# Finding Nash and Earning Cash: Applications of Iterated Dominance to Reporting GPAs on Resumes 

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## 1 Introduction

A job applicant's grade point average (GPA) is often among the most important aspects of his or her resume. The GPA is a quantitative measure of his academic performance, typically at his highest level of education. As such, a person's GPA is important to any potential employer who seeks to evaluate any of the qualities that might lead to a strong academic performance, such as intelligence, discipline, and organization. GPAs are often calculated on a scale of 0.0 to 4.0 , where a 4.0 GPA indicates perfect grades in every class and a 0.0 GPA represents that the applicant failed each one. Many firms have GPA cutoffs, and others place significant weight on the GPA ${ }^{1}$; consequently, applicants have an incentive to maximize their perceived GPAs from the perspective of employers.

GPAs are generally reported as rounded decimals. In particular, they are usually expected to be rounded to one or two decimal places, with three-decimal-place GPAs occasionally occurring. ${ }^{2}$ Sometimes, a firm or university will require an applicant to report the scale on which the GPA is measured, or to convert it to a 4.0 scale. ${ }^{3}$ Rounding conventions hold that a number is rounded to the nearest value of a certain level of precision; if it is equidistant between two values, it is rounded to the higher one.

A GPA can be particularly informative about an applicant's quality if it is rounded to more than one decimal place, as such a GPA is more precise. Many applicants vary in terms of how many digits they round their GPAs

[^0]to, and accordingly how much information they disclose. Given the convention of rounding GPAs to either one or two decimal places, concepts from iterated dominance show that the Nash-equilibrium strategy is for job applicants to nearly always round their GPAs to two digits, and for employers to always assume that a GPA rounded to only one decimal place rounds to the lowest possible two-decimal figure (i.e., a GPA that is rounded to 3.7 on a resume would imply a true GPA of 3.65 , instead of, say, 3.71). Despite the importance of GPAs in securing employment, it seems that many job seekers do not play the best response ${ }^{4}$ to employers' strategies, and instead round to one decimal place when the equilibrium strategy is for them to usually round to two.

This paper begins by explaining why the best response strategy in this particular scenario is for job applicants to usually round to two decimal places. It then explores possible factors that would affect when a job applicant would actually follow the Nash equilibrium strategy. It continues by detailing a theoretical mathematical model that attempts to predict when applicants deviate from the equilibrium strategy. Finally, it compares the predictions of the model to results from the field. The data reveal that job applicants generally do not play the best response strategies given the assumptions made in the model; however, the frequency with which they rounded to multiples of 0.5 suggest possible other strategies.

[^1]
## 2 Literature Review

There is not much available research regarding how people round their GPAs. However, there is a significant amount of literature available about information asymmetry and adverse selection, categories that rounding GPAs would fall into. When an individual applies for a job and reports a rounded GPA, he has private information (the true GPA) that he aims to make appear as high as possible in the eyes of the employer. However, the employer's incentive is to know the applicant's GPAs as accurately as possible in order that she can determine his merits for the job. In such interactions, party A (in this case, the applicant) is incentivized to maximize the expected value of his private information from the perspective of party B (here, the employer), while party B is incentivized to learn as accurately as possible the value of party A's private information. As a result, in equilibrium one of two outcomes occurs: screening or signaling.

Screening occurs when those in party A cannot or do not accurately reveal their private information. For example, Akerlof (1970) provides a classic example of screening in the used car market, a market in which the seller has private information (details about the car's quality) that the potential buyers generally cannot access. In such a market, Akerlof asserts, the cars that are sold are often of unusually bad quality. His reasoning is as follows: since potential buyers cannot tell the difference between a good car and a bad one (a "lemon"), the price buyers are willing to pay reflects the quality of the average car for sale. However, since the pricing of the used cars does not differentiate between cars of good and bad quality, the potential
sellers of the best cars do not want to sell, as they would not get paid a price reflective of their cars' true values. Next, cars of medium quality are driven out of the market: buyers are willing to pay a price that reflects the average quality of the remaining cars, but with the truly good cars gone, the cars of medium quality are the best ones left, and are more valuable than the new price reflects. The owners of these cars, not willing to accept a price that is less than the cars' true value, remove their cars from the market. Ultimately, the only cars left for sale are the lemons; the buyers' prices have screened the good cars from the bad ones.

While building on Akerlof's analysis of screening in the used car market, Viscusi (1978) provides a model of information disclosure in which signaling occurs. In his paper, Viscusi describes how companies disclose information about their products in an iterated manner-the firms with highest-quality products are the first to disclose the information, followed by the firms with the next-highest quality products, and so on. Viscusi's example of firms' disclosure of product information is not entirely analogous to Akerlof's used car example, as Viscusi's firms are able to accurately signal their private information, whereas Akerlof's car sellers are not. However, the examples are similar in that in both instances, an unraveling process occurs, ultimately deterring even those with below-average private information from being associated with those who have not revealed their private information (car sellers who remain in the market and firms that do not reveal the quality of their products). As such, since Viscusi's firms can and do reveal their private information without prompting from the buyers, his model is an example of signaling.

Nonetheless, these theoretical models do not always describe actual behavior. In particular, Camerer (2009), Luca and Smith (2014), and Jin, Luca, and Martin (2015) provide excellent examples of such deviations. For a more detailed analysis of these sources and other, please see the appendix.

There has been no research done as to the strategies job applicants use in rounding their GPAs, and relatively little about the distribution of GPAs. Of the rare literature in this field, Smits et al. (2002) uses existing GPAs to estimate the values of expected grades that students might have earned in classes they did not take. This paper may be able to provide more insight on distributions of GPAs in addition to analyzing rounding strategies.

However, there is a significant amount of research that has been done that shows that people experience a significantly greater amount of utility in barely reaching a reference point than in barely missing one. These articles could help us consider whether applicants reporting their GPAs, for which they often may have reference points, might sometimes choose different strategies of reporting GPAs depending on whether they reached a reference point. Most famously, Kahneman and Tversky (1979) assert that reference points affect utility. More recent research, including that of Mellers, Schwartz, Ho, and Ritov (1997) and Larrick, Heath, and Wu (2009) discuss some of the sources of these reference points, which include expectations and pre-set goals. This paper may be able to provide some additional detail as to how reference points affect GPA rounding strategies.

Simonsohn and Pope (2011) might be able to shed light on an additional reference point that could affect how job applicants report their GPAs: round numbers. The article found that baseball players strategically chose
to miss at-bats in order to finish with batting averages that reached the milestone of .300 , as opposed to barely missing it. Perhaps more importantly as it relates to GPAs, the authors also reported that students are more likely to retake the SAT when they miss a round number (in this case, a multiple of 100) by a small margin than when they do not. It may also be possible that individuals reporting GPAs exhibit a similar bias: they may be more likely to report a GPA that narrowly reaches a milestone than one that barely misses such a figure.

