Majorana Liquids: The Complete Fractionalization of the Electron

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Majorana liquids: the complete fractionalization of the electron

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We describe ground states of correlated electron systems in which the electron fractionalizes into separate quasiparticles which carry its spin and its charge, and into real Majorana fermions which carry its Fermi statistics. Such parent states provide a unified theory of previously studied fractionalized states: their descendants include insulating and conducting states with neutral spin $S = 1/2$ fermionic spinons, and states with spinless fermionic charge carriers. We illustrate these ideas on the honeycomb lattice, with field theories of such states and their phase transitions.

The study of two-dimensional quantum antiferromagnets has proved to be fertile ground for finding many-electron states whose excitations do not carry all the quantum numbers of the electron [1]. This phenomenon is often referred to as "spin-charge" separation. It leads to some of the most non-trivial examples of quantum entanglement at long scales, and is crucial for the understanding of a variety of correlated electron materials, and for designing topological quantum computers.

Upon fractionalizing the electron into its spin and charge, one is faced with the decision of locating its Fermi statistics. In early theories of gapped spin liquid states of insulating, frustrated, antiferromagnets, distinct physical motivations led to two main pictures:
(*i*) A picture of projecting out doubly-occupied and vacant sites from a free electron Slater determinant produced an attachment of Fermi statistics to spin, leading to [2] spin liquids with neutral, spin $S = 1/2$ excitations ('spinons') which are fermions.
(*ii*) A picture of quantum-'disordering' magnetically ordered states produced an attachment of Fermi statistics to charge, leading to [3] spin liquids with bosonic spinons. This dichotomy persists when we consider insulating spin liquids with gapless excitations, and also to metallic or superconducting fractionalized states. Thus in the first picture we have 'algebraic spin liquids' (ASL) with Dirac points or Fermi surfaces of neutral fermionic spinons [4–8]; while in the second picture we have gapless bosonic spinons and/or gapless, spinless fermionic charge carriers [9–11], including 'algebraic charge liquids' (ACL).

This paper will provide a unification of these seemingly divergent pictures. We will argue that the states above descend from a common parent 'Majorana liquid', in which there is complete fractionalization of the electron into components carrying its spin, charge, and Fermi statistics. No choice is made in the parent liquid regarding attachment of Fermi statistics, which is instead carried by real Majorana fermions which carry neither spin nor charge. While we will introduce our approach generally, a specific realization of the theory will be provided on the honeycomb lattice. We will also relate our results to a recent numerical study of the Hubbard model on the honeycomb lattice [12].

We motivate our approach by starting from the second picture [11]. Here, we imagine there is some preferred local magnetic order, and transform the electron (c_σ, $\sigma = \uparrow, \downarrow$) into a rotating reference frame determined by the orientation of the local magnetic order [13]:

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \quad ; \quad \mathcal{R}_z \equiv \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}. \quad (1)$$

Here the spinful bosons z_σ define a SU(2) rotation matrix $\mathcal{R}_z$ determined by the magnetic order, and f_1, f_2 are spinless fermions which carry the charge of the electron. It is also useful to note the various global and gauge symmetries associated with Eq. (1). The global spin rotation SU(2)$_{\text{spin}}$ acts as a left multiplication on $\mathcal{R}_z$, while the global charge U(1)$_{\text{charge}}$ is carried by $f_{1,2}$. In addition there is a local SU(2)$_{s,g}$ gauge invariance: this acts as a right multiplication of $\mathcal{R}_z$ and the doublet (f_1, f_2) transforms as its fundamental.

