0$ on the complement of this set. Doing so is Pareto efficient in the sense that lenders are indifferent and both fiscal and monetary authorities prefer equilibria with lower inflation. In particular, we have:

Lemma 2. *The best (Pareto efficient) monotone equilibrium has* $\Pi(\boldsymbol{b}) = 0$ *for* $\boldsymbol{b} \in \overline{\Omega}^2$ *such that* $\boldsymbol{b}_H \leq \boldsymbol{b}_\pi(\boldsymbol{b}_L)$.

Not all monotone equilibria are characterized by a simple threshold that separates zero and maximal inflation. In particular, it is possible to construct monotone equilibrium with

$\Pi(\boldsymbol{b}) \in (0, \bar{\pi})$ for a non-trivial domain of $\boldsymbol{b}$. These equilibria, however, are Pareto dominated by the threshold equilibrium.

We collect the above in the following proposition:

Proposition 1. *Define $\boldsymbol{b}_\pi(\boldsymbol{b}_L)$ from equation (5) and $\overline{\Omega} = \left[0, \frac{\chi y}{\rho}\right]$. The following is the best monotone equilibrium: For all $(b, \boldsymbol{b}) \in \overline{\Omega}^3$:*

(i) Consumption policy functions:

$$C(b, \boldsymbol{b}) = u(y - \rho b);$$

(ii) Inflation policy function:

$$\Pi(\boldsymbol{b}) = \begin{cases} 0 & \text{if} \quad \boldsymbol{b}_H \leq \boldsymbol{b}_\pi(\boldsymbol{b}_L), \\ \bar{\pi} & \text{if} \quad \boldsymbol{b}_H > \boldsymbol{b}_\pi(\boldsymbol{b}_L); \end{cases}$$

(iii) Interest rate schedule:

$$r(\boldsymbol{b}) = \begin{cases} r^\star & \text{if} \quad \boldsymbol{b}_H \leq \boldsymbol{b}_\pi(\boldsymbol{b}_L), \\ r^\star + \bar{\pi} & \text{if} \quad \boldsymbol{b}_H > \boldsymbol{b}_\pi(\boldsymbol{b}_L); \end{cases}$$

(iv) Value functions:

$$V(b, \boldsymbol{b}) = \begin{cases} \frac{u(y-\rho b)}{\rho} & \text{if} \quad \boldsymbol{b}_H \leq \boldsymbol{b}_\pi(\boldsymbol{b}_L), \\ \frac{u(y-\rho b)-\psi_0\bar{\pi}}{\rho} & \text{if} \quad \boldsymbol{b}_H > \boldsymbol{b}_\pi(\boldsymbol{b}_L); \end{cases}$$

and $J(\boldsymbol{b}) = \eta V(\boldsymbol{b}_H, \boldsymbol{b}) + (1-\eta)V(\boldsymbol{b}_L, \boldsymbol{b})$.

The best monotone equilibrium is graphically depicted in figure 1. We do so for a given $\boldsymbol{b}_L$ and let $\boldsymbol{b}_H$ vary along the horizontal axis. Given the symmetry of the environment, diagrams holding $\boldsymbol{b}_H$ constant and varying $\boldsymbol{b}_L$ have similar shapes, but with thresholds defined by the inverse of $\boldsymbol{b}_\pi$.

A prominent feature of this equilibrium is the discontinuity in the value functions at $\boldsymbol{b}_\pi$. A small decrease in debt in the neighborhood above $\boldsymbol{b}_\pi$ leads to a discrete jump in welfare. The lack of coordination between fiscal and monetary authorities prevents the currency union from exploiting this opportunity. We now discuss this "fiscal externality" in greater detail. For expositional ease, we do so in the case of homogenous debt levels ($\eta = 1$). We then return to the case of heterogeneity to explore the extent of disagreement about policy and composition of the monetary union.

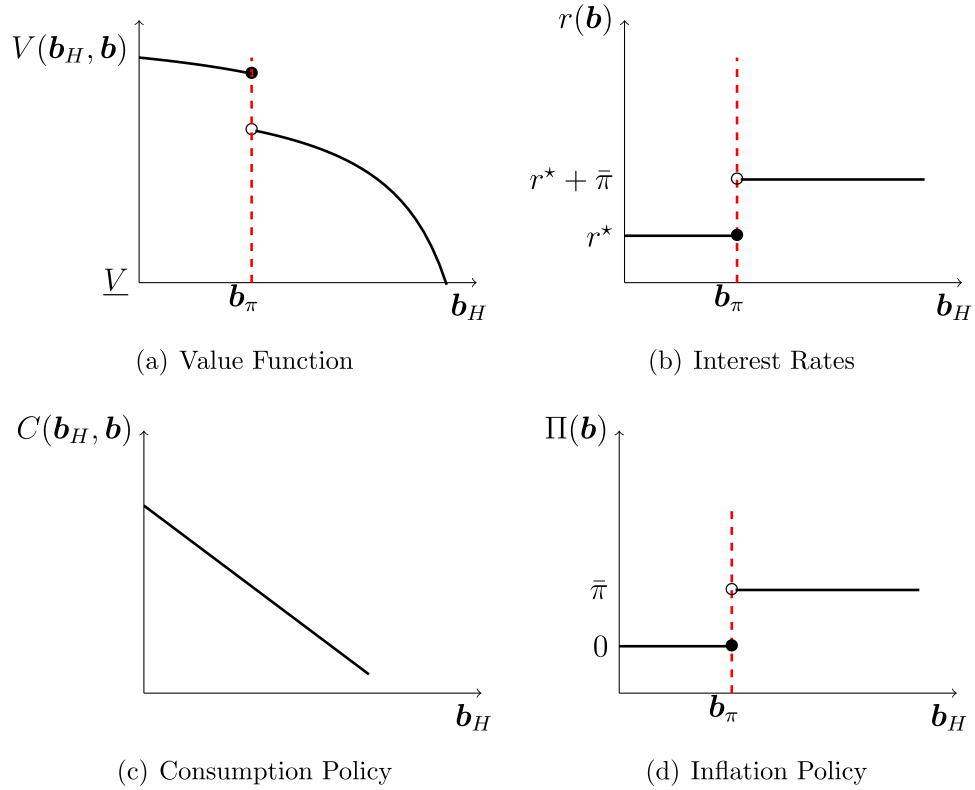


Figure 1: Solution in the Monetary Union with No Crisis

2.3 Fiscal externalities in a monetary union

In this subsection, we assume all members of the monetary union have the same level of debt. In particular, we set $\eta = 1$, let $\boldsymbol{b}$ denote $\boldsymbol{b}_H$ and let $\boldsymbol{b}_\pi$ denote the solution to (5) when $\eta = 1$ (that is, $u'(y - \rho \boldsymbol{b}_\pi)\boldsymbol{b}_\pi = \psi_0$).

The equilibrium described in Proposition 1 reflects the combination of lack of commitment and lack of coordination. With full commitment, the monetary authority would commit to zero inflation in every period.[14] In this equilibrium, nominal interest rates would equal $r^\star$. This generates the same level of consumption, but strictly higher utility for $\boldsymbol{b} > \boldsymbol{b}_\pi$. This is the Ramsey allocation depicted in figure 2, in which $V = u(y - \rho \boldsymbol{b})/\rho$ for all $\boldsymbol{b}$. The figure also depicts the allocation of Proposition 1, which is denoted "MU" for monetary union. Clearly, the Ramsey allocation strictly dominates the monetary union case in the region of high inflation.

This point is reminiscent of the result in Chari and Kehoe (2007), which compares mon-

[14] It could also use commitment to rule out default and borrow above $\chi y/\rho$, but would have no incentive to do so.

etary unions in which the monetary authority has full commitment versus one that lacks commitment. This comparison is enriched by considering the role of coordination in an environment of limited commitment, a point to which we now turn.

Absent commitment, the members of the monetary union cannot achieve the Ramsey outcome at higher levels of debt. However, they may be able to do better than the benchmark allocation by coordinating monetary and fiscal policy, even under limited commitment. As noted above, the discontinuity in the value function at $\boldsymbol{b}_\pi$ represents an unexploited opportunity for a small amount of savings to generate a discrete gain in welfare. With coordinated fiscal and monetary policy, the optimal policy under limited commitment would be to reduce debt in the neighborhood above $\boldsymbol{b}_\pi$. Specifically, coordination makes the monetary union a fiscal union as well, and we can consider the entire region a small open economy (SOE) that faces a world real interest rate $r^\star$. This environment is characterized in detail in Aguiar et al. (2012). Here we simply sketch the equilibrium so as to compare it to the solution of the monetary union (MU) and refer the reader to the paper for the details of the derivation.

Specifically, we consider the same threshold equilibrium defined in Proposition 1.[15] In particular, consider an interest rate schedule $r(\boldsymbol{b})$ defined on $\overline{\Omega}$ which equals $r^\star$ for $\boldsymbol{b} \le \boldsymbol{b}_\pi$ and $r^\star + \bar{\pi}$ for $\boldsymbol{b} > \boldsymbol{b}_\pi$. We now sketch how the centralized fiscal and monetary authority responds to this schedule, and verify that it is indeed an equilibrium. We then contrast the resulting allocation with that depicted in figure 1.

Faced with this schedule, the unified "SOE" government solves the following problem:

$$V_E(\boldsymbol{b}) = \max_{\{\pi(t)\in[0,\bar{\pi}],c(t)\}} \int_0^\infty e^{-\rho t}(u(c(\boldsymbol{b}(t)) - \psi\pi(t))dt, \tag{P3}$$

subject to

$$\dot{\boldsymbol{b}}(t) = c(t) + (r(\boldsymbol{b}(t)) - \pi(t))\boldsymbol{b}(t) - y, \quad \boldsymbol{b}(0) = \boldsymbol{b} \quad \text{and} \quad \boldsymbol{b}(t) \in \overline{\Omega},$$

where the subscript E refers to the value for a small open economy. Unlike the problem in the monetary union, fiscal and monetary policies are determined jointly in P3. Therefore the impact of debt choices on inflation is internalized by the single authority.

At points where the value function is differentiable, the Bellman equation is given by,

$$\rho V_E(\boldsymbol{b}) = \max_{\pi(t)\in[0,\bar{\pi}],c(t)} \left\{ u(c) - \psi_0\pi + V_E'(\boldsymbol{b})\left(c - y + (r(\boldsymbol{b}) - \pi)\boldsymbol{b}\right)\right\}. \tag{6}$$

[15]There are other coordinated SOE equilibria. See Aguiar et al. (2012) for details.

The first order conditions are:

$$u'(c) = -V_E'(\boldsymbol{b}),$$
$$\pi = \begin{cases} 0 & \text{if } \psi_0 \geq -V_E'(\boldsymbol{b})\boldsymbol{b} = u'(c)\boldsymbol{b} \\ \bar{\pi} & \text{if } \psi_0 < u'(c)\boldsymbol{b}. \end{cases}$$

The first condition is the familiar envelope condition that equates marginal utility of consumption to the marginal disutility of another unit of debt. However, such a condition is not satisfied by the monetary authority's value function in the uncoordinated equilibrium, as seen in equation (4). In the coordinated case, there is no disagreement between monetary and fiscal authorities regarding the cost of another unit of debt. In particular, this provides the incentive for the fiscal authority to reduce debt in the neighborhood above $\boldsymbol{b}_\pi$ in the coordinated equilibrium.

