Coherent optical fields provide a powerful tool for manipulating ultracold atoms [1,2]. However, diffraction sets a fundamental limit for the length scale of such manipulations, given by the wavelength of light [3]. In particular, the large period of optical lattices determines the energy scale of the associated many-body atomic states [4–7]. The resulting scaling can be best understood by noting that in the first Bloch band the maximum atomic momentum \( \sim 1/\ell \), where \( \ell \) is the lattice spacing. This sets the maximum kinetic energy to \( h^2/2m\ell^2 \) [8]. For conventional optical lattices, the lattice spacing is set by half the wavelength of the trapping light (\( \sim 500 \text{ nm} \)); this yields corresponding tunneling rates of up to a few tens of kilohertz. Additionally, for atoms in their electronic ground states the interaction associated with nanoplasmonic systems. It allows one to considerably increase the energy scales in the realization of Hubbard models and to engineer effective long-range interactions in coherent and dissipative many-body systems. Realistic imperfections and potential applications are discussed.

We propose to use subwavelength confinement of light associated with the near field of plasmonic systems to create nanoscale optical lattices for ultracold atoms. Our approach combines the unique coherence properties of isolated atoms with the subwavelength manipulation and strong light-matter interaction associated with nanoplasmonic systems. It allows one to considerably increase the energy scales in the realization of Hubbard models and to engineer effective long-range interactions in coherent and dissipative many-body dynamics. Realistic imperfections and potential applications are discussed.

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Nanoplasmonic Lattices for Ultracold Atoms

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spheres in the center of a SiO$_2$ core; the trapping light is red-detuned (704 nm) with respect to the plasmon resonance (682 nm) and applied from above with rotating $x$-$y$ polarized light.

$\delta = \omega - \omega_1$ is the detuning between the atomic transition and the laser. Expanding near the trap minima gives the trapping frequency $\omega_\perp^2 = \frac{\hbar \Omega_0^2}{\delta m^2} \text{Re}(\alpha)^2/\alpha^2 \sim \hbar \Omega_0^2/\delta m a^2$.

The trap depth can be controlled by applying a second field with the opposite polarization, as illustrated in Fig. 1(c). By using this method, the atoms can be loaded into the near field traps by starting with a cold, dense gas of atoms in a large trap and then adiabatically turning on the near field traps.

We now address several practical considerations. First, for alkali atoms there is a large disparity between the natural plasmon resonance and the atomic trapping transitions. For a solid silver sphere, the plasmon resonance occurs near 350 nm [16], compared to 780 nm for the alkali atoms there is a large disparity between the near field traps.

For a solid silver sphere, the plasmon resonance should be smaller than its inverse quality factor $Q = \omega_{sp}/\kappa$, which for silver (gold) nanospheres is the distance of the atom to the sphere center, $r$, and resistivity $\rho$. Then the incoherent transition rate between hyperfine states is $\sim (g_r \mu_B \hbar)^2 k_B T (a/r^3)/\hbar^2 \rho r$, where $r$ is the distance of the atom to the sphere center, $g_r$ is the hyperfine $g$ factor, $\mu_B$ is the Bohr magneton, and $T$ is the temperature [18].

Figures 1(c) and 1(d) show the atomic trapping potential for a single sphere and an array, respectively. We numerically obtained the trapping potential in Fig. 1(c) by using Mie theory, and the vdw potential was obtained by using the methods in Ref. [19]. To solve for the trapping potential in the array in Fig. 1(d), we approximated the scattered field from each nanoshell by a dipole and solved for the total field self-consistently. Using the parameters in Fig. 1(c) for trapping $^{87}$Rb above a silver nanoshell at room temperature with $\Omega_0 = 25$ GHz (corresponding to $\sim 10^8 I_{sat}$, where $I_{sat} = 1.7$ mW/cm$^2$) and $\delta = 25$ THz, we estimate a trap depth of $\sim 25$ MHz and a trapping frequency of $\sim 5$ MHz. Both the magnetic field noise and laser detuning limit the decoherence rate to $\sim 10$ Hz and the heating rate to $\sim 1$ Hz, meaning that the atom can be trapped for $\sim 1$ s.

