Compressed Statistical Testing and Application to Radar

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Abstract

We present compressed statistical testing (CST) with an illustrative application to radar target detection. We characterize an optimality condition for a compressed domain test to yield the same result as the corresponding test in the uncompressed domain. We demonstrate by simulation that under high SNR, a likelihood ratio test with compressed samples at 3.3x or even higher compression ratio can achieve detection performance comparable to that with uncompressed data. For example, our compressed domain Sample Matrix Inversion test for radar target detection can achieve constant false alarm rate (CFAR) performance similar to the corresponding test in the raw data domain. By exploiting signal sparsity in the target and interference returns, compressive sensing based CST can incur a much lower processing cost in statistical training and decision making, and can therefore enable a variety of distributed applications such as target detection on resource limited mobile devices.

Index Terms—compressed statistical testing, likelihood ratio test, Sample Matrix Inversion, pulse-Doppler radar, target detection, space-time adaptive processing (STAP), compressive sensing, compressed sampling, compressed domain processing.

I. INTRODUCTION

Sensor data is growing very fast in many application areas ranging from radar target detection to machine-to-machine (M2M) systems. As the speed of data capture continues to accelerate, it will become less practical for sensor front ends to incorporate significantly more computing, storage, and networking resources in order to keep up with ever-increasing data volume. It would be desirable to quickly compress sensor data and transmit compressed data to remote sites for processing. While it is easy to understand that we should only transmit, process and store needed data, it can be difficult to decide what data to retain.

By exploiting the fact that discrete-time signals in practice are usually sparse in some basis, compressive sensing offers an approach to mitigate this issue by allowing simple and blind data compression without having to comprehend the signal [1], [2]. Given a K-sparse vector x of length N with sparsity $K < N$, we can encode it with an $M \times N$ sensing matrix Φ to obtain a smaller vector of compressive measurements:

$$y = \Phi x \tag{1}$$

with $M < N$. By designing Φ to satisfy certain regularity properties such as the restricted isometry property (RIP), we can reconstruct x from y with high probability, where M can be as small as $M = c(K \log N/K)$ for some small constant $c > 0$.

However, signal reconstruction can be computationally expensive and cause significant delay, especially when N is large. It would therefore be of interest to conduct application-level processing directly on compressed samples. By working on data sets of significantly reduced size, compressed domain operation could greatly accelerate processing. For example, for an application algorithm of $\mathcal{O}(N^3)$ operations, a 2x compression will translate into 8x speedup in processing, since $2^3 = 8$. In this paper, we show that compressed domain processing is feasible for commonly used statistical testing methods such as the Sample Matrix Inversion (SMI) algorithm [8]. We call our approach *compressed statistical testing* (CST).

To illustrate CST, we consider a distributed radar target detection scenario, where sensing front ends transmit compressed raw radar data to back ends or end users for target declaration or other applications. As depicted in Figure 1, sensor front ends could be integrated into airborne or unattended ground systems communicating through an airborne platform, whereas a detection analysis could be performed by back end processors or mobile users on the ground. This scenario is in part motivated by the aforementioned desire to offload processing from sensing front ends in order to control their complexity in the face of growing sensing resolution and processing requirements, while being able to take advantage of computing resources or application contexts available at remote sites. Although sensor data acquisition rates may outpace the bandwidth of a communication link, as shown in Figure 1, by transmitting compressed data we can accommodate even disadvantaged communication links of relatively small bandwidth, such as air link connecting to airborne platforms. This paper further explains attractiveness of this offload approach by showing that the receiver can actually achieve a reduction in processing cost by performing application processing directly in the compressed domain.

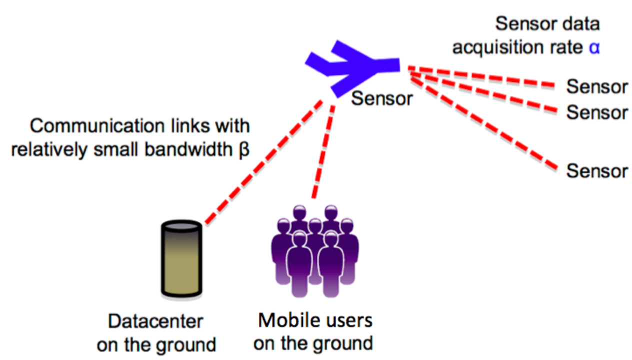

Fig. 1. To control the complexity of sensing front ends, we offload application processing of sensor data to back end datacenters and users on the ground. To address the situation that the sensor data acquisition rate α may outpace the link bandwidth β, we transmit compressed data obtained by using compressive sensing algorithms.

