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Effective Field Theory of Vacuum Tilting

R. Sekhar Chivukula*
Physics Department
Boston University
Boston MA 02215

Howard Georgi†
Lyman Laboratory of Physics
Harvard University
Cambridge, MA 02138

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Abstract

Simple models of topcolor and topcolor-assisted technicolor rely on a relatively strong $U(1)$ gauge interaction to "tilt" the vacuum. This tilting is necessary to produce a top-condensate, thereby naturally obtaining a heavy top-quark, and to avoid producing a bottom-condensate. We identify some peculiarities of the Nambu-Jona-Lasinio (NJL) approximation often used to analyze the topcolor dynamics. We resolve these puzzles by constructing the low-energy effective field theory appropriate to a mass-independent renormalization scheme. We construct the power-counting rules for such an effective theory. By requiring that the Landau pole associated with the $U(1)$ gauge theory be sufficiently above the topcolor gauge boson scale, we derive an upper bound on the strength of the $U(1)$ gauge-coupling evaluated at the topcolor scale. The upper bound on the $U(1)$ coupling implies that these interactions can shift the composite Higgs boson mass-squared by only a few per cent and, therefore, that the topcolor coupling must be adjusted to equal the critical value for chiral symmetry breaking to within a few per cent.

*sekhar@bu.edu
†georgi@physics.harvard.edu

1 Introduction

Simple models of topcolor [1] and topcolor-assisted technicolor [2] rely on chiral-symmetry breaking driven by the combination of an isospin-symmetric top-color gauge interaction and a relatively strong isospin-violating $U(1)$ gauge interaction which couple to the third generation of quarks. The top-color gauge interaction binds a composite Higgs boson which, if the interaction is near critical, is very light compared to its compositeness scale and thus can be described in an effective Lagrangian description with a fundamental scalar field [3, 4, 5, 6, 7, 8, 9, 10]. The $U(1)$ interaction is necessary to "tilt" the vacuum and produce a top-condensate, thereby naturally obtaining a heavy top-quark, but to avoid producing a bottom-condensate. We begin in section 2 by identifying some peculiarities of the Nambu-Jona-Lasinio (NJL) [11] approximation often used to analyze the chiral-symmetry breaking dynamics. In section 3, we construct the low-energy effective field theory appropriate to a mass-independent renormalization scheme and construct the power-counting rules for such an effective theory. The low-energy effective theory allows for an analysis of the dynamics of chiral-symmetry breaking beyond the NJL approximation, and the puzzles found in that approximation are resolved.

Phenomenological constraints [12] require that the mass of the topcolor gauge bosons, M, be substantially larger than the top-quark mass. For this to be possible, the topcolor chiral phase transition must be (at least approximately) second order [13] and the topcolor coupling strength (renormalized at scale M) must be adjusted to be close to the critical value α_c for chiral symmetry breaking. To tilt the vacuum, the $U(1)$ coupling must be strong enough to prevent bottom-quark condensation. In section 4 we use the effective low-energy theory to discuss the relationship between the strength of the $U(1)$ gauge coupling and the amount of "tuning" of the topcolor coupling that is required. In section 5 we show that, because the high-energy theory includes a non-asymptotically free $U(1)$ gauge interaction, some tuning is required. By requiring that the Landau pole [14] associated with the $U(1)$ gauge theory be sufficiently above the topcolor gauge boson scale, we derive an upper bound on the strength of the $U(1)$ gauge-coupling evaluated at the topcolor scale. The upper bound on the $U(1)$ coupling implies that the topcolor coupling must be adjusted to equal the critical value for chiral symmetry breaking to within a few per cent.

2 A Puzzle in NJL

The business end of a topcolor model [1, 2] consists of a set of quarks, including the t and b, that transform under topcolor, a stronger version of color $SU(3)$. There is also a weaker $SU(3)$ gauge interaction that we will call "protocolor". In the NJL approximation, the formation of composite Higgs bosons is driven by a four-fermion operator obtained by integrating out the massive colorons — the gauge bosons of the strong topcolor interactions. The mass, M, of the colorons comes from spontaneous breaking of topcolor cross protocolor, which preserves ordinary color. We want to discuss this spontaneous symmetry breaking in detail, so let us imagine for simplicity that it comes from the VEV of a spinless field that transforms like a $(3,\overline{3})$ under topcolor cross protocolor. All the essentials will be the same if the breaking is done in any simple dynamical way. The coloron mass will then be given by

$$M^2 = (g_{tc}^2 + g_{pc}^2)\, v^2 \tag{1}$$

where g_{tc} and g_{pc} are the topcolor and protocolor couplings at the scale M, v is the VEV, and the coupling of ordinary color at the scale M is then given by