Now let us turn to the first picture [4, 6]. We can view this as a transformation into a rotating reference frame in the pseudo-spin space of particle-hole transformations of the electron [14]. For the Hubbard model on bi-partite lattices at half-filling, there is in fact a global SU(2)$_{\text{pseudospin}}$ which contains U(1)$_{\text{charge}}$ as a subgroup; however our approach only assumes U(1)$_{\text{charge}}$ symmetry in general. Paralleling Eq. (1) we now have [4, 6]:

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow^\dagger \end{pmatrix} = \mathcal{R}_b \begin{pmatrix} f_1 \\ f_2^\dagger \end{pmatrix} \quad ; \quad \mathcal{R}_b \equiv \begin{pmatrix} b_1^* & b_2^* \\ -b_2 & b_1 \end{pmatrix}. \quad (2)$$

Independent of the existence of a global SU(2)$_{\text{pseudospin}}$ symmetry, this parameterization introduces a local SU(2)$_{c,g}$ gauge invariance: this acts as a right multiplication of $\mathcal{R}_b$ and the doublet $(f_1, f_2^\dagger)$ transforms as its fundamental. This SU(2)$_{c,g}$ gauge invariance played a crucial role in the classification of various spin liquid states with fermionic spinons [15, 16].

There is a natural and simple unification of Eqs. (1) and (2). We write the complex fermions in terms of 2 sets of real fermions ζ_a and χ_a, with $a = 1 \ldots 4$, by $c_\uparrow = \zeta_1 + i\zeta_2$ and $c_\downarrow = \zeta_3 + i\zeta_4$ and similarly between $f_{1,2}$ and χ_a. Then we have

$$\zeta = \mathcal{R}\chi \quad (3)$$

where $\mathcal{R}$ is a real SO(4) matrix. This shows that a combination of Eqs. (1) and (2) enjoys a SO(4)$_g$ gauge in-

variance; it will become evident below that $\text{SO}(4)_g \sim \text{SU}(2)_{s,g} \otimes \text{SU}(2)_{c,g}$. As before, the $\text{SO}(4)_g$ gauge invariance acts as a right multiplication of $\mathcal{R}$, χ is a fundamental of $\text{SO}(4)_g$, and global symmetries act as left multiplication of $\mathcal{R}$. For an explicit form of the global symmetries, let us represent the 4×4 real matrices as tensor products of 2 sets of 2×2 Pauli matrices: s^α acting on the $\uparrow, \downarrow$ space ($\alpha = x, y, z$), and ρ^α acting on the $\text{Re}[c], \text{Im}[c]$ space. Then the global $\text{SU}(2)_{\text{spin}}$ symmetry is generated by

$$S^x = s^x \rho^y \quad , \quad S^y = s^y \quad , \quad S^z = s^z \rho^y. \qquad (4)$$

The global pseudo-spin transformations are generated by

$$T^x = s^y \rho^z \quad , \quad T^y = s^y \rho^x \quad , \quad T^z = \rho^y; \qquad (5)$$

here T^z generates $\text{U}(1)_{\text{charge}}$, while $T^{x,y}$ generate the remaining pseudo-spin transformations which need not be a symmetry of the Hamiltonian. The matrices in Eqs. (4) and (5) are the 6 generators of $\text{SO}(4)$; note also that the S^α commute with the T^α, realizing the factorization into $\text{SU}(2) \otimes \text{SU}(2)$. To complete our formulation, we can express $\mathcal{R}$ in terms of the complex bosons in $\mathcal{R}_z$ and $\mathcal{R}_b$. Writing $z_\uparrow = \phi_0^s - i\phi_3^s$, $z_\downarrow = -\phi_2^s - i\phi_1^s$, and $b_1 = \phi_0^c + i\phi_3^c$, $b_2 = -i\phi_2^c + \phi_1^c$ where ϕ_a^c and ϕ_a^s are real scalars, we have

$$\mathcal{R} = Z_s \otimes Z_c$$
$$Z_s = \phi_0^s + i\phi_1^s S^x + i\phi_2^s S^y + i\phi_3^s S^z$$
$$Z_c = \phi_0^c + i\phi_1^c T^x + i\phi_2^c T^y + i\phi_3^c T^z. \qquad (6)$$

Eqs. (3) and (6) contain our general statement of electron fractionalization: the electron ζ decomposes into the bosonic fields Z_s and Z_c which carry its global $\text{SU}(2)_{\text{spin}}$ and $\text{U}(1)_{\text{charge}}$ quantum numbers respectively, and into the Majorana fermion χ carrying the Fermi statistics. The resulting theory has a $\text{SO}(4)_g = \text{SU}(2)_{s,g} \otimes \text{SU}(2)_{c,g}$ gauge invariance: Z_s and χ carry $\text{SU}(2)_{s,g}$ charges, and Z_c and χ carry $\text{SU}(2)_{c,g}$ charges.

