Number-space mapping in human infants

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Abstract

Mature representations of number are built on a core system of numerical representation that connects to spatial representations in the form of a ‘mental number line’. The core number system is functional in early infancy, but little is known about the origins of the mapping of numbers onto space. Here we show that preverbal infants transfer the discrimination of an ordered series of numerosities to the discrimination of an ordered series of line lengths. Moreover, infants construct relationships between individual numbers and line lengths that vary positively, but not between numbers and lengths that vary inversely. These findings provide evidence for an early developing predisposition to relate representations of numerical magnitude and spatial length. A central foundation of mathematics, science and technology therefore emerges prior to experience with language, symbol systems, or measurement devices.
The mapping of number to space is fundamental to measurement, mathematics, and science, and is evident whenever processing of number occurs (de Hevia, Vallar & Girelli, 2008; Hubbard, Piazza & Pinel, 2005). In adults, this mapping takes the form of a ‘mental number line’: an analogue continuum in which numerical magnitude is represented along an oriented axis (Dehaene, 1992; Restle, 1970). Number lines appear to be universal across humans, although there is cultural variability in their direction (oriented left-to-right in most western cultures: Dehaene, Bossini & Giraux, 1993) and spacing (equal intervals for western adults under many conditions, but logarithmically spaced intervals for children and adults in a remote community lacking formal education: Dehaene, Izard, Spelke & Pica, 2008; Siegler & Opfer, 2003). Number lines are activated even when adults perform no relevant numerical task, enhancing their responses to numbers whose value accords with the spatial position of the response (the SNARC effect: Dehaene et al., 1993).

Although adults’ numerical and spatial representations build on systems of number and geometry that are functional in early infancy (Brannon, 2002; Newcombe & Huttenlocher, 2000; Xu & Spelke, 2000), little is known about the developmental origins of the capacity to relate these representations. Some evidence suggests that number-space mappings develop through the acquisition of culture-specific skills and formal education: Children show evidence of an oriented number line only some years after they begin schooling (Berch, Foley, Hill & Ryan, 1999; van Galen & Reitsma, 2008), and the orientation of this representation is modulated by the orientation of the culture’s writing system (Zebian, 2005). Nevertheless, humans may be predisposed to treat space and number as intrinsically related. Adults with little or no formal education map numbers
onto space when asked to place quantities on a horizontal line, revealing their internal organization of magnitude (Dehaene et al., 2008; though see Cantlon, Cordes, Libertus & Brannon, 2009; Dehaene, Izard, Pica & Spelke, 2009). Moreover, western preschool children spontaneously map numbers onto space when presented with a visuo-spatial task (de Hevia & Spelke, 2009), and they even display an intuition of the left-to-right organization of numerical magnitude (Opfer & Thompson, 2006).

Here we hypothesize that the number-space connection originates in early-developing predispositions to relate these two dimensions. We tested this hypothesis by assessing eight-month-old infants’ response to numerical and spatial magnitudes with three methods.

**Experiment 1**

The first experiment used a habituation/novelty preference method to probe infants’ sensitivity to relationships between increasing or decreasing sequences of numbers (presented as visuo-spatial arrays) and lengths (presented as horizontal lines). Infants were habituated to repeating sequences of arrays of visual elements that successively doubled or halved in number (from 4 to 64 or the reverse), while the non-numerical properties of item and array size were controlled. Envelope size was held constant during habituation, as envelope size and density cues were absent at test. After looking time to the sequences declined, infants were presented with sequences of horizontal lines that successively doubled or halved in length on alternating test trials (Figure 1a). If infants naturally map representations of number to space, they should
detect the order of magnitude change in both dimensions and generalize habituation from increasing (or decreasing) number to increasing (or decreasing) length.

Methods

Participants were 24 full-term infants (13 female, mean age 8 months, range: 7 months 16 days to 8 months 15 days). Fifteen more infants were eliminated because of crying (8), experimenter error (3), equipment failure (2), or test trial looking times exceeding 3 standard deviations from the overall group mean (2).1

Materials. Numerical displays were composed of colored circles, squares, or equilateral triangles. For each trial, a single figure and color were presented. The continuous variables that could serve to relate the numerical displays to the line displays (i.e., the summed area occupied by the figures and the total envelope area) were controlled. Summed area was equated across displays by varying item size inversely to number: for arrays of 4, 8, 16, 32, and 64, respectively, circle diameters were 3.8°, 2.7°, 1.9°, 1.3°, and 0.9°, square lengths were 3.4°, 2.4°, 1.7°, 1.2°, and 0.8°, and triangle sides were 4.8°, 3.4°, 2.4°, 1.7°, and 1.2°. Envelope area was equated across displays by positioning items randomly within a fixed area. Test displays consisted of centrally positioned rainbow-colored rectangles of constant height (1.2°) and varying lengths (1.3°, 2.7°, 5.3°, 10.5°, and 20.4°) that changed symmetrically in five successive steps by doubling or halving.