In the region $\boldsymbol{b} \in [0, \boldsymbol{b}_\pi]$, the SOE, like the benchmark, faces an interest rate of $r^\star$ and finds it optimal to set $c = y - \rho\boldsymbol{b}$ and $\pi = 0$. The consumption is optimal as the rate of time preference equals the interest rate and the latter is optimal as –by definition – $\psi_0 \leq u'(y - \rho\boldsymbol{b})\boldsymbol{b}$ for $\boldsymbol{b} \leq \boldsymbol{b}_\pi$. Thus $\pi = 0$ satisfies the first order condition for inflation on this domain.

The distinction between a SOE and the benchmark MU allocation becomes apparent in the neighborhood above $\boldsymbol{b}_\pi$. We start with the allocation at $\boldsymbol{b}_\pi$. At this debt level, $V_E = u(y - \rho\boldsymbol{b}_\pi)/\rho$, which is the value achieved in the MU equilibrium. As in the MU economy, in the neighborhood above $\boldsymbol{b}_\pi$, a small open economy cannot credibly deliver zero inflation, as $\psi_0 < u'(y - \rho\boldsymbol{b})$ for $\boldsymbol{b} > \boldsymbol{b}_\pi$. However, by saving it can do better than the MU allocation. Specifically, the SOE chooses $C_E(\boldsymbol{b}) < y - \rho\boldsymbol{b}$, where C_E denotes the consumption policy function of the coordinated fiscal policy, and thus $\dot{\boldsymbol{b}}(t) < 0$. At this consumption, $\psi_0 > u'(C_E)$, and so the associated inflation remains $\Pi_E(\boldsymbol{b}) = \bar{\pi}$, validating the jump in the equilibrium interest rate.

In the neighborhood above $\boldsymbol{b}_\pi$, the SOE can achieve the value $V(\boldsymbol{b}_\pi)$ by saving a small amount. That is, the SOE value function will be continuous at $\boldsymbol{b}_\pi$. As noted above, the monetary union keeps debt constant in this neighborhood as the idiosyncratic fiscal authorities do not internalize this potential jump in welfare from a small decrease in *aggregate* debt. There is no such externality in the coordinated case.

The precise level of consumption in the neighborhood above $\boldsymbol{b}_\pi$ can be determined by substituting in the envelope condition into (6) and using continuity of V_E. In particular, define $c_E \in (0, y - \rho \boldsymbol{b}_\pi)$ as the solution to:

$$u(y - \rho \boldsymbol{b}_\pi) = u(c_E) - \psi_0 \bar{\pi} - u'(c_E)\left(c_E - y + \rho \boldsymbol{b}\right).$$

This consumption level satisfies the Bellman equation in the neighborhood above $\boldsymbol{b}_\pi$. In this neighborhood, debt is declining and the economy approaches $\boldsymbol{b}_\pi$ from above. Along this trajectory, there is no incentive for the government to tilt consumption as its effective real interest rate is ρ. That is, $C_E(\boldsymbol{b}) = c_E < y - \rho \boldsymbol{b}_\pi = C_E(\boldsymbol{b}_\pi)$ on a domain $(\boldsymbol{b}_\pi, \boldsymbol{b}^*)$, where the upper bound on this domain is given by $y - c_E = \rho \boldsymbol{b}^*$, the debt level at which c_E no longer generates $\boldsymbol{b}'(t) < 0$. For debt above $\boldsymbol{b}^*$, the government prefers not to save towards $\boldsymbol{b}_\pi$ as the length of time required to reach this threshold is prohibitive.

Collecting the above points, we can characterize the SOE allocation, which is depicted in figure 2 alongside the benchmark "MU" and Ramsey economies. For $\boldsymbol{b} \leq \boldsymbol{b}_\pi$, the SOE, Ramsey, and MU economies are identical. For $\boldsymbol{b} > \boldsymbol{b}^*$, the SOE and MU economies are likewise identical, as the SOE economy finds it optimal to set $\dot{\boldsymbol{b}}(t) = 0$ despite having high inflation, as in the benchmark. However, there is a difference for $\boldsymbol{b} \in (\boldsymbol{b}_\pi, \boldsymbol{b}^*)$. Continuity at $\boldsymbol{b}_\pi$ places the SOE value function strictly above the MU case; however, limited commitment places SOE strictly below the Ramsey welfare. More specifically, from the envelope condition, the SOE's flat consumption policy function (panel b) is associated with a constant $V_E'(\boldsymbol{b})$; that is, V_E is linear on $(\boldsymbol{b}_\pi, \boldsymbol{b}^*)$. Moreover, this value function is continuous, and thus the line connects the MU value function at $\boldsymbol{b}_\pi$ to the MU value at $\boldsymbol{b}^*$. This line lies strictly above the MU value function on this domain, representing the welfare loss MU experiences from lack of coordination, but strictly below Ramsey, representing the welfare loss due to limited commitment.

The presence of fiscal externalities rationalizes the imposition of debt ceilings in a monetary union. They can be designed to correct the incentives of individual fiscal authorities and implement the SOE outcome in a monetary union by simply imposing $b(t) \leq \boldsymbol{b}_t^{\mathrm{SOE}}$. Of course the problem is that it is difficult to make such debt ceilings credible in the face of ex-post challenges—as illustrated by the repeated violations of the Stability and Growth pact in the eurozone.

2.4 Heterogeneity absent Crises

We conclude this section by discussing to what extent heterogeneity in debt positions creates disagreement within a monetary union. We are particularly interested in the question of whether existing members disagree about the debt choices of other members (or potential new members). The answer to this question in the current environment contrasts with

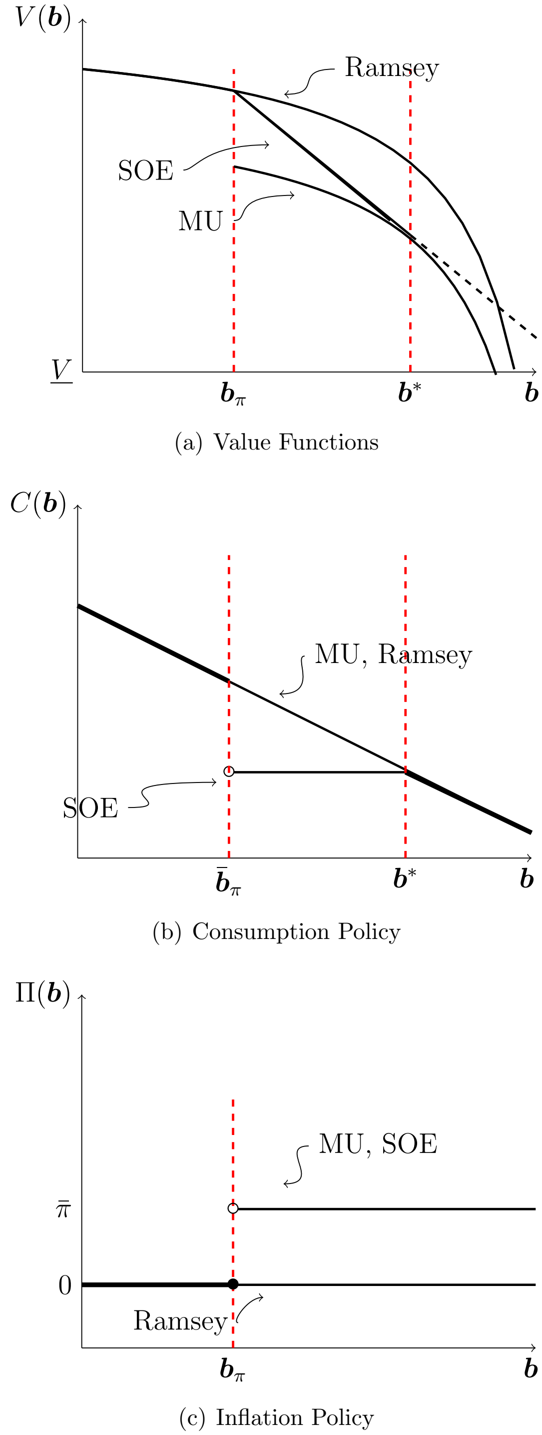


Figure 2: Fiscal Externality, Value of Commitment, Value of Coordination

the answer when rollover crises are possible in equilibrium, and so the discussion in this subsection sets the stage for a key result of the next section.

To do so, we consider $\eta \in (0,1)$, where recall that η is the measure of high-debt members that enter with $\boldsymbol{b}_H > \boldsymbol{b}_L$. From the value functions defined in Proposition 1, all members benefit from a higher $\boldsymbol{b}_\pi$. From the definition of this threshold in equation (5), note that all else equal, $\boldsymbol{b}_\pi$ is decreasing in η if $\boldsymbol{b}_\pi(\boldsymbol{b}_L) > \boldsymbol{b}_L$. This is the relevant domain as otherwise, even low debtors have high enough debt to induce maximal inflation. This implies that even high-debt members would like to see the fraction of low-debt members increase. Although high-debt members trigger high inflation ex post, they would like ex ante commitment to low inflation at the time they roll over their debt, which happens every period. This is accomplished by membership in a low-debt monetary union. In fact, for $\boldsymbol{b}_L < \boldsymbol{b}_\pi(\boldsymbol{b}_L)$, the Ramsey allocation is implemented as $\eta \to 0$. There is also no disagreement among the heterogeneous members that this is welfare improving. The result that high-debt countries benefit by joining a low-debt monetary union is not necessarily true when we introduce rollover crises, the focus of the next section.

3 Rollover Crises

We now enrich our setup to allow for rollover crises defined as a situation where lenders may refuse to roll over debt. This can generate default in equilibrium, unlike the analysis of section 2. The distinction between high and low debtors will be a central focus of the analysis. As in the no-crisis equilibrium from the previous section, in the equilibrium described below, countries that start with low enough debt have no debt dynamics; as we shall see, this is no longer the case for high debtors. To simplify the exposition, we set $\boldsymbol{b}_L = 0$ and drop $\boldsymbol{b}_L$ from the notation, as this state variable is always static in the equilibria under consideration. That is $\boldsymbol{b} = \boldsymbol{b}_H$ is sufficient to characterize the aggregate state in the equilibria described below.

To introduce rollover crises, we follow Cole and Kehoe (2000) and consider coordination failures among creditors. That is, we construct equilibria in which there is no default if lenders roll over outstanding bonds, but there is default if lenders do not roll over debt. In continuous time with instantaneous bonds, failure to roll over outstanding bonds implies a stock of debt must be repaid with an endowment flow. To allow some notion of maturity in a tractable manner, we follow Aguiar et al. (2012) and assume that the fiscal authority is provided with a "grace period" of exogenous length δ during which it can repay the bonds plus accumulated interest at the interest rate originally contracted on the debt. If it repays within the grace period it returns to the financial markets in good standing. If the government fails to make full repayment within the grace period and defaults, it is punished by permanent loss of access to international debt markets plus a loss of output given by the parameter χ. We continue to assume that it is not excluded from the union.

