PARAMETER ESTIMATION FROM IMPROVED MEASUREMENTS OF THE COSMIC MICROWAVE BACKGROUND FROM QUaD

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ABSTRACT

We evaluate the contribution of cosmic microwave background (CMB) polarization spectra to cosmological parameter constraints. We produce cosmological parameters using high-quality CMB polarization data from the ground-based QUaD experiment and demonstrate for the majority of parameters that there is significant improvement on the constraints obtained from satellite CMB polarization data. We split a multi-experiment CMB data set into temperature and polarization subsets and show that the best-fit confidence regions for the $\Lambda$CDM six-parameter cosmological model are consistent with each other, and that polarization data reduces the confidence regions on all parameters. We provide the best limits on parameters from QUaD $EE/BB$ polarization data and we find best-fit parameters from the multi-experiment CMB data set using the optimal pivot scale of $k_p = 0.013$ Mpc$^{-1}$ to be \{$h^2\Omega_b, h^2\Omega_c, H_0, A_s, n_s, \tau$\} = \{0.113, 0.0224, 70.6, 2.29 $\times$ 10$^{-9}$, 0.960, 0.086\}. Key words: cosmic background radiation – cosmological parameters

Online-only material: color figures

1. INTRODUCTION

The cosmic microwave background (CMB) radiation contains fluctuations in temperature and polarization which have specific spectral features that record the evolution and constituent properties of the universe. The radiation is predicted to be polarized at the 10% level due to Thomson scattering in the presence of velocity inhomogeneities in the photon–baryon fluid at last scattering.

The standard cosmological model predicts acoustic peaks in CMB intensity and polarization spectra. Polarized CMB radiation can be decomposed into even-parity $E$-modes which are generated by scalar and tensor perturbations and odd-parity $B$-modes which are generated by gravitational waves and gravitational lensing effects. In this paper, we examine the cosmological implication polarization spectra including the spectra produced by the 2009 improved analysis of QUaD second and third season observations presented in Brown et al. (2009). QUaD improves on the detections of $E$-modes made by the DASI (Kovac et al. 2002), CAPMAP (Hedman et al. 2002), Boomerang (Jones et al. 2006), Wilkinson Microwave Anisotropy Probe (WMAP; Dunkley et al. 2009), and CBI (Readhead et al. 2004) experiments, adding accurate polarization data at small angular scales.

We presented design and optics reports describing the QUaD\textsuperscript{15} experiment in Hinderk\textsuperscript{15} et al. (2009) and O’Sullivan et al. (2008). The first season of QUaD results appeared in Ade et al. (2008). We first reported measurements of temperature and polarization from the second and third season QUaD data in Pryke et al. (2009) and cosmological parameter analysis of the data was carried out in Castro et al. (2009). The second and third season data sets from the QUaD experiment have been reanalyzed using a ground template removal technique rather than field differencing to remove ground contamination. A description of the method and maps and spectra from two parallel and independent pipelines can be found in Brown et al. (2009). The re-analysis has effectively doubled the size of the QUaD field, increasing the precision of the CMB spectra measurements by $\sim$30% and constraining the amplitude of the lensing $B$-modes to $< 0.57 \mu K^2$ at 95% confidence. In Brown et al. (2009), we also presented a cosmological parameter analysis which gauged the QUaD stands for “QUEST and DASI.” In turn, QUEST is “Q & U Extragalactic Survey Telescope” and DASI stands for “Degree Angular Scale Interferometer.”

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\textsuperscript{15} QUaD stands for “QUEST and DASI.” In turn, QUEST is “Q & U Extragalactic Survey Telescope” and DASI stands for “Degree Angular Scale Interferometer.”
effect of using the improved spectra from QUaD in combination with WMAP and ACBAR data, demonstrating that QUaD data add significant power for constraining cosmological models which include tensors and running of the spectral index.

In this paper, we carry out a cosmological parameter estimation in which we explore separately the strengths of CMB temperature data and CMB polarization data including the two year data release by BICEP (Chiang et al. 2010), highlighting the contributions of each to a multi-experiment analysis.