$$g_c^2 = \frac{g_{tc}^2\, g_{pc}^2}{g_{tc}^2 + g_{pc}^2} \tag{2}$$

which is approximately equal to g_{pc}^2 if $g_c^2 \ll g_{tc}^2$. At low energies, coloron exchange can be approximated by a four-fermion coupling of the form

$$\frac{1}{2}\,\frac{(g_{tc}^2\, j_{tc}^\mu - g_{pc}^2\, j_{pc}^\mu)(g_{tc}^2\, j_{tc\mu} - g_{pc}^2\, j_{pc\mu})}{(g_{tc}^2 + g_{pc}^2)^2\, v^2} \tag{3}$$

where j_{tc}^μ and j_{pc}^μ are the topcolor and protocolor currents.

There is something odd about the NJL approximation in a topcolor model. Note that all dependence of (3) on the topcolor coupling goes away in the limit that topcolor is much stronger than ordinary color — which is a good approximation at the topcolor scale. In this limit, the four-fermion interaction is nearly independent of the value of the topcolor gauge coupling, because the coupling appears both in the numerator, in the coupling to the top quark and other relevant fermions, and in the denominator, in M. The way the NJL approximation deals with this peculiarity is to treat M as a fixed cut-off. Then what determines whether composite Higgs bosons are formed is the size of the four-fermion interaction times the cut-off squared — and this is proportional to the topcolor coupling α_{tc} evaluated at the scale M. Thus in the NJL approximation, by adjusting the topcolor coupling we can tune close to the critical "coupling" at which the composite Higgs become massless.

This peculiar behavior returns, however, when we now think about tilting the vacuum with a $U(1)$ coupling. Typically [2], such a tilting interaction is also a spontaneously broken gauge interaction, and in the NJL approximation is again described by a four-fermion operator. The simplest possibility is to have the spinless field or condensate that produces the topcolor breaking transform nontrivially under the strong $U(1)$ as well. Again the details simplify further if we imagine that this tilting $U(1)$ has a much larger coupling than any other $U(1)$ in the theory. Then the low-energy four-fermion operator due to $U(1)$ boson exchange looks like

$$\frac{1}{2}\,\frac{g_1^2\, j_1^\mu\, j_{1\mu}}{m^2} = \frac{1}{2}\,\frac{j_1^\mu\, j_{1\mu}}{Q^2\, v^2}\,, \tag{4}$$

where Q is the $U(1)$ charge of the field that breaks the symmetry, j_1^μ is the $U(1)$ current, and m is the $U(1)$ gauge boson mass, satisfying

$$m^2 \equiv g_1^2\, Q^2\, v^2\,. \tag{5}$$

There are two peculiar things about (4):

1. It does not depend on g_1.

2. It does depend on Q, and the interaction can be made very large by taking Q small (while leaving the couplings to the t- and b-quarks fixed).

Both of these features are physically unreasonable. Do we really believe that the $U(1)$ interaction tilts the vacuum even in the limit when its coupling g_1 is very tiny? Should we believe that for moderate g_1 we can still get very large tilting just by taking Q and thus m very small? We think not, and indeed these puzzling features do not persist in an effective field theory analysis that goes beyond the NJL approximation.

3 Matching in Large N

We consider the effect of a spontaneously broken $U(1)$ on the effective field theory describing the composite Higgs in a topcolor model near the critical coupling. We will work in a large N expansion [15] for the topcolor interactions, but will not assume that the NJL approximation is valid. We will assume instead that the low energy theory is consistently defined in a mass independent renormalization scheme such as dimensional regularization with modified minimal subtraction ($DR\overline{MS}$). We assume that the topcolor theory can be matched at the scale M (the mass of the colorons) onto a theory of massless colored quarks coupled to light scalars (Φ), and study the effect of the massive $U(1)$ gauge boson. While the matching at the scale M is nonperturbative, below the scale M, without the strongly coupled topcolor gauge bosons, the theory can be analyzed perturbatively (until we get down to the QCD scale where ordinary color gets strong). After reviewing some simple consequences of the $1/N$ expansion and effective field theory, we will discuss some general properties of the matching that we can establish simply by requiring that the physics be continuous as we vary the mass of the $U(1)$ gauge boson.