## 3 Unraveling in Resume GPAs

Deciding how many digits to round a GPA to is analogous to Viscusi's paper in that it is an example of signaling. ${ }^{5}$ Let us suppose that all job applicants know the distribution of true GPAs in the applicant pool, and that it is common knowledge between all job applicants and employers that all members of both groups are perfectly rational (we will call rational applicants in such a situation "sophisticates"). Given the convention of rounding GPA to either one or two decimal places, the best response strategy for job applicants given common knowledge of a rational employer is for nearly all applicants to round their GPAs to two digits: at each iteration, the best remaining applicants whose GPAs round to a given one-decimal figure will round to two decimal places, until nearly all applicants have rounded to two decimal

[^2]places. In equilibrium, the only applicants who round to one decimal place are those whose GPAs round to the lowest possible two-decimal GPA given their rounded one-decimal GPA. A formal proof is given in the appendix.

To illustrate this with a real example, suppose an applicant chooses to report a GPA of 3.7. Under this scenario, an employer would have concluded that the applicant's GPA must not have been any higher than 3.70, otherwise he would have reported, say, 3.73 instead of rounding down to 3.7 . The applicant, aware that the employer was expecting a true GPA in the range of 3.65-3.70 when she saw the 3.7 figure, would have reported a 3.70 if that were truly his GPA, so as not to be lumped together with applicants in the 3.65-3.69 range. Of course, the employer would have known this as well, so she would assume that the applicant's GPA must be lower than 3.70, since he would have reported 3.70 instead of 3.7 if that were his true GPA.

Since 3.7 would now represent a GPA of $3.65-3.69$ in the employer's mind, the applicant, knowing this, would have reported a 3.69 if that were his true GPA. Since the applicant didn't report a 3.69 GPA, the employer believes that the applicant's GPA is in the range of $3.65-3.68$. Since the applicant knows this, he would have reported 3.68 if that were his true GPA, so as not to be confused with applicants in the range of 3.65-3.67. Ultimately, this iterated process continues until the only remaining possible GPA for an applicant who reports 3.7 is 3.65 . Thus, no matter where the applicant's 2 decimal GPA falls in the range of 3.66-3.75 (the possible 2 decimal GPAs that round to 3.7 ), the applicant is never worse off, and usually better off, rounding to two digits, making it the Nash equilibrium strategy to always round to two decimal places.

However, if the applicant's two-decimal GPA is 3.65 , but his one-decimal GPA is 3.7 , it is the Nash equilibrium strategy for an applicant to round to only one decimal place. A two decimal GPA of 3.65 represents a GPA in the range of 3.645 to $3.655-\epsilon$, where $\epsilon$ is arbitrarily small. Some of these values round to a one-decimal GPA of 3.6 , and some round to 3.7. Even if an employer assumes that a GPA reported as 3.7 rounds to a two-decimal GPA of 3.65 , the employer knows that the GPA must be at least 3.650 and as high as $3.655-\epsilon$. By contrast, if the applicant reports his GPA as 3.65 , the employer believes that it could be as low as 3.645 . Thus, in equilibrium, any job applicant whose GPA is in the range of 3.650 to $3.655-\epsilon$ will round to one decimal place; consequently, a reported GPA of 3.65 would correspond to an actual GPA in the range of 3.645 to only $3.650-\epsilon$.

## 4 Theoretical Model

However, it seems likely that not all applicants are sophisticates. One reason is that it seems unlikely that all applicants know, or even consider, that all the other applicants are fully rational; it seems equally improbable that very many applicants assume employers to be using the iterated reasoning that the model suggests they do. Furthermore, one would assume that many applicants do not use any iterated reasoning in rounding their GPAs, instead simply rounding their GPAs in a way that maximizes the reported number rather than its expectation from the perspective of a perfectly rational, perfectly informed employer. ${ }^{6}$

[^3]As a result, in the following model we will introduce several other types of applicants beside sophisticates. The first group of applicants, strategic naifs, choose the number of decimal places in their reported GPAs based only on which maximizes the reported value of the GPA, as opposed to its expectation from a rational employer's perspective. ${ }^{7}$ For example, a strategic naif might round his 3.79 GPA to 3.8, whereas a sophisticate from the previous model with the same GPA would prefer to report 3.79. Strategic naifs do not know and do not consider the GPAs and rounding strategies of the other applicants. In addition, there are true naifs. These applicants arbitrarily choose how many decimal places to round their GPAs to; they round to 1 decimal place with probability $p .{ }^{8}$

For the purposes of this model, the sophisticates are aware of the distributions of both the true GPAs and the strategies of all the other applicants. We will assume that applicants have a choice to either round their GPAs to one decimal place or report it exactly. We will also assume that GPAs are continuous, though in reality, they are technically discrete, as they are averages of discrete grades (usually multiples of $1 / 3$ ) over a discrete number of classes (it is rare for applicants to have taken more than, say, 50 classes for credit). ${ }^{9}$ In this model, we will assume that GPAs are uniformly distributed between 0 and 4.

We will assume that applicants' GPAs follow a modified normal distribution (it is impossible to have a GPA less than 0 or greater than a specified

[^4]maximum GPA, often 4.00) with mean $\mu$ and variance $v .{ }^{10}$ Given this assumption, we will further assume that the proportion of job applicants who would get a GPA below 0 in a normal distribution will end up with a GPA of 0 ; similarly, we will assume that the proportion of job applicants who would get a GPA above 4.00 under a normal distribution will end up with exactly a 4.00 . We will also continue to assume that job applicants would like to maximize their perceived GPAs in the eyes of an employer.

Let us suppose that the there are sophisticates, strategic naifs, and true naifs present in the applicant pool, and the proportions of each group are $s, n$, and $t$, for which $s+n+t=1$. For a given one-decimal figure $a$ for $0.0<a<4.0$, the expected value of a one-decimal GPA reported by a true naif is this figure $a .{ }^{11}$ Since a strategic naif will only round her GPA to one decimal place if it is below $a$, the expectation of a one-decimal GPA reported by a strategic naif is $\frac{a+(a-0.05)}{2}=a-0.025$. As such, the expectation of a one-decimal GPA reported by a generalized naif is $t p a+s / 2(a-0.025)$.

For ease of notation, we will let $E_{0}(a(x))$ refer to the expected value of a one-decimal GPA $a(x)$ if only naifs are included. We assume that at the first iteration, sophisticates in the range of GPAs that round to $a(x)$ would want to round their GPAs to $a(x)$ if their GPAs are lower than $E_{0}(a(x))$. We define $E_{1}(a(x))$ as the subsequent expected value of all GPAs given this rounding strategy by the sophisticates; as such, $E_{1}(a(x))=$

[^5]$\left(1-s_{0}\right) E_{0}(a(x))+s_{0}\left(\frac{\int_{c=a \alpha(x)-0.05}^{c=E_{0}(x)} c \frac{\phi(c-\mu)}{\sqrt{v}}}{\int_{c=a(x)-0.05}^{c=E(x)} \frac{\phi(c-\mu)}{\sqrt{v}}}\right)$; this occurs because the expected value of the sophisticates' GPAs is $\frac{\int_{c=a(x)-0.05}^{c=E_{0}(x)} \frac{\phi(c-\mu)}{\sqrt{v}}}{\int_{c=a(x)-0.05}^{c=E} \frac{\phi(c, \mu)}{\sqrt{v}}}$.

