Many emerging market economies oscillate between periods of high and low growth (see Aguiar and Gopinath, 2007). These changes in growth regimes generate business cycles that are markedly different from the ones observed in developed countries: consumption and investment are volatile relative to output, and net exports are strongly counter-cyclical. This volatility is often accompanied by sharp changes in the policy environment as well. For example, Figure 1 shows the relationship between two measures of expropriation and political risk and real GDP for Argentina between 1984 and 2007. For easy comparison to the GDP series, the risk factors are inverted so that an increase in the index corresponds to a decrease in risk, and all series are normalized to 100 in 1984. The risk factor series are highly correlated with output. When GDP is higher than average, the institutions and government policies in Argentina foster growth, as measured by increased political stability, enhanced respect for property rights, and stronger contract enforcement.

In Aguiar et al. (forthcoming), we develop a framework to understand these policy reversals and the associated economic volatility. In particular, we explore the joint dynamics of sovereign debt, investment, and expropriation risk in a small open economy model. Our departing point from previous work was the introduction of two political economy frictions. The first friction is that the government cannot commit to policies, either tax or debt policy. It always faces the temptation to expropriate capital and default on debt. The second friction is that the risk of losing office makes the government impatient relative to the market. We show theoretically that the combination of the government’s impatience (however small) and inability to commit generates perpetual cycles in

\[\text{Figure 1: Argentina: Time path of GDP and Expropriation Risk}\]
both sovereign debt and investment in an environment in which the first best capital stock is a constant. The small open economy dynamics converge to a region where the expected tax on capital varies with the state of the economy and investment is distorted more in recessions than in booms, generating persistent effects from iid shocks.

In this note, we extend our previous work along the following dimensions. First, we explore numerically the comparative statics of the behavior of investment, consumption, output and net exports to different rates of government impatience. Second, we discuss the implications of imposing a balanced budget rule on the government.

In our numerical exercise, we maintain a simple framework with iid shocks such that a government that discounts at the world interest rate would generate zero volatility in investment and consumption in the long run. The question we address is how much volatility and persistence is induced for reasonable reductions in the governmental discount factor. We find that the magnitudes are substantial. If the government discounts at 90 percent of the world interest rate, investment volatility rises to approximately five times that of output. Similarly, consumption volatility rises from zero to forty percent of output volatility. Moreover, the cyclicity of net exports switches signs – it is 1.0 at the world discount factor but declines and turns negative quickly as we increase the government impatience parameter.

We then consider the impact of imposing a balanced budget rule on the government. We prove a version of the folk theorem that states that the first best allocation can be sustained if the government is patient “enough.” In our numerical exercise, we show that the necessary threshold for government patience is extremely low. Specifically, as long as the government’s discount factor is at least 20 percent of the world interest rate, a balanced budget rule will deliver the first best allocation from the domestic agents perspective (who are assumed to discount at the world interest rate). Consequently, a balanced budget rule, as is being followed in countries like Chile and Brazil, can be welfare enhancing for domestic agents.

I. Environment

We study a small open economy populated by private agents and a government. There are two technologies in the economy that produce a single commodity. The traditional technology is associated with an endowment stream \( z \). The second technology has capitalists using domestic labor to produce output according to \( y = A(z)f(k, l) \), where \( A \) is total factor productivity indexed by \( z \), \( k \) is capital, \( l \) is labor, and \( f \) is a constant returns to scale production function. Aggregate output is given by \( F(z, k, l) = A(z)f(k, l) + z \), and \( F \) is strictly increasing in \( z \). We model \( z \in Z \) as a finite state Markov process that is iid over time, and let \( z^t = (z_0, z_1, \ldots, z_t) \) denote the history of shocks through time \( t \). Let \( z_{\text{max}} \) be the highest possible shock.

