Accretion Models of Gamma-Ray Bursts

Citation

Narayan, Ramesh, Tsvi Piran, and Pawan Kumar. 2001. "Accretion Models of Gamma-Ray Bursts." *The Astrophysical Journal* 557 (2): 949–57. <https://doi.org/10.1086/322267>.

Permanent link

<http://nrs.harvard.edu/urn-3:HUL.InstRepos:41384933>

Terms of Use

This article was downloaded from Harvard University's DASH repository, and is made available under the terms and conditions applicable to Other Posted Material, as set forth at <http://nrs.harvard.edu/urn-3:HUL.InstRepos:dash.current.terms-of-use#LAA>

Share Your Story

The Harvard community has made this article openly available.
Please share how this access benefits you. [Submit a story](#).

[Accessibility](#)

Accretion Models of Gamma-Ray Bursts

Ramesh Narayan¹, Tsvi Piran² and Pawan Kumar³

1. Harvard-Smithsonian Center for Astrophysics, Cambridge, MA 02138, USA

2. Racah Institute of Physics, Hebrew University, Jerusalem 91904, Israel

3. Institute for Advanced Studies, Princeton, NJ 08540, USA.

ABSTRACT

Many models of gamma-ray bursts (GRBs) involve accretion onto a compact object, usually a black hole, at a mass accretion rate of order a fraction of a solar mass per second. If the accretion disk is larger than a few tens or hundreds of Schwarzschild radii, the accretion will proceed via a convection-dominated accretion flow (CDAF) in which most of the matter escapes to infinity rather than falling onto the black hole. Models involving the mergers of black hole white dwarf binaries and black hole helium star binaries fall in this category. These models are unlikely to produce GRBs since very little mass reaches the black hole. If the accretion disk is smaller, then accretion will proceed via neutrino cooling in a neutrino-dominated accretion disk (NDAF) and most of the mass will reach the center. Models involving the mergers of double neutron star binaries and black hole neutron star binaries fall in this category and are capable of producing bright GRBs. If the viscosity parameter α in the NDAF has a standard value ~ 0.1 , these mergers can explain short GRBs with durations under a second, but they are unlikely to produce long GRBs with durations of tens or hundred of seconds. If the accretion disk is fed by fallback of material after a supernova explosion, as in the collapsar model, then the time scale of the burst is determined by fallback, not accretion. Such a model can produce long GRBs. Fallback models again require that the accretion should proceed via an NDAF rather than a CDAF in order for a significant amount of mass to reach the black hole. This condition imposes an upper limit on the radius of injection of the gas.

1. Introduction

The fireball model (see Piran 1999, 2000, for reviews) provides a good understanding of conditions within the γ -ray-emitting and afterglow-emitting regions of gamma-ray bursts (GRBs). According to this model, GRBs are produced when relativistic ejecta from a “central engine” are slowed down by interactions, either with an external medium (the external shock model) or among different layers within the ejecta themselves (the internal shock model). In the interactions the kinetic energy in the ejecta is converted to relativistic electrons which produce the observed

radiation. Among the many successes of the model we note the observational confirmation of relativistic motion in the afterglow (Frail et al. 1997; Katz & Piran 1997).

Despite the successes of the fireball model, the nature of the central engine remains a mystery. The problem is that the central engine is hidden from view; no radiation (apart from gravitational radiation and neutrinos that may possibly be detected in the distant future) reaches the observer directly from the engine. For a number of so-called “long bursts” accurate positions have been determined through observations of their afterglows. Based on this, there is circumstantial evidence that these bursts are associated with star-forming regions (e.g. Bloom et al. 2000). There is no information at present on the other class of GRBs, the so-called “short bursts.”

Although we lack direct evidence on the nature of the central engine, it is nevertheless widely accepted that GRBs are the result of cataclysmic events involving either neutron stars or stellar-mass black holes. The arguments in support of this hypothesis are straightforward. (i) Since bursts radiate the bulk of their energy in the γ -ray band, it seems likely that a relativistic object is behind their production. (ii) The energy budget ($\sim 10^{51}$ erg) is comparable to the kinetic energy of ejecta in a supernova explosion. (iii) Most long bursts are highly variable in gamma-rays, and so are many short bursts (Nakar & Piran, 2001a). In particular, the ratio of the total duration of the burst to the variability time scale is large, from which one concludes that the gamma-rays must be produced in internal shocks (Sari & Piran, 1997). A key feature of internal shocks is that the observed gamma-ray variability reflects the variability in the activity of the central engine (Sari & Piran, 1997). Since variability time scales as short as a millisecond are observed, the engine must contain a compact object of no more than a few solar masses (otherwise the light-crossing time would exceed the variability time).

An interesting clue to the nature of GRBs is provided by the durations of bursts. While the fastest variability time scale is under a millisecond, burst durations are usually very much longer. Long bursts have durations ranging from 10–1000 seconds, and even short bursts have a median duration of about 0.3 seconds. Clearly, whatever is the physical mechanism behind GRB production, it acts on a much longer time scale than the fastest dynamical time of the central engine.

Narayan, Paczyński & Piran (1992) suggested that the central engine in GRBs involves the accretion of matter onto a compact star, and that the energy in the burst is provided by the gravitational energy released by the accreting gas. In such a model, the duration of the burst is set by the viscous time scale of the accreting gas. In most accretion flows, the viscous time is significantly longer than the dynamical time, and so the accretion model naturally explains the large difference between the durations of bursts and their fastest variability time.

The formation of an accretion disk is a natural outcome of most popular models of GRBs, e.g. the mergers of double neutron star binaries (Eichler et al., 1989; Narayan, Paczyński & Piran 1992), neutron star black hole binaries (Paczynski, 1991; Narayan Paczyński & Piran 1992), black hole white dwarf binaries (Fryer et al., 1999), black hole helium star binaries (Fryer & Woosley,

1998), and models based on “failed supernovae” or “collapsars” (Woosley, 1993; Paczynski, 1998; MacFadyen & Woosley, 1999). An important exception is Usov’s (1992) model in which the GRB energy is provided by the magnetic and rotational energy of a newly formed rapidly rotating neutron star.

In this paper, we consider a generic accretion model of a GRB in which a certain amount of mass, $m_d M_\odot$, goes into orbit around a relativistic star of mass $3m_3 M_\odot$. We assume that the orbiting mass is initially inserted into a torus at a radius $r_{\text{out}} R_S$, where R_S is the Schwarzschild radius of the central star: $R_S = 2GM/c^2 = 8.85 \times 10^5 m_3$ cm. Starting from the initial toroidal configuration, the mass spreads out by viscosity and becomes an accretion flow extending from $r = 1$ (the horizon of the central black hole) to $r \sim r_{\text{out}}$. (There might also be an outflow at radii $> r_{\text{out}}$ as we discuss below.) We work out in this paper the time scale, t_{acc} , of the accretion flow, the average mass accretion rate, $\dot{M} = \dot{m} M_\odot \text{s}^{-1}$, onto the central star, the amount of mass, $m_{\text{acc}} = t_{\text{acc}} \dot{m}$, accreted by the star, and the accretion efficiency, $\xi = m_{\text{acc}}/m_d$.

