Denial and Punishment in the North Caucasus: Evaluating the Effectiveness of Coercive Counter-Insurgency

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<td>doi:10.1177/0022343312448389</td>
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Denial and Punishment in the North Caucasus: Evaluating the effectiveness of coercive counterinsurgency

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March 20, 2012

Forthcoming in Journal of Peace Research

Abstract

A growing literature on the subnational diffusion of armed conflict rests on the proposition that political violence triggers more violence, in the same locality and elsewhere. Yet state efforts to contain such uprisings remain largely unexplored, theoretically and empirically. Drawing on a mathematical model of epidemics, we formalize the logic of conflict diffusion and derive conditions under which state coercion might limit the spread of insurgent violence. Using a new dataset of insurgent and government violence in Russia’s North Caucasus from 2000-2008, we evaluate the relative effectiveness of four coercive strategies: (1) denial, which manipulates the costs of expanding insurgent activity to new locations, (2) punishment, which manipulates the costs of sustained fighting in contested areas, (3) denial and punishment, which does both, and (4) no action, which does neither. We find denial to be most effective at containing insurgent violence. Punishment is least effective, and even counterproductive. Not only does such a strategy fail to prevent the spillover of violence to new locations, but it may amplify the risk of continued fighting in contested areas. In the Caucasus, denial is found to be the least inflammatory counterinsurgency option for Russia. For it to succeed, Russia should physically isolate centers of insurgent activity from regions of non-violence, avoid the temptation of punitive reprisals, limit the insurgent’s options, and convince him that he cannot succeed.

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*We thank Idean Salehyan, Rich Nielsen, seminar participants at Harvard, and two anonymous reviewers for helpful comments on earlier drafts. Data, R code and supplemental appendix available at http://www.prio.no/jpr/datasets.
A growing body of political science research has shown violence to be contagious, its transmission facilitated by the flow of people, information, materiel and finances between geographically proximate locations. A natural question emerges: can these patterns of diffusion help us understand how outbreaks of insurgent violence can be stopped? In the emerging disaggregated literature on the spread of armed conflict, coercive efforts to contain violent uprisings remain largely unexplored.¹ A state’s ability to limit the diffusion of violence is generally ascribed to static attributes—such as capacity and regime type—rather than strategic choices.

This paper seeks to address these theoretical and empirical gaps, and forge greater integration between debates on coercion, counterinsurgency and the diffusion of conflict. Using a mathematical model of epidemics, a new dataset of violent events in Russia’s North Caucasus, and simulation, we evaluate the relative effectiveness of four strategies states can use to fight insurgents: (1) denial, which operates by physically isolating insurgents and manipulating the costs of expanding fighting to new locations, (2) punishment, which uses offensive operations in contested areas to manipulate the costs of sustained fighting, (3) denial and punishment, which does both, and (4) no action, which does neither. We find a denial strategy to be most effective at containing insurgent violence. Punishment is found to be counterproductive, slowing recovery from periods of violence and doing little to prevent the spread of unrest.

The article proceeds as follows. We first discuss the theoretical gaps and methodological difficulties impeding more rigorous study of the diffusion of insurgent violence. We then review the literature on strategic coercion, and extend its propositions to the containment of insurgent violence. We introduce an epidemiological model of insurgency and counterinsurgency, and a methodology to evaluate it empirically. We fit this model to new disaggregated data on violence and road networks in the North Caucasus, and empirically estimate the parameters of our formal model using simulation. We use these estimates to identify an optimal strategy for containing insurgent violence in the Caucasus. We conclude with a discussion of the broader implications of our findings.

¹Of 90 most widely cited conflict diffusion articles listed in the Web of Science since 1980, 77 are on the cross-national level and only indirectly address the dynamics of state responses to insurgency. Of the 13 subnational studies on the list, seven disaggregate conflict events by combatant. None explores the consequences of state coercion directly. The full list of articles is available in the online appendix.
Insurgency and the diffusion of violence

Research on the diffusion of violence explores the dynamic process by which armed conflict persists and expands: the tendency of one episode of violence to trigger other instances in the same geographical area or in places nearby (Murdoch & Sandler, 2004; Buhaug & Rød 2006; Raleigh et al., 2010; Hegre et al., 2009; Weidmann & Ward, 2010; Buhaug & Gleditsch, 2008; O’Loughlin & Witmer, 2011). The overarching question driving this work centers on how violence feeds itself: once war or insurgency breaks out, how does the fighting unfold and what is the best strategy to make it stop?

Our focus is on political violence within states, understood as the violent resolution of disputes between incumbent governments and insurgent challengers. The diffusion story begins when the two sides fail to reach a bargain that both prefer to war (Fearon, 1995; Reiter, 2003; Toft, 2003; Powell, 1996, 2006) and insurgents resort to the use of force to impose a change in the status quo (e.g. gain greater autonomy, independence, or control over an entire polity). The insurgents’ ability to impose this change depends on the costs of sustaining and expanding the violent campaign. The incumbent government seeks to maintain a monopoly on the use of force within its borders (Tilly, 1985). The more pervasive the insurgent violence becomes, the less credible the monopoly status remains. Responding to the insurgent challenge, the state may concede its monopoly status, or use coercive force to defend it. Should violence emerge between the two sides, the manner in which it unfolds depends on the coercive strategy the state deploys: punish insurgents where they are known to be active, or deny them the opportunity to expand fighting.

Extant literature has proposed numerous theoretical mechanisms to explain the diffusion of violence, specifying the factors or signals that must be transmitted for fighting to diffuse (e.g. information, labor, capital), and the communication channels that enable this transmission (e.g. alliances, institutions, roads). To better inform counterinsurgency theory and practice, however, conflict diffusion research will need to overcome several shortcomings.

2 Consistent with the distinction between coercion and brute force in intra-war bargaining (Schelling, 1966: 2-6), coercion seeks to compel a strategic decision by the adversary to stop fighting, rather than to physically destroy the adversary’s ability to fight. This is accomplished by through the cumulative infliction of costs (Smith, 1998; Filson & Werner, 2002; Smith & Stam, 2004), or by reducing uncertainty over the two sides’ abilities to inflict or absorb these costs (Slantchev, 2003; Powell, 2004).