We construct a crisis equilibrium as follows. We first consider the fiscal authority and monetary authority's problem in the grace period when creditors refuse to roll over outstanding debt. We compute the welfare of repaying the bonds within the grace period and compare that to the welfare from outright default. This will allow us to determine whether or not a rollover crisis is possible. We then define the full problem of the fiscal and monetary authorities under the threat of a roll over crisis and characterize the equilibria.

3.1 The Grace-Period Problem

In this subsection, we characterize the equilibrium response to a rollover crisis. We continue our focus on symmetric equilibrium and thus characterize the problem for an individual country with debt b and the remaining debtors having debt $\boldsymbol{b}$. This will allow us to establish the payoffs to an idiosyncratic deviation. Countries with 0 debt are by definition not subject to roll-over crisis and with $\rho = r^*$ their consumption $c = y$. We therefore focus on high-debt countries. We re-normalize time to zero at the start of the grace period for convenience.

3.1.1 Fiscal authorities

When a fiscal authority is faced with a run on its debt it cannot issue new bonds to repay maturing bonds. It has the option of repaying all debt within the grace period of length δ or defaulting. When making its decision the individual fiscal authority takes the policies of the other fiscal authorities and the monetary authority as given. However the payoff to repayment depends on these other policies, which in turn depends on whether the other fiscal authorities are themselves subject to a rollover crisis. To capture this dependence we index the grace period problem for an individual fiscal authority by the aggregate policy function $\hat{\Pi}(\boldsymbol{b}, \boldsymbol{r}, \delta, t)$.

In the grace period problem, the government is obligated to repay the nominal balance on or before date δ, with interest accruing over the grace period at the original contracted rate r.[16] In equilibrium, $r = r(b, \boldsymbol{b})$ and $\boldsymbol{r} = r(\boldsymbol{b})$, but for now we treat r and $\boldsymbol{r}$ as arbitrary primitives of the grace period problem.

We now state the problem of the fiscal authority with outstanding real debt b at interest rate r when aggregate outstanding real debt is $\boldsymbol{b}$ and aggregate interest rate is $\boldsymbol{r}$. Because a fiscal authority takes inflation as given, it will be useful to define the value of repaying and

[16] As in Aguiar et al. (2012) we impose the pari passu condition that all bond holders have equal standing; that is, the fiscal authority cannot default on a subset of bonds, while repaying the remaining bondholders. Therefore, the relevant state variable is the entire stock of outstanding debt at the time the fiscal authority enters the grace period.

of default net of inflation costs $\hat{V}^G$, given by

$$\hat{V}^G(b, \boldsymbol{b}, r, \boldsymbol{r}, \delta; \hat{\Pi}) = \max_{c(t)} \int_0^\delta e^{-\rho t} u\left(c(t)\right) dt + e^{-\rho\delta}\hat{V}(0), \tag{7}$$

subject to

$$\dot{b}(t) = c(t) - y + (r_0 - \hat{\Pi}(\boldsymbol{b}, \boldsymbol{r}, \delta, t)b(t), \tag{8}$$

$$b(0) = b, \quad b(\delta) = 0, \quad \text{and} \quad \dot{b}(t) \leq -\hat{\Pi}(\boldsymbol{b}, \boldsymbol{r}, \delta, t)b(t).$$

The term $\hat{V}(0)$ in the objective function represents the equilibrium value of returning to the markets with zero debt at the end of the grace period. The constraint $\dot{b}(t) \leq -\hat{\Pi}(\boldsymbol{b}, \boldsymbol{r}, t)b(t)$ imposes that no new nominal bonds be issued, that is $\dot{B}(t) \leq 0$.

The alternative to repayment is default. The value of default (net of inflation costs) in response to a rollover crisis is given by:

$$\hat{\underline{V}} = \frac{u\left((1-\chi)y\right)}{\rho}.$$

We write $C^G(b, \boldsymbol{b}, r, \boldsymbol{r}, \delta, t; \hat{\Pi})$ for the fiscal authority's consumption function.

The best response of the fiscal authority to a run on its debt is determined by $\hat{V}^G \gtrless \hat{\underline{V}}$. The value $\hat{V}^G(b, \boldsymbol{b}, r, \mathbf{r}, \delta; \hat{\Pi})$ is decreasing in b and r. Note that the direct utility costs of inflation do not enter into the decision to default in a crisis. These costs are borne regardless of the individual fiscal authority's decision. However, inflation also enters into the budget constraint (8). Higher inflation relaxes this constraint making it easier for the fiscal authority to repay its debt quickly. This is not offset by a higher (post-crisis) interest rate, as the fiscal authority is not rolling over its debt at a new interest rate. Therefore, a fiscal authority facing a crisis will find repayment relatively attractive the more accommodating is monetary policy. We now turn to the monetary authority's best response to a crisis.

3.1.2 Monetary authority

We now consider optimal monetary policy when the representative fiscal authority with debt $\boldsymbol{b}_0$ faces a rollover crisis. In a symmetric equilibrium, all debtors will be subject to the same crisis and choose the same response (default or repay). In selecting optimal monetary policy during a rollover crisis, the monetary authority takes the crisis and response of the fiscal authorities as given. If the fiscal authorities default, then the monetary authority simply sets $\pi_t = 0$ for all t, as no member of the currency union holds debt after default. If the representative fiscal authority does not default, the monetary authority's problem is:

$$J^G(\boldsymbol{b}, \boldsymbol{r}, \delta) = \max_{\pi(t)\in[0,\bar{\pi}]} \int_0^{\delta} e^{-\rho t} \left(\eta u(C^G(\boldsymbol{b}(t), \boldsymbol{b}(t), \boldsymbol{r}, \boldsymbol{r}, \delta - t, 0; \tilde{\Pi}^G)) + (1-\eta)u(y) - \psi_0 \pi(t)\right) dt + \frac{e^{-\rho\delta}}{\rho} J(0),$$

subject to

$$\dot{\boldsymbol{b}}(t) = C^G(\boldsymbol{b}(t), \boldsymbol{b}(t), \boldsymbol{r}, \boldsymbol{r}, \delta - t, 0; \tilde{\Pi}^G) - y + (\boldsymbol{r} - \pi(t))\boldsymbol{b}(t), \qquad \boldsymbol{b}(0) = \boldsymbol{b}.$$

This yields an inflation function $\Pi^G(\boldsymbol{b}, \mathbf{r}, \delta, t)$. Equilibrium requires that $\tilde{\Pi}^G(\boldsymbol{b}, \mathbf{r}, \delta, t) = \Pi^G(\boldsymbol{b}, \mathbf{r}, \delta, t)$, and as a result $C^G(\boldsymbol{b}(t), \boldsymbol{b}(t), \boldsymbol{r}, \boldsymbol{r}, \delta - t, 0; \Pi^G) = C^G(\boldsymbol{b}, \boldsymbol{b}, \boldsymbol{r}, \boldsymbol{r}, \delta, t; \Pi^G)$. The reason we have introduced the notation $\tilde{\Pi}^G(\boldsymbol{b}, \mathbf{r}, \delta, t)$ is to make explicit which effects are or are not a priori internalized by the monetary authority, which lacks commitment. In Appendix A, we examine whether there are in fact inefficiencies arising from lack of commitment or coordination between fiscal and monetary authorities in the grace period.

The value function $J^G(\boldsymbol{b}, \boldsymbol{r}, \delta)$ is decreasing in $\boldsymbol{b}$ and $\boldsymbol{r}$. We note that the objective function of the monetary authority and the fiscal authority differ because the former maximizes aggregate welfare and recognizes that only a fraction η of countries have positive levels of debt. Consequently the benefits from inflating are restricted to this η fraction of countries. As we will see later, the problem of the monetary union with heterogeneity is isomorphic to the problem with symmetric countries but with the monetary authority facing a perceived cost of inflation $\psi = \psi_0/\eta$ that differs from ψ_0. For a given $(\boldsymbol{b}, \boldsymbol{r})$, the monetary authority is more likely to inflate the larger the fraction of countries with positive debt, i.e. the higher is η.

3.2 Rollover Crises

Having characterized the equilibrium best response to a rollover crisis, we explore how runs occur in equilibrium. To see how a run can be supported in equilibrium, consider an individual fiscal authority with debt b that faces a roll over crisis. Consider the problem of an individual creditor when all other creditors refuse to purchase new bonds from the fiscal authority, conditional on the aggregate state. If $\hat{V}^G < \underline{\hat{V}}$, then the best response of the fiscal authorities to the rollover crisis is to default. An individual creditor that purchases new bonds is not large enough to alter this decision and thus will receive zero in return for any bonds it purchases. Thus, it is individually optimal for the creditor to also refuse to purchase new bonds. On the other hand, if $\hat{V}^G \geq \underline{\hat{V}}$, bondholders receive the contracted nominal payment and thus the fiscal authority's bonds would trade at a strictly positive price. A run can therefore be sustained in equilibrium at a given level of debt as long as the fiscal authority's best response is to default.

While a run may be sustained at a particular level of debt, it is not the only equilibrium outcome possible. Absent a run, the fiscal authority may be willing to service the debt as usual, paying off maturing bonds by issuing new bonds. Moreover, as discussed in the previous subsections, the response of an individual fiscal authority depends on whether the other fiscal authorities are facing a crisis as well. To incorporate this multiplicity and interdependence in a tractable manner, we extend the environment of Cole and Kehoe (2000), which considers the case of a small open economy. Specifically, part of the equilibrium will be a region of the debt state space in which a fiscal authority is vulnerable to a rollover crisis. Following Cole and Kehoe, we shall refer to this region as the crisis region. To characterize a symmetric equilibrium, this region needs to be defined over two dimensions– the idiosyncratic debt of a fiscal authority and the debt of the representative debtor. Also following Cole and Kehoe (2000), we introduce a sunspot that coordinates creditor beliefs. Specifically, if $(b, \boldsymbol{b})$ is in the crisis region of a fiscal authority with debt b, then with Poisson arrival λ creditors refuse to roll over maturing debt and the fiscal authority defaults. We hew as closely as possible to the single-country case of Cole and Kehoe (2000) by considering a simple threshold $\boldsymbol{b}_\lambda$, such that an individual debtor with debt $b \in \overline{\Omega}$ is vulnerable to a crisis if $b > \boldsymbol{b}_\lambda$ *for all* $\boldsymbol{b} \in \overline{\Omega}$. We shall refer to the set $\{b \in \overline{\Omega} | b > \boldsymbol{b}_\lambda\}$ as the "crisis zone," and its complement in $\overline{\Omega}$ as the "safe zone."

3.2.1 Fiscal Authorities

We now state the problem of the government when not in default. We assume the government faces a bond-market equilibrium characterized by an interest rate function $r(b, \boldsymbol{b})$ defined on a domain $\overline{\Omega}^2$, an aggregate interest rate function $r(\boldsymbol{b})$. The parameters defining the duration of the grace period (δ) and crisis arrival probability intensity (λ) conditional on $b > \boldsymbol{b}_\lambda$ are taken as primitives of the environment.