These parameters are constrained by observations. In binary merger models, the durations of bursts should be comparable to the accretion time t_{acc} . Durations are of order a second or less for the class of “short duration GRBs” and in the range 10–1000 seconds for the class of “long duration GRBs.” The energy in long duration bursts is estimated to be $\sim 5 \times 10^{50}$ erg (Panaitescu & Kumar, 2001; Frail et al. 2001). With a reasonable efficiency of converting accretion energy to relativistic flow ($\lesssim 0.01$) this corresponds to $m_{\text{acc}} \gtrsim 0.1 M_\odot$ and to a peak accretion rate $\dot{M} \gtrsim 10^{-2} M_\odot \text{s}^{-1}$. The values of these parameters for short bursts are less certain, as the distance scale to this population is uncertain. However, it is unlikely that they are smaller by more than an order of magnitude.

The mass accretion rate tends to be extremely high for a typical GRB model — it is of order a fraction of a solar mass per second. At such accretion rates, the optical depth of the accreting gas is enormous and radiation is trapped inside the gas. In the normal course, the accretion would proceed via a radiatively inefficient flow, such as an advection-dominated accretion flow or the related convection-dominated accretion flow. We consider such flows in §2. If the accreting gas has a sufficiently high temperature and density, however, it can cool via neutrino emission, leading to a neutrino-dominated accretion flow, as discussed by Popham, Woosley & Fryer (1999) (see also Ruffert & Janka, 1999, for a numerical simulation of a binary merger which included neutrino losses). We discuss this case in §3. In §4 we go beyond the analytical results of §§2,3 and present numerical results, delineating the regions of (r_{out}, m_d) space where different types of accretion occur. We discuss in §5 the implications of the results for various models of GRBs.

2. Radiatively Inefficient Accretion: CDAF

At the mass accretion rates of interest to us, the optical depth of the gas is extremely high, and the radiation is very effectively trapped (we demonstrate this below). The accretion flow then corresponds to an advection-dominated accretion flow, or ADAF (Narayan & Yi 1994, 1995;

Abramowicz et al. 1995; Chen et al. 1995; see Narayan, Mahadevan & Quataert 1998 and Kato, Fukue & Mineshige 1998 for reviews). Radiation-trapped ADAFs were initially discussed by Katz (1977) and Begelman (1978), and later analyzed via height-integrated “slim disk equations” by Abramowicz et al. (1988).

Igumenshchev and co-workers have carried out hydrodynamic simulations of ADAFs and have discovered several interesting properties of these flows (Igumenshchev, Chen & Abramowicz 1996; Igumenshchev & Abramowicz 1999, 2000). They find that, when the dimensionless viscosity parameter α is large, say $\alpha \gtrsim 0.3$, the accretion flow has a strong bipolar outflow, as anticipated in some previous papers (Narayan & Yi 1994, 1995; Blandford & Begelman 1999). On the other hand, when the viscosity is relatively weak, say $\alpha \lesssim 0.1$, the flow has well-developed convection (which was again anticipated in earlier work, cf. Narayan & Yi 1994, 1995). A convective ADAF has quite unusual properties (Stone, Pringle & Begelman 1999; Narayan, Igumenshchev & Abramowicz 2000; Quataert & Gruzinov 2000; Igumenshchev, Abramowicz & Narayan 2000), which arise because convection moves angular momentum inward rather than outward (Narayan et al. 2000; Quataert & Gruzinov 2000). Convective ADAFs have been given the name of convection-dominated accretion flows, or CDAFs.

As in other accretion flows, “viscosity” in radiation-trapped flows is believed to arise from magnetic stresses resulting from the Balbus-Hawley instability (Balbus & Hawley, 1991). Numerical simulations of shearing MHD flows give values of α in the range $\alpha \lesssim 0.1$. We therefore assume that radiation-trapped accretion flows in GRBs also have α in this range (no 3D MHD simulations of these flows have been done so far, but there has been some 2D work by Stone & Pringle 2001). Given the relatively low value of α , we expect the accretion flow to take the form of a CDAF.

Ball, Narayan & Quataert (2001) have given approximate scalings for various fluid variables in a CDAF. The density and velocity scale as

$$\begin{aligned}\rho(r) &\approx \rho_{\text{out}} \left(\frac{r}{r_{\text{out}}} \right)^{-1/2} = 2.97 \times 10^{14} m_3^{-3} m_d r_{\text{out}}^{-5/2} r^{-1/2} \text{ g cm}^{-3}, \\ v(r) &\approx c r^{-3/2} = 3 \times 10^{10} r^{-3/2},\end{aligned}\tag{1}$$

where, in the first equation, we have used $4\pi r_{\text{out}}^3 R_S^3 (H/R) \rho_{\text{out}} = m_d M_\odot$ to relate ρ_{out} in the CDAF to the mass in the accretion flow:

$$\rho_{\text{out}} = 2.97 \times 10^{14} m_3^{-3} m_d r_{\text{out}}^{-3} \text{ g cm}^{-3}.\tag{2}$$

Note the use of 4π rather than $4\pi/3$ in the equation for the mass. This has been done to obtain a better match with the results for an NDAF (see §3).

We assume that the accreting gas consists of photo-disintegrated nuclei with roughly equal numbers of neutrons and protons (and electrons). The optical depth through the flow is

$$\tau_{\text{out}} \approx \frac{\rho_{\text{out}}}{2m_p} \sigma_T r_{\text{out}} R_S = 5.21 \times 10^{19} m_3^{-2} m_d r_{\text{out}}^{-2}.\tag{3}$$

The diffusion time for radiation to leak out of the flow is then

$$t_{\text{diff}} \approx \tau_{\text{out}} \frac{r_{\text{out}} R_S}{c} = 1.54 \times 10^{15} m_3^{-1} m_d r_{\text{out}}^{-1} \text{ s}. \quad (4)$$

This time is very much longer than the accretion time for the parameter ranges of interest to us, namely $m_3 \sim 1$, $m_d \sim 0.1 - 1$, $r_{\text{out}} \sim 10 - 10^4$. Thus we expect radiation to be very effectively trapped within the accretion flow.

Ball et al. (2001) also estimate the isothermal sound speed c_s and the scale height H of the accretion flow. We use slightly different coefficients here which are more appropriate for a radiation-pressure dominated $\gamma = 4/3$ gas (as opposed to the $\gamma = 5/3$ gas that Ball et al. 2001, considered):

$$\begin{aligned} c_s^2 &\approx 0.3 c^2 r^{-1} = 2.70 \times 10^{20} r^{-1} \text{ cm}^2 \text{ s}^{-2}, \\ \frac{H}{R} &= \frac{c_s}{\Omega_K R} \approx 0.77. \end{aligned} \quad (5)$$

These scalings are approximate, but they are likely to be accurate enough for the purposes of this paper. The mass accretion rate in the CDAF is estimated to be $\dot{M} = 4\pi r^2 R_S^2 (H/R) \rho(r) v(r)$, which gives, using the above relation for H/R ,

$$\dot{m} = 3.39 \times 10^4 m_3^{-1} m_d r_{\text{out}}^{-5/2}. \quad (6)$$

(Recall that $\dot{m}$ is defined as the mass accretion rate in units of solar masses per second.)

