Interference Aware-Coordinated Beamforming System in a Two-Cell Environment

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***Abstract*— In this paper, we propose jointly optimized transceiver algorithms called interference aware coordinated beamforming (IA-CBF) for a two-cell system where each base station is equipped with multiple transmit antennas. The case of single stream transmission to two mobile stations with two or more receive antennas is considered. First we propose minimum-mean-square-error IA-CBF using a lower bound on the achievable sum rate. Next we derive optimal (under an assumption of zero other-cell interference) and suboptimal transmit beamforming vectors called zero-forcing IA-CBF. We also investigate the optimality of the proposed IA-CBF algorithms with respect to the number of receive antennas. Analytical and numerical results confirm that the proposed system with two transmit/receive antennas achieves full degrees of freedom (a.k.a. multiplexing gain) of the two-cell multiple-input multiple-output channel while showing a better sum rate performance than the conventional solutions such as non-cooperative eigen-beamforming and interference nulling.**

***Index Terms*— MIMO system, multiuser system, multi-cell system, interference alignment, interference suppression, coordinated beamforming.**

I. Introduction

OVER the last decade, point-to-multi-point (multiuser) multiple-input multiple-output (MIMO) has been extensively investigated to achieve high capacity [1]. It has been shown, however, that the capacity gain obtained by multiuser MIMO processing degrades severely in multi-cell environments [2]. To solve this problem, several MIMO processing strategies have been proposed for multi-cell environments, where multiple base stations cooperate with each others to enhance system performance [3]–[10].

For multi-cell environments, linear pre-processing combined with dirty paper coding (DPC) was proposed using Wyner's infinite linear cell-array model in [3]. More practically, linear and non-linear network coordinated beamforming have been proposed to approach the multi-cell sum capacity in [4]. The strategies in [3], [4] assume that all base stations are connected to a central base station controller that has perfect channel state information and that all base stations know all the messages to be transmitted to the mobile stations. Therefore, these can be interpreted as examples of multiuser MIMO with a total power constraint in a single cell. In [5], joint multi-cell resource allocation has been studied to optimize the network performance. A multi-cell downlink channel with an individual power constraint per base station was analyzed in [6]. The joint transmission sum rate was maximized through an optimization of the mobile stations' input convariance matrices and optimal encoding ordering. Uplink/downlink duality was used to solve the non-convex optimization problem; thus, in practice, the computational complexity could be an issue. More recently, [10] proposed a globally optimized iterative solution, using the Lagrangian duality, to compute the transmit beamforming vectors based on a generalization of uplink-downlink duality to the multi-cell setting.

Consider a scenario that each mobile station wants to receive a desired data stream only from the desired base station, then this multi-cell MIMO channel is the same as the K-user MIMO interference channel.[1] The capacity region of the MIMO interference channel is still an open problem [11]–[13] but the degrees of freedom, a.k.a. multiplexing gain of the MIMO interference channel, has recently been well studied [14], [15]. The authors in [14], [15] derived the degrees of freedom of the MIMO interference channel. For the fully connected K-user interference channel, a new concept called interference alignment has been proposed. In this case, each mobile station needs to perfectly cancel the interference from the undesired base station. The key idea is to align all interference from the undesired transmitters to a small dimension at each receiver. Most prior work on interference alignment focused mainly on the analysis of the degrees of freedom of the MIMO interference channel. Recently, several centralized and distributed transceiver structures have been proposed to achieve full degrees of freedom of the MIMO interference channel [16], [17] but most prior work requires iterative procedures to optimize the transceiver structures.

To the best of our knowledge, most prior work on multi-cell MIMO was i) information theoretical results [3], [5]–[7], [11]–[17], ii) transceiver designs based on impractical assumptions such as perfect data sharing at the base stations [4] and a per frame rate constraint [9], or iii) required high cost to implement due to iterative procedures [10], [16], [17]. In our prior work [18], which was called coordinated beamforming, the optimal transmit beamforming and receive combining vectors under a zero inter-user interference constraint were derived. The result, however, was for the (single cell) MIMO broadcast channel, where only one base station served two

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[1]Throughout this paper, we do not consider macro diversity, where multiple base stations serve one cell-boundary user to increase the link reliability/outage probability.

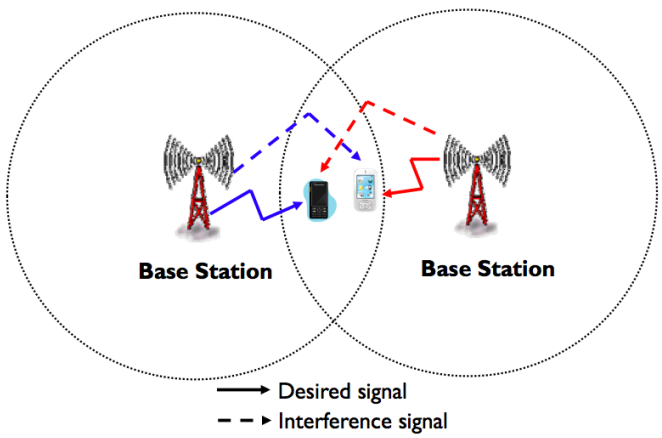

Fig. 1. System model. Two base stations serve two mobile users.

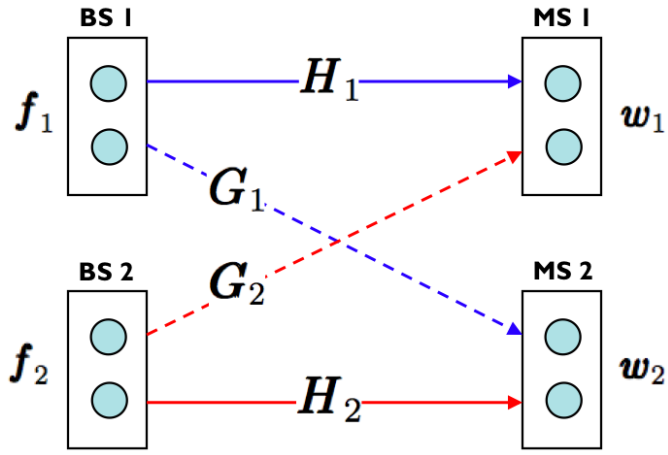

Fig. 2. Equivalent two-user MIMO interference channel. Each base station transmits only one data stream to the desired mobile station. $\boldsymbol{H}_k$ denotes a channel matrix between base station k and mobile station k, where k is 1 or 2. $\boldsymbol{G}_k$ is an interference channel matrix from base station k to the undesired mobile station.

mobile stations. In this paper, a two-cell environment is considered, where two base stations serve two mobile stations as shown in Figs. 1 and 2. A scenario of more than two cells will be considered in our future work.[2] In our system, unlike prior work in [4], each base station has its own data stream to support one mobile station. For this scenario, in this paper, several low complexity linear two-cell MIMO algorithms are designed and investigated. The main contributions of this paper are as follows:

- *Jointly optimized transceiver algorithm*: We propose jointly optimized minimum-mean-square-error (MMSE) and zero-forcing (ZF) MIMO transceiver algorithms called interference aware-coordinated beamforming (IA-CBF) that support two mobile stations by using two base stations. In the proposed IA-CBF, the base stations do not have to know all the messages to be sent to mobile stations unlike several conventional multi-cell MIMO techniques including network coordinated beamforming in [4]. Additionally, unlike prior work on interference nulling algorithms, the proposed algorithm perfectly removes the other-cell interference while also maximizing the desired effective channel gain simultaneously. We show that the proposed ZF IA-CBF algorithm is a sufficient and necessary solution (optimal) under an assumption that other-cell interference is perfectly removed when two antennas are equipped.
- *The optimality of IA-CBF*: We show that the proposed system achieves full degrees of freedom of the two-cell MIMO channel, where the mobile stations have two receive antennas. We also derive the asymptotic expression of the achievable sum rate of the proposed system with respect to the number of receive antennas. We confirm through analytical and numerical results that the sum rate of IA-CBF converges to the sum rate of the point-to-point upper bound while the conventional interference nulling solutions converge to some constant. This constant is only related to signal-to-interference-plus-noise ratio (SINR).

[2]In practice, the mobile station at the cell boundary will most likely have one dominant interference from a neighboring cell so the two-cell assumption seems reasonable.

The organization of this paper is given next. In Section II, we introduce the system model under consideration, specialized for the case of one data stream for each mobile station and propose IA-CBF algorithms. Performance evaluation and conclusion are given in Sections III and IV.

II. Interference Aware-Coordinated Beamforming

In this section, we introduce the notation and describe the system model under consideration. We also propose MMSE and ZF IA-CBF algorithms, and investigate the asymptotic achievable sum rate behavior of the proposed systems.

A. Notation

Throughout this paper, upper case and lower case boldface are used to denote matrices $\boldsymbol{A}$ and vectors $\boldsymbol{a}$, respectively. We use $\boldsymbol{A}^*$, $\bar{\boldsymbol{A}}$ to indicate the conjugate transpose, the complex conjugate of matrix $\boldsymbol{A}$, $\|\boldsymbol{A}\|$ to denote the matrix two-norm, respectively.