Different patterns of breaking the $\text{SO}(4)_g$ gauge invariance and global symmetries lead to a plethora of possible phases, and we present a broad classification:
(I) Phases with conventional excitations are obtained when both Z_c and Z_s are condensed. In such phases, we can always choose a $\text{SO}(4)_g$ gauge in which $Z_c = Z_s = 1$, and then it becomes clear from Eq. (3) that the fermion χ has just the same quantum numbers as the electron ζ. By condensing various fermion bilinears (just as in conventional Hartree-Fock/BCS theory), we can obtain Fermi liquids, semi-metals, antiferromagnets, valence bond solids (VBS), superconductors, or quantum spin Hall states [17, 18].
(II) When Z_s is condensed, we can use the $\text{SU}(2)_{s,g}$ gauge invariance to set $Z_s = 1$. Then Eq. (3) reduces to Eq. (2), and we therefore reproduce the phases of Refs. [4, 15] on the square lattice, and those of Ref. [6] on the honeycomb lattice, including the ASLs. In these phases, the fixing of

the $\text{SU}(2)_{s,g}$ gauge transfers the global $\text{SU}(2)_{\text{spin}}$ quantum numbers from Z_s to χ, while the $\text{U}(1)_{\text{charge}}$ quantum numbers remain on Z_c. Thus these phases have neutral fermionic spinons, and bosonic charge carriers.
(III) A complementary situation is realized when Z_c is condensed. Now we can set $Z_c = 1$, Eq. (3) reduces to Eq. (1), global $\text{U}(1)_{\text{charge}}$ quantum numbers are transferred from Z_c to χ, and so we obtain neutral bosonic spinons and fermionic charge carriers. We reproduce phases of Refs. [10, 11] on the square lattice, and Ref. [19] on the honeycomb lattice, including the ACLs.
(IV) When neither Z_c and Z_s are condensed, we can obtain phases in which the Z_c, Z_s, and χ are separate elementary excitations, carrying the charge, spin, and Fermi statistics of the electron respectively. These are the Majorana liquids of this paper. These elementary excitations all carry gauge quantum numbers, and so the stability of such phases requires that gauge forces not be confining: we will describe specific examples of deconfinement mechanisms below.

The remainder of the paper applies our general theory to the half-filled, extended Hubbard model on the honeycomb lattice. For weak interactions, we have a conventional semi-metal with electronic excitations at 2 Dirac points in the Brillouin zone: this is as realized in graphene. For strong interactions, there is convincing numerical evidence [12] for an insulator with collinear, two-sublattice antiferromagnetism (Néel order). The simplest possibility is that there is a direct transition between the these two category I phases [20]. However, recent numerical studies [12] indicate there may be an intermediate phase.

We begin our analysis by describing the free electron spectrum in the semi-metal phase, associated with the Hamiltonian

$$H_0 = -t \sum_{\langle ij \rangle,} c_{i\sigma}^\dagger c_{j\sigma} \qquad (7)$$