The accretion time scale is not simply equal to $m_d/\dot{m}$. The reason is that much of the mass in a CDAF actually flows out of the system rather than into the central black hole (Stone et al. 1999; Narayan et al. 2000; Quataert & Gruzinov 2000; Igumenshchev & Abramowicz 2000). We therefore proceed as follows.

The random velocities of convective blobs in a CDAF are typically less than the local Keplerian velocity by a factor which is proportional to the viscosity parameter α (see Narayan et al. 2000). We thus write approximately

$$v_{\text{turb}} \approx \alpha v_K = 2.12 \times 10^9 \alpha_{-1} r^{-1/2} \text{ cm s}^{-1}. \quad (7)$$

The residence time of a convective blob at radius r is then

$$t_{\text{res}} \approx \frac{r R_S}{v_{\text{turb}}} = 4.17 \times 10^{-4} \alpha_{-1}^{-1} m_3 r^{3/2} \text{ s}. \quad (8)$$

We make the reasonable assumption that the accretion time is of the same order as t_{res} at $r = r_{\text{out}}$:

$$t_{\text{acc}} \approx t_{\text{res}}(r_{\text{out}}) = 4.17 \times 10^{-4} \alpha_{-1}^{-1} m_3 r_{\text{out}}^{3/2} \text{ s}. \quad (9)$$

We then find that the amount of mass accreted by the black hole is $m_{\text{acc}} = t_{\text{acc}} \dot{m}$, but since this mass cannot exceed the total available mass m_d we write:

$$\begin{aligned} m_{\text{acc}} &= m_d, & r_{\text{out}} &\leq 14.1 \alpha_{-1}^{-1} m_d, \\ &= 14.1 \alpha_{-1}^{-1} m_d r_{\text{out}}^{-1}, & r_{\text{out}} &> 14.1 \alpha_{-1}^{-1} m_d. \end{aligned} \quad (10)$$

Note that, when r_{out} is large, the accreted mass is much less than m_d . The reason is that the bulk of the mass is ejected from the system, flowing out at $r \sim r_{\text{out}}$. The energy for the ejection is provided by convective energy flux from the interior of the flow.

The coefficient 14 in equation (10) is somewhat uncertain since we do not know the exact relation between t_{acc} and t_{res} ; there could well be a numerical factor other than unity relating the two. A different (and more more detailed) derivation of equation (10) is given in the Appendix, where we again find that the coefficient is uncertain. In the rest of the paper we use equation (10) as written, but we should keep in mind that it could be in error by a factor of a few.

Let us now calculate the temperature of the CDAF. The pressure in the accreting gas is given by

$$p = \rho c_s^2 = 8.02 \times 10^{34} m_3^{-3} m_d r_{\text{out}}^{-5/2} r^{-3/2} \text{ erg cm}^{-3}. \quad (11)$$

The pressure has three contributions: radiation pressure, gas pressure, and degeneracy pressure (Popham et al. 1999):

$$p = \frac{11}{12} a T^4 + \frac{\rho k T}{m_p} + \frac{2\pi h c}{3} \left(\frac{3}{8\pi m_p} \right)^{4/3} \left(\frac{\rho}{\mu_e} \right)^{4/3}. \quad (12)$$

The quantity a is the radiation constant, and the factor $11/12$ includes the contribution of relativistic electron-positron pairs (assuming that the temperature is sufficiently above the pair threshold limit). The gas pressure term includes the contributions from non-relativistic particles. We assume that we have an equal mix of protons and neutrons (i.e. we assume that all complex nuclei have been photo-disintegrated). In the degeneracy pressure term we use an electron molecular weight $\mu_e = 2$, assuming an equal mix of protons and neutrons; the ratio of neutrons to protons in the interior of a NS is about 0.2. The contribution of e^-e^+ pairs has been ignored in the degeneracy pressure term. For the calculations presented in §4, we solve for T by using the full expression for p given in equation (12). But here, in order to obtain simple analytic estimates, we simplify the equation and assume that radiation pressure dominates. We then find that

$$T = 1.84 \times 10^{12} m_3^{-3/4} m_d^{1/4} r_{\text{out}}^{-5/8} r^{-3/8} \text{ K}. \quad (13)$$

This estimate of T is valid only if gas pressure is smaller than radiation pressure. The gas pressure is equal to

$$p_{\text{gas}} = \frac{\rho k T}{m_p} = 4.52 \times 10^{34} m_3^{-15/4} m_d^{5/4} r_{\text{out}}^{-25/8} r^{-7/8} \text{ erg cm}^{-3}. \quad (14)$$

For this to be smaller than radiation pressure we require

$$\frac{r}{r_{\text{out}}} < 1.5 m_3^{6/5} m_d^{-2/5}. \quad (15)$$

For our fiducial black hole of mass $3M_\odot$, gas pressure dominates only if the disk is quite massive, $m_d \gtrsim 5$, and if r is close to r_{out} . (The detailed numerical calculations in §4 show that gas pressure dominates for $m_d \gtrsim 1$ rather than 5, but even for such values of m_d , if r is much less than r_{out} , radiation pressure takes over.)

Let us next calculate the rate of cooling of the accreting gas. We showed earlier that cooling via radiative diffusion is negligible. However, at the temperatures found in these flows, cooling via neutrino losses may be important (cf. Popham et al. 1999). The cooling rate per unit volume due to neutrinos, q^- , takes the form

$$q^- = q_{\nu\bar{\nu}}^- + q_{eN}^- \approx 5 \times 10^{33} T_{11}^9 + 9.0 \times 10^{23} \rho T_{11}^6 \text{ erg cm}^{-3} \text{ s}^{-1}, \quad (16)$$

where the first term on the right describes cooling via pair annihilation (the so-called URCA process) and the second term describes cooling via pair capture on nuclei (estimated for $X_{\text{nuc}} = 1$ as appropriate for our fully photo-disintegrated nuclear gas, cf. Popham et al. 1999). For the range of parameters of interest to us, q_{eN}^- invariably dominates over $q_{\nu\bar{\nu}}^-$. We therefore neglect $q_{\nu\bar{\nu}}^-$ hereafter.

We then obtain the following estimate for the cooling rate in the CDAF:

$$q_{eN}^- = 1.04 \times 10^{46} m_3^{-15/2} m_d^{5/2} r_{\text{out}}^{-25/4} r^{-11/4} \text{ erg cm}^{-3} \text{ s}^{-1}. \quad (17)$$

Since the energy density of the radiation-dominated gas is equal to $3p_{\text{rad}} = (11/4)aT^4$, we estimate the cooling time of the gas at radius r to be

$$t_{\text{cool}} = \frac{3p_{\text{rad}}}{q_{eN}^-} = 2.31 \times 10^{-11} m_3^{9/2} m_d^{-3/2} r_{\text{out}}^{15/4} r^{5/4} \text{ s}. \quad (18)$$

One of the important requirements for the accretion flow to behave like a CDAF is that it should be radiatively inefficient. We, therefore, require the cooling time t_{cool} at each radius to be longer than the residence time t_{res} of a convecting blob at that radius. The ratio of cooling to residence time is

$$\frac{t_{\text{cool}}}{t_{\text{res}}} = 5.54 \times 10^{-8} \alpha_{-1}^{7/2} m_3^{-3/2} m_d^{15/4} r_{\text{out}}^{-1/4}. \quad (19)$$

Since the ratio decreases with increasing r , we set $r = r_{\text{out}}$. Then we see that a CDAF is possible only if the following condition is satisfied:

$$r_{\text{out}} > 118 \alpha_{-1}^{-2/7} m_3^{-1} m_d^{3/7}. \quad (20)$$

If the mass in the accretion flow is initially at a radius greater than the above limit, then the flow will become a CDAF at $r = r_{\text{out}}$, and indeed for all $r < r_{\text{out}}$. The scaling relations derived in this section would then be valid. If, however, the initial radius of the gas is below the above limit, then neutrino-cooling will prevail, and we will have a cooling-dominated accretion flow at all radii from r_{out} down to the black hole.