3 Although we use these terms interchangeably, “spread” traditionally refers to the extent of territory affected by hostilities at equilibrium (Braithwaite, 2006: 508), “diffusion” refers to the dynamic process of expansion or relocation by which hostilities reach this extent (Schutte & Weidmann, 2011), and “contagion” refers to a mechanism facilitating this process – in which certain events are transmitted through various forms of physical contact (Iqbal & Starr, 2008: 319).
First, most research on the spread of violence and armed conflict has focused on macro-level phenomena, such as the dynamics of interstate wars (Most & Starr, 1980; Siverson & Starr, 1991; O’Loughlin & Anselin, 1991; Ward & Gleditsch, 2002) or cross-national spillovers of civil war (Lake & Rothchild, 1998; Salehyan & Gleditsch, 2006; Forsberg, 2008; Braithwaite, 2010). The more recent “disaggregated” study of war and insurgency owes much to this older research tradition (O’Loughlin & Witmer, 2011; O’Loughlin et al., 2010; Schutte & Weidmann 2011). However, many applications of geospatial methods to disaggregated conflict data remain descriptive (see review in Raleigh et al., 2010) and most theoretical mechanisms isolated at the macro level – cross-border interactions, alliances and linkage politics – do not travel easily to the micro level, where the propensity of violence to diffuse is more closely tied to military logistics and operational costs (Zhukov, 2012).

Second, the scope of most disaggregated work has been limited to the positive diffusion of violent phenomena across time and space (Most & Starr, 1980: 933). Although cross-national civil war research highlights the importance of state capacity to inhibit and contain the spread of violence (Braithwaite, 2010), similar “counter-diffusion” on the local level has received little attention. Unless the state is a passive actor – and we know it is not – the literature overlooks a potentially decisive interaction between the contagion of insurgent violence and corresponding state efforts to contain it. This omission has not entirely prevented cross-pollination between the literatures on conflict diffusion and counterinsurgency (Lyall, 2009; Kocher et al., 2011), but the state’s ability to shape the geographic spread of violence remains under-explored.

Third, the empirical study of conflict diffusion suffers from several methodological limitations, in part due to the unavailability of data sufficiently detailed to distinguish between key theoretical mechanisms. The spatial context of a conflict zone, for example, has traditionally been defined by measures such as border/grid contiguity or Euclidean (straight-line) distance. These are proxies at best for the real-world transportation and communication networks that we expect to facilitate the diffusion of violence. Such measures have been shown to induce significant bias into estimates of spatial dependence, particularly in rugged areas, where mountains and other impassable terrain violate the core metrics underpinning them (Lu & Chen, 2007; Zhukov, 2012).

Of no less concern is the literature’s treatment of connections between places – and the broader networks through which violence can spread – as static. While physical locations of municipalities and regions may indeed be fixed, the distances between them may not be. Combatants can block roads, close borders, and implement other measures to increase or
curtail accessibility. Capturing the dynamic nature of these networks can illuminate important patterns in the diffusion of violence and the strategic choices that states face as they confront insurgents.

**Counterinsurgency and the containment of violence**

The logic of diffusion forces a conceptual distinction between two types of coercive responses. If the government’s goal is to minimize the geographic scope and temporal persistence of insurgent violence—thereby restoring a monopoly on the use of force—coercion may be used to manipulate the costs of (1) expanding fighting to new areas, or (2) continuing fighting in already-contested areas. We call a strategy oriented around the former “denial,” and a strategy oriented around the latter “punishment”.

Denial as a coercive pathway is an ancient concept, but the most important recent uses come from research on nuclear strategy (Snyder, 1960), strategic bombing (Pape, 1996; Byman & Waxman, 2000; Horowitz & Reiter, 2001; Mueller, 2001), counterterrorism (Pape, 2003) and counterinsurgency (Arreguín-Toft, 2001, 2005). Denial is often associated with “counterforce” or “countermilitary” targeting, which seeks to reduce the perceived capacity of an adversary to affect a political outcome. Denial inflicts costs on insurgents by preventing physical communication between violent and non-violent locations through cordonning, roadblocks, and similar quarantine-like measures. When implemented effectively, denial transforms the conflict zone into a closed system—insurgents are unable to flee to or reinforce operations from adjacent areas—although the government does not take direct action to liquidate local insurgent presence.

By contrast, punishment-based approaches rely more heavily on the infliction of pain to gain coercive leverage, the effectiveness of which depends on a close association of fear with compliance (Schelling & Halperin, 1961; Snyder, 1960; Pape, 1996). Punishment seeks to raise the costs of continued insurgent activity through the use of various kinetic operations, such as search-and-destroy missions, artillery shelling, air strikes and raids at sites of recent violence. These costs are intended to test the insurgents’ resolve and are inflicted across a broad set of “countervalue” targets, potentially including those not normally considered legitimate in war, like noncombatants. However, punishment does not restrict insurgent movement to and from an area in any systematic way.

These strategies are not mutually exclusive. A government may combine the two approaches by blocking the affected area while simultaneously conducting military opera-
tions within it, as in cordon-and-search operations and their Russian variant, the mop-up (zachistka). Alternatively, a government can avoid coercion altogether.

The logic underpinning these strategies has shaped the evaluation of counterinsurgency in Vietnam (Leites & Wolf, 1970; Schultz, 1978; Pape, 1996), Iraq (Petraeus, 2007; Peters, 2007), Chechnya (Arreguín-Toft, 2001, 2005), and the Palestinian Territories (Kaplan et al., 2005). Despite these efforts, deep divisions remain over the relative effectiveness of the two approaches: whether denial can prevent conflict contagion, whether punishment is inflammatory or suppressive, and whether – when deployed together – the effects of one strategy intensify, diminish or reverse the effects of the other.

While there are a number of advocates of denial-based coercive approaches to counterinsurgency (e.g. Galula, 1963) and an increasing number who advocate grievance redress (e.g. Thompson, 1966; Arreguín-Toft, 2001, 2002, 2005, 2007; Nagl, 2002), it is relatively difficult to find advocates of punishment. The most famous articulation of this approach is in Trinquier (1961). More recent incarnations have appeared in Merom (2003), Luttwak (2007) and – on the practitioner side – Peters (2007). Arguments in favor of a punishment-based approach share a common thread: the adversary’s capacity to harm us is beyond our ability to affect directly, and, even if it were not, we cannot reduce to an acceptable level the probability of being physically harmed by this same adversary. Denial is too costly, too gradual, and too passive. The only acceptable policy option is to convince one’s adversary that the costs of resistance are existential. Anything short of such an effort is at a minimum naive or, as Luttwak puts it, “malpractice.”