B. System Model

Consider a two-cell MIMO system as shown in Fig. 1 where two base stations serve two mobile stations equipped with more than one receive antenna. We assume that each base station is equipped with two or more transmit antennas and only one stream is transmitted to each mobile station. In this paper, we do not consider opportunistic scheduling; thus no user selection algorithm is required. Throughout this paper, we assume equal power allocation at each base station for transceiver designs. Sharing each base station's transmit power in practice is unlikely since each base station has its own transmit power amplifiers and is geographically placed far from the other base stations. To study the achievable rate region, we consider a total power constraint, although we note that power allocation is not directly related to our proposed algorithms.

The channel between base station k and mobile station k ($k = 1$ or 2) is represented by $\boldsymbol{H}_k$ of size $N_r \times N_t$ with complex entries, where N_t and N_r are the number of transmit and receive antennas, respectively. A matrix $\boldsymbol{G}_k$ of size $N_r \times N_t$ is used to denote the other-cell interference channel between base station k and mobile station ℓ (k, ℓ=1 or 2, $k \neq \ell$). The received signal at each mobile station with an equal power allocation, i.e., $\frac{P}{2}$ for each base station, is given by

$$y_k = \underbrace{\sqrt{\frac{P}{2}}\boldsymbol{w}_k^*\boldsymbol{H}_k\boldsymbol{f}_k x_k}_{\text{desired signal}} + \underbrace{\sqrt{\frac{P}{2}}\boldsymbol{w}_k^*\boldsymbol{G}_\ell\boldsymbol{f}_\ell x_\ell}_{\text{other-cell interference}} + \boldsymbol{w}_k^*\boldsymbol{n}_k \tag{1}$$

where x_k is the transmitted data signal desired to mobile station k, $\boldsymbol{f}_k$ and $\boldsymbol{w}_k$ are the transmit beamforming and receive combining vectors for mobile station k, and P is the total transmit power at the base stations. $\boldsymbol{n}_k$ is an $N_r \times 1$ additive white Gaussian noise vector with variance σ^2 per entry observed at the mobile station. In this paper, we assume that all channel matrices $\boldsymbol{H}_1, \boldsymbol{H}_2, \boldsymbol{G}_1$, and $\boldsymbol{G}_2$ are available at the base stations so that the base stations can compute jointly optimized transmit beamforming vectors $\{\boldsymbol{f}_k\}_{k=1}^2$ and receive combining vectors $\{\boldsymbol{w}_k\}_{k=1}^2$.

C. Minimum Mean Square Error (MMSE) IA-CBF

In this section, we propose an MMSE IA-CBF algorithm that maximizes the achievable rate especially at the low SINR regime. From (1), the received SINR at the kth user can be expressed as

$$\begin{aligned}\mathsf{SINR}_k &= \frac{\frac{P}{2}\boldsymbol{f}_k^*\boldsymbol{H}_k^*\boldsymbol{w}_k\boldsymbol{w}_k^*\boldsymbol{H}_k\boldsymbol{f}_k}{\frac{P}{2}\boldsymbol{f}_\ell^*\boldsymbol{G}_\ell^*\boldsymbol{w}_k\boldsymbol{w}_k^*\boldsymbol{G}_\ell\boldsymbol{f}_\ell + 1} \\ &= \frac{\frac{P}{2}\boldsymbol{w}_k^*\boldsymbol{H}_k\boldsymbol{f}_k\boldsymbol{f}_k^*\boldsymbol{H}_k^*\boldsymbol{w}_k}{\boldsymbol{w}_k^*(\frac{P}{2}\boldsymbol{G}_\ell\boldsymbol{f}_\ell\boldsymbol{f}_\ell^*\boldsymbol{G}_\ell^* + \boldsymbol{I}_{N_r})\boldsymbol{w}_k},\end{aligned} \tag{2}$$

where $k, \ell = 1$ or 2, $k \neq \ell$. Eq. (2) is known as a Rayleigh quotient and is maximized when the receive combining vector $\boldsymbol{w}_k$ (before normalization) is given by

$$\boldsymbol{w}_k = \left(\frac{P}{2}\boldsymbol{G}_\ell\boldsymbol{f}_\ell\boldsymbol{f}_\ell^*\boldsymbol{G}_\ell^* + \boldsymbol{I}_{N_r}\right)^{-1}\boldsymbol{H}_k\boldsymbol{f}_k. \tag{3}$$

This is the principal singular vector of $\frac{\frac{P}{2}\boldsymbol{H}_k\boldsymbol{f}_k\boldsymbol{f}_k^*\boldsymbol{H}_k^*}{\frac{P}{2}\boldsymbol{G}_\ell\boldsymbol{f}_\ell\boldsymbol{f}_\ell^*\boldsymbol{G}_\ell^* + \boldsymbol{I}_{N_r}}$. The corresponding SINR for user k and the achievable sum rate are, respectively, defined as

$$\mathsf{SINR}_k = \frac{P}{2}\boldsymbol{f}_k^*\boldsymbol{H}_k^*\left(\frac{P}{2}\boldsymbol{G}_\ell\boldsymbol{f}_\ell\boldsymbol{f}_\ell^*\boldsymbol{G}_\ell^* + \boldsymbol{I}_{N_r}\right)^{-1}\boldsymbol{H}_k\boldsymbol{f}_k, \tag{4}$$

$$\mathcal{R}_{\mathsf{MMSE-CBF}} = \sum_{k=1}^{2}\log_2(1 + \mathsf{SINR}_k). \tag{5}$$

Theorem 1: Given the receive combining vectors $\boldsymbol{w}_k$ in (3), the achievable sum rate, $\mathcal{R}_{\mathsf{MMSE-CBF}}$, is lower-bounded by the following transmit beamforming vector:

$$\boldsymbol{f}_k = \boldsymbol{v}_{\max}\left\{\left(\boldsymbol{G}_k^*\boldsymbol{G}_k + \frac{2}{P}\boldsymbol{I}_{N_t}\right)^{-1}\boldsymbol{H}_k^*\boldsymbol{H}_k\right\} \tag{6}$$

where $k = 1$ or 2 and $\boldsymbol{v}_{\max}\{\boldsymbol{A}\}$ is the principal singular vector.

Proof: Since it is not simple to derive optimal MMSE transmit beamforming and receive combining vectors that maximize the achievable sum rate, we use the achievable product rate, which is given by

$$\begin{aligned}\mathcal{R}_{\mathsf{MMSE-CBF}} &= \sum_{k=1}^{2}\log_2(1 + \mathsf{SINR}_k) \\ &> \log_2\prod_{k=1}^{2}\mathsf{SINR}_k = \mathcal{R}_{\mathsf{MMSE-CBF}}^{prod},\end{aligned}$$

$$\begin{aligned}&\mathcal{R}_{\mathsf{MMSE-CBF}}^{prod} \\ &= \log_2\prod_{k,\ell=1,k\neq\ell}^{2}\left(\frac{\frac{P}{2}\boldsymbol{w}_k^*\boldsymbol{H}_k\boldsymbol{f}_k\boldsymbol{f}_k^*\boldsymbol{H}_k^*\boldsymbol{w}_k}{\boldsymbol{w}_k^*(\frac{P}{2}\boldsymbol{G}_\ell\boldsymbol{f}_\ell\boldsymbol{f}_\ell^*\boldsymbol{G}_\ell^* + \boldsymbol{I}_{N_r})\boldsymbol{w}_k}\right) \\ &= \log_2\prod_{k,\ell=1,k\neq\ell}^{2}\left(\frac{P}{2}\boldsymbol{f}_k^*\boldsymbol{H}_k^*\left(\frac{P}{2}\boldsymbol{G}_\ell\boldsymbol{f}_\ell\boldsymbol{f}_\ell^*\boldsymbol{G}_\ell^* + \boldsymbol{I}_{N_r}\right)^{-1}\boldsymbol{H}_k\boldsymbol{f}_k\right) \\ &= \log_2\prod_{k=1}^{2}\left(\frac{P}{2}\boldsymbol{f}_k^*\boldsymbol{H}_k^*\left(\frac{P}{2}\boldsymbol{G}_k\boldsymbol{f}_k\boldsymbol{f}_k^*\boldsymbol{G}_k^* + \boldsymbol{I}_{N_r}\right)^{-1}\boldsymbol{H}_k\boldsymbol{f}_k\right) \\ &\overset{(a)}{\geq} \log_2\prod_{k=1}^{2}\frac{(\boldsymbol{f}_k^*\boldsymbol{H}_k^*\boldsymbol{H}_k\boldsymbol{f}_k)^2}{\boldsymbol{f}_k^*\boldsymbol{H}_k^*\left(\boldsymbol{G}_k\boldsymbol{f}_k\boldsymbol{f}_k^*\boldsymbol{G}_k^* + \frac{2}{P}\boldsymbol{I}_{N_t}\right)\boldsymbol{H}_k\boldsymbol{f}_k} \qquad (7) \\ &\overset{(b)}{\geq} \log_2\prod_{k=1}^{2}\frac{\boldsymbol{f}_k^*\boldsymbol{H}_k^*\boldsymbol{H}_k\boldsymbol{f}_k}{\boldsymbol{f}_k^*\left(\boldsymbol{G}_k^*\boldsymbol{G}_k + \frac{2}{P}\boldsymbol{I}_{N_t}\right)\boldsymbol{f}_k}. \qquad (8)\end{aligned}$$