For completeness, we consider also the case when gas pressure dominates (which requires a large value of m_d , and r close to r_{out} , as shown above). The temperature is then obtained by setting $p = p_{\text{gas}}$. This gives

$$T = \frac{m_p c_s^2}{k} = 3.27 \times 10^{12} r^{-1} \text{ K}. \quad (21)$$

The analysis presented so far assumes that we are given the initial mass in the disk m_d and the initial radius of the gas r_{out} . Another possibility is that we have steady injection of mass at a rate $\dot{m}_{\text{inj}} M_\odot \text{s}^{-1}$ at a circularization radius r_{out} . This is the case, for instance, in the collapsar model, where the material is supplied by fallback from a supernova explosion. We may assume that the accretion flow achieves an approximate steady state in which the rate of injection of mass equals the rate of mass loss from the flow (both inflow and outflow): $\dot{m}_{\text{inj}} = m_d/t_{\text{acc}}$. Therefore, we obtain

$$m_d = 4.17 \times 10^{-4} \alpha_{-1}^{-1} m_3 \dot{m}_{\text{inj}} r_{\text{out}}^{3/2}. \quad (22)$$

This expression, which gives the mapping between m_d and $\dot{m}_{\text{inj}}$, may be substituted into the various relations derived in this section to obtain the corresponding results for the case of steady mass injection.

3. Radiatively Efficient Accretion: NDAF

We saw in the previous section that when the mass is initially injected at an outer radius smaller than the limit given in equation (20), cooling via neutrino emission becomes significant and the flow is no longer radiatively inefficient. We then have a regime of accretion in which the viscous energy dissipation is balanced by neutrino cooling. Popham et al. (1999) named this a neutrino-dominated accretion flow (NDAF) and worked out its properties. We extend their results in this section.

Because the gas cools efficiently, an NDAF behaves like a thin accretion disk and we are entitled to use the basic theory of thin disks (Shakura & Sunyaev 1973; Frank, King & Raine 1992). As before, the pressure of the gas has three contributions and is described by equation (12). The isothermal sound speed c_s and the vertical scale height H are given by $c_s^2 = p/\rho$ and $H = c_s/\Omega_K$, where $\Omega_K = (GM/R^3)^{1/2} = 2.40 \times 10^4 m_3^{-1} r^{-3/2} \text{ s}^{-1}$ is the Keplerian angular velocity. We write the coefficient of kinematic viscosity in the usual form as

$$\nu = \alpha \frac{c_s^2}{\Omega_K}, \quad (23)$$

where α is a dimensionless parameter which we expect to have a value ~ 0.1 . We denote, therefore, $\alpha_{-1} = \alpha/0.1$, and write down our scalings in terms of α_{-1} .

A thin accretion disk satisfies two equations. Angular momentum balance gives

$$\dot{M} = 3\pi\nu\Sigma \left[1 - \left(\frac{R}{R_*} \right)^{-1/2} \right] \approx 6\pi\nu\rho H, \quad (24)$$

where $\Sigma = 2\rho H$ is the surface density and R_* is the radius of the inner edge of the disk ($3R_S$ for a Schwarzschild black hole). The approximation on the right is valid for $R \gg R_*$. The radial velocity of the gas is given by $v = 3\nu/2R$ and so the accretion time is

$$t_{\text{acc}} = \frac{R_{\text{out}}}{v(R_{\text{out}})} = \frac{2R_{\text{out}}^2}{3\nu}. \quad (25)$$

If we write the disk mass as $m_d = t_{\text{acc}} \dot{M}$, then by combining the previous two equations we find that

$$m_d = 4\pi R_{\text{out}}^3 \left(\frac{H}{R} \right)_{\text{out}} \rho_{\text{out}}. \quad (26)$$

Note that we used this relation, with the same coefficient 4π , also for a CDAF.

The condition of energy balance (viscous heating equals radiative losses) gives:

$$\frac{3GM\dot{M}}{8\pi R^3} = q^- H = (q_{eN}^- + q_{\nu\nu}^-) H. \quad (27)$$

This relation closes our set of equations and allows us to solve for ρ , T and other quantities. We show numerical solutions of the full equations in §4.

In the rest of this section we derive analytical scalings by making some approximations. First, the numerical calculations show that the q_{eN}^- term in the cooling law dominates over $q_{\nu\nu}^-$ for all parameters of interest. We will therefore assume this. The calculations also show that for most parameters, either gas pressure or degeneracy pressure dominates. (There is a small region of parameter space where radiation pressure dominates; we ignore this region in the analytical work.)

Let us first assume that gas pressure dominates. Expressing all results in terms of the scaled temperature, $T_{11} = T/10^{11} K$, and substituting in the angular momentum equation, we obtain a relation between ρ and T_{11}

$$\rho = 2.56 \times 10^{13} \alpha_{-1}^{-1} m_3^{-2} \dot{m} r^{-3} T_{11}^{-3/2} \text{ g cm}^{-3}. \quad (28)$$

Substituting this in the energy equation, we can solve for T_{11} , and thereby obtain the various other quantities (Popham et al. have derived similar relations):

$$\begin{aligned} T_{11} &= 0.548 \alpha_{-1}^{1/5} m_3^{-1/5} r^{-3/10}, \\ t_{\text{acc}} &= 2.76 \times 10^{-2} \alpha_{-1}^{-6/5} m_3^{6/5} r_{\text{out}}^{4/5} \text{ s}, \\ \dot{m} &= 36.2 \alpha_{-1}^{6/5} m_3^{-6/5} m_d r_{\text{out}}^{-4/5}, \\ \rho_{\text{out}} &= 2.28 \times 10^{15} \alpha_{-1}^{-1/10} m_3^{-29/10} m_d r_{\text{out}}^{-67/20} \text{ g cm}^{-3}. \end{aligned} \quad (29)$$

In a cooling-dominated thin disk, very little mass is expected to be lost to outflows, so we expect nearly all the mass in the disk to be accreted by the star, i.e.

$$m_{\text{acc}} \approx m_d. \quad (30)$$

The above results are valid provided gas pressure dominates. By comparing gas pressure to degeneracy pressure we can determine the condition for this to be true. We find that we require

$$r_{\text{out}} > 26.2 \alpha_{-1}^{-2/7} m_3^{-46/49} m_d^{20/49}. \quad (31)$$