II. Compressive Sensing and Compressed Statistical Testing (CST): A Systems Perspective

Compared to other compression schemes (see, e.g., [3]), compressive sensing offers a number of system advantages to the sender (encoder) and receiver (decoder), including:

- Simple encoding at the sender. For encoding we can use a simple data-independent sensing matrix Φ consisting of randomly chosen entries, or one that is formed by randomly selected rows of a Discrete Fourier Transform (DFT) matrix. For example, if Φ is chosen to be a random Bernoulli matrix with $+1$ and -1 entries, then encoding can be accomplished with only plus and minus operations.
- Tolerance to packet loss. The receiver can perform detection processing or recover the original signal as long as a sufficient number, M, of compressive measurements have been received. It is not necessary to receive any particular measurements. This property is particularly useful in data transmission over disadvantaged links such as the aforementioned air link, which is inherently low-bandwidth and possibly unreliable.
- "Largest first" information processing. By processing just a few received measurements, the receiver can already reveal the location of the largest nonzero components, or the largest discrepancy or anomalies in the original signal vector. As more measurements are received, anomalies of smaller magnitude are recovered next. This will allow best-effort early detection or signal recovery. See [4] for a discussion of the largest first principle in the context of detecting server stragglers in cloud computing.
- Leveraging "sparsifying" knowledge at the receiver. The receiver can benefit from such knowledge by using a properly chosen sparsifying basis in decoding and detection. This means a smaller number of required measurements and earlier detection or recovery. The "sparsifying" knowledge only needs to be available to the receiver at the decoding time. The sender does not need it at the encoding time. See [5] for an example of exploiting sparsifying knowledge available to the receiver.

It is easy to see these advantages of compressive sensing in the context of distributed radar scenarios such as the one illustrated by Figure 1. CST exploits yet another important advantage of compressive sensing, namely, its support of compressed domain processing. That is, statistical training and detection tests can now be performed on compressed data vectors of length M rather than their original data vectors of length N, with $M \ll N$, resulting in substantial savings in processing cost and detection time. Consider, for example, our scenario depicted in Figure 1. The CST approach can now allow low-cost target detection at analytic back ends or in end-user applications.

III. Review of Space-Time Adaptive Processing in Radar Target Detection

We consider a multi-channel pulse-Doppler radar system to illustrate the compressed statistical testing (CST) approach of this paper. Here we briefly review a space-time adaptive processing (STAP) procedure for discrepancy detection by following the treatment given in [6]. To simplify the description, we assume an environment where interference is homogeneous across range cells.

Consider a data cube consisting of returned radar signals for STAP analysis, as depicted in Figure 2. We view the channel-pulse (#channels $\times$ #pulses) matrix at each range cell i as a multivariate random variable. Let x_i denote a sample of the random variable for range cell i.

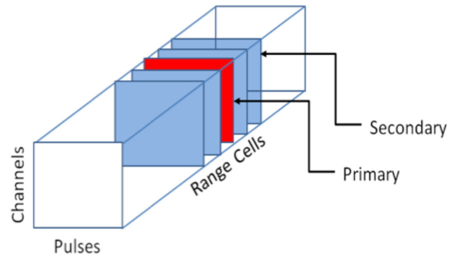

Fig. 2. Data cube for the STAP processing (note guard cells in [6] are omitted here to simplify the drawing and description).

The STAP procedure determines the adaptive beamforming weights for the radar receiver sensor array using a sample covariance matrix, as given by Eqn. (2):

$$\mathbf{w} = \hat{R}^{-1}\mathbf{s} \tag{2}$$

where $\mathbf{s}$ (steering vector) sets the "look direction," or the direction in angle and Doppler being tested for the presence of a target, and $\hat{R}$ (sample covariance matrix) is obtained by averaging over $\mathcal{L}$ secondary data samples chosen from range cells close to the range cell of interest (the primary range cell) as given by Eqn. (3) and illustrated in Figure 2.

$$\hat{R} = \frac{1}{\mathcal{L}}\sum_{i=1}^{\mathcal{L}} \mathbf{x}_i\mathbf{x}_i^H \tag{3}$$

Throughout the paper we assume that the sample covariance matrix $\hat{R}$ is invertible, which is generally true in practice when measurements are noisy.