where i are sites on the honeycomb lattice, and $\langle ij \rangle$ refers to nearest neighbors. We take the low energy limit of H_0 in the standard manner, obtaining two valleys of two-component Dirac fermions. Explicitly, we expand the electron at two Dirac valleys by $d_{1,2} = e^{i\vec{Q}_{1,2}\cdot\vec{r}}c$ (where $\vec{Q}_{1,2} = \pm(\frac{4\pi}{3}, 0)$ are the wavevectors of the valleys), and introduce Pauli matrices τ^α and μ^α which act on the sublattice and valley spaces respectively. Then, after introducing real Majorana fermions ζ_a as the real and imaginary parts of $e^{i\frac{\pi}{4}\tau^x}e^{i\frac{\pi}{4}\mu^x}(d_1, i\tau^y d_2)^t$, we obtain the continuum Lagrangian

$$\mathcal{L}_0 = \sum_{a=1}^{8} \bar{\zeta}_a \gamma_\mu \partial_\mu \zeta_a. \qquad (8)$$

Here μ is a 2+1 dimensional spacetime index, and the Dirac γ matrices are $(\gamma_0, \gamma_1, \gamma_2) = \tau^y, \tau^z, \tau^x$. In contrast

to the single-site Majorana fermion in Eq. (3), here the fermion field ζ has additional components associated with the sublattice and valley spaces. The sublattice index is equivalent to the spacetime Dirac index and has been left implicit, and from now on the 'flavor' index $a = 1\ldots8$ accounts for the spin, pseudospin, and valley indices; thus for each a, ζ_a is now a 2-component Majorana spinor. We can now decompose ζ as in Eq. (3), with χ_a having the same spinor structure as ζ_a, while $\mathcal{R}$ is as in Eq. (6).

We obtain our parent *algebraic Majorana liquid* (AML) when interactions beyond those in H_0 leave both Z_s and Z_c un-condensed and realized as gapped quanta which carry spin and charge respectively. The AML is in our category IV above, and at energies below the spin and charge gaps, it has gapless, relativistic, Majorana fermions ζ_a coupled to emergent SU(2) gauge fields $A^\alpha_{s,\mu}$ and $A^\alpha_{c,\mu}$ associated with the $\mathrm{SU}(2)_{s,g} \otimes \mathrm{SU}(2)_{c,g}$ gauge invariance:

$$\mathcal{L}_{\mathrm{AML}} = \bar{\chi}\gamma_\mu \left(\partial_\mu - iA^\alpha_{s,\mu}S^\alpha - iA^\alpha_{c,\mu}T^\alpha\right)\chi \qquad (9)$$

The stability of the AML requires that the gapless χ fermions suppress monopoles, and so that the $\mathrm{SU}(2)_{s,g} \otimes \mathrm{SU}(2)_{c,g}$ gauge forces are not confining [5, 21]. Such monopole suppression happens for a sufficiently large number of gapless fermion flavors, and it is not known if the 8 real fermion flavors here are sufficient. Assuming deconfinement, the AML has a gap to all spin and charge excitations, and has gapless Majorana fermions which carry only energy.

Whether or not the AML is stable, we can use it to describe a very large number of descendant phases. The rest of the paper will note some interesting or physically relevant examples.

First, we can expect that the $\mathrm{SU}(2)_{s,g} \otimes \mathrm{SU}(2)_{c,g}$ gauge forces lead to the analog of chiral symmetry breaking, and the simplest possibility is the appearance of a O(8) invariant $\bar{\chi}\chi$ condensate. Such a condensate leads to a fermion mass gap, and breaks time-reversal symmetry, leading to a *chiral Majorana liquid*, also in category IV. The fermions generate Chern-Simons terms for the gauge fields, and this leads to deconfinement [22] for the gapped Z_c, Z_s, and χ excitations. There is non-zero spin chirality $\vec{S}_1 \cdot (\vec{S}_2 \times \vec{S}_3) + \cdots$, and also nonzero electrical currents on the lattice. However, the physical current $\bar{\zeta}\zeta$ is suppressed from $\bar{\chi}\chi$:

$$\langle\bar{\zeta}\zeta\rangle \sim \langle\bar{\chi}\chi\rangle\langle Z^\dagger_{s,i}Z^\dagger_{c,i}Z_{s,j}Z_{c,j}\rangle_{\ll i,j \gg}, \qquad (10)$$

i and j are two next nearest neighbor sites, therefore the expectation value $\langle Z^\dagger_{s,i}Z^\dagger_{c,i}Z_{s,j}Z_{c,j}\rangle$ is expected to be small when Z_s and Z_c are both gapped.

