A Turbulent Model of Gamma-Ray Burst Variability

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A turbulent model of gamma-ray burst variability

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ABSTRACT

A popular paradigm to explain the rapid temporal variability observed in gamma-ray burst (GRB) light curves is the internal shock model. We propose an alternative model in which the radiating fluid in the GRB shell is relativistically turbulent with a typical eddy Lorentz factor γ_t. In this model, all pulses in the gamma-ray light curve are produced at roughly the same distance R from the centre of the explosion. The burst duration is $\sim R/c\Gamma^2$, where Γ is the bulk Lorentz factor of the expanding shell, and the duration of individual pulses in the light curve is $\sim R/c\Gamma^2\gamma_t^2$. The model naturally produces highly variable light curves with $\sim \gamma_t^2$ individual pulses. Even though the model assumes highly inhomogeneous conditions, nevertheless the efficiency for converting jet energy to radiation is high.

Key words: radiation mechanisms: non-thermal – gamma-rays: bursts.

1 INTRODUCTION

Our understanding of gamma-ray bursts (GRBs) has improved enormously in the last 15 years, thanks to observations by dedicated γ-ray/X-ray satellites such as *Compton-GRO*, *BeppoSAX*, *HETE-2* and *Swift*, and follow-up observations from the ground in optical and radio (for recent reviews see Mészáros 2002; Piran 2005; Fox & Mészáros 2006; Woosley & Bloom 2006; Zhang 2007). As a result of this work, it is now established that at least some long-duration bursts are produced in the collapse of a massive star (as suggested by Woosley 1993; Paczynski 1998), accompanied by the ejection of a highly relativistic jet. Afterglow observations as well as energy considerations indicate that the jet is well collimated (Rhoads 1999; Frail et al. 2001; Panaitescu & Kumar 2001). The presence of a relativistic jet has also been directly confirmed in the nearby burst GRB 030329 which exhibited 'superluminal' motion in its radio afterglow (Taylor et al. 2004).

GRB light curves are known to be highly variable (Meegan et al. 1992), and this has led to the development of the *internal shock model* (Piran, Shemi & Narayan 1993; Katz 1994; Rees & Meszaros 1994). According to this model, the Lorentz factor of the jet varies with time, and as a result, faster portions of the jet catch up with slower portions. In the ensuing collision, a fraction of the kinetic energy of the jet is converted to thermal energy. The observed radiation is then produced via synchrotron and inverse-Compton processes.

In a seminal paper, Sari & Piran (1997) provided a general argument why GRB light curve variability cannot be explained by simply appealing to a highly inhomogeneous source. They showed that the efficiency for converting jet energy to the observed radiation is extremely poor when variability arises purely from inhomogeneity; the essence of their argument is summarized in Section 2. The argument is both powerful and compelling, and it has led to wide acceptance of the internal shock model. A basic feature of the model is that different pulses within the light curve of a GRB are produced in distinct internal collisions/shocks, generally at different distances from the central explosion.

There are, however, a number of observations as well as theoretical considerations that pose difficulties for the internal shock model. Internal shocks have only a modest efficiency ~ 1–10 per cent for converting jet energy to the radiation observed in the 20 keV–1 MeV band (Kumar 1999; Lazzati, Ghisellini & Celotti 1999; Panaitescu, Spada & Mészáros 1999).[1] Even a ~ 10 per cent radiative efficiency is low compared to the burst efficiency implied by measurements of the jet kinetic energy through modelling GRB afterglow light curves (Panaitescu & Kumar 2002).

Another difficulty with the internal shock model is the large distance from the central explosion that one estimates ($R \gtrsim 10^{16}$ cm) for the γ-ray-producing region in a number of GRBs (Kumar et al. 2007). This distance is significantly larger than is expected in the internal shock model. Moreover, the estimated distance is within a factor of a few of the deceleration radius where the jet begins to interact with the external medium. This coincidence between two unrelated radii is unexpected.

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[1] Beloborodov (2000) and Kobayashi & Sari (2001) reported a much higher efficiency, ~ 100 per cent. However, their estimates were based purely on the kinematics of colliding shells, where shells of Lorentz factor $\gamma \sim 1$ collided with high γ shells. They did not take into consideration the efficiency of the emergent radiation in the commonly observed 20 keV–1 MeV band.

