Environment-Assisted Precision Measurement

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Environment-Assisted Precision Measurement

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We describe a method to enhance the sensitivity of precision measurements that takes advantage of the environment of a quantum sensor to amplify the response of the sensor to weak external perturbations. An individual qubit is used to sense the dynamics of surrounding ancillary qubits, which are in turn affected by the external field to be measured. The resulting sensitivity enhancement is determined by the number of ancillas that are coupled strongly to the sensor qubit; it does not depend on the exact values of the coupling strengths and is resilient to many forms of decoherence. The method achieves nearly Heisenberg-limited precision measurement, using a novel class of entangled states. We discuss specific applications to improve clock sensitivity using trapped ions and magnetic sensing based on electronic spins in diamond.

Precision measurement is among the most important applications of resonance methods in physics. For example, quantum control of atomic systems forms the physical basis of the world's best clocks. Ideas from quantum information science have been used to demonstrate that entanglement can enhance these measurements [1–3]. At the same time a wide range of quantum systems have been recently developed aimed at novel realizations of solid-state qubits. Such systems can be used as quantum measurement devices, such as magnetic sensors [4]. In this Letter we describe a novel technique that employs the local environment of the sensor as a resource to amplify its response to weak perturbations. We will use solid-state sensors and ion clocks as examples.

The purpose of quantum metrology is to detect a small external field coupled to the sensor by a Hamiltonian: $H_b^S = b(t)\kappa S_z$, where S_z is the spin operator of the quantum sensor. Here, $b(t)$ can be an external magnetic field or the detuning of a laser from a clock transition, while κ is the spin's coupling to the field. The working principle of almost any quantum metrology scheme can be reduced to a Ramsey experiment [5,6], where the field is measured via the induced phase difference between two states of the quantum sensor. The figure of merit for quantum sensitivity is the smallest field $\delta b_{\min}$ that can be read out during a total time T. For a single spin $1/2$ the sensitivity is given by $\delta b_{\min} \simeq \frac{1}{\kappa\sqrt{T\tau}}$, if the sensing time is limited to τ (e.g., by environmental decoherence).

In many cases the external field also acts on the sensor's environment, which normally only induces decoherence and limits the sensitivity. Here we show that in some cases the environment can instead be used to enhance the sensitivity. For generality we will illustrate the key ideas using the *central spin model* [Fig. 1(a)]. A central spin (which can be prepared in a well defined initial state, coherently

manipulated and read out) is coupled to a bath of *dark* spins that can be polarized and collectively controlled, but cannot be directly detected. The system is described by the Hamiltonian $H = H_b + H_{\text{int}}$, with

$$H_b = b(t)\left(\kappa S_z + \xi \sum I_z^i\right), \qquad H_{\text{int}} = |1\rangle\langle 1| \sum \lambda_i I_z^i, \quad (1)$$

where λ_i are the couplings between sensor and environment spins, while κ and ξ are couplings to the external field of the central and dark spins, respectively. Here, $|0\rangle$, $|1\rangle$ and S_z refer to the central spin while $|\uparrow\rangle$, $|\downarrow\rangle$, I_z^i to the dark spins (and we set $\hbar = 1$). We consider two cases. In the first one, H_{int} can be turned on and off at will and is much larger than any other interaction (e.g., a laser-mediated ion interaction). In the second case, H_{int} is intrinsic to the system and of the same order of magnitude as the inverse of the relevant sensing time (e.g., dipole-dipole interactions between solid-state spins). In all cases we will assume collective control over the dark spins.

To illustrate the sensing method we consider first the idealized case where the couplings between the central and the dark spins can be turned on and off at will and the dark

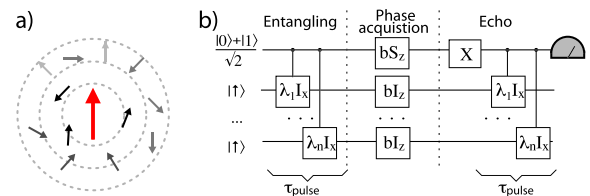FIG. 1 (color online).   (a) A central spin coupled to a spin bath, with couplings varying with distance. (b) Ideal measurement circuit. Gates labeled $\lambda_i I_x$ represent controlled rotations $e^{-i\lambda_i I_x^i}$ of the dark spins, obtained via H_{int}. The gates bI_z represent rotations e^{-ibI_z} due to the external field b. At the end, only the state of the central spin is measured.