After this first level of iteration, all sophisticates with GPAs less than $E_{0}(a(x))$ will round them to one decimal place. However, since the new expected GPA of $a(x)$ is now $E_{1}(a(x))$, the sophisticated applicants with GPAs between $E_{0}(a(x))$ and $E_{1}(a(x))$, representing a proportion $s_{1}$ of the applicants, are now compelled to round them to two or more decimal places, as they are now above the expectation. As a result, the expectation of the one-decimal GPA $a(x)$ moves downward to $E_{2}(a(x))$, leading the proportion $s_{2}$ of applicants with GPAs between $E_{1}(a(x))$ and $E_{2}(a(x))$ to round to two or more digits. This iterated process continues infinitely. In equilibrium, sophisticates whose GPAs round to a given $a$ but are above some $x_{a} *$ will round their GPAs to two or more decimal places, whereas such applicants whose GPAs are below $x_{a} *$ will round them to $a$. Consequently, the equilibrium expectation $x_{a} *$ of the GPAs reported to one decimal place is the value that satisfies the equation $x_{a} *=\left(E_{0}(a(x))\right)\left(1-s_{0}\right)+\frac{\left(s \int_{a(x)-0.05}^{x *} x \Phi \frac{(x-\mu)}{\sqrt{\nu})}\right.}{\left(s \int_{a(x)-0.05}^{a(x+0.05} x \Phi \frac{(x-\mu)}{\sqrt{v}}\right)}$.

For simplicity's sake, let us assume that GPAs that round to the same one-decimal figure $a$ are uniformly distributed, although we will continue to assume that the one-decimal figures themselves are normally distributed. As a result, $E_{0}(a)=a-0.025$, as it is the average of $a(x)$, the highest possible GPA that an applicant would round to $a$, and $a-0.05$, the lowest such GPA. We find that $x_{a} *=\frac{E_{0}(a(x))(1-s)(1 / 2)+s\left(x_{a} *-(a(x)-0.05)\right) / 0.1}{1 / 2(1-s)+s\left(x_{a} *-(a-0.05)\right) / 0.1}$. Solving for $x_{a} *$, we find that $x_{a} *=\frac{10 s+10 s a-1 / 2+\sqrt{\left((10 s a+10 s-1 / 2)^{2}-40 s(10.5 s a-a / 2-41 s / 80+1 / 80)\right)}}{20 s}$.

The model makes the following predictions. First, it forecasts that the
mean of the GPAs reported to two decimal places that round to a onedecimal figure $a$ is greater than $a$. Second, it suggests that there are few GPAs reported to two decimal places that are less than such an $a$. Finally, it predicts that for each one-decimal figure, there are two distinct values slightly below it at which the number of GPAs reported to two decimal places greatly increases as we move downward: at the higher of the two, the naifs switch from rounding from two decimal places to one, and at the lower, the sophisticates do the same. ${ }^{12}$

## 5 Results

I collected 4804 resumes from the careers website indeed.com; of these, 4634 were suitable to be analyzed. ${ }^{13}$ For each resume, I collected the first and last name of the applicant, the reported GPA, the highest level of education attained by the applicant, and the field of the applicant's college major.

### 5.1 Data

The following graph shows the frequency of the amount by which the GPAs with at least two decimal places differ from the one decimal figures that they round to. The subsequent graph shows the distribution of the number of decimal places in the reported GPAs. ${ }^{14}$

[^6]

Figure 1: Difference between GPAs of at least two decimals and the onedecimal figures the GPAs round to


Figure 2: Number of decimal places in the reported GPAs

Our first aim was to test whether GPAs reported to two or more decimal places are greater than the one-decimal figures they round to, and the data displayed in the above chart indicate that this is not the case: the mean difference between the GPAs containing at least two decimal places and the one-decimal figures that they round to is not positive, as we had predicted. If we had indeed found that these GPAs were greater than the one-decimal figures that they rounded to, this result would have been evidence of strategic rounding, as it would have suggested that people were more likely to
round their GPAs to a one-decimal figure if it were higher than their actual GPA. In fact, if anything, we find that the opposite of what we had predicted is more strongly supported by the data: more GPAs of at least two decimals round up to the nearest tenth rather than down. For no level of applicant education in our dataset (High School, College, Graduate, Other) is there a statistically significant difference between the GPAs with at least two decimal places and the one-decimal figures that they round to.

The following two charts show the distribution of GPAs; the first displays the data in bins of 0.1 , whereas the second displays it in bins of 0.01 .


Figure 3: GPA distribution in bins of 0.1


Figure 4: GPA distribution in bins of 0.01

Of the 3143 one-decimal GPAs, there was the following distribution of tenths places.


Figure 5: Distribution of tenths places among GPAs reported to one decimal place

We note the high proportion of GPAs ending in 5 and $0: 55.3 \%$ of the one-decimal GPAs ended in one of these figures.This result is particularly noteworthy because of the fact that of the two-decimal GPAs, only $20.5 \%$ rounded to a one-decimal figure that ended in 0 or 5 , statistically indistinguishable from the $20 \%$ figure that would be expected if each decimal were equally likely.

For the 1335 GPAs with exactly two decimal places, I found the following distribution of hundredths places. ${ }^{15}$

[^7]

Figure 6: Distribution of hundredths places among GPAs reported to more than one decimal place.

As with the tenths place in one-decimal GPAs, a high proportion of the hundredths place digits are 0 or 5.464 of the 1483 GPAs (31.3\%) end in 0 or 5 . Similarly to the tenths digits of the one-decimal GPAs, this result is particularly noteworthy because there is a strong difference between the proportion of hundredths digits of two-decimal GPAs that are 0 or 5 and the proportion of rounded hundredths digits in GPAs with at least three decimal places that are 0 or 5 .

The results lead us to the following conclusions. First, there is no statistically significant difference between the mean of two-decimal GPAs and the one-decimal figures they round to, as the model had predicted; as such, there is seemingly little evidence of sophisticated, or even naive strategic, applicants. Furthermore, the absolute number of students who report GPAs above the one-decimal figure that they round to is equal to the number who report GPAs below this figure. Finally, the proportions of both one- and multiple-digit GPAs that end in 5 or 0 are much higher than what would be expected by chance; this result was certainly not predicted by any of the models earlier in this paper, but seems worthy of further consideration.
likely to exhibit strategic behavior, given their educational attainment. However, this is not the case, as the below table demonstrates.

### 5.2 Discussion

It is noteworthy that the data suggests that few of the applicants seem to behave like sophisticates or strategic naifs. One potential explanation for this behavior is that the difference between, say, a 3.67 GPA and a 3.7 GPA might not feel important to many applicants. If true, this behavior would be consistent with the findings of Camerer et al. (2002), which hold that individuals behave closer to a Nash equilibrium strategy when incentives are strongest.