When the outer radius is below this limit, degeneracy pressure takes over from gas pressure and we obtain a different set of analytical scalings:

$$\begin{aligned}
 \dot{m} &= 355\alpha_{-1}m_3^{-13/7}m_d^{9/7}r_{\text{out}}^{-3/2}, \\
 \rho_{\text{out}} &= 7.32 \times 10^{14}m_3^{-18/7}\dot{m}^{6/7}r_{\text{out}}^{-3} \text{ g cm}^{-3}. \\
 T_{11} &= 0.800\alpha_{-1}^{1/6}m_3^{-13/42}m_d^{1/21}r_{\text{out}}^{-5/12}, \\
 t_{\text{acc}} &= 2.82 \times 10^{-3}\alpha_{-1}^{-1}m_3^{13/7}m_d^{-2/7}r_{\text{out}}^{3/2} \text{ s}.
 \end{aligned} \tag{32}$$

In our analysis of both the gas-pressure-dominated and degeneracy-pressure-dominated regimes, we assumed that the accreting gas is optically thin to its own neutrino emission. This assumption breaks down at sufficiently small radii. We may estimate the neutrino optical depth as

$$\tau_\nu = \frac{q_{eN}^- H}{4\sigma T^4} = 5.20 \times 10^3 \alpha_{-1}^{-2/3} m_3^{-55/21} m_d^{23/21} r_{\text{out}}^{-17/6}. \tag{33}$$

The radius at which the optical depth goes to unity is

$$r_{\tau=1} = 20.5\alpha_{-1}^{-4/17}m_3^{-110/119}m_d^{46/119}. \tag{34}$$

This radius lies within the degeneracy-pressure-dominated zone. Inside this radius, we need to consider neutrino transport in more detail. This is beyond the scope of the paper.

It is interesting to examine whether the NDAF solution is stable. Following Piran (1978) we use the general condition for thermal stability:

$$\left(\frac{d \ln Q^+}{d \ln H} \right)_{|\Sigma} < \left(\frac{d \ln Q^-}{d \ln H} \right)_{|\Sigma}, \tag{35}$$

where $Q^\pm$ are the integrated (over the height of the disk) heating (+) and cooling (−) rates. For an NDAF with viscosity described by the α -prescription, the heating rate goes as $Q^+ \propto H^2$, and the vertically integrated cooling rate (for pair capture on nuclei) goes as $Q^- \propto \Sigma T^6$. For an NDAF in which gas pressure dominates, $T \propto H^2$ and $Q^- \propto \Sigma H^{12}$. The criterion (35) is satisfied easily and we conclude that such an NDAF is thermally stable.

If radiation pressure dominates, $T^4 \propto p \propto \Sigma H$ and $Q^- \propto \Sigma^{5/2} H^{3/2}$. The condition (35) is not satisfied and the NDAF is unstable. This resembles the situation in “conventional” accretion disks whose inner regions become thermally unstable when radiation pressure dominates (Frank et al. 1992).

If degeneracy pressure dominates, the temperature is independent of either H or Σ . The temperature of the disk can then freely adjust such that Q^- balances any Q^+ . The situation is clearly thermally stable.

The condition for viscous stability is:

$$\frac{d\dot{M}}{d\Sigma} > 0. \tag{36}$$

We have $\dot{M} \propto \Sigma$ for the gas pressure case, $\dot{M} \propto \Sigma^7$ for the radiation pressure case, and $\dot{M} \propto \Sigma^{9/7}$ for the degeneracy pressure case. All three cases are viscously stable.

Thus, combining all these results on stability, we find that an optically thin NADF is unstable only if it is radiation pressure dominated. There is a very narrow region of parameter space near the boundary between NDAFs and CDAFs in $(r_{\text{out}}, \dot{m})$ space where radiation pressure does dominate. This unstable region could conceivably play a role in determining the temporal behavior of some bursts. The stability properties of the optically thick regions of the NDAF remain to be worked out.

The disks that we are considering are massive and we should also consider gravitational instabilities. The CDAF zone is always gravitationally stable in our models; the Toomre Q parameter is invariably much greater than unity. For gas pressure dominated NDAFs, we find that the Toomre Q parameter is given by

$$Q = 8.5 \alpha_{-1}^{1/10} m_3^{9/10} m_d^{-1} r^{-9/20} r_{\text{out}}^{4/5}. \quad (37)$$

We see that Q decreases with increasing r , so that the flow is most unstable on the outside. However, even for $r = r_{\text{out}}$, Q is sufficiently larger than unity (for all reasonable values of parameters) that we are guaranteed stability. A similar result applies to radiation pressure dominated NDAFs.

The case of degeneracy pressure dominated NDAFs is more complicated. For most of the parameter space (m_d, r_{out}) we find Q to be greater than unity, but it is only marginally so ($Q \approx 3$). When $r_{\text{out}} < 10$, we find that Q might become less than unity, signifying gravitational instability. But this is also the region in which the disk becomes optically thick to neutrinos and the analysis we have carried out breaks down.

4. Numerical Results

We discuss in this section numerical results which we have obtained by solving the full equations. We assumed that we are given the initial mass of the accretion disk m_d and the initial radius r_{out} . For each choice of r_{out} and m_d , we first assumed that the flow consists of a CDAF and used the relations described in §2 to calculate the properties of the accreting gas. In particular, we used equations (11) and (12), with ρ given by equation (1), to solve for the temperature T as a function of r . We checked the flow at all radii from $r = r_{\text{out}}$ down to $r = 1$ to make sure that the gas is radiatively inefficient at all radii. Specifically, we estimated the cooling time t_{cool} , using the cooling formula given in equation (16), and checked that t_{cool} is longer than the residence time of convective blobs t_{res} at that radius. If $t_{\text{cool}} > t_{\text{res}}$ at all r , then we identified the flow as a pure CDAF and estimated the accretion time t_{acc} and the total mass accreted m_{acc} , using the formulae given in §2.

For a small region of parameter space near the transition between a pure CDAF and a pure NDAF, we found that the flow starts off as a CDAF at $r = r_{\text{out}}$ and switches to an NDAF at a smaller radius. We estimated t_{acc} and m_{acc} for these cases by using a reasonable matching formula

at the transition radius. (We do not provide the details here, since this case is seen for only a small range of parameters).

For r_{out} less than a critical value (whose value depends on m_d), we found that the flow cools too rapidly at $r = r_{\text{out}}$ to be a CDAF. In such cases, the flow becomes an NDAF at $r = r_{\text{out}}$ and remains an NDAF all the way down to $r = 1$. We solved the corresponding set of equations (see §3), and estimated t_{acc} and m_{acc} appropriately.

Figures 1 & 2 show contours of t_{acc} and the accretion “efficiency” $\xi \equiv m_{\text{acc}}/m_d$, plotted in the space of the two principal parameters of the problem, r_{out} and m_d . These models have the “canonical values” $m_3 = \alpha_{-1} = 1$. The boundary between the CDAF and NDAF zones is clearly seen in both figures, but especially in Fig. 1. The numerically determined location of the CDAF-NDAF boundary is fairly close to the analytical approximation given in equation (20) and shown as the right-most dotted line in Fig. 1.

As equation (9) shows, the accretion time scale t_{acc} for a CDAF is independent of m_d and depends only on r_{out} . In contrast, t_{acc} has a more complicated dependence on r_{out} and m_d for an NDAF (eqs. 29 and 32). These behaviors are seen clearly in Fig. 1.