An illustrative example — the Noether current

It is convenient to begin with a calculation that is trivial. Consider the matching of a current,

$$j^\mu = \overline{\psi_L}\,\gamma^\mu\,Q_L\,\psi_L + \overline{\psi_R}\,\gamma^\mu\,Q_R\,\psi_R \tag{6}$$

in the high energy theory onto the low energy theory. This is the conserved Noether current associated with a $U(1)$ symmetry in the high energy theory. It is therefore mapped onto the corresponding Noether current in the low energy theory, which has the form:

$$\tilde{j}^\mu = \overline{\psi_L}\,\gamma^\mu\,Q_L\,\psi_L + \overline{\psi_R}\,\gamma^\mu\,Q_R\,\psi_R + i\,\mathrm{tr}\Big[\Phi^\dagger\ \overleftrightarrow{\partial}^\mu\ (Q_L\,\Phi - \Phi\,Q_R)\Big]\,. \tag{7}$$

Note that the current $\tilde{j}^\mu$ in the low energy theory contains contributions from both the fermions and the composite Higgs, as it must, because both transform under the $U(1)$ symmetry associated with the current.

$U(1)$ gauge boson exchange — leading order in N

Let us now consider the only slightly less trivial calculation of integrating out a $U(1)$ gauge boson, which couples to the L and R topcolored fermions with charges Q_L and Q_R, respectively. If the $U(1)$ gauge boson's mass is greater than M, we integrate out the gauge boson before matching onto the low energy theory to obtain the 4-fermion operator

$$\frac{g_1^2}{2\,m^2}\,j^\mu\,j_\mu \tag{8}$$

where g_1 is the coupling, m is the $U(1)$ gauge boson mass and j^m is the current in (6). In leading order in N, the matching is trivial. Because the currents are invariant under the topcolor gauge transformations, the leading contributions factor [16, 17], and the result is[1]

$$\frac{g_1^2}{2\,m^2}\,\tilde{j}^\mu\,\tilde{j}_\mu \tag{9}$$

This simple result contains an important lesson. In general, the results of matching onto the low energy theory will involve both the fermions and the scalars. We will come back to this momentarily.

If instead, the $U(1)$ gauge boson's mass is less than M, the gauge boson survives into the low energy theory, where it now couples to the low energy current $\tilde{j}^\mu$. Now when the gauge boson is integrated out, we again obtain (9). In this case, continuity of the physics across the boundary $m = M$ is automatically satisfied and does not give us any more information.

Matching a mass term

Now consider the matching of a mass operator

$$\Sigma \equiv \overline{\psi_R}\,\psi_L\,. \tag{10}$$

In this case, we cannot evaluate the matching exactly, but we can use symmetry arguments, N and loop counting, and dimensional analysis to write the dominant contribution in terms of a small number of parameters of order one.[2] The matching produces an operator in the low energy theory of the form

$$\tilde{\Sigma} = A\,\overline{\psi_R}\,\psi_L + B\,\frac{\sqrt{N}}{4\pi}\,M^2\,\Phi + C\,\frac{4\pi}{\sqrt{N}}\,\Phi\,\Phi^\dagger\,\Phi + \cdots\,, \tag{11}$$

where the $\cdots$ are higher dimension terms suppressed by powers of $1/M$ in the low energy theory. The parameters A, B, and C are of order one (at the scale M), but their precise value depend on the details of the strong topcolor physics. The factors of $\frac{\sqrt{N}}{4\pi}$ come from N and loop counting. One can think about them as follows. The coupling of Φ to the fermions must be of order $4\pi/\sqrt{N}$, in order that the Φ kinetic energy term which comes from planar diagrams like Fig. 1 have the standard normalization.

[1] Note that here, renormalization group running plays no role because both operators are renormalized at the the same scale, M.

[2] Another difference between this case and the case of the Noether current is that the mass operator has an anomalous dimension. Such anomalous dimensions are very important for the structure of the low energy theory at scales much smaller than M, because they give rise to large logarithms from renormalization that allow us to calculate some quantities reliably in spite of the nonperturbative nature of the matching [10]. Here, however, we will be discussing matchings at or near the scale M, so there are no large logs.

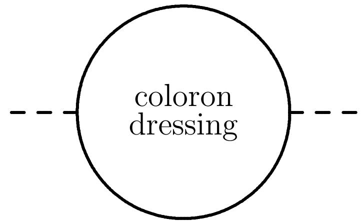

Figure 1: Typical Feynman graph contributing to the Φ kinetic energy term. Such graphs, with arbitrary planar coloron dressing, are proportional to $\frac{N}{16\pi^2}$, and thus the Φ coupling should be of order $\frac{4\pi}{\sqrt{N}}$.