Given a sample $\mathbf{x}_i$ for range cell i, we test for hypothesis H_1 versus H_0 with the following likelihood ratio test (LRT):

$$r = \frac{p(\mathbf{x}_i|H_1)}{p(\mathbf{x}_i|H_0)} \tag{4}$$

where H_1 or H_0 is the hypothesis concerning the presence or absence of a target at range cell i, respectively; and $p(\mathbf{x}_i|H_1)$ and $p(\mathbf{x}_i|H_0)$ is the conditional probability of $\mathbf{x}_i$ given that H_1 or H_0 holds, respectively.

If $\mathbf{x}_i$ has a multivariate Gaussian distribution, then H_1 or H_0 is the hypothesis that $\mathbf{x}_i$ has a multivariate distribution with a nonzero or zero mean, respectively, and covariance equal to $\hat{R}$ given by Eqn.(3). We know from the literature [7] that the test of Eqn. (4) applying a steering vector $\mathbf{s}$ amounts to the following Sample Matrix Inversion (SMI) test [8] for comparing the detector d_i against a discrimination threshold:

$$d_i = \mathbf{s}^H\hat{R}^{-1}\mathbf{x}_i \tag{5}$$

If sample $\mathbf{x}_i$ for range cell i passes the test, then a discrepancy is detected in the cell at the location pointed by $\mathbf{s}$. For simplicity, in this paper we focus on the case where at most one target can be present in any range cell.

IV. Compatible Compression and Compression Ratio

Given that our target detection is commensurate with the granularity of range cells, we compress the channel-pulse sample associated with each range cell separately. That is, let $\mathbf{x}_i$ denote an *original sample* of the channel-pulse matrix at range cell i. We will compress $\mathbf{x}_i$ by computing

$$\mathbf{y}_i = \Phi \mathbf{x}_i \tag{6}$$

where $\mathbf{x}_i$ is a vector of length $N =$ #channels $\times$ #pulses, Φ is an $M \times N$ sensing matrix satisfying some regularity conditions such as the restricted isometry property (RIP) with $M < N$, and $\mathbf{y}_i$ is a vector of length M. We call $\mathbf{y}_i$ the *compressed sample* of $\mathbf{x}_i$ for range cell i. For simplicity, we assume for all range cells, the same Φ is used.

We say this compression of Eqn. (6) is *compatible* in the sense that if a target is present only in $\mathbf{x}_i$ then it also reflects in $\mathbf{y}_j$ but not in any other $\mathbf{y}_j$ with $j \neq i$. We assume in the rest of this paper that compatible compression is used. Note that each $\mathbf{y}_i$ consists of only M compressive measurements with $M < N$, compared to the corresponding x_i which has N components. The resulting *compression ratio* is defined as N/M. For example, if $N/M = 5$, then a $5:1$ or 5x compression ratio is achieved.

V. Compressed Statistical Testing

We now describe our CST approach. Suppose that we are given a compressed sample $\mathbf{y}_i$ computed from the original sample $\mathbf{x}_i$ for range cell i according to Eqn. (6). Corresponding to the LRT of Eqn. (4) in the original domain, we perform the LRT in the compressed domain:

$$\tilde{r} = \frac{p(\mathbf{y}_i|H_1)}{p(\mathbf{y}_i|H_0)} \tag{7}$$

Furthermore, we can perform the compressed domain SMI test that corresponds to the Sample Matrix Inversion (SMI) test of Eqn. (5):

$$\tilde{d}_i = \mathbf{t}^H \hat{G}^{-1} \mathbf{y}_i \tag{8}$$

where $\mathbf{y}_i$=$\Phi\mathbf{x}_i$, $\mathbf{t} = \Phi\mathbf{s}$ and $\hat{G}$ is computed as below:

$$\hat{G} = \frac{1}{\mathcal{L}} \sum_{i=1}^{\mathcal{L}} \mathbf{y}_i \mathbf{y}_i^H \tag{9}$$

A design objective for compressed statistical testing is to have $\tilde{d}_i$ approach d_i in their values.