For a given initial mass m_d , the accreted mass m_{acc} is equal to m_d , independent of r_{out} , in the case of an NDAF, and so $\xi = 1$ (see Fig. 2). However, if the flow becomes a CDAF (which happens for r_{out} greater than a critical value), m_{acc} is significantly less than m_d , and the rest of the mass is ejected from the system; m_{acc} scales as r_{out}^{-1} in this case (cf eq 10). Therefore, ξ becomes significantly less than unity. Figure 2 illustrates this dependence.

In some GRB models, such as those involving a BH-NS merger or the collapse of a very massive star, the mass of the black hole could be larger than $3M_{\odot}$. We have therefore computed results for an $M = 30M_{\odot}$ black hole, i.e. $m_3 = 10$. Equations (20), (31) and (34) show that the various critical radii vary roughly inversely as m_3 (which is equivalent to saying that the physical radii $R = rR_S$ at which the corresponding transitions occur are independent of the black hole mass). We have confirmed this result in the detailed numerical calculations. As a result, for $m_3 = 10$, the NDAF zone shrinks by a factor of 10 in r_{out} , and the various variants of the NDAF (degeneracy pressure dominated zone, optically thick zone) practically disappear. Thus, large black hole masses are not conducive to the formation of an interesting NDAF zone.

In addition, we investigated models with a smaller value of the viscosity coefficient: $\alpha = 0.01$, i.e. $\alpha_{-1} = 0.1$. In this case, the various critical radii become larger by roughly a factor of 2, as expected from equations (20), (31) and (34). More importantly, t_{acc} increases by a factor of ~ 10 (see equation 25).

We also considered the case when the accretion is fed at a constant rate $\dot{m}_{\text{inj}}$ (rather than being initiated with an instantaneous addition of mass m_d as we have assumed so far). This case could be relevant for collapsar models. Figure 3 shows contours of accretion efficiency defined in this case as $\xi = \dot{m}/\dot{m}_{\text{inj}}$, plotted in the space of r_{out} and $\dot{m}_{\text{inj}}$ for the “canonical values” $m_3 = \alpha_{-1} = 1$. The

overall behavior is consistent with the scaling relations derived in §§2,3.

5. Discussion

The starting point for this work is the fact that the accretion flow in a putative GRB central engine can have two very different forms: we expect the accretion to occur as a radiatively inefficient CDAF (convection-dominated accretion flow, cf Narayan et al. 2000, Quataert & Gruzinov 2000) if the mass m_d is introduced at a somewhat large outer radius, namely r_{out} greater than the limit given in equation (20), while we expect the accretion to proceed via a radiatively efficient NDAF (neutrino-dominated accretion flow, cf. Popham et al. 1999) if r_{out} is smaller than this limit. For a narrow zone in parameter space close to the CDAF/NDAF transition, it is possible for the flow to be a CDAF on the outside ($r \lesssim r_{\text{out}}$) and to switch to an NDAF on the inside. But this is rare. By and large, for most choices of r_{out} and m_d , the flow is either a CDAF at all radii or an NDAF at all radii. Since the two kinds of flow are very different from each other, there is a rather large difference in what an observer would see in the two cases.

In most of the region of (r_{out}, m_d) space where the flow is a CDAF, the mass accretion rate $\dot{m}$ and the amount of mass accreted m_{acc} are both very small. This is because in a CDAF, especially when r_{out} is large, much more mass flows out of the system than into the black hole. If, as seems reasonable, a GRB engine requires a relatively large m_{acc} in order to produce a viable burst, then our work suggests that systems which form CDAFs are less likely to produce observable bursts.

In contrast, an NDAF has substantially larger values of $\dot{m}$ and m_{acc} for a given m_d . (In fact, we assume that $m_{\text{acc}} = m_d$ for an NDAF; there is probably some mass loss in a wind, but we expect it to be a small fraction of the total mass.) Therefore, a source with an NDAF is a much more plausible model of a GRB engine. By this argument, neutrino cooling (the key ingredient of an NDAF) is important for an efficient GRB. Note that we are not assuming anything about the actual mechanism of a burst. The fireball may be produced through neutrino-antineutrino annihilation (Eichler et al., 1989), or it could be the result of some other mechanism. Our suggestion is that whatever the mechanism may be, if it is based on accretion, then it probably requires a large m_{acc} to operate efficiently; such an m_{acc} can be achieved only through neutrino cooling of the accreting gas in an NDAF model.

We note a possible caveat to the above conclusion. The energy in long duration GRBs is estimated to be about 5×10^{50} erg (Panaitescu & Kumar, 2001; Frail et al. 2001). This energy could in principle be generated in a CDAF model with $m_{\text{acc}} \lesssim 0.1M_{\odot}$, provided the gravitational energy of the accreted matter is used very efficiently to launch a high Lorentz factor wind. We feel, however, that it is reasonable to hypothesize that the efficiency for converting the gravitational energy of m_{acc} to a GRB-producing relativistic wind is low, say not larger than 1%, and that bursts arise only from systems that form NDAFs and have large m_{acc} .

Figures 1 and 2 show results corresponding to a simple model in which we assume that the

accretion disk is formed instantaneously, e.g. by the disruption of a companion star. We specify the initial state of the disk by giving its mass m_d and initial radius r_{out} . For such a model, we see that the region of parameter space where NDAFs form corresponds to short accretion times t_{acc} of order a few tenths of a second. This suggests that a binary-disruption-based accretion model is capable of producing short GRBs. It is, however, very hard to see how such an accretion system could produce a long burst.

One way to make a long burst from an NDAF is to decrease the value of the viscosity parameter α (see eqs. 29 and 32). Could α be significantly lower, say 0.01? MHD simulations of thin accretion disks generally give α in the range 0.01 upwards, and it is widely agreed that the values obtained are lower limits since the simulations have limited spatial resolution. Empirical estimates of α in cataclysmic variables (obtained by comparing observations of dwarf nova outbursts with model predictions) give $\alpha \sim 0.1$ in the high state (which is most relevant for our models). There is clear evidence that α is smaller, perhaps ~ 0.01 , in the low state of CVs. This is probably the result of the cold gas becoming neutral and losing its coupling to the magnetic field (Gammie & Menou 1998), which is clearly not relevant for our ultra-hot plasma. We feel that $\alpha \sim 0.1$ is a reasonable estimate for the viscosity parameter.

Sakimoto & Coroniti (1981) suggested that α may be lower in radiation-dominated gases because the magnetic pressure may be in equipartition with only the gas pressure rather than the total pressure. Their proposal requires that the gas and the radiation be able to slip past each other (cf. Blaes & Socrates 2001). At the extraordinarily large radiation optical depths found in our models (both CDAFs and NDAFs), the gas and the radiation are extremely tightly coupled. It is therefore very unlikely that the value of α would be modified.

The models we consider have large disk masses. The accreting gas may therefore develop gravitational instabilities (see the discussion near the end of §3) and lose angular momentum via gravitational waves (see Bonnell & Pringle 1995 and references therein). This might lead to an increase in the effective value of α . Another potential uncertainty is that, for some choices of the parameters, electron degeneracy becomes important. It is not understood how the Balbus-Hawley instability (which is thought to produce the shear stresses behind α) behaves in such a gas.

