# Incomplete Contracts and Signaling

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<td>doi:10.2307/2555872</td>
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Incomplete contracts and signalling

Kathryn E. Spier*

This article presents a principal-agent model in which asymmetric information leads to contractual incompleteness. I show that in the presence of transactions costs, incompleteness may act as a signal of the principal's type. Two types of transactions costs are considered: those incurred ex ante (drafting costs) and those incurred ex post (enforcement or verification costs). I prove that in the presence of either of these costs, asymmetric information leads to more contractual incompleteness than full information does.

1. Introduction

The extensive theoretical literature devoted to the characterization of optimal contracts predicts that contracts should be highly intricate.¹ In reality, however, most contracts are surprisingly simple, demonstrating an absence of the fine-tuned behavior predicted by neoclassical economic theory. They also tend to be incomplete, containing gaps that must be filled through renegotiation or legal intervention.

The common justifications given for contractual incompleteness and simplicity are transactions costs and bounded rationality.² The transactions cost approach argues that the costs of contracting on an unlikely contingency may well outweigh the benefits. The bounded rationality approach argues that agents either have a limited ability to evaluate elaborate contingencies or cannot foresee unlikely contingencies.³ This article identifies a third reason for contractual incompleteness: asymmetric information. Specifically, an individual may refrain from including a particular clause in a contract in order to signal his type. The optimal contracts signed under full information may be as elaborate as the standard theory

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This article is adapted from chapter three of my thesis at MIT. I thank James Dana, Oliver Hart, Benjamin Hermalin, Matthew Rabin, Jean Tirole, a referee, and seminar audiences at MIT, Boston University, the Department of Justice, and the Sixth World Congress of the Econometric Society for helpful comments. Financial support from the National Science Foundation and the Olin Program in Law and Economics is gratefully acknowledged.

¹ Hart and Holmström (1987) present a survey of this literature.

² Oliver Williamson (1985) in particular has promoted the economics of transactions costs, which stresses the limitations of contracting. Dye (1985) analyzes a model in which additional contingencies are costly and shows that as these costs decrease, contracts become more complicated. Huberman and Kahn (1988) argue that the ability to renegotiate contracts allows parties to specify less complicated (and hence less costly) contracts. Townsend (1979) analyzes the role of costly state verification in contractual simplicity. Other relevant articles include Allen and Gale (1992), Harris and Holmström (1987), Lewis and Sappington (1989), and Her malin (1988). The economic consequences of contractual incompleteness have been explored by Grossman and Hart (1986) and Hart and Moore (1988), among others.

³ Since it may be costly for an individual to evaluate and foresee events, bounded rationality can be viewed as a transactions cost.
predicts, but when asymmetric information is present they may be incomplete or overly simple.

One of the many examples that provide intuition for this phenomenon is profit sharing. In collective bargaining, unions are often distrustful when management suggests the inclusion of a profit-sharing clause. The union reasons that if management has private information concerning future profits, then it is more likely to suggest such a clause when the company is headed for a downturn. The union’s expected return from the proposed agreement is consequently lower than average, and the contract’s terms must be particularly favorable in order to induce the union to concede. 4

Another illustrative example is supplied by an athlete negotiating a contract with a particular team. The athlete’s agent may advise him to refrain from asking for an injury clause, because the team manager would infer from such a request that the athlete is more accident prone and would make the terms of the contract worse. On a related note, a fellow might hesitate to ask his fiancée to sign a prenuptial agreement, a contract that stipulates the division of property in the event of divorce, because to do so would lead her to believe that the quality of the marriage will be lower—or the probability of divorce will be greater—than she had thought.

This article uses a simple and stylized framework to formalize the notion that contractual incompleteness may be driven by asymmetric information. A risk-averse principal hires a risk-neutral agent to manage a stochastic technology. The principal is one of two types, good or bad, where the good type has higher expected profits. Naturally the principal would like the agent to absorb the risk of production; he would like to pay the agent a high wage when the profits are high and a low wage when the profits are low, thereby making the agent into the residual claimant. 5

A contract is defined to be “complete” if the wage is contingent on the level of profits, and “incomplete” if the wage is insensitive to this level. 6 There are transactions costs associated with adopting complete contracts. If the costs are sufficiently small, they will have no effect on the equilibria; if the costs are very large, then all contracts will be left incomplete. In an intermediate range, some contracts will be complete and others will be incomplete; the exact pattern will depend on the information structure as well as the nature of the transactions costs.

