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Essays on the Role of Banks in the Macroeconomy

A dissertation presented
by

Charles-Henri Weymuller

to

The Department of Economics

in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy
in the subject of
Economics

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Essays on the Role of Banks in the Macroeconomy

Abstract

This dissertation consists of four essays that explore the interaction between banks and the supply of safe assets in the macroeconomy.

The first essay, "Banks as Safety Multipliers: A Theory of Safe Assets Creation", argues that the role of banks is to multiply safety. The optimal risk-sharing arrangement between risk-neutral banks and risk-averse investors is implemented with the bank issuing debt securities. To guarantee their safety, banks hold on their balance sheets long-term public debt, whose return is negatively correlated with macroeconomic shocks. I show that lowering the supply of safe public debt induces banks to shrink their balance sheet, which hurts the supply of safe private debt. The decentralized equilibrium is constrained inefficient, as agents do not internalize the benefits of issuing long-term securities. I document that the supply of bank debt is positively correlated with the supply of public debt in Europe, where risk-averse investors heavily rely on banks to create safe assets.

The second essay, "Leverage and Reputational Fragility in Credit Markets", develops a bargaining model of short-term secured debt markets. It characterizes the equilibrium leverage of borrowers differing in the strength of their reputation and their long-term relationships. I show that borrowers with higher reputation and with more stable funding relationships are more levered and more profitable, but also more fragile to aggregate shocks, as reputation endogenously vanishes when net worth gets closer to zero. Reputation acts as a stabilization mechanism in good times but as an amplification in bad times. I find that these predictions are confirmed on the repo market, a predominant source of financing for US banks.

The third essay, "Measuring Liquidity Mismatch in the Banking Sector", shows that banks issue liabilities that are more liquid than their assets. We define and construct a liquidity mismatch index (LMI) for 2870 banks. The aggregate LMI worsens from -$2 trillion in 2004 to -$5 trillion in 2008, before reversing back to -$2 trillion in 2009. In the cross section, we find that banks with more liquidity mismatch (i) experience more negative stock returns during the crisis, but more positive returns in non-crisis periods; (ii) experience more negative stock returns on events corresponding to a liquidity run, and more positive returns on events corresponding to a government liquidity injection.

The fourth essay, "Optimal Eurobond Design", explores joint-liability issuance by a group of countries. We show that issuing jointly decreases welfare because of a non-cooperative game that leads to over-borrowing. Joint liabilities at best implement a transfer across asymmetric countries. However, if safe assets are scarce in the economy, pooling and tranching sovereign debt cater to a safety premium and then lead to a Pareto improvement.

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To my parents and my grandmother,
who educated us with an enduring thirst for knowledge

Introduction

Even if banks have been extensively analyzed over the course of their longstanding history, the question of their usefulness to society has always led to passionate debates. The mystery of their economic role is partly explained by the fact that their activities have been evolving over time, since the 14th century and the birth of modern banking[1]. From a macroeconomic perspective, one could expect that developed financial markets would have taken the lead in channeling funds from borrowers to lenders. Bank would have then been made redundant. This dissertation argues that the role of modern banks is deeply rooted in their supply of safe assets. It explores the preeminent interactions between the business model of banks and the asset supply in the macroeconomy.

Banks are traditionally defined as financial intermediaries that accept deposits and channel these deposits into lending activities. Nevertheless, classical macroeconomics has considered that such financial intermediation was a wash for the purpose of General Equilibrium analysis. Most macroeconomic models take for granted that savings find their way to investment. They push into the background the mechanics enabling to reach this equality. As a result, macroeconomics and finance grew as two distinct fields in the academic literature. On the one hand, macroeconomics focused on the propagation of aggregate shocks through the development of sophisticated Dynamic Stochastic General Equilibrium models. These models traditionally assume a representative household, which implies there is no credit in equilibrium, hence no need for financial intermediation. Only recently, since the 2007-2009 financial crisis that can be characterized as a purely financial shock, had macroeconomic models have started to explicitly take into account frictions on financial markets[2].

---

[1] See Kindleberger (1978) for a history of the role played by banks in financial crises.

On the other hand, the field of finance has been concerned with the microeconomic analysis of financial interactions. It benefited from the development of contract theory for the analysis of agency and financial frictions[3]. This partial equilibrium approach typically focuses on a specific type of friction, i.e. moral hazard, asymmetric information, or limited enforcement. These models often restrict the timeline to three periods, with no ambition to be calibrated. Several microeconomic theories of banking have been developed this way. These models can be allocated in two categories, depending on which side of banks balance sheets drives their behavior:

- "Asset-side" theories concentrate on the interaction between banks as lenders and firms that borrow from these banks. For instance, the comparative advantage of banks in Diamond (1984) comes from their ability to monitor borrowers. These theories explain what assets banks hold, but are mostly silent about the capital structure of their liability side.
- "Liability-side" theories emphasize the specific structure of banks liabilities[4]. Gorton and Pennacchi (1990) argues that banks provide the economy with "information-insensitive" securities, i.e. claims that are not sensitive to new information about the cash flows collateralizing the claim. In contrast, these theories do not develop the asset side of banks.

This dissertation develops the idea that the special role of banks in the economy arises from their ability to simultaneously decide the composition of the asset side and of the liability side of their balance sheet. In the following essays, both sides are endogenous: the ability of banks to increase their leverage directly influences the extent to which they lend to the economy. In turn, the profitability of their investments in assets feeds back on the level of leverage banks choose in equilibrium.

---

[2] Macroeconomic models concerned with amplification mechanisms arising from financial frictions include Bernanke et al. (1999), Curdia and Woodford (2009), He and Krishnamurthy (2012a) and Brunnermeier and Sannikov (2010).

[3] Seminal references include Townsend (1979), Aghion and Bolton (1992), Shleifer and Vishny (1997). See Tirole (2006) for a comprehensive theory of corporate finance based on agency frictions

[4] Bank runs models following Diamond and Dybvig (1983) belong to this category. In particular, Calomiris and Kahn (1991) and Diamond and Rajan (2001b) argues that the threat of a bank run acts as a disciplining device on banks. However banks in Diamond and Dybvig (1983) cannot hold marketable securities, a the mere presence of such market unravels the optimality of the deposit contract (Jacklin (1987)).

Banks make these joint decisions about assets and liabilities in a General Equilibrium environment. As the recent financial crisis has vividly reminded, the variation of bank leverage has a direct impact on macroeconomic variables. The empirical analysis by Reinhart and Rogoff (2008) and Jorda et al. (2013) show that the correlation between bank leverage and economic growth hold in long-time series. Bank leverage often is a predictor of financial crisis. However the interaction goes in the two ways: the macroeconomic environment influences bank leverage decisions.

This dissertation develops a credit cycle theory based on the scarcity of asset supply in the macroeconomy. Banks mitigate the savings glut of anxious money[5] by supplying riskless claims to the economy. Geanakoplos (2009) develops the view that optimist agents borrow from pessimist agents to invests in the risky assets, and choose optimally their level of leverage. This approach leads to a leverage cycle theory. The main departure from this reference consists in introducing a scarcity of safe and liquid assets. The following essays develop three distinct - but not mutually exclusive - views about how banks mitigate the shortage of safe assets. They have in common that what makes banks special comes from a fundamental interaction between assets and liabilities. Banks create value for the economy by the way they get financed, on their liability side. It can be by issuing safer securities (Chapter 1), it can be by pledging intangible collateral (Chapter 2), or it can be by issuing more liquid securities (Chapter 3).

In Chapter 1, **Banks as Safety Multipliers: A Theory of Safe Assets Creation**, the only defining feature of banks consists in their lower risk-aversion compared to other agents. This essay is motivated by the observation that banks hold significant volumes of seemingly safe and well-scrutinized securities, such as government bonds. Theories of banking revolving around information asymmetries between banks and final investors[6] are not suited to explain such types of holdings. In contrast, the essay argues that the purpose of such assets holdings by banks is to enhance their leverage ability. Banks bundle public bonds with risky assets to enable themselves to issue safer debt securities. In a General Equilibrium environment with endogenous collateral constraints and multiple assets, risk-neutral banks issue debt securities to cater to the safety demand from risk-averse investors. This essay shows that defaultable debt is the optimal risk-sharing agreement between the banks and the final investors. To enhance their ability to create private safe assets (bank debt), banks decide to hold on their balance sheets public safe assets (government debt), whose returns are negatively correlated with macroeconomic shocks. These holdings of public debt give rise to a safety multiplier. Indeed, when heterogeneity in risk aversion between banks and investors is large enough, lowering the exogenous supply of public debt induces banks to shrink their balance sheet, which in turn hurts the supply of private debt. According to this approach, the economic role of banks is to multiply safety.

---

[5] The glut of savings have been extensively commented, see Bernanke (2008). This dissertation insists on the risk-aversion of the investors behind the savings glut.

The positive theory of the business model of banks calls for a normative exploration. Are banks doing too much or too little of private debt issuance? To investigate this question, Chapter 1 makes endogenous the negative correlation of public debt with macroeconomic shocks. In the corresponding dynamic model, both public and private agents are able to choose the maturity of debt securities they issue. In this environment, the expectation of a flight-to-safety tomorrow transforms long-term securities into hedging instruments today. The decentralized equilibrium is constrained inefficient because of an issuance externality. Private agents do not internalize the benefits of the negative correlation of their own long-term liabilities with macroeconomic shocks. As a result, the private economy lacks long-term securities. Public debt is non-neutral, and issuing long-term debt leads to a

---

[6] See for instance Gorton and Pennacchi (1990), DeMarzo and Duffie (1999) and Holmstrom and Ordonez (2013).

Pareto improvement. Nevertheless the optimal level of public securities is finite, given that their hedging properties deteriorate as their supply increases.

Chapter 1 interprets the ongoing European debt crisis as a shortage of public safe assets. In the time series, the correlation between the supply of public debt and the supply of private debt is positive in Europe, contrary to the behavior of these time series in the United States. This difference can be rationalized by the assumption that European final investors are constrained by a more limited participation in risky technologies. As private equity markets are less developed in Europe than in the United States, the self-diversification of risk-investors is more restricted in Europe. As a result, these investors rely more on public debt to find safe vehicles of savings. This amplifies the safety multiplier mechanism in the case of Europe. Besides, as the essay shows, European banks increased their holdings of safe public debt during the crisis, in order to guarantee private debt. The model has asset-pricing implications. Bank leverage can be predicted from the spread between public debt yield and private debt yield. Thus this spread can be used as a macro-prudential indicator.

The open economy version of the environment illustrates how the damages of sovereign risk are exacerbated by the safety multiplier mechanism. Indeed, the risk of default of the sovereign authority deteriorates the safety properties of public debt. Given that the latter is used as a key input in the process of private safety creation, aggregate private leverage sharply decreases in the presence of sovereign risk. At the same time, domestic banks become the natural holders of domestic public debt. Therefore the essay provides a theory establishing an interconnection between sovereign risk and bank risk, but as an endogenous response, and not a cause, to a public debt crisis[7].

Chapter 2, **Leverage and Reputational Fragility in Credit Markets**), develops a complementary theory of banking in the macroeconomy. In this essay, the comparative advantage of banks relies in their ability to use the reputation of their franchise to obtain better refinancing terms. In Chapter 1, banks borrow through unsecured debt contracts, in chapter 2 their borrowing uses secured debt, which is collateralized by an asset the lender can seize if the borrower defaults. Chapter 1 describes commercial banks, whereas Chapter 2 is more suited to describe the behavior of financial intermediaries composing the "shadow banking system".

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[7] The European debt crisis that started in 2010 has led policymakers to point the vicious circle between sovereign risk and bank risk as the main culprit of the difficulties of the Eurozone. It is conventionally assumed that this vicious circle is implied by implicit guarantees provided by sovereign authorities to their bank. This conviction has supported the policy impetus towards the creation of a Banking union, featuring a common supervision and a single resolution mechanism for all European banks.

Specifically, the essay analyzes the equilibrium on secured debt markets where borrowers differ in the strength of their reputation and of their financing relationships. It develops a bargaining model to characterize the leverage chosen by borrows in equilibrium. In this model, debt capacity depends not only on the nature of collateral but also on the reputation of the borrower. Agents with higher reputation secure both more and cheaper credit, i.e. lower haircuts and lower rates. As a result, haircuts and rates are positively correlated in the cross-section of borrowers, a prediction that cannot be delivered in a model where the heterogeneity among borrowers is solely about collateral[8]. The dynamic model endogenizes reputation by identifying it to the continuation value of the borrower in a repeated game of borrower-lender relationships on the market for secured debt. Due to the endogeneity or reputation, leverage is procyclical in this environment: leverage is high and stable when agents are sufficiently capitalized. Nevertheless, as the net worth of a given borrower gets depleted, reputation vanishes in a non-linear way, and leverage becomes restricted and more volatile. Reputation acts as a stabilization mechanism in good times but as an amplification mechanism in bad times. In this dynamic environment, long-term contracts stabilize leverage by avoiding margin calls (i.e. increases in haircuts) in the states in which the borrower has low net worth. Reputation also provides an economic rationale for long intermediation chains, as observed for instance through the rehypothecation of collateral. A long intermediation chain enables to monetize all the franchises of the successive financial intermediaries.

The empirical analysis on a dataset of repo transactions from money market funds confirms the predictions of the model. The repo market has been the main source of financing for US banks. Its market freeze it experienced has been pointed as one of the main transmission channel of the credit crunch during the 2007-2009 financial crisis. In this dataset, financial intermediaries with high reputation indeed write repo contracts with lower haircuts and lower rates. Furthermore, haircuts are more sensitive to borrower's reputation when borrower's net worth is low. Finally, long-term relationships stabilize funding especially in times of liquidity stress. It provides evidence that reputation and bilateral relationships matter even in secured funding markets.

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[8] For instance, borrowers can differ in their beliefs about the uncertain realization of collateral.

Chapter 3, **Measuring Liquidity Mismatch in the Banking Sector** (joint with Jennie Bai and Arvind Krishnamurthy), emphasizes the role of banks as liquidity providers. It argues that banks also create value by running a liquidity mismatch between their illiquid assets and their liquid liabilities. This chapter proposes a theoretical framework to analyze how banks choose the amount of liquidity they provide to the market. When a bank runs a liquidity mismatch between its assets and its liabilities, it earns a liquidity premium, as it is able to refinance cheaply. However, increasing the liquidity mismatch also increases the cost incurred by the bank if a liquidity stress event occurs. This cost function, which summarizes the liquidity need, is assumed to have a recursive structure. The cost function summarizes the liquidity need. By solving for the bank's liquidity choice and for the equilibrium value of this cost function, the essay proposes an analytical expression to the "Liquidity Mismatch Index (LMI)" envisioned by Brunnermeier, Gorton and Krishnamurthy (2011). The LMI measures the mismatch between the market liquidity of assets and the funding liquidity of liabilities. Chapter 3 then constructs this LMI for 2870 bank holding companies during 2002 – 2013, and investigates its time-series and cross-sectional patterns. The aggregate LMI worsens from around -\$2 trillion in 2004 to -\$5 trillion in 2008, before reversing back to -\$2 trillion in 2009. In the cross section, we find that banks with more liquidity mismatch (i) experience more negative stock returns during the crisis, but more positive returns in non-crisis periods; (ii) experience more negative stock returns on events corresponding to a liquidity run, and more positive returns on events corresponding to government liquidity injection. The LMI proves to be a useful a macro-prudential indicator, as its sensitivity to aggregate liquidity shocks can be measured, and can be used to run liquidity stress tests.

Finally, Chapter 4, **Optimal Eurobond Design** (joint with Eduardo Davila), investigates if sovereign authorities can supply safe assets to the economy by applying the same diversification mechanism as banks. It explores the optimal design of a joint-liability arrangement across a group of countries. One of the policy proposals recently discussed to alleviate the ongoing European sovereign debt crisis is the issuance of "Eurobonds". Such joint-liability scheme consists of bundling together a predetermined share of individual sovereign bonds issued by several countries. Chapter 4 shows that higher levels of pooling leads to higher borrowing by the risky countries and lower borrowing by the safe countries. Thus it actually reduces the supply of safe assets to the economy. Indeed with Eurobonds, issuance of sovereign debt becomes a non-cooperative Cournot game between the countries participating in such joint-liability scheme. A negative free-riding externality induces risky countries to over-borrow. Nevertheless, if there is an exogenous demand for safe assets in the economy and if the joint liability can be tranched[9], then this joint-liability scheme can be welfare improving. Chapter 4 explores the conditions to achieve a Pareto improvement with this instrument. The optimal pooling and tranching thresholds are derived from the trade-off the social planner faced between the free-riding externality (cost of the joint-liability scheme) and the safety premium earned on the safe part of the Eurobond (benefit of the joint-liability scheme).

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[9] Pooling and tranching sovereign debt is the essence of the European Safe Bonds ("ESBies") policy proposal by Brunnermeier and al. (2011).

Chapter 1

Banks as Safety Multipliers: A Theory of Safe Assets Creation[1]

1.1 Introduction

Why do banks hold so much public debt that yields so little? Banks hold 15% of their assets in safe securities on their balance sheet (Figure 1.1). This paper is motivated by banks' holdings of large quantities of mundane securities, even at an accounting loss.

Traditional theories of banking usually revolve around the idea that banks are able to mitigate some type of agency frictions. In the Gorton and Pennacchi (1990) tradition, there is asymmetric information about the quality of the projects undertaken by entrepreneurs, and banks exhibit a comparative advantage in screening and monitoring loans to these projects. For instance, Holmstrom and Ordonez (2013) argues that banks business is all about being able to 'keep secret' about these loans, i.e. issue information-insensitive securities against these loans. It is hard to apply this line of argument to universal banks, which hold substantial amounts of highly liquid, marketed and researched securities such as government bonds. For these holdings, banks do not have an intrinsic comparative informational advantage on the market.

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[1] First draft: September 2013. I thank Emmanuel Farhi, Andrei Shleifer, Philippe Aghion and Jeremy Stein for invaluable guidance. For helpful comments and insightful discussions, I thank Eduardo Davila, Oliver Hart, John Campbell, Adi Sunderam, Adrien Auclert, Erik Loualiche, Luis Viceira, Greg Mankiw, Robert Barro, Gita Gopinath, Alberto Alesina, Pol Antras, Vania Stavrakeva, Vivien Lévy-Garboua and especially Alp Simsek, as well as seminar participants at Harvard, HBS, MIT Sloan, NYU Stern and the Federal Reserve Bank of New York. JEL Classification: G12, G23, G32, D8, L14. Keywords: financial intermediation, risk sharing, safe assets, public debt, sovereign risk.

In this paper I develop a view of banks as insurers against aggregate shocks. Banks are in the business of producing safe liabilities, and they use public debt as an *input* to their safety production function. Public debt has two desirable features: a safe terminal payoff, and an interim value that is negatively correlated with aggregate shocks. The output of banking arises on their liability side, whereas their asset side is merely the juxtaposition of inputs that maximize the safety output. This interaction between public debt and private debt, the former being an input to the production of the latter, has crucial positive and normative implications related to the macroeconomic shortage of safe assets.

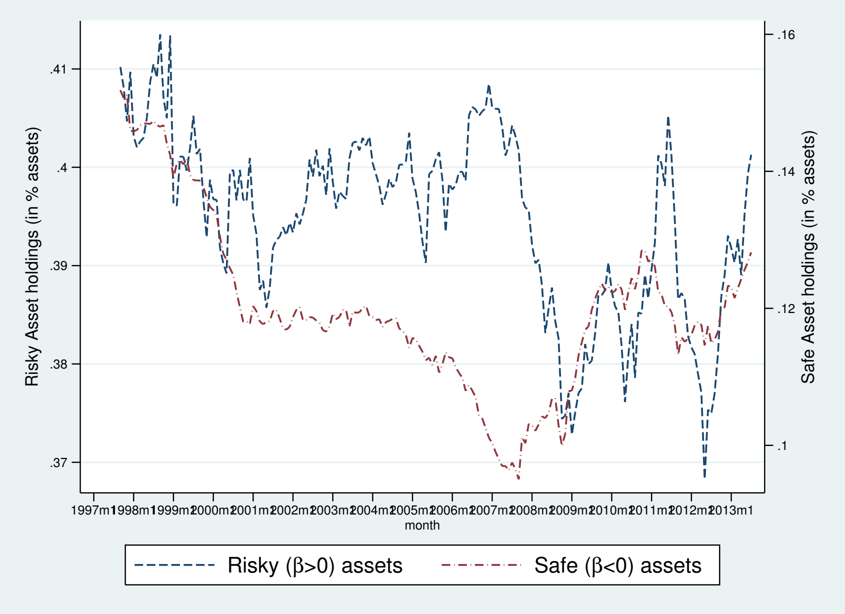

Figure 1.1: *First stylized fact: European banks' portfolio composition, holdings divided according to their β.*

The model of endogenous leverage laid down in this paper aims at capturing three novel stylized facts. The first one concerns banks' balance sheets. Figure 1.1 splits the aggregate balance sheet of the Eurozone financial sector in two categories of assets: loans and fixed assets that are mainly positive beta with the stock market, and securities and holdings such as gold that are mainly negative beta with the stock market ('safe assets'). The figure illustrates that negative beta holdings by European banks are substantial, but also that these holdings did increase with the ongoing Eurozone crisis. This is puzzling, as one could think that in stress times, safe assets ownership gets more concentrated in risk-averse hands, i.e. moves away from banks' balance sheets to household portfolios. Figure 1.2 computes the beta of German and Italian 10 year government bonds with the European stock market index DJ EUROSTOXX 50. As both countries belong to the same monetary union, it controls for expectations about the monetary policy stance, i.e. the Neo-Keynesian channel emphasized in Campbell et al. (2013b). And still, the two assets exhibit radically different behavior during the Eurozone crisis: German public debt exhibits now an even more negative beta, whereas Italian public debt has turned sharply positive. I interpret this as Italian public debt losing its safe asset status, in line with my definition of safe assets.[2]

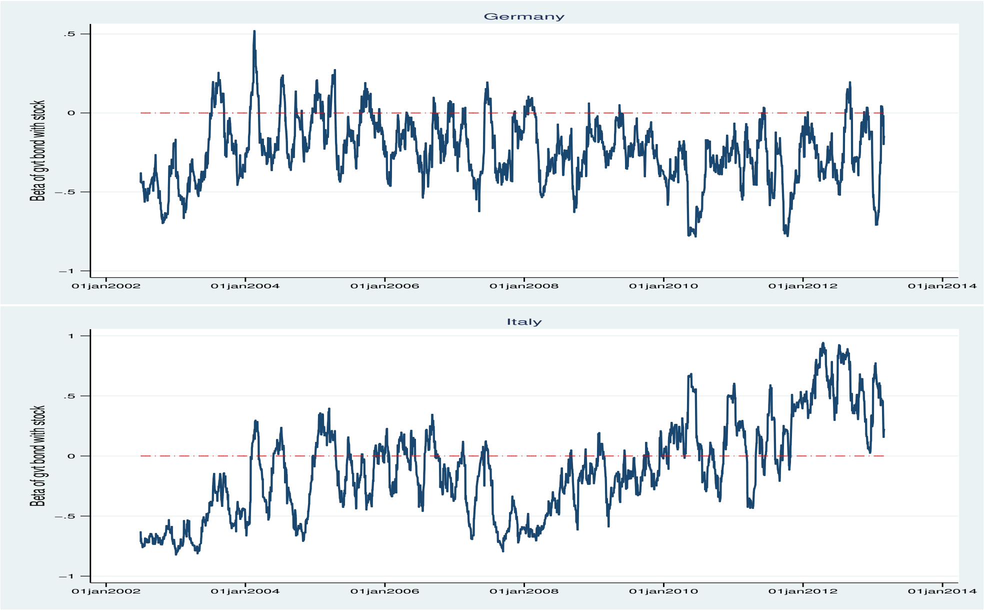

Figure 1.2: *What is a safe asset? Daily betas of government bonds with DJ EUROSTOXX 50, Germany (top) and Italy (bottom).*

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[2] Section 4 of this paper microfounds the beta of public debt in an open economy model, and rationalizes these patterns along with the redomestication of public debt currently at play in Europe.

The second stylized fact is concerned with the pricing of safe assets. I take the view in this paper that no asset is entirely riskless, but that there is an inelastic supply of quasi-safe assets. Critically I draw the distinction between *public* safety (public debt) and *private* safety (private bank debt). This leads me to consider two distinct safe prices and safety premia: one on public debt and one on bank debt. Figure 1.3 computes the spread between these two yields. This figure illustrates that this spread progressively decreased during the run up to the crisis, even turning negative, before sharply bouncing back in the midst of the crisis. This spread is exactly the carry trade banks are doing when they hold public debt and finance these holdings by bank debt. A micro investigation of the same carry trade is shown by Figure 1.20 in the Appendix. In this figure, I additionally subtract from the carry trade the operating expenses of running the bank pro rata the safe asset holdings. This figure shows that the median European bank loses almost ten million euros on its safe asset holdings.

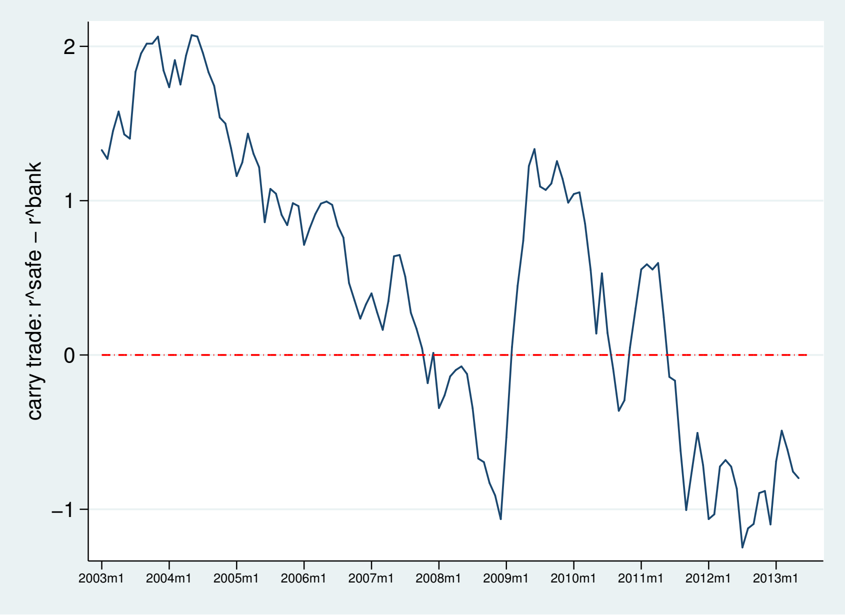

Figure 1.3: *Second stylized fact: negative carry trade by banks on safe public debt:* $SMI = r^{safe} - r^{bank}$.

The third and last stylized fact the model captures is a macroeconomic one. I compute two aggregates measures: one is the stock of safe Eurozone public debt, the other is the stock of Eurozone bank short-term debt. Figure 1.4 shows these two time-series, scaled by GDP. This figure eyeballs a positive comovement of private debt with public debt in Europe. This stands in contrast with the result obtained by Krishnamurthy and Vissing-Jorgensen (2013a) for the United States, in which they show there is a negative comovement between these two aggregates. This stylized fact motivates the intuition that in Europe, for limited participation reasons, the creation of private safe assets is even more needed. This leads to a larger banking system in size and a positive comovement of private debt with public debt.