Each numerical trial consisted of a repeating cycle (9 s in total) that began with the image of a dog moving with a concurrent noise (1000 ms), followed by a blank screen (500 ms) and then by the series of five numerical displays (1200 ms each) separated by a
blank screen (300 ms; total sequence length, 7500 ms). Each line length trial was identical to the numerical trials, except that the displays consisted of the line lengths.

**Design.** Prior to the habituation trials, infants were familiarized with both increasing and decreasing line lengths, in order to avoid a general novelty response during the test trials due to the introduction of displays with new colors and shapes. All infants were presented with two familiarization trials, consisting of the increasing/decreasing line lengths presented at test. Half of the infants, randomly assigned, were then habituated to an ascending numerical sequence and the other half to a descending numerical sequence. The order of familiarization and test trials was counterbalanced across infants within each habituation group.

**Procedure.** Infants were seated on a parent’s lap and faced a screen surrounded by black surfaces and curtains in a softly illuminated room. Parents were instructed to refrain from interacting with the infant, and they closed their eyes during the test sequence. A camera below the screen was directed to the infant’s face, and a display camera was placed behind the infant to record the displays. The video cameras were mixed onto a TV monitor and a VCR in a separate room, where an observer recorded the infant’s looking times with the display portion of the monitor occluded to ensure that observers were blind to the habituation and test conditions. For each infant, two observers coded the data live or from videotape, with an averaged inter-coder reliability of 91%.

At the beginning of each trial, a black occluder was lifted to reveal a white 57 cm x 48 cm screen on which images were back-projected, resulting in an image measuring 21.4° x 25.4° of visual angle at a viewing distance of about 60 cm. Infants first were
presented with two familiarization trials in which each test display was visible until the infant had looked for 20 s. For the remainder of the experiment, each display remained visible until the infant looked for at least 0.5 s, and ended when infants looked away for 2 s continuously (or for a maximum look of 120 s). Habituation trials continued until the infant either received 14 trials or reached the criterion of a 50% decline in looking time on three consecutive trials, relative to the total looking time on the first three trials that summed to at least 12 seconds. If the infant did not meet this criterion after six trials, the displays were cycled in the same order until the end of habituation. Following the habituation sequence, all infants were shown six trials in which the two test displays appeared in alternation.

**Analyses.** Infants’ looking times during test trials were submitted to an ANOVA with habituation condition (increasing vs. decreasing) and test order (familiar first vs. second) as between-subjects variables, and test trial type (familiar vs. novel) as within-subjects variable. All other tests are two-tailed.

**Results and Discussion**

Infants looked longer at the test trials with the reversed ordering of lines (e.g., from increasing numbers of dots to decreasing line lengths), relative both to the test trials with the congruent ordering of lines (12.7 s to reversed order vs. 8.9 s to congruent order, \(t_{23} = 2.37, p = .03, d = .52\)), and to the final habituation trials (\(t_{23} = 2.65, p = .01\); Fig. 1b). The novelty preference was shown by 18/24 infants \(z = 2.24, p = .02\), sign test; \(z = 2.48, p = .01\), Wilcoxon signed-rank test), and influenced infants’ test trial preferences \(F_{1,20} = 4.65, p = .04, \eta_p^2 = .19\) independently of the ordering of numbers presented during habituation \(F < 1, \text{n.s.}\). No other effects or interactions were significant. Infants
therefore generalized an increment or decrement in number to an increment or decrement in length.

Because the first experiment presented numbers and lines that halved or doubled repeatedly in a predictable order, it is possible that these events called infants’ attention to the numerical and spatial orderings, yielding an amodal transfer of ordinal meaning from one dimension to the other. The next experiment aimed to replicate and extend the findings from Experiment 1 under conditions that precluded such ordinal transfer. We tested whether infants would learn a specific, positive relationship between numbers and lengths and generalize the relationship to new numerical and spatial values.