I consider two types of transactions costs: an ex ante cost (which could be, for example, the cost of drafting a complicated contract) and an ex post cost (which could be the legal fees necessary to enforce the contract in certain states of nature). This second cost is very different from the first because it is incurred only when contractual renegotiation is unsuccessful. For each kind of transactions cost I analyze how the full-information equilibria

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4 This intuition has been noted by Greenwald and Stiglitz (1987, p. 450): “If firms offered (and workers were willing to accept) equity shares in future output in lieu of current wages, then layoffs could, in theory, be avoided. However, the firms willing to do this are likely to be those whose future equity values are low relative to their observable characteristics. And workers . . . ought to avoid such offers.” And hence informational asymmetries are “likely to interfere with employment contracts that distribute risk.”

It is interesting to note that in response to this, a union may demand that the profit-sharing clause appear in future contracts as well. In other words, although the union has received a bad signal concerning the firm’s short-term profitability, it is willing to accept the plan (which functions as a wage reduction) conditional upon sharing in company profits thereafter. (For a discussion of this phenomenon, see Hoerr (1983).)

5 This framework is not meant as a literal model of profit sharing. In our model, the principal benefits from contractual completeness because of the allocation of risk; in reality, the management’s benefits come through other channels (increased productivity, for example). The intuition for both, however, is the same.

6 This definition of contractual incompleteness is by no means standard in the literature. Here, the incomplete contract covers the state space since it specifies transfers for each state of nature. This definition captures the essence of contractual simplicity. Under an alternative definition of incompleteness there exist actual “gaps” in the contract that must be filled in later. In other words, there may exist states of nature for which no action or transfer is specified.
change when asymmetric information about the principal’s type is introduced. In particular, I show that under asymmetric information the good type of principal offers an incomplete contract more frequently—she signals her type through incompleteness.

This result is more general than the simple risk-sharing example would suggest. Consider a scenario in which the principal and the agent have payoffs that are functions of both the contract and the principal’s type. Furthermore, suppose that if the contract is incomplete, then the agent’s payoff is independent of the principal’s type. The profit-sharing story told earlier fits into this framework: the union would be satisfied contracting for the same flat wage regardless of its beliefs about the firm’s future profits, but if management wants to implement profit sharing, then the future profitability of the firm is of great concern.7

In this more general framework, the introduction of asymmetric information will not constrain the principal’s choice within the set of incomplete contracts. The principal is likely, however, to be constrained within the set of complete contracts. Since the agent’s preferences are type-dependent within this set, there may be an incentive for one type of principal to mimic another. For separating equilibria, this would (weakly) reduce the principal’s benefit of offering a complete contract versus an incomplete one, and hence asymmetric information will lead to separating equilibria with more contractual incompleteness.8

Section 2 presents the model and the assumptions. Section 3 considers the case of ex ante transactions costs, and Section 4 considers the case of ex post costs. Concluding remarks are offered in Section 5.

2. The model

- A risk-averse principal owns a productive asset that, with the help of an agent, yields a stochastic level of output. The output, whose price is normalized to one, takes on one of two values $Q \in \{Q_L, Q_H\}$, where $Q_H > Q_L$. The principal’s expected utility function is assumed to be concave in income, $x$, linear in transactions costs, $y$, and separable: $V(x) - y$.9 The principal is one of two types, good ($g$) or bad ($b$); the proportion of good types in the economy, $\pi$, is strictly between zero and one. The good type has, on average, a higher level of output; letting $p$ denote the probability that output is high, we have $p_g > p_b$. The principal observes her own type and the level of output.

The agent is risk neutral and has reservation wage $W_0$. He does not observe the output level or the principal’s type (although I suppose that the distribution of types is common knowledge). The court may verify the level of output but not the principal’s type, and it follows that the output level is contractible while the principal’s type is not. Under these conditions, contracts may at best specify wage payments to be made for each realization of output: $\{W_H, W_L\}$.10 In the absence of transactions costs, the principal would like the agent to provide insurance through a contract that specifies a lower wage when output is low and a higher wage when output is high.

Definition. A contract is complete (incomplete) if it specifies different (identical) wages for each level of output.

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7 The prenuptial agreement example does not strictly satisfy these assumptions, since the happiness of the bride will depend on the type of the groom even in the absence of a prenuptial agreement. Nevertheless, the example dramatically conveys the intuition of the model.

8 If the agent’s payoff were type dependent within the set of incomplete contracts as well, then signalling concerns could potentially lead to more contractual incompleteness.

9 The separable specification is quite natural if one views the transactions costs as costs of effort. Analogous results may be derived for monetary costs.