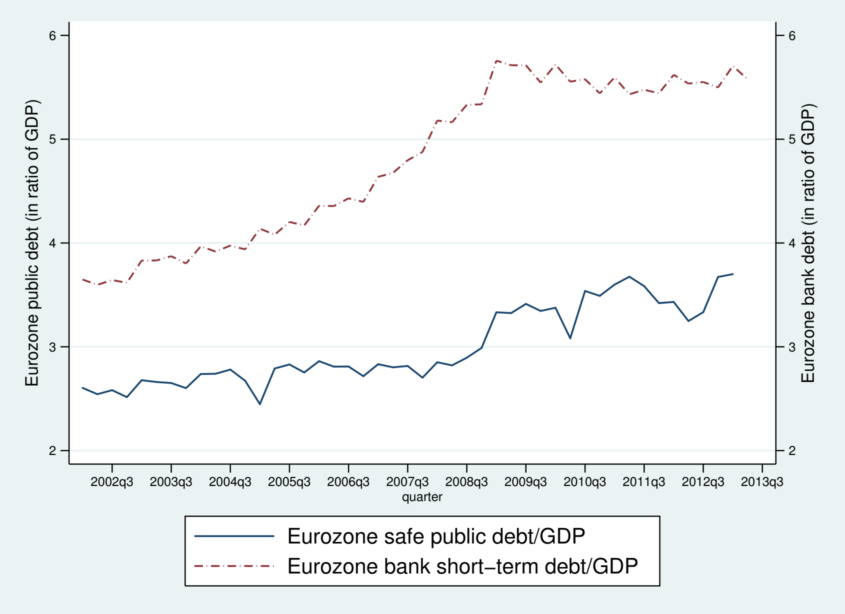

Figure 1.4: *Third stylized fact: comovement of public safe assets and private safe assets in Europe.*

The three stylized facts, respectively related to the financial sector balance sheet, the financial sector income statement and to monetary aggregates, are rationalized in the model of private safety creation developed in this paper. Banks produce private safe assets in a general equilibrium environment. The macro inspiration comes from Caballero and Farhi (2013), which emphasizes the shortage of safe assets as a key macroeconomic imbalance.

The authors make the point that public debt, being a bearish asset, plays a central role in mitigating this shortage by a mechanism that they call *safety multiplier*. The present paper microfounds this mechanism by putting banks at the heart of the creation of private safe assets: increasing the supply of public debt enables banks to lever up more, and this increases the endogenous supply of private debt. The dynamic version of the model also microfounds why public debt has a negative beta. In my environment, a shortage of public safe assets triggers a recession, not through a New-Keynesian demand channel as in Caballero and Farhi (2013), but through a supply channel caused by bank deleveraging. The diversification motive has some commonality with Gennaioli et al. (2013b) model of shadow banking, but applied to banks in general. In Gennaioli et al. (2013b), by diversifying away idiosyncratic risk, securitized debt is made entirely riskless. In my macro environment, there are only two assets, so the law of large numbers is ineffective to create safety. It is the endogenous correlation properties of public debt that enable banks to produce safe assets.

The safety multiplier mechanism critically relies on two ingredients: risk-aversion heterogeneity and incomplete markets. Risk-aversion heterogeneity is a parsimonious way to capture the distinction between active wealth (risk-neutral banks) and passive wealth (risk-averse investors). Only with incomplete markets banks' leverage is determinate and depends on the supply of public debt. If markets were complete, risk-neutral banks would be able to fully insure risk-averse investors, so equilibrium leverage would always be equal to the net worth of risk-averse investors. There would be no safety multiplier.

Putting banks at the heart of the creation of safe assets has two key normative implications. The private competitive equilibrium without public debt is constrained inefficient. Banks under-provide insurance when the economy lacks long-dated securities. The issuance of long-term public debt improves welfare by facilitating intragenerational risk-sharing. Thanks to their negative beta properties, long-term securities exert a *positive* externality. They are an attractive input for safety production, but potential issuers of long-term debt do not internalize it. Nevertheless, issuing too much public debt destroys its own hedging properties, and this can eventually hurt welfare. As a result, there exists a finite optimal level of public safe assets in the economy.

The second normative implication of the model relates to the economic role of universal banks. Narrow banking regulations such as Glass-Steagall, which call for a split between banks' securities arm and retail arm, are harmful from a safety-creation standpoint. Indeed, such a split prevents banks from leveraging the hedging properties of public debt, implying a lower level of bank debt, and a lower level of private safe assets in the economy. Under Glass-Steagall, banks under-provide insurance to the risk-averse investors.

Finally, an open-economy version of the model introduces heterogeneity in sovereign risk. Public debt is then priced according to its relative sovereign risk compared to other public debts. This open economy environment rationalizes Figure 1.2, in which the *relatively* safer public debt exhibits a negative beta whereas the relatively riskier public debt has positive beta. This environment proposes a pure asset pricing, moral-suasion and moral-hazard free, perspective for why domestic debt gets redomesticated on domestic banks' balance sheets in sovereign crises. The leveraging ability of domestic banks is determined at the margin by the flight to safety of domestic investors. This implies that domestic banks become the natural holders of domestic debt under sovereign risk heterogeneity.

Model and theoretical results I develop a model of endogenous leverage based on risk aversion heterogeneity and endogenously incomplete markets. I do not resort to heterogeneity in beliefs disagreement as Geanakoplos (2009), as I explicitly introduce multiple assets than can be used as collateral for recourse debt. It is hard to discipline the beliefs of different agents on different assets.[3] Compared to asset pricing models with heterogeneity, such as Dumas (1989), I introduce equilibrium default through a limited liability constraint.[4] Risk-neutral banks then partially insure risk-averse lenders against macroeconomic shocks. I embed this rationale for endogenous leverage in a general equilibrium environment in which the supply of public debt is perfectly inelastic and there are no deep pockets investors. In this economy, too many savings are chasing too few *safe* assets On the other hand, the supply of risky assets is perfectly elastic. Through their leverage decision, banks control the endogenous supply of private safe assets. The partial equilibrium effects of the agents' portfolio choices interact with the general equilibrium channel of a Walrasian market for public debt.

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[3] It is unclear whether optimists on the stock market should also be optimists on negative beta assets.

[4] Without limited liability, risk-neutral banks would fully insure risk-averse investors and the safety multiplier disappears. This could be seen as a particular case of the generic result of Krishnamurthy (2003) of irrelevance of balance sheet recession under complete contracting (see Di Tella (2013) for a continuous-time formulation). In my environment, limited liability is key to endow banks with a role in portfolio construction.

Under low supply of public safe assets and high heterogeneity in risk-aversion, the economy features a *safety multiplier* mechanism. It is optimal for risk-neutral banks to hold public safe assets and bundle them with risky assets. Doing so creates more safety in the economy than the endowment of public debt would do alone. When public safe assets become scarce, they can become so expensive that banks prefer to delever. This impairs the supply of private safe assets, as well as less investment in the real economy. This credit crunch is caused by a shortage of public debt. In such situation, the proportion of safe assets on banks' balance sheets rise, as they are best used in risk-neutral hands to trigger the safety multiplier. Hence there is a *pecking order* in public debt ownership: having them on the balance sheets of banks not only hedges risky investment, but at same time enables private safety creation. When the risk averse agents have limited access to the risky technology (Europe), equilibrium leverage is higher and the parameter region under which the economy features a safety multiplier is wider.

The welfare analysis is carried out in a stochastic environment of overlapping long-lived generations. The negative beta of long-term securities is microfounded be the expectations of a flight-to-safety in low aggregate states. The government and private agents are facing the exact same maturity choice between short-term debt and long-term debt. The decentralized equilibrium is constrained inefficient because private agents do not issue enough long-term securities. The reason is that they do not fully internalize the positive effects of having a high supply of negative beta assets in the economy in order to crowd-in aggregate investment.

Asset pricing implication: the Safety Mismatch Index The model delivers endogenous closed-form solutions for two safety premia: one on public debt and one on private debt.

The spread between the two is the carry trade made by the banks on public debt. I show this carry trade increases with public debt beta and decreases with leverage. I argue that this spread is a relevant welfare and financial stability indicator. It reveals the fragility of banks' balance sheets, as well as the extent to which the economy is exposed to a sovereign debt crisis that feeds into a banking crisis. I call this spread the Safety Mismatch Index and show its predictive power on the Eurozone sovereign crisis. The empirical analysis confirms the key predictions of the model on monetary aggregates. I interpret the European crisis as a shortage of *public* safe assets, which deprives banks of their leveraging ability.

Key related literature My paper connects two strands of literature: models of endogenous leverage a la Geanakoplos (1997) and general equilibrium macro models a la Caballero and Farhi (2013). The latter also studies the effect of a fixed supply of safe assets, but does not feature optimizing banks. Taking into account the inelastic supply of risk free assets is a key departure from the standard asset pricing approach of Campbell and Viceira (2002), which focuses on exogenous changes of risk preferences to pin down the risk free rate. My paper also contributes to the banking literature about what banks do. It suggest a view of banks as private safety creators, alternative to the agency view of the firm: e.g. Diamond (1984), Diamond and Rajan (2001b), and Tirole (2003) for a unified theory of banking relying on agency frictions. Furthermore, I argue that treating bank debt holders as risk-neutral is counterfactual with the vision that these holders are 'passive money.' As a result, I focus on the main heterogeneity between active money and passive money emphasized in Caballero and Farhi (2013) and Gennaioli et al. (2013b), and I introduce it in an asset-pricing model with endogenous leverage. Kashyap et al. (2002) sees banks as liquidity economizers, whereas I see them as safety multipliers. Diamond (1984) banks engage in idiosyncratic diversification, whereas my financial intermediaries engage in diversification of aggregate shocks. My model can also be seen as providing microfoundation to the view in Philippon (2012) of banks as service providers to households.[5] On the creation of safe assets, all the previous theoretical models are assuming *substitutability* between public safe assets and private safe assets: Gorton and Metrick (2012), Gourinchas and Jeanne (2012), Sunderam (2012) and Greenwood et al. (2010). On the contrary, my model exhibits crowding-in. Empirically, I document the stylized fact of a positive comovement between US Treasuries and bank debt supplies in Europe, whereas Krishnamurthy and Vissing-Jorgensen (2013a) shows there is substitutability in the US. Compared to Campbell et al. (2013b), I do not emphasize the *nominal* properties of bonds, but their relative *safety* properties, in order to analyze their negative beta. On the normative side, contrary to Stein 2010 who argues there is too much private safe assets, my model hints at a lack of public safe assets. The stochastic OLG model used for the normative analysis is reminiscent of Ball and Mankiw (2007), but allows for within-generations heterogeneity and maturity choices. Compared to Woodford (1990) and Gale (1990), my environment features both intergenerational risk sharing (between generations) *and* intra-generational risk-sharing (within a generation). The interplay between the two is at the core of the constrained inefficiency result.

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[5] In his empirical investigation, Philippon (2012) uses exogenous weights to value the services provided by banks to households. Similarly, DeAngelo and Stulz (2013) have in mind a liquidity premium on bank debt. My

The paper is organized as follows. Section 2 presents the environment in which banks create safe assets. Section 3 solves for the decentralized equilibrium, taking the supply of public debt as given. Section 4 carries out the normative analysis: it shows the private equilibrium is constrained inefficient, and how public debt issuance can achieve a Pareto improvement. Section 5 analyzes the open economy extension. Section 6 turns to the empirical analysis. Section 7 discusses the results in light of the literature and concludes.

1.2 A Model of Private Safety Creation

I develop a model of endogenous leverage in an environment of risk aversion heterogeneity, multiple assets and limited liability. Even if it shares some flavors with models of beliefs disagreement (Geanakoplos 1997, Simsek (2013)), I choose to work with risk aversion heterogeneity. This is motivated by the focus of this paper on asset multiplicity, and the

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model provides rigorous microfoundations to their insights, by pinning down banks production function of safety to their portfolio choice.

view of banks as macro insurers.[6]

1.2.1 The safety multiplier argument in a nutshell

Before developing the dynamic CARA-normal environment, I start by a static four-states example to illustrate the safety multiplier argument. Why would banks ever hold the public safe assets on their own balance sheets instead of letting these safe assets being held in risk-averse hands?

Consider only two dates: $t = 0, 1$, four equally plausible states. Assume that a $t = 1$ risky payoff for the technology and consider a public security that is imperfectly negatively correlated with the technology:

$$technology = s_K = \begin{bmatrix} 8 \\ 6 \\ 4 \\ 2 \end{bmatrix} \text{ and } public\,debt = s_B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 3 \end{bmatrix}$$

By bundling the two assets ('self-diversification'), a risk-averse investor would get:

$$aggregate = \begin{bmatrix} 9 \\ 8 \\ 7 \\ 5 \end{bmatrix}$$

The mean of this payoff is 7.25 and its volatility is 2.19. But actually, a risk-neutral bank can help improving the Sharpe ratio effectively faced by the risk-averse investor. Assume the risk-neutral bank and the risk-averse investor enter 7 units of a financial contract that promises $\bar{s} = 1$ at $t = 1$, and that the risk-neutral agent holds both assets on its balance sheet. The state-contingent value of the risk-neutral agent portfolio is *aggregate*. Thus he defaults on its contract if and only if one of the two lowest states realize. As a result, the state-contingent payoff faced by the investor, by the means of the financial contract ('delegated diversification'), is now:

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[6] Fostel and Geanakoplos (2008) argue that a reasonable assumption consists in treating optimists for one asset as also optimists for the other assets, and this is how they obtain contagion. On the contrary, I do not want to take a stand on who is more optimistic on a given class of assets, so I claim that risk-aversion heterogeneity is a more parsimonious modeling device.

$$contract = \begin{bmatrix} 7 \\ 7 \\ 7 \\ 5 \end{bmatrix}$$

This state-contingent payoff has a mean of 6.5 and its volatility is 0.75. By bundling the two assets and issuing risky debt against it, the bank has been able to significantly improve the Sharpe ratio faced by the risk-averse agent. This asset will be traded in equilibrium. Formally, the equilibrium is defined by the risk-neutral maximization of the bank and the mean-variance maximization of the investor, under their respective budget constraints, as well as two market clearing conditions, where B is the exogenous supply of public debt (price q_B) and D is the endogenous supply of private debt (price q_D):

$$x^A_{\hat{B}} + x^P_{\hat{B}} = B \text{ and } x^P_{1^A} = y^A$$

The portfolios $\left\{i^A, x^A_{\hat{B}}, y^A\right\}$, $\left\{i^P, x^P_{\hat{B}}, x^P_{1^A}\right\}$, and the two prices q_B and q_D are the endogenous variables. Denoting $D = q_D y^A$ the value of private debt, Appendix B.1 shows that this economy features a safety multiplier:

$$\frac{\partial D}{\partial B} > 0$$

This example shows that the only ingredients needed are limited liability and risk-aversion heterogeneity. From a situation of market incompleteness (2 assets and 4 states), agents endogenously decide to *partially complete* the markets. Defaultable debt is a 3rd asset that enables to attain the constrained efficient allocation. The net worth n^A of the risk-neutral bank gives an additional rationale for leverage (i.e. cross-subsidization), but does not play a role in the safety multiplier mechanism.

What critically misses on this example is the endogeneity of the default threshold, which is actually a choice variable for the bank. The full-fledged model below captures it. This makes the price p_B depend on the supply B. The safety multiplier result carries through, as long as the supply B is small enough.

1.2.2 General environment

The model is a stochastic overlapping generations model under aggregate technology risk. Contrary to the canonical OLG, risk is not on endowments, but on the technology in perfectly elastic supply. It is cast in a discrete infinite horizon framework, each period is indexed by t.[7]

Agents' preferences Each generation is populated by a continuum of agents of two types. There is a mass 1 of banks (type A: active) and a mass 1 of investors (type P: passive). Banks are risk-neutral whereas investors are risk-averse, with a coefficient of absolute risk aversion of γ^P.[8] For banks, the risk-neutrality assumption captures enhanced sophistication, diversification opportunities or bailout expectations.[9] Risk-neutrality should be seen as a normalization, as what matters is the *differential* of risk aversion between the two populations of agents. The CARA parameter of investors captures this differential. CARA preferences are more constraining than HARA or Epstein-Zin, often used to shed light on the risk-free rate puzzle. Agents' types are common knowledge. At birth, the mass of risk-neutral agents A is endowed with n^A of numeraire, whereas the mass of risk-averse investors P is endowed with n^P.

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[7] The purpose of the dynamic model is to endogenize the beta of safe assets. Microfounding the beta of public debt paves the way to the welfare analysis. The model is the exact repeated sequence of the static game considered in the example above, with a broader set of available contracts to allow for a maturity choice of agents.

[8] I have also solved a version of the model with risk-averse banks, with CARA coefficient γ^A. The results of this paper are robust to this extension, as long as $\gamma^A < \gamma^P$. The model generalizes to a continuum of types γ^i, and then features assortative matching in the competitive equilibrium: the surplus of each bilateral match is endogenous to agents outside options. Therefore the least risk averse agent is matched with the least risk-averse agent above the endogenous cutoff $\bar{\gamma}$ that decides who the lenders are: $[\bar{\gamma}; \gamma^{max}]$.

[9] All these reasons point towards value maximization by banks. For the latter microfoundation, it will amplify the destructive effects of sovereign risk shown in the model extension, as arguably sovereign risk would in this case hurt bailout ability of the sovereign, hence increasing banks' risk aversion.

Demographics There are overlapping generations of such agents. Each generation lives three periods.[10] The heterogeneity within generations is kept constant over time. Every new born of type A is endowed with n^A of numeraire, agents of type P with n^P.[11] Not having the wealth distribution as a state variable makes the model bloc-recursive. The notation G_t^i with $i \in \{A, P\}$ refers to an agent of type i that belongs to the generation that was born in period $t-2$ and dies in period t.

Technology There is only one exogenous asset to invest in: a risky linear technology. This technology is short-term: investing one unit at t yields an uncertain dividend at $t+1$: s_{t+1}. I assume that this payoff follows a random walk:[12] $s_{t+1} \sim N(s_t, \sigma_1)$. Thus s_t is the aggregate state of the economy. All agents have the same beliefs of the shock distribution. The technology is in perfectly elastic supply, so it has an exogenous linear cost p_K in numeraire.[13] The ratio p_K/s_t should be thought as the time-varying Tobin-q of the model. There is *no* riskless storage technology.

Government A government $\hat{B}$ finances public spending by raising taxes and issuing public debt. The latter plays the role of a second asset from private agents' perspective, albeit endogenous and in fixed supply.

Public debt securities are non state-contingent assets, which promise to one unit of numeraire at maturity. Public debt can be long-term: it can be issued at period t and pay back only at period $t+2$. The government then issues public debt at different maturities $h = 1, ..., H$. Taking $H = 2$ exactly matches the horizon of private agents, and therefore do not endow the government with undue advantage on agents.[14] Denote B_t^h the outstanding stock, at date t, of public debt maturing at date $t+h$. Government consumption g_t is exogenous. To avoid asymmetric tax treatment across generations, only the old are taxed.[15] There can be asymmetric tax treatment within generation, but I assume the tax schedule $\left(\tau^A, \tau^P\right)$ is not optimized upon by the government, perhaps because of informational frictions on the types (A, P), which renders fiscal policy less agile than public debt issuance policy. Fiscal policy then merely tracks the public debt issuance policy by balancing the government budget.

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[10] Beyond breaking the First Welfare Theorem, the OLG structure then mutes undesirable discounting and long-run wealth effects.

[11] It is within-generation heterogeneity in preferences, not in endowments as Sargent-Wallace (1982) and Smith (1988).

[12] Qualitative results apply to general Markov chains.

[13] The price ϕ can be normalized to 1. Results are robust to decreasing return to scale technology, i.e. a partially elastic offer curve. On the other hand, having an inelastic supply of capital (supply of one Luca's tree) renders the quantity of aggregate risk in the economy constant, and this shuts down the crowding-in effects of public debt. A weaker version of the safety multiplier and constrained inefficiency results can be recovered in the latter case when allowing for young consumption.

For each residual maturity of public debt $h = 1, ..., H$, a Walrasian market opens at each period t. All agent of all generations have access to these markets, and primary debt and secondary debt is fungible: a government promise for date $t+h$ has the same price, because it is traded on the same market, which clears at price $\left\{\hat{q}_t^h\right\}_h$. As a h period promise at t becomes a $h-1$ period promise at $t+1$, the government budget writes:

$$g_t + \sum_h \hat{q}_t^h \left(B_{t-1}^h - B_t^h\right) \quad \leq \quad \tau_t^A + \tau_t^P \tag{1.1}$$

A *riskless financial policy* is a collection $\left(\left\{B_t^h\right\}_h, \tau_t^A, \tau_t^P\right)$ that satisfies the government budget constraint at each history s^t. Nevertheless, I also allow for the possibility of endogenous government default.[16] To see this, consider the case of an extremely bad aggregate shock (or an extremely high public spending shock). Given the fiscal policy choice of only taxing the old, the proceeds the government can raise in this case is null. Moreover, as will become clear through the safety multiplier mechanism elicited in this paper, the economy is in a Laffer regime for public debt, hence the government cannot raise additional revenue by issuing more public bonds. The government is then forced into default. On other words, the government has endogenously limited fiscal capacity.

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[14] I restrict the government ability to issue long-term debt to only two periods in order to respect the spirit of constrained efficiency. In the theory of the 2nd-Best, the government has the same instruments as private agents. In my environment, this means allowing the government for the exact same maturity choice as the agents.

[15] Whether to tax the young, the middle or the old generation is innocuous, as wealth effects do not play a role in the safety multiplier.

[16] The technical motivation is to rule out a trivial strategy for the social planner to issue unlimited level of public short-term debt. The economic motivation is to analyze in Section 5 the interplay between sovereign credit risk and bank credit risk in an open economy environment.

When a long-term public debt is issued at t, it carries *interest rate* risk: at period $t+1$, its price will be determined mark-to-market, and this price is uncertain from t perspective.[17] An investor with a one-period ahead horizon faces a portfolio choice in three stochastic potentially correlated assets: the technology, the short-term public debt and the long-term public debt.[18]

In a second-moments approximation, the prices at $t+1$ of the technology and long-term public debt are multivariate normal:

$$\begin{bmatrix} s_{t+1} \\ q(s_{t+1}) \end{bmatrix} \sim N\left(\begin{bmatrix} s_t \\ \mathbb{E}_t\left[q(s_{t+1})\right] \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma_K^2 & \rho\sigma_K\sigma_B \\ \rho\sigma_K\sigma_B & \sigma_B^2 \end{bmatrix} \right)$$

where σ_K is the exogenous volatility of technology, σ_B is the endogenous volatility of interim price of long-term debt, and ρ is their correlation. One should think of the long-term debt volatility as low ($\sigma_B < \sigma_K$) and its correlation with the technology as negative ($\rho < 0$), as is shown in the recursive equilibrium. The expectation on technology is the aggregate state: $\mu_K = s_t$, whereas the expectation on long-term debt is the expectation of its market price tomorrow: $\mu_B = \mathbb{E}_t\left[q(s_{t+1})\right]$.

Private financial contracts At the same time, agents can borrow or lend to each other by trading on securities markets. One unit of security is a promise to pay $\bar{s}$ of numeraire at maturity $t+h$. To rule out Ponzi schemes, private agents can issue promises only at the horizon of their life span: one- or two-period ahead when young, and only one-period ahead when middle-aged. Contrary to Geanakoplos and Zame (2013), this contract does not specify a given level of collateral. This intends to capture risky recourse debt. *Risky* implies that agents can default on their contract, and they will do so as long as the payoff on their portfolio (their asset side) is realized below the sum of all the promises contracted by the agent. *Recourse* implies that a borrower cannot pledge only a part of its balance sheet. When the borrower defaults, the lender has recourse to seize the entire balance sheet assets of the borrower. This matches the empirical fact that, in practice, most short-term debt is recourse.[19]

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[17] Contrary to the safe asset literature (Gourinchas and Jeanne (2012), Gorton and Ordonez (2013)), and in line with Caballero and Farhi (2013), my paper insists on a defining characteristics of public debt: its negative beta with macroeconomic shocks.

[18] As a consequence of potential sovereign default, short-term public debt is not fully riskless.

Each security $\bar{s}$ is traded on a Walrasian market, which clears at a price $q_{\bar{s}}$. An agent selling y units of such contract is therefore able to raise $D = q_{\bar{s}} * y$ at $t = 0$, against the promise of paying back $\bar{S} = \bar{s} * y$ at $t = 1$. Define the rate of return on bank debt as the ratio of the promise to its price: $r_{\bar{s}} = \frac{\bar{s}}{q_{\bar{s}}} = \frac{\bar{S}}{D}$.[20] Primary and secondary debt are fungible, i.e. any promise issued by a given agent G^{θ}_{t+j} at a given maturity $t+h$ trades on the same Walrasian market, which clears at price $\left\{ q_{\substack{G^{\theta}_{t+j} \\ \bar{s}_{t+h}}} \right\}_{h \leq j}$. y denotes a short-term promise (issued one period before maturity), whereas $y^{primary}$ denotes a long-term promise (issued two periods before maturity).

Timeline The sequence of actions is as follows.

- At each period t, the government rolls over its debt according to its financial policy. The government issues $B^{h}_{t-1} - B^{h}_{t}$ units of new public bonds with the residual maturity

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[19] Geanakoplos (2009) features secured non-recourse debt. In practice, most of the so-called secured debt such as repurchase agreement contracts (repo) includes an additional claim to the balance sheet of the issuer in case of collateral shortfall, which makes them in effect recourse. In case of default, the lenders seize the whole balance sheet of the agent in default. See Weymuller (2013b) for an analysis of the idiosyncratic drivers of the market for secured debt.

[20] This General Equilibrium approach is not equivalent to the Principal-Agent approach where the borrower and the lender bargain over the loan contract. Theorem 1 of (Simsek, 2013) (equivalence with full bargaining to the borrower) does not apply due to the absence of a riskless technology. Nevertheless, appealing to the first theorem of welfare, both environment are constrained efficient, so they trace the same Pareto frontier. The Walrasian equilibrium is therefore equivalent to a Principal-Agent economy with a specific bargaining power. The Walrasian treatment is more transparent, as it restricts the space of ex-ante transfers that could be achieved through the terms of the contract.

$t+h$.

- At each period t, with macro state s_t, a new generation of agents $G_{t+2}^{\theta\in\{A,P\}}$ is born with endowments $\left(n^A, n^P\right)$. The young generation make portfolio choice decisions by investing in the technology: agents of type $\theta \in \{A,P\}$ invest $i_t^{G_{t+2}^{\theta}}$ in the technology. At the same time they trade on the Walrasian markets for legacy promises: public debt and claims on former generation balance sheets, as well in the primary markets for new promises issued at t, whether within or between generations. We denote $\bar{s}_{t+h}^{G_{t+j}^{\theta}}$ the number of units of promise they buy on market for a given promise $\bar{s}_{t+h}^{G_{t+j}^{\theta}}$. They also create primary markets for promises on their own balance sheets by *issuing* long-term debt and short-term debt. We denote these sort positions in their own long-term and short-term promises $y_{\bar{s}_{t+2}^{G_{t+2}}}$ and $y_{\bar{s}_{t+1}^{G_{t+2}}}$ respectively.
- At the following period $t+1$, the same agents becomes middle-aged. The risky technology then pays off s_{t+1}, and their portfolio of promises (long and short) can be mark-to-market with the Walrasian markets that open at period $t+1$. Maturing promises are settled by the actual payment of the promise or by the issuer defaulting. In the latter case, any holder of a promise seizes the total balance sheet $\{technology\ payoff + residual\ promises\}$ of the agent in default, pro-rata the promise. Subsequently, these middle-aged agents make new decisions of investment in the short-term risky technology $i_{t+1}^{G_{t+2}^{\theta}}$ and rebalance their portfolio of promises, and can open primary markets for promises by issuing short-term debt.
- Finally, at period $t+2$, the agents of this generation $G_{t+2}^{\theta\in\{A,P\}}$ become old. Before their death, they receive the payoff s_{t+2} of the technology and they liquidate their portfolio of promises, and consumes these proceeds.