Let $T \in (0, \infty]$ denote the first time loans are called (i.e., a rollover crisis occurs). From the fiscal authority's and an individual creditor's perspective, T is a random variable with a distribution that depends on the path of the state variable. In particular,

$$Pr(T \leq \tau) = 1 - e^{-\lambda \int_0^\tau 1_{b_t > \boldsymbol{b}_\lambda} dt}.$$

The realization of T is public information and it is the only uncertainty in the model.

Given that an individual country takes the monetary policy as given, it is helpful to define $\hat{V}(b)$ to be the utility function without the inflation costs for a country that has an amount of debt b. The government's problem can then be defined to be:

$$\hat{V}(b) = \max_{c(t)} \int_0^\infty e^{-\rho t - \lambda \int_0^t 1_{\{b_s > \boldsymbol{b}_\lambda\}} ds} u(c(t)) dt + \int_0^\infty e^{-\rho t - \lambda \int_0^t 1_{\{b_s > \boldsymbol{b}_\lambda\}} ds} \lambda \underline{\hat{V}} dt \tag{P3}$$

subject to

$$\dot{b}(t) = c(t) - y + (r^\star + \lambda 1_{\{b_t > \boldsymbol{b}_\lambda\}}) b(t), \quad b(0) = b \quad \text{and} \quad b(t) \in \overline{\Omega}.$$

where $\underline{\hat{V}} = u((1-\chi)y)/\rho$ (that is, the default value function excluding the inflation costs). Note, we have used that in equilibrium the nominal interest rate the government faces is $r^\star + \lambda 1_{\{b_t > \boldsymbol{b}_\lambda\}} + \pi(t)$. It follows then that the real rate that the government faces at any time t is independent of the government's choices at time t and equal to $r^\star + \lambda 1_{\{b_t > \boldsymbol{b}_\lambda\}}$.

The solution to this problem delivers a consumption function $C(b)$.

3.2.2 Monetary Authority

The problem of the monetary authority is given by:

$$J(\boldsymbol{b}) = \max_{\pi(t)} \int_0^\infty e^{-\rho t - \lambda \int_0^t 1_{\boldsymbol{b}_s > \boldsymbol{b}_\lambda} ds} \left(\eta u(C(\boldsymbol{b}(t)) + (1-\eta) u(y)) \right) dt + \int_0^\infty e^{-\rho t - \lambda \int_0^t 1_{\boldsymbol{b}_s > \boldsymbol{b}_\lambda} ds} \lambda \underline{V} dt - \int_0^\infty e^{-\rho t} \psi_0 e^{-\lambda \int_0^t 1_{\boldsymbol{b}_s > \boldsymbol{b}_\lambda} ds} \pi(t) dt, \tag{P4}$$

subject to

$$\dot{\boldsymbol{b}}(t) = C(\boldsymbol{b}(t)) - y + (r(\boldsymbol{b}(t)) - \pi(t)) \boldsymbol{b}(t) \quad \text{and} \quad \boldsymbol{b}(0) = \boldsymbol{b}.$$

This generates an inflation function $\Pi(\boldsymbol{b})$.

3.3 Crisis Equilibrium

We now state the definition of equilibrium with crisis:

Definition 2. *A* Recursive Competitive Equilibrium with crisis *specifies an aggregate interest rate schedule* $\boldsymbol{r}(\boldsymbol{b})$*, an individual interest rate schedule* $r(b, \boldsymbol{b})$*, a consumption function* $C(\boldsymbol{b})$*, an inflation function* $\Pi(\boldsymbol{b})$*, value functions* $\hat{V}(b)$ *for a fiscal authority and* $J(\boldsymbol{b})$ *for the monetary authority, a value function (net of inflation costs) for a fiscal authority in the grace period* $\hat{V}^G(b, \boldsymbol{b}, r, \boldsymbol{r}, t; \hat{\Pi})$*, and finally a threshold* $\boldsymbol{b}_\lambda$*, such that:*

(i) $\hat{V}(b)$ *is the value function for the solution to the fiscal authority's problem and* $C(b)$ *is the maximizing policy function for consumption;*

(ii) $J(\boldsymbol{b})$ *is the value functions for the solution to the monetary authority's problem and* $\Pi(\boldsymbol{b})$ *is the maximizing policy function for inflation;*

(iii) Bond holders earn a real return r^**, that is* $r(b, \boldsymbol{b}) = r^* + \Pi(\boldsymbol{b}) + \lambda 1_{b > \boldsymbol{b}_\lambda}$ *and* $r(\boldsymbol{b}) = r^* + \Pi(\boldsymbol{b}) + \lambda 1_{\boldsymbol{b} > \boldsymbol{b}_\lambda}$*;*

(iv) $\hat{V}(b) \geq \underline{\hat{V}}$*;*

(v) if $\boldsymbol{b} \leq \boldsymbol{b}_\lambda$ *and* $b > \boldsymbol{b}_\lambda$ *then* $\hat{V}^G(b, \boldsymbol{b}, r(b, \boldsymbol{b}), \boldsymbol{r}(\boldsymbol{b}), \delta; \Pi) < \underline{\hat{V}}$*; if* $\boldsymbol{b} > \boldsymbol{b}_\lambda$ *and* $b > \boldsymbol{b}_\lambda$ *then* $\hat{V}^G(b, 0, r(b, \boldsymbol{b}), \boldsymbol{r}(\boldsymbol{b}), \delta; \Pi) < \underline{\hat{V}}$*, where with a slight abuse of notation, we write* $\hat{\Pi} = \Pi$ *for the equilibrium inflation policy function with* $\Pi(\boldsymbol{b}, \boldsymbol{r}, t)$ *given by* $\Pi(\boldsymbol{b}, \boldsymbol{r}, t) = \Pi(\boldsymbol{b}(t))$ *where* $\boldsymbol{b}(t)$ *is defined by the following differential equation* $\dot{\boldsymbol{b}}(t) = C(\boldsymbol{b}(t)) - y + (r(\boldsymbol{b}) - \Pi(\boldsymbol{b}(t)))\boldsymbol{b}(t)$ *with* $\boldsymbol{b}(0) = \boldsymbol{b}$*.*

Condition (v) stipulates that an individual country facing a rollover crisis prefers to default when its debt b at the start of the crisis exceeds $\boldsymbol{b}_\lambda$ regardless of what the debt $\boldsymbol{b}$ of the representative debtor is at the start of the crisis. The inflation policy chosen by the monetary authority varies depending on whether the representative debtor country with debt $\boldsymbol{b}$ is in the safe zone or in the crisis zone. If it is in the crisis zone inflation policies correspond to inflation choices with $\boldsymbol{b} = 0$ because the representative country defaults in the crisis area.

The solution to the problem of the fiscal authority with positive debt level in the safe zone is exactly the same as that of problem (P1) described in Section 2. The consumption policy function is the steady state solution $C(\boldsymbol{b}) = y - \rho \boldsymbol{b}$.

The solution to the problem of the monetary authority (P4) in the safe zone, is also the same as the solution to problem (P2) in the no-crisis equilibria. The first order condition for inflation as before requires comparing ψ_0 to $-J'(\boldsymbol{b})\boldsymbol{b}$ where now:

$$-J'(\boldsymbol{b}) = \eta u'(y - \rho \boldsymbol{b}) + \frac{\psi_0 \Pi'(\boldsymbol{b})\boldsymbol{b}}{\rho} \tag{9}$$

The cut off level of debt $\bar{\boldsymbol{b}}_\pi$ above which there will be high inflation in the safe zone is therefore determined by the condition:

$$u'(y - \rho \bar{\boldsymbol{b}}_\pi)\bar{\boldsymbol{b}}_\pi = \frac{\psi_0}{\eta} \tag{10}$$

This equation follows intuitively. As the fraction of countries with positive debt increases the monetary authority is more tempted to inflate as it perceives more countries benefiting from the reduction in the real value of debt. In this sense as long as $\eta < 1$ there is a difference between the cost/benefit of inflation as perceived by the monetary authority ψ_0/η and the private cost of inflation ψ_0.

3.4 Threshold for the safe zone

There are many recursive threshold equilibria corresponding to different thresholds $\boldsymbol{b}_\lambda$. We now propose a particular equilibrium selection, which leads to a specific determination of the threshold for the safe zone $\boldsymbol{b}_\lambda$. We then motivate our equilibrium selection.

Consider a recursive threshold equilibrium. Let $\hat{V}^G(b, \boldsymbol{b}, r, \boldsymbol{r}, \delta; \hat{\Pi})$ be the value function (net of inflation costs) for a fiscal authority in the grace period, and let $\Pi^G(\boldsymbol{b}, \boldsymbol{r}, \delta, t)$ be the inflation function for the monetary authority in the grace period when it satisfies the consistency requirement $\Pi^G = \tilde{\Pi}^G$. Define $\underline{\boldsymbol{b}}_\lambda$ and $\overline{\boldsymbol{b}}_\lambda$ by:

Definition 3. *Let*

$$\underline{\boldsymbol{b}}_\lambda \equiv \sup\left\{ \boldsymbol{b} \le \frac{(1-e^{-r^\star\delta})y}{\rho} \middle| \hat{V}^G(\boldsymbol{b}, \boldsymbol{b}, r^* + \bar{\pi}, r^* + \bar{\pi}, \delta; \Pi^G) \ge \underline{\hat{V}} \right\};$$

$$\overline{\boldsymbol{b}}_\lambda \equiv \sup\left\{ \boldsymbol{b} \le \frac{(1-e^{-r^\star\delta})y}{\rho e^{-\bar{\pi}\delta}} \middle| \hat{V}^G(\boldsymbol{b}, \boldsymbol{b}, r^*, r^*, \delta; \Pi^G) \ge \underline{\hat{V}} \right\}.$$

These two thresholds correspond to the maximal debt the government is willing to repay within the grace period if the interest rate is $r^\star + \bar{\pi}$ and $r^\star$, respectively. Note that we have only to consider these two interest rates because we are firstly focusing on threshold equilibria where inflation takes the two value 0 or $\bar{\pi}$ and secondly because there is no rollover crisis in equilibrium in the safe zone.

From the fiscal authority's problem described in Section 3.1, we have $\underline{\boldsymbol{b}}_\lambda < \overline{\boldsymbol{b}}_\lambda$. This follows from the fact that $\hat{V}^G(\boldsymbol{b}, \boldsymbol{b}, \boldsymbol{r}, \boldsymbol{r}, \delta; \Pi^G)$ is strictly decreasing in $\boldsymbol{r}$. The equilibrium threshold for a rollover crisis $\boldsymbol{b}_\lambda$ lies in $\in [\underline{\boldsymbol{b}}_\lambda, \overline{\boldsymbol{b}}_\lambda]$, the exact value within this interval being determined by optimal inflation.

We motivate our equilibrium selection as follows. In Appendix *A*, we show formally that that in the grace period there is no fiscal externality. That is, if all countries are symmetric and if fiscal and monetary decisions for all countries are delegated to a central authority, then in the grace period, this authority would implement exactly the same allocation as that reached in an equilibrium with independent fiscal authorities.