spins are initialized in a pure state $|\Uparrow \ldots \uparrow\rangle$. Consider the circuit in Fig. 1(b). First, the central spin is prepared in an equal superposition of the two internal states $\sim |0\rangle + |1\rangle$. Then H_{int} is rotated to the x axis and applied for a time τ. This induces controlled rotations of the dark spins, resulting in an entangled state

$$(|0\rangle| \uparrow \ldots \uparrow\rangle + |1\rangle|\varphi_1 \ldots \varphi_N\rangle)/\sqrt{2}, \qquad (2)$$

where $|\varphi_i\rangle \equiv \cos(\varphi_i)| \uparrow\rangle - i \sin(\varphi_i)| \downarrow\rangle$, with $\varphi_i = \lambda_i \tau$. This state is then used to sense the magnetic field. For now we neglect the coupling of the central spin to the external field ($\kappa = 0$) and define $\theta_d = \xi \int_0^\tau dt b(t)$.

Under the action of the magnetic field, up to a global phase, the state evolves to $\frac{1}{\sqrt{2}}(|0\rangle| \uparrow \ldots \uparrow\rangle + |1\rangle|\psi_1 \ldots \psi_N\rangle)$, with

$$
\begin{aligned}
|\psi_i\rangle &= \cos\varphi_i| \uparrow\rangle_i - i \sin\varphi_i e^{i\theta_d/2}| \downarrow\rangle_i \\
&= [\cos^2(\varphi_i) + e^{i\theta_d}\sin^2(\varphi_i)]|\varphi_i\rangle \\
&\quad + \sin\left(\frac{\theta_d}{2}\right)\sin(2\varphi_i)|\varphi_i^\perp\rangle \approx e^{i\theta_d\sin^2\varphi_i}|\varphi_i\rangle \\
&\quad + \frac{1}{2}\theta_d \sin(2\varphi_i)|\varphi_i^\perp\rangle,
\end{aligned}
$$

where $\langle\varphi_i|\varphi_i^\perp\rangle = 0$ and the last equation holds for small fields (to first order in θ_d). The central spin is then flipped by a π pulse and another control operation with H_{int} along x is applied, yielding the final state $|\Psi_f\rangle = \frac{1}{\sqrt{2}}(|1\rangle|\varphi_1 \ldots \varphi_N\rangle + |0\rangle|\psi_1 \ldots \psi_N\rangle)$. If the field were zero ($\theta_d = 0$) the net result of the protocol would be to decouple the sensor spin from the environment—as in a spin-echo—since in this case $|\psi_i\rangle = |\varphi_i\rangle$. At the end of the sequence, the signal is read out by a single measurement of the S_y component of the central spin. For nonzero field the signal is given by $S = \langle\Psi_f|(S_y \otimes \mathbb{1})|\Psi_f\rangle = \text{Im}\{\prod_i\langle\varphi_i|\psi_i\rangle\} \approx \text{Im}\{\prod_i e^{i\theta_d\sin^2\varphi_i}\}$. The effect of a small field is to introduce a phase difference $\Phi \approx \theta_d\sum_i\sin^2(\varphi_i)$ between the states of the dark spins, depending on the central spin state. This phase yields a nonzero signal $S \approx \Phi + O(b^2)$, while terms $\propto |\varphi_i^\perp\rangle$ lead to a second order contribution to the signal. We can further add to the measured signal the phase acquired by the sensor spin alone, $\theta_s = \kappa \int_0^\tau dt b(t)$, (since H_b^S commutes with the rest of the Hamiltonian) to obtain $S \approx \Phi + \theta_s$.