Markets At any period t, there are 3 markets open for private promises.[21]

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[21] The identity of the issuer generation has to kept track of, due to the recourse feature of the promises. A priori it would be $3 * card\,(\theta)$ potential markets, but given the one dimensional heterogeneity within-generation,

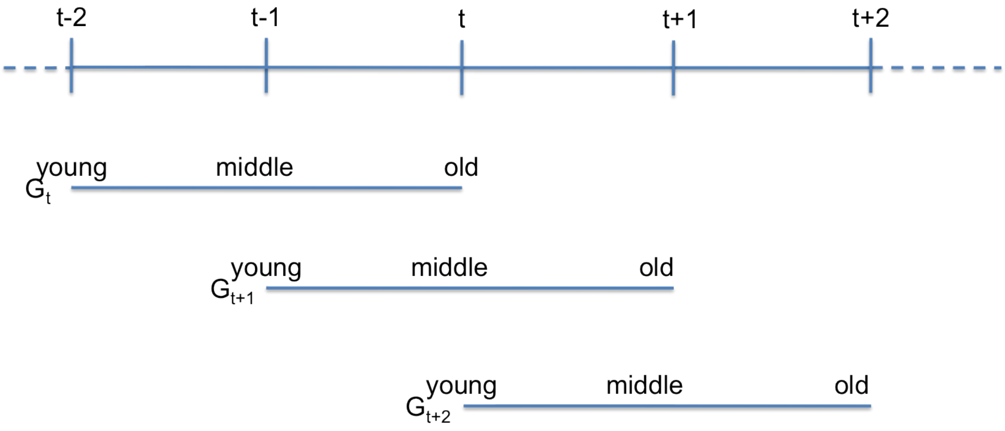

Figure 1.5: *Environment: demographics and timeline.*

- **Secondary market for long-term debt**, i.e. promise $\bar{s}^{G^\theta_{t+1}}_{t+1}$ issued by the middle, clearing at price $q_{\bar{s}^{G^\theta_{t+1}}_{t+1}}$:[22]
$$x^{G^{\theta'}_{t+1}}_{\bar{s}^{G^\theta_{t+1}}_{t+1}} + x^{G^\theta_{t+2}}_{\bar{s}^{G^\theta_{t+1}}_{t+1}} = y_{\bar{s}^{G^\theta_{t+1}}_{t+1}} + y^{primary}_{\bar{s}^{G^\theta_{t+1}}_{t+1}} \tag{1.2}$$
- **Primary market for short-term debt**, i.e. promise $\bar{s}^{G^\theta_{t+2}}_{t+1}$ issued by the young, clearing at price $q_{\bar{s}^{G^\theta_{t+2}}_{t+1}}$:
$$x^{G^\theta_{t+1}}_{\bar{s}^{G_{t+2}}_{t+1}} + x^{G^\theta_{t+2}}_{\bar{s}^{G_{t+2}}_{t+1}} = y_{\bar{s}^{G^\theta_{t+2}}_{t+1}} \tag{1.3}$$
- **Primary market for long-term debt**, i.e. promise $\bar{s}^{G^\theta_{t+2}}_{t+2}$ issued by the young, clearing at price $q_{\bar{s}^{G^\theta_{t+2}}_{t+2}}$:
$$x^{G^\theta_{t+1}}_{\bar{s}^{G_{t+2}}_{t+2}} + x^{G^\theta_{t+2}}_{\bar{s}^{G^\theta_{t+2}}_{t+2}} = y^{primary}_{\bar{s}^{G^\theta_{t+2}}_{t+2}} \tag{1.4}$$

On the other hand, for government promises, there are only two markets, as the primary market for short-term public debt and the secondary market for long-term public debt are fungible. Despite being called the market for short-term public debt, it includes the legacy

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promises are traded only in one direction, hence only 3 markets are actively traded.

[22] I allow for $y^{G_{t+1}}_{\bar{s}^{G_{t+1}}_{t+1}} < 0$, which corresponds to the buy back of the legacy stock $y^{G_{t+1}\,primary}_{\bar{s}^{G_{t+1}}_{t+1}}$ of long-term promises. The stock $y^{G^\theta_{t+1}\,primary}_{\bar{s}^{G_{t+1}}_{t+1}}$ is held at the beginning of the period by the old generation G_t. They are selling in order to consume before dying.

long-term public debt that matures next period.

- **Market for short-term public debt**, i.e. for public promise of $1^{\hat{G}}_{t+1}$ at $t+1$, which clears at price $\hat{q}_{t+1}$:

$$x^{G^{\theta}_{t+1}}_{1^{\hat{G}}_{t+1}} + x^{G^{\theta}_{t+2}}_{1^{\hat{G}}_{t+1}} = \hat{B}^1_t \tag{1.5}$$

- **Market for long-term public debt**, i.e. for public promise of $1^{\hat{G}}_{t+2}$ at $t+2$, which clears at price $\hat{q}_{t+2}$:

$$x^{G^{\theta}_{t+1}}_{1^{\hat{G}}_{t+2}} + x^{G^{\theta}_{t+2}}_{1^{\hat{G}}_{t+2}} = \hat{B}^2_t \tag{1.6}$$

1.3 Decentralized Equilibrium

I first analyze on the environment abstracting from any government optimization. In this case, the financial policy $\left(\left\{\hat{B}^h_t\right\}_h, \tau^A_t, \tau^P_t\right)$ is taken as exogenous.[23] I define all the Walrasian recursive equilibria of this economy. I focus on stationary Markov equilibria. Given the bloc-recursive structure of the environment, these equilibria can be defined with a unique state variable: the aggregate shock $s^t \leftarrow (s_{t-1}, s_t)$.

Definition 1. A stationary Markov equilibrium is a collection in each history s^t of portfolio investments $\left\{ i_t^{G^{\theta}_{t+j'}}\left(s^t\right), x^{G^{\theta}_{t+j'}}_{\bar{s}^{G^i_{t+j}}_{t+h}}\left(s^t\right) \right\}_{1 \leq h \leq j, j' \leq H}$, primary issuances $\left\{ y^{G^{\theta}_{t+j'}}_{\bar{s}^{G^i_{t+j}}_{t+h}}\left(s^t\right) \right\}_{1 \leq h \leq j, j' \leq H}$, a vector of public debt prices $\left\{\hat{q}_{t+h}(s^t)\right\}_{1 \leq h \leq H}$ and a vector of private debt prices $\left\{ q_{\bar{s}^{G^i_{t+j}}_{t+h}}\left(s^t\right) \right\}_{1 \leq h \leq j \leq H}$ such that:

i) All agents of all generations optimize.

iii) Markets for private promises clear at each residual maturity.

iv) Markets for public promises clear at each residual maturity.

These equilibria have the flavor of the collateral-constrained equilibria of Geanakoplos and Zame (2013),[24] but the financial assets traded are different, as borrowers' debt here is *recourse*. Another difference is that there is a priori a continuum of contracts that could be traded: one for each state s of the continuum. Markets therefore are complete.[25] Although a priori, an infinite set of securities $(q_{\bar{s}}, \bar{s})$ is available to agents, only one will be traded in the equilibrium of interest given the low level of heterogeneity (only two types of agents).[26] The economy features endogenous market incompleteness: despite having a complete spanning of financial assets, agents' positions are restricted by their endogenous collateral constraints arising from limited liability. However, due to the recourse feature of unsecured debt, my economy is 'more complete' than the Geanakoplos one. As a result, this environment can be seen as an intermediate case between the Arrow-Debreu and the Geanakoplos economies, less complete than the former but more complete than the latter.

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[23] I allow for irresponsible financial policies: the financial policy does not have to be riskless in the sense of satisfying the government flow of funds at each history s^t. Hence the equilibrium features non-zero probability of government default.

The full equilibrium is solved by backward induction. First I characterize the solution of the portfolio choices of the middle-aged generation, taking the supply of legacy long-term debt as given. Then I move backward to characterize the joint decision of portfolios and maturity choices by the young generation.

1.3.1 Middle-aged agents portfolio choice

I consider here the generation G_{t+1}: the middle-aged at period t. Its agents do not face any maturity choice: they only can issue short-term debt. However they can invest in all 3 active markets beyond the technology: within-generation short-term promise[27], next-generation short-term promise and next-generation long-term promises. Without loss of generality, I analyze a decentralized equilibrium with zero supply of public debt.

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[24] It is closer to the liquidity-constrained equilibrium than the debt-constrained equilibrium of Kehoe and Levine (2001). In the latter, as in Hellwig and Lorenzoni (2009), default is strategic and the existence of equilibrium hinges on the self-enforcement of debt. These two papers do not feature equilibrium default.

[25] As markets are complete, there is no need to engage in market design as Athanasoulis and Shiller (2001).

[26] This is fortunate, as it circumvents the possibility of a discontinuity in agents budget sets as in Hart (1975). Restricting agents from consuming at $t = 0$ mutes down consumption smoothing and conveniently avoids difficulties on equilibrium existence.

[27] It is the same as buying back long-term promises if this investment is negative.

I conjecture a *contract equilibrium* in which, within each generation, risk-neutral banks borrow from risk-averse investors, and all agents have non-degenerate portfolio holdings in all assets that are not internal to the generation (the technology and the next-generation short-term and long-term promises). The CARA-normal environment enables to derive the equilibrium closed-form, even with equilibrium default.

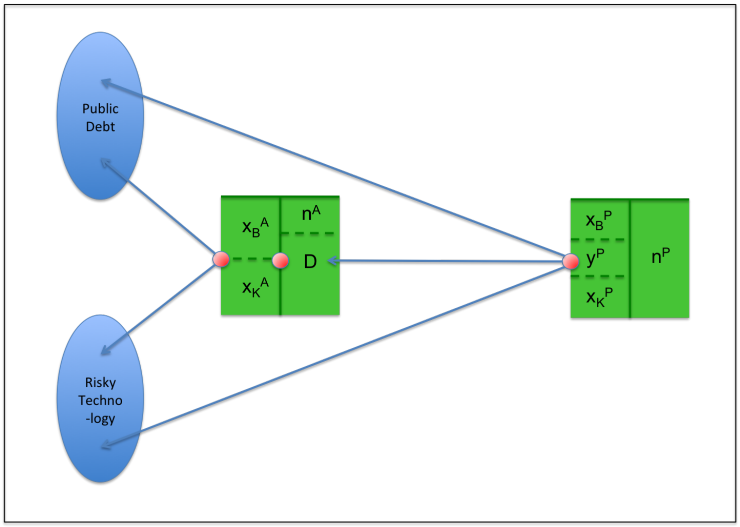

Figure 1.6: *Contract equilibrium representation. Within-generation agents are in green and assets are in blue. Red bullets indicate, from left to right: bank portfolio choice, bank leverage choice and lender portfolio choice.*

The net worth of middle-aged agents is the result of their young portfolio decisions and of the realization of the aggregate state t. Middle-aged agents can be thought as liquidating their entire portfolios, including of long-term debt, at period t market prices, before entirely reinvesting the proceeds. Therefore the post-liquidation net worth $n_t^{G^\theta_{t+1}}(s_t)$ of the middle-aged is the state variable that encodes all the previous decisions of the generation.

Program of middle-aged risk-neutral agents Denote $\bar{S}$ the sum of all promises and its portfolio choice X^{A_m}:

$$\bar{S} = \int \left(y_{\bar{s}_{t+1}^{G^A_{t+1}}} + y^{primary}_{\bar{s}_{t+1}^{G^A_{t+1}}} \right) \bar{s} d\bar{s}$$

$$X^{A_m'} = \left[\begin{array}{ccc} i_t^{G_{t+1}^A} & x_{\bar{s}_{t+2}^{G_{t+2}}}^{G_{t+1}^A} & x_{\bar{s}_{t+1}^{G_{t+2}}}^{G_{t+1}^A} \end{array} \right]$$

The bank pre-default portfolio realization u_{t+1} is:

$$u_{t+1} = X^{A_m'} S_{t+1} - \bar{S} - \tau_{t+1}^A \tag{1.7}$$

The program of the bank then writes:

$$\begin{aligned} \underset{\{X^A, \bar{S}\}}{Max}\ W^{G_{t+1}^{A_m}} \left(n_t^{G_{t+1}^A}(s_t), s_t; y_{\bar{s}_{t+2}^{G_{t+2}^A}}^{primary}(s_{t-1}) \right) &= \mathbb{E}_t \left[u\, 1_{\{u \geq 0\}} \right] \qquad (1.8) \\ s.t. \qquad X^{A_m'} P \leq n_t^{G_{t+1}^A}(s_t) &+ \left\{ \bar{S} - \int y_{\bar{s}_{t+1}^{G_{t+1}^A}}^{primary} \bar{s} d\bar{s} \right\} \frac{q_{\bar{s}_{t+1}^{G_{t+1}^A}}}{\bar{s}} \end{aligned}$$

Out-of-generation trades are with a representative agent of the other generation as counterpart: G_{t+2}. Indeed, the equilibrium can be broken down into two sequential (but interacting) problems: risk-sharing between generations, then risk-sharing within the generation. The legacy stock of long-term promise becomes fungible with short-term promises. P denotes the price vector at t of assets and S their $t+1$ realization. For the technology, price is ϕ and realization s_{t+1}. For securities, 'prices' are Walrasian prices at t and 'realizations' are prices at $t+1$ in history s_{t+1}. The realization of the short-term promise on the outside generation G_{t+2} is risky debt payoff: $Min\left(X^{A_m'} S, x_{\bar{s}_{t+1}^{G_{t+2}}}^{G_{t+1}^A} \bar{s}_{t+1}^{G_{t+2}} \right)$. The key feature of the full equilibrium is that long-term promises issued by generation G_{t+2} are negative beta, thus appealing in the portfolio choice of generation G_{t+1} agents.

In the multivariate normal approximation, u verifies:

$$u \sim N\left(\mu_u, \sigma_u^2\right)$$

with $\mu_u = X'\mu - \bar{S} - \tau_{t+1}^A$ and $\sigma_u^2 = X'\Sigma X$. The objective function of the bank writes:

$$W^{G_{t+1}^{A_m}}(s_t) = \mu_u \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \sigma_u \phi\left(\frac{\mu_u}{\sigma_u}\right)$$

Banks expected utility increases in a convex fashion with the mean of the pre-default payoff mean, and it increases with its variance: this risk-shifting motive arises from the limited liability friction.

Program of middle-aged risk-averse investors Due to the recourse feature of the debt contract, the investor seizes the entire balance sheet of the bank when the bank is in default. Such banks can belong to generation G_{t+1} (within generation) or generation G_{t+2} (cross-generation). Denote by $\bar{S}^{G_{t+2}}$ the sum of promises by banks of generation G_{t+2}, it is seen as an out-of-the-generation risky payoff. The investment in within-generation short-term promise $x^{G^P_{t+1}}_{\bar{s}^{G^A_{t+1}}_{t+1}}$ is captured by $\bar{S}$.

Denote the investor's portfolio choice X^{P_m}:

$$X^{P'_m} = \left[\begin{array}{ccc} i_t^{G^P_{t+1}} & x^{G^P_{t+1}}_{\bar{s}^{G_{t+2}}_{t+2}} & x^{G^P_{t+1}}_{\bar{s}^{G_{t+2}}_{t+1}} \end{array} \right]$$

The investor pre any-default portfolio realization v_{t+1} is:

$$v_{t+1} = X^{P'_m} S_{t+1} + \bar{S} - \tau^P_{t+1}$$

The program of the investor then writes:

$$\begin{aligned} \underset{\{X^P,\bar{S}\}}{Max} \; W^{G^{P_m}_{t+1}}\left(n_t^{G^P_{t+1}}(s_t), s_t\right) &= -\mathbb{E}_t\left[e^{-\gamma^P(u+v)}\, 1_{\{u<0\}} + e^{-\gamma^P v}\, 1_{\{u\geq 0\}}\right] \qquad (1.9) \\ s.t. \qquad X^{P'_m} P + \bar{S}\frac{q_{\bar{s}^{G^A_{t+1}}_{t+1}}}{\bar{s}} &\leq n_t^{G^P_{t+1}}(s_t) \end{aligned}$$

In the multivariate normal approximation, this objective function can be written:

$$\begin{aligned} & W^{G^{P_m}_{t+1}}(s_t) = -e^{-\gamma\mu_v + \frac{1}{2}\gamma^2\sigma_v^2} \\ & \quad \left\{ e^{-\gamma\mu_u + \frac{1}{2}\gamma^2\left(\sigma_u^2 + 2\rho_{uv}\sigma_v\sigma_u\right)} \left\{ 1 - \Phi\left(\frac{\mu_u}{\sigma_u} - \gamma\left(\rho_{uv}\sigma_v + \sigma_u\right)\right)\right\} + \Phi\left(\frac{\mu_u}{\sigma_u} - \gamma\rho_{uv}\sigma_v\right)\right\} \qquad (1.10) \end{aligned}$$

1.3.2 Young agents portfolio and maturity choices

We now turn to generation G_{t+2}. Agents optimize twice over the course of their life: when young and when middle-aged. When young, they face a meaningful maturity choice. The analysis in section 3.1. enables to derive the indirect utilities of the two middle-aged agents as a function of history s^t: $V^{G^{Am}_{t+1}}\left(n_t^{G^A_{t+1}}(s_t), s_t; y^{primary}_{\bar{s}^{G^A_{t+2}}_{t+2}}(s_{t-1})\right)$ and $V^{G^{Pm}_{t+1}}\left(n_t^{G^P_{t+1}}(s_t), s_t\right)$.[28] These give the marginal values of wealth for each agent of the generation G_{t+1} at t in history s^t. But when young, agents optimize their expected utility over payoff *two* periods ahead, at $t+2$.[29] The middle-aged optimization is not a sideshow as, at period t, the market clearings *jointly* involve the portfolio choices of the two generations.

Program of young risk-neutral agents Denote $\bar{S}^{ST}$ the sum of all short-term promises, $\bar{S}^{LT}$ the sum of all long-term promises:

$$\bar{S}^{ST} = \int y_{\bar{s}^{G^A_{t+2}}_{t+1}} \bar{s} d\bar{s}$$

$$\bar{S}^{LT} = \int y^{primary}_{\bar{s}^{G^A_{t+2}}_{t+2}} \bar{s} d\bar{s}$$

The only assets they can invest in are the technology and the short-term promises of the middle-aged G_{t+1} agents. Its portfolio choice is then $X^{A'_y}$:

$$X^{A'_y} = \begin{bmatrix} i_t^{G^A_{t+2}} & x^{G^A_{t+2}}_{\bar{s}^{G_{t+1}}_{t+2}} \end{bmatrix}$$

The portfolio choice $X^{A'_y}$ and the short-term promises only impact the value function through the interim net worth of the bank at $t+1$:

$$n_{t+1}^{G^A_{t+2}}(s_{t+1}) = X^{A'_y} S_{t+1} - \bar{S}^{ST} \tag{1.11}$$

Whereas the long-term promises $\bar{S}^{LT}$ only impact the value function through the long-run payoff u_{t+2}.

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[28] The problem is not stationary in n^B and n^B_{mid}: the first is exogenous, the second is state-contingent (s_{t-1}, s_t) by bloc-recursivity.

[29] Allowing for consumption even when young and middle-aged beyond when old is innocuous. It then suffices to collapse the time and state dimensions in one same dimension. With CARA preferences, intertemporal elasticity of substitution and risk-aversion are equal, hence result about state-smoothing generalizes to consumption-smoothing. Epstein-Zin relaxation is left for further research.

The program of the bank then writes:

$$\begin{aligned} \underset{\left\{X^A,\bar{S}^{ST},\bar{S}^{LT}\right\}}{Max} \; W^{G^{A_y}_{t+2}}\left(s_t\right) &= \mathbb{E}_t\left[V^{G^{A_m}_{t+2}}\left(n^{G^A_{t+2}}_{t+1}(s_{t+1}), s_{t+1}; y^{primary}_{\bar{s}^{G^A_{t+2}}_{t+2}}(s_t)\right)\right] \qquad (1.12) \\ s.t. \qquad & X^{A_y'}P \leq n^A + \bar{S}^{ST}\frac{q_t^{\bar{s}^{G^A_{t+2}}_{t+1}}}{\bar{s}} + \bar{S}^{LT}\frac{q_t^{\bar{s}^{G^A_{t+2}}_{t+2}}}{\bar{s}} \end{aligned}$$

Program of young risk-averse investors The only assets they can invest in outside the generation also are the technology and the short-term promises of the middle-aged G_{t+1} agents. Its portfolio choice $X^{P_y'}$:

$$X^{P_y'} = \left[\begin{array}{cc} i_t^{G^P_{t+2}} & x^{G^P_{t+2}}_{\bar{s}^{G_{t+1}}_{t+2}} \end{array}\right]$$

The out-generation portfolio choice $X^{P_y'}$ and the long positions in *both* the short-term and long-term promises only impact the value function through the interim net worth of the investor at $t+1$:[30]

$$n^{G^P_{t+2}}_{t+1}(s_{t+1}) = X^{P_m'}S_{t+1} + Min\left(\bar{S}^{ST}, n^{G^A_{t+2}}_{t+1}(s_{t+1})\right) + \bar{S}^{LT}\frac{q_{t+1}^{\bar{s}^{G^A_{t+2}}_{t+2}}}{\bar{s}} \qquad (1.13)$$

The program of the investor then writes:

$$\begin{aligned} \underset{\left\{X^P,\bar{S}^{ST},\bar{S}^{LT}\right\}}{Max} \; W^{G^{P_y}_{t+2}}\left(s_t\right) &= \mathbb{E}_t\left[V^{G^{P_m}_{t+2}}\left(n^{G^P_{t+2}}_{t+1}(s_{t+1}), s_{t+1}\right)\right] \qquad (1.14) \\ s.t. \qquad & X^{P_y'}P + \bar{S}^{ST}\frac{q_t^{\bar{s}^{G^A_{t+2}}_{t+1}}}{\bar{s}} + \bar{S}^{LT}\frac{q_t^{\bar{s}^{G^A_{t+2}}_{t+2}}}{\bar{s}} \leq n^P \end{aligned}$$

Comparing the programs of the young banks and the young investors, we observe that the choice variables $\bar{S}^{ST}$ and $\bar{S}^{LT}$ are not redundant: even if they are part of the same debt raising at t, given that their payoff happen at different times (t and $t+1$) and that agents have heterogeneous marginal values of wealth, the maturity choice is well defined.

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[30] Due to the presence of Walrasian markets, the investors sell all their long-term promises before reinvesting at $t+1$ (mark-to-market).

1.3.3 Existence of a contract equilibrium

I now prove the existence of a recursive (i.e. time-homogeneous) Markov equilibrium. Such stationary equilibrium satisfies the following properties: ergodicity, conditional spotlessness, and compatibility with arbitrary initial conditions.

Lemma 1. *There exists a recursive Markov equilibrium for any given financial policy* $(\hat{B}^h, \tau^A, \tau^P)$.

Proof. The proof is a direct application of Duffie et al. (1994). $\square$

Despite the existence of the equilibrium, the above lemma does not ensure a non-degenerate equilibrium in which assets that pay no dividends have non-zero value. As a matter of fact, Duffie et al. (1994) notes that *"we do not know whether coexistence of with- and without-dividend assets is possible in a stochastic economy without population growth, either with or without ergodicity"*.[31] The following proves that there exists an equilibrium with coexistence of risky and safe assets in the stochastic economy. In such equilibrium risk-averse agents lend to risk-neutral agents and in which long-term debt has non-zero value.

To show existence of such contract equilibrium, I proceed in two steps. First I take the portfolio and maturity choices of young as given, and solve for the equilibrium leverage (risk-sharing agreement) within the middle-aged generation. Second, I use this portfolio choice to compute middle-aged value functions, before fully solving for the young portfolio and maturity choice.

Benchmark case: no long-term debt

I start by solving for the equilibrium when the technology is the only potential investment: the supply B of next generation promises (as well as public debt) is set to zero. This is the case if there is no maturity choice. The following lemma shows that the optimal contract is risky debt.

---

[31] Scheinkman (1980) shows there does not exist such equilibrium in deterministic economies. Spear and Srivastava (1986) and Spear et al. (1990) entirely characterize the structure of equilibrium in stochastic OLG models.

Lemma 2. *When there is no outside asset in the economy beyond the technology, the risk-averse investors lend all their wealth to the risk-neutral banks through one financial contract: risky debt.*

The proof in Appendix B.1 makes use of the first theorem of welfare. The problem can be broken on one hand on optimal level of aggregate investment and on the other hand on the optimal risk-sharing between risk-neutral and risk-averse agents. For aggregate investment, the resource constraint pins it down, as, in this benchmark, there are no other assets to invest in:

$$p_K\left(i_t^A + i_t^P\right) \leq n^A + n^P$$

As for risk-sharing, the constrained efficient allocations are such that risk-averse agents enjoy a constant consumption as long as the technology shock realizes above this consumption level. When the shock realizes below this threshold, the risk-averse agents consume all the $t = 1$ wealth of the economy. Any of such allocations is implemented by a risky debt contract, with face value the desired constant consumption level.

Equilibrium within the middle-aged generation

I first characterize the solution of the two above program and the market clearings taking the next generation issuance quantities as given. The latter act as an out-of-generation supply of assets, from generation G_{t+1} perspective. I am interested in the equilibrium leverage (i.e. within-generation risk-sharing), and how it varies with the supply of negative beta assets (i.e. the supply of long-term promises by generation G_{t+2}). There is a safety multiplier when the two quantities covary positively.

Introducing out-of-the generation long-term debt, taken for now as an exogenous fixed supply B,[32] breaks the proof of Lemma 1. Indeed, the ex ante investment depends now on the endogenous price $q_{B,t}$ of long-term debt. Indeed, the resource constraint of the economy is now:

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[32] In the notation of the full-model, we have: $B = y^{primary}_{\bar{s}_{t+2}^{G_{t+2}^A}}$ and $q_t^B = q_t^{\bar{s}_{t+2}^{G_{t+2}^A}}$.

$$p_K\left(i_t^A+i_t^P\right)+q_{B,t}B\leq n^A+n^P$$

Thus the endogenous total value of the public safe asset $(q_{B,t}B)$ crowds out private investment in the technology. If there is a **Laffer** effect, i.e. if the value $(q_{B,t}B)$ decreases with respect to the supply B, then issuing more long-term debt B actually crowds in investment.

The within-generation equilibrium is defined by 8 endogenous variables: for each of the two agents, their investment in the technology i_t^θ, their investment in the long-term debt $x_{B,t}^\theta$ and their position in the intrageneration risk-sharing contract $\bar{S}_t^\theta$, as well as the endogenous price for long term-debt $q_{B,t}$ and the endogenous price of the risk-sharing contract $q_{\bar{s},t}$. And the equilibrium is characterized by the 8 independent equations: for each of the two agents, one portfolio choice and one leverage choice, as well as two budget constraints and the two market clearings for long-term debt and for the risk-sharing contract. Appendix B.2 solves for the equilibrium fully closed-form.