With the result, (11), we can immediately write down the result for matching of a 4-fermion operator of the form $\mathrm{tr}(\Sigma\,\Sigma^\dagger)$. In leading order in the $1/N$ expansion, this just goes into $\mathrm{tr}(\tilde{\Sigma}\,\tilde{\Sigma}^\dagger)$. This follows immediately from factorization for large N. This result and (11) might seem slightly strange to a reader steeped in the NJL approximation. In the NJL approximation, (11) smacks of double counting, because the mass operator Σ plays much the same role as the scalar field Φ. In particular, one might worry that the $\mathrm{tr}(\Sigma\,\Sigma^\dagger)$ term in $\mathrm{tr}(\tilde{\Sigma}\,\tilde{\Sigma}^\dagger)$ would produce additional contributions to the Φ dependence from loop graphs like those shown in Fig. 2. But in the low energy theory, these graphs vanish identically. They are relevant in the NJL approximation because of the use of a momentum space cut-off of order M. In some way, the use of the momentum space cut-off is supposed to mock up the nonperturbative physics of the strong coloron exchange near the critical coupling. But in our language, these contributions (for zero momentum transfer through the loops) are simply eliminated by $DR\overline{MS}$

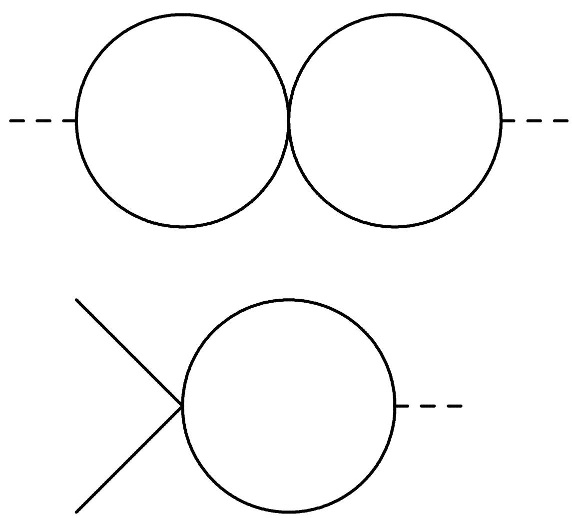

Figure 2: These graphs vanish in the low energy theory at zero momentum transfer.

Dimensional Analysis

As in any composite Higgs theory we can systematically estimate the size of any term by dimensional analysis [18, 19]. The intrinsic scale of the compositeness interactions is the topcolor scale, M. Hence we expect derivatives to be suppressed by a factor of M. If the amplitude for the creation of a composite Higgs field is proportional to f (the analog of f_π in chiral Lagrangian dimensional analysis in QCD [20, 21]), each field Φ comes suppressed by a factor of f. In a strongly interacting theory, we expect there are no additional relevant parameters [18] in the low-energy theory. Requiring that the kinetic energy terms of the fields Φ and ψ have the canonical normalization then determines that the overall size of each term in the Lagrangian is of order $M^2 f^2$.

A priori, the dimensionless ratio M/f is undetermined. The effective Lagrangian contains an infinite series of terms of arbitrary dimension. As in the QCD chiral Lagrangian [20, 21], loop diagrams involving lower dimension operators will require counterterms of higher dimension. Since the renormalization point associated with the logarithms in the results of the loops is undetermined to $\mathcal{O}(1)$, the *smallest* size of the coefficients of these higher dimension operators is of order the typical size of the coefficient of the chiral log for which the operator is a counterterm. This consistency condition implies [22]

$$\frac{M}{f} \sim \frac{4\pi}{\sqrt{N}} \,, \tag{12}$$

and the results given in the previous section immediately follow. In general, the N counting implicit in (12) will give the dependence of the *leading* contribution in large-N and should only be taken as a guide. As we will see below, there are cases in which the leading contribution vanishes and we will need to adjust the N-dependence to correspond to the diagrams which would be evaluated in matching the full and effective theories.

Using these rules to estimate the generic size of the composite Higgs mass term in the effective theory, we find $m_\Phi^2 \propto M^2$. This is the hierarchy problem [23]. In the absence of some other symmetry not accounted for in these rules, some adjustment is required to obtain $m_\Phi^2 \ll M^2$.

