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Low Porosity Metallic Periodic Structures with Negative Poisson’s Ratio

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The Poisson’s ratio, ν, defines the ratio between the transverse and axial strain in a loaded material [1]. For isotropic, linear elastic materials, ν cannot be less than 0.5 nor greater than 0.5 due to empirical, work-energy, and stability considerations leading to the conditions that the shear modulus and bulk modulus have positive values [2]. Although materials with negative Poisson’s ratio can exist in principle, most materials are characterized by ν > 0 and contract in the directions orthogonal to the applied load when they are uniaxially stretched. The discovery of materials with negative Poisson’s ratio (auxetic materials, that counter intuitively expand in the transverse direction under tensile axial load) is relatively recent [3, 4]. Auxetic response has been demonstrated in a number of natural systems, including metals with a cubic lattice [5], zeolites [6], natural layered ceramics [7] and ferroelectric polycrystalline ceramics [8]. Furthermore, following the pioneering work of Lakes [9], several periodic 2-D geometries and structural mechanisms to achieve a negative Poisson’s ratio have been demonstrated. In all of these cases, careful design of the microstructure has lead to effective Poisson’s ratio (auxetic materials, that counter intuitively expand in the transverse direction under tensile axial load) is relatively recent [3, 4]. Auxetic response has been demonstrated in a number of natural systems, including metals with a cubic lattice [5], zeolites [6], natural layered ceramics [7] and ferroelectric polycrystalline ceramics [8]. Furthermore, following the pioneering work of Lakes [9], several periodic 2-D geometries and structural mechanisms to achieve a negative Poisson’s ratio have been demonstrated. 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where $L_0$ denotes the size of the RVE (see Fig. 1-a). In fact, when $\nu$ is plotted as a function of $L_{\text{min}}/L_0$ as shown in Fig. 1-b all data remarkably collapse on a single curve, which can be used to effectively design structure with the desired values of Poisson’s ratio and porosity. Thus, the ligament length appears to be the essential parameter controlling the auxetic response of these structures.

Next, we proceed by attempting to reproduce this auxetic behavior experimentally. In particular, we focus on two extreme cases and investigate the response of structures with porosity $\psi = 5\%$ and aspect ratios of $a/b = 1$ and $a/b = 30$. The experiments were performed on a $300 \text{ mm}$ by $50 \text{ mm}$ by $0.4046 \text{ mm}$ aluminum (6061 alloy) cellular plate (see Fig.2), which were manufactured using the CNC milling process described in Materials and Methods. The gage section of the samples were patterned with circular holes with radius of $a = b = 3.154 \text{ mm}$ (see Fig.2-top) and elliptical holes with major and minor axis $a = 33.27 \text{ mm}$ and $b = 1.16 \text{ mm}$ (see Fig.2-bottom), respectively. Note that due to the size of the end mill (with a diameter of $0.397 \text{ mm}$) required to manufacture the holes, the tips of exact ellipsoidal shapes were not produced, resulting in slightly lower aspect ratios, $a/b = 28.7$.

The samples are tested under uniaxial tension in an Instron biaxial testing machine equipped with a $10 \text{ kN}$ load cell (pictures of the experimental set up are shown in the Supporting Information). Similarly to previous studies where optical methods have been used to characterize the deformation in auxetic foams [34–37], here the displacements within the samples are captured in detail using Digital Image Correlation (DIC). DIC is a technique by which displacements can be measured by correlating (via software) the pixels in several digital images taken at different applied loads [38]. In order for multiple frames to be correlated, every single part of the image must be uniquely detectable. This requires the surface of the samples to be covered in a non-repetitive, isotropic, and high-contrast pattern. In addition, the pattern must be fine enough to capture the desired displacement details consistent with the cameras and lenses used to capture the images. In our experiments, the surfaces of the samples are painted white with a fine distribution of black speckles using a Badger 150 airbrush and water-based paint leading to a density of approximately 3-6 pixels per speckle. The deformation of the sample is monitored using a high-resolution digital camera (1.3 MPixel Retiga 1300i with a Nikon optical lens system) and given our experimental setup the displacement accuracy is estimated to be approximately $0.05 \text{ mm/s}$ [39]. The samples are loaded via displacement control at a rate of $0.05 \text{ mm/s}$ with the camera synchronized using the software Vic-Snap (Correlated Solutions) to capture images at a rate of 1 frame per second. Quantitative estimates of the deformation of the gage section of the sample are made using the image correlation software, Vic-2D (Correlated Solutions).