As banks are risk-neutral, the equilibrium price $q_{\bar{s},t}$ of the risk-sharing contract (bank debt) does not directly depend on the quantity traded in this contract $\bar{S}_t$. It is illustrated in Figure 1.7, where the slope of bank supply curve is constant and equal to this equilibrium price $q_{\bar{s},t}$. The rate on bank debt is given by:

$$r^{bank}=\frac{\bar{s}}{q_{\bar{s}}}=\frac{\mu_B}{q_B}\frac{1-\frac{\mu_K}{\mu_B}\frac{\sigma_B}{\sigma_K}X\left(i_t^A,x_{B,t}^A;\rho\right)}{1-\frac{p_K}{q_B}\frac{\sigma_B}{\sigma_K}X\left(i_t^A,x_{B,t}^A;\rho\right)} \tag{1.15}$$

where X is an endogenous measure of bank balance sheet correlation: $X\left(i_t^A,x_{B,t}^A;\rho\right)=\frac{\rho+\frac{\sigma_B x_{B,t}^A}{\sigma_K i_t^A}}{1+\rho\frac{\sigma_B x_{B,t}^A}{\sigma_K i_t^A}}$. This metrics increases with asset correlation ρ if and only if $\frac{\sigma_B x_{B,t}^A}{\sigma_K i_t^A}<1$. In this case, X increases with ρ from -1 to 1. The ratio $\frac{\sigma_B x_{B,t}^A}{\sigma_K i_t^A}$ controls the concavity: when it tends to zero, the mapping $X\left(\rho\right)$ is linear.

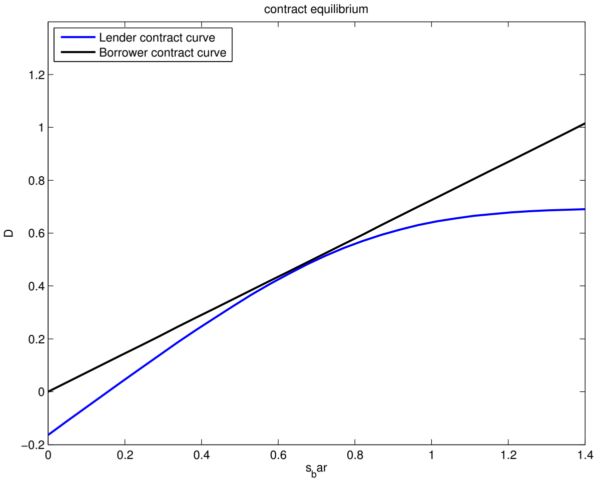

Figure 1.7: *Equilibrium on the market for bank debt: leverage D w.r.t. promise $\bar{s}$.*

Appendix B.2 characterizes the equilibrium in terms of only two endogenous variables (μ_u, σ_u): the mean and the volatility of the pre-default bank payoff. The solution strategy is as follows. The bank budget constraint and optimality conditions form a quadratic system in bank asset holdings $(i_t^A, x_{B,t}^A)$. Solving this system delivers (x_K^A, x_B^A) as non-linear functions of (μ_u, σ_u). The definition of σ_u then delivers a functional $F_{MVF}(\mu_u, \sigma_u) = 0$: a bank mean-variance frontier. In parallel, the equilibrium on the debt market delivers a debt-pricing functional $F_{debt}(\mu_u, \sigma_u) = 0$.

From bank's perspective, its risk-shifting motive deters from holding any negative beta assets. However there is a countervailing force: holding negative beta asset makes its balance sheet less risky, which relaxes its endogenous collateral constraint, hence enabling to lever more. Consider banks portfolio choice condition:

$$p_K \sigma_B \left(\sigma_B x_{B,t}^A + \rho \sigma_K i_t^A \right) - q_B \sigma_K \left(\sigma_K i_t^A + \rho \sigma_B x_{B,t}^A \right) = \left(\frac{\mu_K}{p_K} - \frac{\mu_B}{q_B} \right) p_K q_B \sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)}$$

As long as the endogenous the long-term debt rate $r_{safe} = \frac{\mu_B}{p_{B,t}}$ is lower than the risky rate $r_K = \frac{\mu_K}{p_K}$, the right-hand side is positive. It implies that $\frac{\sigma_B x_B^A}{\sigma_K x_K^A} < 1$ can be satisfied only if

$\pi > 1$ where $\pi = \frac{p_K \sigma_B}{p_B \sigma_K}$. And in that case, the portfolio condition imposes:

$$1 > \frac{\sigma_B x^A_{B,t}}{\sigma_K i^A_t} > \frac{1 - \rho\pi}{\pi - \rho}$$

The right hand side decreases with ρ: a low correlation restricts more the portfolio choice due to the risk-shifting motive. Having $\pi = \frac{c_K \sigma_2}{p_B \sigma_1} > 1$ does not prevent $\sigma_B < \sigma_K$, as long as assets expectations are chosen such that $r_K > r_{safe}$ but $p_K > q_B$.

Assumption 1. *I make the following PE parameter restriction:*

$$\frac{\sigma_B}{\sigma_K} > \frac{q_B}{p_K} > \frac{\mu_B}{\mu_K}$$

It puts a range on the safe asset price q_B, which translates on bounds on the supply of public safe asset B in general equilibrium.

Lemma 3. *Partial Equilibrium existence*

There exists a contract equilibrium if and only if the safe asset volatility σ_B verifies Assumption 1. In this equilibrium, risky debt is the optimal contract and is the only traded financial contract.

Leverage then is determinate: Modigliani-Miller fails without resorting to any agency frictions. Despite complete markets, the limited liability frictions shapes the optimal contract to be risky debt. Hence equilibrium features limited risk-sharing and equilibrium default.[33]

The volatility of long-term debt must be high enough for the contract equilibrium to exist. If $\sigma_B = 0$ (i.e. a riskless storage technology such as money), passive agents all fly to money, and do not find attractive to lend to banks. In the full equilibrium, long-term debt volatility comes from interest rate risk.[34]

The mean-variance frontier of the bank $F_{MVF}(\mu_u, \sigma_u) = 0$ is non-degenerate despite banks being risk-neutral. The MVF implicit mapping $\sigma_u \overset{MVF}{\mapsto} \mu_u$ is concave, whereas the *debt* implicit mapping $\sigma_u \overset{debt}{\mapsto} \mu_u$ (from $F_{debt}(\mu_u, \sigma_u) = 0$) is an increasing first-order linear

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[33] Default happens in the low aggregate states, and not in high-income states, a counterfactual feature of Alvarez and Jermann (2000).

[34] Equilibrium is then unique: there are *not* two equilibria, one with cheap debt, high leverage and good diversification, and another one with expensive debt, low leverage and poor diversification.

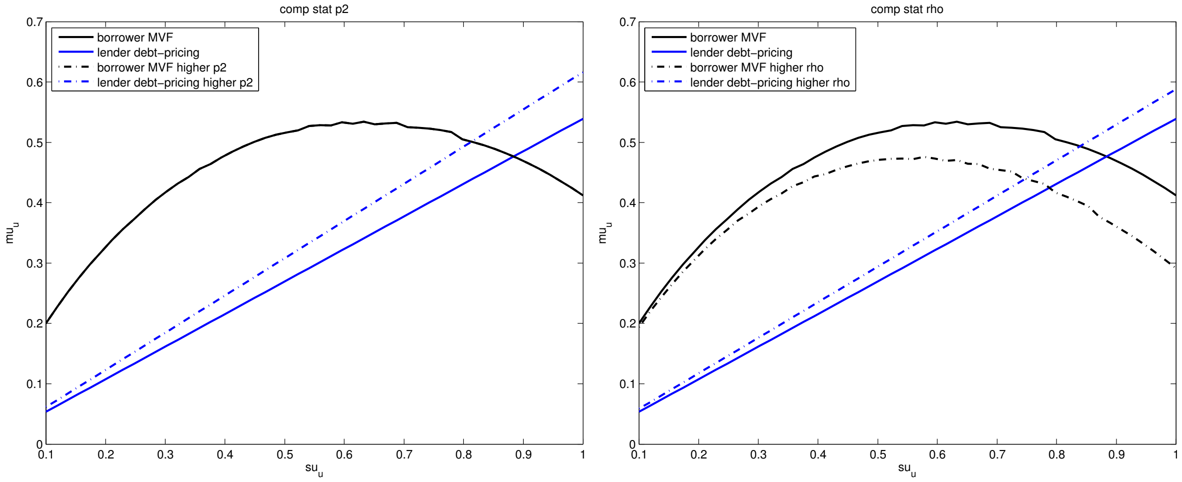

Figure 1.8: *Long-term debt as a hedge: impact of its price and its correlation on bank leverage.*

function. Endogenous default makes the banks effectively risk-averse. The equilibrium variables have the following comparative statics with respect to correlation and price of long-term debt:

$$\mu_u = f\left(\underset{(-)}{\rho}\ ,\ \underset{(+)}{q_B}\right) \text{ and } \sigma_u = g\left(\underset{(-)}{\rho}\ ,\ \underset{(-)}{q_B}\right)$$

Lemma 4. *A lower safe asset price and lower beta increases the probability default of the bank.*

That is, when the safe asset is a cheaper and better hedge, banks choose to lever up more and to take more risk. Lower ρ make bank lever up and take more risk, whereas lower q_B make bank lever up more and take less risk.

Effect of a negative correlation ρ Banks have enhanced leverage ability when ρ is low. The safe asset holdings of banks are thus pinned down by the trade-off between the traditional risk-shifting motive (dislikes low ρ) and the debt pricing by investors (likes low ρ). Hedging properties of public debt help the within-generation risk-sharing agreement.

The General Equilibrium endogenizes the safe asset price q_B through the market clearing:

$$x_B^A + x_B^P = B$$

Combining the market clearing condition with the two budget constraint eliminates equilibrium leverage D and recovers the resource constraint:

$$q_B B + p_K \left\{ x_K^A \left(\mu_K ; p_B \right) + x_K^P \left(\mu_K ; p_B \right) \right\} = n^A + n^P \tag{1.16}$$

Thus the safe asset price q_B depends on the equilibrium only through the level of aggregate investment $x_K^A + x_K^P$. This is the heart of the safety multiplier: more expensive long-term debt can deter investment through a GE effect that overcomes the portfolio choice. Assumption 1 and the resource constraint imply a general equilibrium parameter restriction on B for the contract equilibrium to exist:

$$\frac{\mu_K}{\mu_B} \left\{ \frac{n^A + n^P}{p_K} - \left(x_K^A + x_K^P \right) \right\} > B > \frac{\sigma_K}{\sigma_B} \left\{ \frac{n^A + n^P}{p_K} - \left(x_K^A + x_K^P \right) \right\}$$

A necessary condition is:

$$\frac{B}{n^A + n^P} < \frac{\mu_K}{\mu_B p_K}$$

The closed-form expression for safe asset price q_B enables to prove the following corollary.

Corollary 1. *Existence in General Equilibrium*

There exists a within-generation contract equilibrium if and only if the out-generation safe asset supply B is low enough with respect to aggregate wealth $n^A + n^P$.

The existence does not need any short sale constraints. Limited liability implies an endogenous collateral constraint. Only under a contract equilibrium the aggregate wealth $n^A + n^P$ and the wealth distribution n^A / n^P are priced in the safe asset q_B. A 'safe asset shortage' should qualify a situation in which long-term debt supply is very low with respect to passive wealth n^P: a savings glut of anxious wealth.

Full equilibrium characterization

I move now backward to the program of the young and the inter-generational full equilibrium at period t. The interaction between the young and the middle aged generations adds two features to the model: endogenous $t+1$ price functional for long-term debt, and endogenous long-term debt supply through the maturity choice of the young. I analyze these two equilibrium features sequentially.

Long-term debt endogenous price functional In the time-homogeneous Markov equilibrium, the price $q^{interim}_{\bar{s}^{G_{t+2}}_{t+2}}\left(s^{t+1}\right)$ of long-term debt is fully endogenous. This key feature of the model enables to derived a formula for the endogenous correlation of long-term debt with aggregate risk (its 'beta'). I solve for the fixed point in the long-term debt price functional, using a heuristic approach drawing on the 'static' pricing by the middle-aged derived in the above section 3.3.2. I still take here the supply of long-term debt as exogenous $B = x^{G_{t+2}\,primary}_{\bar{s}^{G_{t+2}}_{t+2}} + \hat{B}$ (private and public long-term debt).

By fungibility, the realized marked-to-market price at $t+1$ of long-term is the same as one of a short-term promise issued by the same risk-neutrals, the ones of generation G_{t+2}. At $t+1$, such promise can be bought by the middle-aged G_{t+2} risk-averse agents, or by the young G_{t+3} young risk-averse agents. Let first focus on the first type of buyers, the within generation risk-averse agents. In this case, the $t+1$ price on the market for this promise is given by the debt market equilibrium solved in the above section 3.3.2:

$$q^{interim}_{\bar{s}^{G_{t+2}}_{t+2}}\left(s^{t+1}\right) = q^{t+1}_{\bar{s}^{G_{t+2}}_{t+2}}\left(s^{t+1}\right) \tag{1.17}$$

So in effect we have to mappings that relate the long-term debt functional with the short-term debt functional: the one that gives the price of long-term debt at t as a function of the price of short-term debt at $t+1$ (equation 1.17), and the one that gives the price of short-term debt at t as a function of the price of short-term debt at $t+1$ (equation 1.15), which can be written formally:

$$q^{t}_{\bar{s}^{G_{t+1}}_{t+1}}\left(s^{t}\right) = g\left(\left\{q^{interim}_{\bar{s}^{G_{t+2}}_{t+2}}\left(s^{t+1}\right)\right\}_{s^{t+1}}\right) \tag{1.18}$$

The heuristic solution goes as follows. I take the bank debt price functional $q_{\bar{s}}\left(\mu_K\right)$ that solves the static model, and develop it in two orders of μ_K. I then analytically compute the multivariate second moments of this functional with respect to the underlying shock $s_t \equiv \mu_K$: its mean $\hat{\mu_B}$, its volatility $\hat{\sigma_B}$ and its correlation $\hat{\rho}$. This leads to three equations in the three unknowns $(\hat{\mu_B}, \hat{\sigma_B}, \hat{\rho})$, whose fixed point gives the second moments of the fixed point functional $q_{\bar{s}^{G_{t+2}}_{t+2}}\left(s^t\right)$. This is tantamount to working locally to make the following multivariate normal (2nd-order moments) approximation valid:

$$\left[s^t, q_{\bar{s}^{G_{t+2}}_{t+2}}\left(s^t\right)\right] \sim N([\mu_K \hat{\mu}_B], [\sigma_K \hat{\sigma}_B \hat{\rho}])$$

This heuristic approach uses the implicit characterization of the safe asset price functional from the resource constraint:

$$p_K x_K\left(q_{\bar{s}^{G_{t+2}}_{t+2}}\left(s^t\right); s^t\right) + q_{\bar{s}^{G_{t+2}}_{t+2}}\left(s^t\right) B = n^A + n^P$$

where $x_K\left(s^t\right) = x_K^A\left(s^t\right) + x_K^P\left(s^t\right)$ is aggregate investment. A second-order expansion in s^t of the equilibrium value of investment in the static model leads to an implicit expression of the endogenous beta:

$$\hat{\rho} = -1 + 4\left(s^t\right)^2\left(\left(\frac{\partial_{\mu_K} x_K}{\frac{B}{p_K} + \partial_{p_B} x_K}\right)^2 \frac{\partial_{p_B p_B} x_K}{\partial_{\mu_K} x_K} - 2\frac{\partial_{\mu_K p_B} x_K}{\frac{B}{p_K} + \partial_{p_B} x_K}\right)^2$$

Appendix B.3 shows the existence of a triplet $(\hat{\mu_B}, \hat{\sigma_B}, \hat{\rho})$ satisfying the fixed point of this equation that defines correlation, as well two additional equations from the definition of long-term debt mean and volatility: $\hat{\mu_B}$ and $\hat{\sigma_B}$. It leads to the solution in the second-order approximation for the endogenous beta of public debt in the recursive equilibrium:

$$\hat{\rho} = -1 + \left(\frac{B}{n^A}\frac{\left[\hat{\sigma}_B^2 p_K^2 + \sigma_K^2 \hat{q}_B^2\right]^2}{2\sigma_K^4 p_K^2 \hat{q}_B^2}\right)^2 \tag{1.19}$$

This expression for the endogenous beta of public debt is interesting in many respects. First, the beta is indeed negative for low levels of B. In the recursive equilibrium, the flight to safety enjoyed by public debt *endogenously endows* this security with an hedging property. It is the expectation of a flight to safety in the low states of tomorrow that endows public debt with negative beta. Ex ante, this enables (within-generation) safety creation. In the canonical Samuelsonian treatment, money is valued today if people expect it to have value tomorrow. This is a deterministic argument. In contrast, public debt has value in my environment because of its endogenous hedging properties.

This flight to safety is amplified by the safety multiplier. A higher level of bank net worth n^A commands a stronger safety multiplier effect. In this regime of high n^A, in the states s^t of low technology productivity, not only the bank does rebalance aggressively away from technology towards the public debt and at the same time delevers. Private safe assets supply then dwindles and investors also rebalance towards the public debt. Thus when n^A is high, the two portfolio rebalance compounds towards a flight to the public debt. Beta of public debt thus decreases with bank net worth.

Second, the following lemma characterizes the dependence of the negative beta to the supply of public debt. This result is key for the normative analysis.

Lemma 5. *In the stationary Markov equilibrium, the beta of public debt increases with the supply of long-term debt B.*

A scarce supply of public debt makes the flight to safety it enjoys more aggressive. Subsequently, the hedging properties of public debt are enhanced by its scarcity. The candidate heuristic equilibrium derived above is shown to be an equilibrium, using this property of public debt beta. It leads to the dynamic counterpart of the static contract equilibrium.

Lemma 6. *A stationary Markov equilibrium in which risk-averse agents lend to risk-neutral agents (contract equilibrium) exists only if B is low enough with respect to aggregate wealth.*

Proof. The sketch goes as follows. A low enough B creates imply a highly negative beta

ρ^{endo} through the flight to safety. It also implies volatility on the safe asset $\hat{\sigma}_B$. We then appeal to Lemma 3. $\square$

A corollary of this lemma is that, in the dynamic case, the comovement of private debt supply with public debt supply is ambiguous. On the one hand, increasing public debt supply triggers the safety multiplier mechanism described in the static model, and this creates a positive comovement force. On the other hand, the increase of public debt supply also destroys its hedging properties. The latter leads banks to choose a lower equilibrium leverage, thus a lower endogenous supply of private debt. This trade-off is characterized below, in the context of the normative analysis.

Private maturity choice: endogenous supply of long-term debt The last element of the environment to endogeneize is B: the supply of long-term debt. This is carried out by considering the maturity choice at t of the young generation. The following lemma shows that when facing their maturity choice, risk-neutrals agents (banks) choose more short term debt than long term debt. The inefficiency ('too much' short term) is only shown in Section 4.

Lemma 7. *Banks face a meaningful maturity choice: both short-term debt and long-term debt are issued.*

The basic intuition goes as follows. The banks of G_{t+2} will issue the two types of securities at t, as there always is an endogenous price for the two. However, the two securities cater two different types of lenders. Short-term debt is sold within the generation, to cater to the risk-averse of this generation. Long-term debt is sold to the other generation active in trading, the middle-aged one G_{t+1}, *as an outside-generation* hedging asset. Young investors and middle-aged agents do not have the same one-period ahead risk-sharing needs, therefore the two contracts are not redundant.[35] As a middle-aged bank, a negative beta asset is of particular interest, as the only out-of-the-generation security is short-term debt, which is entirely bought up by the young risk-averse investors (they outbid the middle-aged bank).

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[35] They do not have the same payoff profile: short-term debt has $Min\left(\bar{S}, X'S(s_{t+1})\right)$, whereas long-term debt has $q^{interim}_{\bar{s}^{G_{t+2}}_{t+2}}\left(s^{t+1}\right)$.

The maturity choice is driven in the current environment by the design of two different risk-sharing contracts and the catering to two distinct populations of lenders. It is a different mechanism from He and Xiong (2011) and Diamond and He (2013), in which the maturity choice is driven by the non-stationarity of the exogenous shock. In He and Xiong (2011) long-term debt then is always dominated by either short-term debt or cash hoarding. If optimists are very optimistic, they use short-term debt because leverage incentives overwhelm rollover risk. If optimists are not that optimistic, they prefer to hoard cash in order to wait for a degradation of the state. This behavior strongly hinges on a *mean-reversion* assumption, engineered through the beliefs structure. On the contrary, my environment features persistent shocks. In this case, the cash hoarding strategy is always dominated by leverage, and both short-term and long-term debt are issued.

1.3.4 Results

The safety multiplier

The model explains why risk-neutral banks would ever hold negative beta assets: they have an *endogenous collateral value*, which depends on their correlation with the rest of bank's balance sheet. Long-term debt holdings on bank balance sheet makes bank short-term debt less risky through their hedging property. It decreases the equilibrium default threshold, and this is efficient given the risk aversion of investors. By bundling long-term safe assets with risky assets, banks are able to create more private short-term safe assets. Doing so, it satisfies the risk-averse demand for safety. The first comparative statics captures the macroeconomic puzzle highlighted on Figure 1.4: a positive comovement of long-term (public) safe assets B and private short-term safe assets D.

Proposition 1. *Complementarity between private safe assets and public safe assets*

When the safe asset supply B is low enough, the supply of private short-term debt D comoves with the supply of long-term debt: $\exists B^* | \forall B < B^*,\ \frac{\partial D}{\partial B} > 0$.

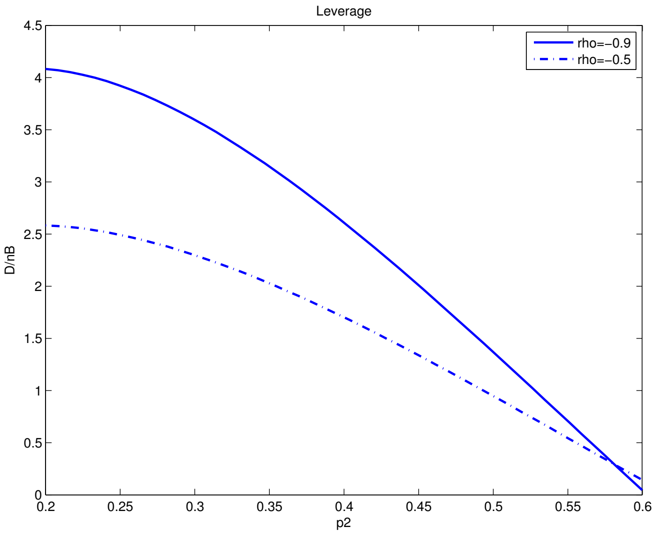

Figure 1.9: *The safety multiplier: positive comovement of public and private debt, in partial equilibrium.*

There is a safety multiplier when public debt supply B is low enough. In that case, a shortage of public debt leads banks to delever, because the input 'government debt' is too expensive for the safety production function of banks. Proposition 1 shows there is *crowding-in* of private safety by public safety.

The intuition goes as follows. Banks leverage decision trades off the benefit of leverage with its cost. The latter is determined by the lender's outside option, which itself depends on the price of the safe asset. When the latter is high, the lender prefers to lend to the bank. This is a crowding-out effect: lower supply of public debt calls for higher supply of private debt. However, when B is low enough, this effect is overturned by a crowding-in effect. From banks perspective, an expensive public debt input makes them scale down safety production, i.e. less leverage. They decide to lever less as soon as the increase in input price (public debt) swamps the increase in output price (private debt).

The effect can be seen in partial equilibrium by decomposing leverage, where $\bar{S}$ is banks total short-term promises:

$$D\left(q_B\right) \quad = \quad \underset{(+)}{\frac{1}{r_{bank}}} * \underset{(-)}{\bar{S}}$$

A more expensive public debt induces banks to diversify less. Bank debt is then made riskier, which makes it more expensive from banks view. In turn, it leads banks to issue less promises $\bar{S}$. If the latter endogenous response is strong enough, the combination of the two effects leads to less equilibrium $t = 0$ leverage D. Finally, the safety multiplier can be seen coming from the role of volatility dampener of banks. The ratio of volatilities $\sigma\left(bank\,debt\right)/\sigma_B$ is less than one. However, when public debt is expensive, this ratio increases. Banks are hindered in their volatility transformation function.

Effect of risk aversion heterogeneity This safety multiplier mechanism is stronger for a high degree risk aversion heterogeneity.

Corollary 2. *Higher investors risk-aversion leads to higher equilibrium leverage and a safety multiplier for a larger set of the parameter B:* $B^* = f\left(\underset{(+)}{\gamma^P}\right)$.

The first part is counter-intuitive, as it seems to say that risk-averse agents invest in a bank debt that is riskier when they are more risk averse. The reason is that the risk-aversion parameter γ^P captures the *differential* of risk attitudes among agents. The optimal risk-sharing agreement features a larger flat part when this differential is higher.

The second part of the corollary comes from the fact that, given that equilibrium leverage is high under high risk aversion, the economy is then more responsive to the safety multiplier mechanism. The scarcity of public safe assets activates more the crowding-in than the crowding-out forces.

Effect of wealth distribution $\left(n^A, n^P\right)$ The safety multiplier is also stronger when banks are badly capitalized: n^A/n^P low. This comes from the fact that leverage is slightly *procyclical* in the present model: $\frac{\partial\left(D/n^A\right)}{\partial n^A} > 0$. The first order is linear but second order terms of D are convex in n^A. In economic terms, the bank caters even more the safety demand of risk-averse investors when they relatively less capitalized. Similarly, the cutoff B^* broadens when passive wealth n^P os abundant: $B^* = f\left(\underset{(+)}{n^P}\right)$. The safety multiplier mechanism is stronger when there is an *anxious-savings glut.*

Real economy implication: a safe-asset driven credit crunch

The second and third comparative statics are related to the portfolio composition of banks: real investment in the technology vs. holdings of safe assets. In this economy with endogenous leverage, banks do not risk shift in stress times. In these stress times of low supply of public safe assets, the latter are so expensive for banks that they decide to lever up *less*. Total risk-bearing capacity is hindered. The collateral damage on their asset side is an overall crunch of investment in the technology.

Proposition 2. *Non-conventional credit crunch due to a shortage of public debt*

Lowering the supply of public debt decreases aggregate investment in the risky technology: $\frac{\partial\left(x_K^A + x_K^P\right)}{\partial B} > 0$.

That a lower level of public debt in the system triggers a credit crunch is not a priori straightforward.[36] Indeed, it makes the public debt more expensive and induces the banks to rebalance their portfolio toward the other asset, the risky asset. Crowding-in arises when the need of the hedging properties of public debt for leverage purposes dominates the portfolio rebalancing force. Appendix B.3 also shows that the mapping $x_1(B)$ is increasing concave: the crunch is exacerbated when B shrinks close to 0.

Proposition 2 can be seen as a beneficial **Laffer** effect of public debt issuance. In the regime of interest, increasing public debt supply B decreases its ex ante value $p_B B$, and the resource constraint (1.16) then implies crowding-in of aggregate investment. The credit crunch has a counterpart on bank safe assets holdings.

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[36] My channel of a credit supply crunch is complementary to Caballero and Farhi (2013) safety trap, in which the recession is engineered through a New-Keynesian demand channel.