While the time scale of the accretion t_{acc} depends fairly sensitively on α , the size of the NDAF zone is insensitive to α (cf eq. 20). Thus, Figs. 1 and 2 give fairly reliable limits on the radius inside which the mass needs to be introduced if we wish to have an NDAF. We may use this information to deduce a few interesting results on GRB models.

NS-NS and BH-NS merger models, with $(r_{\text{out}}, m_d) = (10, 0.1)$ and $(10, 0.5)$ (see Popham et al. 1999), are well inside the NDAF zone and, according to our calculations, are capable of producing GRBs. However, this is only if the black hole is small (few $M_\odot$). If the black hole is larger than $\sim 10M_\odot$ its Schwarzschild radius becomes too large and there is not enough “room” for an NDAF solution around it. Moreover, the neutron star in this case is swallowed whole by the BH and it is not tidally disrupted to create an accretion disk. On the other hand, as already noted, unless

the viscosity is much smaller than what we have assumed (which we consider unlikely), such disks cannot produce long bursts lasting hundreds or even tens of seconds. This suggests that NS-NS mergers and BH-NS mergers with smallish BH masses produce the class of short duration GRBs, but not the long duration GRBs.

Other merger models, specifically the BH-WD and the BH-He star merger models, would appear not to be viable GRB engines. As the secondaries in these systems are not compact, they would form accretion flows with large values of r_{out} . For instance, Popham et al. (1999) estimate $r_{\text{out}} \sim 3000$ for a BH-WD binary and $r_{\text{out}} \sim 5000$ for a BH-He star binary. At these radii, the accretion flow will be a very extended CDAF and hardly any mass will be accreted. Although the time scales of these models are consistent with long bursts, the extremely small value of m_{acc} suggests that these models do not produce GRBs of any kind. It is interesting to speculate what kind of observable events these binaries might produce (as undoubtedly they do merge in nature).

All of the discussion so far is concerned with binary mergers, where we have imagined that a certain fixed amount of mass is instantaneously input into the accretion flow. The popular collapsar model (MacFadyen & Woosley 1999) corresponds to a different scenario in which mass is steadily fed over a period of time by fallback from the supernova explosion. MacFadyen and Woosley (1999) show that the time scale of the GRB is set by the physics of fallback rather than by accretion. Further, the time scales they obtain are consistent with observations of long GRBs.

While the time scale may be set by fallback, the efficiency of the burst still depends on the nature of the post-fallback accretion. Let us assume that fallback supplies mass at a certain injection rate $\dot{m}_{\text{inj}}$ at a characteristic radius r_{out} ; the latter depends on the specific angular momentum of the material. Figure 3 shows the numerical results. As expected, efficient accretion, where most of the fallback material reaches the black hole, is possible only if r_{out} is small and falls within the NDAF zone. If collapsars have a distribution of r_{out} , then our calculations suggest that only those systems that have $r_{\text{out}} \lesssim 100\alpha_{-1}^{-2/7}m_3^{-1}$ will make bursts. Systems with larger specific angular momentum, and hence larger r_{out} , will form CDAFs and will eject most of the mass. Such systems may make very interesting supernova explosions, but if at all they make GRBs the bursts are likely to be very weak.

We have assumed in this paper that the energy of a GRB is proportional to the total mass accreted on the central compact object. For NS-NS and NS-BH binaries, which give rise to short duration bursts, the disk mass is expected to be nearly constant and hence the total energy might be roughly the same for all bursts. However, for long bursts, which we assume result from the collapse of a massive star, the energy release depends on the angular momentum of the stellar core (which determines r_{out}) and the mass of the stellar envelope that falls back on the collapsed core. Observations of long duration bursts indicate that the energy does not vary much from one burst to another (Panaiteescu and Kumar 2001, Frail et al. 2001). This means that, for some reason, the total accreted mass on the central object depends only weakly on the properties of the progenitor star.

In our calculations we neglected photo-disintegration of nuclei, which Popham et al. (1999) and MacFadyen & Woosley (1999) show to be an important coolant of the accreting gas. If we include this effect, the boundary between CDAFs and NDAFs will move to somewhat higher values of r_{out} , perhaps by a factor of a few. However, it will not change our key conclusions.

We are grateful to Eliot Quataert and Andrew MacFadyen for useful discussions, and Martin Rees for comments on the manuscript. We thank the organizers of the Jerusalem Winter School in Physics for hospitality while part of this research was carried out. This work was supported in part by NSF grant AST 9820686 and by a US-Israel BSF grant 98-00225.

A. Mass Accretion versus Ejection in CDAF Models

We provide an alternate “derivation” of equation (10), and discuss the relevant uncertainties in the result.

In a CDAF, there is a flow of energy from the inside to the outside as a result of convection. Let us write the convective luminosity as

$$L_c = \epsilon_c \dot{M}_{\text{in}} c^2,$$

where $\dot{M}_{\text{in}}$ is the mass accretion rate onto the black hole. Since the convective luminosity is proportional to α (Quataert & Gruzinov 2000; Narayan et al. 2000), we may write $\epsilon_c = \alpha\eta$, with η roughly a constant. Ball et al. (2001) computed a global model of a CDAF with $\alpha = 0.03$, and found that $\epsilon_c = 0.0045$. This suggests that $\eta \approx 0.15$.

In our GRB model, there is practically no radiative emission from the gas. Therefore, the entire convective luminosity L_c must be converted into thermal and mechanical energy of the gas on the outside. This energy will cause the gas at $r \sim r_{\text{out}}$ to move to larger radii.

How much energy do we need to evaporate gas from $r = r_{\text{out}}$? Since the gas in a CDAF is hardly bound to the black hole — the Bernoulli parameter is close to zero or even positive (cf Quataert & Gruzinov 2000; Narayan et al. 2000) — very little energy is needed to drive a mass outflow to infinity. Let us write the energy carried away per unit mass of escaping gas as $\zeta GM/r_{\text{out}} R_S$. The rate at which mass is ejected is then given by

$$\dot{M}_{\text{out}} = \frac{L_c r_{\text{out}} R_S}{\zeta GM} = \frac{0.2\eta\alpha_{-1}}{\zeta} r_{\text{out}} \dot{M}_{\text{in}}.$$

Thus we have

$$\frac{m_{\text{acc}}}{m_d} = \frac{\dot{M}_{\text{in}}}{\dot{M}_{\text{out}}} = \frac{5\zeta}{\eta} \alpha_{-1}^{-1} r_{\text{out}}^{-1}.$$

Equation (10) of the main paper (derived by a different approach) has the same scaling, but with a coefficient equal to 14.

By the Bernoulli parameter argument mentioned earlier, we expect ζ to be significantly less than unity; we would arbitrarily guess that $\zeta \sim 0.1 - 0.3$. Thus we expect $5\zeta/\eta \sim 10$ (compared to 14 in the main text). However, the coefficient is uncertain since ζ could be much smaller than our assumed value, or conceivably much larger. In the latter case, the convective luminosity must be carried away by a small amount of gas that is accelerated to a speed much greater than the escape velocity. This possibility is not supported by the numerical simulations carried out to date. Our estimate of η is also uncertain since it is based on a single global model calculated by Ball et al. (2001). A better estimate could be obtained from full-scale numerical simulations.