Another possible explanation is that some applicants might believe that they should always round down when rounding to one decimal place. As a result, someone with, say, a 3.67 GPA might be likely to report it, as she would prefer to report this figure than a 3.6. If this rounding behavior did occur in some people, we would expect to see more two-decimal GPAs that ended in high digits like 8 or 9 , and fewer such GPAs that ended in 1 or 2 .

The data suggest that this explanation may be valid. There are slightly more two-decimal GPAs that end in $6,7,8$, or 9 than end in $1,2,3$, or 4 . Although this result is not statistically significant under nearly any model, it may be weakened by a proportion of applicants rounding their GPAs strategically and in the conventional manner; such applicants would be more likely to report a second decimal of $1,2,3$, or 4 than one of $6,7,8$, or 9. Therefore, this explanation seems plausible, although the data cannot confirm that it truly has an effect on rounding strategies.

It is significant that the average GPA is approximately 3.53 , closer to an A- than to any other grade. The fact that it is so high suggests that there may in fact be adverse selection among job applicants who do not post GPAs on their resumes. If true, the fact that those who do report their GPAs tend to have higher GPAs is an important source of error, as it would lead to a sampling bias that could affect the proportions of naive and sophisticated strategic rounders in the dataset.

One piece of evidence that there is a sampling bias among individuals who report GPAs is that mean GPAs in the United States are lower than those in our dataset for each level of education for which there was available data. In our dataset, the average GPAs for high school ${ }^{16}$ and college ${ }^{17}$ students are both greater than available recent figures by a margin that is significant at $p=0.000001$. While it is possible that job seekers have higher GPAs than the general population, one might assume that people with higher GPAs are more likely to already be employed; however, there is little direct evidence. ${ }^{18}$ As such, it appear that the GPAs on Indeed.com are not truly reflective of students' GPAs nationwide, as those with higher GPAs are disproportionately likely to report theirs.

Another possible source of error is that the resumes on indeed.com do not come from applicants from similar backgrounds all applying to the same sort of jobs. Applicants differ in many respects, including the quality of the

[^8]schools they attended, the highest level of education they attained, and the competitiveness of the jobs they wish to obtain. An applicant, for example, who believes that her resume is particularly strong beside her GPA may be more likely to round to one decimal place as opposed to posting a twodecimal GPA that is slightly below the one-decimal figure, believing that her strength in other areas of her resume might suggest to employers that her GPA is relatively high given the one-decimal figure it rounds to.

Furthermore, although I only included GPAs greater than 1.0 and less than or equal to 4.0, it is possible that some of these GPAs are calculated on a scale other than 4.0. For example, some schools set A+ as the highest possible grade and weight it as 4.33 GPA points. ${ }^{19}$ As such, students who are graded on this scale could easily receive GPAs less than 4.0 and still show up in the data set. If so, these applicants could skew the results.

### 5.2.1 Multiples of . 5 and . 05

Of potential interest is the fact that multiples of so-called "round numbers" are overrepresented in the data. For example, of the 4626 reported GPAs that fit the criteria, an incredible 2202 ( $47.6 \%$ ) round up to multiples of .5 , far above the $20 \%$ that would be expected by chance. Looking only at GPAs rounded to two or more decimal places, we see a similar result: out of 1483 GPAs, $464(31.8 \%)$ round to a multiple of 0.05 when rounded to the nearest hundredths place. While a possible explanation would be that both of these results are due to GPAs rounding to friendly numbers being more common, this does not seem to be the case: other GPAs that would seem

[^9]to be particularly common due to being close to common fractions, such as 3.33 , are not unusually common, and if the 4.0 GPAs are removed from both the overall and two-decimal groups, the results are still significant at the $\mathrm{p}=0.000001$ level assuming a uniform distribution. Thus, it is possible that job applicants are being strategic in frequently reporting GPAs that barely reach these round numbers.

Why would job applicants be incentivized to report GPAs that just barely reach milestone numbers? One possible reason for the frequency of GPAs ending in 0 and 5 is that applicants may believe that employers may exhibit round number bias: that there is a significant difference between how employers perceive a GPA that has barely reached a round number and how they perceive a nearly identical GPA that has barely missed this threshold. ${ }^{20}$ Regarding this bias, Lacetera et al. (2012) did research as to whether used car customers were influenced by milestone numbers, such as a used car reaching 10,000 miles. They found that customers were influenced in this way; specifically, they found that there were significant decreases in sale prices for cars that had reached a multiple of 10,000 miles on the odometer. There was also a lesser effect for cars that reached a multiple of 1,000 miles.

If true, excluding the GPAs ending in 0 and 5 , we might expect fewer GPAs that barely missed a milestone GPA (i.e. 2.9) than those that were not barely miss one (i.e. 2.1). This selection bias could occur if students exhibited round number bias, and did not have a good conception of the

[^10]true distribution of GPAs. For example, a student with a GPA that barely missed a round number, such as 2.9, might believe that this GPA was around average, while the student with the 2.1 GPA might also believe that this GPA was about average. However, the student with the 2.1 GPA might overweight it, as it barely reached a round number, whereas the student with the 2.9 GPA might underweight it. As such, the student with the 2.1 GPA might be more likely to report it than the student with the 2.9 GPA would.

To determine the validity of this explanation, I first collected data as to whether job applicants were frequently withholding their GPAs from their resumes. I compiled 1910 resumes from the careers website Recruiting.com using a generic search term. Of the resumes, 1835 ( $96 \%$ ) withheld GPAs; only $75(4 \%)$ showed them. Thus, it is clear that many are willing to withhold their GPAs from online career websites.

I then looked at whether there were more GPAs ending in 1 than in 9 to determine if this effect of favoring GPAs that barely reach a milestone holds. There are far more GPAs reported ending in 9 for both one- and two-decimal GPAs; as a result, the data strongly contradicts this explanation. ${ }^{21}$

However, even if this effect were real, it is not obvious why someone would view a GPA ending in .5 as a round number. If true, a possible explanation is that many jobs have GPA cutoffs than are typically themselves multiples of milestone numbers. For example, many companies require that

[^11]applicants have GPAs of at least 3.5 in order to apply. ${ }^{222324}$ An applicant whose resume is otherwise fairly unimpressive might worry about believability if they reported a GPA that is too high. As such, many applicants might simply report GPAs that barely reached the thresholds.

It is also possible that applicants are not being dishonest, but simply do not remember their GPAs. Such individuals would conceivably be more likely to approximate their GPAs with a round number, as they might believe that it is more honest to report a heavily rounded GPA than a more precise one that may or may not be accurate. If this were true, it would be reasonable to anticipate a positive correlation between years since graduation and probability of reporting a GPA that is a multiple of 0.5 or 0.05 , as older applicants, with more years since receiving their GPAs, would probably be more likely to forget their GPAs. To test this hypothesis, I collected additional data from the career website recruiting.com. In particular, I compiled data on how many years had elapsed since an applicant had graduated from the institution at which (s)he had received the reported GPA, and searched for whether there was a positive correlation between years since graduation and probability of reporting a GPA ending in 0 or 5 . The data show that there is no such correlation, suggesting that forgetting is not a major explanation for the frequency of GPAs ending in these digits.