10 More complicated contracts in which the transfers depend upon messages sent between the parties are ruled out.
Throughout the article I assume that it is costly for the parties to write complete contingent contracts; the costs may be incurred either at the contract formation stage or during the enforcement/verification stage. The former, which I refer to as *ex ante* costs and denote by \( k \), may be interpreted as the costs of drafting an unambiguous contract that is enforceable by the court system. The latter, which I refer to as *ex post* costs and denote by \( c \), are incurred only if the dispute is resolved by the court. The court can observe \( Q \in \{ L, H \} \) and enforce the contract \( \{ W_H, W_L \} \) at cost \( c \) to the principal.\(^{11}\) If the parties successfully renegotiate the wage contract without the intervention of the court, however, then no *ex post* cost is incurred by the principal.

The timing of the game is as follows. The principal privately observes her type, \( i \in \{ g, b \} \), and offers a contract to the agent, \( \{ W_H, W_L \} \). The agent may either accept or reject her offer. If he rejects, the principal gets nothing and the agent receives his reservation wage. If he accepts, then the agent contributes a contractible labor input, after which output is realized. Next, the parties enter a renegotiation phase. The bargaining takes place under incomplete information, since the agent does not directly observe the output level. I assume that bargaining takes the following form: The principal makes a take-it-or-leave-it offer to the agent. If the agent rejects this final offer, then the dispute goes to court.

Both the contracting phase and the pretrial negotiation phase are signalling games, and it is well known that such games are plagued with multiple equilibria. For the contracting phase, I shall focus on the sequential equilibria (Kreps and Wilson (1982)) that survive the "intuitive criterion," the refinement proposed by Cho and Kreps (1987). This refinement is quite powerful here, and it isolates a unique equilibrium I shall refer to as the "CK equilibrium." The separating equilibrium selected by this refinement corresponds to the "least-cost-separating allocation" that has been emphasized in many other studies of signalling games (Riley (1979) and Spence (1974), to name two) and is also the unique allocation chosen by the techniques discussed in Maskin and Tirole (1992).\(^{12}\) I postpone discussion of the equilibrium outcome of the pretrial negotiation game until Section 4.

Two polar cases are examined: in Section 3 I consider the case of *ex ante* transactions costs \( (k > 0, c = 0) \), and in Section 4 I consider the case of *ex post* transactions costs \( (k = 0, c > 0) \). For each case I compare the full-information outcome (in which the principal's type is observed by the agent) to the asymmetric-information outcome (in which the principal has private information about her type). In this way I am able to identify the roles that transactions costs and asymmetric information play in determining contractual incompleteness. I conclude that for a range of parameter values, contractual incompleteness is caused by asymmetric information and not by the transactions costs per se.

### 3. Transactions costs incurred *ex ante*

- It is intuitively clear why *ex ante* transactions costs can lead to contractual incompleteness: although complete contracts may offer desirable risk sharing, the cost of incompleteness may outweigh these benefits. Asymmetric information reduces the benefits of complete contracts without altering the costs. When transactions costs are negligible, the good type of principal will signal that she is good through a complete contract that gives less insurance; if the costs are sufficiently large, she will prefer to signal that she is good through an incomplete contract. In this section I demonstrate that this phenomenon occurs for an intermediate range of *ex ante* transactions costs.

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\(^{11}\) The court is not a strategic player; it simply observes the state of nature and enforces the contract. It is assumed that the court cannot punish the players for lying during their private negotiations, and that the players cannot contract upon the punishments.

\(^{12}\) Myerson (1983) considers the informed-principal problem using cooperative game theory, while Maskin and Tirole (1992) present a noncooperative analysis. In the latter article, the informed principal offers menus of contracts, thereby giving the signalling game screening properties.
To start, if verification is costless \((c = 0)\), then the pretrial negotiation game can be ignored, for every sequential equilibrium has the obvious outcome that the wages \(W_H\) and \(W_L\) are enforced. The principal never offers more than the true wage because she can pay the prescribed amount in court at no extra cost, and the agent refuses offers that give him less than his expected wage in court. Therefore, \(W_H\) is paid in state \(Q_H\), and \(W_L\) is paid in state \(Q_L\).

Assuming that the agent accepts the contract, the principal’s payoff can be represented as a function of the contract \(\{W_H, W_L\}\) given \(p\) and \(k\):

\[
E_Q V(W_H, W_L; p, k) = \begin{cases} pV(Q_H - W_H) + (1 - p)V(Q_L - W_L) - k & \text{if } W_H \neq W_L \\ pV(Q_H - W_H) + (1 - p)V(Q_L - W_L) & \text{if } W_H = W_L. \end{cases}
\]  

(1)

The function is discontinuous because the principal incurs the \textit{ex ante} cost only when the contract is complete.