Note that for CST it is possible to use a smaller set of samples for training due to the reduced dimensionality. For presentation simplicity we use the same $\mathcal{L}$ as in Eqn.(3) in computing $\hat{G}$ in Eqn. (8).

VI. Relationship between Compressed and Original Statistical Testing

We examine how testing in the compressed domain is related to that in the original raw data domain. Recall that N or M is the size of the original sample $\mathbf{x}_i$ or the corresponding compressed sample $\mathbf{y}_i$ derived by Eqn. (6), respectively. We show that the following statements hold:

- **Statement S1**

  Suppose that $\mathbf{x}_i$ is K-sparse and M is sufficiently large (relative to K) to allow exact recovery of $\mathbf{x}_i$ from $\mathbf{y}_i$. Then with high probability, the compressed domain LRT of Eqn. (7) yields the same result as the original domain LRT of Eqn. (4).
- **Statement S2**

  Under the assumption that $\mathbf{x}_i$ is multivariate Gaussian, the compressed domain LRT of Eqn. (7) is the same as the compressed domain SMI test of Eqn. (8).
- **Statement S3**

  Under the assumption that $\mathbf{x}_i$ is multivariate Gaussian and the assumption of S1 about sufficiently large M, the SMI test of Eqn. (8) based on compressed samples is an optimal compressed domain test for CFAR performance. (This statement corresponds to the well-known result that the SMI test is an optimal test for CFAR performance under the Gaussian assumption [9].) Moreover, for any desired CFAR performance assured by the SMI test in the original domain, there is a compressed domain SMI test that can achieve the same performance under compression with a sufficiently large $M \leq N$.
- **Statement S4**

  The sample covariance matrix $\hat{G}$ of Eqn. (9) formed in the compressed domain has the following property:

  $$\hat{G} = \Phi \hat{R} \Phi^H \tag{10}$$

  where $\hat{R}$ is the sample covariance matrix in the original domain.

Notice that, unlike the other statements, statement S1 holds without the Gaussian assumption. In addition, note that regularity conditions on the sensing matrix Φ such as RIP are needed only for assuring exact recovery with high probability (or approximate recovery, in a generalized result) of $\mathbf{x}_i$ from $\mathbf{y}_i$. Other parts of these statements hold without such regularity conditions.

A. *Proof for S1*

When M is sufficiently large, $\mathbf{x}_i$ is uniquely determined by $\mathbf{y}_i$ with high probability. Thus following hold with high probability:

$$p(\mathbf{y}_i|H_0) = p(\mathbf{x}_i|H_0) \tag{11}$$

$$p(\mathbf{y}_i|H_1) = p(\mathbf{x}_i|H_1) \tag{12}$$

By Eqn. (11) and (12),

$$\frac{p(\mathbf{y}_i|H_1)}{p(\mathbf{y}_i|H_0)} = \frac{p(\mathbf{x}_i|H_1)}{p(\mathbf{x}_i|H_0)} \tag{13}$$

B. Proof for S2

If $\mathbf{x}_i$ is multivariate Gaussian, so is $\mathbf{y}_i$, as the latter is a linear projection of the former according to Eqn. (6). That is, if

$$p(\mathbf{x}_i|H) \sim \begin{cases} H_0 : \mathcal{N}(0, R) \\ H_1 : \mathcal{N}(\mu, R) \end{cases} \tag{14}$$

for a certain mean μ and covariance R, then

$$p(\mathbf{y}_i|H) \sim \begin{cases} H_0 : \mathcal{N}(0, \Phi R \Phi^H) \\ H_1 : \mathcal{N}(\Phi\mu, \Phi R \Phi^H) \end{cases} \tag{15}$$

Since $\mathbf{y}_i$ is multivariate Gaussian, the test under steering vector $\mathbf{t} = \Phi\mathbf{s}$ is equivalent to the SMI test of Eqn. (8), as described in [8].

C. Proof for S3

As noted earlier, when $\mathbf{x}_i$ is Gaussian, the LRT in the original domain is the same as the SMI test. By statement S1 we know that when M is sufficiently large relative to K, the compressed domain LRT of Eqn. (7) yields the same result as the original domain LRT of Eqn. (4) with high probability. This means that the SMI test using compressed samples will be an optimal test in the compressed domain for CFAR performance. In addition, for any desired CFAR performance assured by the SMI test in the original domain, there is a compressed domain SMI test for assuring the same performance under compression with a sufficiently large $M \leq N$.