where (a) follows by the fact $\boldsymbol{a}^*\boldsymbol{Q}^{-1}\boldsymbol{a} \geq \frac{(\boldsymbol{a}^*\boldsymbol{a})^2}{\boldsymbol{a}^*\boldsymbol{Q}\boldsymbol{a}}$, where $\boldsymbol{Q}$ is any nonsingular Hermitian positive definite matrix and $\boldsymbol{a}$ is any vector [19]. In (8), (b) results from $\frac{\boldsymbol{a}^*\boldsymbol{Q}\boldsymbol{a}}{\boldsymbol{a}^*\boldsymbol{a}} \leq \lambda_{\max}(\boldsymbol{Q}) = \|\boldsymbol{Q}\|$, where $\|\boldsymbol{Q}\| = \boldsymbol{f}_k^*\boldsymbol{G}_k^*\boldsymbol{G}_k\boldsymbol{f}_k + \frac{2}{P}$. Using the lower bound of the achievable product rate in (8), we want to find the transmit beamforming vectors as follows:

$$\{\hat{\boldsymbol{f}}_1, \hat{\boldsymbol{f}}_2\} = \arg\max_{\boldsymbol{f}_k, \|\boldsymbol{f}_k\|=1}\log_2\prod_{k=1}^{2}\frac{\boldsymbol{f}_k^*\boldsymbol{H}_k^*\boldsymbol{H}_k\boldsymbol{f}_k}{\boldsymbol{f}_k^*\left(\boldsymbol{G}_k^*\boldsymbol{G}_k + \frac{2}{P}\boldsymbol{I}_{N_t}\right)\boldsymbol{f}_k}. \tag{9}$$

Since $\boldsymbol{f}_k$ is only a function of $\boldsymbol{H}_k$ and $\boldsymbol{G}_k$, Eq. (9) can be decoupled as

$$\begin{aligned}\hat{\boldsymbol{f}}_1 &= \arg\max_{\boldsymbol{f}_1:\|\boldsymbol{f}_1\|=1}\frac{\boldsymbol{f}_1^*\boldsymbol{H}_1^*\boldsymbol{H}_1\boldsymbol{f}_1}{\boldsymbol{f}_1^*\left(\boldsymbol{G}_1^*\boldsymbol{G}_1 + \frac{2}{P}\boldsymbol{I}_{N_t}\right)\boldsymbol{f}_1}, \\ \hat{\boldsymbol{f}}_2 &= \arg\max_{\boldsymbol{f}_2:\|\boldsymbol{f}_2\|=1}\frac{\boldsymbol{f}_2^*\boldsymbol{H}_2^*\boldsymbol{H}_2\boldsymbol{f}_2}{\boldsymbol{f}_2^*\left(\boldsymbol{G}_2^*\boldsymbol{G}_2 + \frac{2}{P}\boldsymbol{I}_{N_t}\right)\boldsymbol{f}_2}.\end{aligned}$$

This is known as a generalized Rayleigh quotient and the transmit beamforming vector maximizing the product rate is given by (6) and this concludes the proof. ■

As can be seen from the proof, the lower bound of the achievable sum rate is achieved using the proposed solution in *Theorem* 1. Thus this implies that the proposed solution might not maximize the objective function $\mathcal{R}_{\mathsf{MMSE-CBF}}$ over all SINR values. As will be shown in Section III, however, this solution gives good sum rate performance especially at the low SINR regime.

D. Zero-forcing IA-CBF with Maximum Ratio Combining

In Section II-C, simple transmit beamforming and receive combining vectors are derived based on the lower bound of the achievable sum rate. This solution, however, does not guarantee to maximize the achievable sum rate since the lower bound of the achievable sum rate, i.e., achievable product rate, is used. In this section, we propose a ZF IA-CBF algorithm under a zero other-cell interference constraint. Throughout this section, we assume that maximum ratio combining (MRC) is used at each mobile station [20]. This is a reasonable choice to maximize the achievable rate since we can guarantee zero other-cell interference by using the proposed transmit beamforming vectors. Then, the received signal at each mobile station can be rewritten as

$$\begin{aligned} y_k &= \sqrt{\frac{P}{2}}\boldsymbol{w}_k^*\boldsymbol{H}_k\boldsymbol{f}_k x_k + \sqrt{\frac{P}{2}}\boldsymbol{w}_k^*\boldsymbol{G}_\ell\boldsymbol{f}_\ell x_\ell + \boldsymbol{w}_k^*\boldsymbol{n}_k \\ &= \underbrace{\sqrt{\frac{P}{2}}\frac{\boldsymbol{f}_k^*\boldsymbol{H}_k^*\boldsymbol{H}_k\boldsymbol{f}_k}{\|\boldsymbol{H}_k\boldsymbol{f}_k\|}x_k}_{\text{desired signal}} + \underbrace{\sqrt{\frac{P}{2}}\frac{\boldsymbol{f}_k^*\boldsymbol{H}_k^*\boldsymbol{G}_\ell\boldsymbol{f}_\ell}{\|\boldsymbol{H}_k\boldsymbol{f}_k\|}x_\ell}_{\text{other-cell interference}} + \frac{\boldsymbol{f}_k^*\boldsymbol{H}_k^*\boldsymbol{n}_k}{\|\boldsymbol{H}_k\boldsymbol{f}_k\|}. \end{aligned} \tag{10}$$

Then the design goal is to maximize the desired signal term and to remove the other-cell interference term found in (10). Thus we propose an ZF IA-CBF with MRC algorithm that satisfies the following condition:

$$\begin{aligned} \boldsymbol{w}_1^*\boldsymbol{G}_2\boldsymbol{f}_2 &= 0 = \boldsymbol{w}_2^*\boldsymbol{G}_1\boldsymbol{f}_1 \\ \Leftrightarrow \boldsymbol{f}_1^*\boldsymbol{H}_1^*\boldsymbol{G}_2\boldsymbol{f}_2 &= 0 = \boldsymbol{f}_2^*\boldsymbol{H}_2^*\boldsymbol{G}_1\boldsymbol{f}_1, \end{aligned} \tag{11}$$

which implies that the other-cell interference term in (10) is perfectly removed; at the same time, the proposed system maximizes the desired effective channel gain $|\boldsymbol{w}_k^*\boldsymbol{H}_k\boldsymbol{f}_k|^2$ by using MRC, $\boldsymbol{w}_k = \frac{\boldsymbol{H}_k\boldsymbol{f}_k}{\|\boldsymbol{H}_k\boldsymbol{f}_k\|}$. Note that no inter-user interference is guaranteed thanks to the transmit beamforming vectors $\{\boldsymbol{f}\}_{k=1}^2$. Since we are considering a two-cell environment, this can be interpreted as the two-user MIMO interference channel illustrated in Fig. 2.

Theorem 2: Under a zero other-cell interference constraint in (11), the sufficient and necessary beamforming vectors with MRC for mobile station ℓ (where k, ℓ are 1 or 2, $\ell \neq k$) are generalized eigenvectors of $\boldsymbol{H}_k^*\boldsymbol{G}_\ell$ and $\boldsymbol{G}_k^*\boldsymbol{H}_\ell$.

Proof: From the zero other-cell interference constraint, we have the following conditions:
for user k,

$$\boldsymbol{f}_k^*\boldsymbol{H}_k^*\boldsymbol{G}_\ell\boldsymbol{f}_\ell = 0 \;\Leftrightarrow\; \boldsymbol{H}_k\boldsymbol{f}_k \perp \boldsymbol{G}_\ell\boldsymbol{f}_\ell \;\Leftrightarrow\; \boldsymbol{f}_k \perp \boldsymbol{H}_k^*\boldsymbol{G}_\ell\boldsymbol{f}_\ell \tag{12}$$

and for user ℓ,

$$\boldsymbol{f}_\ell^*\boldsymbol{H}_\ell^*\boldsymbol{G}_k\boldsymbol{f}_k = 0 \;\Leftrightarrow\; \boldsymbol{H}_\ell\boldsymbol{f}_\ell \perp \boldsymbol{G}_k\boldsymbol{f}_k \;\Leftrightarrow\; \boldsymbol{f}_k \perp \boldsymbol{G}_k^*\boldsymbol{H}_\ell\boldsymbol{f}_\ell \tag{13}$$

where $\perp$ denotes the perpendicular. From (12) and (13), we have

$$\boldsymbol{H}_k^*\boldsymbol{G}_\ell\boldsymbol{f}_\ell \parallel \boldsymbol{G}_k^*\boldsymbol{H}_\ell\boldsymbol{f}_\ell \;\Leftrightarrow\; \boldsymbol{H}_k^*\boldsymbol{G}_\ell\boldsymbol{f}_\ell = \lambda_1\boldsymbol{G}_k^*\boldsymbol{H}_\ell\boldsymbol{f}_\ell. \tag{14}$$