There are also a large number of possible Higgs phases. One interesting example is the Higgs condensate of the vector H^α:

$$H^\alpha = \bar{\chi}\,\rho^y S^\alpha\,\chi. \qquad (11)$$

If H^α had involved bilinears of the original electron ζ, its condensate would break spin rotation invariance and lead to a quantum spin Hall phase [18]. In the present situation, the H^α in Eq. (11) does not carry any global quantum numbers. and its Higgs condensate does not break any global symmetries. The resulting phase is in fact a Z_2 *Majorana liquid*, and is in category IV, as we now argue. The H^α condensate breaks the $\mathrm{SU}(2)_{s,g} \otimes \mathrm{SU}(2)_{c,g}$ gauge invariance to $\mathrm{U}(1)_{s,g} \otimes \mathrm{U}(1)_{c,g}$; if we choose $H^\alpha \propto (0,0,1)$, then the U(1)'s are generated by S^z and T^z. Thus the low energy theory of this phase is

$$\mathcal{L}_{Z_2} = \bar{\chi}\gamma_\mu \left(\partial_\mu - iA^z_{s,\mu}S^z - iA^z_{c,\mu}T^z\right)\chi + m\,\bar{\chi}\rho^y S^z\chi, \qquad (12)$$

where the fermion mass term is induced by the H^α condensate. We can further integrate out the massive χ fermions, and then using the analog of the arguments in Ref. [18], we find that the physics is controlled by a mutual Chern-Simons term:

$$\mathcal{L}_{cs} = \frac{2i}{2\pi}\epsilon_{\mu\nu\rho}A^z_{c,\mu}\partial_\nu A^z_{s,\rho}. \qquad (13)$$

As discussed elsewhere [24], with such a term, the gauge forces are quenched, and the matter fields carry only Z_2 gauge charges. The Z_2 gauge field endows mutual statistics between excitations with S^z and T^z charges, for instance between the spin- and charge-carrying bosons, Z_s and Z_c, as in Ref. [23].

The last two category IV phases above, the chiral and Z_2 Majorana liquids, are attractive candidates for the intermediate state in Ref. [12] between the semi-metal and the Néel insulator. They have gapped Z_c, Z_s, and χ excitations, and we have demonstrated that the charge, spin, and Fermi statistics they carry remain deconfined. These phases treat the charge and spin excitations at an equal footing (unlike the proposal in Ref. [19]), and so they are appropriate for the vicinity of the metal-insulator transition. The chiral Majorana liquid has weak spontaneous spin chirality and electrical currents, which have not been detected so far. The Z_2 Majorana liquid has no broken symmetry, and so remains compatible with all existing computations.

Let us now turn to category II. As noted earlier, such phases have $\langle Z_s \rangle \neq 0$, and on the honeycomb lattice our theory reduces to that of Hermele [6]. He found an $\mathrm{SU}(2)_{c,g}$ ASL insulator: at low energies, this is described by the theory of the AML in Eq. (9), but with the $\mathrm{SU}(2)_{s,g}$ gauge fields $A^\alpha_{s,\mu} = 0$ because of the Z_s condensate. As with the AML, this is stable only for sufficiently large number of fermion flavors. The χ fermions now carry spin (explained in (II)), and so the spin excitations are gapless. The intermediate phase of Ref. [12] has a spin gap in addition to the charge gap, and this is not compatible with an ASL.