$U(1)$ gauge boson exchange — beyond leading order in N

Now finally we are ready to discuss the effect of $U(1)$ gauge boson exchange beyond the leading order in N. This is important, because it is just such a nonleading effect that produces the "tilting" of the vacuum alignment that is crucial to the phenomenology of topcolor models [1, 2, 24]. We begin by discussing the situation for $m > M$ where we first integrate out the $U(1)$ gauge boson to obtain the 4-fermion operator (8). In particular, the interesting effect involves both the left-handed and right-handed parts of the currents:

$$\frac{g_1^2}{m^2}\left[\overline{\psi_L}\,\gamma^\mu\,Q_L\,\psi_L\right]\,\left[\overline{\psi_R}\,\gamma_\mu\,Q_R\,\psi_R\right] \tag{13}$$

This gives rise to symmetry-breaking matching contributions to the Φ mass and Yukawa coupling from arbitrary planar coloron dressings of the diagrams in Figs. 3 and 4, where the dotted line connects the left-handed and right-handed gauge invariant parts of the 4-fermion operator in (13) (note that the 4-fermion contribution is already included in (9)). These give following contributions to the Φ mass and Yukawa coupling

$$- D\,\frac{g_1^2\,M^4}{16\pi^2 m^2}\,\mathrm{tr}\Big[Q_L\,\Phi\,Q_R\,\Phi^\dagger\Big] + E\,\frac{g_1^2\,M^2}{16\pi^2 m^2}\,\frac{4\pi}{\sqrt{N}}\,\overline{\psi_L}\,Q_L\,\Phi\,Q_R\,\psi_R + \text{h.c.} + \cdots \tag{14}$$

where D and E are of order 1.[3] Naive application of dimensional analysis would yield an additional factor of N. However, these contributions (see Fig. 3 and 4) are formally nonleading in N because the fermion lines connect the two gauge invariant parts of the operator (13). This can also be seen directly from the diagrams: the factors of $1/\sqrt{N}$ in the Φ coupling cancel against the N from the single fermion loop. Note that "planar" in this case includes graphs in which the coloron lines go through the dotted line in Fig. 3 and 4. Thus this contribution does not factorize, and we cannot relate it to (11). Note also that the $U(1)$ gauge coupling goes like $1/\sqrt{N}$ for large N [24] so the contribution of (14) to the Φ mass goes to zero as $N \to \infty$.

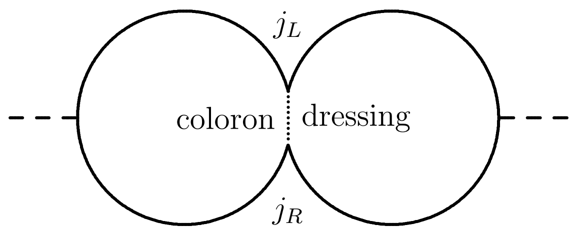

Figure 3: Symmetry breaking matching contribution to the Φ mass. The dotted line connects two gauge invariant parts of the 4-fermion operator. The matching contribution incorporates the effect of the coloron dressing which is not present in the low energy theory.

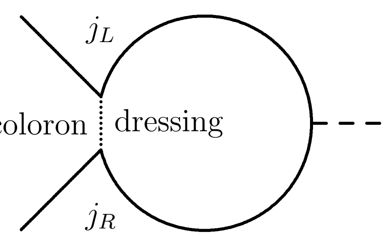

Figure 4: Symmetry breaking matching contribution to the Φ Yukawa coupling. The dotted line connects two gauge invariant parts of the 4-fermion operator. The matching contribution incorporates the effect of the coloron dressing which is not present in the low energy theory.

Note that the diagrams in Figs. 3 and 4 are really the same graphs as those in Fig. 2, except that they contain coloron dressing. It is the coloron effects that produce the matching contribution, and the scale M that is required to give a nonzero result.

[3] In the NJL approximation, D is positive.

Now what happens if $m < M$? Now there are a number of contributions that arise from the diagrams in Figs. 5 and 6, both with and without coloron dressing. There are effects from integrating out the colorons at the scale M; these contributions to the Φ mass and Yukawa coupling have the form

$$- D' \frac{g_1^2}{16\pi^2} M^2 \operatorname{tr}\Big[Q_L \Phi Q_R \Phi^\dagger\Big] + E' \frac{g_1^2}{16\pi^2} \frac{4\pi}{\sqrt{N}} \overline{\psi_L} Q_L \Phi Q_R \psi_R + \text{h.c.} + \cdots . \tag{15}$$

There also are effects from from integrating out the massive $U(1)$ gauge boson at the scale m in the low energy theory; these contributions to the Φ mass and Yukawa coupling have the form

$$- D'' \frac{g_1^2}{16\pi^2} m^2 \operatorname{tr}\Big[Q_L \Phi Q_R \Phi^\dagger\Big] + E'' \frac{g_1^2}{16\pi^2} \frac{4\pi}{\sqrt{N}} \overline{\psi_L} Q_L \Phi Q_R \psi_R + \text{h.c.} + \cdots . \tag{16}$$

