Learning from Comparison in Algebra

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Alex and Morgan were asked to solve $45y + 90 = 60y$

**Alex’s “combine like terms” way**

1. I first combined like terms on the left side of the equation.
2. Then I subtracted both sides by $60y$.
3. Then I divided both sides by $75$ to get the answer.

**Morgan’s “combine like terms” way**

1. First I subtracted $45y$ on either side; $60y - 45y$ is $15y$.
2. Then I divided both sides by $15$ to get the answer.

**Take-away page**

Like terms contain the same variable or group of variables raised to the same power. In order for two or more terms to be “like terms,” their coefficients can be different, but the terms need to have the same variables raised to the same powers. Unlike terms cannot be combined by addition or subtraction.

* How did Alex solve the equation?
* How did Morgan solve the equation?
* Why did Alex combine the terms on the left as a first step?
* Why did Morgan subtract $45y$ as a first step?
* Which way is correct, Alex’s or Morgan’s way? How do you know?
* Can you state a general rule about combining like terms that describes what you have learned from comparing Alex’s and Morgan’s ways of solving this type of problem?
Alex and Morgan were asked to simplify $16^{\frac{3}{4}}$.

**Alex's "rewrite the exponent" way**

- I rewrote the fractional exponent as $3 \times \frac{1}{4}$.
- I expanded the expression in the parentheses.
- I got $4096$. Then I applied the exponent.

This is my answer.

**Morgan's "use perfect squares to rewrite the exponent" way**

- I rewrote the fractional exponent as $\frac{1}{4} \times 3$.
- I simplified the expression in the parentheses. Since $2^4$ is $16$, I know that $16^{\frac{1}{4}}$ is $2$.
- I got $4096$. Then I applied the exponent.

This is my answer.

* How did Alex simplify the expression?
* How did Morgan simplify the expression?
* What are some similarities and differences between Alex's and Morgan's ways?
* Which strategy do you think is more efficient for this problem? Why?
Alex and Morgan were asked to simplify the expression
3x(5x + 2) + 4(5x + 2)

Alex's way

Morgan's way

First I expanded the expression using the distributive property.

3x(5x + 2) + 4(5x + 2)

Then I simplified the expression.

15x² + 6x + 20x + 8

(3x + 4)(5x + 2)

15x² + 26x + 8

First I factored the expression.

15x² + 6x + 20x + 8

Then I expanded the expression.

15x² + 26x + 8

Hey Alex, what did we learn from comparing these two ways?

Like expressions enclosed by grouping symbols, such as parentheses, can be combined as like terms are combined.
How do they differ?

Alex was asked to graph the equation $y = 2x$  
and Morgan was asked to graph the equation $y = -2x$.

**Alex's "graph $y = 2x$" way**

- I rewrote the equation in $y = mx + b$ form.
- I graphed the $y$-intercept, $(0,0)$ and counted up 2, right 1 and down 2, left 1 to plot other points on the line. I connected the points to draw the graph of the line.

**Morgan's "graph $y = -2x$" way**

- I rewrote the equation in $y = mx + b$ form.
- I graphed the $y$-intercept, $(0,0)$ and counted down 2, right 1 and up 2, left 1 to plot other points on the line. I connected the points to draw the graph of the line.

In the slope-intercept form of a line ($y = mx + b$), the coefficient of $x$, which is $m$, indicates the slope.

Changing the sign of $m$ changes the slope, or the steepness, of the line. When a line has a positive slope, its height increases from left to right. When a line has a negative slope, its height decreases from left to right.

* How did Alex graph the line given by his equation? How did Morgan graph the line given by her equation?
* Can you think of another way that Alex and Morgan could have used to find the graphs of their lines?
* What are some similarities and differences between Alex's and Morgan's problems?
* What are some similarities and differences between Alex's and Morgan's graphs?
* How does changing the sign of $m$ affect the graph of a line?