Similar questions over the utility of punishment have emerged in research on the relationship between government repression and the diffusion of violence in rebellions and civil wars. Some scholars have found that a state can prevent the diffusion of violence through the threat or actual use of force (Toft, 2003; Weidmann, 2009; Braithwaite, 2010), especially if a state is particularly strong or repressive (Weyland, 2009, 2010; Beissinger, 2007). Others have found the opposite relationship, where state repression escalates fighting by creating a pro-insurgent backlash that actually reduces the recruitment and logistical costs of continuing insurgent violence (Francisco, 2004; Saxton & Benson, 2008). Still others highlight a more complicated picture, where moderate levels of repression inflame violence, but high and low levels dampen it (Hegre et al., 2001; Salehyan & Gleditsch, 2006; Buhaug & Gleditch, 2008).

As with much literature on conflict diffusion, repression research has kept its focus mostly on cross-national comparisons rather than subnational dynamics of violence. Countries’ use
of repression has traditionally been measured indirectly through static attributes – like regime type (Hegre et al., 2001; Salehyan & Gleditsch, 2006; Iqbal, 2007; Buhaug & Gleditsch, 2008; Braithwaite, 2010) – which may shape insurgents’ expectations of government strategy, but are poor proxies for whether, where and when a certain coercive strategy is actually employed.

While there is general agreement over “ideal-type” definitions, the literatures on diffusion, coercion and repression have largely avoided systematic empirical study of the relative effectiveness of denial and punishment in counterinsurgency. At a minimum, we should expect a more rigorous effort to clarify the causal logic of each approach, and empirically evaluate each set of claims to establish not whether either “works,” so much as to bracket the conditions under which each approach is apt to be more or less successful.

An epidemic model of insurgency and counterinsurgency

To evaluate the effectiveness of different coercive strategies, we begin with a model of how insurgency spreads – absent any government countermeasures. Following the classic statement of the war diffusion hypothesis by Most & Starr (1980: 933), we distinguish between two interrelated processes: the impact of violent events on the likelihood of future fighting in the same location, and their impact on future fighting in other locations. The first of these, which we call recovery, determines how quickly a location transitions from violence back to peace. Where the costs of continuing operations, recruiting local personnel and procuring supplies are high, insurgent activity is more difficult to sustain, and locations recover from violence at a faster rate. The second process, transmissibility, determines how quickly insurgents can reinforce or expand the fighting to other areas. Where the costs of transporting personnel, ammunition and materiel are high, violence is transmitted from one location to another at a slower rate. These parameters operate in opposite directions: insurgency is most pervasive when recovery from violence is slow but transmissibility is fast.

This framework describes several dynamics of violence identified in the literature on conflict diffusion (Cohen & Tita, 1999; Schutte & Weidmann, 2011; Zhukov, 2012). Slow recovery and fast transmissibility facilitate an escalation of violence – fighting persists in its location of origin, while expanding to neighboring areas. When recovery and transmissibility are both fast, a relocation of violence is more likely – the fighting shifts to new areas while abandoning old ones. Slow rates of recovery and transmissibility produce hot spots – persistent violence in certain locations, but little spillover to neighboring areas. Fast recovery
and slow transmissibility limit the fighting to rare, isolated events.

Faced with insurgent violence, a government responds with a bundle of coercive strategies. Punishment seeks to increase the costs of sustained insurgent activity by inflicting casualties in contested areas, but does little to limit the expansion of insurgent activity. Denial seeks to increase the costs of expanding insurgent activity through cordons, roadblocks and other obstructions, but avoids direct engagement with the opponent. The first approach is effective if it accelerates the recovery of locations from violence. The second is effective if it slows the transmission of violence to new areas. The government may implement these strategies separately, jointly, or not at all.

These dynamics are illustrated in Figure 1a and formalized as follows:

\[ V' = (\beta - d)VC - (\alpha + p)V \]
\[ C' = -(\beta - d)VC + (\alpha + p)V \]  (1)

where
- \( V \) = proportion of units experiencing insurgent violence at \( t \),
- \( C \) = proportion of units experiencing no violence at \( t \),
- \( V', C' \) = time derivatives of \( V, C \),
- \( \beta \) = rate of transmissibility in the absence of denial,
- \( \alpha \) = rate of recovery in the absence of punishment,
- \( d \) = offsetting impact of denial on transmissibility,
- \( p \) = offsetting impact of punishment on recovery.

Assuming \( V + C = 1 \), \( \beta, \alpha > 0 \), \( |p| < \alpha \) and \( |d| < \beta \) this system has two equilibria:

\[ V_{eq} = 0 \quad C_{eq} = 1 \quad \text{(non-violent)} \]  (2)
\[ V_{eq} = 1 - \frac{\alpha + p}{\beta - d} \quad C_{eq} = \frac{\alpha + p}{\beta - d} \quad \text{(violent)} \]
In the first equilibrium, every unit is non-violent and the government enjoys a monopoly on the use of force. In the second, non-violent and violent units coexist at levels determined by the recovery and transmissibility parameters. These solutions are plotted in Figure 2 in the absence of any offsetting coercive measures ($p = 0, d = 0$), with higher proportions of $V_{eq}$ shown in darker colors. The dynamics depend on the basic reproduction number $R_0$,

$$R_0 = \frac{\beta - d}{\alpha + p}$$

(3)

The violent equilibrium becomes possible only where transmissibility is faster than recovery ($R_0 > 1$), as in the area below the diagonal in Figure 2. Everywhere else ($R_0 < 1$), the system will converge to a non-violent equilibrium.\(^4\) The government’s counterinsurgency objective is to make $R_0$ as small as possible.