Finally, we turn to category III, where we have $\langle Z_c \rangle \neq 0$. These were discussed in Ref. [11] for the square lattice,
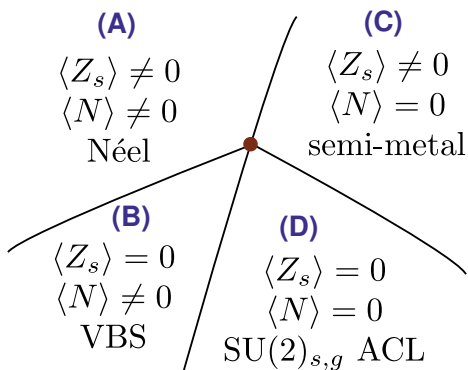
FIG. 1: Some phases of the half-filled honeycomb lattice. All phases above have the condensate $\langle Z_c \rangle \neq 0$, and so can be described by Eq. (1). Phase D is category III and is described by Eq. (9) but with $A^\alpha_{c,\mu} = 0$: it has a spin gap, and gapless, spinless critical fermions at the Dirac points carrying the electronic charge. Phases A and C are conventional and so in category I: C has gapless electron-like excitations at the 2 Dirac points, which are gapped in A. Phase B is described by Eq. (15), which undergoes monopole-induced confinement to VBS order.

and we generalize the discussion here to the honeycomb lattice. As in category II above, we begin with the AML in Eq. (9), and now derive a SU(2)$_{s,g}$ ACL obtained by setting the SU(2)$_{c,g}$ gauge fields $A^\alpha_{c,\mu} = 0$. This phase has a spin gap, but the fermions χ now form gapless excitations which carry charge (as explained in (III)), again incompatible with the phase of Ref. [12]. As for the AML, the ACL can lead to a 'chiral charge liquid' with a $\bar{\chi}\chi$ condensate.

Other phases which descend from the SU(2)$_{s,g}$ ACL are shown in Fig. 1. The simplest of these is the semi-metal phase C: this is the category I phase with both Z_c and Z_s condensates, as noted earlier. For the remaining phases in Fig. 1 we have to consider a Higgs condensate of the field

$$N^\alpha = \bar{\chi}\, \mu^y S^\alpha\, \chi. \tag{14}$$

This choice is motivated by the fact that if we replace χ by the electron ζ, then Eq. (14) is the conventional Néel order; this replacement is permitted when both Z_c and Z_s are condensed. This is the case in phase A, which is then a category I phase with Néel order. The transition between phases C and A involves the order parameter and the gapless Dirac electrons, but no gauge fields; it is in the class of field theories studied in Ref. [20].

Now we consider the remaining phase B in Fig. 1. By a gauge transformation, we orient the N^α condensate along $(0,0,1)$; such a condensate breaks the SU(2)$_{s,g}$ gauge invariance down to U(1)$_{s,g}$, leaving only the $A^z_{s,\mu}$ gauge field active. The neutral Z_s spinons are gapped, and so the low energy theory of phase B (following Eq. (12)) is

$$\mathcal{L}_{\rm B} = \bar{\chi}\gamma_\mu \left(\partial_\mu - iA^z_{s,\mu}S^z\right)\chi + m\,\bar{\chi}\mu^y S^z\chi. \tag{15}$$

As after Eq. (12), we can integrate out the massive fermions, but now find only a Maxwell term for the $A^z_{s,\mu}$ gauge field. Thus the low energy theory of phase B has a gapless, relativistic U(1)$_{s,g}$ photon. In the absence of gapless matter, it is known that the monopoles in such a gauge field condense, and lead to long-range order determined by the quantum numbers of the monopole [25]. We will describe the computation of monopole quantum numbers elsewhere, showing that a kekulé type valence bond solid (VBS) order develops. The same conclusion is reached by approaching phase B from phase A [25]. Thus, while phase B started out in category III, it ultimately becomes category I VBS state.

This paper has unified two previously divergent approaches to the study of the electron fractionalization: those with fermionic [2, 4–8, 15, 16] versus bosonic [9–11] spinons. We have applied the theory to the honeycomb lattice and predicted new phases of possible relevance to recent numerical results [12].

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