**Experiment 2**

Infants viewed a series of displays of an array of dots above a horizontal line, in a quasi-random, unordered sequence. Across trials, the dots varied in number and the line varied in length, such that longer lines accompanied greater numbers of dots. Following familiarization, infants were shown two test trials presenting new numbers and line lengths, paired positively or inversely (shorter lines accompanying greater numbers of dots) (Fig. 2a). To ensure that infants learned relations between lengths and numbers rather than between lengths and item sizes, array sizes, or filled display area, the latter spatial variables were controlled (see below). If infants extract the rule that positively relates number to line length, they should apply that rule to the test exemplars and display a preference for the new pairings conforming to the extracted rule, despite the absence of any correlated spatial variable common to the familiarization and test displays.

**Methods**
Participants were 20 infants (five female, mean age 8 months 3 days, range, 7 months 18 days to 8 months 19 days). Twelve more infants were eliminated because of crying (5), parental interference (2), or test trial looking times more than 3 standard deviations from the overall group mean (5).

Materials. The familiarization displays consisted of three of the lines and element arrays of Exp. 1: the largest, smallest, and middle values. In the numerical displays, the total overall area of the visual elements was kept constant across numerosities, at half the size of the elements in Exp. 1. Summed area and array size were equated by varying item size inversely to number (number therefore was correlated with item size during familiarization). To ensure that infants responded to number, these spatial properties of the numerical displays changed between familiarization and test. In the test displays, item size and array size were equated (number therefore was correlated only with summed area: a dimension that did not covary with length during familiarization). The test displays consisted of the remaining lines of Exp. 1: i.e., 2.7° and 10.5°, paired with arrays of 8 or 32 dots that were systematically altered so as to keep constant the elements’ size (dot’s diameter size for numerosities ‘8’ and ‘32’ was 1.3°). The numerical and spatial values presented during test therefore were novel but lay within the range of the values presented during familiarization.

For each familiarization trial, three different images for each number-line pairing were arranged in a pseudorandom order, so that consecutive numerosities did not follow any predictable order (e.g., 16, 64, 4, 16, 4, 64, 16, 4, 64). For each test trial, three different images for each number-line pairing were presented in alternation, starting with ‘8’. Element position varied across trials. Each trial consisted of a repeating cycle
beginning with the presentation of a numerical display centered in the upper half of the screen (1000 ms), joined by the line centered on the lower half of the screen (1000 ms), followed by a blank screen (500 ms), and the next display. Each cycle lasted 22.5 s during familiarization and 15 s during the test; cycles were looped until the end of each trial.

**Design.** Infants were familiarized with the number-line pairings following a positive rule. Half the infants were tested first with the positive number-length pairing and half with the inverse pairing.

**Procedure and analyses.** Once infants were seated, learning trials began, following the same procedure as Experiment 1. After the learning sequence, all infants were given two test trials. For 19 of 20 infants, two observers coded the data live or from videotape, with an averaged inter-coder reliability of 95%. Test trial looking times were submitted to an ANOVA with trial order as the between-subjects variable, and trial type (positive vs. inverse pairing) as the within-subjects variable. All other tests are two-tailed.

**Results and Discussion**

Infants received on average 8.3 familiarization trials, exhibiting habituation from the first three (16.6 s) to the last three trials (5.9 s) \(t_{19} = 7.8, p < .0001\). Relative to the last three familiarization trials, infants showed no dishabituation to the positive \(t_{19} = -1.51, \text{n.s.}\) or inverse test displays \(t_{19} = 1.28, \text{n.s.}\). Nevertheless, infants looked longer at the positive test display (7.9 s to positive pairing vs. 4.8 to inverse pairing; \(t_{19} = 3.12, p = .005, d = .77; \text{Figure 2b}\)). This effect was shown by 15/19 infants (one infant looked
equally; \( z = 2.29, p = .02 \), sign test; \( z = 2.77, p = .005 \), Wilcoxon signed-rank test) and was the only tested variable affecting looking times \( (F_{1,18} = 9.48, p = .006, \eta^2_p = .34) \).

These findings provide evidence that infants learned the number/length relationship in the familiarization displays and generalized this relationship to the new numbers and lengths in the test displays. Because all the spatial variables that can covary with number in visual arrays were equated during familiarization or test, infants’ generalization must have depended on abstraction of a relationship between line length and element number. Thus, infants were sensitive to the positive number-length mapping.

**Experiment 3**

Experiment 2 reveals that infants detect a positive relationship between number and length, but it does not indicate whether this relationship is favored over an equally consistent inverse relationship. From birth, infants are able to learn arbitrary relationships between events (Fiser & Aslin, 2002; Siqueland & Delucia, 1969). Thus, Experiment 3 used the same methods and displays, except that numbers and lengths were inversely related, with longer lines accompanying smaller numbers of visual elements (Figure 2a).