![Figure 2: Equilibrium levels of insurgent violence ($V_{eq}$) in the absence of government coercion ($p, d = 0$). Darker shades indicate greater prevalence of insurgency.](image)

\(^4\)Proof of this stability condition is in the online appendix.
A strategy bundle \( \{d, p\}_k \) from choice set \( k \in S \) is considered optimal if \( R_0(k) < R_0(j) \forall j \neq k \in S \) — meaning that the prevalence of insurgent violence under strategy \( k \) is lower than it would have been under any other strategy. Punishment and denial actions are considered successful if they increase the recovery rate and reduce the transmissibility rate \( (p > 0, d > 0) \), respectively. The empirical record shows, however, that coercion does not always operate as intended. It is possible that either or both of these measures prove counterproductive \( (p < 0, d < 0) \), inflaming local grievances through repression or increasing the flow of illicit goods due to corruption and bribery at checkpoints. In such cases, successes in one area may be offset by failures in others, producing a null or even deleterious effect on overall levels of violence.

An empirical estimate for \( R_0 \) can be calculated by modeling the deterministic system in Figure 1a as the two-state finite Markov Chain shown in Figure 1b. Following Amemiya (1985) and Jackman (2000), a logit link function can be used to relate covariates to the probability of transitioning from one state to the other:

\[
Pr_{it} = Pr_{i,t-1}(V)Pr_{i,t-1}(V|V) + Pr_{i,t-1}(C)Pr_{i,t-1}(V|C) \\
= \log^{-1}[y_{i,t-1}(x_{it}\phi_V) + (1 - y_{i,t-1})(x_{it}\phi_C)]
\] (4)

where \( y_{i,t-1} \) is a binary variable coded 1 if unit \( i \) is experiencing insurgent violence at time \( t - 1 \), and 0 otherwise. The covariates \( x \) include a government’s counterinsurgency strategy choice and various local demographic, socioeconomic and geographic risk factors. \( \phi_V \) and \( \phi_C \) are sets of regression coefficients that capture the conditional effects of the covariates under the two possible prior states (violent and non-violent).

Predicted probabilities from this model can be used to find empirical estimates of the transmissibility and recovery rates \( (\beta \) and \( \alpha) \), the offsetting impacts of punishment and denial \( (p \) and \( d) \), and the reproduction number \( (R_0) \),

\[
\hat{R}_0 = V_0^{-1}\frac{\ln \left( Pr(C|C) \right)}{\ln \left( Pr(V|V) \right)} \propto \log_{Pr(V|V)} Pr(C|C)
\] (5)

where \( Pr(C|C) \) is the mean predicted probability of continuing non-violence, \( Pr(V|V) \) is the mean predicted probability of continuing insurgent violence and \( V_0 \) is the initial proportion of locations experiencing violence (a constant).\(^5\)

The epidemic model has three advantages for strategic evaluation. First, by permitting

\(^5\)Full derivation of this statistic is provided in the online appendix.
the effects of strategy choices to vary depending on the prior state of a locality, the empirical model in (4) mirrors the logic of the theoretical model in (1), which also postulates different theoretical mechanisms behind new versus recurring incidents of insurgent violence (i.e. transmissibility versus recovery). Second, the model allows us to estimate theoretical quantities of interest like the basic reproduction number directly from the data, account for uncertainty, and compare these estimates across various hypothetical scenarios. Third, the model makes no assumptions about the effectiveness of counterinsurgency measures: punishment may have inflammatory or suppressive effects, just as denial may succeed or fail to increase communication costs. The model assumes only that these measures are in a counterinsurgent’s arsenal, which is the case for countries as disparate as Russia, the U.S. and Pakistan. Whether these measures operate as intended, and whether a different strategy might produce better results, is a matter for empirical assessment.

Data and measurement

To apply the epidemic model to a real world case, we use an original dataset of violent events in Russia’s North Caucasus region, assembled from incident reports maintained by the independent Memorial Center. We aggregated individual events of violence to monthly indicators at the municipal level in nine regions of southern Russia: Dagestan, Chechnya, Ingushetia, North Ossetia, Kabardino-Balkaria, Karachaevo-Cherkessiya, Adygea, Krasnodar Kray and Stavropol Kray. Our overall sample size is 773,568 (7,584 municipalities × 102 months between July 2000 and December 2008). For the purpose of out-of-sample prediction, we withheld a randomly selected 10 percent of the sample. The spatio-temporal structure of the data is that of a dynamic network, where the municipalities are nodes connected to each other by shortest-path road distances. These road distances vary over time, depending on the government’s decision to block or keep open certain routes.

Automated dictionary-based event coding was used to classify reports into the categories defined below. A fuzzy matching script was used to geocode the events against the 7,584 municipalities included in the National Geospatial Intelligence Agency’s GEOnet Names Server (GNS) for the North Caucasus region. Of 38,789 records in Memorial’s timeline, 9953 were reports of a historical nature, press statements, and other entries not addressing specific incidents of violence or their geographical locations. Of the remaining 28,836, we were able to geocode 73% at the municipality level, 79% at the rayon (district) level and 94% at the oblast (province) level. Additional details on the data and how variables were measured are provided at http://www.prio.no/jprdatasets.

The term “municipality” refers to a single populated and named locality, such as a city, settlement, town, village or hamlet.
Our dependent variable, insurgent violence, is binary.\textsuperscript{8} It is coded 1 if at least one incident of insurgent violence was reported in a given municipality-month and 0 if no insurgent violence was reported. Insurgent violence includes all actions where members of an “unlawful armed group” (NVF) engaged in at least one of the following tactics: terrorist attack, hostage-taking, firefight, bombing, ambush or hit-and-run attack.\textsuperscript{9} This definition excludes events initiated by government forces and non-political acts of violence – such as those resulting from unambiguously criminal activity like burglary and armed robbery. Of the 773,568 cases in our sample, insurgent violence was recorded in 3,202 (0.41%). Our set of explanatory variables includes a counterinsurgency strategy portfolio and a set of local controls. Recognizing that strategic intent is not always easy to infer from event data, we defined such choices by the government actions we felt were consistent with one of our ideal types.

**Punishment strategy**

Incidents consistent with an punishment strategy include physically lethal operations – such as search and destroy missions, artillery and air strikes, and raids – conducted within the boundaries of a given municipality by the Russian armed forces, security services (FSB), internal troops (MVD) or their regional and local affiliates. Where at least one such incident took place in a municipality-month, but was not accompanied by denial actions, we assigned the punishment indicator a value of 1. Our sample includes 1,505 (0.19%) such cases.