Here, $\parallel$ denotes parallelity between two complex vectors. This is known as a generalized eigen-problem where λ_1 is the generalized eigenvalue. Therefore, the transmit beamforming vector $\boldsymbol{f}_\ell$ for user ℓ is the generalized eigenvectors of $\boldsymbol{H}_k^*\boldsymbol{G}_\ell$ and $\boldsymbol{G}_k^*\boldsymbol{H}_\ell$. ■

From *Theorem* 2, if the number of transmit antennas, N_t, is two and the matrices $\boldsymbol{H}_k^*\boldsymbol{G}_\ell$ or $\boldsymbol{H}_\ell^*\boldsymbol{G}_k$ are invertible, the closed form expression of the transmit beamforming vector can be derived as follows:

$$\boldsymbol{f}_\ell \in \left[\frac{\boldsymbol{t}_\ell}{\sqrt{(a-b)^2+d_{21}}}, \frac{\boldsymbol{t}_k}{\sqrt{(a+b)^2+d_{21}}}\right] \tag{15}$$

where

$$\boldsymbol{A} = \boldsymbol{H}_k^*\boldsymbol{G}_\ell = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},$$

$$\boldsymbol{B} = \boldsymbol{G}_k^*\boldsymbol{H}_\ell = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix},$$

$$\boldsymbol{C} = \begin{pmatrix} A_{22} & -A_{12} \\ -A_{21} & A_{11} \end{pmatrix}, \;\; \boldsymbol{D} = \boldsymbol{C}\boldsymbol{B} = \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix},$$

$$a = (d_{11} - d_{22})/2, \quad b = \sqrt{(a^2 + d_{21}d_{12})},$$

$$\boldsymbol{t}_k = \begin{pmatrix} a-b \\ d_{21} \end{pmatrix} \quad \text{and} \quad \boldsymbol{t}_\ell = \begin{pmatrix} a+b_{12} \\ d_{21} \end{pmatrix}.$$

This is because the generalized eigenvectors of $\boldsymbol{H}_k^*\boldsymbol{G}_\ell$ and $\boldsymbol{G}_k^*\boldsymbol{H}_\ell$ are the same as eigenvectors of $(\boldsymbol{G}_k^*\boldsymbol{H}_\ell)^{-1}\boldsymbol{H}_k^*\boldsymbol{G}_\ell$ or $(\boldsymbol{H}_k^*\boldsymbol{G}_\ell)^{-1}\boldsymbol{G}_k^*\boldsymbol{H}_\ell$. We omit the detailed derivations here due to space limitations. Once all channel matrices are given, $\boldsymbol{f}_\ell$ can be simply obtained through *Theorem 2*.

Theorem 3: Given $\boldsymbol{f}_\ell$, where each base station has two transmit antennas, the sufficient and necessary beamforming vector (unique up to complex multiplications) for mobile station k, $\boldsymbol{f}_k$, can be expressed as

$$\boldsymbol{f}_k = \mu\begin{pmatrix} -\bar{z}_2 \\ \bar{z}_1 \end{pmatrix} \quad \text{or} \quad \boldsymbol{f}_k = \mu\begin{pmatrix} \bar{z}_2 \\ -\bar{z}_1 \end{pmatrix} \tag{16}$$

where

$$\boldsymbol{z} = \boldsymbol{H}_k^*\boldsymbol{G}_\ell\boldsymbol{f}_\ell = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

and $\bar{z}_2$ is the complex conjugate of z_2.

Proof: Let the unit-norm transmit beamforming vector $\boldsymbol{f}_k = (f_{k,1},\ f_{k,2})^T$ and from the condition (12), $\boldsymbol{f}_k \perp \boldsymbol{H}_k^*\boldsymbol{G}_\ell\boldsymbol{f}_\ell$, we have

$$f_{k,1}\bar{z}_1 + f_{k,2}\bar{z}_2 = 0 \quad \text{and} \quad f_{k,1}^2 + f_{k,2}^2 = 1 \tag{17}$$

$$\Leftrightarrow \boldsymbol{f}_k = \mu\begin{pmatrix} -\bar{z}_2 \\ \bar{z}_1 \end{pmatrix} \quad \text{or} \quad \boldsymbol{f}_k = \mu\begin{pmatrix} \bar{z}_2 \\ -\bar{z}_1 \end{pmatrix} \tag{18}$$

since $\boldsymbol{z}^*\boldsymbol{f}_k = 0$ ($\Leftrightarrow \boldsymbol{f}_k \perp \boldsymbol{z}$). In (18), μ is a non-zero normalization constant. Note that the solutions in (18) give the same sum rate performances since we are interested in maximizing $|\boldsymbol{f}_k^*\boldsymbol{H}_k^*\boldsymbol{H}_k\boldsymbol{f}_k|$. Note that this solution is unique up to complex multiplication and optimal (sufficient and necessary solution) when each base station has two transmit antennas. ■

From *Theorems* 2 and 3, we realize that the proposed ZF IA-CBF with MRC is a sufficient and necessary solution; i.e., i) if (non-zero) transmit beamforming vectors $\boldsymbol{f}_1$, $\boldsymbol{f}_2$ satisfy the zero inter-cell interference constraints in (11), then satisfy (14)

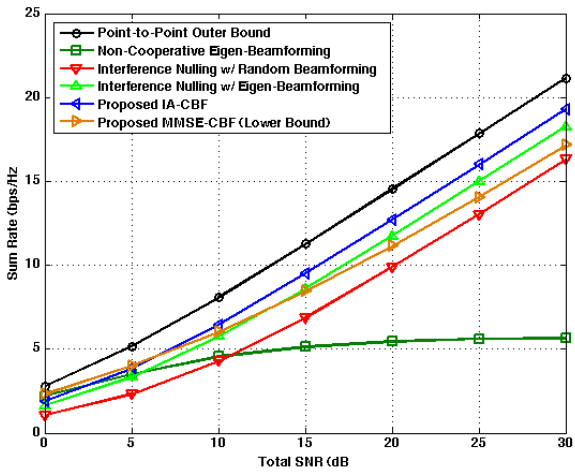

Fig. 3. Sum Rate Comparisons vs. SNR, where $N_t = 2$ and $N_r = 2$.

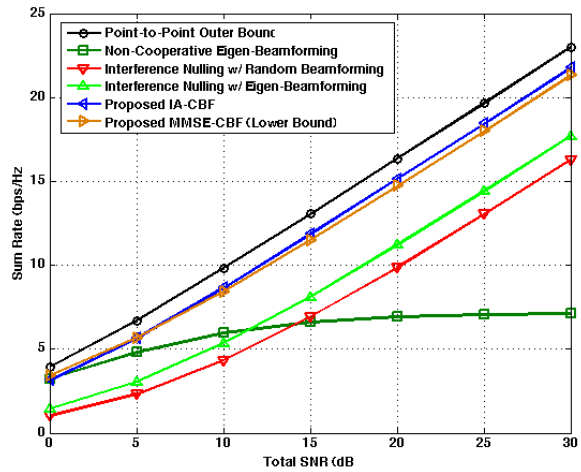

Fig. 4. Sum Rate Comparisons vs. SINR, where $N_t = 2$ and $N_r = 4$.

and (18), ii) any vector set from *Theorems* 2 and 3 satisfy the zero inter-cell interference constraint in (11).

Theorem 4: For more than two transmit antennas at each base station, if the transmit beamforming vector is given by $\boldsymbol{f}_\ell = \mu\left(\bar{z}_1^{-1}\ \bar{z}_2^{-1}\cdots(1-N_t)\bar{z}_{N_t}^{-1}\right)^T$, no other-cell interference is obtained, i.e., $\boldsymbol{f}_k \perp \boldsymbol{H}_k^*\boldsymbol{G}_\ell\boldsymbol{f}_\ell$ and $\boldsymbol{f}_k \perp \boldsymbol{G}_k^*\boldsymbol{H}_\ell\boldsymbol{f}_\ell$.