Risk neutral capitalists have an opportunity cost \( r + \delta \), where \( r \) is the exogenous world risk free rate and \( \delta \) is the depreciation rate. Capital is installed at the end of the previous period, before the shock is realized. Let \( k_{t-1} = k(z^{t-1}) \) denote capital installed at the end of period \( t - 1 \) after history \( z^{t-1} \) and operated in period \( t \). Firms face a competitive spot labor market and pay wages \( w_t = w(z^t) \) in period \( t \). Profits (gross of rent and depreciation) are denoted \( \Pi(z) = A(z_f)(k_{t-1}, l_t) - w_t l_t \). Profits are taxed at a rate \( \tau_t = \tau(z^t) \), which is set after capital is installed and the shock is realized. We limit the tax rate at 1, so that the government cannot take more than 100 percent of the capital income. Taking as given the equilibrium path of taxes, the firms’ optimality conditions for capital and labor are therefore:

\[
\begin{align*}
\mathbb{E}_{t-1}(1 - \tau_t)A(z_t)f_t(k_{t-1}, l_t) &= r + \delta \\
A(z_t)f_t(k_{t-1}, l_t) &= w_t.
\end{align*}
\]

The notation \( \mathbb{E}_{t-1} \) refers to expectation conditional on history through \( t - 1 \). For future ref-

4The existence of the traditional technology is needed to bound the autarky value from below (as will be shown below). An alternative assumption would be to impose a bound on the capital tax rate that is less than unity.

5In particular, we can think of capitalists as foreign based who can completely diversify the shocks hitting the small open economy.
ference, let \( k^* \) denote the first best capital. That is, \( \mathbb{E}A(z_t) f_t(k^*, 1) = r + \delta \).

The primary actor in the model is a government, which taxes capital and labor income, and borrows and lends a non-contingent bond with the rest of the world. The absence of contingent debt is without loss of generality, as taxes are assumed to be state contingent and can be used to trade resources across states of nature. Let \( b_t \) denote the debt contracted in period \( t \) and due in \( t+1 \). The government’s period budget constraint is given by

\[
\tau_t \Pi_t + b_t = T_t + (1 + r)b_{t-1},
\]

where \( T_t \) denotes lump sum transfers to the workers.

Workers do not have access to financial markets directly (or, equivalently for our purposes, the government has enough instruments to control their consumption/savings decisions). The representative worker’s budget constraint is

\[
c_t = w_t + t + z_t.
\]

As the government controls the worker’s consumption, we can combine and rearrange their budget constraints into a single aggregate resource constraint:

\[
F(z_t, k_{t-1}, l_t) - (1 - \tau_t) F_k(z_t, k_{t-1}, l_t) k_{t-1} + b_t = c_t + (1 + r)b_{t-1}.
\]

This equation states simply that consumption and debt payments (the right hand side) must equal total output minus equilibrium payments to capital plus new debt.

The government’s objective function is to maximize the present discounted value of utility of the workers: \( \mathbb{E}\sum_{t=0}^{\infty} \beta^t u(c_t) \). The interpretation is as follows: \( u(c_t) \) refers to the workers flow utility. Workers and capitalists are assumed to discount at the same rate, \( \frac{1}{1+r} \). However, \( \beta \) represents the government’s discount factor, which may be smaller than the agents’.

This captures a government that may lose office and therefore prefers consumption to occur sooner rather than later.\(^6\)

The government chooses a sequence of taxes to maximize this objective function subject to the aggregate resource constraint and the firms’ first order conditions. If the government could commit to a tax plan, we show in Aguiar et al. (forthcoming) that it would set taxes such that \( k = k^* \) every period. That is, it would not distort capital. While this implies taxes are zero “ex ante”, they are not zero ex post, as taxes and transfers with capitalists are useful to insure the worker’s consumption.