D. Proof for S4

Consider the sample covariance matrix of Eqn. (9) formed by compressed samples $\mathbf{y}_i$. We note that

$$\begin{aligned} \frac{1}{\mathcal{L}} \sum_{i=1}^{\mathcal{L}} \mathbf{y}_i \mathbf{y}_i^H = & \ \tfrac{1}{\mathcal{L}} \textstyle\sum_{i=1}^{\mathcal{L}} (\Phi\mathbf{x}_i)(\Phi\mathbf{x}_i)^H \\ = & \ \tfrac{1}{\mathcal{L}} \Phi \mathcal{L} \hat{R} \Phi^H = \Phi \hat{R} \Phi^H \end{aligned} \tag{16}$$

Thus Eqn. (10) holds. As shown in the next section, we can derive an optimality condition for the compressed domain detector $\tilde{d}$ based on Eqn. (10).

VII. Characterizing Optimal Compressed Domain Detector and High Performing Steering Vectors

We characterize a condition under which the compressed domain detector $\tilde{d}_i$ will yield the same value as the original detector d_i on the raw data domain. We establish this result by characterizing those steering vectors $\mathbf{s}$ for which d_i and $\tilde{d}_i$ are the same for any sample $\mathbf{x}_i$ for range cell i. We then apply this result to the design of the optimal sensing matrix Φ.

Consider an $N \times K$ matrix $\mathbf{S}$ with its columns being K steering vectors of interest. Suppose that

$$\Phi = \mathbf{S}^H \hat{R}^{-1} \tag{17}$$

For any sample $\mathbf{x}_i$ in the original domain, let

$$\mathbf{y}_i = \Phi \mathbf{x}_i \tag{18}$$

Corresponding to Eqn. (5) and (8), recall that for any column vector $\mathbf{s}$ in the matrix $\mathbf{S}$,

$$d_i = \mathbf{s}^H \hat{R}^{-1} \mathbf{x}_i \tag{19}$$

$$\tilde{d}_i = (\Phi \mathbf{s})^H \hat{G}^{-1} (\Phi \mathbf{x}_i) \tag{20}$$

By Eqn. (10) and (17),

$$\begin{aligned}(\Phi \mathbf{x}_i)^H \hat{G}^{-1} (\Phi \mathbf{S}) =& (\Phi \mathbf{x}_i)^H (\Phi \hat{R}^{-1} \Phi^H)^{-1} (\Phi \mathbf{S}) \\ =& (\Phi \mathbf{x}_i)^H \left(\Phi \hat{R}^{-1} (\mathbf{S}^H \hat{R}^{-1})^H \right)^{-1} (\Phi \mathbf{S}) \\ =& (\mathbf{x}_i^H \hat{R}^{-1} \mathbf{S})(\Phi \hat{R} \hat{R}^{-1} \mathbf{S})^{-1} (\Phi \mathbf{S}) \\ =& \mathbf{x}_i^H \hat{R}^{-1} \mathbf{S} \end{aligned} \tag{21}$$

By Eqn. (19), (20) and (21),

$$d_i = \tilde{d}_i \tag{22}$$

for any column vector $\mathbf{s}$ of $\mathbf{S}$.

We see that detector $\tilde{d}_i$ in the compressed domain attains value d_i when Eqn. (17) holds, or when the following holds:

$$\mathbf{S}^H = \Phi \hat{R} \tag{23}$$

Thus we call those steering vectors which are column vectors of $(\Phi \hat{R})^H$ *high performing* steering vectors. Note that for a high performing steering vector $\mathbf{s}$, equality of Eqn. (22) holds for any sample $\mathbf{x}_i$. Thus d_i and $\tilde{d}_i$ have the same distribution.