Proof: From the same conditions as in (17), the transmit beamforming vector $\boldsymbol{f}_k = (f_{k,1}\cdots f_{k,N_t})^T$ has to satisfy the following conditions.

$$\begin{aligned} f_{k,1}\bar{z}_1 + f_{k,2}\bar{z}_2 + \cdots + f_{k,N_t}\bar{z}_{N_t} = 0, \\ f_{k,1}^2 + f_{k,2}^2 + \cdots + f_{k,N_t}^2 = 1. \end{aligned} \tag{19}$$

The solution $\boldsymbol{f}_k$ satisfying (19) can be interpreted as the intersection of an N_t-dimensional plane and an N_t-dimensional hypersphere. Therefore, there exist infinite solutions that satisfy (19) and it can easily be shown that $\boldsymbol{f}_\ell = \mu\left(\bar{z}_1^{-1}\ \bar{z}_2^{-1}\cdots(1-N_t)\bar{z}_{N_t}^{-1}\right)^T$ is one of the solutions. Note that the proposed beamforming vector is not optimal when the number of transmit antennas is greater than two. ■

Given all channel matrices, the base stations can easily compute the transmit beamforming vectors $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ through (15) and (18). Note that there are M $(= \min(N_t, N_r))$ generalized eigenvectors if all channel matrices have full rank. Therefore, we choose the transmit beamforming vectors that maximize the achievable sum rate as follows:

$$\{\hat{\boldsymbol{f}}_1, \hat{\boldsymbol{f}}_2\} = \arg \max_{\|\boldsymbol{f}_1\|=1, \|\boldsymbol{f}_2\|=1: \boldsymbol{f}_1, \boldsymbol{f}_2 \in \left\{\boldsymbol{f}_1^{(m)}, \boldsymbol{f}_2^{(m)}\right\}_{m=1}^{M}} R^{(m)} \tag{20}$$

where the achievable sum rate $R^{(m)}$ is defined as

$$\begin{aligned} R^{(m)} = \log_2\left(1 + \frac{\frac{P}{2}|\boldsymbol{w}_1^{(m)*}\boldsymbol{H}_1\boldsymbol{f}_1^{(m)}|^2}{\frac{P}{2}|\boldsymbol{w}_1^{(m)*}\boldsymbol{G}_2\boldsymbol{f}_2^{(m)}|^2 + 1}\right) \\ + \log_2\left(1 + \frac{\frac{P}{2}|\boldsymbol{w}_2^{(m)*}\boldsymbol{H}_2\boldsymbol{f}_2^{(m)}|^2}{\frac{P}{2}|\boldsymbol{w}_2^{(m)*}\boldsymbol{G}_1\boldsymbol{f}_1^{(m)}|^2 + 1}\right). \end{aligned} \tag{21}$$

Thus the base stations need to compute (21) with the beamforming vector candidate set and find the transmit beamforming vectors $\{\hat{\boldsymbol{f}}_k\}_{k=1}^2$ through (20). Therefore, the achievable sum rate of the proposed system is given by

$$\begin{aligned} \mathcal{R}_{\mathsf{ZF-CBF}} = \log_2\left(1 + \frac{\frac{P}{2}|\hat{\boldsymbol{w}}_1^*\boldsymbol{H}_1\hat{\boldsymbol{f}}_1|^2}{\frac{P}{2}|\hat{\boldsymbol{w}}_1^*\boldsymbol{G}_2\hat{\boldsymbol{f}}_2|^2 + 1}\right) \\ + \log_2\left(1 + \frac{\frac{P}{2}|\hat{\boldsymbol{w}}_2^*\boldsymbol{H}_2\hat{\boldsymbol{f}}_2|^2}{\frac{P}{2}|\hat{\boldsymbol{w}}_2^*\boldsymbol{G}_1\hat{\boldsymbol{f}}_1|^2 + 1}\right) \end{aligned} \tag{22}$$

where, $\hat{\boldsymbol{w}}_1 = \frac{\boldsymbol{H}_1\hat{\boldsymbol{f}}_1}{\|\boldsymbol{H}_1\hat{\boldsymbol{f}}_1\|}$ and $\hat{\boldsymbol{w}}_2 = \frac{\boldsymbol{H}_2\hat{\boldsymbol{f}}_2}{\|\boldsymbol{H}_2\hat{\boldsymbol{f}}_2\|}$.

E. Zero-forcing IA-CBF with Maximum Ratio Transmission

So far, we have assumed MRC at each mobile station. In this case, the role of the transmit beamforming vector $\boldsymbol{f}_k$ is to nullify the other-cell interference while the role of the receive combining vector $\boldsymbol{w}_k$ is to maximize the desired channel link. Suppose that we use the receive combining vector $\boldsymbol{w}_k$ to nullify the other-cell interference (in this case, we name $\boldsymbol{w}_k$ the interference nulling vector); i.e.,

$$\boldsymbol{w}_1^*\boldsymbol{G}_2\boldsymbol{f}_2 = 0 \quad \text{and} \quad \boldsymbol{w}_2^*\boldsymbol{G}_1\boldsymbol{f}_1 = 0. \tag{23}$$

Let the transmit beamforming vector $\boldsymbol{f}_k = \boldsymbol{H}_k^*\boldsymbol{w}_k$, which is maximum ratio transmission (MRT) to maximize the desired signal strength at the base stations. Then we want to find the interference nulling vectors as follows:

$$\begin{aligned} \{\hat{\boldsymbol{w}}_1, \hat{\boldsymbol{w}}_2\} = \arg \max_{\boldsymbol{w}_1, \boldsymbol{w}_2, \|\boldsymbol{w}_1\|=1, \|\boldsymbol{w}_2\|=1} \\ \left\{\log_2\left(1 + \frac{\frac{P}{2}|\boldsymbol{w}_1^*\boldsymbol{H}_1\boldsymbol{H}_1^*\boldsymbol{w}_1|^2}{\frac{P}{2}|\boldsymbol{w}_1^*\boldsymbol{G}_2\boldsymbol{H}_2^*\boldsymbol{w}_2|^2 + 1}\right)\right. \\ \left. + \log_2\left(1 + \frac{\frac{P}{2}|\boldsymbol{w}_2^*\boldsymbol{H}_2\boldsymbol{H}_2^*\boldsymbol{w}_2|^2}{\frac{P}{2}|\boldsymbol{w}_2^*\boldsymbol{G}_1\boldsymbol{H}_1^*\boldsymbol{w}_1|^2 + 1}\right)\right\} \end{aligned} \tag{24}$$

and the zero other-cell interference constraint in (23) can be rewritten as

$$\boldsymbol{w}_1^*\boldsymbol{G}_2\boldsymbol{H}_2^*\boldsymbol{w}_2 = 0 = \boldsymbol{w}_2^*\boldsymbol{G}_1\boldsymbol{H}_1^*\boldsymbol{w}_1. \tag{25}$$

Theorem 5: Under a zero other-cell interference constraint in (23), the sufficient and necessary beamforming vectors with

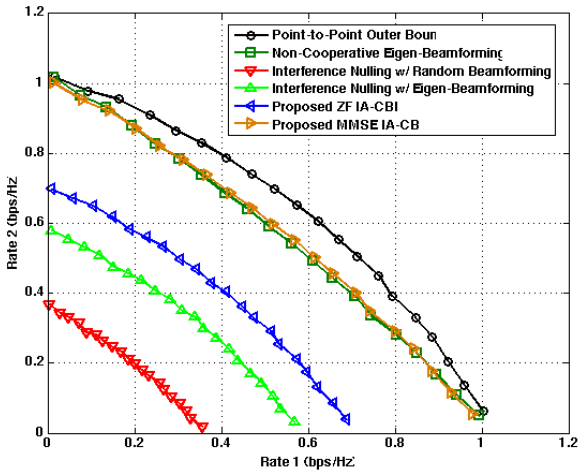

Fig. 5. Comparisons of Achievable Rate Region. Total Power is -5 dB and $N_t = 2$, $N_r = 2$.

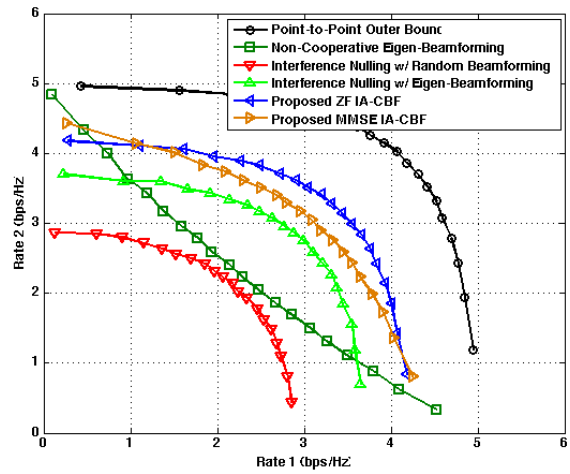

Fig. 6. Comparisons of Achievable Rate Region. Total Power is 10 dB and $N_t = 2$, $N_r = 2$.

MRT for mobile station ℓ (where, k, ℓ are 1 or 2, $\ell \neq k$) are generalized eigenvectors of $\boldsymbol{G}_\ell \boldsymbol{H}_\ell^*$ and $\boldsymbol{H}_k \boldsymbol{G}_k^*$.