First, let us suppose that the principal’s type is observed by the agent. The full-information contract is the solution to the following program:

\[
\max_{W_H, W_L} E_Q V(W_H, W_L; p, k) \tag{2}
\]

subject to

\[
(IR) \quad pW_H + (1 - p)W_L \geq W_0.
\]

The solution maximizes the expected utility of the principal subject to an individual rationality (IR) constraint, which specifies that the expected wage be at least as large as the agent’s reservation wage.

We can think about this program in two steps. First, the best incomplete contract for the principal is clearly \(\{W_0, W_0\}\); she economizes on transactions costs but receives no insurance. Second, the best complete contract that the principal could offer is

\[
\{W_0 + (1 - p)(Q_H - Q_L), W_0 - p(Q_H - Q_L)\}. \tag{3}
\]

This contract fully insures the principal, since \(Q_H - W_H = Q_L - W_L\), and it has an expected value equal to the agent’s reservation wage. The principal will choose the contract that yields the highest payoff.

If the \textit{ex ante} costs are small, then both types of principal offer complete contracts, and if the \textit{ex ante} costs are sufficiently large, then both types offer incomplete contracts. Provided that \(p_b\) and \(p_g\) are large, the good type of principal offers an incomplete contract and the bad type offers a complete contract for \textit{ex ante} costs in some middle range. This result is quite intuitive: the \textit{ex ante} transactions cost is analogous to the cost of obtaining insurance, and the bad type of principal faces a riskier level of output than the good type and thus has a higher risk premium.\(^{13}\)

\textbf{Proposition 1.} Under full information with \textit{ex ante} transactions costs and for \(p_b\) and \(p_g\) sufficiently large, there exist parameter values \(k_b > k_g \geq 0\), such that if

\[
k < k_g, \quad \text{then the good and bad types offer complete contracts},
k \in [k_g, k_b], \quad \text{then the good type offers an incomplete contract and the bad type offers a complete contract},
k > k_b, \quad \text{the good and bad types offer incomplete contracts}.
\]

\textbf{Proof.} The principal is indifferent between the best incomplete contract \(\{W_0, W_0\}\) and the full-insurance contract defined in (3) when

\(^{13}\) If, on the other hand, the probabilities are small, then the good type is facing greater risk and the results are reversed.
\[ V[pQ_H + (1 - p)Q_L - W_0] - k = pV(Q_H - W_0) + (1 - p)V(Q_L - W_0). \] (4)

Let \( k(p) \) be the value of \( k \) such that equation (4) holds; \( k(p) \) may simply be interpreted as the principal’s risk premium. \( k(0) = k(1) = 0 \), since the principal is facing no risk in these two cases. Solving for \( k(p) \) and taking the derivative yields

\[ k'(p) = V''[pQ_H + (1 - p)Q_L - W_0](Q_H - Q_L) - V(Q_H - W_0) + V(Q_L - W_0). \] (5)

\( k'(p) \) is positive when \( p \) is small and negative when \( p \) is large. Equation (5) implies that \( k''(p) < 0 \), and therefore \( k(p) \) is strictly positive on \((0, 1)\) and achieves a unique maximum.

If \( p_g \) and \( p_b \) are greater than the argmax of \( k(p) \), then we have

\[ k_g = k(p_g) < k(p_b) = k_b. \] \( Q.E.D. \)

An incentive problem arises when the principal privately observes her type, and in general the contracts described in Proposition 1 are not sustainable. In particular, if \( k \leq k_g \), then it is easy to see that the bad type strictly prefers the good type’s full-information contract to her own. When \( k > k_g \), however, there is no incentive problem: when \( k_g < k \leq k_b \), the good type offers the incomplete contract under full information, and by revealed preference the bad type has no desire to mimic her behavior. Similarly, when \( k > k_g \), both types offer incomplete contracts. Our main task, therefore, is to characterize the asymmetric information equilibria for \( k \leq k_g \).

In our signalling equilibrium, the bad type of principal always offers her optimal full-information contract, while the good type is constrained to offer a contract that would give the bad type of principal weakly less than her full-information payoff (the incentive compatibility (IC) constraint). The contract generated by this method is unique, and it is the only sequential equilibrium to satisfy the intuitive criterion (the CK equilibrium).14 It follows that the good type’s contract in the CK equilibrium is the solution to the following program, where \( V^*(p, k) \) is the principal’s full-information payoff given \( p \) and \( k \):

\[ \max_{W_H, W_L} E_QV(W_H, W_L; p_g, k), \] (6)

subject to

(IR) \[ p_g W_H + (1 - p_g) W_L \geq W_0 \]

(IC) \[ E_QV(W_H, W_L; p_b, k) \leq V^*(p_b, k). \]

Since the good type’s optimization problem under full information, (2), was not constrained by (IC), the good type receives a weakly lower payoff when asymmetric information is present. Recall that under full information, when \( k = k_g \), the good type is indifferent between offering \( \{W_0, W_0\} \) and her best complete contract. If the principal’s type is not observed, however, then (IC) binds, and when \( k = k_g \), the good type strictly prefers to offer the incomplete contract. The range of costs that lead to incompleteness has consequently expanded.