**Denial strategy**

Actions consistent with a denial strategy include government efforts to physically disrupt lines of communication connecting a municipality to other locations. This definition goes beyond routine road obstructions like vehicle checkpoints and includes only larger-scale operations like efforts to establish a cordon around a whole village or town. At the event level, this variable was coded in a manner mutually exclusive from punishment, such that no denial event included an action where local kinetic operations were reported to have also

\textsuperscript{8}The binary coding decision is theory-driven. The expressions in (1) and (4) require that each municipality be grouped into one of two discrete states – violent (V) or nonviolent (C). Although the model can be extended to accommodate more detailed measures of violence intensity, little information is lost in this dichotomization: the mean number of insurgent attacks in village/months in state V was 1.65 (SD: 1.85), with a median of 1.

\textsuperscript{9}NVF is a Russian legal term, which applies to any armed group, militia, guerilla or terrorist organization, formed outside the frameworks of existing laws and operating outside the command and control structure of the Russian state.
taken place. If both types of events were observed, the resulting interaction was called denial + punishment. We recorded 333 (0.04%) cases where only denial was employed and 97 (0.01%) cases where both actions were used in the same month and locality. Since denial actions are employed far less frequently than punishment, one may question whether the two strategies are indeed substitutes. The establishment of a cordon is more logistically complex and resource-intensive than a raid, and one may expect such measures to be employed in response to higher levels of insurgent activity. If denial is an escalation from punishment, however, the motivation for such escalation is not apparent from the data. Villages where Russian forces employed denial actions experienced, on average, 0.90 (standard deviation 1.87) insurgent attacks in the previous month, compared to 0.89 (standard deviation 2.37) for villages where punishment was used. In addition, the data reveal that government strategy is not driven by demographic and topographic conditions. Differences in means across the two groups were not statistically significant for any pre-coercion variables.

While punishment is modeled as a binary indicator, we accounted for denial by modifying the network topology of our dataset. For each observation, we calculated the road distance to the closest municipality where an act of insurgent violence was reported during the previous month. This value was based on an origin-destination matrix of 57,517,056 shortest-path road distances connecting our 7,584 municipalities, and time-lagged values of the dependent variable. Where a denial action was reported, we modified the matrix to reflect that location’s temporary inaccessibility due to road obstructions – effectively treating the municipality as geographically isolated with no road connections heading in or out.

Control

In addition to a government’s strategic choices, rates of transmissibility and recovery may be mitigated or amplified by local and regional factors. Disaggregated research on civil wars (Raleigh & Hegre, 2009; Weidmann, 2009; Balcells, 2011) has found areas with a high population density to be at greater risk of insurgent violence due to the abundance of potential targets for insurgent attack and a potentially large pool of manpower to sustain the violence. Absent a village-level census of insurgent supporters – data not available to intelligence ser-

\footnote{Municipalities where denial was used had an average population density of 2158 people/km2, elevation of 349 meters, slope of 2.22 degrees, and were 30 km by road from the nearest military or MVD base. The same statistics for punished villages were 1942 people/km2, 318 meters, 2.19 degrees and 34 km.}

\footnote{The use of road distances to capture the mutual accessibility of municipalities to insurgents follows Zhukov (2012), who examines the role of road networks in the diffusion of conflict in the North Caucasus, and finds that the use of road distances in spatial lags leads to superior model fit and prediction accuracy than Euclidean distance.}
vices, much less to social scientists – a geographically concentrated population can serve as a proxy for insurgents’ opportunities to mobilize, overcome coordination problems and sustain fighting (Toft, 2003; Weidmann, 2009). We measure population density as the number of people residing within one square kilometer of territory. This value ranges from 0 to 25,181, with a median of 13.

A further factor shaping the transmissibility of violence is local terrain. Although cross-national research has long seen mountainous areas as conducive to insurgent activity (Fearon & Laitin, 2003; Hegre & Sambanis, 2006), disaggregated studies have revealed this relationship to be more complex (O’Loughlin & Witmer, 2011; O’Loughlin et al., 2010; Balcells, 2011). Remote areas may offer ideal conditions for base camps and sanctuaries, but they rarely offer a rich set of targets for attack. Municipalities located in flat, low-lying terrain are more accessible to insurgents than those high in the mountains, where the costs of fighting at high altitude and inclement weather reduce opportunities for expansion. We account for terrain with two measures: (1) elevation, in meters above sea level, which ranges from -31 (lowest) to 2,818 (highest), with a median of 239, and (2) slope of elevation, in degrees, which ranges from 0 (flattest) to 40.02 (steepest), with a median of 1.16.

Some locations may also be of greater political significance than others. In contrast to cross-national findings that civil violence emerges where state capacity is lacking (Hegre et al., 2001; Mueller, 2003; Braithwaite, 2010), recent research has shown that conflict – particularly conflict over governance – tends to cluster around capital cities (Buhag & Rød, 2006; Hegre et al., 2009) and local hubs of political power (O’Loughlin & Witmer, 2011). Since attacks near political centers are likely to garner significant media visibility and policy impact, we expect regional capitals to attract a disproportionate share of insurgent resources. Similarly, areas in close proximity to major military installations are likely to attract a high share of attacks, due to their salience as the government’s strategic center of gravity and control. We calculated the road network distance from each municipality to the nearest major military base, defined as the headquarters of army, MVD and FSB units equivalent to battalion level or higher. This variable, in kilometers, ranges from 0 (closest) to 208 (farthest), with a median of 55.5.

A further explanation of political violence is found in local socioeconomic conditions. Areas of high unemployment may become hotspots of violence due to discontent arising from perceived relative deprivation (Gurr, 1970), the lack of legitimate economic options, and relatively low opportunity costs associated with participation in rebellion (Becker, 1968). Measured as the size of the idle share of a region’s working-age population, in thousands,
unemployment ranges from 1.9 (lowest) to 376.5 (highest), with a median value of 24.9.\textsuperscript{12}

Finally, the likelihood of new and renewed insurgent violence may be driven by a host of potentially unobserved factors, like local culture, traditions and norms, which vary across space but not necessarily time. Some of these local idiosyncrasies might shape both government strategy choices and insurgent target selection, making it difficult to consistently estimate the effect of one on the other. To account for this long-term spatial heterogeneity, we include a nonparametric thin-plate spatial spline, which fits a smooth surface as a function of each municipality’s geographical coordinates. This surface is interpreted as baseline risk of insurgent violence. We fit the resulting semiparametric specification as a Generalized Additive Model (GAM) with a logit link (Wood, 2006).