Despite the absence of the traditional fiscal externality in the grace period, there remains a "default externality." The default externality arises because there may be more than one equilibrium best response from the monetary union. If the fiscal authorities default, the monetary authority will not inflate thus making repayment difficult. Conversely, an alternative equilibrium response may exist in which fiscal authorities repay within the grace period, aided by accommodative monetary policy.

While the default externality may be of interest in some contexts, it is not robust to a straightforward coordination of beliefs among members of the monetary union. In Appendix *A*, we show that if all countries are symmetric and if default, fiscal and monetary decisions for all countries are delegated to a central authority, then faced with a rollover crisis, the allocation implemented by this authority can also be reached in an equilibrium when default and fiscal decisions are made by independent fiscal authorities. That is, the monetary union can achieve the single-decision-maker outcome in the symmetric equilibrium by coordinating beliefs on the preferable equilibrium response.[17]

Our equilibrium selection imposes the requirement that if there exists an equilibrium best response in which the monetary authority comes to the rescue of the fiscal authorities in a crisis by generating high inflation, the fiscal authorities proceed as if they will be rescued. This selection is appealing given the plausibility that beliefs within the union can be coordinated in this way.

Given this requirement, our equilibrium is the monotone threshold equilibrium with the largest possible crisis zone, i.e. the lowest possible value of $\boldsymbol{b}_\lambda$.[18]

3.5 Crisis vulnerability and the composition of debt

In this section we determine how the threshold for the safe zone $\boldsymbol{b}_\lambda$ varies with the fraction of countries with positive debt η. In our environment we can perform this analysis without solving the problem of the crisis zone.

In figure 3 we graph the thresholds $\underline{\boldsymbol{b}}_\lambda$, $\bar{\boldsymbol{b}}_\lambda$ and $\bar{\boldsymbol{b}}_\pi$ as a function of $1/\eta$. From the problem of the monetary authority in the grace period, Section 3.1.2, we have that the monetary authority is more likely to inflate the higher is η. $\hat{V}^G(\boldsymbol{b}, \boldsymbol{b}, \boldsymbol{r}, \boldsymbol{r}, \delta; \Pi^G)$ excludes the direct cost of inflation (as fiscal authorities ignore the impact of their decisions on inflation) but includes the indirect benefit that arises from higher consumption when debt is inflated away. Since the monetary authority is more likely to inflate the higher is η we have $\hat{V}^G(\boldsymbol{b}, \boldsymbol{b}, \boldsymbol{r}, \boldsymbol{r}, \delta; \Pi^G)$

[17]Note that this is very different than the fiscal externality in section 2.3. In that case, there was not a consistent set of equilibrium beliefs that resolved the fiscal externality. Indeed, this result stems from the fact that there is no fiscal externality in the grace period as the interest rate on debt in arrears is constant.

[18]In the safe zone a unique monotone threshold equilibrium will always exist. However, as we discuss below for some values of ψ_0 monotone threshold equilibria may not exist in the region where crisis is an equilibrium possibility.

(weakly) increasing in η and accordingly $\underline{\boldsymbol{b}}_\lambda$ and $\overline{\boldsymbol{b}}_\lambda$ are weakly decreasing in η^{-1}. As drawn in figure 3 there is a flat segment initially which allows for the possibility that for a range of very low η^{-1} the monetary authority chooses to inflate to the maximal level and therefore $\hat{V}^G$ is independent of η over this range. The inflation threshold $\overline{\boldsymbol{b}}_\pi$ is increasing in η^{-1}, which follows straightforwardly from equation (10).

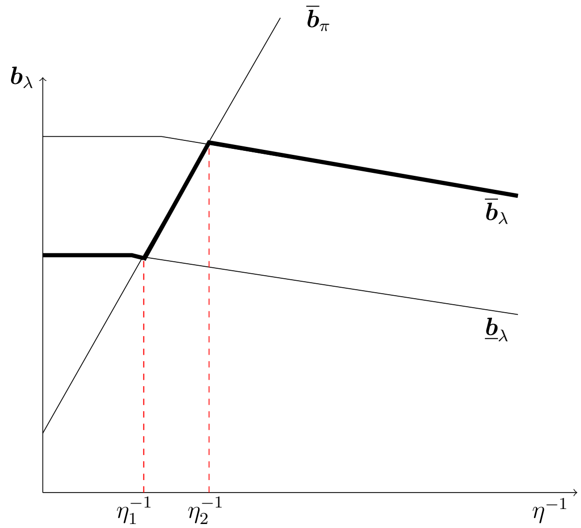


Figure 3: Crisis threshold and debt composition

In the range $\eta^{-1} < \eta_1^{-1}$, $\boldsymbol{b}_\lambda > \overline{\boldsymbol{b}}_\pi$, that is inflation jumps from 0 to $\bar{\pi}$ in the safe zone. This implies that the relevant threshold for the crisis zone is $\underline{\boldsymbol{b}}_\lambda$. When $\eta_1^{-1} < \eta^{-1} < \eta_2^{-1}$, $\overline{\boldsymbol{b}}_\pi \in [\underline{\boldsymbol{b}}_\lambda, \overline{\boldsymbol{b}}_\lambda]$ and therefore the jump in inflation triggers a crisis and the crisis threshold $\boldsymbol{b}_\lambda$ tracks $\overline{\boldsymbol{b}}_\pi$. To the right of η_2^{-1} a crisis is triggered even when inflation is low and accordingly $\boldsymbol{b}_\lambda$ tracks $\overline{\boldsymbol{b}}_\lambda$. The crisis threshold evolves non-monotonically with η^{-1}.

When there are a large number of debtor countries, very low η^{-1}, the monetary authority inflates all the time, both in tranquil times and in response to a roll-over crisis. Since all of the inflation is priced into interest rates there is no gain from inflating in crisis times. At the other extreme, when η^{-1} is very high, there are so few countries with positive debt that the monetary authority never inflates, neither in tranquil times nor in response to a crisis. For intermediate levels of debtor nations and therefore intermediate levels of aggregate debt the monetary authority is able to keep inflation low in tranquil times and therefore interest rates are low in tranquil times and have surprise inflation in response to a crisis. This ability to generate surprise inflation reduces the real value of debt the country with positive debt owes and increases the region of debt over which there is no rollover crisis.

In Section 4 we analytically characterize the full solution to the crisis problem.

4 Full Solution for Crisis Equilibria

4.1 Fiscal Authorities

We have already described the solution to the problem of the fiscal authority in the safe zone. In this section we turn to the solution over the entire domain of debt, including the crisis region, which is defined by the threshold $\boldsymbol{b}_\lambda$. The fiscal authority's value given its idiosyncratic debt as well as union-wide debt is $V(b, \boldsymbol{b})$. However, aggregate debt only enters the fiscal authority's problem due to the inflation costs, which is additively separable and – from the perspective of the fiscal authority – independent of fiscal policy. We can therefore consider the fiscal authority's problem net of inflation costs. This net value is denoted $\hat{V}(b)$. Note that this separability also exploits the fact that the crisis threshold faced by an idiosyncratic fiscal authority, $\boldsymbol{b}_\lambda$, is independent of aggregate debt.

At points of differentiability $b > \boldsymbol{b}_\lambda$ of the value function, the HJB of the planning problem is:

$$(\rho + \lambda)\hat{V}(b) = \max_c u(c) + \hat{V}'(b)[(\rho + \lambda)b + c - y] + \lambda\underline{\hat{V}}.$$

The first-order condition for consumption and the envelope condition are simply

$$u'(c) = -\hat{V}'(b),$$

$$\hat{V}''(b)[(\rho + \lambda)b + c - y] = 0.$$

We continue to assume $\rho = r^*$, however it is no longer the case that the solution for the fiscal authority in the crisis zone for all levels of debt is the stationary solution. This is because while fiscal authorities do not internalize the effect of their decisions on inflation, they do internalize the effect of their debt choices on the individual interest rate they face as it depends on the country's default probability. The stationary level of consumption in the crisis zone will be $C(b) = y - (\rho + \lambda)b$ and consequently the stationary solution for $\hat{V} = u(y - (\rho + \lambda)b)/(\rho + \lambda)$. This $\hat{V}$ is discontinuously lower to the right of $\boldsymbol{b}_\lambda$, the crisis threshold, when compared to the $\hat{V} = u(y - \rho b)/\rho$ in the safe zone. Just as in Cole and Kehoe (2000), the fiscal authority therefore has an incentive to save to the right of $\boldsymbol{b}_\lambda$ so as to exit the crisis zone, trading off lower consumption in the transition for higher steady state consumption. There is an optimal level of debt $b_{\max} > b^* > \boldsymbol{b}_\lambda$ such that for $b > b^*$ the fiscal authority prefers the stationary solution, the "staying zone", as it is too costly in terms of foregone consumption to save out of the crisis zone.

Over the "saving zone" we have $(\rho + \lambda)b + c - y < 0$. Satisfying the envelope condition requires that $\hat{V}''(b) = 0$, that is the value function is linear over the range of debt where the country saves. As a result $\hat{V}'(b)$ and consumption $u'(c) = -\hat{V}'(b)$ are constant. The solution for the constant consumption in the saving zone, $C_\lambda(b_\lambda)$ is determined by value matching using the HJB at $\boldsymbol{b}_\lambda$. Smooth pasting at b^* imposes that this constant consumption level $C_\lambda(b_\lambda) = y - (\rho + \lambda)b^*$.

The equilibrium solution for consumption, given the crisis threshold $\boldsymbol{b}_\lambda$ is then given by:

$$C(\boldsymbol{b}) = \begin{cases} y - \rho b & \text{if } b \leq \boldsymbol{b}_\lambda, \\ C_\lambda(\boldsymbol{b}_\lambda) & \text{if } \boldsymbol{b}_\lambda < b < b^*, \\ y - (\rho + \lambda)b & \text{if } b^* \leq b \leq b_{max}, \end{cases}$$

where $C_\lambda(\boldsymbol{b}_\lambda)$ is defined implicitly by

$$\frac{u(y - \rho \boldsymbol{b}_\lambda)}{\rho} - \frac{u(C_\lambda(\boldsymbol{b}_\lambda))}{\rho + \lambda} - \frac{\lambda}{\rho + \lambda}\frac{u(\chi y)}{\rho} + u'(C_\lambda(\boldsymbol{b}_\lambda))\frac{C_\lambda(\boldsymbol{b}_\lambda) - y + (\rho + \lambda)\boldsymbol{b}_\lambda}{\rho + \lambda} = 0,$$

and $\boldsymbol{b}^*$ by $C_\lambda(\boldsymbol{b}_\lambda) = y - (\rho + \lambda)\boldsymbol{b}^*$. Note that the consumption policy function depends on η through its impact on the equilibrium determination of $\boldsymbol{b}_\lambda$.