Proposition 3. *Bank safe asset holdings*

Lowering the supply of public debt increases bank holdings of safe assets: $\frac{\partial x_B}{\partial B} < 0$.

This might be the most surprising result: an increase in the price of the public safe asset leads banks to *increase* their holdings in this asset. This comes from a General Equilibrium effect. An exogenous decrease in B makes it a scarce and sought-after asset with desirable hedging properties. Proposition 3 shows that the marginal buyers for such asset actually are banks, who needs it for a double purpose: hedge their risky investment and relax their collateral constraints. Hence, in times of safe assets shortage, public safe assets are more valuable on banks balance sheets than on passive balance sheets.

Proposition 3 puts forward a *pull* theory of banks holdings of public debt: banks are asking for this public debt, as an input in their safety production function. It is alternative to the *push* theory of financial repression, in which banks are forced to hold public debt by moral suasion from the Treasury. The negative comovement of banks holdings of public safe assets with their aggregate supply is confounded by the two theories. However, the pull theory I develop also predicts a positive comovement of banks holdings of public debt with banks leverage, whereas the push theory of financial repression predicts the contrary. Stylized facts documented in section 6 provide support to the former. In basic supply-demand framework, the fact that both price q_B and quantity x_B^A increased in the safe-asset credit crunch shows that the demand curve did move up.[37]

Finally, the ratio $\frac{x_B^A}{D}$ captures the bank role in the safety multiplier. Contrary to conventional wisdom about the Liquidity Coverage Ratio, banks should not see holdings of safe assets as a constraint, but as an economic force that constitutes an integral input in their macroeconomic role of safety producer.

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[37] Zhang (2013) uses households risky assets ratio to predict returns in the US. Theoretically, we can formulate a 'Jacklin critique' to the financial repression argument. With anonymous trading of public debt, the financial repression argument does not hold: optimists will always find it profitable to sell this public debt to risk-averse agents. Contrary to the Jacklin argument, it is not between patient and impatient households shortcutting the bank, but between the bank and the investor shortcutting the government. Here is a profitable deviation which is to circumvent the bank (HH lending directly to the government). So financial repression cannot explain government holdings for sovereigns with deep secondary markets. Now, a long-term contract between sovereign and private agents is sustainable. As the value of this security increases when the value of the risky asset decreases, the optimists will now find it profitable to hold to it in its portfolio, so no more profitable deviation. It is an interesting case of *contagion of commitments*: the government endogenously do not default, preserving a high price for safe debt, diminishing the default threshold of banks.

Asset pricing implications: the two safety premia

As there is not an elastic supply of riskless asset in the environment, a safe rate can only be defined in relative terms. There are two endogenous safe rates: the yield on public debt and the yield on bank debt. I define safety premia taking as reference the exogenous rate of return r_K on the risky technology. The *public safety premium* and the *private safety premium* are:

$$\Sigma_{public} = r_K - \frac{\mu_B}{p_B} \text{ and } \Sigma_{private} = r_K - \frac{\bar{S}}{D}$$

I define the Safety Mismatch Index as the spread between the two premia:

$$SMI = r_{safe} - r_{bank} = \Sigma_{private} - \Sigma_{public}$$

The SMI is the opposite of the endogenous credit spread on banks. It can also be seen as the spread between the *Liquidity Value* and the *Collateral Value* in this collateral constrained economy, using the language of Geanakoplos and Zame (2013).

Proposition 4. *The Safety Mismatch Index.*

Under Assumption 1, the Safety Mismatch carry trade decreases with public debt supply B and increases with public debt beta ρ. Furthermore:

$$r_{safe} - r_{bank} < 0 \Leftrightarrow \rho < -\frac{\sigma_B x_B^A}{\sigma_K x_K^A}$$

The model delivers a negative carry trade on public safe asset: the collateral value dominates the liquidity value. As long as the correlation of public debt with the stock market is low enough, reach for yield is stronger on public safe assets than on private safe assets, despite lower payoff volatility of the latter. This is due to the double role played by the public safe asset when held by banks: hedge the risky investment and back private debt.

The carry trade SMI depends on equilibrium balance sheet quantities only through the correlation metrics X that captures the diversification of banks balance sheets. Under

Assumption 1, X is an increasing monotic transformation of asset correlation ρ and of $\frac{\sigma_B x_B^A}{\sigma_K x_K^A}$. Leverage is high when ρ and $\frac{\sigma_B x_B^A}{\sigma_K x_K^A}$ are low, hence X is a sufficient statistics that captures high equilibrium leverage and higher default probability when low. This translates into low SMI. The latter therefore is a macroprudential market-based tool that reveals aggregate leverage. It is the default risk counterpart of LMI for liquidity risk. It can also be used to track the effect of public debt supply experiments on private bank leverage D.

Relation to bank profitability Banks expected profits increases both in μ_u and σ_u. In times of low SMI, banks' expected profits are higher.

Corollary 3. *Bank profits are higher for a higher supply of public safe assets.*

This is a direct implication of the safety multiplier mechanism. The same force, higher public safe asset price, that leads to a lower equilibrium leverage in safe asset shortage also leads to lower equilibrium bank expected profits.

1.3.5 Discussion

Limited liability required for a safety multiplier The very parsimonious friction of non-negative consumption at $t = 1$ leads to an economy featuring a safety multiplier. There is no need of any market incompleteness a la Allen and Gale (1994).[38] Agents can trade in a full set of Arrow-Debreu securities, but in equilibrium, only one contract is traded, risky debt: markets are endogenously incomplete, but are a priori complete. Without the limited liability friction, private would be riskless and public debt issuance would therefore have no traction on private debt issuance[39]. As banks are essentially doing pooling and tranching in my model (pooling public safe and risky asset, and tranching to issue the private safe asset), a natural question is to ask whether the asset pricing implications are just an application of Modigliani-Miller. This intuition is incorrect because of the limited liability friction (non-negative consumption), which breaks full insurance. Compared to the standard Arrow-Debreu economy, the Inada condition is relaxed by assuming risk neutrality on bankers.

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[38] Their chapter in Gale (1990) investigates the efficient design of public debt. Early contributions that risk-sharing could be facilitated by public debt trace back to Weiss (1979). However, all these papers do not entertain the mechanism of public debt as an input to private debt, which is at the core of the present paper.

[39] Full insurance can never be attained under the limited commitment friction as long as $\rho > -1$ and $n^B < \infty$. In that case, bank debt is never entirely riskless, even with the promise $\bar{s}$ arbitrarily close to 0. Therefore, private bank debt can never dominate public debt.

Compared to Holmstrom and Tirole (1998), the CARA-normal environment discards the need of a 3-period timing with a liquidity shock at $t = 1$. In their environment, public debt is purely a store of value. Therefore, with an exogenous liquidity shock, it is intuitive that 'public debt' (i.e. cash) should be held by active wealth (entrepreneurs/banks). What my model shows is that when public debt is at tension between two needs: production insurance *and* safety consumption, there exists a *pecking order* of public debt ownership: first bank-entrepreneurs should hold it on their balance sheet, before being held directly in passive hands. By holding public debt on their balance sheets, bank-entrepreneurs fulfill two roles: they are able to insure their technology (the macro shock) and at the same time their private debt then synthetically provides safety and therefore can act as an (imperfect) substitute to public safety. Counter-intuitively, this arrangement strictly dominates having the public debt owned directly by passive hands.

Finally, I differ from Diamond and Dybvig (1983) by focusing on aggregate shocks and not idiosyncratic liquidity shocks. In the latter, banks and depositors enter an optimal contract. Intermediaries both have a liquidity pooling and liquidity insurance role. But the intertemporal liquidity insurance role is not robust to asset spot markets (Jacklin critique, as formulated in Farhi et al. (2009)): as long as there is a spot market for the long-term asset, depositors prefer to invest directly in the risky long-term technology than entering the deposit contract (bank 'long-term' debt). It completely unravels the role of intermediaries in liquidity provision: financial intermediaries would not exist, and all the assets, including the risky long-term ones, would be in the hands of households. In my environment of *safety provision* and not liquidity provision, the spot market does not unravel the role of intermediaries: it is robust to the Jacklin critique.

Why a safety multiplier in Europe and not in the US: limited participation The model is solved with full participation of all agents in all markets. Risk-averse agents can carry out some diversification themselves, by directly bundling in their own hands public debt and the risky technology. However, as long as the public debt beta $\rho > -1$, they cannot perfectly hedge the macro shock through their own portfolio choice of the technology and public debt. As a consequence the flat part of bank debt still has its appeal. The three assets: public debt, private bank debt and the technology are jointly held by risk-averse investors. Bank leverage is therefore determinate.

In the case of limited participation, i.e. when the risk-averse agents are prevented from investing directly in the risky technology, the safety multiplier mechanism is strenghtened. Having the risk-averse investors doing directly some diversification dampens the safety multiplier. The cutoff B^* in Proposition 1 is determined by the tension between two forces: lender portfolio choice which tilts towards crowding-out, and debt safety creation which tilts towards crowding-in. Relaxing the limited participation constraint strengthens the portfolio choice force, as now, the synthetic asset {technology+public debt} can exist and is a better substitute to {private debt} than {public debt} alone. The debt safety creation motive, which is entirely driven by bank portfolio and leverage choice, is not affected by limited participation. As a consequence the cutoff with direct access of passive wealth to the technology is lower than the cutoff in the limited participation environment, i.e. there is a larger parameter region with a safety multiplier under limited participation than under full participation:

$$B^{*\,full\ participation} < B^{*\,limited\ participation}$$

This comparative statics helps rationalize why Europe behaves differently than the US, i.e. why there is empirically a safety multiplier in Europe and not in the US. As a consequence, it reconciles my empirical stylized fact of positive comovement of private debt with public debt in Europe, whereas Krishnamurthy and Vissing-Jorgensen (2013a) shows that private debt and public debt negatively comove in the United States. I argue that this can be explained by applying my limited participation environment to Europe and applying the full participation environment just described to US. Indeed, it is extremely well documented that disintermediated instruments such as Private Equity and Venture Capital are much more developed in the US than in Europe. As a consequence, Europe is much more of the limited participation environment, and I just showed that in this environment there is a safety multiplier for a larger parameter region: crowding-in of private debt by public debt. The higher equilibrium leverage in that case is consistent with the pervasive role played by European banks in the financing of the real economy: 80% of the financing is intermediated by banks and not the corporate bond market (instead of 20% in the US).

Moreover, the Appendix derives:

$$B^*\left(\underset{(-)}{\rho}\,,\underset{(+)}{\sigma_K}\right)$$

The first comparative statics rationalizes the time series: public and private debt comove more when public debt is actually negative beta (it is a recent phenomenon). The second comparative statics also helps to rationalize the cross-country Europe vs. US: public debt and private debt comove more when the risky technology is riskier.

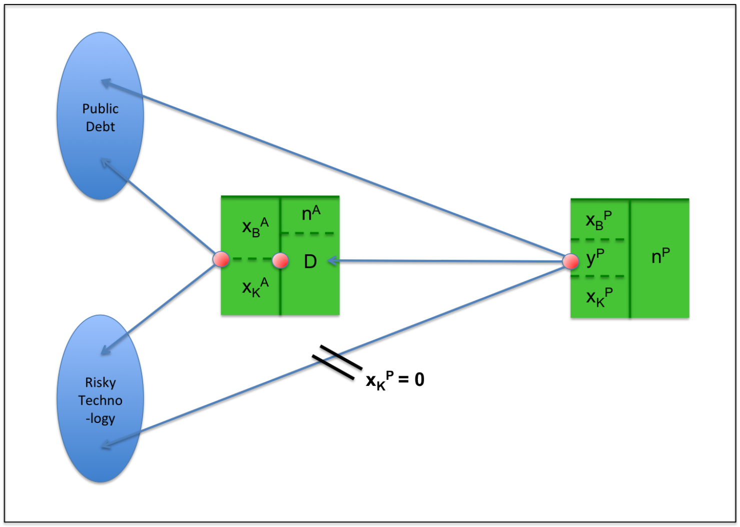

Figure 1.10: *Contract equilibrium representation under limited participation. Agents are in green and assets in blue. Red bullets indicate, from left to right: bank portfolio choice, bank leverage choice and lender portfolio choice.*

Maturity of bank debt In practice, banks create safety on their liability side at different maturities. The model endogenously endows banks with a maturity transformation role. In equilibrium they decide to issue liabilities of shorter maturity than the one of their asset holdings.

One could argue that deposits exhibit long-term liability aspects, given their stickiness. However, the overall cost of funding is weighted average of deposits costs and wholesale funding costs.[40] The marginal cost of funding is pinned down in the latter. On this wholesale funding it is clear that empirically, banks are in the business of creating short-horizon safety. Furthermore, it can be argued that a bulk of securities holdings by banks is not mark-to-market in practice. This is not an issue for the relevance of the model, as, as long as there is some short-term debt to be repaid, banks will in effect mark to market their balance sheet by getting out and rolling over their holdings. As these debt instruments are unsecured as argued in the static model, negative beta holdings have an input role on the asset side of banks.

The dynamic model predicts that risk-neutral agents hold the long-end of public debt and risk-averse agents hold the short-end. Indeed, in line with the view of a scarcity of public safe assets (i.e. their inelastic supply), the model delivers a pecking order in the ownership of public debt. Risk-neutral and risk-averse agents compete for public debt ownership on Walrasian markets. Short-term public debt (T-bills) is the dominating safe asset in the economy. The risk-averse agents value it the most. Therefore in equilibrium, T-bills are owned by risk-averse investors and they yield the lowest. What comes next on the safety ladder is long-term public debt (T-bonds). As these ones are endowed with the negative beta property, they enjoy a 'double coincidence of will' when put on the balance sheet of the risk-neutral agents. At the same time this hedges their investment in the risky technology, and, by relaxing the endogenous collateral constraint, this enables the creation of private short-term debt. Banks do reach for yield in the sense that, contrary to the risk-averse investors, they prefer T-bonds (with higher yield) on T-bills.

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[40] Figure 1.15 shows that, for the Eurozone, the deposit base is $10trn$€ and the debt from wholesale funding is $2trn$€.

Wealth effects The recent macro-finance literature analyzes the non-linearities due to financial frictions: Krishnamurthy and He (2013) and Brunnermeier and Sannikov (2010) using an agency friction, Mendoza (2010) using an exogenous collateral constraint, and Cao (2013) using beliefs disagreements. Adrian and Boyarchenko (2012) is the only model solved closed-form, using a VaR friction. All these papers feature the wealth distribution as states variables. Thus they are all after the interaction of wealth effects with the financial constraint. On the contrary, my environment analyzes how safe asset shortage interacts with limited liability constraints. Their interaction jointly explains the negative beta of public debt and high bank leverage. Amplifying mechanisms arising with wealth effects do not make any normative case as they are constrained efficient environments (no market failure), whereas my environment does make a normative point, as fleshed out in the next section.

Even if wealth effects are not a key ingredient of the interesting dynamics, the endogenous beta formula 1.19 shows that the flight to safety is more aggressive when n^B increases faster than n^L, which implies lower beta and higher bank leverage. Thus, when keeping track of wealth effects, the dynamic model would be able to generate leverage cycles at business cycle frequencies.

1.4 Optimal supply of public debt

In this normative analysis, I explore if issuing public debt can lead to a Pareto improvement compared to the decentralized equilibrium without public debt. In order to endow public debt with a welfare role, the competitive equilibrium needs to be shown constrained inefficient.[41]

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[41] i.e. whether the planner does better than the decentralized equilibrium, using the same instruments as the market.

1.4.1 Constrained inefficiency

Constrained efficient allocations As the environment features only two types of agents (risk-neutral A and risk-averse P), constrained efficient allocations trace a Pareto frontier in the space of the indirect utilities $\left(V^{G^{Ay}_{t+2}}, V^{G^{Py}_{t+2}}\right)$. Under constrained efficiency, the planner directly chooses consumption allocations. In the spirit of Rawls and Ball and Mankiw (2007), I consider a Social Planner that treats all the future generations, as of period $t = 0$, under the veil of ignorance. I also assume it weights all generations uniformly. However, the within-generation heterogeneity is known to the planner, and let denote β^A the weight on risk-neutral agents and β^P the weight on risk-averse agents. Therefore the Social Planner chooses the state-contingent history of consumptions and history of aggregate investment to maximize the following welfare function under the resource constraints at each history s^t and the non-negativity of consumption and investment:[42]

$$\begin{array}{rl}
\underset{\left\{\{c^\theta(s_t)\}_{s^t},\{i(s_t)\}_{s^t}\right\}}{Max} & \sum_{t\geq 0}\left(\beta^A\mathbb{E}_0\left[c^{G^A_{t+2}}(s_t,s_{t+1},s_{t+2})\right]+\beta^P\mathbb{E}_0\left[-e^{-\gamma^P c^{G^P_{t+2}}(s_t,s_{t+1},s_{t+2})}\right]\right)\\
s.t. \quad (\lambda_{t+2}) & c^{G^A_{t+2}}+c^{G^P_{t+2}}+i_{t+2}\leq n^A+n^A+\dfrac{s_{t+2}}{p_K}i_{t+1}\\
\left(\mu^\theta_{t+2}\right) & 0\leq c^{G^\theta_{t+2}}\\
\left(\nu^\theta_{t+2}\right) & 0\leq i_{t+2}
\end{array}$$

Appendix proves the following lemma.

Lemma 8. *Constrained efficient allocations are characterized by efficient risk-sharing within generation and a level of aggregate investment* $i^*_t = a\ s_t$ *where the optimal rule a is defined.*

The Pareto frontier is then traced by deriving the indirect utilities, as of date 0 of the two agents of the first generation G_2. The ratio β^P/β^A controls the risk aversion of the Social Planner. As long as it is not zero the planner has some willingness to redistribute wealth across states.

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[42] Recall that agents only consume when old. In the private equilibrium, consumption depends on the three aggregate shocks endured by the agent over its lifespan: $c^{G^\theta_{t+2}}(s_t,s_{t+1},s_{t+2})$.

Undeprovision of long-term securities in the decentralized equilibrium The stochastic OLG structure of the model brings in the classic violation of the First Welfare Theorem, caused by the infinite value of the aggregate endowment. What is novel in the present environment is a constrained inefficiency when agents face a maturity choice. Even when allowed to share risk with one period-ahead generation, they do not issue the same securities the planner would.

To see this, first observe that the planner is able to engineer Pareto improvement by manipulating the level of investment. This opens the avenue to increase *both* the expected returns and the level of within-generation risk-sharing, hence weakly enhancing the indirect utility of the two agents.

There is some long-term debt issuance in the decentralized equilibrium. It cannot be zero, as there must be the market 1.4 for this debt. But there is not enough of it. This is due to the fact the issuance of long-term debt exhibit strategic complementarities: an **issuance externality**. In terms of allocations, beyond the intragenerational risk-sharing analyzed in the static model, there is willingness to share risk inter-generationally: generation G_t is exposed to shocks s_{t-1} and s_t, generation G_t is exposed to shocks s_t and s_{t+1} and generation G_t is exposed to shocks s_{t+1} and s_{t+2}. So there is some willingness to smooth risk across these periods, beyond smoothing across states. The first market, the secondary market 1.2 for long-term debt, is used to share risk between G_{t+1} and G_{t+2} of their risk at the $t+2$ horizon, whereas the two other markets, the primary market 1.4 for long-term debt and the market 1.3 for short-term debt issued by young, are used to share risk between G_{t+1} and G_{t+2} of their risk at $t+1$ horizon.

The externality arises from the fact that G_{t+1} and G_{t+2} share too much $t+1$ risk but not enough $t+2$ risk. In the choice of maturity when young, i.e. does the young bankers issue $x^{G_{t+1}\,primary}_{\bar{s}^{G_{t+1}}_{t+1}}$ or $x^{G_{t+1}}_{\bar{s}^{G_{t+1}}_{t}}$. When they do so they do not internalize the fact that their own decision on the primary market at $t-1$ will impact the market clearing 1.2 on the secondary market of its own public debt (the same as its new issuance). The market clearing on the primary market 1.4 for long-term debt takes care of its Walrasian role, but does not internalize at the subsequent market clearing on the secondary market 1.2 for long-term debt. The pecuniary Walrasian role works well only to equate the *concomitant* quantity choices: $y^{G_{t+1}}_{\bar{s}^{G_{t+1}}_{t+1}} + x^{G_{t+2}}_{\bar{s}^{G_{t+1}}_{t+1}} + y^{G_{t+2}}_{\bar{s}^{G_{t+1}}_{t+1}}\ \mathit{and}\ x^{G_{t+1}}_{\bar{s}^{G_{t+1}}_{t+1}}$, but not in a retroactive way.

All 3 markets clear, so there will be a bit of all debt, but *not enough* long-term debt, due to the non-internalization of the safety multiplier mechanism $\frac{\partial\left(x^A_K + x^P_K\right)}{\partial x^{G_{t+2}\,primary}_{\bar{s}^{G_{t+2}}_{t+2}}} > 0$. This what the planner takes into account: in the f.o.c. for $x^{G_{t+1}\,primary}_{\bar{s}^{G_{t+1}}_{t+1}}$ it does plug in the market clearing condition *1.2*.

Proposition 5. *The private competitive equilibrium is constrained inefficient: the decentralized equilibrium under-provides long-term debt:* $x^{G_{t+1}\,primary}_{\bar{s}^{G_{t+1}}_{t+1}} < \left[x^{G_{t+1}\,primary}_{\bar{s}^{G_{t+1}}_{t+1}}\right]^{planner}$.

Proof. By inspection of the f.o.c.: the planner would like to have more investment: both $xi^{G_{t+1}}_{t-1}$ and $xi^{G_{t+1}}_{t}$ (i.e. at the two periods $t-1$ and t). Especially at t, investment is crowded-in with low price $q_{\bar{s}^{G_{t+2}}_{t+2}}$ by the resource constraint $\left(x^A_K + x^P_K\right) c_K + x^{G_t\,primary}_{\bar{s}^{G_t}_{t}} q_{\bar{s}^{G_{t+2}}_{t+2}} \leq n^P + n^A$. The price $q_{\bar{s}^{G_{t+1}}_{t+1}}$ on *1.2* is too high. On the other hand, the planner takes into account this supply effect: $\frac{\partial\left(q_{\bar{s}^{G_{t+1}}_{t+1}} x^{G_t\,primary}_{\bar{s}^{G_t}_{t}}\right)}{\partial x^{G_t\,primary}_{\bar{s}^{G_t}_{t}}} < 0$. □

The basic intuition is that the planner can engineer a Pareto improvement by *crowding-in* investment in the current period by borrowing from *two* periods ahead. Compared to the competitive equilibrium, he makes risk-neutrals happier by increasing levered returns *and* risk-agents happier by lowering the state of default. Banks prefer to issue short-term debt than long-term debt, and doing so they starve the economy from negative-beta assets. Banks do not internalize the hedging properties of long-term debt, which could have been used as an input by another agent to create more safety. In other words, the appealing risk characteristics of its own long-term liabilities are not internalized by the bank.

Discussion: the source of the issuance inefficiency The inefficiency rises not from an overinvestment in the risky asset (Lorenzoni (2008)), nor from an overinvestment in the safe asset (Hart and Zingales (2011)), but from a too short maturity structure of private claims. I coin this externality an *issuance externality*, which is a bit different from the terms-of-trade vs. collateral externalities in Davila (2011) topography of pecuniary externalities.[43] It is akin to the latter, except that the externality does not arise from a direct 'price in the contraint' kind of effect: my model therefore does not feature direct pecuniary externalities. Indeed, the only friction, limited liabilty $c \geq 0$, does not feature any 'price in the constraint'. The effective endogenous collateral constraint is the result of the combination of limited liability and bank portfolio. The issuance externality I uncover here also comes from a pecuniary effect, but less direct. Consider the ex-ante resource constraint of the economy:

$$q_{\bar{s}_{t+1}^{G_{t+1}}}\left(s^t\right) x_{\bar{s}_{t+1}^{G_{t+1}}}^{G_{t+1}\,primary} + c_K x_K\left(s^t\right) \leq n^A + n^P$$

When choosing to supply the economy with $x_{\bar{s}_{t+1}^{G_{t+1}}}^{G_{t+1}\,primary}$ legacy long-term debt, the primary issuers of generation G_{t+1} do not take internalize the crowding-in role that their own long-term liabilities play through the safety multiplier mechanism. Increasing $x_{\bar{s}_{t+1}^{G_{t+1}}}^{G_{t+1}\,primary}$ decreases the value $q_{\bar{s}_{t+1}^{G_{t+1}}}\left(s^t\right) x_{\bar{s}_{t+1}^{G_{t+1}}}^{G_{t+1}\,primary}$ by the **Laffer** effect analyzed in the static model. As can be seen on the resource constraint, issuing more long-term securities $x_{\bar{s}_{t+1}^{G_{t+1}}}^{G_{t+1}\,primary}$ would crowd-in real investment $x_K\left(s^t\right)$, leading to a Pareto improvement, but private agents do not take this aggregate channel into account in their private maturity choice.

The government then becomes the natural provider of long-term securities. This is *not* just driven by the superior taxation power of the government. It is driven by the fact that, in a competitive equilibrium, long-term securities are used as input in the production of short-term securities. This leads to the pecking order: government issues long-term, private sector issues short-term. Public debt improves welfare because its supply impacts the creation of private safe assets. The government is able to manipulate bank leverage through its public debt issuance policy. Constrained inefficiency means that welfare improvement can be achieved by endowing the social planner with the *exact* same issuance capability than the private agents: the available maturities are the same for public debt as for private debt[44].

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[43] Eduardo initially coined his terms-of-trade externality 'risk-sharing', but the mechanism he has in mind is non-equality of MRS, which can hold even in a deterministic environment. On the contrary, my issuance externality entirely hinges on the stochastic environment and equilibrium default, so it can be seen as a risk-sharing externality.

Market incompleteness is the feature of the economy that breaks the Scheinkman (1980) generic efficiency result. Indeed, when there is no public debt, the risk-neutral banks underprovide insurance to the risk-averse investors. This is due to a lack of well-diversified collateral on their balance sheet. Having endogenous leverage arising from limited commitment breaks his Modigliani-Miller-like neutrality theorem.[45]

This *intragenerational* inefficiency has nothing to do with the intergenerational inefficiency due to overaccumulation of capital (dynamic inefficiency) of Diamond (1965) or Gale (1990).[46] The intergenerational inefficiency arises from intergenerational limited participation: all generations cannot trade with each other. Whereas in my setup, generations do trade with each other on the secondary market for public debt. In my model of limited commitment, public debt enhances intragenerational risk-sharing.

1.4.2 Implementation: optimal issuance of public debt

As the positive part of the dynamic model just showed, by playing around with the supply B of public debt, the government is able, even in the decentralized economy, to manipulate the price $q_{\bar{s}_{t+2}}^{G_{t+2}}$, hence the leverage i.e. the risk-sharing decision between the two agents. Public debt enables the planner to move aggregate wealth *across states*, and not only across periods as stressed out by the OLG literature. Recall the resource constraint:

$$p_K\left(i_t^A + i_t^P\right) + q_{B,t}B \leq n^A + n^P$$

By issuing more long-term public debt B, through the Laffer effect it decreases the value $q_{B,t}B$, hence crowding in aggregate investment. The dynamic model tells us that $\mathbb{E}\left[\left\{r^{safe}\left(s_t\right)-r^K\left(s_t\right)\right\}\right]<0$ and $Cov\left[r^K\left(s_t\right),\left\{r^{safe}\left(s_t\right)-r^1\left(s_t\right)\right\}\right]<0$. So issuing public debt can improve the risk profile, but there is a cost which is to crowd out investment. The cost is parametrized by $q_{\bar{s}_{t+2}^{G_{t+2}}}$ whereas the gains are parametrized by:

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[44] Allowing the government to issue state-contingent debt would obviously lead to even greater welfare improvements.