2. DATA AND METHOD

The QUaD data set consists of 143 selected days of data measured during the Austral winters of 2006 and 2007, at 100 GHz and 150 GHz, in a region of approximately 60 deg$^2$, and has produced polarization spectra of unprecedented quality (Pryke et al. 2009; Brown et al. 2009). The QUaD experiment is the first experiment with the sensitivity to detect multiple acoustic oscillations in the E-mode spectrum and TE spectrum up to $\ell = 2000$ in addition to providing the lowest upper limits on B-mode detection (Brown et al. 2009). The spectra we used for cosmological data analysis, Temperature–Temperature (TT), Temperature–E-mode (TE), E-mode–E-mode (EE), and B-mode–B-mode (BB), are an optimally weighted combination of three sets of spectra: 100 GHz, 150 GHz, and 100 GHz–150 GHz cross, each with 23 band power values in the range $200 < \ell < 2000$.

Our CosmoMC-based parameter analysis is broadly similar to the Monte Carlo Markov chain analysis of Castro et al. (2009), hereafter Parameter Paper 1. A single, constant covariance matrix was estimated from simulations within the QUaD pipeline. The covariance matrix is populated only in the diagonal, 1st off-diagonal, and 2nd off-diagonal terms of each of the sub-blocks of the spectra. TT–TE and TE–EE have no 2nd off-diagonal terms, and TT–EE and EE–BB have covariance terms up to the 12th and 4th band powers, respectively. Beyond these regions the covariance matrix would be dominated by noise from the numerical simulations. We also generated offset-lognormal x-factors from noise-only simulations. This enables us to model the offset-lognormal likelihood for our parameter estimation as suggested by Bond et al. (2000).

We make use of templates of the Sunyaev–Zel’dovich (SZ) amplitude derived from Komatsu & Seljak (2002), which model a frequency-dependent contribution to the temperature power spectrum from the thermal SZ effect. We apply a cut of spectral power above $\ell = 2000$ to avoid the effects of residual point-source contamination and analytically marginalize over the SZ amplitude. The justification of a fit to SZ parameters on scales below $\ell = 2000$ has not been confirmed by the high multipole temperature data obtained by QUaD (Friedman et al. 2009).

We estimate likelihoods from our CMB distribution using the publicly available CosmoMC (Lewis & Bridle 2002) Monte Carlo Markov chain algorithm. The theoretical CMB model at every stage is obtained from the publicly available CAMB Boltzmann code (Lewis et al. 2000). The choice of baseline parameter set and the shape of the priors on parameters fitted concurrently will impact on the one-dimensional (1D) marginalized parameters calculated using the Monte Carlo Markov chain. For the standard ΛCDM six-parameter cosmological model, we limit this effect by using the baseline parameter combination comprising the baryon density, cold dark matter density, acoustic peak scale which records the expansion history of the universe, scalar spectral amplitude, scalar spectral index, and optical depth: $[h^2 \Omega_b, h^2 \Omega_c, \theta, \ln(10^{10} A_s), n_s, \tau]$; the use of ln$(10^{10} A_s)$ also considerably increases the rate of convergence; $H_0 = 100 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$ is the Hubble constant. This is the default parameter set of CosmoMC (Lewis & Bridle 2002). When presenting two-dimensional (2D) plots of likelihood space evaluated from a CMB spectrum that does not have information in the angular range below the first acoustic peak, we use the combined $A_s \exp(-2\tau)$ parameter, a measure of the overall amplitude of the fluctuations in the spectra optimally sampled in the analysis using the baseline parameter set. The pivot scale used to evaluate the amplitude and spectral index is $k_p = 0.05 \, \text{Mpc}^{-1}$ in the case where we use QUaD data exclusively. The pivot scale used for multi-experiment CMB spectra analysis is $k_p = 0.013 \, \text{Mpc}^{-1}$ in common with our cosmological parameter results of Brown et al. (2009). We impose flat priors on the baseline parameter set (Table 1), make the assumption that the universe is flat, and include in all analyses the effects of weak gravitational lensing.