The controlled patterning of arrays of metallic nanoparticles can be done lithographically in a top-down approach or through the controlled self-assembly of metallic nanoparticles in a bottom-up approach [20–23]. In any nanofabricated system, one must contend with disorder; the relevant disorder in this system occurs in the particle positioning and particle formation. In lithographic approaches, one can control the particle formation at the level of 1–2 nm [21]. In bottom-up, self-assembly approaches, it is possible to create large regions of well-ordered crystal with a finite density of point and line defects, much like a conventional solid [23]. Because of the local nature of the traps, the disorder in the particle positioning will not affect the trapping. Errors in the particle formation can influence the trap by shifting the plasmon resonance and the field enhancement of each particle. To achieve consistent traps, the fractional error in the plasmon resonance should be smaller than its inverse quality factor $Q = \omega_{sp}/\kappa$, which for silver (gold) nanospheres...
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ment where the ground state uncertainty becomes compa-
rable to the free-space scattering length. For two atoms in a
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solved exactly, leading to an effective scattering length $a_{eff}(\omega_T)$, which depends on the confinement energy
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Disorder in the lattice will also affect the Hubbard
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show that the rms of the disorder potential is given by
$U_{dis} = \frac{\sqrt{2}^2}{28} \left( \frac{c_T}{a_0^2} \right) Q^2 \eta / \omega_{sp}$, where $\eta$ is the rms error in the plasmon resonance. If we take $\eta / \omega_{sp} \sim 5\%$, then for a
wide range of parameters, including those in Fig. 1(d), we
find that $U_{dis}$ can be made smaller than, or comparable to,
the maximum tunneling. In addition, since the disorder is
static, one can reduce it by using the techniques described
in Ref. [29]. The effect of disorder on the single-particle

The cavity QED figure of merit $g^2/\kappa \gamma$ with changing system size assuming the atom is trapped
at a distance of twice the sphere radius. We show the scaling for
both silver and gold nanoshells with a $Q$ of 80 and 20, respec-
tively. (Inset) Single atom trapped above a nanosphere acts as a
cavity QED system with atomic and cavity losses $\gamma$ and $\kappa$,
respectively, and a coherent coupling $g$. (b) Fidelity for generat-
ing a ground state singlet state between two atoms on the lattice
with their separation after optimization. The entanglement is
generated through interaction with the collective plasmon
modes, where we took the metal losses of bulk silver. (Inset)
Scalable cavity QED array of atoms and plasmons.
lattice [38,39]. We calculate the interaction of two atoms through these modes in a 1D chain of nanospheres. For each sphere in the chain, we can write the self-consistent equation for their dipole moments as [40]

$$p_n = \alpha(\omega)(E_n + N_{nm}p_m),$$

where $p_n$ is the induced dipole moment of the $n$th nanoparticle, $E_n$ is the incident field, and $N_{nm}$ is the $3 \times 3$ matrix that gives the dipole field at site $n$ due to the dipole at site $m$. In 1D, there are two sets of transverse modes where the dipoles are oriented perpendicular to the chain and one set of longitudinal modes for parallel orientation. Defining $\tilde{p}_q$ to be the $q$th eigenvector of $N_{nm}$ with eigenvalue $D_q$, then the effective polarizability of the $q$th mode is $\alpha_q^{-1} = \alpha^{-1} - D_q$, i.e., $\tilde{p}_q = \alpha_q E_q$. For a Lorentzian polarizability, the real part of $D_q$ gives the shift in the resonance frequency of the $q$th mode, and the imaginary part gives the change in the linewidth. $N_{nm}$ is diagonalized by Fourier transform, and if we neglect all but nearest neighbor terms, $D_q = 2N_{01} \cos q - i k^3/6\pi \epsilon_0$, where $N_{01} = \text{Re}(N_{q0})$.

Let us consider atoms trapped above the 1D array of spheres. The plasmonic modes can be adiabatically eliminated by using standard methods in quantum optics [41]. For two-level atoms polarized parallel to the 1D chain, the atomic density matrix evolution is

$$\dot{\rho} = -\frac{i}{2} \sum_n [\sigma_n^+, \rho] - \frac{i}{2} \sum_{nm} \delta \omega_{nm} [\sigma_n^+ \sigma_m^-, \rho] - \frac{1}{2} \sum_{nm} \gamma_{nm} (\{\sigma_n^+ \sigma_m^-, \rho\} - 2 \sigma_m^- \rho \sigma_n^+).$$