[45] Modiglina-Miller-like neutrality results all comes from a redundancy in the linear space spanned by the assets.

[46] A key difference with Gale (1990) is that his budget constraints at the time of trade already involve the returns on the securities. In effect, he rules out limited commitment and default. On the contrary, I make clear that, at the trading period, the budget sets are bounded by net worth, whereas the securities promises arrive only at the ex post period. Compared to Fischer (1983) and Peled (1985), I do not have risk on the endowments, but on the assets.

$$Cov\left[r^K\left(s_t\right),\left\{r^{safe}\left(s_t\right)-r^K\left(s_t\right)\right\}\right]<0$$

There will be a Pareto improvement as long as $B\,q_{\bar{s}_{t+2}^{G_{t+2}}}$ decreases and still:

$$Cov\left[r^K\left(s_t\right),\left\{r^{safe}\left(s_t\right)-r^K\left(s_t\right)\right\}\right]<0$$

I now derive the optimal financial policy $\left(B,\tau^A,\tau^P\right)$ of the government, and show how it can implement a welfare improvement. The welfare criterion I use here is the indirect utility of the risk-averse investors (the 'grandmas').[47] Puuting all the Pareto weight on risk-averse agents enables to focus on the safety creation role of banks. The government maximizes with respect to its financial policy their indirect utility under the competitive equilibrium:[48]

$$\underset{\{B\}}{Max}\;\bar{W}_t^P\left(x_K^P\left(s^t\right),\left\{x_{B,h}^P\left(s^t\right)\right\}_h,\left\{y_{\bar{s}_{t+h}}^P\left(s^t\right)\right\}\right)\;s.t.\,equilibrium$$

The government takes as a constraint its own budget constraint (flow of funds). In a stationary recursive equilibrium we have the identity: $\hat{B}_{t-1}^h=\hat{B}_t^{h-1}$. Appendix B.7 derives the indirect utility and investigates its comparative statics in the supply of long-term public debt S^K. The following expression signs the welfare criterion with respect to an increase in public debt supply, taking the beta as given:

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[47] I do not take into account distributional concerns and mutes any active role of the tax scheme. Bhandari et al. (2013) solves for the optimal policy trading-off redistribution and tax distortions. Their Ramsey problem exhibits a Ricardian irrelevance of the level of public debt. My stance for public debt is even stronger, as I show that the optimal policy features high levels of public debt. This said, I share with them the emphasis on *who* holds the public debt. The safety multiplier makes it more valuable in risk-neutral hands.

[48] Lorenzoni and Werning (2013), in the tradition of Calvo (1988), claims that the government policy choice is the ex ante value of debt ($\hat{q}^h B$) and not the ex post value (B). However, in practice, auctions ran by treasuries always announce a *face value* of government debt to raise, which is tantamount to choosing the ex post value B.

$$\bar{W}_t^P \propto \underset{(+)}{r_t^{safe}} n^P + \underset{(+)}{\left\{ r_t^{bank} - r_t^{safe} \right\}} \underset{(+)}{D_t} \tag{1.20}$$

$$+\frac{1}{2}\underset{(-)}{\mu_{u,t}} - \frac{1}{2}\sqrt{\frac{2}{\pi}}\underset{(+)}{\sigma_{u,t}} \tag{1.21}$$

The effect of increasing public debt supply is broken down in different channels. The beneficial effect on the first term $r_t^{safe} n^L$ is a purely net worth effect. It is a **wealth channel**. This benefit is entirely muted by the necessary tax adjustments it implies. Satisfying the government flow of funds with a lower price $\hat{q}^h$ of public debt forces to increase taxes. I shut down any redistributional role of taxes. This is tantamount to *not allowing* for any ex-ante transfers between agents. I choose $\tau^A = 0$ by consistency with the welfare criterion. I shut down any role of fiscal policy: monetary dominance, the fiscal policy is here only to balance government flow of funds. In this case, the tax burden exactly undoes the wealth effect of the increase in public debt supply.[49] This is tantamount to the Woodford neutrality critique of the portfolio rebalance theory.

Nevertheless, public debt still is able to improve welfare through its *indirect* effect on the creation of private safe assets. This effect is captured by the second term of 1.21: a **safety multiplier channel**. In the language of Weitzman (1974), public debt supply has a price effect on the SMI spread and a quantity effect on bank leverage. The latter is beneficial due to the safety multiplier result (Proposition 1). The former price effect comes from the result that SMI decreases with public debt supply (Proposition 4), and that investor welfare 1.21 features the opposite of SMI (i.e. bank credit spread). The last two terms are adjustment terms that capture the cost of bank default from the investor perspective. They penalize the private debt, which traces back to public debt through the safety multiplier. This **default channel** is not inocuous, and is reminiscent of the traditional fire-sale externality which leads to cost of overborrowing. But here, they are only the third channel happening in conjunction with the wealth channel and the safety multiplier channel. The latter completely overturns the case of overborrowing by the private economy. That public debt leads to a Pareto improvement shows that the private equilibrium features a marginal inefficiency, in the sense that it does not rely on Pareto ranked multiple equilibria. The Pareto improvement can be thought as: public debt does crowd in real investment (Proposition 2). This makes *both* banks and investors happier, the first for an expected return reason, the second for a safety supply reason. This is how *ex-post* Pareto improvements can be achieved, a much more demanding task than not ex-ante ones (from the sum of utilities, but this then leaves room to arguing about the Pareto weights).

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[49] This Ramsey problem of public debt issuance implicitly takes into account the *cost* for the sovereign of having a lower price for the public debt: from this perspective, having low public debt price $\hat{q}^h$ is harmful. Shutting down the redistritibutional effects of taxation mutes Krueger and Perri (2010) mechanism of using taxation (public risk sharing) to overcome imperfect private risk sharing. The latter finds it hard to obtain crowding-in of private insurance by public insurance, whereas my model does feature such crowding-in for a large parameter region.

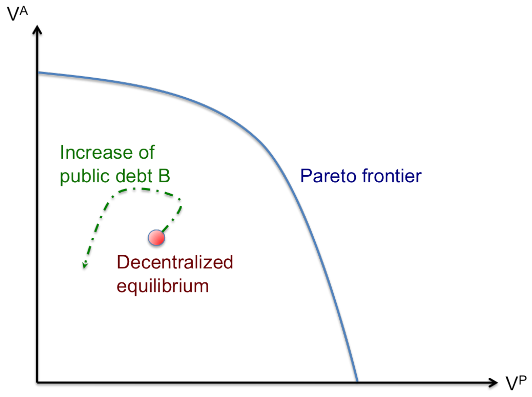

Figure 1.11: *Pareto-improving issuance of public debt.*

Finally, there is a countervailing negative effect of public debt arising in the endogeneity of public debt beta. This force goes on the exact *opposite* direction than the static effect just described: increasing public debt supply increases its beta, hence it decreases bank leverage D, and also increases SMI, hence decreases bank credit spread $\left\{r_t^{bank} - r_t^{safe}\right\}$. In economic terms, flooding the economy with public debt hurts its hedging property. The final proposition of the paper qualifies the exact dependence of welfare with respect to the supply of long-term public debt.

Proposition 6. *Optimal level of long-term public debt*

In the dynamic model, there exists an interior optimal level of public debt $B^{optimal}$.

Proposition 7 endows public debt with a powerful role in regulating leverage. As pointed out by Davila (2011), when no transfers are allowed, capital regulation does *not* lead to a Pareto improvement (the traditional discussion misses accounting for agents heterogeneity). In contrast, public debt issuance is shown here to lead to a Pareto improvement, by manipulating leverage.

However, the dynamic harmful effect of issuance ends up by overwhelming this static beneficial effect. Intuitively, the static effect is bounded upper. The safety multiplier mechanism is concave: the resource constraint 1.16 imposes that private leverage cannot grow as fast as public debt supply. Hence the static beneficial effects of public debt will start to fade away. On the other hand, the dynamic negative effects of issuance do not fade away, as the beta of public debt increases monotically with its supply.

The beneficial role of public debt works through a *market mechanism*, in which banks freely decide how much to issue of private safety (bank debt). This is different from Holmstrom and Tirole (1998) in which the government has the taxation power of circumventing the exogenous collateral constraints of private agents (not a constrained inefficiency). The decomposition 1.21 also helps to distinguish the beneficial role of public debt from what Gale (1990) has in mind for intergenerational risk-sharing. In my model public debt leads to welfare improvement not only when real rates are negative (safe price above 1). Finally, the Pareto improvement relies on the investment of private agents. The government is able to manipulate the level of investment merely by issuing long-term debt. Therefore, it can be argued that the Pareto improvement engineered this way requires less public intervention than the social security system designed in Ball and Mankiw (2007), in which the government engages himself in investment, or even than their lighter implementation, which involves safe debt holdings by the government and time-varying social benefits.[50]

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[50] They conclude that "negative indexation and government ownership of capital seem to be the only mechanisms that allow current capital risk to be shared optimally with future generations." Long-term public debt issuance should be thought as an effective third avenue to enhance risk allocation.

The present argument in favor of public long-term debt is novel. Indeed, the strategy of Angeletos (2002) is inoperational here given the continuum of states. It is also an interesting counterpart to the Aguiar and Amador (2013) argument for short-term debt for incentive reasons. Taken together, the two results makes clear that long-term debt is beneficial for *hedging purposes*, but at the expense of government repayment incentives. Arellano and Ramanarayanan (2012b) also features a hedging-incentive trade-off in the issuance of long-term debt. However, their hedging motive is partial equilibrium in nature as they do not consider a closed economy. On the contrary, my hedging motive is desirable even in GE as it crowds-in investment.

Compared to Greenwood et al. (2010) gap-filling theory (which stresses the crowding out), my theory advocates for the issuance of *long-dated* public securities. This normative recommendation on public debt maturity choice is in line with the empirical supply of Eurozone public debt. The short-end of public debt is quantitatively much smaller than the long end ($1 trn$€ vs. $7.5 trn$€). If agents were long-lived, they would reach for short-term public debt to avoid interim volatility. They would still underprovide long-term negative beta securities.

As Corollary 3 informs us that bank expected utility also increases with the supply of public debt (due to an increase in equilibrium expected returns), the public debt financial policy of Proposition 5 does lead to a Pareto improvement. Everyone takes advantage of a higher supply of public debt in the economy. The model does point towards a *positive externality* of long-term securities, which private agents do not internalize. Issuing long-term public debt is a constrained efficient way to achieve Pareto improvements.

This dynamic model with endogenous leverage leads to policy recommendations in line with Caballero and Farhi (2013), but more nuanced. The present model helps to qualify their prescriptions. First, Proposition 5 can be interpreted as Quantitative Easing being good only up to a certain extent.[51] That SMI also reveals the welfare impact of public debt enables the

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[51] Long-term government bonds can also help on the monetary side (see Sheedy (2013)). So the two objectives, debt to GDP stabilization and safety creation are not orthogonal.

Treasury to conduct experiments of debt issuance and by tracking the response on SMI, can calibrate the effective level of optimal public debt. Second, Operation Twist (i.e. removing long-duration assets from the economy to prop down long-term real rates) has negative welfare effects through the safety multiplier by starving the economy from long-term public debt. This is contrary to Stein (2010), where there is an exogenous collateral constraint which gives room to a pecuniary externality due to exogenous credit constraints and this implies *too much* private safe asset creation. In the safety multiplier mechanism, the private equilibrium does *not* produce enough private safe assets.[52]

Finally, public debt is *not* as any other asset. It is *not* collateralized, so it can be seen as the *ultimate collateral*, the very starting point of any collateral chain. This is why the government should not take over the banking sector, and to the private safety creation itself: for the model to be efficient, there needs to be two distinct sectors (government and banks), for the liabilities of the former to be used as a hedge by the latter. As such, even if the economy is exposed as a whole to only one univariate shock, having the government sector and banking sector enables the synthetic second asset, public debt, to be used as a hedge by the banks in their safety production.

1.4.3 Financial regulation: the cost of narrow banking

The safety multiplier beneficial effect uncovered in the previous section critically needs risk-neutral banks to be able to issue unsecured debt. The interesting comparative statics here is with respect to the *size* of the banking sector.

In the model it is captured by net worth of banks n^A. In the limit of no banks: $n^A = 0$ the safety multiplier is entirely muted, and risk-averse investors invest all their wealth in the aggregate portfolio. It is straightforward to see that the welfare of the risk-averse agents increases with the size of the risk-neutral sector. Even without accounting for the welfare of banks, the economy benefits from the presence of banks for their private safety creation role.[53] The welfare improvement coming from the presence of risk-neutral banks is equal to:

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[52] This result is also different from Barro (1974) and Angeletos (2002), in which the benefits of public debt are traded off against the costs of distortionary taxation. My perspective is pure asset pricing, in which the value of public debt relies in its hedging property. It also complements Cochrane's view that Treasury should go long to hedge its interest rate risk: insurance property of long maturities. My model takes the perspective of the private agents, and argues they need long-term Treasuries to produce safety.

$$\Delta W^P = \left(r^{bank} - r^{safe}\right) D + \gamma^P \frac{1}{2}\sigma_B^2 \frac{r^{safe}}{\mu_B} D - (0.4\sigma_u - 0.5\mu_u)$$

Corollary 4. *Universal banks are beneficial to risk-averse investors:* $\bar{W}_t^P \left(\underset{(+)}{n^A}\right)$.

This corollary isolates the beneficial role of private safety creation of banks. It echoes Gennaioli et al. (2013a) finance as preservation of wealth. With asset pricing and endogenous leverage, my model describes *how* the financial system is able to create private money by transforming risk. A natural implication of Corollary 4 is that Glass-Steagall act type regulations have a cost. By breaking up the banks and splitting their positive beta from their negative beta part of the balance sheet, these regulations prevent the bank to create private safe asset by taking advantage of the hedging property of public safe assets[54]. Thus, according to this theoretical treatment, the universal banking model is welfare improving[55]. As a result, for safety creation, narrow banking and bank structural reforms[56] can harm social welfare in the exact same way as the Glass-Steagall act. It prevents banks from issuing safe bank debt. Empirically, Berger et al. (2013) show that capital injections and regulatory interventions have a costly persistent negative effect on liquidity creation, in line with the theory developed here.

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[53] My model offers a more benign view of leverage than recent macroprudential academic and policy. The ex ante macroprudential regulation literature insists on the negative pecuniary externality of leverage (fire sales), so concludes to the necessity to curb leverage. This is an artefact of having an exogenous collateral constraint. This argument misses that bank leverage does fulfill an economic role of safe asset supply. This is made clear with the present environment, in which the only friction is limited liability.

[54] This bright side of universal banking must be traded off against the moral hazard cost of too big to fail.

[55] A reminiscent intuition is present in the Jacques de Larosiere report on financial stability. An easy complement normative investigation should quantify the cost of market incompleteness. If markets were complete, banks would never default and therefore their debt would be riskless and risk-averse investors would fully invest in it. This is the first-best. It is the other limiting case, opposite to the world without banks $n^B = 0$.

[56] Volcker and Vickers proposals, as well as a number of bank reforms such as the French bank law voted in July 2013, aim at separating market-based activities from bank deposits. However different banking structural reforms have different levels of restrictions on market activities. If the ring-fencing is too constraining, it can prevent universal banks from playing their economic role of diversification and supplier of private safe assets.

A final argument to consider is why the government would not take over the whole banking system and private public and former private safety by doing the tranching itself. The answer is negative and the argument relies in the endogenous hedging properties of government debt. From the government perspective, if it merges its own balance sheet with the balance sheet of the financial sector, it faces only one univariate shock: the macroeconomic shock. The government is therefore unable to effectively hedge this shock, as it does not have access to any other asset. Whereas as long as the financial sector balance sheet is different from the government balance sheet, government liabilities become the **support** of the next period flight to safety. By combining this endogenous claim with the risky technology, an autonomous financial sector is able to create synthetic safety.

1.5 Extension: open economy

The goal of this extension is two-fold. First, it investigates the impact of sovereign risk on the safety multiplier mechanism elicited in the main model. That is, I investigate the *supply side* of public safe assets, after having analyzed in detail its demand side in the previous section. Second, it opens the avenue to analyze the open economy environment counterpart of the closed economy model, in which two countries only differ by their fiscal capacity (i.e. sovereign risk).

The key insight of this section is that sovereign risk hurts the safety multiplier by introducing a force towards higher endogenous beta ρ. The following qualifies this intuition. The dynamic model with endogenous beta is therefore critically required to address this issue.

1.5.1 Introducing sovereign risk

I model sovereign default through a notion of fiscal capacity, and not through strategic default: default is *suffered*, not strategic. This is a realistic assumption, having the Eurozone debt crisis in mind[57]. As a result, there is now in the economy private *and* public equilibrium defaults.

Precisely, I extend the dynamic infinite horizon model in the following way. I model fiscal capacity as default threshold $\bar{\bar{s}}$ on the macro TFP shock below which the country defaults. At each period $t > 0$, there is sovereign default if and only if:

$$\textit{public default at period t i.if. } s^t < \bar{\bar{s}}$$

In the states of the world in which of public default, the consol becomes worthless. To compensate the holders of public debt, the troika manages to secure to them an amount that is linear to the macro TFP shock: κs^t. κ is a measure of troika efficiency. The payoff of the public debt is, and illustrated on Figure 1.12:

$$\tilde{p}_B^t = \kappa s^t \; 1_{\{s^t < \bar{\bar{s}}\}} + p_B^t \; 1_{\{s^t \geq \bar{\bar{s}}\}}$$

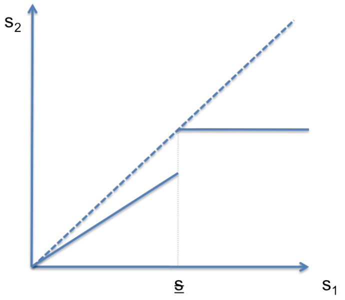

Figure 1.12: *Payoff of public debt with respect the aggregate state* s^t.

Until now, fiscal capacity was taken as fixed and very high. In the general case, Appendix B.8. derives that in this extension, the endogenous beta of public debt is equal in a second-order approximation, not to Equation 1.19 anymore, but to:

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[57] The shutdown drama of fall 2013 in the US is another illustration that sovereign defaults are driven by *non-strategic* factors in developed countries. The fact that it made public debt long-term yields go up and short-term public debt go down reinforces the safety pecking order exhibited in the previous section: sovereign risk only hurts long-term public debt, not the short end, due to the overall scarcity of safe assets.

$$\tilde{\rho} = -1 + \frac{\left(\frac{B}{n^A} \frac{\left[\hat{\sigma}_B^2 p_K^2 + \sigma_K^2 \hat{p}_B^2 \right]^2}{2\sigma_K^4 p_K^2 \hat{p}_B^2} \right)^2 + \left(1 + \kappa \frac{1}{\hat{p}_B} \left(s^t - \frac{B}{n^A} \frac{\left[\hat{\sigma}_B^2 p_K^2 + \sigma_K^2 \hat{p}_B^2 \right]^2}{2\sigma_K^4 p_K^2 \hat{p}_B^2} \right) \right) \Phi \left(\frac{\bar{\bar{s}} - s^t}{\sigma_K} \right)}{1 + \kappa \frac{1}{\hat{p}_B} \frac{B}{n^A} \frac{\left[\hat{\sigma}_B^2 p_K^2 + \sigma_K^2 \hat{p}_B^2 \right]^2}{2\sigma_K^4 p_K^2 \hat{p}_B^2} \Phi \left(\frac{\bar{\bar{s}} - s^t}{\sigma_K} \right)}$$

The increasing dependence of $\tilde{\rho}$ to sovereign risk $\bar{\bar{s}}$ and the comparative statics of the main model with respect to ρ yield the following lemma.

Lemma 9. *Fragility of the safety multiplier to sovereign risk*

In the closed economy, leverage (private safety D) and real economy lending (x_K) decrease with sovereign risk $\bar{\bar{s}}$.

The intuition is that sovereign risk not only increases the volatility of public debt (and lowers its expected return), but more interestingly increases the endogenous correlation of public debt with the macro TFP shock. As a result, it destroys the hedging properties of public debt, and doing so it hinders the safety multiplier mechanism. Sovereign risk also alters the Safety Mismatch Index.

An interesting aspect of introducing sovereign risk in an endogenous leverage model is that it goes *against* the expropriation channel of contagion from sovereign risk to corporate risk. Bai and Wei show that the expropriation contagion channel holds for the general corporate sector. My model shows that, as far as the *banking sector* is concerned, the contagion can go the other way: higher sovereign risk can lead banks to delever so much that they become *less* risky.

1.5.2 Environment

There are two countries: North and South. I introduce one unique source of heterogeneity between the two: their fiscal capacity. North has a larger fiscal capacity: $\bar{\bar{s}}^{South} > \bar{\bar{s}}^{North}$. The risky technology is the same for both countries. As a result, there are three assets: the risky technology, North public debt and South public debt. Due to sovereign risk we have in equilibrium: $\sigma_B^{South} > \sigma_B^{North}$, $\rho^{South} > \rho^{North}$, $p_B^{South} < p_B^{North}$. The agents are homogeneous in both countries: same endowments and same preferences. The only degree of specificity is the limited participation assumptions: risk-averse investors have access only to their local bank debt and to their local bond market[58]. This local investor base will endogenously drive heterogeneous behavior between North bank and South bank in terms of leverage and portfolio choice.

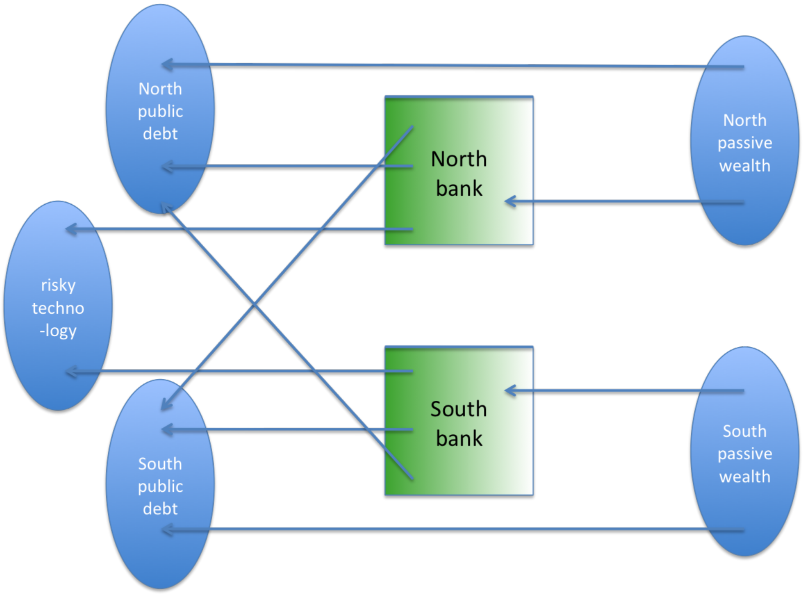
North public debt
North bank
North passive wealth
risky techno -logy
South bank
South passive wealth
South public debt

Figure 1.13: *Open economy environment.*

1.5.3 Equilibrium

Consider a South bank. Label Asset 2 the South bond and Asset 3 the North bond: we have $\rho_3 < \rho_2$. The balance sheet correlation metric $X(\rho_2, \rho_3)$ is now:

$$X_{OE} = \frac{(x_2\sigma_2 + \rho_2 x_1 \sigma_1)}{(x_1\sigma_1 + \rho_2 x_2 \sigma_2 + \rho_3 x_3 \sigma_3)}$$

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[58] This is empirically relevant despite a slight quiet run on Greek banks from Greek investors. The assumption of special access to the local base can be relaxed. What matters is that *banks* have better access to the risky technology and the other bond market. It is empirically verified that cross-border holdings of sovereign debt are mainly owned by foreign *banks*, not final investors.

This balance sheet correlation measure is generically higher than in the closed-no sovereign economy. As a result the carry trade on South sovereign bond:

$$r^{bond} - r^{bank} = \left(r_K - r^{bond}\right) \frac{\mu_K}{c_K} \left(\frac{1}{1 - \frac{c_K}{p_B} \frac{\sigma_B}{\sigma_K} X_{OE}} - 1 \right)$$

can now turn positive for the South bank, as long as ρ_2 high enough such that the carry trade increases with X_{OE} (therefore with ρ). This positive carry trade *for South banks* is at the source of the following proposition.

Proposition 7. *Redomestication of public debt.*

In the open economy, South banks hold more South bonds: $x_B^{South} > x_B^{North}$*. The heterogeneity increases with sovereign risk differential.*

Proof. See Appendix B.8. □

We can interpret this proposition as sovereign risk strengthening the risk shifting channel and weakening the safety multiplier channel (debt pricing channel). What is really interesting is that sovereign risk strengthens the risk shifting motive and weakens the endogenous leverage mechanism: i.e. higher sovereign risk, due to the outside option of investor local base makes bank debt cheaper so that bank decides to lever up *more*, and they do so using the risky public debt (which is cheaper than the North public debt): a version of risk shifting. The economic intuition can be grasped with a perturbation argument. Start with a symmetric equilibrium, with same level of sovereign risk and increase by ϵ the sovereign risk of the South country. South bond has worse hedging properties, banks should fly to North bond. But: South bond has higher volatility, disliked by the local investors. Thus South investors have higher need of bank debt. Thus South banks have access to cheap credit; they lever up. As South bond is marginally cheap compared to North bond, *they use the South bond* to hedge its additional risky investment.

As a result, my model captures redomestication of South sovereign debt in Eurozone. The stylized fact is document in the open economy section of the appendix, which also shows that bank leverage is more stable for banks of the periphery than for bank of the core countries. The redomestication of sovereign debt is also documented by Acharya and Steffen (2013), although they do not explain why *South banks* are better positioned to do this greatest carry trade). Uhlig (2013) is a moral hazard model of the South free-riding on bailout expectations from the North (through the ECB). The present model is a pure Asset Pricing perspective with endogenous leverage, in which, due to the General Equilibrium, South banks are the natural marginal holders of South public debt. I do not need any financial repression type of argument. The domestication is an efficient asset pricing outcome. The fragmentation of government bond markets is aligned with fragmentation of bank debt markets. Global banks are also portfolio *choosers* between domestic debt and foreign debt. So foreign banks have a key role in the safety multiplier applied to domestic debt. There is an eviction effect due to foreign debt in the safety creation. I thus microfound a home bias in government bond holdings: domestic debt is actually a better hedge of domestic equity than foreign debt.[59] Redomestication is due to the apparition of sovereign risk heterogeneity which made Eurozone switch from the closed economy model to the open economy model. My model of private safety creation rationalizes why banks hold public debt on their balance sheet, and as a result it microfounds the diabolic loop: the increased sensitivity of bank default risk to sovereign risk.[60] In normal times, banks help the sovereigns to refinance cheaply (high public debt price). However, due to the contagion of default risks from public to private (a contagion of commitments), this financing pact did break up.

1.5.4 Normative implications in open economy

A narrative of the Eurozone crisis as a closed economy can treat sovereign risk as a neglected risk on which investors did load and which, when it did come to mind, destroyed the safety multiplier mechanism and triggered deleverage and credit crunch. Figure A.3 illustrates how the beta of South public debt did turn sharply positive during the Eurozone crisis. This sheds light on a dark side of the safety multiplier: private safety creation incentivizes investors to neglect sovereign risk.