Proof: From the zero other-cell interference constraint with MRT, we have the following conditions:
for user k,

$$\boldsymbol{w}_k^* \boldsymbol{G}_\ell \boldsymbol{H}_\ell^* \boldsymbol{w}_\ell = 0 \;\Leftrightarrow\; \boldsymbol{G}_\ell^* \boldsymbol{w}_k \perp \boldsymbol{H}_\ell^* \boldsymbol{w}_\ell \;\Leftrightarrow\; \boldsymbol{w}_k \perp \boldsymbol{G}_\ell \boldsymbol{H}_\ell^* \boldsymbol{w}_\ell \tag{26}$$

and for user ℓ,

$$\boldsymbol{w}_\ell^* \boldsymbol{G}_k \boldsymbol{H}_k^* \boldsymbol{w}_k = 0 \;\Leftrightarrow\; \boldsymbol{G}_k^* \boldsymbol{w}_\ell \perp \boldsymbol{H}_k^* \boldsymbol{w}_k \;\Leftrightarrow\; \boldsymbol{w}_k \perp \boldsymbol{H}_k \boldsymbol{G}_\ell^* \boldsymbol{w}_\ell. \tag{27}$$

From (26) and (27), we have

$$\boldsymbol{G}_\ell \boldsymbol{H}_\ell^* \boldsymbol{w}_\ell \parallel \boldsymbol{H}_k \boldsymbol{G}_\ell^* \boldsymbol{w}_\ell \quad \Leftrightarrow \quad \boldsymbol{G}_\ell \boldsymbol{H}_\ell^* \boldsymbol{w}_\ell = \lambda \boldsymbol{H}_k \boldsymbol{G}_\ell^* \boldsymbol{w}_\ell \tag{28}$$

where, λ_2 is the generalized eigenvalue. Therefore, the interference nulling vector $\boldsymbol{w}_\ell$ for user ℓ is the generalized eigenvectors of $\boldsymbol{G}_\ell \boldsymbol{H}_\ell^*$ and $\boldsymbol{H}_k \boldsymbol{G}_k^*$. ■

Given $\boldsymbol{w}_\ell$, we can also compute the interference nulling vector for user k, $\boldsymbol{w}_k$, through *Theorems* 3 and 4 by changing the parameter $\boldsymbol{z} = \boldsymbol{G}_\ell \boldsymbol{H}_\ell^* \boldsymbol{w}_\ell$. Since this is straightforward, we omit the details. Note that we have the same conclusion as in ZF IA-CBF with MRC; i.e., for two antennas, the proposed algorithm is a sufficient and necessary solution and for more than two antennas, the proposed solution is suboptimal.

F. Degrees of Freedom of the Proposed IA-CBF

In Sections II-C-II-E, three IA-CBF algorithms were proposed. In this section, we investigate the degrees of freedom of the proposed IA-CBF.

Lemma 6: The total number of degrees of freedom for the two-cell MIMO system with two transmit and receive antennas under consideration is equal to 2.

Proof: As previously discussed, the two-cell MIMO system under consideration matches the two-user MIMO interference channel. For the generalized K-user $N_r \times N_t$ MIMO interference channel, the degrees of freedom (DoF) is given by

$$\begin{aligned} \text{DoF} &= \min(N_t, N_r) \frac{T}{T+1} K \quad &\text{if} \quad K > T \\ \text{DoF} &= \min(N_t, N_r) K \quad &\text{if} \quad K \leq T \end{aligned}$$

where $T = \frac{\max(N_t, N_r)}{\min(N_t, N_r)}$ [15]. If $N_t = N_r = K = 2$, therefore, the degrees of freedom of the two-cell MIMO channel is 2. ■

The total number of degrees of freedom that the proposed IA-CBF achieves is also 2, as each base station transmits only one data stream to each mobile station, i.e,

$$\lim_{\mathsf{SINR} \to \infty} \frac{\mathcal{R}_{\mathsf{ZF-CBF}}(\mathsf{SINR})}{\log_2 \mathsf{SINR}} = 2. \tag{29}$$

This means that the proposed IA-CBF is the full-degrees-of-freedom-achieving solution for a two-cell MIMO channel where each mobile station has two receive antennas.

III. Performance Evaluation

We compare the achievable sum rate of the proposed IA-CBF algorithm with the following: simple point-to-point outer bound, other cell interference nulling with random- or eigen-beamforming, and non-cooperative eigen-beamforming.

A. Reference Models

1) Simple point-to-point outer bound [12]: Let us assume there is no other-cell interference and each base station transmits the signal to the desired mobile station through the best eigen-mode. In this case, the achievable sum rate of a simple point-to-point system is given by

$$\mathcal{R}_{\mathsf{pp}} = \log_2 \left(1 + \frac{P |\boldsymbol{u}_1^* \boldsymbol{H}_1 \boldsymbol{v}_1|^2}{2} \right) + \log_2 \left(1 + \frac{P |\boldsymbol{u}_2^* \boldsymbol{H}_2 \boldsymbol{v}_2|^2}{2} \right)$$

where $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ are the principle left side singular vectors, and $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are the principle right side singular vectors of $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$, respectively. Note that this sum rate cannot be

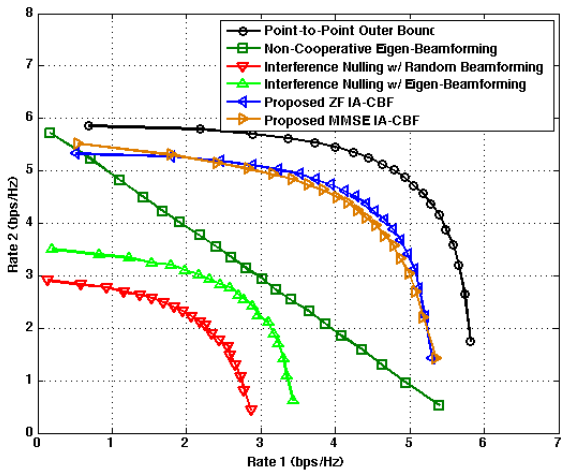

Fig. 7. Comparisons of Achievable Rate Region. Total Power is 10 dB and $N_t = 2$, $N_r = 4$.

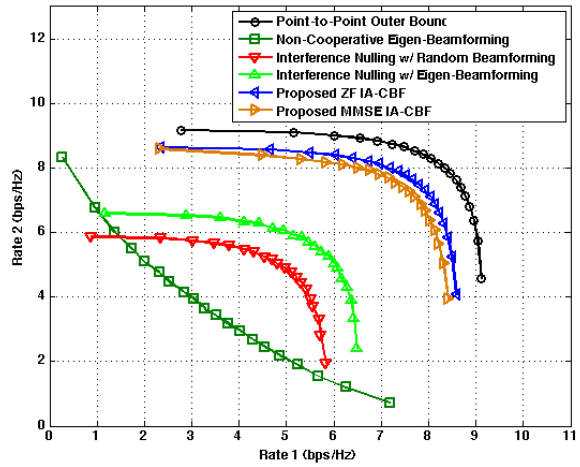

Fig. 8. Comparisons of Achievable Rate Region. Total Power is 20 dB and $N_t = 2$, $N_r = 4$.

achieved unless $\boldsymbol{G}_1$ and $\boldsymbol{G}_2$ are zero matrices. We use this outer-bound, however, for performance comparison purposes since the capacity region of the interference channel is not known yet [12].

2) Non-cooperative other-cell interference nulling: We also consider the interference nulling techniques for performance comparison purposes. In this technique, the role of the receive combining vector is only to eliminate the other-cell interference. We consider two different transmit beamforming vectors combined with interference nulling: i) random beamforming, and ii) eigen-beamforming. Given the transmit beamforming vectors $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$, we need to find the receive combining vectors that satisfy the condition of no other-cell interference, which is given as:

$$\boldsymbol{w}_1^* \boldsymbol{G}_2 \boldsymbol{f}_2 = 0 \quad \text{and} \quad \boldsymbol{w}_2^* \boldsymbol{G}_1 \boldsymbol{f}_1 = 0. \tag{30}$$

Note that $\boldsymbol{w}_k$ ($k = 1$ or 2) is not a function of $\boldsymbol{H}_k$ and $\boldsymbol{f}_k$ but a function of $\boldsymbol{G}_\ell$ and $\boldsymbol{f}_\ell$, where $k \neq \ell$. As an example of two transmit antenna systems, from (30), the receive combining vectors are given by

$$\boldsymbol{w}_1 = \mu_1 \begin{pmatrix} -\bar{g}_{2,2} \\ \bar{g}_{2,1} \end{pmatrix} \quad \text{and} \quad \boldsymbol{w}_2 = \mu_2 \begin{pmatrix} -\bar{g}_{1,2} \\ \bar{g}_{1,1} \end{pmatrix}$$

where $\boldsymbol{g}_1 = \boldsymbol{G}_1 \boldsymbol{f}_1 = (g_{1,1}, g_{1,2})^T$ and $\boldsymbol{g}_2 = \boldsymbol{G}_2 \boldsymbol{f}_2 = (g_{2,1}, g_{2,2})^T$, and μ_1 and μ_2 are non-zero normalization constants. We can simply generalize this approach for the case $N_r > 2$ by changing the size of $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$. Since we can perfectly remove the other-cell interference terms in (1), these approaches are also full-degrees-of-freedom-achieving solutions.