We are, however, interested in the case when the government cannot commit to a tax plan. We look for self-enforcing taxes such that the government has no incentive to deviate along the equilibrium path. These equilibria are supported by trigger strategies such that any deviation is punished by autarky, that is zero investment and no access to financial markets.\(^7\) If the government deviates, it transfers all the output that period to the workers and then lives off the endowment thereafter.\(^8\) Therefore, the value of deviation is: \( u(F(z, k, l)) + \beta V_{aut} \), where \( V_{aut} \equiv \mathbb{E}u(z)/(1 - \beta) \) is the government’s value function in autarky. A sequence of taxes and debt positions must satisfy the following participation constraints at every history:

\[
\mathbb{E}_z \sum_{s=0}^{\infty} \beta^s u(c_{t+s}) \geq u(F(z_t, k_{t-1}, l_t)) + \beta V_{aut}.
\]

Faced with these constraints, we assume that the government will pursue a sequence of taxes and debt positions to maximize its objective function.

II. Equilibrium Allocations

The equilibrium allocation can be solved using standard recursive techniques. We consider maximizing payments to debt holders conditional on delivering a particular utility to the government. This is the dual of the primal problem in which the government maximizes utility given an outstanding stock of debt. The state variable is the promised utility of the government, \( v \). Denote the net present value of payments to bond holders conditional on delivering

\(7\)In Aguiar and Amador (2008) we discuss conditions under which autarky is the worst self-enforcing equilibrium and so the particular equilibrium we study is on the Pareto frontier.

\(8\)More generally, the country could also consume the existing capital stock, or operate the capital for a period of time. The important assumption is that the utility from deviation is increasing in capital. The current formulation is a useful technical simplification.

\(^6\)This is a special case of the more general political economy preferences studied in Aguiar and Amador (2008).
v to the government as \( B(v) \).

Net payments to bond holders in a period is
\[
F(z_t, k_{t-1}, l) - (1 - \tau_t) F_k(z_t, k_{t-1}, l) k_{t-1} - c_t,
\]
which is total output minus payments to capital and domestic consumption. As debt is non-contingent, net payments are independent of the particular realization of \( z_t \). We therefore can average over the realizations \( z_t \), and as \( z \) is \( iid \) over time the expectation will be independent of the history through \( t - 1 \). Taking expectation over \( z_t \), the expected net payments are
\[
\mathbb{E} \{ F(z_t, k_{t-1}, l) - (r + \delta) k_{t-1} - c_t \},
\]
where we use the fact that expected payments to capital equal the opportunity cost in equilibrium.

The recursive problem can be expressed:

\[
B(v) = \max_{(u(z), \omega(z), k) \in \Omega} \sum_{z \in Z} \pi(z) \left[ F(z, k) - c(u(z)) \right.
\]
\[
- (r + \delta) k + \frac{1}{1 + r} B(\omega(z)) \right],
\]
subject to:

\begin{align}
\sum_{z \in Z} \pi(z) [u(z) + \beta \omega(z)] & \leq v \label{eq:1} \\
U(F(z, k)) + \beta V_{\text{out}} & \leq u(z) + \beta \omega(z), \quad \forall z \in Z. \label{eq:2}
\end{align}

The choice variables are capital, state contingent government utility \( u(z) \), and state contingent continuation utility \( \omega(z) \), which will be the next period’s state variable. The function \( c(u) \) is the inverse utility function, that is, the amount of consumption need to deliver utility \( u \). The first constraint is the promise keeping constraint and the second constraint is the participation constraint. The choice variables are taken from a compact set \( \Omega \), whose boundaries are chosen so as not to constrain the equilibrium allocations (See Aguiar et al., forthcoming, for details).

We studied the solution to this problem in detail in Aguiar et al. (forthcoming). Here, we summarize the key analytical results and refer the reader to that paper for proofs. We start with the optimal policy for capital. Capital is strictly increasing in promised utility \( v \) until \( k = k^* \). Specifically, \( k = k^* \) for \( v \geq v^* \equiv U(F(k, z_{\text{max}})) + \beta V_{\text{out}} \) and \( k < k^* \) for \( v < v^*. \) Capital is never larger than the first best. Considering the primal problem, a high \( v \) is equivalent to a low stock of outstanding debt. The results imply that debt crowds out capital. The intuition for this result is that a large amount of debt makes autarky relatively attractive. Therefore, the government cannot credibly accommodate a large amount of investment in the presence of the strong incentive to deviate. This translates into a high expected tax on capital.