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[59] Maybe also perhaps because of a relational contract between domestic banks and domestic sovereign, the outside option being to invest in foreign debt and accepting foreign investors as debt holders (this would be in a model of strategic default).

[60] It is from sovereign risk to bank risk (Greek crisis), so my model is alternative toGennaioli et al. (forthcoming). It does not feature the reverse causality, from bank risk to sovereign risk (Ireland crisis) through bailout expectations (Acharya et al. (2011b), Broner et al. (2013)). I stick to a pure asset pricing model without resorting to any moral hazard friction and shows how it can rationalize all the stylized facts of the Eurozone crisis.

Make public debt cheap or expensive? The last step is to keep in mind that, through bailout expectations, there exists a feedback loop from banks to sovereigns: i.e. an increasing causality from bank leverage D to sovereign risk $\bar{\bar{s}}$. In a recent tribune in the Financial Times (October 1, 2013), Jens Weidmann (Bundesbank) argues that the banks holdings of public debt are dangerous should be 'taxed' through increased capital requirements or a large counterparty exposure regulation. His normative thinking is concerned with this feedback loop and its crowding out effect on real investment. On the contrary, I argue that there are benefits of these relatively safer holdings by banks, through the crowding-in mechanism of this paper. As an implication, Weidmann argues for *expensive* public debt (low yields). On the contrary, the model developed here, based on a scarcity of safe assets, argues for a *cheap* public debt.

Also, as described in the positive model, the safety multiplier mechanism increases the interconnection between banks and sovereigns (diabolic loop). The full-fledged normative analysis should also keep in mind that there is another cost to public debt issuance due to the safety multiplier: exposure to the sovereign neglected risk. The normative analysis in the Open Economy model raises a flag at the Outright Monetary Transactions (OMT)[61]: they have the undesirable effect of propping up the price of public debt p_B^{South}, which in consequence hurts private leverage.

Eurobonds In the open economy environment, Eurobonds is a beneficial policy, as this increases the supply of low sovereign risk public debt disproportionately more than it increases the sovereign risk of the junior part of the South bond. Contrary to Hellwig and Philippon (2011) who advocate for Eurobills and not Eurobonds, the present environment argues for issuance of long-term maturities. See Davila and Weymuller (2013) for a security design approach to the optimal amount of Eurobond issuance. EFSF and ESM are doing exactly the same from an economic standpoint. They wash away the idiosyncratic sovereign risk in order to enhance the supply of public safe assets B. Furthermore, splitting the Eurozone has negative consequence on the welfare of risk-averse investors as long that this break up implies a hindered access of banks to foreign public debt.

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[61] http://www.ecb.europa.eu/press/key/date/2013/html/sp130902.en.html

1.6 Empirical analysis

I investigate in this section how far my simple theory of private safety creation brings us to rationalize the main monetary aggregates in Europe an in the US. The essential insight of the model is that private debt quantities are driven by the risk characteristics (i.e. the beta) of public debt. A calibrated version of the model captures the aggregate patterns of private money in Europe.

The theory has rich empirical predictions regarding the balance sheet of the financial sector. I focus on how safe asset holdings and leverage comove with the supply of public safe assets in the economy, and argue these comovements are consistent with the safety multiplier mechanism and not with alternative theories of financial repression, bailout expectations and term premium trades.

1.6.1 Calibration of the safety multiplier on the Eurozone

Measure of the supply of public safe assets S^b

Among all assets available to investors, only the ones that are negative beta are included in the measure of supply of safe assets. The candidates for assets with such property are public debt and gold. Within public debt, I apply the negative beta criterion: as soon as the public debt beta turns positive, it is excluded from the S^b measure. Conceptually, a positive beta asset joins back the risky technology status: it is close to being just one more risk asset as any other. I measure the supply of safe assets by accounting for all the Eurozone public debt that is negative beta with the DJ EUROSTOXX 50 (proxy for the common risky technology).

The daily betas of 10-year government bonds of all 17 Eurozone countries with the European stock index (DJ EUROSTOXX 50) are computed on a 30-day rolling window[62]:

$$\beta_k = \frac{cov(R_t^{b\,k}, R_t^e)}{\sigma(R_t^{b\,k})\sigma(R_t^e)}$$

Daily betas are graphed in the Appendix (Figures A.2 and A.3) for the public debt of four Eurozone countries. These stylized facts illustrate a divergence of behavior between North and South public debt that goes beyond the divergence in the yields (Figure 1.18). The betas figures show that North betas became even more negative over the crisis, whereas South betas turned sharply positive, in accordance with the open economy version of the model.

In the spirit of the model, I then construct a composite measure of public safe asset supply by summing all the public debt that is negative beta:

$$S^b = \sum_{Eurozone\,country\,k} 1_{\{\beta_k<0\}}\, S^k$$

where S^k is the nominal stock of public debt of country k. A smoother version of the measure penalizes the positive beta public debts without completely excluding them: $S^b = \sum_{Eurozone\,country\,k} S^k \left(1 - e^{-Max(0,\beta_k)}\right)$.

Measure of the balance sheet of the European financial sector

Similarly on the demand side of assets, I split the assets side of the financial sector balance sheet in two categories: the positive beta asset (the risky technology) and the negative beta assets (the safe assets holdings). Data come from the Monetary ECB reports, which enable to construct an aggregate balance sheet for the financial sector of the Eurozone. The universe I am considering are all Monetary and Financial Institutions of the Euro area (17 countries), excluding the Central Banks of the Eurosystem. In the case of Europe, as the central bank does not implement its monetary policy through open market operations[63] but through reverse repo operations, ECB balance sheet works as the negative of a bank balance sheet. I therefore subtract its balance sheet from the European financial sector balance sheet.

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[62] I rely on daily quoted yields for the 17 MU countries. The daily returns of stock and bonds are computed as $r_t = ln(\frac{P_t}{P_{t-1}})$ where P_t is the price of the stock index or the price implied by the yield on 10 year government bond. Source: Global Financial Database.

The typology of assets is carried out with respect to their beta: loans, shares and other equity are positive beta, whereas securities and remaining assets (such as gold) are negative beta. Fixed assets are zero beta.

Assets side All loans except loans to governments are treated as risky assets. For securities, I only have the breakdown between securities to government and to other euro area residents. I treat both categories as safe assets, keeping in mind it is an approximation for securities to non-government (but even for the latter category, beta would be overall negative). Also for government securities, there is a split between $\beta < 0$ and $\beta > 0$[64]. Loans and securities and shares to MFIs are contracts internal to the financial sector, therefore I do not take them into account to avoid double counting. External assets/liabilities are net assets and are therefore included on the asset side, as a risky asset.

In the end, the risky asset holdings measure includes:

$$\begin{aligned} x_1 &= Loans\ to\ euro\ area\ residents\ (excluding\ MFI\ and\ gvt) + shares\ and\ other\ equity \\ &\quad + securities\ to\ gvt\ whose\ \beta > 0 + net\ external\ assets \end{aligned}$$

Whereas, the safe assets holdings measure includes:

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[63] In this case, as for the FED, its balance sheet should be subtracted from the *public* safe asset supply.

[64] As a robustness test, I also use data from stress tests ran by the European Banking Authority, which annually disclosed sovereign debt holdings of the 91 largest European banks. This enables to compute an alternative time-series for x_2.

$$\begin{aligned} x_2 &= Loans\ to\ gvt + securities\ to\ gvt\ whose\ \beta < 0 \\ &\quad -deposits\ of\ gvt\ in\ MFIs + securities\ to\ euro\ area\ residents\ (excluding\ MFI\ and\ gvt) \end{aligned}$$

The measures of x_1 and x_2 constructed are given by Figure 1.14. These measures document that the volume x_2 of safer assets comparatively increased more during the European debt crisis.

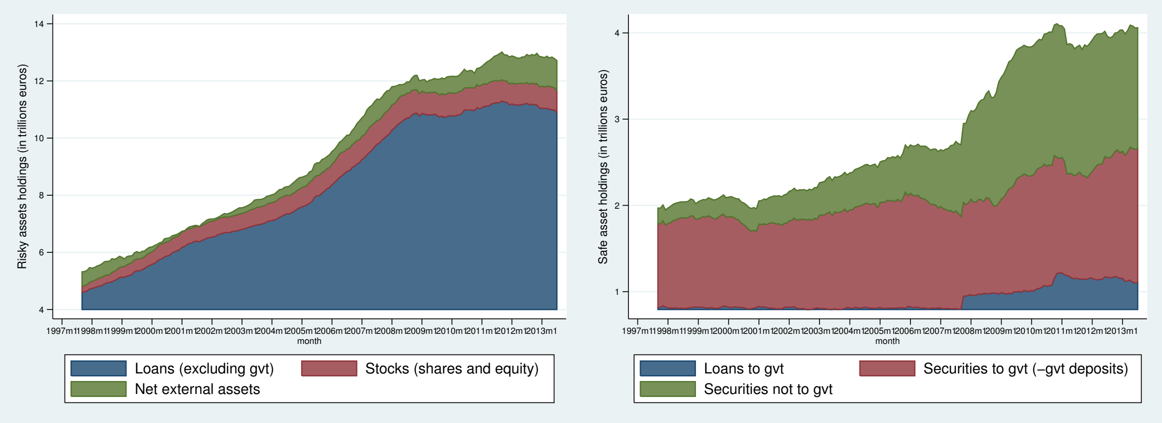

Figure 1.14: *Composition of assets holdings of Eurozone banks: risky assets holdings ($\beta > 0$) on the left, and safe assets holdings ($\beta < 0$) on the right. Data in trillion €.*

Liabilities side On the liabilities side, all deposits count as private money (M1) and is therefore included in the private money measure D. From deposits I only exclude MFIs deposits (which is double counting) and central government deposits (which are negative position of the financial sector in central government liabilities). MMF shares are net liabilities and are therefore included in D. Remaining assets/liabilities are slightly net liabilities, so are also counted as D. Finally, debt securities[65] up to 2 years are counted in D, whereas debt securities issued over 2 years are counted as bank net worth (sticky liabilities). Debt securities include Commercial Paper (CP). Capital and reserves are naturally counted also as bank net worth n^B (bank equity). Compared to KVJ, I do *not* distinguish between backed (M1) and unbacked (M3-M1) bank debt, as I argue that given the recourse feature of most secured funding (e.g. repo), the two types of securities are more substitutes than what is thought. I treat fixed assets as negative bank net worth.

---

[65] There should not be double-counting of debt securities held by other MFIs.

$$\begin{aligned} D &= \mathit{Deposits\,of\,euro\,area\,residents\,(excluding\,MFI\,and\,gvt) + net\,MMF\,shares} \\ &\quad \mathit{+debt\,securities\,and\,CP\,up\,to\,}2\,\mathit{years + net\,remaining\,liabilities} \end{aligned}$$

Whereas bank net worth ('risk-neutral wealth') is:

$$n^B = \mathit{Capital\,and\,reserves + debt\,securities\,and\,CP\,over\,}2\,\mathit{years - fixed\,assets}$$

euros and collection is end of period. All quantities will be scaled by Eurozone GDP that quarter (which was € 2.2trn in 2011q3).

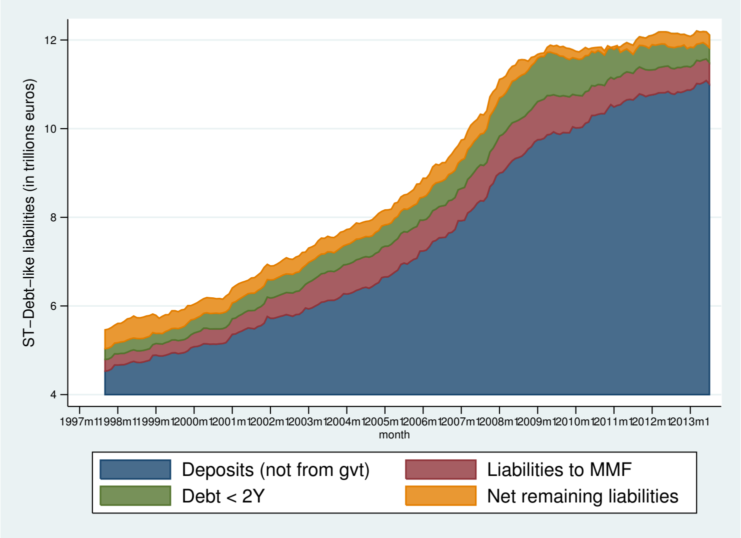

Figure 1.15: *Short-term debt issuance of Eurozone banks. Amounts in trillion €.*

Calibration

This section presents a calibration of the theoretical model based on the European aggregates just calculated. All quantities in trillions of euros[66]. A complication arises from the treatment of the corporate sector balance sheet. The asset side of the corporate balance sheet is a juxtaposition of cash holdings and actual risky $\beta > 0$ investments. I net out the former by carving out the cash part, and merge it with risk-averse net worth.

Table 1.1 gives the exogenous parameters estimated from available sources, and the equilibrium variables delivered by the calibrated model. The success of the calibration to fit observable portfolios provides support for the safety multiplier view of banking developed in this paper.

1.6.2 Time-series tests

In this subsection I provide suggestive evidence for the empirical prediction of the model: when the supply of safe assets shrinks, bank leverage shrinks, real lending shrinks and bank holdings of safe assets increases. As a motivating evidence that banks act as insurers to risk-averse investors, the mark-to-market value ($p_1x_1 + p_2x_2$) of the overall EU bank balance sheet is remarkably stable.

Positive comovement of safe public and private debt

I argue here that the European banking crisis (deleveraging) has been caused by a *shortage of public safe debt*, following the loss of safe asset status of Southern Europe government bonds. This tests for the safety multiplier comparative statics of Proposition 1: $\frac{\partial D}{\partial S^b} > 0$, with public safe assets S^b and private debt D computed as just described.

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[66] Estimations for the wealth of Monetary and Financial Institutions and of households wealth are computed from the ECB dataset (http://sdw.ecb.europa.eu/reports.do?node=100000161).

Table 1.1: *Calibration of the static model on the Eurozone economy.*

All quantity variables are given in trillion €. Price variables for public debt are from German 10 year bond. The probability of bank default is inferred from the CDS of the 90 largest Eurozone banks.

Parameters		
Bankers wealth	$n^B = 5$	
Investors wealth	$n^L = 15$	
Investors risk-aversion	$\gamma_L = 2$	
Risky technology volatility	$\sigma_1 = 0.4$	
Risky technology return	$r^1 = 0.12$	
Public debt beta	$\beta = -0.3$	
Public debt volatility	$\sigma_2 = 0.2$	
Public debt supply	$S^b = 3.5$	
Equilibrium variables		
Bank portfolio expectation	$\mu_u = 0.3$	
Bank portfolio volatility	$\sigma_u = 0.2$	
Observable quantity variables		
	Model	Data
Bank risky assets holdings	$x_1 = 13$	$x_1 = 15$
Bank safe assets holdings	$x_2 = 2$	$x_2 = 4$
Bank debt	$D = 13.5$	$D = 14$
Bank leverage	$D/n^B = 2.7$	$D/n^B = 2.8$
Investors direct risky assets holdings	$y_1 = 0$	$y_1 = 0.5$
Investors direct safe assets holdings	$y_2 = 1.5$	$y_2 = 2$
Observable price variables		
Public debt yield	$r^{safe} = 2\%$	$r^{safe} = 1.8\%$
Bank debt yield	$r^{bank} = 5\%$	$r^{bank} = 4\%$
Bank probability of default	$P^{def} = 6.7\%$	$P^{def} = 4.5\%$

This test aims at answering the following question: when the supply of public safe assets in Europe shrank, did privately safe asset creation stepped in, as an equilibrium outcome, in order to satiate the demand for safety, as hinted by Gorton and Metrick (2012)? Or in the contrary, did it even more crunch private safety, according to my safety multiplier mechanism?

I provide suggestive evidence of the quantity comovement between public safety and private safety in Europe, hence putting forward the safe asset shortage as a key cause of the Eurozone current recession. The empirical tests of the model revolve around the impact of public debt beta on private leverage. Not only more public debt, but also 'better' hedging qualities of public debt under the form of lower beta, enables the financial sector to sustain high leverage. I use sovereign risk as an instrument to identify this negative beta channel of public debt on private debt.

It replicates what happened in Europe over the last decade: a decrease in the supply of safe public debt. It triggers a non-conventional deleveraging of banks and a credit crunch due to the safety multiplier. There are three regimes in the time period: increase in safe asset supply (2001-2007), even more so with Keynesian stimulus plans (2008-2010). Then for 2010-2013, there is a split: the total amount of Euro public debt increases, but the amount of it which is safe (no-GIPSI) actually decreased. On a longer time horizon, my model could also shed light the double leverage cycle: public debt and private debt (both domestic and external) documented in Reinhart and Rogoff 2008 and Jorda et al. 2013. One is used as an input to the production of the other, this is why they comove positively.

Beyond the suggestive evidence of positive comovement from 1.16, the model calls the following specification:

$$D_t = \delta + \gamma_1 \, S_t^b + \gamma_2 \, \beta_t + \epsilon_t$$

and the new empirical predictions are: $\gamma_1 > 0$ and $\gamma_2 < 0$. All quantities variables are scaled by GDP. Results of this specification are given in Table 1.2 and illustrated in Figure 1.16. I always include a trend regressor to absorb the economy growth effect.

Table 1.2: *Quantities tests: regression of leverage D and risky holdings x_1 on the supply S^b, the beta β of public safe assets and the interaction between the two $S^b * Max(-\beta, 0)$. All quantities are scaled by EU GDP.*

	(1) D b/se	(2) D b/se	(3) D b/se	(4) x1 b/se	(5) x1 b/se	(6) x1 b/se
S_b/GDP	1.749***			1.889***		
	(0.16)			(0.17)		
Germany β		-0.404			-0.265	
		(0.87)			(0.94)	
$S_b\ \beta_{GER}$			0.275			0.203
			(0.35)			(0.38)
Constant	-0.464	4.763***	4.641***	-0.593	5.098***	4.981***
	(0.48)	(0.25)	(0.26)	(0.51)	(0.27)	(0.28)
N	42	42	42	42	42	42
R-squared	0.76	0.01	0.01	0.76	0.00	0.01

* p<0.10, ** p<0.05, *** p<0.01

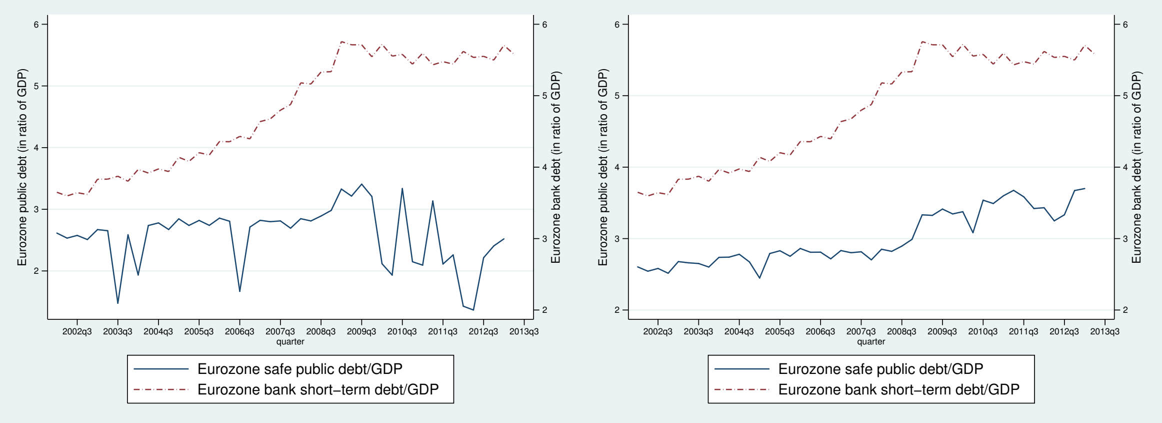

Figure 1.16: *Public and Private safety positive comovement. The one on the left uses the rigorous measure of public safe asset supply: $S^b = \sum_{Eurozone\,country\,k} 1_{\{\beta_k<0\}} S^k$. Whereas the one on the right uses a smoother version of the measure: $S^b = \sum_{Eurozone\,country\,k} S^k \left(1 - e^{-Max(0,\beta_k)}\right)$.*

Sovereign risk as an instrument to the causality of beta on private leverage Naturally, the above specification is plagued with reverse causality and omitted variable issues, especially on γ_2[67]. To mitigate these concerns, I instrument public debt beta with sovereign CDS. This empirical strategy is inspired by the new result of the model: sovereign risk should impact private leverage only through public debt beta[68].

Therefore the IV regression I run is, on a first stage, public debt beta on sovereign CDS (as a measure of sovereign risk):

$$\beta_t = \delta + \hat{\gamma}_1 \, sov \, CDS_t + \epsilon_t$$

And then use the resulting $\hat{\beta}_t = \hat{\gamma}_1 \, sov \, CDS_t$ in the second stage:

$$D_t = \delta + \gamma_2 \, \hat{\beta}_t + \epsilon_t$$

The model predicts $\hat{\gamma}_1 > 0$ and $\gamma_2 < 0$. Sovereign CDS is obtained from Markit. These results help to distinguish my theory from moral hazard theories of the sovereign-bank diabolic loop, who have no predictions on bank leverage. The IV strategy takes advantage of the exogenous variations in the supply of safe assets in Europe[69] in order to identify the impact of public debt beta on bank leverage. It could also be used, in an IO approach, to trace down the demand curve for safe assets. Here a key issue is the sovereign risk heterogeneity within Europe. This calls to refine the IV specification, taking into account the open economy. It is tantamount to a test of the open economy model.

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[67] For γ_1, causality of S^b on D is more reasonable, as long as stay in a Greek style crisis and not an Irish style crisis.

[68] This is a test of the fragility of the safety multiplier to sovereign CDS. I abstract from the reverse feedback loop of implicit guarantees (the Irish style sovereign crisis) to focus on Greek style sovereign crisis: sovereign risk is causal to bank risk.

[69] cf. "Global pool of triple A status shrinks 60% (Financial Times, March 26 2013).

Effect of public debt supply on the composition of banks portfolios

Credit crunch Proposition 3 says that lending to the real economy decreases when the supply of safe assets S^b shrinks and its hedging properties get worse (higher β). I therefore compute x_1 from ECB data as the sum of all Monetary and Financial Institutions loans to the real sector (and HH – should be fixed) as described above, and then run the following specification:

$$x_{1,t} = \delta + \gamma_1 \, S_t^b + \gamma_2 \, \beta_t + \epsilon$$

where the model predicts: $\gamma_1 > 0$ and $\gamma_2 < 0$. Results are given in Table 1.2.

Banks safe asset holdings The last quantity test concerns Corollary 3: banks safe asset holdings increase when safe asset supply shrinks. I compute a measure of safe asset holdings x_2 by banks as the sum of bank holdings of public debt. I also do the same for risk-averse investors (money market funds) to compute y_2. I then run the following specification[70]:

$$x_{2,t} = \delta + \gamma_1 \, S_t^b + \gamma_2 \, \beta_t + \gamma_3 \, \beta_t * S_t^b + \epsilon$$

where the model predicts: $\gamma_1 < 0$ and $\gamma_2 > 0$. The coefficient $\gamma_2 > 0$ on beta indicates that having better hedging properties (low β) make banks to need less of public debt in order to ensure the safety of their debt, hence levering up. This is a defining test of my theory against other diabolic loop theories (financial repression, bailout expectations). *asset pricing* properties of public debt drive these holdings, not *preferences* from investors. Uhlig (2013) and Acharya-Drechsler-Schnabl (2013)'s Irish style bank to sovereign crisis do not have prediction relating asset pricing properties of public debt to asset holdings. Similarly, I reject the financial repression hypothesis for France and Germany[71].

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[70] Another specification could focus on safe asset holdings by risk-averse investors (i.e. non banks): y_2 in the model. E.g. insurance companies are large holders of government debt for anti-transformation purposes.

[71] Deep secondary markets exist for these government bonds. Moral hazard and bailout expectations by banks do not seem to hold.

I also test the additional predictions of the model on banks asset holdings. When public debt beta is low enough, banks are able to ensure the safety of their private debt even with a small amount of safe assets on their balance sheet. Therefore the banks safe assets holdings negatively comove with the hedging properties of public debt. I test this prediction, along with the safe asset driven credit crunch prediction: real lending positively comoves with the hedging properties of public debt. The model predicts that in crises times, banks become the natural holders of government debt. The empirical analysis identifies this reconcentration from risk-averse agents (insurance companies, pension funds) to risk-neutral agents (banks) in Europe.

A confounding explanation could be that safe asset holdings are just liquidity hoarding by banks, driven by a precautionary motive, or requirements coming from regulation[72]. Indeed the regulation of liquidity that is progressively introduced requires banks to hold High Quality Liquid Assets (HQLA) in order to satisfy a Liquidity Coverage Ratio. However, negative beta assets holdings substantially exceed these requirements, which are therefore not binding. These holdings could also be induced by capital requirements, which weight each type of assets according to its risk. Nevertheless, capital requirements would be innocuous, as banks would anyway be willing to hold safe assets to hedge the positive beta part of their balance sheet.

1.6.3 Cross-sectional test: safe assets holdings and refinancing costs

This section tests in the cross-section of banks the mechanism of the paper, i.e. that government debt is used as an input in safety creation. It aims at proving that South banks did redomesticate due to moral suasion, whereas North banks played their role of diversification. To do this, I run cross-sectional tests of banks easiness to refinance (bank CDS) on a measure of the quality of the diversification of their balance sheet: $\frac{\sigma_2 x_2}{\sigma_1 x_1}$.

For these cross-sectional tests, I use data on sovereign debt holdings by banks from the

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[72] It should be noted that the safe asset holdings of banks are much more important than the cash holdings of the corporate sector.

European Banking Authority. They provide granular snapshots of the balance sheet of the 90 largest European banks. In these cross-sectional tests, in order to compute x_1 and x_2 from EBA data, I use *Exposure at Default* (EAD). For on-balance-sheet transactions, EAD is identical to the nominal amount of exposure. For off balance sheet, it is modeled by the bank itself. This is the relevant measure of $p_1 x_1$ and $p_2 x_2$.

I explore here how the holdings of sovereign debt impacts funding costs *in the cross-section* of banks. For examplle, the French bank BNP holds 15% of its total EAD in its portfolio of sovereign debt. I want to show that the more diversified this one is, the better its refinancing cost will be (i.e. low CDS). For each bank i, I compute the holdings of safe public debt:

$$x_{2,i} = \sum_{Eurozone\, country\, k} 1_{\{\beta_k < 0\}}\; x_{k,i}$$

Figure 1.17 explores the explanatory power in the cross-section of bank CDS of either risky public debt holdings and safe public debt holdings. It illustrates that *safe asset holdings* have far more explanatory power in the cross-section than risky asset holdings. Table **??** provides the results of the corresponding regression:

$$CDS_i = \delta + \gamma_1\, x_{2,i} + \epsilon_t$$

The results show that the cross-section of bank CDS is primarily driven by *safe public debt holdings* (i.e. the object of interest of this paper), and not the *risky debt holdings* . This result is not inconsistent with the narrative developed in Acharya and Steffen (2013) that banks did engage in a carry trade on risky debt. Nevertheless, it shows the cross-section of banks refinancing costs is better explained by holdings of public debt.