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These difficulties, along with the problem of avoiding excessive baryon loading, motivate us to consider an alternative to the internal shock model. We show in Section 3 that the argument of Sari & Piran (1997) against an inhomogeneous source can be successfully overcome if we consider source inhomogeneities that move randomly with *relativistic velocities*. In Section 4, we calculate a model light curve corresponding to such relativistic turbulence and demonstrate that it is consistent with observations of GRBs. We conclude in Section 5 with a discussion.

2 INTERNAL SHOCK MODEL

We consider an idealized model of a GRB in which a spherical shell is located at radius R with respect to the centre of the explosion and expands ultra-relativistically outward with a bulk Lorentz factor Γ. We define a second length-scale,

$$r \equiv R/\Gamma. \tag{1}$$

For simplicity, we ignore cosmological redshift. In the frame of an external observer, the time since the explosion is $\sim R/c$, while in the comoving frame of the expanding shell, the corresponding time is $\sim R/c$. The causal horizon around any point in the fluid is thus a sphere of radius $\sim r$. We assume causal contact in the radial direction. The radial width of the shell in the fluid frame is then $\sim r$, and the radial width in the observer frame is $\sim r/\Gamma = R/\Gamma^2$.

The radiation from any fluid element in the shell is expected to be isotropic in the fluid frame, but it is beamed within a cone of half-angle $\sim 1/\Gamma$ in the observer frame. Therefore, for a given distant observer, most of the received radiation comes from a circular patch of transverse radius $\sim R/\Gamma = r$ on the shell, that is from a single causal volume in the radiating fluid.

Consider the radiation emitted from the outer surface of the shell. As seen by the observer, the time delay between the radiation received from the centre of the visible patch and that from the edge of the patch is $\sim R/\Gamma^2 c$ (we ignore factors of order unity in this paper). Assuming the shell is homogeneous, this is the shortest time-scale over which the observed signal can vary. Since the radial width of the shell in the observer frame is $\sim R/\Gamma^2$, the smoothing time due to the finite radial width of the source is also of the same order. Thus, in this model of a GRB, which we refer to as the *standard model*, the variability time-scale is given by

$$\text{Standard model}: \qquad t_{\text{var}} \sim R/\Gamma^2 c. \tag{2}$$

It is natural to associate the time-scale t_{var} with the duration of individual pulses (spikes) in the γ-ray light curve. However, for a typical long GRB, the total burst duration t_{burst} is several tens of times, and sometimes even a couple of hundred times longer than t_{var}. To explain the longer time-scale t_{burst}, the standard model invokes a long-lived central engine with significant power output over a time $\sim t_{\text{burst}}$. Furthermore, the engine is postulated to be highly variable and to eject a large number of successive shells with different Lorentz factors. These shells collide with one another in internal shocks, each shock producing a pulse in the light curve of duration t_{var}.

Instead of having multiple shells and internal shocks, could the burst variability be explained via inhomogeneity in the radiating fluid? For instance, could t_{burst} be equal to $R/\Gamma^2 c$ and could the observed rapid variations in the light curve be the result of bursts of radiation from tiny active blobs within the radiating fluid? Sari & Piran (1997) gave the following simple and powerful argument against such a model.

Let us define the variability parameter $\mathcal{V} \equiv t_{\text{burst}}/t_{\text{var}}$; a typical value is $\mathcal{V} \sim 100$. For most bursts, $\mathcal{V}$ is roughly equal to the number of pulses in the light curve, i.e. the pulses fill the light curve with a duty cycle of order unity. If we wish to set t_{burst} equal to $R/\Gamma^2 c$, then a blob that produces any single pulse in the light curve must have a radial extent no larger than $\sim r/\mathcal{V}$. Assuming that the blobs are roughly spherical in shape (in the comoving frame of the fluid), this means that there must be $\sim \mathcal{V}^3$ independent blobs within a causal volume ($\sim r^3$) of the fluid. However, the number of pulses observed in the GRB light curve is no more than $\mathcal{V}$. Also, each pulse must be produced on average by only one blob since the intensity varies by order unity across a pulse. We thus conclude that, out of $\sim \mathcal{V}^3$ blobs, only $\mathcal{V}$ blobs radiate.[2] That is, only one out of every $\mathcal{V}^2 \sim 10^4$ blobs radiates, and ~ 99.99 per cent of the fluid is silent.