3) Non-cooperative eigen-beamforming (MRT-MRC) [21]: This approach just maximizes its own effective channel gain through the best eigen-mode after treating the other-cell interference as noise. Therefore, the base station uses the right-side principle singular vector, and the mobile station uses the left-side principle singular vector of the desired channel matrix $\boldsymbol{H}_k$, respectively. This non-cooperative eigen-beamforming method does not take into account the other-cell interference. Therefore, we expect this approach to be unsuitable for cell-boundary users. It may, however, be useful for cell-interior users who receive very weak other-cell interference. The achievable sum rate of the non-cooperative eigen-beamforming can be expressed as

$$\begin{aligned} \mathcal{R}_{\text{nc}} = &\log_2 \left(1 + \frac{\frac{P}{2} |\boldsymbol{u}_1^* \boldsymbol{H}_1 \boldsymbol{v}_1|^2}{\frac{P}{2} |\boldsymbol{u}_1^* \boldsymbol{G}_2 \boldsymbol{v}_2|^2 + 1} \right) \\ &+ \log_2 \left(1 + \frac{\frac{P}{2} |\boldsymbol{u}_2^* \boldsymbol{H}_2 \boldsymbol{v}_2|^2}{\frac{P}{2} |\boldsymbol{u}_2^* \boldsymbol{G}_1 \boldsymbol{v}_1|^2 + 1} \right). \end{aligned} \tag{31}$$

It is noteworthy that the non-cooperative eigen-beamforming maximizes the numerator terms (desired effective channel gain) while the interference nulling minimizes the denominator terms (other-cell interference).

In Table I, we address the channel matrices and the beamforming vectors required to compute the transmit/receive beamforming vectors. As can be seen from Table I, MMSE IA-CBF requires less channel information at the base stations than ZF IA-CBF with MRC since each base station needs its own channel matrix $\boldsymbol{H}_k$ and interference matrix $\boldsymbol{G}_k$ to the undesired user. On the other hand, all channel matrices are required at base station 1 (or base station 2) for ZF IA-CBF with MRC. Instead, ZF IA-CBF with MRC requires only the effective channel vector, $\boldsymbol{H}_k \boldsymbol{f}_k$ at the mobile station, which can be estimated through the dedicated pilot channel [22], [23]. Therefore, there is a complexity tradeoff between MMSE IA-CBF and ZF IA-CBF with MRC. As reciprocity, ZF IA-CBF with MRT requires less information at the base stations and more information at the mobile stations. Note that the conventional interference nulling requires the least channel information but in general does not show good achievable rate performance.

B. Numerical Results

We illustrate the achievable sum rate of the proposed method when the base stations are equipped with two transmit

TABLE I

CHANNEL MATRICES AND BEAMFORMING VECTORS REQUIRED TO COMPUTE THE TRANSMIT BEAMFORMING AND RECEIVE COMBINING VECTORS AT THE BASE STATIONS AND THE MOBILE STATIONS.

	MMSE IA-CBF	ZF IA-CBF with MRC	ZF IA-CBF with MRT	Interference Nulling
Base Station 1	$\boldsymbol{H}_1, \boldsymbol{G}_1$	$\boldsymbol{H}_1, \boldsymbol{H}_2, \boldsymbol{G}_1, \boldsymbol{G}_2$	$\boldsymbol{H}_1^*\boldsymbol{w}_1$	$\boldsymbol{H}_1$
Base Station 2	$\boldsymbol{H}_2, \boldsymbol{G}_2$	$\boldsymbol{H}_2, \boldsymbol{G}_1, \boldsymbol{f}_1$	$\boldsymbol{H}_2^*\boldsymbol{w}_2$	$\boldsymbol{H}_2$
Mobile Station 1	$\boldsymbol{f}_1, \boldsymbol{f}_2, \boldsymbol{G}_2, \boldsymbol{H}_1$	$\boldsymbol{H}_1\boldsymbol{f}_1$	$\boldsymbol{H}_2, \boldsymbol{G}_2. \boldsymbol{w}_2$	$\boldsymbol{G}_2\boldsymbol{f}_2$
Mobile Station 2	$\boldsymbol{f}_1, \boldsymbol{f}_2, \boldsymbol{G}_1, \boldsymbol{H}_2$	$\boldsymbol{H}_2\boldsymbol{f}_2$	$\boldsymbol{H}_2, \boldsymbol{G}_2, \boldsymbol{H}_1, \boldsymbol{G}_1$	$\boldsymbol{G}_1\boldsymbol{f}_1$

antennas. There are two active users equipped with more than one receive antenna in the network. For numerical simulations, Figs. 3–9, we model the elements of each mobile station's channel matrix as independent complex Gaussian random variables with zero mean and unit variance $\mathcal{CN}(0,1)$. *Note that the proposed algorithm is not directly related to the channel model*. Once the base stations know all channel matrices, the transmit beamforming and receive combining vectors can be computed through *Theorems* 1 and 2.

Fig. 3 shows the achievable sum rates of: i) point-to-point bound, ii) non-cooperative eigen-beamforming, iii) interference nulling at the mobile stations and random beamforming at the base stations, iv) interference nulling at the mobile stations and eigen-beamforming at the base stations, and v) the proposed MMSE and ZF IA-CBF algorithms. Since ZF IA-CBF with MRC and ZF IA-CBF with MRT show the same sum rate performances, we plot only one result. To illustrate this figure, we assume that each mobile station is equipped with two receive antennas; i.e., $N_r = 2$. As can be seen from the figure, all approaches except the non-cooperative eigen-beamforming achieve the same degrees of freedom.

One may be tempted to argue that the gap between the proposed system and the interference nulling with eigen-beamforming is not very big (about 1 bps/Hz). This gap, however, increases as N_r increases. In Fig. 4, we compare the sum rates where $N_r = 4$. Notice the gap is no longer marginal. That is because the transmit beamforming and the receive combining vectors for interference nulling are not jointly optimized. Here, the role of the transmit beamforming vector is only to nullify the other-cell interference. In addition, the transmit beamforming is computed only to maximize the dedicated channel gain without considering the receive combining vector. Note also that in this scenario we do not argue that the proposed solution is optimal since the full degrees of freedom for the two-user MIMO interference channel with $N_r = 4$ is four. We claim that the proposed system achieves much better performance than the other solutions when the system has a one stream per user constraint. We leave the problem of multi-stream transmission to future work.

So far, we assumed an equal power allocation and now consider a sum power constraint to understand behavior of point-to-point outer bound and IA-CBF algorithms. Figs. 5-8 illustrate the achievable rate regions under various system environments. In Fig. 5, we can observe that the proposed MMSE IA-CBF shows a better achievable rate performance than other solutions including the proposed IA-CBF, interference nulling techniques. With a higher total transmit power as in Figs. 5-8, the proposed ZF IA-CBF is better than MMSE IA-CBF over all power combinations. From these numerical results, we realize that the proposed solutions show good achievable rate performance under any power constraint.

$$\begin{aligned}
\mathcal{R}_{\mathsf{pp}}^{Asymp} &= \lim_{N_r\to\infty} \max_{\{p_k:p_k\geq 0,\sum_k p_k\leq P\}} \left\{\log_2\left(1+p_1|\boldsymbol{u}_1^*\boldsymbol{H}_1\boldsymbol{v}_1|^2\right)+\log_2\left(1+p_2|\boldsymbol{u}_2^*\boldsymbol{H}_2\boldsymbol{v}_2|^2\right)\right\} \\
&\overset{(c)}{=} \lim_{N_r\to\infty} \max_{\{p_k:p_k\geq 0,\sum_k p_k\leq P\}} \{\log_2(1+p_1N_r)+\log_2(1+p_2N_r)\} \\
&= \lim_{N_r\to\infty} \max_{\{p_k:p_k\geq 0,\sum_k p_k\leq P\}} \left\{\log_2 N_r\left(\frac{1}{N_r}+p_1\right)+\log_2 N_r\left(\frac{1}{N_r}+p_2\right)\right\} \\
&= \lim_{N_r\to\infty} \max_{\{p_k:p_k\geq 0,\sum_k p_k\leq P\}} \{2\log_2 N_r+\log_2 p_1+\log_2 p_2\} \\
&= \lim_{N_r\to\infty} \left\{2\log_2 N_r\right\}+2\log_2\left(\frac{P}{2}\right),
\end{aligned} \tag{32}$$

$$\begin{aligned}
\mathcal{R}_{\mathsf{IA-CBF}}^{Asymp} &= \lim_{N_r\to\infty} \max_{\{p_k:p_k\geq 0,\sum_k p_k\leq P\}} \left\{\log_2\left(1+\frac{p_1|\boldsymbol{w}_1^*\boldsymbol{H}_1\boldsymbol{f}_1|^2}{p_2|\boldsymbol{w}_1^*\boldsymbol{G}_2\boldsymbol{f}_2|^2+1}\right)+\log_2\left(1+\frac{p_2|\boldsymbol{w}_2^*\boldsymbol{H}_2\boldsymbol{f}_2|^2}{p_1|\boldsymbol{w}_2^*\boldsymbol{G}_1\boldsymbol{f}_1|^2+1}\right)\right\} \\
&\overset{(d)}{=} \lim_{N_r\to\infty} \max_{\{p_k:p_k\geq 0,\sum_k p_k\leq P\}} \left\{\left(\log_2\left(\frac{1}{N_r}+p_1\right)+\log_2 N_r\right)+\left(\log_2\left(\frac{1}{N_r}+p_2\right)+\log_2 N_r\right)\right\} \\
&= \lim_{N_r\to\infty} \left\{2\log_2 N_r\right\}+2\log_2\left(\frac{P}{2}\right),
\end{aligned} \tag{33}$$

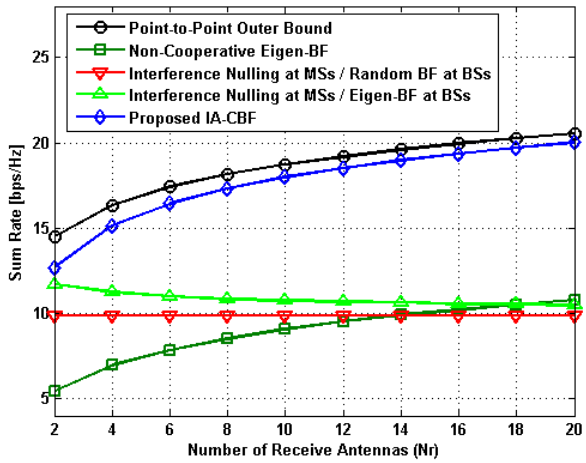

Fig. 9. Sum Rate Comparisons vs. the Number of Receive Antennas.