The characterization of Eqn. (23) has implications in the design of the optimal sensing matrix Φ. For example, for the case when $\hat{R}$ is an identity matrix or close to it, if Φ is a subset of rows of a DFT matrix, then $\tilde{d}_i$ is maximized for those steering vectors corresponding to these rows. This means if certain steering vectors are known to be of interest a priori, then we should choose rows of the DFT matrix accordingly to form the sensing matrix Φ. On the other hand, if such knowledge about interested steering vectors is unavailable at design time, then use of a random Φ could be appropriate. A full treatment of this subject is beyond the scope of this paper. We plan to address this in a future paper.

VIII. Performance Evaluation with Numerical Simulation

We have simulated compressed statistical testing for a "simple single target" radar detection scenario. In the simulation the dimension of $\mathbf{x}_i$ for each range cell is $N = 500$. To measure detection performance, we use the receiver operating characteristic (ROC) plot of the detection rate vs. false alarm rate.

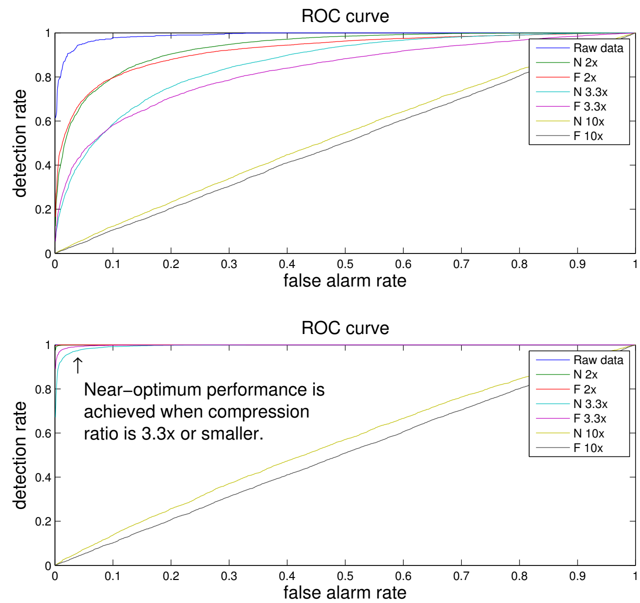

Fig. 3. ROC performance comparison between SMI test in the original and compressed domains under two different levels of interference (upper plot: SNR ~ 3.3 and lower plot: SNR ~ 8.7). Comparison results under various compression ratios are reported. Compression with the sensing matrix based on the DFT matrix or random Gaussian matrix is flagged by $\mathcal{F}$ or $\mathcal{N}$, respectively.

A. *Compressed Domain SMI Test Performance*

Figure 3 compares detection performance of the SMI test performed in the original vs. compressed domain under various compression ratios and two different interference levels. A randomly selected steering vector **s**, which is not a high performing one as defined by Eqn. (23), is used in the test. The SNR of the detector d_i or $\tilde{d}_i$ is defined as:

$$SNR = \frac{\mu}{\sigma} \tag{24}$$

where μ is the mean of the signal and σ is the standard deviation of the noise in the detector. Results of Figure 3 show that compressed domain SMI test has better performance under higher SNR.

We notice from Figure 3 that compressed domain performance increases when M increases or, equivalently, compression ratio decreases. In practice, we may want to weigh the tradeoff between the achieved compression ratio and the CFAR performance based on application needs.

The blue curve in Figure 4 shows the SNR of the detector under the same setting as the lower plot in Figure 3. For any desired CFAR detection, we can always find some $M \leq N$ that achieves the desired performance. In this example, we see that with a random Gaussian sensing matrix, compressed SMI test can achieve greater than 90% detection and less than 5% false alarm at a compression ratio of 3.3x (see the $\mathcal{N}$: 3.3x curve in lower plot of Figure 3). This means computing the SMI test of Eqn. (8) in the compressed domain would be about $3.3^3 \sim 36$ times faster than that of Eqn. (5) in the raw data domain, assuming that matrix inversion is an $\mathcal{O}(N^3)$ computation.

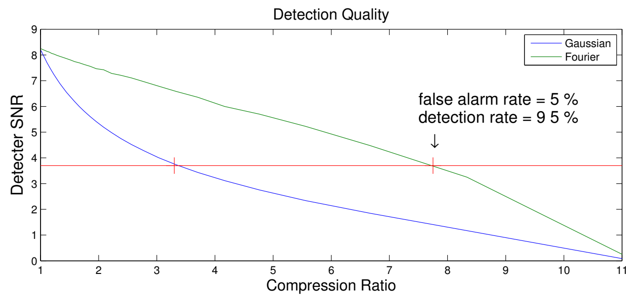

Fig. 4. SNR of the detector $\tilde{d}_i$ in Eqn. (8) with varying compression ratios under a random Gaussian sensing matrix (blue). SNR for a high performing steering vector under the Fourier sensing matrix (green).