\section*{Finding the optimal strategy}

Table I shows parameter estimates for four GAM logit specifications. Model 1 regresses the incidence of insurgent violence in a municipality-month on variables capturing a government’s strategic choice (kinetic operations for punishment, road distance to nearest attack for denial), controls for population density and elevation, and the spatial spline. Models 2 and 3 also consider the impacts of proximity to major military installations and regional capitals. Model 4 examines the impact of unemployment in the reduced sample. We also ran an additional four models with identical specifications apart from the substitution of slope for elevation. Since those results were virtually identical to those in Table I, we report them in the online appendix.

The parametric portion of each model is presented in two columns. The first set of regression coefficients ($\phi_C$) reports the determinants of transitions from a non-violent to a violent state $Pr(V|C)$. The second ($\phi_V$) reports the determinants of remaining in a violent state $Pr(V|V)$.

To evaluate whether observed conflict dynamics were more likely to have been generated by an epidemic process than a strictly additive one, we tested the null hypothesis that the determinants of violence are constant irrespective of the previous state of a unit ($\phi_V = \phi_C$). This hypothesis was rejected at the $p < 0.001$ level by likelihood ratio tests of all models against additive specifications with the same covariates, suggesting that epidemic models

\textsuperscript{12}Unlike our other controls, which have universal coverage over the population of cases, unemployment statistics contain a large number of missing records, mostly in Chechnya during the most active phase of the conflict, 2000-2005. Because these values are not missing at random, we include this variable in a separate model on a reduced sample for which we have complete data.
provide a more empirically plausible fit. Areas under the curve of receiver-operator characteristic plots (AUC) further suggest that all models exhibit excellent prediction accuracy, with in-sample AUC’s ranging between .93 and .94, and out-of-sample statistics between .91 and .93. The best-performing model, with the lowest Akaike Information Criterion (AIC) and highest AUC, is Model 3. Unless otherwise indicated, the parameters of this model are used for inference and simulations.\footnote{AIC statistics for Model 4 are not comparable to the rest, as they use a different sample size.}

All models suggest that punishment actions have a strong inflammatory effect on new and recurring cases of insurgent violence. In previously non-violent municipalities, the use of punishment produces a relative risk of new violence equivalent to 10.64, or 964 percent higher than in a location where punishment is not employed, with a 95 percent confidence interval of (CI: +817, +1,023).\footnote{Relative risk and confidence interval estimates are based on 1,000 simulations using parameters from Model 3.} In municipalities already experiencing insurgent violence, the impact is a smaller, but still formidable 96 percent (CI: +52, +102).

Denial, meanwhile, appears to have a suppressive effect on new cases of violence. A municipality located 100 km from a location where insurgent violence recently occurred is 44.6 percent less likely (CI: $-38.7, -46.6$) to transition from non-violence to violence than a location located less than 1 km away. The impact on recurring violence is statistically indistinguishable from zero, indicating that road closures restrict opportunities for conflict to spread to new locations, but have little influence on the duration of violence after it erupts. This heterogeneity is consistent with the epidemic model: denial strategies operate by changing transmissibility rates, not recovery rates.

Beyond these core findings, several ancillary results are worth noting. As expected, municipalities in areas of high population density are at greater risk of both new and recurring acts of insurgent violence. The impact of other structural conditions, however, is mixed. Although villages located in difficult terrain are more likely to remain non-violent than counterparts in flat, low-lying areas, they also take longer to recover from violence.\footnote{Municipalities at an elevation of 500 meters are 46.83 percent less likely to experience new violence than those at sea level (CI: $-46.81, -46.85$), but are also 6.01 percent more likely to remain in a state of conflict once attacked (CI: 3.5, 8.6).} Similarly, road distance from a major military installation appears to have opposite effects on the occurrence and re-occurrence of conflict events.\footnote{Municipalities 100 km from such a facility are 75.1 percent less likely to experience new violence than those less than a kilometer away (CI: $-70.9, -76.4$), but are 19.6 percent more likely to experience continued fighting (CI: +8.7, +86.1).} The same considerations that turn municipalities near military bases into attractive insurgent targets may also make fighting...
there less sustainable, due to greater local government capacity. Finally, higher unemploy-
ment has a slight positive impact on new cases of insurgent violence, but no effect on areas
already experiencing violence.

What are the implications of these empirical patterns for strategic choice? While iso-
lation from areas of insurgent activity may prevent the spread of violence, this observation is
by itself insufficient to show that denial strategies – implemented consistently and system-
atically – outperform punishment. The impact of road closures on communications depends
on where in the network they are implemented – in remote, poorly connected areas, or in
centrally-positioned transit hubs. Comparing average probabilities of violence at different
hypothetical road distances overlooks this complexity. Counterfactual statements about the
effectiveness of denial require iterative re-specification of the network matrix to account for
road closures, and the re-calculation of inter-municipality road distances under the new net-
work structure. Conventional simulation-based statistical inference programs such as Clarify
or Zelig (King et al., 2000, Imai et al., 2008) skip the additional computational steps associ-
ated with dynamic networks, preventing us from extracting many theoretically meaningful
quantities – such as parameter estimates for our formal epidemic model, transition proba-
bilities, and forecasts of insurgent activity under different strategic scenarios. Nevertheless,
these quantities can still be calculated through specialized simulation techniques.

To facilitate the necessary inferences and find an optimal strategy bundle that minimizes
the basic reproduction number $R_0$ in (3), we ran four sets of simulations in which insurgents
attempted to stage a series of attacks and government forces attempted to contain them.
Each simulation begins with an insurgent offensive. An insurgent attack is carried out in
each municipality with probability $Pr_{it}(V)$, as defined in the stochastic epidemic model (4).
The parameters of Model 3 and initial conditions from December 2008 are used to calculate
starting values for $Pr_{it}(V)$, and a series of Bernoulli random draws determines if each mu-
nicipality transitions to violence.