As a technical aside, we have characterized the fiscal authority's problem as the solution to the HJB where differentiable. Note that the fiscal authority's optimal consumption policy involves a discontinuity at $\boldsymbol{b}_\lambda$. This reflects the desire to save out of the crisis region. Once the safe region is reached, consumption jumps to the stationary-debt level. From the first order condition, $u'(c) = -\hat{V}'(b)$, this discrete jump in consumption implies that $\boldsymbol{b}_\lambda$ is a point of non-differentiability. While the HJB cannot hold in the classical sense at this point, it does satisfy the conditions for a "viscosity" solution, which is the appropriate generalized solution.[19]

4.2 Monetary Authority

Having already described the solution for the problem of the monetary authority in the safe zone, we evaluate the problem in the crisis zone. The HJB for the monetary authority in the crisis zone, where differentiable is given by:

$$(\rho + \lambda)J(\boldsymbol{b}) = \max_{\pi \in [0, \bar{\pi}]} \eta u(C(\boldsymbol{b})) + (1 - \eta)u(y) - \psi_0 \pi + J'(\boldsymbol{b})[(r(\boldsymbol{b}) - \pi)\boldsymbol{b} + C(\boldsymbol{b}) - y] + \lambda \underline{V}.$$

[19] See Aguiar et al. (2012) for a discussion of viscosity solutions in a related context.

The first order condition and envelope condition are given by:

$$\Pi(\boldsymbol{b}) = \begin{cases} 0 & \text{if} \quad \psi_0/\eta \geq -J'(\boldsymbol{b})\boldsymbol{b}, \\ \bar{\pi} & \text{if} \quad \psi_0/\eta < -J'(\boldsymbol{b})\boldsymbol{b}. \end{cases}$$

$$J'(\boldsymbol{b})\Pi'(\boldsymbol{b})\boldsymbol{b} + J''(\boldsymbol{b})\left((\rho+\lambda)\boldsymbol{b} + C(\boldsymbol{b}) - y\right) = 0. \tag{11}$$

The full solution will depend on the value of η that in turn determines whether the jump in inflation takes place in the safe zone or the crisis zone. As a counterpart to $\bar{\boldsymbol{b}}_\pi$, which is the debt threshold above which the monetary authority chooses high inflation when faced with the interest rate ρ, define $\tilde{\boldsymbol{b}}_\pi$ as the maximum debt threshold above which the monetary authority will choose high inflation when faced with the interest rate $\rho+\lambda$. The equilibrium inflation threshold is denoted by $\boldsymbol{b}_\pi$.

Before providing the full analytical characterization of the solution for different values of η in section 5.1, we describe using an example the circumstances under which a high-debt country can be better off in a monetary union with an appropriate number of high debtors than one with only low-debt countries. This relates to the discussion in the introduction about optimal currency areas in the presence of self-fulfilling crises.

5 Optimal composition of a currency union

In the case without self-fulfilling crises a country with high debt is strictly better off when every other member has low debt, ($\eta = 0$) as discussed in Section 2. This composition of the currency union endogenously lowers the benefit of inflation for the monetary authority thus enabling it to deliver the commitment outcome of zero inflation. However, this conclusion changes when countries are exposed to roll-over risk. In this case a country with high debt may be better off when there is an appropriate measure of high debtors, i.e. η is sufficiently greater than zero, as it can lower the vulnerability of the country to self-fulfilling crises, if that pushes the central bank to inflate in response to a roll-over crisis but not in tranquil times.

To illustrate this, consider a currency union where every country has low debt i.e. $\eta = 0$. In this case the perceived cost of inflation, ψ_0/η goes to infinity and the monetary authority never inflates, neither in tranquil times nor in crisis times. Define $\boldsymbol{b}_{\lambda_1}$ as the crisis threshold in this case. Now consider increasing the fraction η in the currency union such that it leads to an increase in $\boldsymbol{b}_\lambda$ (as shown earlier) to $\boldsymbol{b}_{\lambda_2}$ with the jump in inflation occurring to the right of $\boldsymbol{b}_{\lambda_2}$. A comparison of the two cases demonstrates that a country with a debt level $\boldsymbol{b}_{\lambda_1} < b < \boldsymbol{b}_{\lambda_2}$ is necessarily better off joining the currency union with a few high debtors (the second case), than in a currency union with only low debtors ($\eta = 0$). The two value functions are depicted in figure 4. The argument is simple. The value functions at $\boldsymbol{b}_{\lambda_1}$ are equal and the slope $-u'(C_\lambda(\boldsymbol{b}_{\lambda_1}))$ of the value function with $\eta = 0$ is strictly more negative than the slope $u'(y - \rho\boldsymbol{b}_{\lambda_1})$ with $\eta > 0$ since $C_\lambda(\boldsymbol{b}_{\lambda_1}) < y - \rho b_{\lambda_1}$.

There is therefore a range of high-debt over which welfare is higher for a high-debt member in a monetary union with a larger number of high-debt members than with too few high debt members. When η is high the monetary authority is credibly able to keep inflation low in tranquil times but inflate in response to a crisis. This greater use of inflation in the grace period increases the size of the safe zone and increases the welfare of the country. Given our assumption on inflation costs, low-debt members are not worse off following the increase in η because inflation happens off-equilibrium.[20]

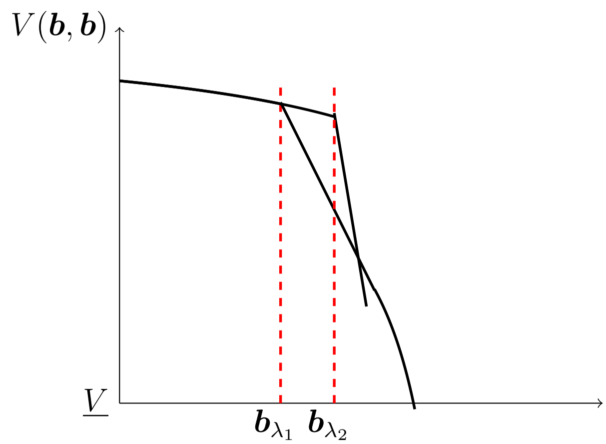


Figure 4: Welfare and debt composition

We now describe analytically the different possible configurations of equilibrium thresholds and their implications for welfare.

5.1 Cases

<u>**Case 1:**</u> $\bar{\boldsymbol{b}}_\pi < \underline{\boldsymbol{b}}_\lambda$

This is the case when the fraction of countries with positive debt is so high that the jump in inflation takes place in the safe zone. The solution is depicted in figure 5.

[20] Alternative cost specifications might lead to an increase in the equilibrium inflation level, but the point that the impact on the crisis threshold depends on off-equilibrium inflation remains.

(i) Consumption policy function:

$$C(\boldsymbol{b}) = \begin{cases} u(y - \rho \boldsymbol{b}) & \text{if} \quad \boldsymbol{b} \leq \underline{\boldsymbol{b}}_{\lambda}, \\ C_{\lambda}(\underline{\boldsymbol{b}}_{\lambda}) & \text{if} \quad \underline{\boldsymbol{b}}_{\lambda} < \boldsymbol{b} < \boldsymbol{b}^{*}, \\ u(y - \rho \boldsymbol{b}) & \text{if} \quad \boldsymbol{b}^{*} \leq \boldsymbol{b} \leq \boldsymbol{b}_{max}. \end{cases}$$

(ii) Inflation policy function:

$$\Pi(\boldsymbol{b}) = \begin{cases} 0 & \text{if } \boldsymbol{b} \leq \bar{\boldsymbol{b}}_{\pi}, \\ \bar{\pi} & \text{if } \bar{\boldsymbol{b}}_{\pi} < \boldsymbol{b} \leq \boldsymbol{b}_{\max}. \end{cases}$$

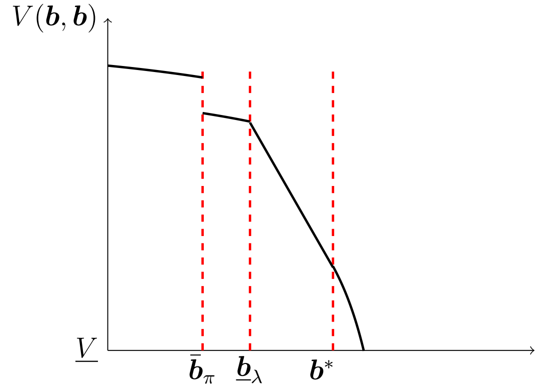
(iii) Interest rate schedule:

$$r(\boldsymbol{b}) = \begin{cases} \rho & \text{if} \quad \boldsymbol{b} \leq \bar{\boldsymbol{b}}_{\pi}, \\ \rho + \bar{\pi} & \text{if} \quad \bar{\boldsymbol{b}}_{\pi} < \boldsymbol{b} \leq \underline{\boldsymbol{b}}_{\lambda}, \\ \rho + \bar{\pi} + \lambda & \text{if} \quad \underline{\boldsymbol{b}}_{\lambda} < \boldsymbol{b} \leq \boldsymbol{b}_{max}. \end{cases}$$

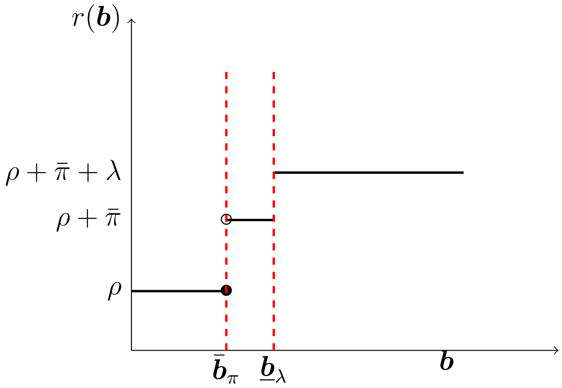
(iv) Value functions:

$$V(\boldsymbol{b}) = \begin{cases} \frac{u(y-\rho \boldsymbol{b})}{\rho} & \text{if } \boldsymbol{b} \leq \bar{\boldsymbol{b}}_{\pi}, \\ \frac{u(y-\rho \boldsymbol{b})-\psi_0 \bar{\pi}}{\rho} & \text{if } \bar{\boldsymbol{b}}_{\pi} < \boldsymbol{b} \leq \boldsymbol{b}_{\lambda}, \\ V(\underline{\boldsymbol{b}}_{\lambda}) - u'(C_{\lambda}(\underline{\boldsymbol{b}}_{\lambda}))(\boldsymbol{b} - \underline{\boldsymbol{b}}_{\lambda}) & \text{if } \boldsymbol{b}_{\lambda} < \boldsymbol{b} \leq \boldsymbol{b}^{*}, \\ \frac{u(y-(\rho+\lambda)\boldsymbol{b})-\psi_0 \bar{\pi}}{\rho+\lambda} + \frac{\lambda}{\rho+\lambda} \underline{V} & \text{if } \boldsymbol{b}^{*} < \boldsymbol{b} \leq \boldsymbol{b}_{max}. \end{cases}$$

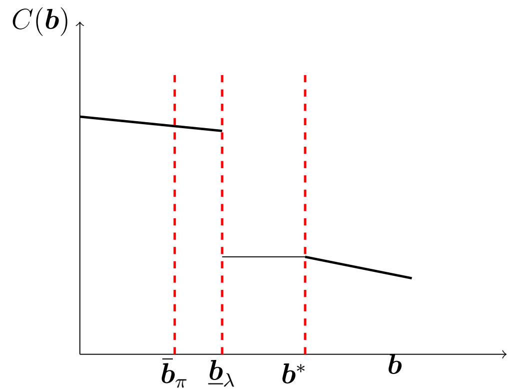
Case 2: $\bar{\boldsymbol{b}}_{\lambda} < \tilde{\boldsymbol{b}}_{\pi} < \boldsymbol{b}^{*}$ This is the case when the jump in inflation takes place within the saving zone. In this case a monotone threshold equilibrium may not exist for certain values of η as we discuss below. When it exists, the solution is as described below and depicted in figure 6.