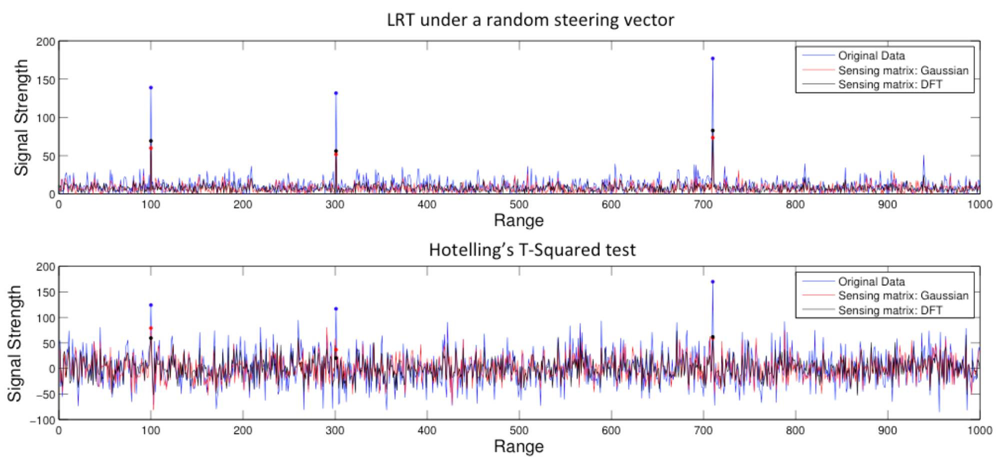

Fig. 5. Detection performance using original samples and compressed samples under 2x compression ratio for the likelihood ratio test (LRT) and Hotelling's T-Squared test. (Upper figure: SNR $\sim$ 17.6 and lower figure: SNR $\sim$ 16.2.)

B. Compressed Domain SMI Test Performance for High Performing Steering Vectors

In this simulation, we compare the performance of a normal steering vector and a high performing steering vector as defined in Section VII. For the high performing steering vector, we use a vector $\mathbf{s}$ with the property that $\mathbf{s}^H$ is a column of a Fourier sensing matrix.

The detector SNR under these two steering vectors are shown in Figure 4. We note that with the Fourier sensing matrix the SNR is much higher when the steering vector $\mathbf{s}$ is a high performing steering vector. In this case, the compressed domain SMI test could achieve the same high CFAR performance as in the case of the Gaussian sensing matrix at a much higher compression ratio ~ 7.8x.

C. Hotelling's T-Squared Test Under Compressed Domain

In Figure 5 we compare performance of compressed vs. original SMI test for the likelihood ratio test (LRT) and Hotelling's T-Squared test at 2x compression ratio. For the latter test, we compare $|\mathbf{x}_i{}^H \hat{R}^{-1} \mathbf{x}_i|$ obtained in the original domain and $|\mathbf{y}_i^H \hat{Q}^{-1} \mathbf{y}_i|$ obtained in the compressed domain. We see that when SNR is relatively large, compressed domain LRT test achieves good detection performance. Note that Hotelling's T-Squared test does not rely on directivity gain achieved through application knowledge of steering vectors. It exhibits less detection performance in the simulation result, as one would expect.

IX. Concluding Remarks and Future Work

We have described a novel compressed statistical testing (CST) approach, characterized a condition when CST achieves optimal performance, and demonstrated by simulation that it can achieve performance comparable to testing in the raw data domain at compression ratios as large as 3.3x or even higher. This paper focuses only on the SMI test due to space limitation. As shown in Section VIII-C, similar results hold for other tests such as the Hotelling's T-squared test. By integrating compressive sensing reconstruction algorithms, our work can be extended to simultaneous detection of multiple targets, and to estimate the sparsity of a signal. Other future work includes generalizing the notion of compatible compression to arbitrary slides of the data cube beyond range cells, and use of possibly different sensing matrices for these slides.

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