**Proposition 2.** Under asymmetric information with ex ante transactions costs there exists a unique CK equilibrium. For \( p_g \) and \( p_b \) sufficiently large, there exists \( k \in (0, k_g] \) such that the equilibrium has the feature that if

\[ \]
$k < \hat{k}$, then the good and bad types offer complete contracts,
$k \in [\hat{k}, k_b]$, then the good type offers an incomplete contract and the bad type offers a complete contract,
$k > k_b$, then the good and bad types offer incomplete contracts,

where $k_b$ and $k_g$ are parameters defined by Proposition 1.

Proof. If $p_b$ and $p_g$ are sufficiently large, then by Proposition 1 we have $k_g < k_b$.

Let the best separating complete contract for the good type of principal be denoted $\{W_{\hat{H}}, W_{\hat{L}}\}$. When $k > k_g$, it is clear that the good type offers her full-information contract. When $k \leq k_g$, however, the (IC) constraint in (6) strictly binds, and $\{W_{\hat{H}}^g, W_{\hat{L}}^g\}$ gives the principal a lower payoff than does the full-insurance complete contract given in equation (3). Therefore, when $k = k_g$, the good type of principal strictly prefers to offer the incomplete contract. If $k = 0$, then clearly the good type prefers to offer a complete contract (because it provides her with some insurance). Since the principal’s payoff from offering a complete contract is strictly decreasing in $k$, it follows that there exists a cutoff $0 < \hat{k} < k_g$ such that when $k \geq \hat{k}$, the good type offers an incomplete contract, and when $k < \hat{k}$, the good type offers a complete contract. $\hat{k}$ is the solution to

$$p_g V(Q_{\hat{H}} - W_{\hat{H}}^g) + (1 - p_g) V(Q_{\hat{L}} - W_{\hat{L}}^g) - \hat{k}$$

$$= p_g V(Q_{\hat{H}} - W_{0}) + (1 - p_g) V(Q_{\hat{L}} - W_{0}). \quad Q.E.D. \quad (7)$$

In the range $[\hat{k}, k_g]$, the good type offers an incomplete contract in order to signal that she is good; under full information she would offer a complete contract. This proposition confirms our intuition that contracts may be left incomplete for signalling reasons in the presence of transactions costs. The next section considers the more subtle case of ex post transactions costs.

4. Transactions costs incurred ex post

This section is motivated by the idea that incompleteness may result from costly contractual enforcement. In the context of our analysis, when a contract specifies more contingencies, there is a greater likelihood that the parties will disagree about the wage, and these disagreements will lead to costly litigation, despite the opportunity for renegotiation.

The story develops a new and interesting twist: the bad type of principal will be involved in disputes more often and therefore must pay a higher expected price for insurance than the good type. Even though the bad type has a higher risk premium, there is a range of costs under full information for which the good type of principal offers a complete contract and the bad type offers an incomplete contract (Proposition 3). This stands in stark contrast to the result from the previous section, in which the opposite pattern prevailed. When asymmetric information is introduced, however, the result from the previous section is surprisingly reestablished: there exists a range of costs for which the good type offers an incomplete contract—thereby signalling that she is good—and the bad type offers a complete contract (Proposition 4).

In order to solve for the equilibrium contracts, I first select and characterize a particular sequential equilibrium of the pretrial bargaining/contract renegotiation subgame. Working backwards, I use this outcome to find the equilibrium of the entire game.

Pretrial negotiation phase. When it is costly to resolve a contractual dispute in court, the parties will strive for a private solution: they will bargain with the outside option of using the court to enforce the contract should negotiations fail. Models of pretrial bargaining under one-sided incomplete information have a number of general properties. First, if the cost of using the court is small, then negotiations will sometimes break down. Second, there
is a predictable bias in the kinds of cases that proceed to trial.\textsuperscript{15} In our framework, the principal who owes the agent a low wage will necessarily go to trial more often than the principal who owes a high wage. This bias is important for our analysis, since it implies that the bad type of principal must pay a higher expected price for insurance than the good type.