where the parameters D', E', D'' and E'' are of order 1. In addition, there are calculable effects from the renormalization group running in the low energy theory that causes these coupling to evolve with the energy scale. The interesting difference between (14) in the high energy theory and (15) and (16) in the low energy theory is the $1/m^2$ dependence in (14). There is no way that either the coloron effect at the scale M, or the running and matching below M can produce the $1/m^2$ dependence. At M, the typical momentum in the matching calculation is of order M, so the $U(1)$ gauge boson propagator does not produce a $1/m^2$. Below M, we can never get an m^2 in the denominator of the mass term or Yukawa coupling (or the Φ^4 coupling, which we have not written), because it would have to be compensated by factors of M^2 in the numerator, and there is no way that such factors can arise in the low energy theory. They can originate in the matching at the scale M from the nonperturbative physics of the coloron exchange, but new factors of M cannot occur in the low energy theory.

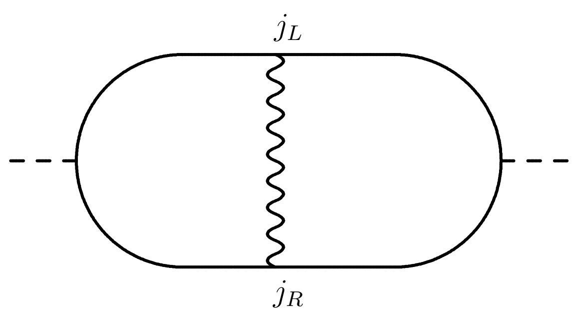

Figure 5: Symmetry breaking contribution to the Φ mass in the low energy theory.

Now we can impose the constraint that the physics should be continuous at the boundary $m = M$. Clearly, this implies

$$D = D' + D'' , \qquad E = E' + E'' . \tag{17}$$

This determines the Yukawa coupling below the scale m because it is only the sum $E' + E''$ that contributes.

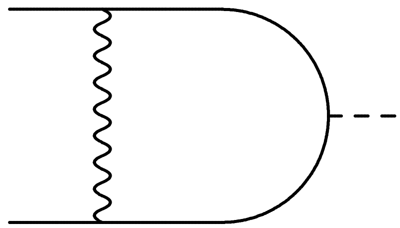

Figure 6: Symmetry breaking contribution to the Φ Yukawa coupling in the low energy theory.

In fact, though we cannot prove it using these methods, we think it likely that the most important symmetry breaking contribution to the Φ mass comes from integrating out the $U(1)$ gauge boson, in which case we would conclude

$$D'' \stackrel{?}{\gg} D' \,. \tag{18}$$

This would have the interesting consequence that the $U(1)$ interaction is most efficient at tilting the vacuum when $m = M$. Then the contribution to the symmetry breaking mass of the Φ would be

$$\Delta m_\Phi^2 \propto \begin{cases} m^2 & \text{for } m < M\text{, and} \\ \dfrac{M^4}{m^2} & \text{for } m > M. \end{cases} \tag{19}$$

But whether we can erase the question mark in (18) or not, it is clear that the tilting effect of a $U(1)$ gauge boson as a function of m cannot be much larger that the effect at $m = M$.

We have now resolved the NJL puzzles posed in the previous section. When g_1^2 is small, all the contributions to tilting are proportional to g_1^2, and we cannot get large tilting by taking Q and thus m very small.

4 Tuning and Tilting

Now we apply the results of the previous section to discuss the issue of fine tuning in topcolor models. The fine tuning we have in mind is the tuning required to make the composite Higgs bosons much lighter than the scale M. First, let us turn off the $U(1)$ interaction. Then the scalar fields described by the field Φ have a common mass, μ. For sufficiently small μ, we expect μ to be determined by a relation of the form

$$\mu^2 = F\left(\frac{\alpha_c - \alpha_{tc}(M)}{\alpha_c}\right) M^2 \,, \tag{20}$$

where F is a positive constant of order 1 and α_c is the critical value of the topcolor coupling for chiral symmetry breaking. In the NJL approximation, we can compute F and α_c. In the more general effective field theory description, we cannot calculate them reliably, but the form of the relation should be true more generally so long as the chiral symmetry breaking transition is second order. We will say that this is a fine tuning of order [25]

$$T = \frac{\alpha_c - \alpha_{tc}(M)}{\alpha_c}\,. \tag{21}$$

We now consider the $U(1)$ interaction. For simplicity, we will consider the simple situation in which only the t and b carry topcolor. Then the field Φ consists of four light complex scalar fields which have the quantum numbers of two independent 2-component "Higgs" fields (ϕ_t & ϕ_b). The issue then is whether we can tilt the vacuum so that the ϕ_t mass is negative and the ϕ_b mass is positive, so that a top condensate will produce a large t mass, but no b condensate will form.