In addition, we make use of the CosmoMC utility to analytically marginalize over “nuisance” parameters, which is implemented for beam and calibration uncertainties (Bridle et al. 2002). The effective beam size for the combined QUaD spectra is 4.1 arcmin. The beam error is a function dominated by the sidelobes and varies with scale. The calibration uncertainty is 6.8% in power.

We provide parameter constraints on these baseline parameters and additionally include plots and constraints in the more traditional format of $[h^2 \Omega_b, h^2 \Omega_c, H_0, A_s, n_s, \tau]$. We present 68% and 95% confidence regions for cosmological parameters and 2D and 1D marginalized plots using our independent statistics code on 10 or more Monte Carlo Markov chains with at least 100,000 converged steps. Tests of convergence were carried out using the Gelman–Rubin statistic (Gelman & Rubin 1992). We also made use of the Getdist package, which is bundled with CosmoMC, as a consistency check of our results.

We carried out a parameter fit from the products of both QUaD data analysis pipelines to confirm consistency.

2.1. Other Data

We combine QUaD with the following CMB data sets: ACBAR (Reichardt et al. 2009) including the 2% beam error on the 5 arcmin beam and calibration error 4.6% in power; CBI

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**Table 1** Baseline and Derived Parameters and Flat Priors on Baseline Parameter Set

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Prior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baryon density</td>
<td>$\Omega_b h^2$</td>
<td>0.001–0.999</td>
</tr>
<tr>
<td>Cold dark matter density</td>
<td>$\Omega_c h^2$</td>
<td>0.001–0.999</td>
</tr>
<tr>
<td>Acoustic peak scale</td>
<td>$\theta$</td>
<td>0.3–12</td>
</tr>
<tr>
<td>Scalar fluctuation amplitude</td>
<td>$\ln(10^{10} A_s)$</td>
<td>2.7–4</td>
</tr>
<tr>
<td>Scalar fluctuation index</td>
<td>$n_s$</td>
<td>0.01–2</td>
</tr>
<tr>
<td>Optical depth</td>
<td>$\tau$</td>
<td>0.01–0.8</td>
</tr>
<tr>
<td>Age</td>
<td>$\text{Age (Gyr)}$</td>
<td>10–20</td>
</tr>
<tr>
<td>Dark energy density</td>
<td>$\Omega_{\Lambda}$</td>
<td>...</td>
</tr>
<tr>
<td>Matter density</td>
<td>$\Omega_m$</td>
<td>...</td>
</tr>
<tr>
<td>Reionization depth</td>
<td>$z_{\text{ef}}$</td>
<td>...</td>
</tr>
<tr>
<td>Hubble constant</td>
<td>$H_0$</td>
<td>40–100</td>
</tr>
<tr>
<td>Linear mass perturbation</td>
<td>$\sigma_8$</td>
<td>...</td>
</tr>
</tbody>
</table>

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16 SZ templates are available at http://lambda.gsfc.nasa.gov.
17 Our analysis in Parameter Paper 1 did not include marginalization over an SZ amplitude or offset-lognormal factors.
QUaD shares its entire survey region with Boomerang, therefore we do not include Boomerang spectra in CMBmany.

(A color version of this figure is available in the online journal.)
The best-fit spectral index value obtained from the QUaD EE/BB spectra subset is 0.53. We show in Figure 3 that the likelihood surface is skewed over the range of scalar spectral index values. We overplot the likelihood for WMAP TE to demonstrate the best-fit regions for the spectral index overlap. To supplement our QUaD EE/BB only parameter fits, we also carried out an analysis where we implement a Gaussian prior on the Hubble constant with the latest independent limits obtained by the Hubble Space Telescope (HST; Riess et al. 2009) $74.2 \pm 2.6$ km s$^{-1}$ Mpc$^{-1}$. The Hubble constant primarily impacts the width and separation of the spectral peaks, while the spectral index is evaluated from the tilt of the data. Applying
the information we have on the Hubble constant improves our parameter confidence ranges, while not preferentially impacting the spectral index signal in the data. These contours are also plotted in Figure 3, confirming the robustness of the $EE/BB$ analysis. The parameter constraints from QUaD $EE/BB$ spectra improve on the corresponding constraints we presented in Parameter Paper 1.