It is highly unlikely that the GRB energy is localized inside just $\sim 10^{-4}$ of the volume of the fluid in the shell. It is more likely that the energy from the explosion is spread uniformly over the entire shell. But if this is the case, then the prompt GRB emission must be highly inefficient, with only $\sim 10^{-4}$ of the available energy being radiated during the GRB. Such extreme inefficiency is unpalatable. For instance, after correcting for beaming, the energy release in gamma-rays in a typical long-duration GRB is found to be of order 10^{51} erg (Frail et al. 2001). With an inhomogeneous model in which the efficiency is only ~ 0.01 per cent, the true energy release would be $\sim 10^{55}$ erg, which is larger by a factor of $\sim 10^4$ than the kinetic energy of relativistic ejecta in GRBs as determined from multiwavelength modelling of their afterglow light curves (Wijers & Galama 1999; Panaitescu & Kumar 2002).

We are thus compelled to give up the idea of variability arising from inhomogeneity, and forced to accept the standard internal shock model. According to this model, the burst duration t_{burst} is equal to the lifetime of the central engine, variability is produced by a large number of random internal shocks among independent shells ejected from the engine, and the variability time t_{var} is given by equation (2).

3 TURBULENT MODEL

We now describe an alternative model – the *turbulent model* – in which we assume that the fluid in the GRB shell is relativistically turbulent. In the shell frame, let the typical Lorentz factor of an energy-bearing eddy be γ_t. As mentioned earlier, the lifetime of the system in the shell frame is $\sim r/c$. In the frame of an eddy, this corresponds to a lifetime $\sim r/\gamma_t c$. Therefore, by causality, we expect the maximum size of an eddy in its own frame to be $\sim r/\gamma_t$. Let us make the reasonable assumption that the energy-bearing eddies have roughly this size. Thus, the size of an eddy in its own frame is

$$r_e \sim r/\gamma_t \sim R/\Gamma\gamma_t. \tag{3}$$

Each eddy has a volume $\sim r_e^3$, so we expect the total number of eddies in a causal volume of the shell to be

$$n_e \sim (r/r_e)^3 \sim \gamma_t^3. \tag{4}$$

In the shell frame, an eddy has a size $\sim r_e$ in a plane perpendicular to its velocity vector, and a Lorentz-contracted size $\sim r_e/\gamma_t$ parallel to its motion.

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[2] Sari & Piran (1997) considered a somewhat different geometry where they took the radial width of blobs to be the same as the shell width, and thus concluded that $\mathcal{V}$ out of a total of $\mathcal{V}^2$ blobs radiate or that the radiative efficiency is $\sim \mathcal{V}^{-1}$.
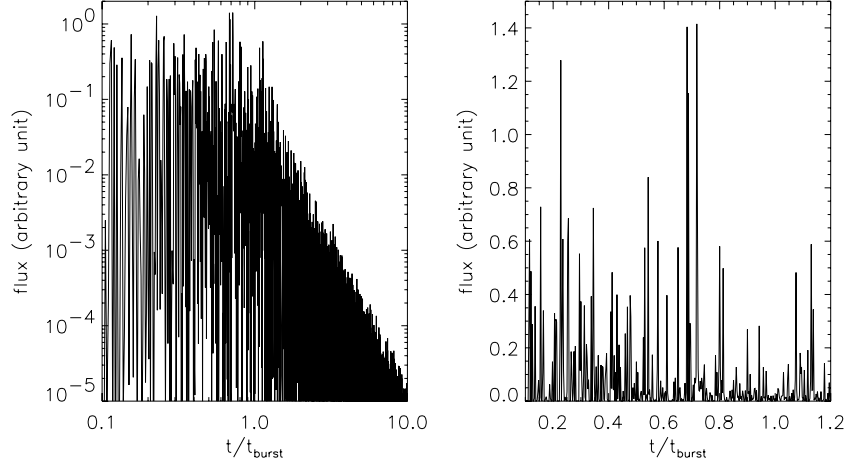
Figure 1. Typical GRB light curve, calculated using the relativistic turbulent model. The two panels correspond to logarithmic and linear scales, respectively. The bulk Lorentz factor of the expanding GRB shell is taken to be $\Gamma = 500$ and the turbulent Lorentz factor of the eddies to be $\gamma_t = 10$. Time is scaled by the burst duration as defined in equation (5). Note the high degree of variability during the main burst ($t \lesssim t_{burst}$), and the rapid decrease of the flux at late times due to off-axis emission.