While the signal is enhanced by a factor $\propto \Phi$, the quantum projection noise remains the same as we still read out one spin only. The minimum field that can be measured in a total time T is then:

$$\delta b_{\text{min}} = \sqrt{\frac{\tau}{T}}\frac{1}{\Phi + \theta_s} \approx \frac{1}{n\xi\sqrt{T\tau}}, \qquad (3)$$

where n is the total number of dark spins. The linear scaling in n of the phase Φ can be achieved in principle for any distribution of λ_i's, since we can always choose a duration τ such that $\langle\sin^2(\lambda_i\tau)\rangle \geq \frac{1}{2}$, leading to order one contribution from each spin [7]. Thus we are able to perform Heisenberg-limited spectroscopy despite the fact that the precise form of the entangled state (2) is uncontrolled, and may not even be known to us [8]. This

considerably relaxes the requirements for entanglement enhanced spectroscopy as compared to known strategies involving squeezed or GHZ-like states (for examples of nonsymmetric and generalized entanglement applications in metrology, see [9,10]). Specifically, we note that in contrast to prior work [10], our scheme does not involve creating, or projecting onto, the GHZ or NOON states at any point during the protocol.

We now present two experimental implementations that approximate this idealized scheme: quantum clocks with trapped ions and spin-based magnetometry.

To reach high precision in quantum clocks, the ions must posses several characteristics: a stable clock transition, a cooling cycling transition, good initialization and reliable state detection. It is then convenient to use two species of ions in the same Paul trap [11]. The *spectroscopy* ions (e.g., $^{27}\text{Al}^+$) provide the clock transition while the *logic* ion (e.g., $^9\text{Be}^+$) fulfills the other requirements. Although inspired by ideas similar to our proposal, experiments using two ion species have so far been limited to just one spectroscopy ion [11]; our method allows increasing the number of spectroscopy ions.

Specifically, we can implement the Hamiltonian H_{int} [Eq. (1)] using multichromatic gates [12], which are known to be much more robust to heating noise than Cirac-Zoller gates used so far [13]. Then, our scheme can map the phase difference due to the detuning of several spectroscopy (dark) ions onto a single logic ion, which can be read out by fluorescence. This achieves Heisenberg-limited sensing of the clock transition without individual addressability of the spectroscopy ions and without creating GHZ states. Importantly, we accomplish this even if the spectroscopy ions have different couplings to the logic ion, as it is the case in a trap with different ion species due to the absence of a common center of mass mode.

We briefly discuss the effects of decoherence. It has been argued [14] that the coherence time reduction for a n-spin entangled state, when each spin undergoes individual Markovian dephasing, reduces the sensitivity to roughly that of spectroscopy performed on n individual spins. The sensitivity scales then as $\sqrt{n}$, as opposed to the ideal scaling $\propto n$ derived here (see Ref. [15] for a different scaling in the case of atomic clocks). Since our method allows reading out n spectroscopy ions using only a single logic ion, even in this unfavorable case we would obtain an improvement $\propto \sqrt{n}$ with respect to current experimental realizations where only a single ancillary ion is possible [11]. Furthermore, this decoherence model is not so relevant in present setups, as technical noise during the gates and imperfect rotations are dominant for traps with many ions. Our method is highly robust to static repetitive imperfections during the gates, leading to further improvement depending on the exact noise model.

In many physical situations short bursts of controlled rotations, as used above, are not available. Instead, the couplings between the central and dark spins are always on and their exact strength is unknown. This situation is

encountered in solid-state spin systems used for magneto-
metry [4]. Even for these systems it is possible to achieve
nearly Heisenberg limited metrology.

Specifically, we consider magnetic sensing using a
single nitrogen vacancy (NV) center in diamond [16],
surrounded by *dark* spins associated with Nitrogen elec-
tronic impurities [17,18]. We focus on NV centers since
their electronic spins ($S = 1$) can be efficiently initialized
into the $S_z = 0$ state by optical pumping and measured via
state selective fluorescence. By applying a static magnetic
field that splits the degeneracy between $S_z = \pm 1$ states and
working on resonance with the $(0 \leftrightarrow 1)$ transition, the NV
center can be reduced to an effective two-level system [4].
Then, the system comprising one NV center and several
Nitrogen spins is well described by Hamiltonian (1).