The pretrial negotiation subgame is quite complicated for several reasons. First, the principal has (as many as) two dimensions of private information: the output level and her type. Second, the equilibrium may be very sensitive to the history of the game, since the principal has the opportunity to signal her type at an earlier date. Finally, the bargaining specification in which the informed principal makes a take-it-or-leave-it offer is a signalling game that gives rise to many sequential equilibria. Our focus, however, will be on a particularly simple sequential-equilibrium outcome.

Consider the continuation game that begins after the contract has been signed and output has been realized. Let $p$ represent the agent’s updated beliefs about the probability of high output (this value summarizes his beliefs about the likelihood that the principal is good or bad), let \{\( W_H, W_L \)\} be the contract, and let \( W^* \) denote the principal’s settlement offer. The following bargaining outcome is what survives within the class of sequential equilibria when we restrict attention to pure strategies and select a separating equilibrium when one exists.\textsuperscript{16}

\textit{Assumption 1} (pretrial bargaining outcome). Suppose that \( W_H > W_L \).\textsuperscript{17}

(i) If \( c \leq V(Q_L - W_L) - V(Q_L - W_H) \), a separating equilibrium results:

\begin{itemize}
  \item If \( Q = Q_H \), then \( W^* = W_H \), and the agent accepts.
  \item If \( Q = Q_L \), then \( W^* \leq W_L \), and the agent rejects.
\end{itemize}

(ii) If \( c > V(Q_L - W_L) - V(Q_L - W_H) \), a pooling equilibrium results:

\[ W^* \in \{ pW_H + (1 - p)W_L, W_H \} \], and the agent accepts.

If the contract is incomplete, then the outcome is described by (ii) above: the parties will settle out of court at the contracted wage. A separating equilibrium is obtained when the contract is complete and \( c \) is small (i). The principal makes a high offer (which is accepted) if the state is high, but she would rather make a low offer and be taken to court if the state is low. If the contract is complete and \( c \) is high, however, then this separating equilibrium cannot be supported, as the principal would prefer to pay a high wage in both states of nature because the cost of going to court is large. Rather, a pooling equilibrium results in which the parties settle out of court.

\textbf{Contracting phase.} A contract specifying wages that are very close together is, for all practical purposes, incomplete. We know that the court will not be used to resolve the wage dispute, because the benefit to the principal from paying a lower wage is small relative to her cost of using the court. Rather, the parties will negotiate a settlement out of court, and the same wage will be paid in both states of nature. Since the parties can perfectly anticipate this renegotiated wage, \( W^* \), they can generate exactly the same outcome through an in-


\textsuperscript{16} In a pure strategy separating equilibrium, when \( Q = Q_H \) it must be that \( W^* = W_H \) and the agent accepts. (If \( W^* < W_H \), the agent would prefer to go to court, and if \( W^* > W_H \), the principal would reduce the offer.) When \( Q = Q_L \), the principal offers \( W^* \leq W_L \) and incentive compatibility for the principal implies that the agent rejects. When \( c \) is small, this outcome is supportable as a sequential equilibrium for any interim beliefs \( p \in (0, 1) \). This is not the most efficient sequential equilibrium, for there is another where the agent mixes between accepting and rejecting the low offer. The probability of acceptance is intricately linked to the agent’s subjective beliefs and the wage contract, hence adopting such an outcome would greatly complicate the analysis.

\textsuperscript{17} If \( W_H < W_L \), simply switch the variables.
complete contract that specifies the associated wage $W^*$ for both states of nature. Therefore, any complete contract that falls into category (ii) is weakly dominated at the initial contracting stage.

In what follows, I consider only complete contracts that lead to separating equilibria in the pretrial bargaining game, and I call such complete contracts “implementable.” (If the principal faced a small ex ante cost, then in equilibrium every complete contract would be implementable.)

Definition. A complete contract, $\{W_H, W_L\}$, is implementable (IMP) if

$$c \leq V(Q_L - W_L) - V(Q_L - W_H).$$

Assumption 2. Complete contracts offered by the principal are implementable.

Since all complete contracts must fall into category (i), the bargaining outcome exhibits a very special property: the equilibrium strategies are independent of $p$. Therefore the history of the game is irrelevant in the pretrial negotiations. This property is very attractive from a modelling perspective because it implies that the analysis may proceed by backwards induction. In particular, under Assumption 2 the principal’s payoff at the contracting stage may be represented as a simple discontinuous function of the wage contract:

$$E_Q V(W_H, W_L; p, c)$$

$$= \begin{cases} 
pV(Q_H - W_H) + (1 - p)V(Q_L - W_L) - (1 - p)c & \text{if } W_H \neq W_L \\
pV(Q_H - W_H) + (1 - p)V(Q_L - W_L) & \text{if } W_H = W_L. \end{cases} \quad (8)$$

Proposition 3 states that under full information there is a range of $c$ for which the bad type offers an incomplete contract and the good type offers a complete contract. The fact that the bad type has a higher risk premium is outweighed by the feature that the bad type also faces a larger cost of insurance than the good type does.