**Methods**

The methods were the same as in Experiment 2, except as follows. Participants were 20 infants (eight female, mean age 8 months 2 days, range, 7 months 17 days to 8 months 26 days). Nine more infants were eliminated because of crying (6), parental interference (1), or excessive test trial looking times (2). The familiarization arrays from Experiment 2 were presented in a consistent inverse relationship, such that the shortest
line accompanied the largest numerosity. For 18 of 20 infants, two observers coded the data live or from videotape, with an averaged inter-coder reliability of 95%.

**Results and Discussion**

Infants received on average 7.9 familiarization trials, exhibiting habituation from the first three (19 s) to the last three trials (6.6 s) \( t_{19} = 7.5, p < .0001 \). Again, no dishabituation effects were observed for the new displays showing the positive pairing \( t_{19} < 1, \text{n.s.} \) or the inverse pairing \( t_{19} = -1.24, \text{n.s.} \). Infants also showed no preference between the test displays (7.4 s to positive pairing vs. 8.2 to inverse pairing; \( t_{19} = -0.55, \text{n.s.; Figure 2b} \). The rule-consistent preference was shown by only 11/19 infants (one infant looked equally; \( z = 0.45, \text{n.s., sign test; } z = 0.56, \text{n.s., Wilcoxon signed-rank test} \), and did not affect infants’ test trial looking times \( (F < 1) \). Thus, infants did not generalize the inverse mapping between number and length to the new test displays.

Further analyses compared infants’ looking patterns across Experiments 2 and 3. Infants showed similar looking times across the two experiments on the first three and last three familiarization trials (each \( t_{38} < 1, \text{n.s.} \), and they reached the habituation criterion after similar numbers of trials \( t_{38} < 1, \text{n.s.} \). On the test trials, in contrast, the interaction between Experiment (2 vs. 3) and Test Display (positive vs. inverse pairing) was significant \( (F_{1,38} = 5.03, p = .03, \eta^2 = .12) \). Looking times to the positive test pairing in Experiment 2 (7.9 s) were comparable to those for both the positive and the inverse pairings in Experiment 3 (7.4 and 8.2 s, each \( t < 1, \text{n.s.} \).

Might infants have a baseline preference for the positive number-length pairing that masked their successful learning of the inverse relation and boosted their apparently
successful learning of the positive relation? The final experiment tested for this possibility.

Experiment 4

Method

Participants were 20 infants (11 female, mean age 8 months 6 days, range 7;16-8;21). Three additional infants were replaced for excessive looking. The method was the same as Experiments 2 and 3 except that no familiarization sequence was presented and the two test displays appeared three times in alternation. Looking times were analyzed by an ANOVA with test trial pair (1-3) and test display (positive vs. inverse) as within-subjects variables and test order (positive first vs. second) as a between-subjects variable. Further ANOVAs and two-tailed t tests compared looking patterns across Experiments 2-4.

Results

Infants showed greater looking overall at the positive pairing ($F_{1,18} = 12.95, p = .002, \eta_p^2 = .32$), but this effect was complicated by a significant three-way interaction of test trial pair x test display x test order ($F_{2,36} = 8.61, p < .001, \eta_p^2 = .32$). LSD post hoc tests revealed a preference for the positive over the inverse pairing only on the first trial pair for infants who received the positive pairing first ($p < .0001$; all other $ps > .13$):

Thus, infants looked longest on the first trial and at the positive pairing.

We next compared the averaged percent looking time to the positive pairing in Experiment 4 to that in Experiments 2 and 3. Because of the order effect, separate one-way ANOVAs were conducted for each trial pair of Experiment 4. The difference in looking preferences across experiments was only marginally significant for the first trial
pair of Exp. 4, $F_{2,57} = 2.84, p = .06, \eta_p^2 = .09$, but was reliable for the second pair, $F_{2,57} = 3.14, p = .05, \eta_p^2 = .10$, and for the third pair, $F_{2,57} = 3.87, p_{rep} = .02, \eta_p^2 = .12$. Looking preferences in Experiment 2 differed significantly from those shown on the third pair of the baseline experiment ($t_{18} = 2.59, p = .01$). Looking preferences in Experiment 3 did not differ from baseline on any trial pair (all $p > .14$; Figure 3).