Given the relationship of capital to the state variable \( v \), the next question is the evolution of promised utilities. Risk aversion implies that the full commitment solution equalizes consumption (or utility) across states. However, states with high \( z \) have a particularly attractive deviation payoff, so perfect insurance may not satisfy all the participation constraints.

If \( v \geq v^* \), then participation constraints are not binding and utility is equalized across states in the following period. If \( v < v^* \), then consumption is not equalized across states. In particular, high \( z \) states must have high consumption to satisfy participation. The incentive to smooth consumption over time implies that the continuation values \( \omega(z) \) are also higher following high shocks. Therefore, we have a spreading out of continuation values, with high shocks generating high continuation values and low shocks generating low continuation values. Given the policy functions for capital, this generates a positive correlation between the shock and investment, despite the iid nature of the shock. That is, the ability to smooth intertemporally, by using debt, induces persistence in the effect of \( z \) on output.

If the government discounts at the risk free rate, so that \( \beta = 1/(1 + r) \), then a stock of assets is built over time until enough is accumulated so that first best capital and perfect insurance is sustained in the long-run. However, if \( \beta < 1/(1 + r) \), the government is too impatient to sustain the first best. In fact, the economy converges to a unique, non-degenerate ergodic distribution for \( k \) whose support lies strictly below \( k^* \); that is, the economy will converge to a region where some participation constraint is always binding. The fact that this ergodic distribution is unique implies that transfers, such as a debt forgiveness policy, have only temporary effects. The fact that the distribution is non-
degenerate implies that volatility is a permanent feature of the economy. An economy governed by an impatient government is destined for lower income and more volatility than one who discounts at the market interest rate.

III. Numerical Analysis

While in Aguiar et al. (forthcoming) we show that in theory even a small amount of government impatience results in distortions in investment and consumption and volatile cycles in the long-run, it does not shed light on the quantitative magnitude of these distortions. We proceed now to study the model numerically to shed light on this question and to more generally evaluate the comparative static effects of government impatience on economic variables.

Specifically, we vary $\beta$ and evaluate the volatility, persistence and cross-correlation of some key economic indicators. The goal of this exercise is not a full calibration of a model economy. The purpose is to isolate the impact of government impatience and obtain an estimate of whether the effect is quantitatively significant. To that end, we maintain the simple framework of iid shocks in which the economy produces zero long run volatility in investment or consumption when $\beta = 1/(1 + r)$. The question at hand is how much volatility is induced by alternative discount factors.

For the exercise, we assume that the consumption of the capitalists takes place abroad (that is, they are foreign based). The parameters of the model are set as follows.9 A period in the model is one year. Utility is represented by the standard constant relative risk aversion utility function with the coefficient of relative risk aversion set to 2. Total output is given by $F(z,k) = zk^\alpha + z$ with $\alpha = 1/3$. $z$ take two values, with the high shock set to 1.0 and the low shock set to 0.9, and each state occurs with probability 1/2 capturing the iid nature of shocks. These values of $z$ generate a variance of 0.05, which is approximately the variance of HP-filtered log output for Argentina.10 We set both the risk free rate and the depreciation rate to 0.05.

We first consider mean effects. Figure 2 plots mean capital divided by $k^*$ as a function of $\beta(1 + r)$. Recall that at $\beta(1 + r) = 1$, capital is at the first best, so the ratio of mean capital to $k^*$ is one. As $\beta$ decreases, we see that mean capital is distorted down. Specifically, at $\beta(1 + r) = 0.9$, mean capital is distorted by 5 percent and at $\beta(1 + r) = 0.8$ the distortion approaches 10 percent.