Finally, it is possible that some applicants are actually rounding their GPAs to the nearest multiple of 0.5 or 0.05 . The lines may blur between

[^12]dishonesty and rounding strategies, as anecdotal evidence seems to indicate that such a strategy does occur, but is frowned upon. ${ }^{25}$ Nonetheless, if this were true, and if applicants were rounding their GPAs strategically, one might expect there to be fewer GPAs that are slightly above a multiple of 0.5 (i.e. 3.6) or slightly above a multiple of 0.05 (i.e. 3.36) than slightly below these milestones. Therefore, it is ambiguous as to whether these rounding strategies are "small cheats", or are simply the result of an unorthodox rounding format done in good faith.

The data are inconclusive in this regard. Of the 482 GPAs from indeed.com that were reported to at least two decimal places and for which the rounded hundredths place is $1,4,6$, or 9 , for $231(48 \%)$ this digit is 1 or 6 (slightly above a multiple of 0.05 ), and for $251(52 \%)$, it is 4 or 9 . Assuming a uniform distribution of the hundredths place, this result is not statistically significant. Meanwhile, of the 625 one-decimal figures that end in $1,4,6$, or $9,275(44 \%)$ end in 1 or 6 , and $350(56 \%)$ end in 4 or 9 . Whether this result is statistically significant depends on our assumptions about the distribution of the tenths place of GPAs; however, it is clearly not uniform. Therefore, the data cannot show whether these rounding strategies occur.

### 5.2.2 Distribution of GPAs

The distribution of reported GPAs from our data does not closely match any well-known distribution. However, if we limit the data to only GPAs rounded to more than one decimal place, we find that it somewhat resembles

[^13]a normal distribution. ${ }^{26}$ A Q-Q plot of the two-decimal GPAs and a normal distribution with mean 3.83 and variance 0.25 and an accompanying regression reveal an $R^{2}$ value of 0.901 , suggesting that the two-decimal GPAs are distributed somewhat normally.

In fact, if we include the one-decimal GPAs as well and modify the data according to certain assumptions, we find even stronger results. Suppose that we assume that many students round their GPAs to the nearest multiple of .5 . We are assuming that the amount by which the number of GPAs reported as $y^{27}$ exceeds the average of $y-.1$ and $y+.1$ is the number of students who decided to round their GPA to $y$ because it was the nearest multiple of .5. If we distribute these GPAs evenly among the five onedecimal figures that round to $y$, we find that the new distribution very closely resembles normality, with $R^{2}=0.966$.

Nonetheless, we cannot conclude from this information alone that GPAs are normally distributed. One major reason is that it is not clear that the data is limited to GPAs measured on a 4.0 scale; some, for example, might be measured out of 4.3. Perhaps more significantly, the central limit theorem, which holds that the mean of a large group of i.i.d. random variables is approximately normally distributed, likely does not apply in this situation. While a GPA is the mean of a student's grades, it is unlikely that grades are independent of one another: a student who receives an A+ in sophomore year math would presumably be likely to do better than the average student in junior year math.
${ }^{26}$ One should note that the model does not predict a normal distribution.
${ }^{27} \mathrm{We}$ are rounding GPAs with multiple digits to the nearest multiple of .1.

### 5.3 Opportunities for Further Research

There is significant room for further study of strategic behavior in reporting GPAs. First, it would be useful to examine the explanations for why GPAs ending in multiples of .5 and .05 are so common. Furthermore, it might be beneficial to evaluate the strategies job applicants use in determining whether to report any GPA at all. The same sort of unraveling process that explains when students should round their GPAs to one decimal place or two could be used to show that students should always report a GPA when given a choice to, and perhaps some of the explanations for why students report one-decimal GPAs more frequently than they should in equilibrium could be used to show why they withhold their GPAs too often from this perspective.

Additionally, it could be of interest to determine whether employers are playing the best-response strategies given the behavior of applicants. Given that employers often read hundreds or even thousands of resumes a year, compared to job applicants who may only write one version of their resume, it seems possible that their strategies about responding to submitted GPAs might involve a higher degree of strategy than those of job applicants. If their evaluations of resumes do exhibit some understanding of the unraveling process explained in this paper, it could also be of interest to determine whether they consciously are thinking about the unraveling process.

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## 6 Appendix

### 6.1 Data from Indeed.com

### 6.1.1 Data Collection

I collected a sample of 4804 resumes with one- or two-decimal GPAs from the careers website indeed.com.Indeed.com is the most popular jobs website by traffic in the United States. On the website, job seekers can search for available jobs and connect directly with employers. Job seekers are prompted to post their resumes, although no particular format is required. Likewise, employers can use a variety of filters to search the applicant pool for potential employees. The accounts and resumes of all job seekers are publicly available to anyone with a valid employer account. I did no screening other than finding resumes using the search term "GPA". I included all the results that fulfilled my criteria of having a GPA measured on a 4.0 scale. When a user searches for resumes using a keyword, as I did, ties are broken by location. I recorded the name, reported GPA, number of decimal places, field, and level of education of the various applicants. I made no ex ante predictions about whether level of education or field affect GPA.

I skipped over all of the resumes that did not contain GPAs. In order to measure GPAs on a 4.0 scale, I only included GPAs greater than 1.0 and less than or equal to 4.0. While some reported GPAs less than 1.0 could theoretically be measured on a 4.0 scale, in all likelihood, these GPAs were measured on a 0-1 scale. As evidence, there were no GPAs greater than 1 and less than 2 , but there were 8 less than or equal to 1 , several of which
were reported as percentages. In addition, I did not include any of the 8 GPAs that were rounded to 0 digits (i.e., reported as 4 instead of 4.0). I did this to prevent GPAs which were multiples of 1 from being overrepresented in the data.

In examining the proportion of applicants that were strategic in rounding, I omitted two-decimal GPAs that ended in 5 , as it is not clear which one-decimal GPAs these figures would round to. For the same purpose, I also omitted GPAs above 3.95, as these GPAs would round up to 4.0 when rounded to one decimal place, whereas there are no GPAs that could round down to 4.0 , potentially causing misleading results regarding the proportion of GPAs that were above the one-decimal GPA they rounded to.

### 6.1.2 Overall Data

The below table displays the frequency with which GPAs of at least two decimal places differed from the one-decimal figures that they round to.

| Difference | -0.04 | -0.03 | -0.02 | -0.01 | 0 | 0.01 | 0.02 | 0.03 | 0.04 | $+/-0.05$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 136 | 147 | 126 | 107 | 172 | 93 | 138 | 130 | 142 | 292 |

The below table shows the distribution of the number of decimal places in reported GPAs.

| Decimal Places | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Observations | 8 | 3143 | 1335 | 140 | 8 |

Below is a summary of the values of the GPAs.