In Fig. 2 we introduce a control sequence that yields an
effect equivalent to the circuit in Fig. 1(b). The pulse
sequence can be best understood using the well known
equivalence between Ramsey spectroscopy and Mach-
Zehnder interferometry [5,6], where the interferometer
arms describe the central spin state. It is sufficient to
consider the evolution of each arm separately, replacing
S_z by its eigenvalues $m_s = \{0, 1\}$ and describing the evo-
lution in the interaction frame defined by the control pulses
[19]. Hamiltonian (1) becomes time dependent, with dark
spins alternating between I_z^i and I_x^i as shown in Fig. 2.
Then, for different halves of the spin-echo sequence, the
coupling Hamiltonian in each arm is zero ($m_s = 0$) while
for the other halves it has identical forms. In the absence
of a magnetic field the evolution is thus the same along
each arm of the interferometer. Adding an external field
creates a phase shift between the two arms. For small field
strengths we can then evaluate the phase difference be-
tween the two arms as a perturbation. For a finite polariza-
tion P of the dark spins we find

$$\Phi = \overline{\theta_s}\left[1 + 2P\sum \frac{\overline{\theta_d}}{\overline{\theta_s}}\sin^2(\varphi_i/4)\right], \quad (4)$$

with $\overline{\theta_s} = \frac{\kappa}{\tau}[\int_0^{\tau/2} dt b(t) - \int_{\tau/2}^{\tau} dt b(t)]$, $\overline{\theta_d} = \frac{\xi}{\tau} \times$
$\int_{\tau/2}^{3\tau/4} dt b(t)$. Compared to spin-echo based magnetometry
[4], the signal is increased by the factor in the square
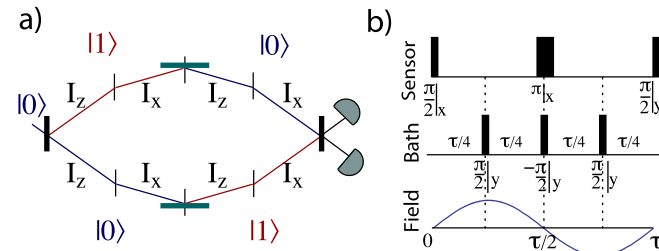bracket while the measurement noise is the same, as we

FIG. 2 (color online). Environment-assisted magnetometry
with NV centers. (a) Mach-Zehnder interferometer, showing
the central spin state and the orientation of the dark spins in
the toggling frame of the external pulses. (b) Pulse sequence
acting on the sensor and dark spins.

still read out one spin only. Note that all the dark spins
contribute positively. For values of the couplings such that
$|\lambda_i\tau| \gtrsim \pi$, or *strongly coupled* dark spins, we obtain a
contribution $\propto 2n_{\rm sc}\langle\sin^2(\frac{\lambda_i\tau}{4})\rangle \approx n_{\rm sc}$. Each of the $n_{\rm sc}$
strongly coupled spins thus gives a contribution of order
one, irrespective of the sign or exact value of the coupling.
Weakly coupled dark spins ($\lambda_i\tau \lesssim 1$) contribute instead
with a factor $(\lambda_i\tau)^2/8$ and we obtain a total phase $\Phi \approx$
$\overline{\theta_s}[1 + \frac{\overline{\theta_d}}{\overline{\theta_s}}P(n_{\rm sc} + \frac{1}{8}\sum'(\lambda_i\tau)^2)]$, where the primed sum is
on the weakly coupled spins only. In general the sensitivity
enhancement scales at least as $\sim Pn_{\rm sc}$ [20]. We thus
achieve nearly Heisenberg-limited sensing of the external
field [14].