$$\delta \omega_{nm} = -\frac{3\epsilon^3}{8k^3 \epsilon_0^2} \Gamma_0 \text{Re} \left( \frac{\epsilon^*}{\sin^*} \right) e^{-q^*|n-m|},$$

$$\gamma_{nm} = \frac{3\epsilon^3}{8k^3 \epsilon_0^2} \Gamma_0 \text{Im} \left( \frac{\epsilon^*}{\sin^*} \right) e^{-q^*|n-m|},$$

where $z$ is the position of the atoms above the sphere and $q^* = q^*_i + i q^*_f$ is the resonant wave vector such that $\alpha^{-1}(\omega_n) = 0$. The first line in Eq. (4) describes the coherent evolution, and the second line describes the collective dissipation. Here we have neglected the contribution to the interaction from free-space radiative modes.

The coherent and dissipative contributions to Eq. (4) are equally strong when the atom and plasmon are near resonance. Working far off resonance, however, results in purely coherent dynamics, which can be used to implement long-range interacting spin models including frustration [42,43]. Alternatively, the collective dissipative dynamics can be used to prepare correlated atomic states [44]. As an example, we now show how to directly prepare a ground state singlet between two atoms separated by large distances on the lattice. We take two ground states $|g\rangle$ and $|s\rangle$ and an excited state $|e\rangle$ which is coupled to $|g\rangle$ via an external field and only decays via the plasmons back to $|g\rangle$ [see the inset in Fig. 3(a)]. An external microwave field mixes the two ground states. To prepare the singlet state $|S\rangle = |gs\rangle - |sg\rangle$ we use a similar approach to Ref. [45], whereby the singlet state is engineered to be the steady state of a driven, dissipative evolution. We take a separation $n$ such that $\cos \alpha q^*_n = 1$ and

$$\dot{\rho} = -\gamma_0 \mathcal{D}(|\sigma_1^{ge} + \sigma_2^{ge}|) - \delta \gamma_n \mathcal{D}(|\sigma_1^{ge}| + \mathcal{D}(|\sigma_2^{ge}|)) \rho,$$

where $\mathcal{D}[c] \rho = 1/(2[c^+ c, \rho] - c^* c \rho)$ and $\mathcal{D}[c] \mathcal{D}[c^+] \rho = \mathcal{D}[c^+] \mathcal{D}[c] \rho$. The dynamics can be mapped to a cavity QED system by identifying $\gamma_0$ with the collective decay $\gamma^2/\kappa$ and $\delta \gamma_n$ with the free-space decay $\gamma$. The two excited states $|ge\rangle$ and $|ge\rangle$ split into a superradiant state $|ge\rangle + |ge\rangle$ and a subradiant state $|ge\rangle - |ge\rangle$ with decay rates $2\gamma_0 + \delta \gamma_n$ and $\delta \gamma_n$, respectively.

The singlet preparation proceeds as follows. First, we selectively excite the subradiant transition $|ge\rangle$ to $|ge\rangle$ by driving with a weak external laser field $\Omega - \delta \gamma_n \ll \gamma_0$, which we take to have a $\pi$ phase difference on the two atoms. Second, in order to make the singlet state a unique steady state, we apply a global microwave field to mix the triplet ground states without affecting the singlet state. In the resulting dynamics, the pumping rate into the singlet state is $\Omega^2/\delta \gamma_n$, while the pumping rate back into the triplets is $\Omega^2/\gamma_0$ [26]. The steady state of this process gives the singlet state with fidelity $F = \langle S|\rho|S\rangle \sim 1 - 1/P'$, where $P' = \gamma_0/\delta \gamma_n$. Figure 3(b) shows the fidelity for two atoms with variable separation obtained from numerical simulation of Eq. (4).

To measure the correlations in this system, an all-optical approach could be realized by making the nanoparticle array in the near field of a solid immersion lens, which enhances the resolution beyond the diffraction limit by a factor of $n$, the index of refraction of the solid immersion lens [46]. Combining a solid immersion lens with, e.g., superresolution microscopy techniques would allow one to reach the requisite resolution of $\sim 50$ nm at optical wavelengths [47].

Our analysis shows that combining cold atom techniques with nanoscale plasmonics reaches new regimes in controlling both the collective motion of atoms and atom-photon interactions. Combining excellent quantum control of isolated atoms with nanoscale localization may open up exciting new possibilities for quantum control of ultracold atoms.

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