Consider a model with $U(1)$-charge y_L for the left-handed $t-b$ doublet, and charges y_{t_R} and y_{b_R} for the right-handed t and b, respectively. Using the results of the previous section, we conclude that the contribution of the $U(1)$ to the Φ mass cannot be larger than

$$-\,D\,\frac{g_1^2}{16\pi^2}\,M^2\ \mathrm{tr}\Big[Q_L\,\Phi\,Q_R\,\Phi^\dagger\Big] \tag{22}$$

for D of order 1. In terms of the $U(1)$ charges, this can now be written as

$$-\,D\,\frac{g_1^2}{16\pi^2}\,M^2\ \mathrm{tr}\left[y_L\,\begin{pmatrix}\phi_t & \phi_b\end{pmatrix}\begin{pmatrix}y_{t_R} & 0\\ 0 & y_{b_R}\end{pmatrix}\begin{pmatrix}\phi_t^\dagger\\ \phi_b^\dagger\end{pmatrix}\right] \tag{23}$$

Combining (23) with the common mass μ gives mass terms for ϕ_t and ϕ_b of order

$$m_{\phi_b}^2 \approx \mu^2 - D\,\frac{g_1^2}{16\pi^2}\,M^2\,y_L\,y_{b_R}\,,\qquad m_{\phi_t}^2 \approx \mu^2 - D\,\frac{g_1^2}{16\pi^2}\,M^2\,y_L\,y_{t_R}\,. \tag{24}$$

To produce a t condensate, but no b condensate, we must have a negative mass squared for ϕ_t and a positive mass squared for ϕ_b, thus we want

$$\mu^2 - D\,\frac{g_1^2}{16\pi^2}\,M^2\,y_L\,y_{b_R} > 0\,,\qquad \mu^2 - D\,\frac{g_1^2}{16\pi^2}\,M^2\,y_L\,y_{t_R} < 0\,, \tag{25}$$

which using (20) and (21) can be written as

$$\frac{D}{F}\,\frac{g_1^2}{16\pi^2}\,y_L\,y_{b_R} < T < \frac{D}{F}\,\frac{g_1^2}{16\pi^2}\,y_L\,y_{t_R}\,. \tag{26}$$

This relation says that the larger T is — that is the less severe the fine tuning to the critical coupling — the larger g_1^2 (times the charges) must be. D/F is unknown, but is expected to be $\mathcal{O}(1)$. In the appendix, we show that $D/F = 4$ in the NJL approximation.

5 Some Tuning Required...

We will now show that T cannot be too large. Because the high-energy theory includes a non-asymptotically free $U(1)$ coupling, it too must be only an effective theory below some cutoff energy Λ. The scale Λ must be lower than the energy scale where the $U(1)$ gauge theory becomes strongly coupled – *i.e.* lower than the potential Landau pole [14]. Consider the one-loop β-function

$$\beta = \mu \frac{dg_1(\mu)}{d\mu} = \frac{b\, g_1^3}{24\pi^2} + \ldots \tag{27}$$

where $b = \sum_f y_f^2$ and the sum runs over all left- and right-handed $U(1)$ charges. Using the first term as an estimate, we find an upper bound on the strength of the $U(1)$ coupling at scale M

$$g_1(M) \lesssim \frac{12\pi^2}{b \, \log \frac{\Lambda}{M}} \ , \tag{28}$$

From eqn. (26) we then obtain

$$|T| < \frac{3\,|y_L|\,|y_R|^{\max}}{4\,b\,\log\left(\frac{\Lambda}{M}\right)} \frac{D}{F} \ , \tag{29}$$

where $|y_R|^{\max} = \max(|y_{t_R}|, |y_{b_R}|)$. Note that this relation makes physical sense: only the *ratios* of $U(1)$-charges appear.

Since the top- and bottom-quarks *must* couple to the $U(1)$, we know

$$b \gtrsim N\,(2|y_L|^2 + (|y_R|^{\max})^2). \tag{30}$$

The ratio of $U(1)$ charges in (29) has a maximum value:

$$\frac{|y_L||y_R|^{\max}}{2|y_L|^2 + (|y_R|^{\max})^2} \leq \frac{1}{2\sqrt{2}} \ . \tag{31}$$

This yields the model-independent bound

$$T < \mathcal{O}(4\%) \tag{32}$$

for $\Lambda/M \geq 10$. That is, the topcolor coupling renormalized at scale M must be tuned equal the critical value for chiral symmetry breaking to $\sim 4\%$! Furthermore, this implies that the $U(1)$ coupling cannot be particularly strong: it can provide a shift of only $\mathcal{O}(4\%)$ of M^2 to the composite Higgs boson mass-squared.