GPA Summary Statistics

| Mean | 3.53215382 |
| :--- | :--- |
| Median | 3.5595 |
| St. Dev. | 0.379157441 |
| Variance | 0.143760365 |
| Observations | 4634 |

Below is the distribution of the tenths place of one-decimal GPAs.

| Tenths Place | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 1105 | 64 | 160 | 120 | 189 | 633 | 211 | 205 | 295 | 161 |

The below table shows the hundredths place of all GPAs with exactly two decimal places.

| Hundredths Place | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 161 | 80 | 125 | 102 | 125 | 283 | 126 | 132 | 109 | 92 |

The below table shows the rounded hundredths place of all GPAs with more than two decimal places.

| Rounded Hundredths Place | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 10 | 14 | 14 | 26 | 17 | 10 | 11 | 17 | 12 | 17 |

The below table shows the rounded hundredths place of all GPAs with at least two decimal places.

| Rounded Hundredths Place | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 171 | 94 | 139 | 128 | 142 | 293 | 137 | 149 | 121 | 109 |

### 6.1.3 Results of Regressions



Figure 7: $Q-Q$ plot of $G P A s$ with at least two decimal places and a normal distribution with mean 3.83 and variance 0.25


Figure 8: $Q-Q$ plot of all GPAs with number of observations adjusted for rounding strategies and a normal distribution with mean 3.83 and variance 0.25

### 6.1.4 High School GPAs

## GPA Values

Mean 3.427

Median
3.5

St. Dev.
0.414

St. Dev. of Mean 0.014
Variance
0.171

Observations 904
Nationwide Average 3.00

## Number of Decimal Places

Mean 1.271

Median
1
St. Dev.
Variance
0.539
0.291

Observations 904


Figure 9: GPA distribution in bins of 0.1


Figure 10: GPA distribution in bins of 0.01

### 6.1.5 College GPAs

| GPA Values |  |
| :--- | :--- |
| Mean | 3.511 |
| Median | 3.5 |
| St. Dev. | 0.359 |
| St. Dev. of Mean | 0.008 |
| Variance | 0.129 |
| Observations | 2081 |
| Nationwide Average | 3.00 |
| Number of Decimal Places |  |
| Mean | 1.374 |
| Median | 1 |
| St. Dev. | 0.548 |
| Variance | 0.301 |
| Observations | 2081 |



Figure 11: GPA distribution in bins of 0.1


Figure 12: GPA distribution in bins of 0.01

### 6.1.6 Graduate GPAs

## GPA Values

Mean
3.673

Median 3.75
St. Dev. 0.329
Variance 0.129
Observations 461
Number of Decimal Places
Mean
1.374

Median
1
St. Dev.
0.548332137

Variance
0.300668133

Observations
461


Figure 13: GPA distribution in bins of 0.1


Figure 14: GPA distribution in bins of 0.01

### 6.1.7 GPAs for Other Levels of Education

## GPA Values

Mean 3.595
Median 3.67
St. Dev. 0.376
Variance 0.142
Observations 1188
Number of Decimal Places
Mean 1.334
Median 1
St. Dev. 0.529
Variance 0.280
Observations 1188


Figure 15: GPA distribution in bins of 0.1


Figure 16: GPA distribution in bins of 0.01

### 6.2 Data from Recruiting.com

### 6.2.1 Graduation Years and GPA Rounding

I collected data from the careers website Recruiting.com in order to determine whether there was a positive correlation between year of graduation and likelihood of posting a GPA that ended in 0 or $5 .{ }^{28}$ I collected 343 results using the search term "gpa 'graduated in""; this search term was

[^14]chosen due to its presumed likelihood of providing resumes with both GPAs and graduation years. Of the 343 results, 18 were rejected from this analysis for lacking one or both of these required items, and another 17 were rejected for containing a GPA not measured on a 4.0 scale.

The data show that job applicants for whom more time has passed since graduation are no more likely to report a GPA ending in 0 or 5 than the applicant pool in general. There is actually a slight negative correlation between time since graduation and likelihood of reporting a GPA ending in 0 to 5 ; nonetheless, the tiny $R^{2}$ value of 0.006 demonstrates that it is not significant.


Figure 17: Scatterplot of Graduation Year and Proportion of GPAs Ending in 0 and 5

### 6.2.2 Willingness to Reveal GPAs

In order to better determine whether there existed a selection bias among the applicants who chose to report their GPAs, I collected data on the proportion of applicants that reported any GPA at all. I used the keyword "the" to search for resumes, as I felt it was a neutral keyword with regard
to whether applicants would report a GPA. Out of 1910 resumes, 75 (3.9\%) reported a GPA; the other 1835 (96.1\%) did not.

### 6.3 Semi-Sophisticates

Suppose that some applicants exhibit $k$ levels of unraveling (assuming that strategic naifs exhibit 1 level of unraveling); we will call such applicants "semi-sophisticates". Let us assume that applicants can either round their GPAs to one decimal place, or report them accurately, and that GPAs are uniformly distributed between 0 and 4 . In an applicant pool consisting of only semi-sophisticates, applicants will round their GPAs to a one-decimal figure $a$ only if their GPAs are below $a-0.05+0.05 / 2^{k}$. In an applicant pool consisting of both semi-sophisticates and naifs, although the rounding behavior of semi-sophisticates cannot be expressed neatly by an integral for large $k$, it is almost identical to that of sophisticates in an applicant pool containing naifs and sophisticates for $k \geq 1$.

In an applicant pool containing both sophisticates and semi-sophisticates, the semi-sophisticates will still round their GPAs to a one-decimal figure $a$ only if their GPAs are below $a-0.05+0.05 / 2^{k}$. As a result, for any one-decimal figure $a$, the GPA value $x_{a} *$ below which sophisticates will round their GPAs to $a$ is the one that solves the equation $x_{a} *=$ $\frac{E_{0}(a(x))(1-s)(1 / 2)+s\left(x_{a} *-(a(x)-0.05)\right) / 0.1}{1 / 2(1-s)+s\left(x_{a} *-a(x)+0.05\right) / 0.1}$, where $E_{0}=a+0.05 * 2^{k}$. We find that $x_{a} *=\frac{10 s+10 s a-1 / 2 s+\sqrt{(10 s+10 s a-1 / 2)^{2}-40 s\left(-1 / 2 a+10 s a-s / 2-\frac{s}{5\left(2^{k+3}\right)}+\frac{1}{5\left(2^{k+3)}\right.}\right.}}{20 s}$.

### 6.4 Generalized Proof of Sophisticates Model

Theorem 6.1. Suppose that there is a pool of sophisticated job applicants who each know the GPAs of everyone else and who intend to maximize their GPAs in the eyes of their potential employers, whom they believe to be playing the empirical best response to the data. All applicants, given the choice between rounding their GPAs to one decimal place and reporting it accurately, will report them accurately in equilibrium, with the exception of the applicants who have the very lowest GPA that rounds to the one-decimal figure that it does; these applicants are indifferent.