Eddies are not likely to travel along perfectly straight lines. Rather, we expect their velocities to change on approximately the causal time, which is $\sim r/c$ in the shell frame. Alternatively, an eddy may move nearly ballistically for a distance $\sim r_e$ in the shell frame, which is the mean-free path for collisions. It might then collide with another eddy and continue ballistically for another distance $\sim r_e$, etc., for a total of $\sim \gamma_t$ independent ballistic trajectories. The results we describe below are valid in either picture.

Consider the radiation from an eddy as viewed in the shell frame. At any instant, the radiation is beamed into a cone of half-angle $1/\gamma_t$. During the life of the eddy, the orientation of the beam wanders by a few radians as a result of turbulent acceleration or collisions. Thus each eddy illuminates a total solid angle $\sim 1/\gamma_t$ in the shell frame in the course of its motion. Boosting to the observer frame, the illuminated solid angle from each eddy is $\sim 1/\Gamma^2 \gamma_t$. Summing over all n_e eddies in a causal volume, the total solid angle illuminated by all the eddies is $\sim \gamma_t^2/\Gamma^2$. All of this radiation is beamed within a solid angle $\sim 1/\Gamma^2$. Therefore, each observer receives radiation from $\sim \gamma_t^2$ eddies.

An observer receives radiation from the entire collection of eddies (inside one causal volume) over a time $\sim R/\Gamma^2 c$. In a major departure from the standard model, let us associate this time with the burst duration t_{burst}. The radiation received from a single eddy then corresponds to an individual pulse in the GRB light curve. To estimate the duration of a pulse, we note that the thickness of an eddy in a direction parallel to its beamed radiation is $\sim r/\gamma_t^2$ in the shell frame or $\sim R/\Gamma^2 \gamma_t^2$ in the observer frame. Thus, an observer receives radiation from a single eddy for a time[3] $\sim t_{burst}/\gamma_t^2$. As we showed above, an observer receives on average $\sim \gamma_t^2$ pulses. Thus, in the turbulent model, we have the following results:

$$t_{burst} \sim R/\Gamma^2 c, \tag{5}$$

Turbulent model :
$$t_{var} \sim R/\Gamma^2 \gamma_t^2 c, \tag{6}$$

$$n_{pulse} \sim \gamma_t^2, \tag{7}$$

where n_{pulse} is the mean number of pulses in the burst. Some of these scalings were obtained by Lyutikov (2006) in a different context.

4 SAMPLE LIGHT CURVE

Equations (5)–(7) show that $n_{pulse} t_{var} \sim t_{burst}$. This has two implications. First, it means that pulses typically fill the entire duration of the burst, i.e. the duty cycle of the pulses is of order unity, as observed in GRBs. Secondly, an observer receives radiation on average from only one eddy at any given time. Thus, we expect order unity variations in the observed γ-ray flux, again consistent with observations. These features are illustrated in the sample light curve shown in Fig. 1.

Fig. 1 was computed by considering a GRB shell expanding outward with a large Lorentz factor $\Gamma = 500$ (the precise value is unimportant). The shell is randomly filled with a population of eddies with turbulent Lorentz factor $\gamma_t = 10$. It is assumed that there are γ_t^3 eddies per volume r^3 in the frame of the shell and that there is a probability $1/\gamma_t$ that the relativistically boosted beam of any given eddy will sweep past the observer during the eddy lifetime. The spectral index of the radiation is taken to be $\beta = 1$ and the observed flux in a fixed energy band is computed using standard relativistic transformations.