Figure 3 plots the standard deviation of log investment normalized by the standard deviation of log income, where standard deviations are those of the ergodic distribution. At $\beta(1 + r) = 1$, the ergodic distribution of investment is a singleton at the first best. As $\beta$ declines, investment volatility increases markedly. For example, at $\beta(1 + r) = 0.9$, investment volatility is five times output volatility. This is a significant increase in volatility in that it is solely due to contracting frictions given the iid nature of the shocks. Note that the effect of $\beta$ is non-

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9The computer code used for the simulations is available from the authors' website. The numerical method used value function iteration on a 10,000 point grid (for each $\beta$ in a range). With the optimal policies computed, the artificial economies were each simulated for $10^6$ periods to generate the moments of the ergodic distribution.

10Specifically, using annual log output from 1960-2007 we extract a trend using an HP filter with smoothing parameter 100. The standard deviation of detrended output is 5.9%. A smoothing parameter of 6.25 yields a standard deviation of 3.6%.
monotonic. As $\beta$ approaches zero, no capital can be supported in equilibrium and investment is again constant.

Figure 4 plots the relative volatility of consumption. Again, the benchmark is perfect insurance if $\beta = 1/(1+r)$. However, a more impatient government generates increased volatility in consumption. The magnitudes are large: a discount factor equal to 90 percent of the market interest rate generates a consumption volatility of 40 percent of output, compared to zero at $\beta(1+r) = 1$.

Figure 5 indicates how the interaction of sovereign debt and investment induces persistence in output in an environment with iid shocks. In particular, it depicts the autocorrelation of output in the ergodic distribution. In the first best, this autocorrelation is zero. However, as $\beta$ falls, we see that output becomes autocorrelated. While the magnitude of the autocorrelation is not large in itself, recall that the benchmark is zero. Relative to this benchmark, governmental impatience adds 0.04 to the autocorrelation when $\beta$ is 90 percent of $1/(1+r)$.

Finally, figure 6 plots the correlation of net exports $(F - C - I)$ with output. At the first best, capital is constant and consumption perfectly insured. This implies a positive correlation between net exports and output, a standard result of insurance in an open economy. How-
ever, as we increase impatience, the correlation of net exports with output turns negative. This highlights how government impatience both limits risk sharing and induces fluctuations in investment.

IV. Balanced Budget Rules

The key source of volatility in the model is the interaction of debt and investment. High levels of debt displace capital due to the government’s inability to commit. This force, combined with the cyclicity of debt, induces persistence and investment volatility in an economy subject only to \( iid \) shocks. A necessary condition for volatility in the ergodic distribution is that the government is impatient relative to the market interest rate; otherwise, the economy will build up enough foreign assets to achieve the first best. If governmental impatience simply reflects private agents’ preferences, the volatile allocation is optimal and imposing an additional constraint cannot improve welfare. On the other hand, if governmental impatience reflect political economy frictions rather than the true preferences of private agents, access to debt induces excess volatility, and might be welfare reducing.

Under the assumption that private agents discount at the international risk free rate, the first best allocation features constant consumption and constant investment. We ask whether a balanced budget rule can deliver this first best allocation despite the presence of an impatient government. To answer this question, we first prove a version of the folk theorem, which states in this instance that if the government is “patient enough”, the first best can be sustained. We then use our numerical model to quantitatively characterize the range of discount factors for which the theorem is relevant. We leave aside the question of why private agents can force a government to follow a balanced budget, but not force it to implement the full commitment allocation directly. One can envision environments in which the transparency of a balanced budget rule make it feasible, while the complexities of a full state-contingent rule may not be feasible. However, fully modeling such an environment is a more ambitious undertaking. In this note, we restrict ourselves to the positive and normative implications of a balanced budget rule and leave aside the issue of implementation.