**Proposition 3.** Given Assumptions 1 and 2, under full information with ex post transactions costs there exist parameter values $c_b, c_g > 0$, and $c_b < c_g$ such that if

- $c < c_b$, then the good and bad types offer complete contracts,
- $c \in [c_b, c_g]$, then the bad type offers an incomplete contract and the good type offers a complete contract,
- $c > c_g$, then the good and bad types offer incomplete contracts.\(^{18}\)

Before presenting the proof of Proposition 3, I shall state and prove a lemma that will be used later to simplify the optimization problem.

**Lemma.** Consider a complete contract $\{W_H, W_L\}$ with $W_H \geq W_0$. If for some type $p$ we have $E_Q(W_H, W_L; p, c) > E_Q(W_0, W_0; p, c)$, then $\{W_H, W_L\}$ is implementable.

**Proof.** Taking the preferences in (8) as given, suppose that $\{W_H, W_L\}$ is preferred to $\{W_0, W_0\}$ by some principal of type $p$:

$$pV(Q_H - W_H) + (1 - p)V(Q_L - W_L) - (1 - p)c$$

$$\geq pV(Q_H - W_0) + (1 - p)V(Q_L - W_0). \quad (9)$$

Rearranging this expression shows that

$$V(Q_L - W_L) - V(Q_L - W_H) - c \geq V(Q_L - W_0) - V(Q_L - W_H)$$

$$+ \left( \frac{p}{1 - p} \right) [V(Q_H - W_0) - V(Q_H - W_H)]. \quad (10)$$

\(^{18}\) Note that this result does not depend on the magnitudes of the probabilities.
When \( W_H \geq W_0 \) the right-hand side is positive, so (IMP) is satisfied. \( \quad Q.E.D. \)

**Proof of Proposition 3.** When \( p \) is known to both parties, the principal selects the contract that maximizes her expected utility (8) subject to the individual rationality constraint for the agent (IR) and also subject to the constraint that a complete contract is implementable (IMP). The best incomplete contract is clearly \( \{ W_0, W_0 \} \). The best complete contract is the solution to the following program:

\[
\max_{W_H, W_L} pV(Q_H - W_H) + (1 - p)V(Q_L - W_L) - (1 - p)c \quad (11)
\]

subject to

\[
\begin{align*}
(\text{IR}) & \quad pW_H + (1 - p)W_L \geq W_0 \\
(\text{IMP}) & \quad V(Q_L - W_L) - V(Q_L - W_H) \geq c.
\end{align*}
\]

First I show that in equilibrium, (IMP) does not bind. Ignoring (IMP), the best complete contract fully insures the principal and is given by equation (3). Since full insurance implies that \( W_H > W_0 \), the lemma establishes that the full-insurance contract is implementable (and hence optimal) if it is preferred to \( \{ W_0, W_0 \} \). The principal is indifferent between the full-insurance contract and the incomplete contract \( \{ W_0, W_0 \} \) when

\[
V[pQ_H - (1 - p)Q_L - W_0] - (1 - p)c = pV(Q_H - W_0) + (1 - p)V(Q_L - W_0). \quad (12)
\]

Define \( c(p) \) to be the value of \( c \) that makes a principal of type \( p \) indifferent. \( c(p) \) may be interpreted as the risk premium divided by \( (1 - p) \). Solving for \( c(p) \) and differentiating shows that \( c'(p) > 0 \) for all \( p \). Therefore \( c_b = c(p_b) > c(p_b) = c_b \). \( \quad Q.E.D. \)

If \( p \) is privately observed by the principal, then the full-information equilibrium is, in general, not sustainable. As before, I select the sequential equilibrium in which the bad type of principal offers her full-information optimal contract and the good type is constrained to offer a contract that would make the bad type weakly worse off. When the (IC) constraint binds, the good type’s payoff from the best complete contract is reduced relative to her full-information payoff, while the best incomplete contract stays the same; asymmetric information consequently expands the range of incompleteness.

Somewhat surprisingly, the full-information pattern described in Proposition 3 is totally reversed: there is a range for \( c \) in which the bad type offers a complete contract while the good type offers an incomplete contract. In this middle range, an incomplete contract signals that the type is good. These results are stated formally in the following proposition.