**Discussion**

The findings of Experiment 4 reveal that infants have an intrinsic preference for positive over inverse pairings of numbers and lengths. Nevertheless, this preference does not account either for the positive findings of Experiment 2 or for the negative findings of Experiment 3. Once infants’ long looking at the early baseline trials diminished, the infants in the baseline condition showed significantly less preference for the positive pairing than those in Experiment 2, providing evidence that the infants in Experiment 2 showed successful learning and generalization of the positive pairing between numbers and lengths. Moreover, the looking preferences of infants in the baseline experiment never differed from those of the infants in Experiment 3, who were given the opportunity to learn the negative pairing. Infants therefore learned to relate numbers to lengths when they varied positively but not inversely.

**General Discussion**

Human infants are sensitive to relationships between number and length, and they revealed this sensitivity in three ways. First, infants generalized from an increasing (or decreasing) sequence of numbers to an increasing (or decreasing) sequence of line lengths. Second, infants abstracted a specific positive (but not inverse) relationship between number and length from a small number of examples, and they generalized the
relationship to new values. Third, infants showed an intrinsic preference for numbers and lengths that are positively rather than inversely related.

These abilities were revealed through three contrasting patterns of looking time. When infants were habituated to an increasing or decreasing sequence of numbers, their interest in the sequence declined, yielding lower looking times at new line lengths that exhibited the same ordinal change (Experiment 1). When infants were allowed to learn a specific relation between number and length from a few randomly ordered examples, they generalized the relation to new examples and showed heightened interest in new numbers and lengths that exhibited the relation (Experiment 2). Finally, when infants were presented with numbers and lengths that were paired with no previous familiarization, they look longer at positive than at inverse pairings (Experiment 4). All three tendencies reflect and likely enhance infants’ predisposition to relate number to space in a productive way.

A current controversy concerns the mechanisms of number-space mappings: do these mappings reflect the intrinsic cognitive architecture of numerical and spatial representations (Dehaene et al., 2008; 2009) or domain-general processes of analogical reasoning (Cantlon et al., 2009)? Adults can use space to represent similarity relations among non-numerical entities such as faces or colors (Munsell, 1912; Shepard, 1987; Valentine, 199; see Stevens & Marks, 1965); it is possible that infants also can map spatial variables to a variety of entities. Nevertheless, infants’ mapping of number to space has a specific direction: infants who viewed an increasing sequence of numbers related that series to a sequence of lengths that increased rather than decreased (Experiment 1), and infants who were exposed to randomly ordered number-length
pairings learned the pairings when greater numbers accompanied longer lengths (Experiment 2) but not shorter ones (Experiment 3). Thus, the ability to relate representations of number and space has a privileged structure, with larger numbers corresponding to greater horizontal lengths for infants, as for adults (de Hevia, Girelli, Bricolo & Vallar, 2008; Restle, 1970). Nevertheless, it is possible that infants would be able to learn an inverse mapping if they were given longer or more varied exposure to the mapping.

Although important aspects of the number-space relation are modulated by experience and education (Dehaene, et al., 1993; Dehaene, et al., 2008; Siegler & Opfer, 2003; Zebian, 2005), the present findings reveal that the human brain is predisposed to treat number and space as related. Human infants form and use relationships between number and space prior to the acquisition of language and counting, and prior to encounters with visual symbols, rulers, or other measurement devices. Mathematics, science and technology therefore build in part on a cognitive propensity with deep roots in human development.
References


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Figure Legends

**Figure 1.** Infants’ generalization of habituation from increasing or decreasing number to increasing or decreasing length. a, Displays presented in Exp. 1: Examples of the numerical displays used during habituation (top), and of the arrays of line lengths used during test (bottom), in increasing order (left) and decreasing order (right). b, Mean looking times (seconds) for the first three and last three habituation trials, and for the test displays with the novel and familiar orderings of lengths. Error bars represent ± s.e.m.

**Figure 2.** Infants’ learning of positive vs. inverse relationships of number to length. a, Displays presented in Exp. 2 and 3: Examples of the displays containing a numerical array and a line used during familiarization (top) and test (bottom), with positive pairings (left) and inverse pairings (right). b, Mean looking times (seconds) for the first three and last three familiarization trials, and test trials displaying a positive vs. inverse pairing, for the infants in Exp. 2 (left) and Exp. 3 (right). Error bars represent ± s.e.m.

**Figure 3.** Infants’ looking time (%) for positive pairings across infants familiarized to the positive rule, familiarized to the inverse rule, or tested without familiarization on three pairs of trials (baseline).
Footnotes

1. For this and all succeeding experiments, the findings are not changed if these subjects are included.

2. We are grateful to Justin Halberda for suggesting this possibility.
Figure 1

a.

b.

- Familiar order
- Novel order
Figure 2

a.

b.

![Figure 2](image-url)
Figure 3