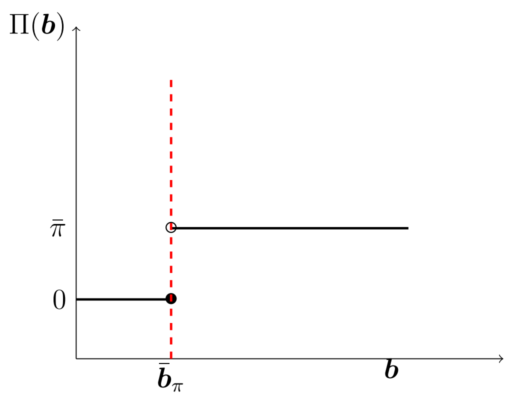
(a) Value Function



(b) Interest Rates



(c) Consumption Policy



(d) Inflation Policy

Figure 5: Solution in the case when $\bar{\boldsymbol{b}}_\pi < \underline{\boldsymbol{b}}_\lambda$

(i) Consumption policy function:

$$C(\boldsymbol{b}) = \begin{cases} u(y-\rho\boldsymbol{b}) & \text{if } \boldsymbol{b} \leq \bar{\boldsymbol{b}}_\lambda, \\ C_\lambda(\bar{\boldsymbol{b}}_\lambda) & \text{if } \bar{\boldsymbol{b}}_\lambda < \boldsymbol{b} < \boldsymbol{b}^*, \\ u(y-\rho\boldsymbol{b}) & \text{if } \boldsymbol{b}^* \leq \boldsymbol{b} \leq \boldsymbol{b}_{max}. \end{cases}$$

(ii) Inflation policy function:

$$\Pi(\boldsymbol{b}) = \begin{cases} 0 & \text{if } \boldsymbol{b} \leq \tilde{\boldsymbol{b}}_\pi, \\ \bar{\pi} & \text{if } \tilde{\boldsymbol{b}}_\pi < \boldsymbol{b} \leq \boldsymbol{b}_{\max}. \end{cases}$$

(iii) Interest rate schedule:

$$r(\boldsymbol{b}) = \begin{cases} \rho & \text{if } \boldsymbol{b} \leq \bar{\boldsymbol{b}}_\lambda, \\ \rho+\lambda & \text{if } \bar{\boldsymbol{b}}_\lambda < \boldsymbol{b} \leq \tilde{\boldsymbol{b}}_\pi, \\ \rho+\bar{\pi}+\lambda & \text{if } \tilde{\boldsymbol{b}}_\pi < \boldsymbol{b} \leq \boldsymbol{b}_{max}. \end{cases}$$

(iv) Value functions:

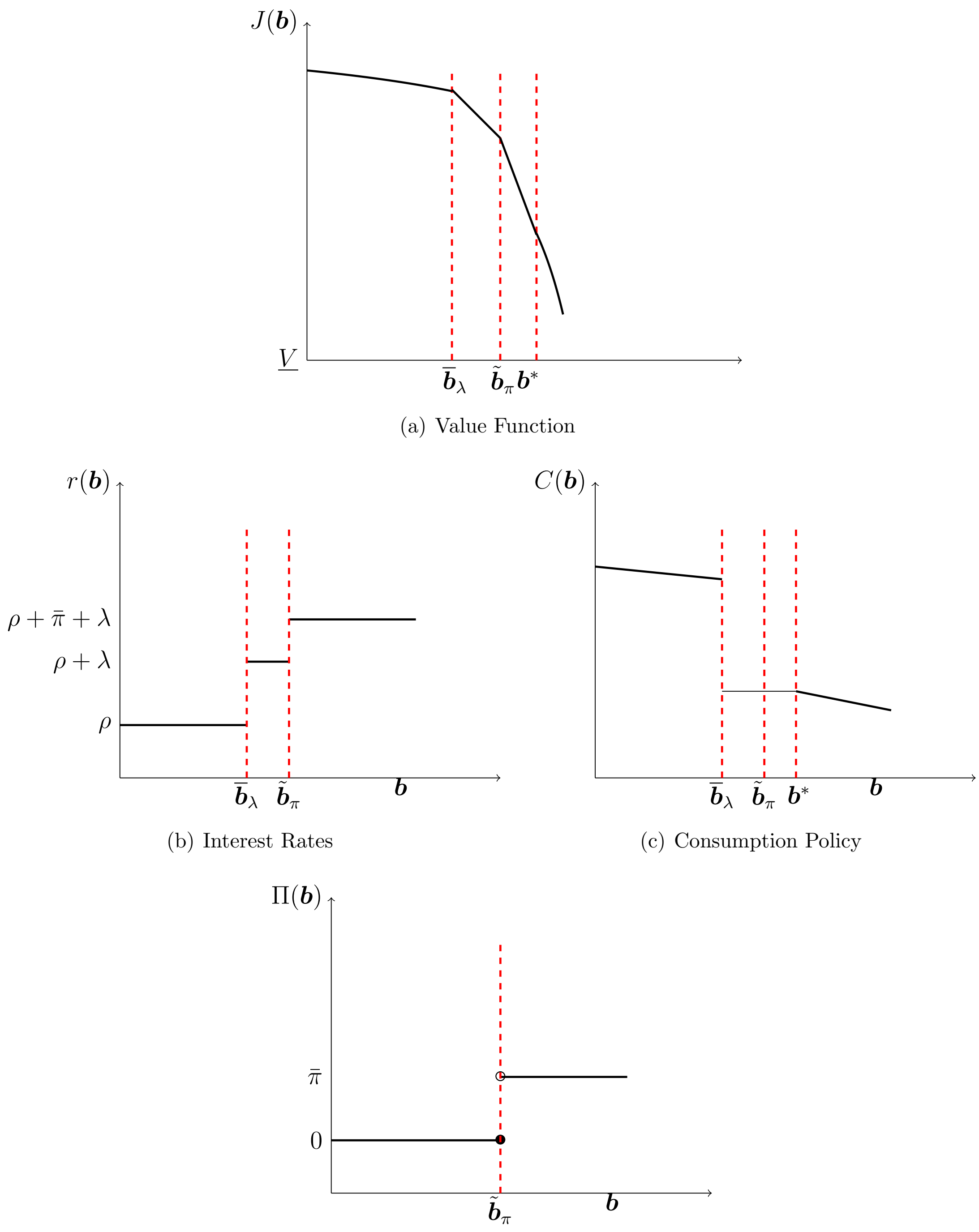
$$V(\boldsymbol{b}) = \begin{cases} \frac{u(y-\rho\boldsymbol{b})}{\rho} & \text{if } \boldsymbol{b} \leq \bar{\boldsymbol{b}}_\lambda, \\ V(\bar{\boldsymbol{b}}_\lambda) - u'(C_\lambda(\bar{\boldsymbol{b}}_\lambda))(\boldsymbol{b}-\bar{\boldsymbol{b}}_\lambda) & \text{if } \bar{\boldsymbol{b}}_\lambda < \boldsymbol{b} \leq \tilde{\boldsymbol{b}}_\pi, \\ V(\tilde{\boldsymbol{b}}_\pi) - [u'(C_\lambda(\bar{\boldsymbol{b}}_\lambda)) + \frac{\psi_0\pi}{(\rho+\lambda)(\boldsymbol{b}^*-\tilde{\boldsymbol{b}}_\pi)}](\boldsymbol{b}-\tilde{\boldsymbol{b}}_\pi) & \text{if } \tilde{\boldsymbol{b}}_\pi < \boldsymbol{b} \leq \boldsymbol{b}^*, \\ \frac{(u(y-(\rho+\lambda)\boldsymbol{b})-\psi_0\bar{\pi})}{\rho+\lambda} + \frac{\lambda}{\rho+\lambda}\underline{V} & \text{if } \boldsymbol{b}^* < \boldsymbol{b} \leq \boldsymbol{b}_{max}. \end{cases}$$

The value function in this solution has a concave kink at $\tilde{\boldsymbol{b}}_\pi$ and a convex kink at $\boldsymbol{b}^*$. To ensure that a monotone threshold equilibrium exists, that is inflation does not jump down to the right of $\boldsymbol{b}^*$, we require that the following condition holds: $u'(C_\lambda(\bar{\boldsymbol{b}}_\lambda))\boldsymbol{b}^* - \psi_0 \geq 0$.[21]

<u>**Case 3:**</u> $\boldsymbol{b}_\lambda < \boldsymbol{b}^* < \tilde{\boldsymbol{b}}_\pi < \boldsymbol{b}_{max}$

This is the case when the jump in inflation takes place within the staying zone. The

[21] We can prove that $[u'(C_\lambda(\bar{\boldsymbol{b}}_\lambda))\boldsymbol{b}^* + \frac{\psi_0\pi}{(\rho+\lambda)(1-\frac{\tilde{\boldsymbol{b}}_\pi)}{\boldsymbol{b}^*}}] - \psi_0 \geq 0$ is satisfied, but this is not sufficient. As $\tilde{\boldsymbol{b}}_\pi \to \boldsymbol{b}^*$, the second term can get large.



(a) Value Function

(b) Interest Rates

(c) Consumption Policy

(d) Inflation Policy

Figure 6: Solution in the case when $\bar{\boldsymbol{b}}_\lambda < \tilde{\boldsymbol{b}}_\pi < \boldsymbol{b}^*$

solution described below is depicted in figure 7.