Proof. Let us suppose that in a pool comprised only of sophisticated applicants, there exists a job applicant $a p p_{j}$ whose GPA $j$ rounds to a one-decimal figure $a$ and that there is another applicant whose GPA $k$ is such that $k$ rounds to $a$ and $j>k$. Assume that the expectation of $j$ in equilibrium is higher if rounded to $a$ than if reported accurately. One can infer that $j$ is not the highest GPA that rounds to $a$. If it were, and there were exactly $n$ applicants with GPA $j$, then the expected value of a GPA $j$ rounded to $a$ would be $\frac{n j+k}{n+1}$, which would be less than $j$. As such, $a p p_{j}$ would prefer to report $j$ faithfully than to round it to $a$, which would be a contradiction.

Therefore, there must be at least one other applicant with a GPA higher than $j$ but that rounds to $a$, and who rounds his GPA to $a$. Of these applicants, let us call the one with the highest such GPA $a p p_{q}$; his GPA we will call $q$. Since the expectation of $a$ is calculated as the mean of all the GPAs that round to $a$, there is at least one GPA lower than $q$ that rounds to $a$, and there are no GPAs higher than $q$ that round to $a, q$ must be greater than the
expectation of a GPA rounded to $a$. However, we had already established that $a p p_{q}$ is rounding $q$ to $a$; therefore, he is not sophisticated. This contradicts our assumption that all the applicants in the pool are sophisticated. Thus, all applicants will report their GPAs accurately in equilibrium, with the exception of the applicants who have the very lowest GPA that rounds to the one-decimal figure that it does.

Looking at the applicants who have the lowest GPA in the applicant pool that rounds to a one-decimal figure $a$, since we know that these applicants are the only ones who will round their GPAs to $a$, the expectation of a GPA rounded to $a$ from the perspective of a rational employer playing the empirical best response to the data is the same as their GPA. Thus, they are indifferent between reporting their GPA faithfully and rounding it to $a$.

### 6.5 Proof of Sophisticates Model from this Paper

Theorem 6.2. Suppose that there is an arbitrarily large pool of job applicants comprised only of sophisticates who each know the GPAs of everyone else and who intend to maximize their GPAs in the eyes of their potential employers, whom they believe to be playing the empirical best response to the data. Assuming that applicants are given the choice of rounding their GPAs to one or two decimal places, and assuming GPAs are normally distributed, all applicants will round their GPAs to two decimal places in equilibrium, except those applicants whose GPAs round to the lowest possible two-decimal figure given the one-decimal figure their GPAs round to; excluding boundary cases, these applicants will always round their GPAs to one decimal place.

Proof. Suppose that there is at least one applicant who prefers to round her GPAs to a one-decimal figure $a$ instead of the two-decimal figure $b$ that her GPA rounds to, for which $b$ is not the lowest two-decimal figure that could round to $a$. From the group of applicants who round their GPAs to $a$, let us choose one who has the highest GPA, $j$, among the people in this group. Since there is at least one applicant who could round her GPA to a two-decimal GPA that is not the lowest possible one $c$ given that the GPA rounds to $a$, we know that $j$ rounds to a two-decimal figure $d$ that is greater than $c$. The GPAs that round to $a$ but not to $d$ must be lower than any GPAs that round to $d$, as $d$ is the greatest two-decimal GPA that a GPA rounding to $a$ could round to.

As such, the expectation of a GPA rounded to $d$ is greater than that of one rounded to $a$. However, since we have established that there is at least one applicant who has rounded her GPA to $a$ despite having the choice to round it to $d$, we contradict our assumption that all the applicants are sophisticated. Thus, all applicants who could round their GPAs to a twodecimal GPA that is not the lowest possible one given the possible onedecimal GPA will round it to two decimal places.

We now claim that of the applicants who have a choice between rounding their GPAs to the lowest possible two-decimal GPA given the one-decimal figure their GPAs round to, all will round to one decimal place in equilibrium, excluding boundary cases. ${ }^{29}$ As proof, we have already established that the applicants who could round their GPAs to a two-decimal GPA that is not

[^15]the lowest possible one given the one-decimal GPA that the GPA rounds to will round to two decimal places. As such, the applicants who have GPAs that could either round to a one-decimal figure $p$ or the highest possible two-decimal figure $q$ that their GPAs could round to conditional on also being able to round to $p$ will round them to $q$.

However, $q$ is also the lowest possible two-decimal GPA that a GPA able to round to the one-decimal figure $a+0.1$ could be rounded to. Although in equilibrium, the expectation of a GPA reported as $a+0.1$ is the mean of the GPAs that round to both $q$ and $a+0.1$, the mean of the GPAs that are reported as $q$ includes the lower GPAs that could be rounded to $a$. Therefore, the expectation of a GPA reported as $a+0.1$ is higher than that of one reported as $q$. Thus, applicants whose GPAs round to the lowest possible two-decimal figure given the one-decimal figure their GPAs round to will, in equilibrium, always round their GPAs to one decimal place. ${ }^{30}$

### 6.6 Model of Alternate Rounding Convention

Suppose that job applicants have a choice to either round their GPAs to the nearest multiple of .5 or report them to the nearest multiple of .1. If all applicants are sophisticates and are assuming that employers are playing the empirical best response to the data, then in equilibrium, all the applicants, except those whose GPAs round to the least multiple of .1 that rounds to the multiple of .5 that their GPA rounds to, as with the earlier sophisticates model, nearly all applicants will report their GPAs to the greatest level of precision allowed, in this case rounding to two decimal places. Only

[^16]those applicants whose have the lowest possible one-decimal GPA given the multiple of .5 they round to might round their GPA to a multiple of .5. We assume that applicants whose GPAs round to one-decimal GPAs that are multiples of 0.5 are able to signify this. ${ }^{31}$

However, unlike in the previous model in Section 3, the sophisticates in this model with the lowest possible precise GPA ${ }^{32}$ given the less precise GPA they could round to ${ }^{33}$ do not strictly prefer the less precise figure to the more precise one; instead, they are indifferent.

To see this, we observe that the same unraveling process occurs as in the aforementioned model, leading nearly all applicants to reveal their GPAs to the greatest level of precision possible under the model. However, in this model, a GPA that rounds to the lowest possible one-decimal figure $s-0.2$ given the multiple of $.5 s$ that it rounds to can only round to $s$, and not $s-0.5 .{ }^{34}$ As a result, the expectation of a GPA reported as $s-0.2$ is the same as that of one reported as $s$.

### 6.7 Literature Relating to Deviations from Equilibrium Information Disclosure Strategies

Jin, Luca, and Martin (2015) demonstrate that laboratory subjects are less likely to disclose negative information than they ought to under a Nash equilibrium strategy. Receivers of information are less likely to interpret a

[^17]lack of reported information as indicative of negative information than would be expected in equilibrium. According to the paper, only when receivers of information are given data about the relation between a sender's quality and his willingness to disclose information about his quality do receivers punish to a greater degree senders who do not disclose information. Meanwhile, Rabin (1996) discusses systematic ways in which participants in information games deviate from the equilibrium.