Where the transition occurs, the government implements one of four strategies: denial,
punishment, denial + punishment or no action. We simulate the strategy choice by changing
the underlying variables in the model’s design matrix. For example, if violence breaks out
in a new municipality, we simulate implementation of the punishment strategy by changing
the value of the variable “punishment actions” in that municipality from 0 to 1. We sim-
ulate the implementation of the denial strategy by re-specifying the road network matrix,
severing road links between the attacked village and all other locations in the system, and
re-calculating distances to nearest sites of recent insurgent violence. These new values are
then plugged into Model 3 to generate a new set of predicted probabilities $Pr_{i,t+1}(V)$. Another round of Bernoulli draws determines the locations of new or recurring acts of insurgent violence, and the process is repeated until the distribution of violence converges to a stable equilibrium.

We ran the simulations for 24 consecutive time periods (measured in months) and repeated the process 100 times for each of the four strategy choices. We chose six months as a burn-in period while computing summary statistics of simulation results.\footnote{We discard the initial iterative period of the Markov Chain sample (burn-in) to minimize the influence of initial values on posterior inference.}

The maps in Figure 3 illustrate the impacts of each strategy after one time period (times 0 to 1), with black circles marking municipalities randomly selected to be in a state of conflict at time zero. All simulations started with the same distribution of violence based on model predictions from December 2008: 12 locations initially in a state of violence ($V$) and 7,572 municipalities in a state of non-violence ($C$). The mean $Pr_{it}(V)$ at time zero was .002.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{strategy_simulation.png}
\caption{Geospatial representation of simulation results. Values report changes in predicted probability of insurgent violence between times 0 and 1.}
\end{figure}

Once government strategies were applied, a general pattern began to emerge. Denial produced the most favorable outcome, reducing the risk of insurgent violence by more than 0.001 in 834 villages and increasing the risk in just 11, most of which were the blocked villages where a successful attack was recently carried out. When punishment was used, the risk of violence increased by over 0.001 in 148 villages – most of which were previously non-violent.
– while reducing the risk in only 60.

Figure 4 shows a time plot of $Pr_{jt}(V)$ averaged over all municipalities. By the third time period, a strategy of denial had reduced the mean probability of attack by a quarter, from 0.002 to 0.0015, while a strategy of punishment increased this risk to 0.0025. While violence was a rare event in each case, this difference accounted for an average of 8 additional municipalities entering or remaining in a state of violence. Denial + punishment had a suppressive effect (0.0018), but not as strong as denial. Finally, a strategy of no action yielded minimal change from the initial distribution of violence (0.0021), but still significantly outperformed punishment.\textsuperscript{18}

\textbf{Figure 4: Equilibrium probabilities of insurgent violence.}

For a deeper look at how each strategy operates, we disaggregated the marginal probabilities of insurgent violence $Pr(V)$ into their underlying components $Pr(V|V)$, $Pr(C|V)$, $Pr(C|C)$, $Pr(V|C)$, and plotted the resulting transition diagrams in Figure 5. One pattern common to all strategy simulations is that new incidents of violence are quite rare. When no actions are taken, a municipality in a state of non-violence will remain in that state with

\textsuperscript{18}A series of two-sample Kolmogorov-Smirnov tests rejected the null hypothesis that the four sets of posterior probabilities were drawn from the same distribution at the $p < 0.001$ level or better.
a probability of 0.998, and will experience insurgent violence two times out of a thousand. In the Caucasus, this risk level will generate 16 new cases of violence per month on average. When a denial strategy is implemented – by itself, or in concert with punishment – this transition probability is cut in half to 0.001, the equivalent of 8 fewer new cases.

\[
\begin{align*}
\text{Denial + Punishment} & : 0.38 \to 0.62 \to 0.999 \\
\text{Punishment} & : 0.32 \to 0.68 \to 0.998 \\
\text{Denial} & : 0.25 \to 0.75 \to 0.999 \\
\text{No Action} & : 0.20 \to 0.80 \to 0.998
\end{align*}
\]

Figure 5: Violence to non-violence transition plots. Transition probabilities reported. 95% CI’s shown in brackets.

The use of punishment has no discernible impact on non-violent areas, but significantly increases the risk of continued violence in municipalities already in violence. When no action is taken, a location in a state of violence is likely to remain in that state with probability 0.20 and become non-violent with probability 0.80. The probability of remaining violent increases to 0.32 when punishment is applied separately, and to 0.38 when applied along with denial.\(^{19}\)

Using these transition probabilities, we calculated estimates of the basic reproduction number \(R_0\). Recall that the government’s optimal strategy is one that minimizes the predicted value of \(R_0\). Figure 6 reports point estimates and 95 percent confidence intervals for this statistic under each of the four scenarios explored in our simulations.

Consistent with what we have already seen, a denial strategy produces the lowest \(R_0\), punishment produces the highest, while denial + punishment and no action both lie somewhere in the middle. With the initial condition \(V_0 = 12/7,584\), a strategy of denial is expected to bring the system to a non-violent equilibrium (\(R_0 < 1\)), and punishment is much more likely to produce a violent equilibrium (\(R_0 > 1\)). This result implies that \(d > 0\) (denial is an effective containment measure) and \(p < 0\) (punishment is counterproductive). Because

\(^{19}\) This result, of course, holds principally for cases where government strategies are implemented in a responsive rather than pre-emptive fashion. In mixed strategy cases – where some non-violent villages are given the same treatment as violent ones, while some attacked villages are skipped in the government's response – the inflammatory effects would not be limited to villages in state \(V\).
the rank ordering of the four reproduction numbers is invariant to the scaling factor $1/V_0$, the relative levels of $R_0$ are even more significant than this threshold. Whatever the scale of the initial insurgent offensive, denial will always outperform its alternatives. Punishment will always fare worst.

**Conclusion**

Following Most & Starr’s (1980) classic statement of the war diffusion hypothesis, we evaluate counterinsurgency effectiveness by formalizing the theoretical distinction between the recurrence of fighting in the same location and its displacement to new areas. We distinguish between two types of counterinsurgency operations, which may be implemented jointly or separately: punishment actions intended to hasten recovery from violence, and denial actions directed toward slowing the transmissibility of violence. Using a mathematical model of epidemics and new disaggregated data on Russia’s North Caucasus, we show that punishment systematically produces the opposite of its intended effect, but denial can be an effective containment measure.