**Proposition 4.** Given Assumptions 1 and 2, under asymmetric information with ex post transactions costs there is a unique CK equilibrium. There exists \( \hat{c} \in (0, c_b) \) such that this equilibrium has the feature that if

\[
\begin{align*}
& c < \hat{c}, \quad \text{then the good and bad types offer complete contracts}, \\
& c \in [\hat{c}, c_b), \quad \text{then the good type offers an incomplete contract and the bad type offers a complete contract}, \\
& c \geq c_b, \quad \text{then the good and bad types offer incomplete contracts},
\end{align*}
\]

where \( c_b \) is the parameter defined in Proposition 3.

**Proof.** The best implementable complete contract that the good type can offer is the solution to the following program:

\[
\max_{W_H, W_L} p_g V(Q_H - W_H^g) + (1 - p_g)V(Q_L - W_L^g) - (1 - p_g)c \quad (13)
\]
subject to

(IR) \[ p_g W^g_H + (1 - p_g) W^g_I \geq W_0, \]

(IC) \[ p_b V(Q_H - W^g_H) + (1 - p_b)V(Q_L - W^g_L) - (1 - p_b)c \leq V^*(p_b, c), \]

(IMP) \[ V(Q_L - W^g_L) - V(Q_L - W^g_H) \geq c, \]

where \( V^*(p, c) \) represents the principal’s full-information payoff given \( p \) and \( c \).

First, let us check that the incentive constraint is binding when \( c \leq c_g \). Suppose the incentive constraint does not bind. By Proposition 3, the good type offers the full-insurance contract. When \( c \leq c_b \), this violates the (IC), since the good type’s full-insurance contract yields a higher certain payoff than the bad type’s. Now suppose that \( c_b < c \leq c_g \). For full information, in this range the bad type offers the incomplete contract and the good type offers the complete contract:

\[
V[p_g Q_H + (1 - p_g) Q_L - W_0] - (1 - p_g)c \\
\geq p_g V(Q_H - W_0) + (1 - p_g)V(Q_L - W_0).
\]  

(14)

Since by assumption the (IC) constraint is satisfied for the bad type, from (14) we have

\[
\frac{d}{dp} [pV(Q_H + W_0) - (1 - p)V(Q_L - W_0) + (1 - p)c_g] < 0,
\]

(15)

which implies

\[
V(Q_H - W_0) - V(Q_L - W_0) - c_g < 0.
\]

(16)

The value of \( c_g \) (which can be found by making (14) an equality) shows that this is false, and we conclude that the (IC) constraint binds for all \( c \leq c_g \).

It is easy to see that the best complete contract satisfies \( W^g_H \geq W_0 \). If this were not so, then the (IR) constraint would imply that for the bad type,

\[
p_b W^g_H + (1 - p_b) W^g_I > W_0,
\]

(17)

but this contradicts the fact that the (IC) binds, since the bad type would never want to offer such a contract (by revealed preference under full information). Therefore, \( W^g_H \geq W_0 \), and from the previous lemma we know that (IMP) will not bind at the optimum whenever the complete contract is chosen.

When \( c \geq c_b \), the bad type must be indifferent between the best complete contract and the incomplete contract:

\[
p_b V(Q_H - W^g_H) + (1 - p_b)V(Q_L - W^g_L) - (1 - p_b)c \\
= p_b V(Q_H - W_0) + (1 - p_b)V(Q_L - W_0).
\]

(18)

Since \( W^g_H \geq W_0 \), we must in fact have \( W^g_H > W_0 \) for this to be satisfied. Analyzing this expression for \( p > p_b \) verifies that the incomplete contract is strictly preferred by the good type.

To prove the existence of a cutoff between zero and \( c_b \) at which the good type is indifferent, we simply note that the good type’s payoff from a complete contract is declining in \( c \) in this range, and that when \( c = 0 \) the good type prefers the complete contract. \( Q.E.D. \)

5. Conclusion

This article explores the idea that strategic considerations during contract formation may lead to contractual incompleteness. It was shown in a stylized principal-agent model with transactions costs that there exists a range of parameter values for which the good type
of principal signals that she is good through an incomplete contract, even though under full information she would have offered a complete contract. The intuition for this result is straightforward. Since the agent’s payoff was assumed to be independent of the principal’s type within the class of incomplete contracts, signalling concerns make complete contracts relatively more expensive for the good type of principal. Consequently, she is likely to forgo completeness for the less expensive option. Further research could consider more general preferences for the agent, and thereby explore the possibility of signalling through complete contracts.

References


Hoerr, J. “Why Labor and Management are Both Buying Profit Sharing.” Business Week, January 10, 1983, p. 84.