The constraint obtained on the acoustic peak scale, $\theta$, is as tight from QUaD $EE/BB$ only as it is from the QUaD temperature spectrum and the constraint on $h^2\Omega_b$ is tighter.

The parameter ranges arrived at by using the complete QUaD data set agree with the values from the CMBmany data set within the 95% contour limits, as demonstrated in Figure 4. The preferred parameter range for the baryon density we presented in Parameter Paper 1 from the full Pryke et al. (2009) QUaD data set was $\sim50\%$ higher than that we now provide. The Hubble constant was also $\sim30\%$ higher than the value we return from the analysis of the new non-field-differenced data. The data analysis pipeline which produced the Brown et al. (2009) spectra differs from the Pryke et al. (2009) analysis in the implementation of a new noise-removal strategy and in modeling a new beam shape. This results in a material difference in the spectra in addition to a reduction of cosmic variance. The scalar spectral amplitude of Parameter Paper 1 (which we quoted as part of a combined $A_s$, $\exp(-2\tau)$ parameter) was significantly shifted relative to our current preferred range, the corollary being in extended parameter analysis isocurvature ratios were more constrained. However, as our parameter analysis has evolved, the confidence limits presented in Parameter Paper 1 cannot be directly compared to the limits in this paper. Repeating the parameter analysis of the second and third season QUaD data of Pryke et al. (2009), using our current CosmoMC-based pipeline, we find 68% confidence regions for $A_s$, $\exp(-2\tau)$ and the Hubble constant that are $30\%$ greater than the corresponding values obtained using the improved data set of Brown et al. (2009).

### 3.2. Parameter Constraints Combining Data Sets

We continue our parameter analysis by investigating the effect of adding QUaD data to the CMBmany temperature and polarization ensemble data set.

In order to investigate the influence of polarization in our cosmological analysis, we carried out cosmological parameter fits using the $TT$ and $TE/EE/BB$ subsets of the QUaD+CMBmany data set. The resulting best-fit regions, presented in Table 3 and Figure 5, are consistent, although the polarization spectra prefer lower values for the Hubble constant and acoustic peak scale as well as a lower spectral index. The multi-experiment $TE/EE/BB$ data set contributes information on all six cosmological parameters.
4. CONCLUSIONS

We present best-fit regions for the standard ΛCDM six-parameter model using QUaD CMB spectra alone, and QUaD data in combination with CMBmany (BICEP, ACBAR, CBI, and WMAP).

QUaD data provide an unprecedented amount of independent information over four spectra in a multipole range of $200 < \ell < 2000$. This improves the ensemble of CMB data sets specifically providing independent cohesive information over an angular range broad enough to span the sparsely populated “hinge” multipole regions of the CMBmany data set.
Figure 5. 2D marginalized contours $h^2 \Omega_b$ against $\theta$ from separate temperature and polarization combined QUaD and CMBmany data sets. QUaD polarization+CMBmany polarization constraints: red/orange dotted; QUaD temperature+CMBmany temperature constraints: blue/green dashed; constraints from QUaD+CMBmany: black solid lines.

(A color version of this figure is available in the online journal.)

Data from the QUaD experiment, as a single, discerning data set can provide excellent independent measures of the baryon density in the universe, the cold dark matter density, and the acoustic scale, which measures the universe’s evolution history.

QUaD $TE/EE/BB$ spectra provide considerably tighter constraints on four of the six standard $\Lambda$CDM parameters than can be obtained from 5 year WMAP polarization data. We present the best confidence ranges obtained from QUaD $EE/BB$ spectra alone. The QUaD temperature spectrum alone also provides good parameter constraints in excellent agreement with the WMAP 5 year best-fit parameters.

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REFERENCES