Surveying the many battlefields of Russia’s North Caucasus, it is difficult to identify a unified strategic approach. By themselves, denial actions have accounted for a relatively small share of counterinsurgency operations (17 percent). The strategic emphasis now –
as it has been for much of the region’s history – remains “killing the enemy” rather than protecting non-violent areas from spillovers. Our analysis shows that such an approach is counterproductive. It does little to prevent outbreaks of violence in non-violent areas, while only amplifying the risk of continued fighting in contested areas. Denial remains Russia’s best option: physically isolate centers of insurgent activity from centers of non-violence, avoid the temptation of punitive reprisals, limit the insurgent’s options, and convince him that he cannot succeed.

The same prescription, of course, may not hold for every counterinsurgent. The epidemic model assumes only that a government seeks to contain insurgent violence, and employs some combination of denial and punishment to achieve this end. The model specifies the logic by which each strategy is intended to operate – but does not assume that the strategy operates as intended. Given the diversity of empirical findings on the effectiveness of repression and indiscriminate violence (Davenport, 2007; Kocher et al., 2011; Lyall, 2009), a model with such broad scope conditions has been in demand for some time. This flexibility enhances the model’s applicability as a counterinsurgency evaluation tool, but also invites caution about extrapolating results from any one case.

One difficulty with a denial strategy, for instance, is that it takes time (Arreguín-Toft, 2005). Different regimes may be differentially vulnerable to delays between the application of military force and visible progress toward ex ante stated political objectives. As noted by Mack (1975), prosperous and large democratic states are apt to be more vulnerable to delays on the battlefield, and although a denial approach may work best for Russia, others may find its success hard to demonstrate in the short term.

Nevertheless, our analysis of two approaches to coercive counterinsurgency in a single theater of operations suggests that if we cannot definitively say what we should do to quell insurgency and advance toward non-violent resolution of political disputes, we can still benefit from a much better understanding of what not to do in the pursuit of peace and stability.
Table 1: Markov transition model. Dependent variable: incidence of insurgent violence in village $i$ at time $t$ ($V_{i,t}$).

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
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</thead>
<tbody>
<tr>
<td>Punishment actions</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kinetic ops ($t-1$)</td>
<td>$\phi_C$ = 2.4073, $\phi_V$ = 0.8276</td>
<td>$\phi_C$ = 2.3917, $\phi_V$ = 0.8233</td>
<td>$\phi_C$ = 2.3702, $\phi_V$ = 0.6728</td>
<td>$\phi_C$ = 2.7565, $\phi_V$ = 0.5448</td>
</tr>
<tr>
<td>Denial actions</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Road distance to nearest attack ($t-1$)</td>
<td>$\phi_C$ = -0.1316, $\phi_V$ = 0.0531</td>
<td>$\phi_C$ = -0.118, $\phi_V$ = 0.0518</td>
<td>$\phi_C$ = -0.1279, $\phi_V$ = 0.0338</td>
<td>$\phi_C$ = -0.0765, $\phi_V$ = 0.0069</td>
</tr>
<tr>
<td>Population density</td>
<td>$\phi_C$ = 0.0004, $\phi_V$ = 0.0003</td>
<td>$\phi_C$ = (2e-05)<em><strong>, $\phi_V$ = (3e-05)</strong></em></td>
<td>$\phi_C$ = (2e-04)<em><strong>, $\phi_V$ = (3e-04)</strong></em></td>
<td>$\phi_C$ = (2e-04)<em><strong>, $\phi_V$ = (3e-04)</strong></em></td>
</tr>
<tr>
<td>Elevation</td>
<td>$\phi_C$ = -0.0015, $\phi_V$ = -0.0002</td>
<td>$\phi_C$ = (2e-04)<em><strong>, $\phi_V$ = (3e-04)</strong></em></td>
<td>$\phi_C$ = (2e-04)<em><strong>, $\phi_V$ = (3e-04)</strong></em></td>
<td>$\phi_C$ = (2e-04)<em><strong>, $\phi_V$ = (3e-04)</strong></em></td>
</tr>
<tr>
<td>Road distance to nearest mil. base</td>
<td>$\phi_C$ = -0.373, $\phi_V$ = -0.1249</td>
<td>$\phi_C$ = (0.0375)***, $\phi_V$ = (0.0695)</td>
<td>$\phi_C$ = (0.0399)***, $\phi_V$ = (0.0772)</td>
<td>$\phi_C$ = (0.0443)***, $\phi_V$ = (0.0829)</td>
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<td>Regional capital</td>
<td>$\phi_C$ = 1.803, $\phi_V$ = 1.652</td>
<td>$\phi_C$ = (2e-04)<em><strong>, $\phi_V$ = (3e-04)</strong></em></td>
<td>$\phi_C$ = (2e-04)<em><strong>, $\phi_V$ = (3e-04)</strong></em></td>
<td>$\phi_C$ = (2e-04)<em><strong>, $\phi_V$ = (3e-04)</strong></em></td>
</tr>
<tr>
<td>Unemployment (t-1)</td>
<td>$\phi_C$ = -9.0727, $\phi_V$ = -7.1078</td>
<td>$\phi_C$ = (0.9394)<em><strong>, $\phi_V$ = (0.9417)</strong></em></td>
<td>$\phi_C$ = (0.360)<em><strong>, $\phi_V$ = (0.411)</strong></em></td>
<td>$\phi_C$ = (0.3534)<em><strong>, $\phi_V$ = (0.4198)</strong></em></td>
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<tr>
<td>N</td>
<td>688,315 (in-sample)</td>
<td>688,315 (in-sample)</td>
<td>688,315 (in-sample)</td>
<td>666,529 (in-sample)</td>
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<td>AIC</td>
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<td>15,331.28</td>
<td>15,224.95</td>
<td>12,930.89</td>
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<td>AUC (prediction accuracy)</td>
<td>In-sample: 0.93, In-sample: 0.93</td>
<td>In-sample: 0.93, In-sample: 0.93</td>
<td>In-sample: 0.93, In-sample: 0.93</td>
<td>In-sample: 0.93, In-sample: 0.93</td>
</tr>
<tr>
<td>LRT $\chi^2$ (vs. additive)</td>
<td>101.59***</td>
<td>112.61***</td>
<td>139.55***</td>
<td>163.93***</td>
</tr>
</tbody>
</table>

Significance levels: *p<.05, **p<.01, ***p<.001
References


Shultz, Richard (1978) Breaking the will of the enemy during the Vietnam War: The operationalization of