In addition to material testing, numerical investigations were performed on the experimental sample geometries using the nonlinear finite element code ABAQUS. Each mesh was constructed using ten-node, quadratic tetrahedral elements (ABAQUS element type C3D10) applied to the CAD model used to fabricate the samples. In order to accurately predict the material response near the voids, automatic adaptive mesh refinement is used, resulting in approximately $162,000$ elements for the sample with circular voids and $119,000$ elements for the sample with elliptical voids. The material is modeled as linearly elastic and perfectly plastic with a Young’s modulus of $70 \text{ GPa}$ and a Poisson’s ratio of $0.35$ [33]. The yield stress is taken to be $275 \text{ MPa}$ based on the experiments and is in agreement with available material data [39]. The applied experimental loading is approximated by fixing the translation at one edge and specifying a static displacement at the opposite edge. The remaining boundaries are traction free.

In Fig. 3 we present both experimental (left) and numerical (right) results for the case of circular (top) and elliptical (bottom) pores. The specimen with circular pores is shown at an applied strain of $0.34\%$, while the specimen with elliptical holes is shown at an applied strain of $0.07\%$. Note that the applied strain is chosen to ensure the horizontal displacements are large enough to be accurately detected by DIC. As described later, the different deformation mechanisms taking place in the two structures considered in this study result in two different values of applied strain (additional results highlighting the effect of the applied strain on $\nu$ are shown in the Supporting Information). To minimize the effect of boundaries, we focus on the central portion of the specimen ($50 \text{ mm} \times 50 \text{ mm}$, see dashed red box in Fig. 2) and report boundary maps for the horizontal ($u_x$) and vertical ($u_y$) component of the displacement field. First, an excellent agreement is observed between simulation and experimental results. Moreover, the displacement maps reported in Fig. 3 clearly show that the hole aspect ratio $a/b$ strongly affect the mechanism by which the structure deforms. For the case of circular holes, the pores are found only to locally perturb the displacement field, so that the displacement field typical of the bulk material can be easily recognized (i.e. linear distribution of $u_x$ and $u_y$ in horizontal and vertical direction, respectively). In contrast, the array of elliptical holes is found to significantly affect the displacement field, completely distorting the linear distribution of $u_x$ and $u_y$ typical of the bulk material. Finally, it is worth noticing that the nature of the displacement contours is not affected by the level of applied strain, as shown in the Supporting Information.

Focusing on $u_x$ we can clearly see that the material is contracting laterally for the case of circular pores. In contrast, significant lateral expansion is observed for the samples with elliptical holes demonstrating auxetic behavior. To quantify the lateral deformation we compute the effective Poisson’s ratio for these structures. In both the numerical and experimental results, we sample the displacement at 8 points along each of the four boundaries of the central regions shown as a dashed box in Fig. 2. Each set of 8 points are averaged (arithmetic mean) to compute the average displacements at the boundaries: $(u_x)_L$, $(u_x)_T$, $(u_y)_R$, $(u_y)_B$, where superscript $L$, $R$, $T$, and $B$ denote the left, right, top, and bottom boundaries, respectively. These average displacements are used to compute local strain averages

$$\langle \epsilon_{xx} \rangle = \frac{(u_x)_T - (u_x)_L}{L_0}, \langle \epsilon_{yy} \rangle = \frac{(u_y)_R - (u_y)_B}{L_0},$$

$$L_0 = 50 \text{ mm},$$

where $\langle \epsilon_{xx} \rangle$ and $\langle \epsilon_{yy} \rangle$ are the average strain between the top/bottom and left/right boundaries in the undeformed configuration. The local strain averages are then used to calculate an effective Poisson’s ratio $\nu$ as

$$\nu = -\frac{\langle \epsilon_{xx} \rangle}{\langle \epsilon_{yy} \rangle}.$$
In Fig. 4 we report numerical results to further highlight the effect of the pore aspect ratio $a/b$ on the deformation of the material. In Fig. 4-a we show the deformed configuration of central region of the samples superimposed over the unloaded configuration, with the displacement field in the deformed image scaled by a factor of 100. For the circular void sample, the deformation mechanism is stretching much the same as a similar void-less structure would be. By contrast, as perviously shown in the analytical study by Grima and Gatt [27], the deformation mechanism in the elliptical void sample is mostly due to rotation, leading to a negative Poisson’s ratio. Finally, the completely different mechanism by which the two structures carry the load results in very different stress distributions within the material, as shown in Fig. 4-b, where we report the contour map for the von Mises stress. In the structures with circular pores there are crosses that are highly stretched. These regions are yielded and highlighted by the grey color in the contour map. In the case of elliptical holes, most of the structure experiences low values of stress and the deformation is found to induce rotation of the domain between holes. Stress is concentrated around the tips of the ellipses, but these can be easily reduced by carefully designing the tips to minimize the curvature.