However, it should be noted that there is also a significant amount of literature detailing when people do behave in more rational ways when interpreting a certain willingness to disclose data. As one example, John, Barasz, and Norton (2016) find some evidence of people forming negative impressions of those who do not disclose information, as would be expected in equilibrium.

There has been significant research done regarding how and when people deviate from the equilibrium strategies in iterated games such as these. Camerer (2009) demonstrates that movies that are not released to critics before opening generate about $15 \%$ more revenue than those that are, when controlling for quality and other factors. Such a finding is inconsistent with rational behavior, as unraveling occurs: if a producer is strategic (and given the millions of dollars on the line, he presumably is), he should certainly reveal the best-quality movies to reviewers, who are able to distinguish these from lower-quality films. Since only moderate-to-low quality films remain inaccessible to reviewers, the producers of moderate-quality films will allow those to be reviewed, leaving only the low-quality movies unreviewed. However, moviegoers' attendance patterns suggest that many did not view
the producers' unwillingness to reveal the quality of their films (by allowing critics to review them beforehand) as an indicator of the movies' presumably low quality.

In addition, Camerer (2003) provides an excellent summary of several findings about such deviations. In particular, he describes Nagel's (1995) findings that repeating iterated games leads to a higher proportion of players following the equilibrium strategy, presumably due to learning. Additionally, he cites the findings of Camerer et al. (2002) that when the stakes were higher, laboratory participants behaved more closely to the Nash equilibrium strategy.

On a related note, Luca and Smith (2014) describe how business schools not only strategically choose whether to disclose information, but are strategic about which information to disclose. Specifically, other than the top institutions, schools that perform well in the prestigious U.S. News rankings are more likely to reveal their rankings than schools that do not. ${ }^{35}$ However, the institutions that do not reveal their U.S. News rankings generally do reveal other rankings, often decreasing the specificity of their rankings or providing vague descriptions of the methodology in order to appear as prestigious as possible.

[^18]
[^0]:    ${ }^{1}$ http://www.forbes.com/sites/susanadams/2013/12/06/do-employers-really-care-about-your-college-grades/\#fbea9f61b690
    ${ }^{2}$ http://econ.duke.edu/dfe/resources/faqs\#Q2\%20Rounding\%20GPA
    ${ }^{3}$ http://www.admissions.iastate.edu/apply/gpa_calc.php

[^1]:    ${ }^{4}$ The best response of an applicant is the one that maximizes her payoff (in this case GPA) given the interpretation strategies of employers.

[^2]:    ${ }^{5}$ In addition to signaling, one could argue that screening also occurs in GPA rounding. An example of screening with GPA rounding is that some employers require GPAs rounded to two decimals; graduate schools require transcripts, where cumulative GPAs are often rounded to three decimals (and the list of all grades received in individual classes is also available).

[^3]:    ${ }^{6}$ For the purposes of this model, a perfectly informed employer is one who knows the exact distribution of GPAa and strategy types.

[^4]:    ${ }^{7}$ I added true naifs to the model after seeing the data.
    ${ }^{8}$ It should be noted that some applicants exhibit $k$ levels of iteration; a model of their behavior is in the appendix.
    ${ }^{9}$ One could theoretically argue that the unequal weightings of classes could make this function continuous if there are infinitely many possible such weightings.

[^5]:    ${ }^{10}$ This paper does not claim that GPAs are normally distributed, and in fact there is little evidence to suggest that this is the case, but a model needed to be selected.
    ${ }^{11}$ For boundary cases 0 and 4 , this is not the case; for example, the expected value of a 4.0 reported by a true naif is 3.975 , as it is impossible to have a GPA above 4.0 under the model.

[^6]:    ${ }^{12}$ These predictions were all made before seeing the data.
    ${ }^{13}$ See Appendix.
    ${ }^{14}$ For the purposes of this analysis, we are not considering GPAs that differ from the one-decimal figure that they round to by $+/-0.05$, with the reasoning being that it is unclear whether they round up or down.

[^7]:    ${ }^{15}$ For GPAs with at least two decimal places, I rounded to the nearest hundredth.

[^8]:    ${ }^{16}$ http://nces.ed.gov/nationsreportcard/pdf/studies/2011462.pdf
    ${ }^{17}$ http://www.usatoday.com/story/news/nation/2013/11/21/college-grade-inflation-what-does-an-mean/3662003/
    ${ }^{18}$ While there is readily available data suggesting that job applicants are more likely to get jobs if they have higher GPAs, and as such those who are still applying for jobs presumably have lower GPAs than people who already have them, there is little data on how job applicants compare on GPA to those who have left the job market altogether.

[^9]:    ${ }^{19} \mathrm{http}: / /$ www.ryerson.ca/currentstudents/essr/gradescales_ugrad/

[^10]:    ${ }^{20}$ An employer I spoke to confirmed that she often paid little attention the digits after the first one in a GPA.

[^11]:    ${ }^{21}$ One could argue that it is more logical to compare the frequency of GPAs ending in 1 to those ending in 8, rather than to those ending in 9: while 9 is the highest possible digit, 1 is not the lowest. The significant margin by which there are more GPAs ending in 8 than 1 also contradicts this theory of round number bias in rounding GPAs.

[^12]:    ${ }^{22}$ http://macaulay.cuny.edu/community/now/2015/09/summer-2016-summer-internship-program-finance-paid-apollo-global-management-expires-1120/
    ${ }^{23}$ https://www.facebook.com/seisenberg/posts/10151875980010116
    ${ }^{24}$ http://www.nytimes.com/2006/12/31/jobs/31gpa.html?pagewanted=all\&_r=0

[^13]:    ${ }^{25} \mathrm{https}: / /$ www.quora.com/Is-it-acceptable-to-round-your-GPA-on-your-resume

[^14]:    ${ }^{28}$ For GPAs with more than two decimal places, I treated the rounded hundredths place as the final decimal.

[^15]:    ${ }^{29} \mathrm{~A}$ student with a choice of rounding her GPA to 0.0 or 0.00 will be indifferent, although would presumably be unlikely to report any GPA unless required.

[^16]:    ${ }^{30}$ Consequently, the expectation of a GPA reported as $q$ further decreases.

[^17]:    ${ }^{31}$ For example, an applicant with a 3.5 GPA is able to signify that she actually received this GPA instead of, say, a 3.3 that she rounded up.
    ${ }^{32}$ Here, a multiple of .1 ; in the previous model, a multiple of . 01
    ${ }^{33}$ Here, a multiple of .5 ; in the previous model, a multiple of .1
    ${ }^{34}$ For example, the lowest possible one-decimal figure that rounds to $3.5,3.3$, cannot round to 3.0.

[^18]:    ${ }^{35}$ Luca and Smith note, however, that the most highly ranked business schools generally abstain from revealing their rankings. They do so because their perceived quality is already high enough that their choice to not reveal any any ranking, even a presumably stellar one, could suggest to prospective applicants familiar with their strong reputations a degree of confidence in the other available information about their quality.