(i) Consumption policy function:

$$C(\boldsymbol{b}) = \begin{cases} u(y - \rho \boldsymbol{b}) & \text{if } \boldsymbol{b} \leq \bar{\boldsymbol{b}}_\lambda, \\ C_\lambda(\boldsymbol{b}_\lambda) & \text{if } \bar{\boldsymbol{b}}_\lambda < \boldsymbol{b} < \boldsymbol{b}^*, \\ u(y - \rho \boldsymbol{b}) & \text{if } \boldsymbol{b}^* \leq \boldsymbol{b} \leq \boldsymbol{b}_{max}. \end{cases}$$

(ii) Inflation policy function:

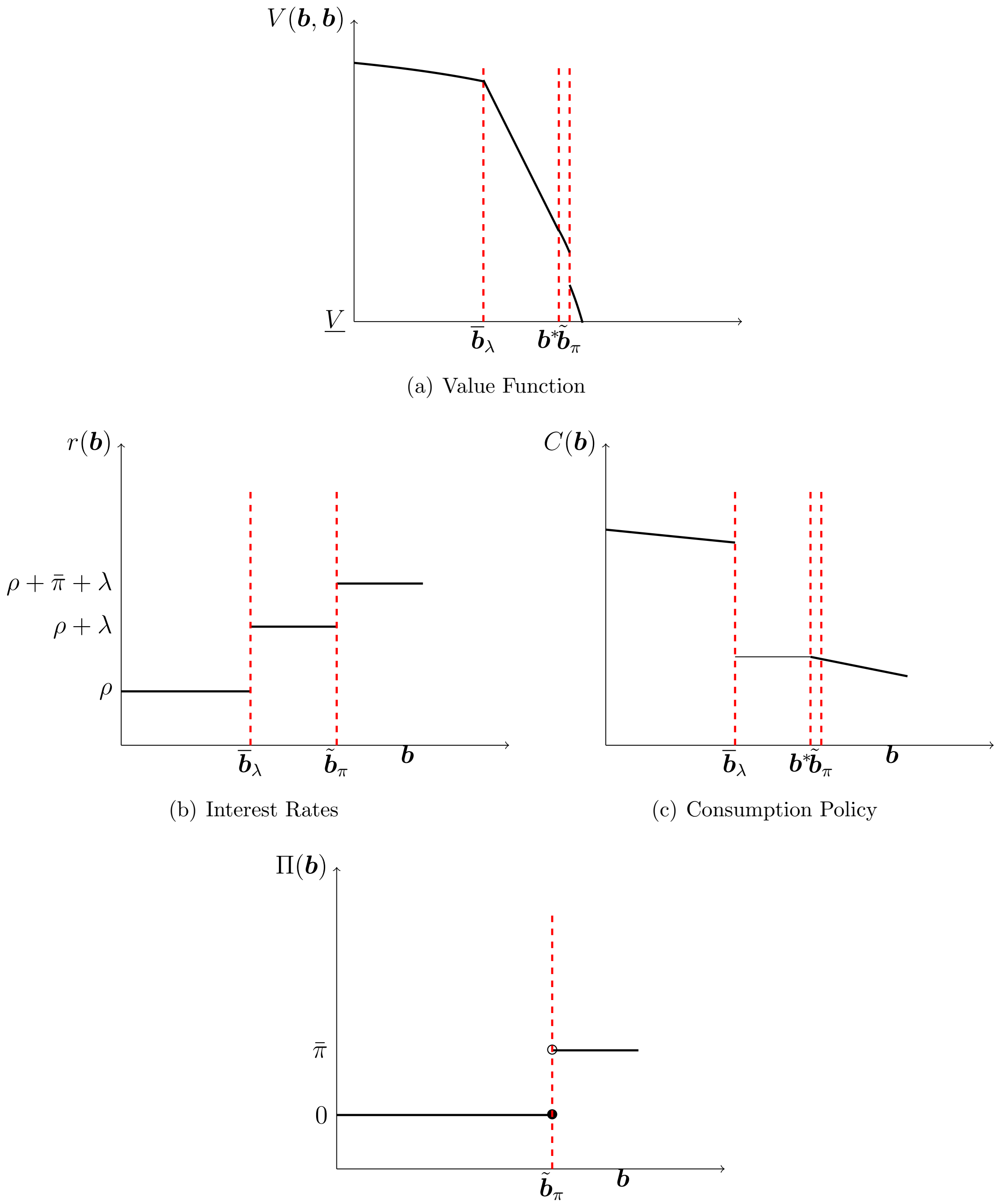
$$\Pi(\boldsymbol{b}) = \begin{cases} 0 & \text{if } \boldsymbol{b} \leq \tilde{\boldsymbol{b}}_\pi, \\ \bar{\pi} & \text{if } \tilde{\boldsymbol{b}}_\pi < \boldsymbol{b} \leq \boldsymbol{b}_{\max}, \end{cases}$$

(iii) Interest rate schedule:

$$r(\boldsymbol{b}) = \begin{cases} \rho & \text{if } \boldsymbol{b} \leq \bar{\boldsymbol{b}}_\lambda, \\ \rho + \lambda & \text{if } \bar{\boldsymbol{b}}_\lambda < \boldsymbol{b} \leq \tilde{\boldsymbol{b}}_\pi, \\ \rho + \bar{\pi} + \lambda & \text{if } \tilde{\boldsymbol{b}}_\pi < \boldsymbol{b} \leq \boldsymbol{b}_{max}. \end{cases}$$

(iv) Value functions:

$$V(\boldsymbol{b}) = \begin{cases} \frac{u(y - \rho \boldsymbol{b})}{\rho} & \text{if } \boldsymbol{b} \leq \bar{\boldsymbol{b}}_\lambda, \\ V(\bar{\boldsymbol{b}}_\lambda) - u'(C_\lambda(\bar{\boldsymbol{b}}_\lambda))(\boldsymbol{b} - \bar{\boldsymbol{b}}_\lambda) & \text{if } \bar{\boldsymbol{b}}_\lambda < \boldsymbol{b} \leq \boldsymbol{b}^*, \\ \frac{u(y - (\rho + \lambda)\boldsymbol{b}}{\rho + \lambda} + \frac{\lambda}{\rho + \lambda} \underline{V} & \text{if } \boldsymbol{b}^* < \boldsymbol{b} \leq \tilde{\boldsymbol{b}}_\pi, \\ \frac{u(y - (\rho + \lambda)\boldsymbol{b}) - \psi_0 \bar{\pi}}{\rho + \lambda} + \frac{\lambda}{\rho + \lambda} \underline{V} & \text{if } \tilde{\boldsymbol{b}}_\pi < \boldsymbol{b} \leq \boldsymbol{b}_{max}. \end{cases}$$



(a) Value Function

(b) Interest Rates

(c) Consumption Policy

(d) Inflation Policy

Figure 7: Solution in the case when $\boldsymbol{b}^* < \bar{\boldsymbol{b}}_\pi < \boldsymbol{b}_{max}$

6 Conclusion

The ongoing euro zone crisis has brought to the fore front the inherent tensions in a monetary union where individual countries have control over fiscal decisions but where monetary decisions are made by a union wide monetary authority that maximizes welfare of the union as a whole. It is a familiar argument that individual countries in a union are worse off when there is limited synchronization in business cycles across countries, as a common monetary policy for the union can be inconsistent with the needs of different countries. Here we highlight another tension that arises when countries are subject to roll-over risk in debt markets.

The monetary authority may be able to use surprise inflation to reduce the real value of debt owed and thus eliminate a roll-over crisis. Whether it will choose to do so and whether it can effectively do so depends on the *aggregate* level of debt in the union. If the aggregate level of debt in the union is low the monetary authority will choose never to inflate, neither in tranquil nor in crisis time. At the other extreme, if the aggregate debt in the union is high, the monetary authority uses inflation all the time and consequently fails to generate surprise inflation. On the other hand when there is an intermediate level of aggregate debt the monetary authority chooses low inflation in normal times and high inflation in crisis times, thus generating surprise inflation and helps prevent a roll-over crisis. An indebted country in the union therefore gets no help from the monetary authority in preventing self-fulfilling crises when everyone else in the union is as indebted as it is or when no one in the union is like it. A "Greece" is better off in a monetary union with some "Germany", but not all "Germany". This composition gives "Greece" both low inflation and eliminates its exposure to self-fulfilling crisis. Importantly, this can take place without any loss of welfare to "Germany" if the use of inflation is done off-equilibrium.[22]

Clearly, debt crises disappear when a country's debt is low enough. However, we demonstrate the existence of a fiscal externality that limits individual countries incentive to reduce their debt. This arises because they fail to internalize the impact of their debt on the union monetary authorities incentive to inflate. Consequently they end up with higher debt than if they were an independent country with control over both fiscal and monetary policy.

[22] We have described an environment where the debt of members of the union are held outside the union. In reality, as in the case of the euro zone, a significant fraction of the debt is held by members of the union. In our environment this would have similar effects to reducing η and therefore the incentive to inflate.

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A Fiscal and Default Externalities in Rollover Crises

The motivation for our equilibrium selection in Section 3.4 is that in response to a rollover crisis, the optimal coordinated fiscal and monetary policies are a symmetric equilibrium best response. In this section, we formalize this arguments. We break it down into two related results. First we establish that in the grace period problem, the optimal coordinated fiscal and monetary policies are a symmetric equilibrium best response. Second we establish that in response to a rollover crisis, the optimal coordinated default, fiscal and monetary policies are a symmetric equilibrium best response.

We start by showing that there is no fiscal externality in the grace period. To see this formally, it is useful to consider the best response to a grace period when monetary and fiscal policies are chosen by the same decision maker during the grace period. This is the scenario of the small open economy considered in section 2.3:

$$V_E^G(\boldsymbol{b}, \boldsymbol{r}, \delta) = \max_{\{\pi(t) \in [0, \bar{\pi}], c(t)\}} \int_0^\delta e^{-\rho t}(\eta u(c(t)) + (1-\eta)u(y) - \psi_0 \pi(t))dt + e^{-\rho\delta}\frac{u(y)}{\rho}, \quad (12)$$

subject to

$$\dot{\boldsymbol{b}}(\boldsymbol{t}) = c(t) + (\boldsymbol{r} - \pi(t))\boldsymbol{b}(t) - y$$
$$\boldsymbol{b}(0) = \boldsymbol{b}, \quad \boldsymbol{b}(\delta) = 0,$$

where as before $\boldsymbol{r}$ is the contracted interest rate at the start of the crisis (which is the equilibrium rate of the decentralized monetary union). For this problem, consumption and inflation are coordinated. We have the following result.

Result 1. *When the representative country is in the grace period, the optimal coordinated fiscal and monetary policies are the only symmetric equilibrium best response.*

This follows from the discussion of fiscal externalities in Section 2.3. Fiscal externalities arise because fiscal authorities fail to internalize the impact of their debt choices on the interest rates they face through its impact on inflation. In the grace period, the nominal interest rate r is fixed at r_0, independent of the level of aggregate debt $\boldsymbol{b}(t)$. As a result, fiscal authorities correctly internalize all the effects of their debt decisions. In other words, there is no fiscal externality.

Despite the absence of the traditional fiscal externality in the grace period, there remains a "default externality." The default externality arises because there may be more than one equilibrium best response from the monetary union. If the fiscal authorities default, the monetary authority will not inflate thus making repayment difficult. Conversely, an alternative equilibrium response may exist in which fiscal authorities repay within the grace period, aided by accommodative monetary policy.

While the default externality may be of interest in some contexts, it is not robust to a straightforward coordination of beliefs among members of the monetary union. To see this formally, consider the best response to a rollover crisis when monetary and fiscal policies are chosen by the same decision maker. The unified decision maker can decide to repay, in which case optimal monetary and fiscal policies are determined as above in equation (12). The unified decision maker can also choose default and receive $\underline{\hat{V}}_E = \eta \frac{u((1-\chi)y)}{\rho} + (1-\eta)\frac{u(y)}{\rho}$. The coordinated best response leads to a value $V_E(\boldsymbol{b}, \boldsymbol{r}, \delta) = \max\left\{V_E^G(\boldsymbol{b}, \boldsymbol{r}, \delta), \underline{\hat{V}}_E\right\}$. We have the following result.

Result 2. *When the representative country is subject to a rollover crisis, the optimal coordinated default, fiscal and monetary policies are a symmetric equilibrium best response.*

This result shows that the default externality is not robust to simple coordination.