The simulations reported by Igumenshchev & Abramowicz (2000) may be used to obtain a direct estimate of $\dot{M}_{\text{in}}/\dot{M}_{\text{out}}$. For their model L ($\gamma = 4/3$, $\alpha = 0.03$, $r_{\text{out}} \sim 7000$), they estimate

$\dot{M}_{\text{in}}/\dot{M}_{\text{out}} \sim 0.003$, whereas equation (10) of the present paper predicts a value of 0.007. The two estimates agree to within a factor of about 2, which is reassuring. On the other hand, for $\alpha = 0.1$ (Model I), they obtain $\dot{M}_{\text{in}}/\dot{M}_{\text{out}} \sim 0.02$ which is much larger than our formula would predict. A possible explanation is that Model I does not strictly correspond to a CDAF. The flow appears to be a transition case where turbulent convection is replaced by a large scale circulation. In the case of MHD simulations (Stone & Pringle 2000) the mass outflow rate is again found to be much larger than the mass accretion rate on the central object. Therefore, the main results discussed here should still be valid.

REFERENCES

- Abramowicz, M., Chen, X., Kato, S., Lasota, J.P., & Regev, O., 1995, ApJ 438, L37
- Abramowicz, M., Czerny, B., Lasota, J.P., & Szuszkiewicz, E., 1988, ApJ 332, 646
- Balbus, S. A. & Hawley, J. F. 1991, ApJ, 376, 214.
- Ball, G., Narayan, R., and Quataert, E., 2001, to appear in ApJ, astro-ph/0007037
- Begelman, M., C., 1978, MNRAS 243, 610
- Blaes, O., & Socrates, A., 2001, ApJ, submitted (astro-ph/0011097)
- Bloom, J.S., Kulkarni, S.R., and Djorgovski, S.G., 2000, AJ, submitted (astro-ph/0010176)
- Bonnell, I. A., & Pringle, J. E., (1995), MNRAS, 273, L12
- Chen, X., Abramowicz, M., Lasota, J.P., Narayan, R., and Yi, I., 1995, ApJ 443, 61
- Eichler, D. Livio, M.; Piran, T. & Schramm, D. N., 1989, Nature, 340, 126.
- Frail, D. A., Kulkarni, S. R., Nicastro, L., Feroci, M., & Taylor, G. B. 1997, Nature, 389, 261
- Frail, D.A. et al., 2001, Nature, submitted (astro-ph/0102282)
- Frank, J.; King, A.; Raine, D., 1992, Accretion Power in Astrophysics, Cambridge Univ. Press
- Fryer, C. L.; Woosley, S. E. 1998, ApJ, 502L.
- Fryer, C.L.; Woosley, S. E.; Herant, M.; Davies, M.B. 1999, ApJ, 520, 650.
- Gammie, C. F., & Menou, K., 1998, ApJL, 492, L75
- Igumenshchev, I.V., Chen, X., and Abramowicz, M.A., 1996, MNRAS 278, 236
- Igumenshchev, I.V., and Abramowicz, M.A., 1999, MNRAS 303, 309
- Igumenshchev, I.V., and Abramowicz, M.A., 2000, ApJS 130, 463
- Igumenshchev, I.V., Abramowicz, M.A., and Narayan, R., 2000, ApJ 537, L271
- Kato, S., Fukue, J., & Mineshige, S., 1998, in "Black-hole accretion disks", Edited by Shoji Kato, Jun Fukue, and Sin Mineshige; Publisher: Kyoto University Press
- Katz, J., I., 1977, ApJ 215, 265
- Katz, J., I., & Piran, T., 1997, ApJ 490, 772
- MacFadyen, A. I. & Woosley, S. E., 1999, ApJ, 524, 262

- Nakar, E., & Piran, T., 2001a, MNRAS, submitted (astro-ph/0103192)
- Narayan, R., Igumenshchev, I.V., and Abramowicz, M.A., 2000, ApJ 539, 798
- Narayan, R., Mahadevan, R., and Quataert, E., 1998, "The Theory of Black Hole Accretion Discs", eds. M. A. Abramowicz, G. Bjornsson, and J. E. Pringle
- Narayan, R., Paczynski, B., & Piran, T., 1992, ApJ, 395L, 83
- Narayan, R., and Yi, I, 1994, ApJ 428, L13
- Narayan, R., and Yi, I, 1995, ApJ 444, 231
- Paczynski, B. 1991, Acta Ast, 41, 257
- Paczynski, B., 1998, ApJ, 494L, 45
- Panaiteanu, A., and Kumar, P., 2001, ApJ, in press (astro-ph/0010257)
- Piran, T., 1978, ApJ 221, 652
- Piran, T., 1999, Phys. Reports, 314, 575
- Piran, T., 2000, Phys. Reports, 333, 529
- Popham, R.; Woosley, S. E.; Fryer, C., 1999, ApJ, 518, 356
- Quataert, E., and Gruzinov, A., 2000, ApJ 539, 809
- Ruffert M., & Janka H.-Th., 1999, A & A, 344, 573
- Sakimoto, P. J., & Coroniti, F. V., 1981, ApJ, 247, 19
- Sari, R., Piran, T., 1997, ApJ, 485, 270.
- Shakura, N. I.; Sunyaev, R. A. 1973, A&A, 24, 337.
- Stone, J.M., and Pringle, J.E., 2000, MNRAS 322, 461
- Stone, J.M., Pringle, J.E., and Begelman, M.C., 1999, MNRAS 310, 100
- Usov, V. V. 1992, Nature, 357, 472.
- Woosley, S. E. 1993, ApJ, 405, 273.

Fig. 1.— Contours of t_{acc} in the r_{out} (in units of R_S) m_d (in solar masses) plane, for $m_3 = 1$ and $\alpha_{-1} = 1$. The contours are equally spaced in log. The lowest contour is at $\log(\text{quantity}) = -1.2$, and succeeding contours are shifted by $+0.2$ up to a maximum value of $+1$. Two contours are highlighted: the contour at -1 is shown as a bold solid line and the contour at 0 is shown as a bold dashed line. The three dotted lines correspond to three important transitions, computed with the analytical approximations of §§2,3. Starting from the right, the lines correspond to: (i) the transition from a CDAF to an NDAF (calculated via equation 20), (ii) the transition from a gas-pressure-dominated NDAF to a degeneracy-pressure-dominated NDAF (equation 31), and (iii) the transition from an NDAF that is optically thin to neutrino emission to one that is optically thick (equation 34). Our calculations are not reliable to the left of the leftmost dotted line since we have not allowed for optical depth effects in the model.

Fig. 2.— Contours of accretion efficiency $\xi = m_{\text{acc}}/m_d$ in the $r_{\text{out}}-m_d$ plane for $m_3 = 1$ and $\alpha_{-1} = 1$. The contour levels correspond to $\log \xi = 0, -0.2, -0.4, \dots, -1$ (solid lines) and $-1.2, \dots, -1.8$ (dotted lines). The white space on the left corresponds to the NDAF zone, where $\log \xi = 0$.

Fig. 3.— Contours of accretion efficiency $\xi = \dot{m}/\dot{m}_{\text{inj}}$ in the $r_{\text{out}}\text{-}\dot{m}_{\text{inj}}$ (in solar masses per second) plane. The contour levels and spacing are the same as in Fig. 2.