The resulting light curve depends to some extent on the precise assumptions we make. However, the model assumptions described above are reasonable. As Fig. 1 shows, this model gives a large number ($\sim \gamma_t^2$) of pulses during the main burst,[4] with a duty cycle not very different from unity. The rapidly declining late-time flux is the result of off-axis emission. Note that this emission continues to show some residual variability. This feature of the model may be worth verifying through observations. However, given the rapid decline in the flux and the limited sensitivity of detectors, one is generally forced to time-average the late-time data and this will cause the variability to be washed out.

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[3] Note that, even if the eddy points towards the observer for an extremely short time, the observed pulse is still smeared in time by the radial dimension of the eddy.

[4] In the spirit of this paper, we have ignored numerical factors of order unity in our definition of t_{burst} (equation 5). The burst duration is probably closer to $R^2/2\Gamma^2 c$.

5 DISCUSSION

The turbulent model described in this paper has several attractive features.

(i) The model naturally produces a highly variable GRB light curve with large amplitude variations across individual pulses and a duty cycle of order unity (Section 4, Fig. 1).

(ii) According to equation (5), the quantity $R/\Gamma^2 c$ is equal to t_{burst} rather than t_{var} (equation 2). The turbulent model can thus accommodate much larger values of R and smaller values of Γ compared to the standard model. This eliminates a problematic constraint which leads to difficulties when attempting to fit GRB observations using equation (2) from the standard model (e.g. Kumar & Narayan 2009).

(iii) The larger value of R obtained with the turbulent model is compatible with ideas described in Lyutikov & Blandford (2003), according to which the jet energy is primarily in the form of Poynting flux. This energy is converted to radiation through plasma instabilities near the deceleration radius ($\lesssim 10^{17}$ cm). The same instabilities may also produce the relativistic turbulence invoked in our model.[5]

(iv) The turbulent model neatly avoids the efficiency argument of Sari & Piran (1997). Any particular observer receives radiation from only a fraction $\sim 1/\gamma_t$ of the available eddies. However, the additional relativistic boost of the received radiation because of turbulent motion makes up for the missing eddies. Thus, each observer receives a fair share of the emission from the shell, and there is no radiative inefficiency in the model.

(v) The model has a clear prediction for the variability parameter $\mathcal{V}$:

$$\mathcal{V} \sim \gamma_t^2, \qquad \gamma_t \sim \mathcal{V}^{1/2}. \tag{8}$$

Since a typical long GRB has $\mathcal{V} \sim 100$, the turbulent eddy Lorentz factor γ_t needs to be ~ 10.

For simplicity, we have assumed in this paper that the eddy motions are isotropic in the frame of the GRB shell. This is, however, not essential. We could, for instance, have random relativistic motions which are concentrated primarily in a plane perpendicular to the radius vector of the shell, for example parallel to the local tangential magnetic field. Such a model would give qualitatively similar results, though some of the scalings may be a little different; the flux decline at the end of the burst, for instance, would be steeper than $t^{-2-\beta}$ when the turbulence is anisotropic (where β is spectral index). Also, we have simplified matters by assuming that all eddies have the same Lorentz factor, which is unlikely in a real turbulent medium. In fact, a likely scenario is that a part of the fluid moves relativistically with a range of Lorentz factors, and a part resides in a (more-or-less) stationary intereddy medium which is produced when eddies collide with one another in shocks. This will slightly modify the radiative properties of the medium (Kumar & Narayan 2009), but it will not change the key features of the model as described in the present paper. The relevant energy-bearing eddies may not be as large as their comoving causality size but may be smaller by a numerical factor, and the solid angle swept by an eddy in the shell frame may be different from $\sim 1/\gamma_t$ (for instance, each eddy may move ballistically for the entire lifetime of the shell, with hardly any acceleration). These effects will modify equations (5)–(7). However, the results will remain qualitatively the same. Finally, depending on the details of the initial acceleration and final deceleration of eddies, and the intervening dynamics of eddies, it may be possible to explain various asymmetries and correlations that have been observed in GRB pulses (e.g. Norris et al. 1996; Ramirez-Ruiz & Merloni 2001; Nakar & Piran 2002).

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[5] Note that the relativistic turbulence that we invoke in our model does not imply 'turbulence' in the strict sense of the word. We merely require random relativistic motions of independent clumps in the radiating medium.