Stronger bounds can be obtained in specific models. In the "minimal" topcolor assisted technicolor model [2], the strong $U(1)$ couples only to the third generation of quarks and leptons with charges proportional to hypercharge. Here $b = 40/3$ and one finds $T < \mathcal{O}(1\%)$. This can severely restrict the relevant parameter space in topcolor models (for example, see [26]).

In a more realistic model, which tries to accommodate intergenerational mixing, prevent large amounts of isospin violation, and avoid the presence of light pseudo-Goldstone bosons [27, 28], the number of fermions coupling to the $U(1)$ is likely to be much larger. As noted by [27], the constraints will likely be even stronger. For example from eqn. (18) of ref. [28] (with the additional assumption that $z_1 = 1.0$) we find $b \approx 117$ and $y_L \approx y_R^{\max} \approx 1$. This yields $T < \mathcal{O}(0.2\%)$.

6 Conclusions

In this paper we have investigated the chiral-symmetry breaking dynamics in models of topcolor and topcolor-assisted technicolor which rely on a relatively strong $U(1)$ gauge interaction to “tilt” the vacuum. We identified some peculiarities of the Nambu-Jona-Lasinio (NJL) approximation often used to analyze the topcolor dynamics. We then resolved these puzzles by constructing the low-energy effective field theory appropriate to a mass-independent renormalization scheme and by constructing the power-counting rules appropriate to such an effective theory. Requiring that the Landau pole associated with the $U(1)$ gauge theory be sufficiently far above the topcolor gauge boson scale, we derived an upper bound on the strength of the $U(1)$ gauge-coupling evaluated at the topcolor scale. The upper bound on the $U(1)$ coupling implies that this interaction can only shift the composite Higgs boson mass-squared by a few per cent and, therefore, that the topcolor coupling must be adjusted to equal the critical value for chiral symmetry breaking to within a few per cent.

In much of this work the assumption that N is large does not play a crucial role. It restricts the form of various terms, but the basic structure of the effective field theory arguments remains the same, assuming that the transition remains second order [29, 30]. The one place where large N is important is in guaranteeing that it is the strong topcolor interactions that are driving the formation of the composite Higgs state, Φ, while the $U(1)$ interaction acts as a $1/N$ perturbation [24]. Because of asymptotic freedom, there is a plausible physical picture of the critical transition driven by topcolor. It results from the fact that the strong topcolor interactions get strong at a scale $\Lambda_{tc} \approx M$.

If N cannot be treated as large, then perhaps this can be turned on its head, but we admit that we do not understand the very different scenario in which the $U(1)$ interaction is nearly critical and the topcolor interaction is weak. It is much less obvious what the effective field theory of a nearly critical U(1) is like — if such a thing exists at all. We have nothing to say about this, except that there is no chance of it’s making sense unless N is small.

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Appendix: Tuning in the NJL Approximation

Consider a model with $U(1)$-charge y_L for the left-handed $t-b$ doublet, and charges y_{t_R} & y_{b_R} for the right-handed t and b respectively. In the NJL approximation, assuming the masses of the topcolor and hypercharge gauge bosons are comparable, the interactions of the third generation of quarks are approximated by the four-fermion operators

$$\mathcal{L}_{4f} = -\frac{g_{tc}^2(M)}{2\,M^2}\left[\overline{\psi}\gamma_\mu \frac{\lambda^a}{2}\psi\right]^2 - \frac{g_1^2(M)}{2\,M^2}\left[y_L\overline{\psi_L}\gamma_\mu\psi_L + y_{t_R}\overline{t_R}\gamma_\mu t_R + y_{b_R}\overline{b_R}\gamma_\mu b_R\right]^2 \; . \tag{33}$$

Here ψ represents the top-bottom doublet, and the $g_{tc}^2(M)$ and $g_1^2(M)$ are respectively the top-color and $U(1)$ gauge-couplings squared evaluated at scale M. The usual NJL gapequation condition [31] for top-condensation but not bottom-condensation is

$$g_{tc}^2(M) + \frac{2}{N} y_L y_{b_R} g_1^2(M) \lesssim g_c^2 = \frac{8\pi^2}{N} \lesssim g_{tc}^2(M) + \frac{2}{N} y_L y_{t_R} g_1^2(M) \ . \tag{34}$$

From this we see that

$$\frac{g_1^2(M)}{4\pi^2} y_L y_{b_R} \lesssim T \lesssim \frac{g_1^2(M)}{4\pi^2} y_L y_{t_R} \ . \tag{35}$$

This corresponds to eqn. (26) with $D/F = 4$.

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