C. Asymptotic Behavior

In this section, we investigate the asymptotic sum rate behavior of the proposed IA-CBF algorithm with respect to the number of receive antennas. As a reference model, we first derive the asymptotic sum capacity by simple point-to-point outer bound, which is given by (32) on the bottom of the page, where equality (c) results from the fact that $\lim_{N_r\to\infty} \boldsymbol{H}_k^*\boldsymbol{H}_k = N_r\boldsymbol{I}_{N_t}$. As $N_r \to \infty$, the generalized eigenvectors of $(\boldsymbol{G}_k^*\boldsymbol{G}_k,\ \boldsymbol{H}_k^*\boldsymbol{H}_k)$ asymptotically converge to $\boldsymbol{f}_k = [\boldsymbol{I}_{N_t\times N_t}]_k$, where the notation $[\cdot]_k$ is the kth column of a matrix. With the transmit beamforming vector $\boldsymbol{f}_k$, we can now compute the asymptotic achievable sum rate of IA-CBF [see (33) on the bottom of the page], where (d) follows by the property that $\lambda_{\max}(M\boldsymbol{I}) = M$, where $\lambda_{\max}$ denotes the largest singular value and M is any arbitrary constant.

We illustrate the achievable sum rates as a function of the number of receive antennas, N_r in Fig. 9. It is quite interesting that the proposed system converges to the point-to-point outer bound while the interference nulling approaches converge to some constant. The sum rate performance of interference nulling approaches is even worse than the non-cooperative eigen-beamforming at the high N_r regime. This is because the transmit beamforming vector becomes asymptotically independent of the receive combining vector. Therefore, as $N_r \to \infty$, we have the following asymptotic sum rate equation, which is the capacity expression of a single-input single-output (SISO) Rayleigh fading channel.

$$C = 2 \cdot e^{2/P} \log_2(e) E_1\left(\frac{2}{P}\right),$$

where $E_1(a)$ is the exponential integral of order of one and is given by

$$E_1(a) = \int_1^\infty e^{-ax}x^{-1}dx, \quad \mathrm{Re}\{a\} > 0.$$

If $P = 20\ dB$, then $C = 9.8752$, which confirms the simulation result in Fig. 9. In this paper, we do not argue that we have to employ as many receive antennas as possible. Rather we analyze the optimality of the IA-CBF algorithms with respect to the number of receive antennas and show that in practice IA-CBF is very attractive since it works quite well even with a small number of receive antennas. Although practical systems will not be able to employ a large number of antennas at the mobile station, we expect that our results will provide guidance in the design of practical systems as well as insights on the impact of the number of receive antennas.

IV. Conclusion

In this paper, we proposed sufficient and necessary (for two antennas), and sufficient (for more than two antennas) interference aware-coordinated beamforming (IA-CBF) algorithms for a two-cell environment where two mobile stations are served through two base stations with multiple transmit antennas. Based on the assumption of single stream per mobile station, a closed-form expression for the transmit beamforming vector was derived. We also investigated the optimality of the proposed IA-CBF and the conventional interference nulling solutions. We confirmed through numerical and analytical results that the proposed IA-CBF significantly outperformed the existing algorithms for a multi-cell environment. For future work, we will consider multi-stream transmission to each mobile station to achieve the full degrees of freedom for any antenna configurations.

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References

[1] D. Gesbert, M. Kountouris, R. W. Heath, Jr., C.-B. Chae, and T. Salzer, "Shifting the MIMO paradigm: From single user to multiuser communications," *IEEE Sig. Proc. Mag.*, vol. 24, no. 5, pp. 36–46, Oct. 2007.

[2] J. G. Andrews, W. Choi, and R. W. Heath, Jr., "Overcoming interference in spatial multiplexing MIMO cellular networks," *IEEE Wireless Comm. Mag.*, vol. 14, no. 6, pp. 95–104, Dec. 2007.

[3] S. Shamai (Shitz) and B. M. Zaidel, "Enhancing the cellular downlink capacity via co-processing at the transmitting end," *Proc. IEEE Veh. Technol. Conf.*, pp. 1745–1749, May 2001.

[4] C.-B. Chae, S. Kim, and R. W. Heath, Jr., "Network coordinated beamforming for cell-boundary users: Linear and non-linear approaches," *to appear in IEEE Jour. Select. Topics in Sig. Proc.*, Dec. 2009.

[5] D. Gesbert, S. G. Kiani, A. Gjendemsjo, and G. Oien, "Adaptation, coordination, and distributed resource allocation in interference-limited wireless networks," *Proc. of IEEE*, vol. 95, no. 12, pp. 2393–2409, Dec. 2007.

[6] S. A. Jafar, G. J. Foschini, and A. J. Goldsmith, "PhantomNet: Exploring optimal multicellular multiple antenna systems," *Eurasip Jour. Applied Sig. Proc.*, vol. 2004, no. 5, pp. 591–604, 2004, doi:10.1155/S1110865704312072.

[7] S. Jing, D. Tse, J. B. Soriaga, J. Hou, J. E. Smee, and R. adovani, "Multicell downlink capacity with coordinated processing," *Eurasip Jour. Wireless Comm. and Networking*, pp. 1–19, 2008, Article ID 586878.

[8] H. Huang and M. Trivellato, "Performance of multiuser MIMO and network coordination in downlink cellular networks," *Proc. of Int. Symp. on Model. and Opt. in Mobile, Ad Hoc, and Wireless Networks*, pp. 85–90, April 2008.

[9] M. K. Karakayali, G. J. Foschini, and R. A. Valenzuela, "Network coordination for spectrally efficient communication in cellular systems," *IEEE Wireless Comm. Mag.*, vol. 13, no. 4, pp. 56–61, Aug. 2006.

[10] H. Dahrouj and W. Yu, "Coordinated beamforming for the multi-cell multi-antenna wireless system," *Proc. of Conf. on Info. Scien. and Systems*, pp. 429–434, March 2008.

[11] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Info. Th.*, vol. 27, no. 1, pp. 49–60, Jan. 1981.

[12] R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Info. Th.*, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.

[13] G. Bresler and D. Tse, "The two-user Gaussian interference channel: A deterministic view," *Europ. Trans. on Telecomm.*, vol. 19, pp. 333–352, April 2008.

[14] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. Info. Th.*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[15] T. Gou and S. A. Jafar, "Degrees of freedom of the K user $M \times N$ MIMO interference channel," *submitted for publication*, 2008, available at http://arxiv.org/ abs/0803.0099.

[16] S. Peters and R. W. Heath, Jr., "Interference alignment via alternating minimization," *Proc. IEEE Int. Conf. Acoust., Speech and Sig. Proc.*, pp. 1–5, April 2009.

[17] K. Gomadam, V. R. Cadambe, and S. A. Jafar, "Approaching the capacity of wireless networks through distributed interference alignment," *Proc. IEEE Glob. Telecom. Conf.*, vol. 19, pp. 1–6, Nov. 30-Dec. 4 2008.

[18] C.-B. Chae, D. Mazzarese, N. Jindal, and R. W. Heath, Jr., "Coordinated beamforming with limited feedback in the MIMO broadcast channel," *IEEE Jour. Select. Areas in Comm.*, vol. 26, no. 8, pp. 1505–1515, Oct. 2008.

[19] R. A. Horn and C. R. Johnson, *Matrix Analysis*, University of Cambridge Press, New York, 4th edition, 1990.

[20] C.-B. Chae, D. Mazzarese, T. Inoue, and R. W. Heath, Jr., "Coordinated beamforming for the multiuser MIMO broadcast channel with limited feedforward," *IEEE Trans. Sig. Proc.*, vol. 56, no. 12, pp. 6044–6056, Dec. 2008.

[21] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*, Cambridge University Press, 2003.

[22] 3GPP Long Term Evolution, "Physical layer aspects of UTRA high speed downlink packet access," *Technical Report TR25.814*, 2008.

[23] "IEEE P802.16m-2007 draft standards for local and metropolitan area networks part 16: Air interface for fixed broadcast wireless access systems," *IEEE Standard 802.16m*, 2007.

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