Under a balanced budget rule, the government’s external debt is constant, which we take to be zero. There is therefore no state variable that links periods given our \( iid \) shock process. We can represent the government’s welfare, \( V \), in recursive form as

\[
V = \max_{k,c,z} \mathbb{E}[u(c(z)) + \beta V],
\]

subject to

\[
\mathbb{E}[F(k, z) - c(z)] - (r + \delta)k = 0,
\]

and

\[
u(c(z)) + \beta V \geq U(F(k, z)) + \beta \text{aut}, \forall z \in Z.
\]

The solution will feature a constant, maybe distorted, investment. Therefore, under the balanced budget assumption, either the first best is attainable immediately or it is never sustainable. Whether the first best is sustainable depends on the government’s discount factor. In this environment, the folk theorem states that with a patient enough government, the full commitment solution is sustainable:

**Proposition 1:** There exists a \( \beta^* \in (0, 1) \) such that for all \( \beta \geq \beta^* \) the full commitment solution is sustainable, and it is not sustainable for \( \beta \in [0, \beta^*) \). In particular, if \( \beta \geq \beta^* \), then
restricting the government to a balanced budget achieves the first best level of capital, $k^*$, and constant consumption.

The proof of this proposition is in the appendix. While the proposition establishes the existence of $\beta^*$, it provides no guidance about how patient the government must be. We shed light on this question using the same numerical model studied in the previous section. Specifically, we vary $\beta$ and solve for the difference between the value function of the government at the first best allocation and the payoff for deviation. We plot this difference for the high $z$ shock in Figure 7, where the horizontal axis is $\beta(1 + r)$ as before. For high $\beta$’s, this difference is positive, implying that the first best is sustainable.\footnote{Note that if the incentive constraint is slack for the high shock, it must be slack also for the low shock.} The value of $\beta^*$ relative to $1 + r$ can be found at the point where the difference crosses the horizontal axis. For this particular parametrization, $\beta^*$ is roughly $0.18/(1 + r)$.

That is, the government needs a discount factor more than 80 percent smaller than the market rate before a balanced budget rule fails to deliver the full commitment allocation. While admittedly stylized, these numbers indicate that balanced budget rules can realistically help stabilize investment without significantly distorting investment.

Appendix

Proof of Proposition 1:

Note that the full commitment and the deviation allocations are independent of the value of $\beta$. Let $c^*$ denote consumption under commitment: $c^* = \mathbb{E}[F(z, k^*) - (r + \delta)k^*]$. Define the difference in the present discounted value of utility under the commitment allocation and autarky as $\Delta(\beta)$:

$$\Delta(\beta) \equiv \mathbb{E} \sum_{s=0}^{\infty} \beta^{s+1} [u(c^*) - u(F(z,0))] = \frac{\beta(u(c^*) - \mathbb{E}u(F(z,0)))}{1 - \beta}.$$  

Note that $u(c^*) > \mathbb{E}u(F(z,0))$, as $k^* > 0$ and $c^*$ is the optimal plan. Therefore, the value in the numerator is strictly positive. This implies that $\Delta(\beta)$ is strictly increasing in $\beta$, is equal to zero when $\beta = 0$, and approaches infinity as $\beta$ approaches one. We can write the participation constraints at the commitment allocation as

$$u(c^*) - u(F(z,k^*)) \geq -\Delta(\beta).$$

As the right-hand side of (11) is strictly decreasing in $\beta$, and the left-hand side does not vary with $\beta$, if this constraint is satisfied at $\beta$, then it is satisfied at any $\beta' > \beta$. When $\beta = 0$, the right-hand side of (11) is zero and the constraint will not hold for some $z$. When $\beta \to 1$, the right-hand side of (11) approaches minus infinity, implying there is a $\beta^* < 1$ for which all the participation constraints are satisfied at the full commitment allocation for $\beta \geq \beta^*$, and at least one constraint is violated at the full commitment allocation for $\beta < \beta^*$.

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