We next compare the sensitivity achievable with the
proposed pulse sequence to that obtained with a spin-echo
sequence. We consider the same system (a sensor spin
surrounded by the same spin bath) and include the effects
of decoherence resulting from external perturbations, the
interaction with the dark spins as well as decoherence of the
dark spins themselves. Once the central spin looses phase
coherence due to interactions with the bath, it is no longer
possible to use it for magnetometry. This limits the sensing
time and consequently the magnetometer sensitivity. Spin
echo (as well as more sophisticated decoupling techniques
[19,21]) can be used to prolong the phase coherence of the
central spin. Under realistic assumptions the coherence
time for the pulse sequence presented here is on the same
order of the sensor coherence time under spin echo, T_2^s.
Indeed, the decoherence rate of an entangled state of the
form (2) is dictated by the internal evolution of the strongly
coupled spins, and the relevant decoherence time is thus the
time it takes before any of these spins have decohered. The
same is, however, the case for spin-echo sequences: if a
central spin is strongly coupled to $n_{\rm sc}$ dark spins, a spin flip
of a single dark spin will lead to different evolutions in the
two halves of the spin-echo sequence, thus decohering the
state of the central spin. As the source of both decoherence
processes is the dipole-dipole interaction among dark spins,
the coherence times of spin-echo and our procedure are on
the same order and the signal amplification obtained with
our strategy [Eq. (4)] yields a sensitivity enhancement [22].

To be more quantitative, we analyze the evolution due
to dipole-dipole couplings in the bath, described by the
Hamiltonian

$$H = |1\rangle\langle1|\sum \lambda_i I_z^i + \sum_{i<j}\kappa_{ij}(3I_z^i I_z^j - \vec{I}_i \cdot \vec{I}_j). \quad (5)$$

A short time expansion shows that the state fidelity
at the end of the spin-echo pulse sequence is given
by $1 - \frac{\tau^4}{8}\cdot\sum_{i<j}(1 - P^2)\kappa_{ij}^2(\lambda_i - \lambda_j)^2 + \dots$, and by
$1 - \frac{\tau^4}{128}\sum_{i<j}\kappa_{ij}^2 \times (7(\lambda_i^2 + \lambda_j^2) + 12\lambda_i\lambda_j + P^2(\lambda_i + 2\lambda_j)\times$
$(2\lambda_i + \lambda_j)) + \dots$, for the modified pulse sequences. For
experimentally relevant values of polarizations, similar
reductions in fidelity occur for both pulse sequences [23].
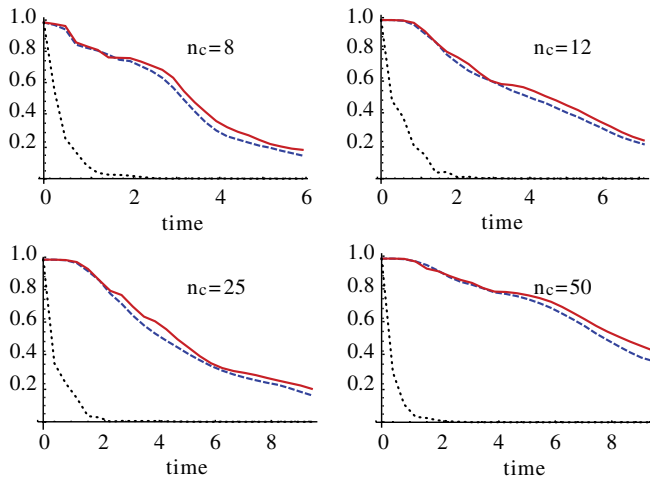To analyze longer time behavior we simulate the signal

FIG. 3 (color online). Simulations. Normalized signal decay for the proposed sequence (red, solid lines), spin-echo (blue, dashed lines) and no control (black, dotted lines). We used a leading-order cluster expansion [24] with perfect delta pulses and simulated WAHUHA sequences with n_c cycles per echo interval. 20 dark spins were randomly placed in a cube of side-length $\sqrt[3]{20}$ with the sensor spin at the center and we averaged over 10 dark spin distributions. We set $P = 0$ and $g\mu_B \equiv 1 \, [\text{m}]^{3/2} [\text{s}]^{-1/2}$.

decay for both spin-echo and the proposed pulse sequence (see Fig. 3) and compare these results to the signal decay when no decoupling is applied (the same environment then induces dephasing with decay time T_2^*). The dark spins are spin 1/2 paramagnetic impurities, undergoing a WAHUHA sequence (which refocuses the dipole-dipole coupling of the environment spins, but leaves the coupling to the external field [19]).