In summary, our findings demonstrate a fundamentally new way of generating low porosity 2-D materials with negative Poisson’s ratio. We show that the effective Poisson’s ratio can be effectively tuned by adjusting the aspect ratio of an alternating pattern of elliptical voids. We have used numerical modeling to gain insight into the design of these structures as well as the underlying rotational mechanism causing the auxetic behavior at high void aspect ratios. These models have been verified using material testing with DIC, which has conclusively shown auxetic behavior in thin aluminum plates. We note that the structures investigated here exhibit two-dimensional cubic symmetry, meaning that the effective response is anisotropic.

Further investigation of material symmetry is outside the scope of the current work, it is an important design issue. Thus, this work provides not only a guide for the simple design of auxetic materials but serves as a basis for future investigations into such areas as material symmetry and void shape optimization to minimize stress concentrations while maintaining a desired Poisson’s ratio.

### Materials and Methods

#### Materials

Test samples were cut from 6061 Aluminum alloy 0.404mm thick plates. The bulk material has a Young’s modulus of 70GPa, Poisson’s ratio of 0.35, and yield stress of 275M Pa. In the numerical simulations its behavior is modeled as linearly elastic and perfectly plastic.

#### Fabrication

Test samples were fabricated using a Haas OM-2A CNC Machine together with a 2-flute ultra-duty coated (TiCo) carbide end mill (diameter = 0.396mm, $R=0.5$). Samples were fed into the mill at a speed of 2 inches per minute with a 0.003 inch depth of cut. Samples were designed in Solidworks CAD software (Dassault Systems) and imported to the CNC machine via SolidCAM.

#### Testing

Samples were tested at the MIT Impact and Crashworthiness Lab. Prior to testing, samples were coated in white water-based paint using a Badger150 airbrush. Contrast was added by spraying a fine pattern of black water-based paint to the recorded surface using the same airbrush. For the tests, samples were loaded into an Instron bi-axial testing machine equipped with a 10 kN load cell using 5cm wedge grips. The grips were secured using a steel block with aluminum spacers screwed together at a fixed torque resulting in constant contact pressure. Samples were illuminated uniformly in situ by means of tripod mounted diffused isotropic lighting. A Retiga 1300i camera (Nikon optical lens system) was mounted to a tripod and focused on a 80x100 mm rectangle in the central gage section of the samples providing digital imagery through the tests. The original image was then cropped to the 50x50 mm rectangle in central dimensions used in this study. The camera has a 1.3 MPixel resolution resulting in a pixel size of 6.75 μm by 6.75 μm. Samples were tested in tension at a rate of 0.05 mm/s with cameras triggered externally (via Vic-Snap) to capture 1 frame per second in synchronization with the applied Instron loading.

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Fig. 1: (a): Results of the numerical investigation on the effect of the hole aspect ratio $a/b$ for an infinite periodic square array in an elastic matrix. Four different values of porosity are considered. The RVE considered in the analysis is shown as an inset. (b): All data collapse on a single curve when $\bar{\nu}$ is plotted as a function of $L_{\text{min}}/L_0$.

Fig. 2: Samples comprising of a square array of (top) circular and (bottom) elliptical (with $a/b \approx 30$) holes in the undeformed configuration. The dashed rectangle represents region over which we perform the ensemble averaging. (Scale bar: 25 mm)
Fig. 3: Contour maps for the horizontal ($u_x$) and vertical ($u_y$) component of the displacement field. Numerical (left) and experimental (right) results are quantitatively compared, showing excellent agreement. In (a) and (b), the applied strain is 0.34%. In (c) and (d), the applied strain is 0.07%. Note that gray areas on experimental results show regions where DIC data could not be obtained.
Fig. 4: Effect of the pore aspect ratio $a/b$ on the deformation of the structure. (a) Deformed configuration superimposed over the unloaded configuration, with the displacement field in the deformed image scaled by a factor of 100. (b) Von Mises stress distribution with plastified areas colored gray. The applied strain is 0.34% and 0.07% for the structure with circular and elliptical holes, respectively.