A different limitation on the sensing time τ is set by the fact that the orientation of the dark spin will not be static as assumed. Dipole-dipole couplings among dark spins during each $\tau/4$ period of free evolution rotate each spin away from the initial direction. This rotation means that the spins cease to build up a phase difference between the two arms of the interferometer for time scales comparable to the correlation time of the dark spin bath τ_c^d. The optimum sensing time is thus $T \sim \min\{T_2^s, \tau_c^d\}$. Since in most systems $\tau_c^d \gtrsim T_2^s$ [19], the optimum sensing time of this pulse sequence is comparable to that of spin-echo based magnetometry and the sensitivity enhancement is roughly the same as the signal strength enhancement.

In conclusion, we proposed a scheme to enhance precision measurement by exploiting the possibility to coherently control the ancillary qubits. In solid-state implementations we are able to exploit dark spins in the bath while preserving roughly the same coherence times as in spin-echo based magnetometry. Thus signal enhancement leads directly to sensitivity enhancement. For trapped ion implementations we can use imperfect phase gates and still achieve Heisenberg-limited sensitivity. Our method has the potential to be applied more generally, using different systems and more sophisticated pulse sequences. It opens the possibility to use a broad class of partially entangled states to achieve Heisenberg-limited metrology, even in the presence of disordered couplings, partial control, and decoherence.

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[1] S. L. Braunstein *et al.*, Ann. Phys. (Leipzig) **247**, 135 (1996).
[2] P. O. Schmidt *et al.*, Science **309**, 749 (2005).
[3] M. H. Schleier-Smith, I. D. Leroux, and V. Vuletic, Phys. Rev. Lett. **104**, 073604 (2010).
[4] J. M. Taylor *et al.*, Nature Phys. **4**, 810 (2008).
[5] D. J. Wineland *et al.*, Phys. Rev. A **50**, 67 (1994).
[6] B. Yurke S. L. McCall, and J. R. Klauder, Phys. Rev. A **33**, 4033 (1986).
[7] It is still necessary to calibrate the quantum sensor by measuring a known external magnetic field to obtain an estimate of the sum $\sum_i \sin^2(\lambda_i \tau)$.
[8] P. Cappellaro *et al.*, Phys. Rev. Lett. **94**, 020502 (2005).
[9] A. Kuzmich and T. A. B. Kennedy, Phys. Rev. Lett. **92**, 030407 (2004).
[10] F. W. Sun *et al.*, Europhys. Lett. **82**, 24001 (2008).
[11] T. Rosenband *et al.*, Phys. Rev. Lett. **98**, 220801 (2007); D. Leibfried *et al.*, Nature (London) **422**, 412 (2003); Science **304**, 1476 (2004).
[12] A. Sørensen and K. Mølmer, Phys. Rev. A **62**, 022311 (2000).
[13] J. I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995).
[14] S. F. Huelga *et al.*, Phys. Rev. Lett. **79**, 3865 (1997).
[15] A. Andre, A. S. Sørensen, and M. D. Lukin, Phys. Rev. Lett. **92**, 230801 (2004).
[16] R. J. Epstein *et al.*, Nature Phys. **1**, 94 (2005).
[17] R. Hanson *et al.*, Phys. Rev. Lett. **97**, 087601 (2006).
[18] T. Gaebel *et al.*, Nature Phys. **2**, 408 (2006).
[19] U. Haeberlen, *High Resolution NMR in Solids Selective Averaging* (Academic Press Inc., New York, 1976).
[20] The signal in Eq. (4) will depend in practice on the average polarization (as P could vary in different experimental runs). We expect the noise from these variations to be smaller than the quantum projection noise.
[21] P. Mansfield J. Phys. C **4**, 1444 (1971).
[22] Although our pulse sequence is more sensitive to dark spin dephasing (as it creates superposition states of the dark spins), this noise source is dominated by the noise due to single species dipole-dipole interactions [13].
[23] For $P \approx 1$ spin echo achieves longer coherence times since flip-flops are quenched in a fully polarized bath, while they would still be allowed with our method. However, the spin-echo coherence time scales poorly as $\sim \sqrt[4]{1 - P^2}$ and decoupling can be used to reduce the flip-flop effects.
[24] W. M. Witzel and S. Das Sarma, Phys. Rev. B **74**, 035322 (2006).