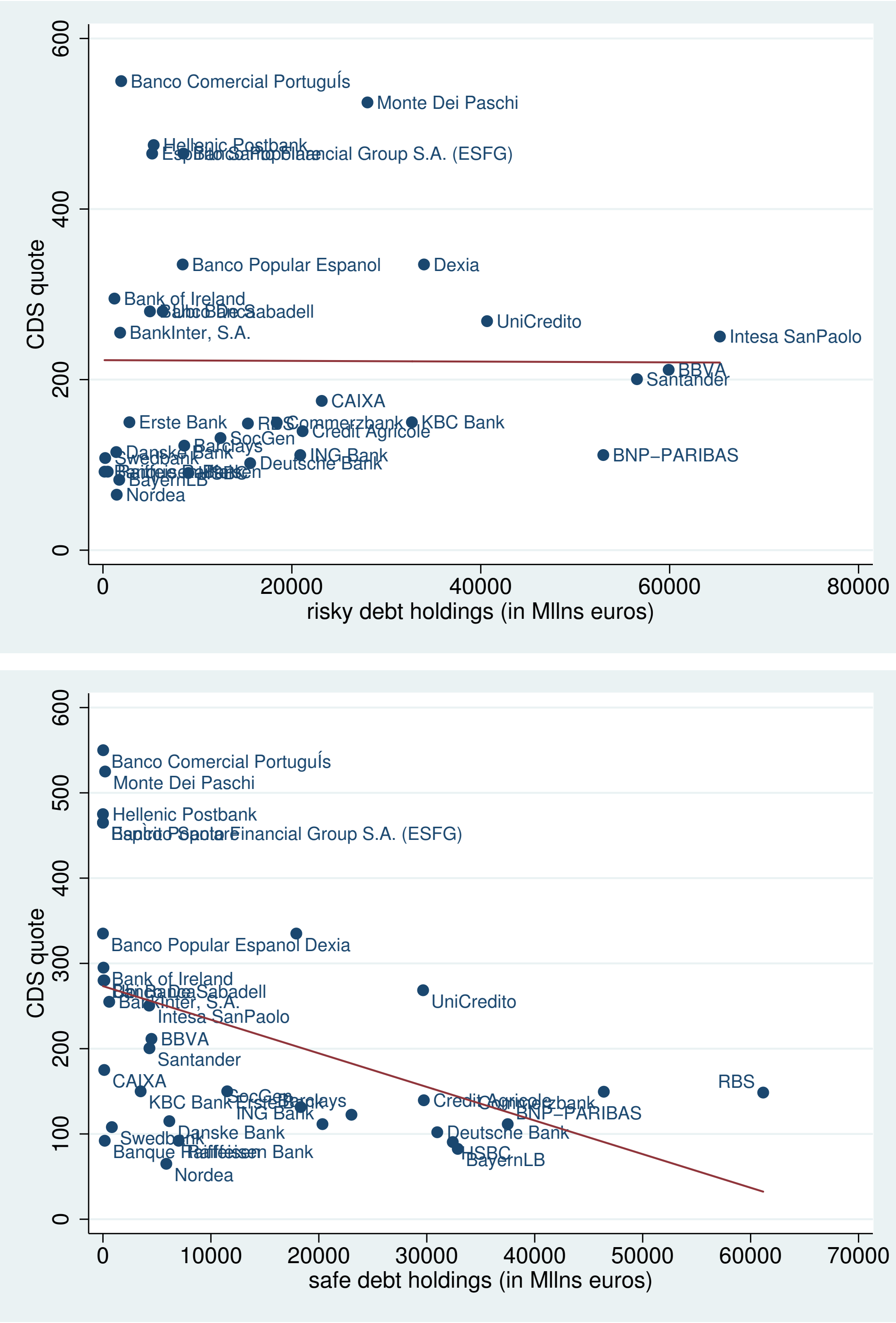


Figure 1.17: *CDS quotes vs. bank holdings of risky (top) and safe (bottom) public debt:* $x_{2,i} = \sum_{Eurozone\,country\,k} 1_{\{\beta_k<0\}}\; x_{k,i}$. *Public debt is assigned in the risky and safe categories according to their beta at the stress test date, 2011q3. Risky public debt* ($\beta > 0$) *are: Belgium, Greece, Ireland, Italy, Portugal, Spain. Safe public debt* ($\beta < 0$) *are: Austria, Cyprus, Finland, France, Germany, Luxembourg, Malta, Netherlands, Slovakia, Slovenia.*

Table 1.3: *Regression of bank CDS on safe public debt holdings.*

Explanatory power of public debt holdings $x_{2,i} = \sum_{Eurozone\,country\,k} 1_{\{\beta_k<0\}}\ x_{k,i}$ in the bank CDS cross-section. Safe_debt_all and Risky_debt_all includes all the countries listed in the former figure, whereas Safe_debt_core is only France and Germany, and Risky_debt_core is only Spain and Italy. All quantities are in billion € and CDS in basis points.

	(1) CDS b/se	(2) CDS b/se	(3) CDS b/se	(4) CDS b/se
safe_debt_all	-3.940***			
	(1.38)			
safe_debt_core		-4.437**		
		(1.71)		
risky_debt_all			-0.042	
			(1.32)	
risky_debt_core				0.444
				(1.44)
Constant	273.331***	265.650***	222.775***	215.866***
	(28.43)	(28.05)	(33.65)	(31.89)
N	33	33	33	33
R-squared	0.21	0.18	0.00	0.00

* p<0.10, ** p<0.05, *** p<0.01

1.6.4 Asset pricing tests

The model has sharp empirical predictions on the price of the public safe asset (government debt) and private safe asset (bank debt). This focus on the safe asset price is a departure from the literature, which mainly focus on the risky asset price, i.e. the equity premium (see Zhang (2013) for a recent example).

Public safety premium

The model predicts that the price of the public safe asset should increase with its hedging properties on the risky asset, and the more so the more the safety mechanism D is at work. As a result I run the following specification on German government yields:

$$r_t^{safe} = \delta + \gamma_1 \beta_t + \gamma_2 \beta_t * D_t + \epsilon$$

where the model predicts: $\gamma_1 < 0$ and $\gamma_2 < 0$. Results of this specification are given in Table 1.4 using yields at the left hand side variable, and these yields are graphed in Figure 1.18.

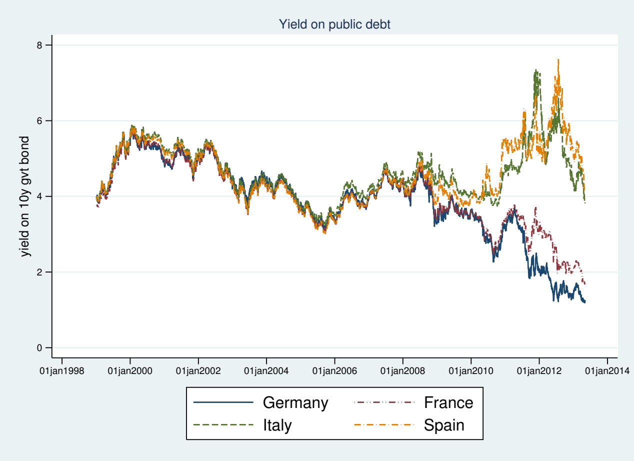


Figure 1.18: *10-year government bond yields in the Eurozone.*

This specification enables to estimate the safety premium on Euro public debt. The interaction term $\gamma_2 \beta_t * D_t$ helps to capture the complementarity effect of public safe assets, through the safety multiplier mechanism. It also implies that the safety premium is even more important in countries where the safety multiplier is at work, such as in Europe.

I can also test how this safety premium depends on *who* owns the debt, banks or households, i.e. run the specification:

$$r_t^{safe} = \delta + \gamma_1 \, n^B + \gamma_2 \, n^L + \epsilon$$

where n^B is bank net worth and n^L is risk-averse net worth. Results of this specification are also given in Table 1.4.

Table 1.4: *Determinants of the public safety premium: r^{safe}.*

	(1) rsafe b/se	(2) rsafe b/se	(3) rsafe b/se	(4) rsafe b/se	(5) rsafe b/se	(6) rsafe b/se
S^b/GDP	0.455**				0.435*	
	(0.22)				(0.22)	
S^bGER/GDP		-5.908***				
		(0.54)				
$Germany * \beta$			0.763		1.171	
			(0.92)		(0.83)	
$Italy * \beta$				-1.917***		
				(0.21)		
n_B/GDP						-1.955***
						(0.29)
Constant	2.388***	8.362***	3.683***	3.370***	2.709***	7.126***
	(0.59)	(0.44)	(0.24)	(0.08)	(0.62)	(0.54)
N	41	41	42	42	41	42
R-squared	0.10	0.76	0.02	0.67	0.14	0.53

* p<0.10, ** p<0.05, *** p<0.01

Safety spread and predictive regression

An insight of this paper is that is not one but two safety premia to consider: one on public debt and one on private debt. I focus here on the spread between the public safety premium and the private safety premium, which is equal to $r^{safe} - r^{bank}$ and which can be called a safety spread or a Safety Mismatch Index (SMI). Figure 1.19 gives the time-serie of this safety spread.

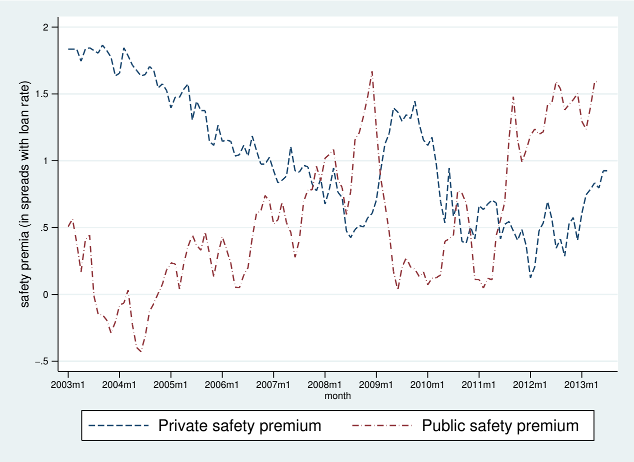


Figure 1.19: *Private and public safety premia.*

I use for r^{safe} the weighted yield on all the public debt that is deemed as safe according in the definition employed in section 5.1.1. of construction of the stock of public safety. Here again, I emphasize that the focus of this study is *safe public debt* holdings, and not *risky public debt holdings* (which is Acharya-Steffen focus).

The empirical counterpart of r^{bank} is computed as the weighted average of the deposit rate (source: ECB) and the wholesale funding rate (computed as the sum of the deposit rate and the CDS spread). Weights are notional amounts of deposits and wholesale funding of the Eurozone banking balance sheet.

The model derives an analytical expression for the SMI, which shows that SMI decreases with public debt beta β, and the more so the more the bank balance sheet is initially diversified: $\frac{\sigma_2 x_2}{\sigma_1 x_1}$ high. I consequently run the following specification:

$$SMI = r^{safe} - r^{bank} = \delta + \gamma_1\, \beta_t + \gamma_2 \left(\frac{\sigma_2 x_2}{\sigma_1 x_1}\right)_t + \gamma_3\, \beta_t * \left(\frac{\sigma_2 x_2}{\sigma_1 x_1}\right)_t + \epsilon$$

where the model predicts: $\gamma_1 > 0$, $\gamma_2 < 0$ and $\gamma_3 < 0$. Results of this specification are given in Table 1.5 and are consistent with the model.

Table 1.5: *Determinants of the safety spread (SMI):* $-SMI = r^{bank} - r^{safe}$

	(1) SMI b/se	(2) SMI b/se	(3) SMI b/se	(4) SMI b/se	(5) SMI b/se	(6) SMI b/se	(7) SMI b/se
S^b/GDP	0.254				0.217		
	(0.28)				(0.27)		
$S^b_G ER/GDP$		-5.453***					
		(0.98)					
Germany β			1.901*		2.222**		17.117
			(1.01)		(1.01)		(19.29)
Italy β				-1.409***			
				(0.36)			
$\frac{\sigma_2 x_2}{\sigma_1 x_1}$						-1.278	-10.668
						(6.16)	(16.39)
interact							-50.732
							(64.43)
Constant	-0.165	4.921***	0.888***	0.355***	0.443	0.830	4.058
	(0.74)	(0.81)	(0.27)	(0.13)	(0.75)	(1.80)	(4.82)
N	41	41	42	42	41	42	42
R-squared	0.02	0.44	0.08	0.28	0.13	0.00	0.10

* p<0.10, ** p<0.05, *** p<0.01

$\gamma_1 > 0$ says that better hedging properties of the public safe asset (low β) induces banks to lever up more, hence a low SMI. $\gamma_2 > 0$ says that a improperly diversified bank balanced sheet ($\frac{\sigma_2 x_2}{\sigma_1 x_1}$ closer to 0 than 1) also implies a low SMI. $\gamma_3 > 0$ says that these two effects reinforce each other. The γ_2 effect can also be interpreted as saying that the *concentrated ownership* of public debt is priced in SMI: low bank safe asset holdings implies improperly diversified bank balance sheets, hence low SMI.

Finally, I run the predictive regression:

$$D_t = \delta + \gamma_1 \, SMI_{t-1} + \epsilon$$

According to the model, the safety spread SMI should be a strong predictor of leverage. Table 1.6 gives the results of the regression and confirms that SMI predicts leverage.

Figure 1.3 suggested this property, and it is confirmed in Table 1.6.

Table 1.6: *Predictive regression of the safety spread (SMI):* $r^{safe} - r^{bank}$ *for bank leverage D.*

	(1) DGDP b/se	(2) DGDP b/se	(3) x1GDP b/se	(4) x1GDP b/se
SMI	0.564***		0.658***	
	(0.08)		(0.08)	
r^{safe}		-0.497***		-0.578***
		(0.11)		(0.12)
Constant	5.156***	6.643***	5.506***	7.232***
	(0.08)	(0.41)	(0.08)	(0.42)
N	42	42	42	42
R-squared	0.54	0.32	0.65	0.38

* p<0.10, ** p<0.05, *** p<0.01

Impact on bank profitability

Finally, low SMI hits negatively banks expected profitability. It is in the spirit of the Fed board paper about bank profitability to interest rate innovations. I therefore run as final specification, where the model predicts: $\gamma_1 > 0$:

$$\Pi_t^B = \delta + \gamma_1 \, SMI_t + \epsilon$$

Banks profitability is computed from Bankscope data as the recurrent income:

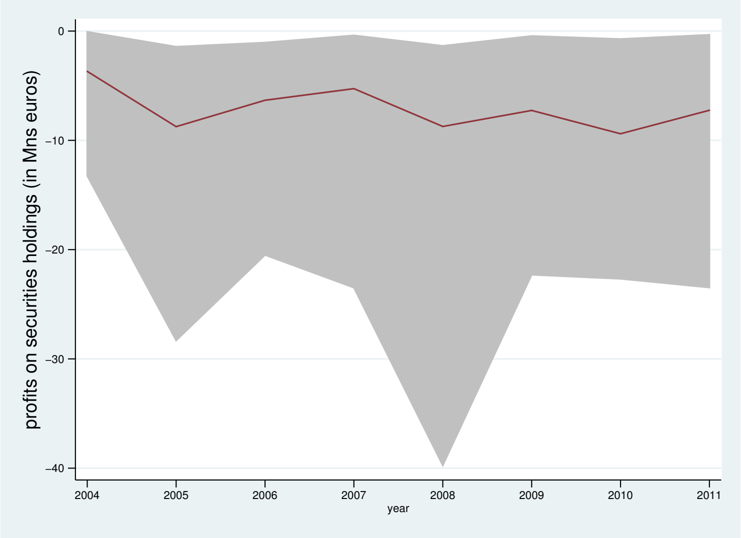
$$\Pi^B = interest\, income - refinancing\, cost - operating\, expenses$$

Refinancing costs are using data from Bankscope ('interest expense' entry) and banks' CDS (in bps) as a better measure of refinancing cost. I am matching *dividend gains* and not *capital gains*. I also explicitly take into account the operating costs of running a bank. The model is able to handle constant returns to banking, i.e. I do not need to rely on a convex cost of banking to pin down endogenous leverage. Therefore adding operating costs is innocuous in the model.

These assumptions enable to estimate the part of banks profits that comes from safe asset holdings:

$$\begin{aligned} \Pi^B | safe\, assets &= yield * securities\, holdings \\ &\quad - refinancing\, cost * securities\, holdings / total\, assets \\ &\quad - operating\, expenses * securities\, holdings / total\, assets \end{aligned}$$

Figure 1.20 illustrates the distribution of profits from safe assets holdings. It shows that most banks are loosing money on these holdings, consistent with the fact that the carry trade on these holdings is negative.



Profits from safe assets computed as: *yield * securities holdings - (refinancing costs+operating expenses) * securities holding/total assets* (source: Bankscope). The shade area gives the 25th and 75th percentiles.

Figure 1.20: *Distribution of profits from safe assets holdings.*

1.7 Discussion

1.7.1 Comparison with existing literature

Role of banks

The traditional banking literature sees the role of banks as mitigating agency frictions: Diamond 1984, Diamond and Rajan 2001b, or more recently as mitigating the adverse selection due to information acquisition: Holmstrom and Ordonez 2013. This literature is not suited to explain the large holdings of liquid, publicly traded and ’safe’ securities by banks.[73] Theories of banking on information asymmetry and banks superior ability to “keep secrets” do not apply to bank holdings of highly liquid and scrutinized assets such as government debt.

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[73] The large holdings of government securities also challenge the view that financial integration helped global banks economize on their liquid assets holdings (Castiglionesi et al. forthcoming).

DeAngelo and Stulz 2013 and Philippon 2012 insist on the liquidity/safety creation role of banks. However they do not model for it, even though Philippon 2012 features production function in reduced-form for the supply of safe assets[74]). These articles do not consider public debt as an already safe asset and its role in safety creation, and do not feature risk-aversion. More generally, macroeconomic models take the cost of financial intermediation (the spread between lending and borrowing rates) as exogenous[75]. Landier et al. 2012 and Begenau et al. 2012 insist on the interest rate exposure as the key feature of banks business. However they do not have a model of banks optimizing behavior, the action still comes from the asset side and not the liability side of banks. They do not explain *why* banks have this interest rate exposure, and what the implications are for monetary policy. In my model interest rates are equilibrium outcomes, not causal variables. Bianchi and Bigio (2013) takes the role of banks as granted (loan supply) and do not model a safety mismatch. The literature on liquidity hoarding views cash as a commitment device. Most recently, Calomiris et al. (2013), following Calomiris and Kahn (1991), lays down a theory of cash holdings of banks as a discipline device against a moral hazard friction. The present model does not rely on an agency friction to explain 'safe asset' holdings by banks. Generally, the theoretical literature has found it puzzling to explain why banks would hold assets yielding lower returns than the deposit rate.

In the present model, the role of banks is to create safety by portfolio construction. They choose their asset holdings as inputs to safety creation. Therefore the key engine of banking is diversification. The role of banks is *pooling* and *tranching* in a General Equilibrium environment, with an enforcement friction: banks cannot commit to repay debt. Contrary to Gennaioli et al. 2013b, I analyze banks in general and not shadow banking, and I consider macro shocks and not only micro idiosyncratic shocks that are washed away by a law of large numbers. In my model, bundling a bearish asset with a bullish asset enhances the value of bank collateral. When the safe asset is government securities holdings, sovereign risk acts the neglected risk. It is also reminiscent of Gennaioli et al. (2013a): finance helps to preserve wealth. My model does microfound *how* banks are able to preserve wealth by issuing securities with quasi-flat payoff, which caters to risk-averse investors. This role of banks as *safety creators* is alternative to Kashyap et al. 2002 model of banks as *liquidity providers*. It has the distinctive prediction of positive comovement of public and private safety. It insists not on the *immediacy* of government debt holdings but on its *hedging* value. Similarly, it insists on safety transformation by banks, less on their liquidity transformation. Liquidity (deposits, repo, MMF) is a bit different as safety: liquidity is immediacy (promise of cash redemption), as in Diamond and Dybvig (1983). Similarly, Gale and Ozgur (2005) studies capital structure when banks insure against liquidity shocks, which is different from the role they have in my environment of insuring against macro shocks. I do not need to resort to preference shocks, which are hard to map to a primitive. On the contrary it has similarities with Gorton and Pennacchi 1990 whose purpose of banking is liquidity creation, but the latter is in an asymmetric information partial equilibrium environment. I share with Geanakoplos 1997 and Simsek 2013[76] the focus on General Equilibrium effects of heterogeneity and the analysis of the determinants of leverageintroduce a second asset, which I call 'safe asset', in exogenous fixed supply. I also depart from beliefs heterogeneity, and I use instead risk-aversion heterogeneity to obtain endogenous leverage. Such environment is more tractable to handle multiple assets.

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[74] He exogenously posits an intermediation requirement for government bonds of 1/10.

[75] Such as in Curdia and Woodford (2009).

Sovereign debt

Tradiional literature on sovereign debt focuses on issues such as sustainability, reputation and default (Hellwig-Lorenzoni (2012), Cole-Kehoe (2002), Krueger-Perri (2010) and Aguiar et al. (2013)). It mostly ignores the key role of banks portfolio choice in their pricing. I argue that banks are key players that decide of the price of domestic debt by pinning down the price of safety. Moreover, in an international environment, global banks adds an additional layer of endogeneity through their portfolio choice between domestic debt and foreign debt. On the other hand, the recent literature on global banks (Schnabl (2009), Ivashina et al. 2012, Bruno and Shin 2012) does not focus on the role of their sovereign debt holdings.

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[76] Cao (2013) is a dynamic version of the original Geanakoplos environment. However the equilibrium is solved numerically.

Recent literature linking sovereign debt and bank debt focuses on bailout expectations to microfound the interconnection between bank risk and sovereign risk. Sovereign debt is solely treated as a *store of value*: Acharya et al. (2011b), Mengus (2012) and Gennaioli et al. (forthcoming). These papers feature a 3-period model in which real investment is prohibited in the first period and sovereign debt is used to transfer wealth from the first to the second period. On the contrary, I do not introduce bailout expectations but I simply treat public debt as a given asset class. I circumvent the ad hoc 3-period timeline of public debt models, as my agents' simultaneous hold government debt and risky projects. Furthermore, in Gennaioli et al. (forthcoming), government debt features procyclical returns, which is counterfactual with the negative beta of public debt. Bolton and Jeanne (2011) exogenously posits the collateral value of government debt, whereas I microfound from its hedging properties. Arellano and Ramanarayanan (2012a) emphasizes the role of public debt as an insurance against interest rate fluctuation *from the government perspective.* Broner et al. (2013) is a model of crowding out of real investment whose mechanism relies on the presence of secondary markets for sovereign debt. They assume that foreign investors do not have *investment*access to local equity market, whereas I restrict them from *funding* access in the local market. Reinhart and Sbrancia 2011 puts forward financial repression as the key reason of government debt holdings by banks. Angeletos et al. (2013) is a normative analysis of the liquidity role of public debt for debt issuance, but as in Bolton and Jeanne (2011) they take the collateral value of public debt as exogenous. Martin and Ventura (2012) construct a model that has a similar feature to mine, that a bubble emerges for its collateral properties. However, they do not relate its value to the value of the risky asset and its impact on leverage. Broner et al. (2010) insists on the role of secondary markets for public debt to discipline the sovereign, I insist on their role to provide safe assets.

On the empirical side, Acharya and Steffen (2013) documents that European banks invested heavily in sovereign bonds of countries of the Eurozone periphery and interpret this as moral hazard. They also document a redomestication of domestic debt ownership, and interpret it as an effect ECB funding collateral requirements. However, this last interpretation does not explain why foreign banks could not do the same type of carry trade. On the contrary, in my open economy environment, heterogeneity in holdings is driven by special access to local funding. Bofondi et al. (2013) documents that foreign banks substituted to the credit crunch of domestic banks in Italy. They do not analyze the portfolio holdings of securities of these global banks. Uhlig (2013) is a moral hazard model of the South free-riding on the expectation of a North bailout(through the ECB). Its model presents however the counterfactual feature that in equilibrium, North banks hold zero South banks, at odds with Acharya and Steffen (2013) evidence. In contrast, the pure asset pricing perspective developed here enables to rationalize cross-border holdings of sovereign debt.

Safe assets

Traditionally, the literature treats safe assets as stores of value. Woodford (1990), Holmstrom and Tirole (1998), Caballero and Krishnamurthy (2008), Kocherlakota (2007) and Kocherlakota (2009) are all environments in which the government improves welfare by relaxing through public debt exogenous collateral constraints imposed on the private sector. On the contrary, exogenous financial frictions are absent from my environment. The government improves welfare through the safety multiplier. The rational bubble literature insists on the scarcity of asset supply in general. On the contrary, I insist on the scarcity of *safe* assets. My environment endows public debt with an even more crucial liquidity role, through the safety multiplier. More recently, (Yared, 2013) introduces liquidity shocks in a Woodford (1990) and obtains substitutability between public debt and private debt. On the contrary, I obtain complementarity between the two.

In the macroeconomic literature, Caballero and Farhi 2013 emphasizes the shortage of safe assets as a key macroeconomic force that can lead to a safety trap. They develop a valuation framework to assess the policies remedies to this safety trap. In contrast, my model features optimizing banks with endogenous leverage. Bank leverage is jointly determined at equilibrium with the safe asset price. Compared to Gorton and Metrick (2012), I emphasize the negative beta property of safe assets, whereas in the latter safe assets are defined as keeping a constant value over time. My 'safety' definition, i.e. assts with negative beta, is similar to the one used by Maggiori (2013) for the analysis of the dollar currency in an international context. That the government should issue negative beta securities is an idea mentioned in Pagano (1988). On the crowding out of private debt by public debt, Krishnamurthy and Vissing-Jorgensen (2013a), Gorton and Metrick (2012), Gorton and Ordonez 2013[77] and Gourinchas and Jeanne 2012 emphasize the substitutability between private and public safe assets. My crowding-in comes from endogenous leverage and public debt being used as input to private debt creation. Sunderam 2012 and Greenwood et al. 2010 microfounds safety demand through money in the utility, whereas I stick to parsimonious risk aversion heterogeneity. Regarding normative implications, Stein 2010 argues there are too much safe debt due to pecuniary externality, Caballero and Farhi 2013 argues there are not enough safe assets. I argue there are not enough *public safe assets* due to a lack of negative beta assets in the economy.

Asset pricing

I relax the standard assumption in standard portfolio allocation theory of Campbell and Viceira (2002) that the risk free asset is in perfectly elastic supply. In my environment, the price of the safe asset results from its scarcity, and in turn it impacts private leverage. In the field of closed economy asset pricing, Campbell et al. (2013b), Campbell et al. 2013a and Backus and Wright 2007 explain changing betas of US government bonds by changes in inflation expectations (monetary policy stance). They develop Neo-Keynesian models to emphasize the nominal nature of government bonds.

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[77] This paper exogenously assumes that private collateral is more information-sensitive than public collateral.

In open economy asset pricing, Coeurdacier and Gourinchas 2013 and Bhamra et al. (2013) insist on the role of bonds to hedge real exchange rate risk. Models in this literature feature complete markets, hence Modigliani-Miller holds and leverage is not determinate.

1.7.2 Conclusion

In this paper I develop a banking model in which banks' economic role is to provide insurance to risk-averse investors against macroeconomics shocks. I analyze a model of safe assets creation that features risk aversion heterogeneity and incomplete markets. Public debt has an endogenous negative beta by anticipation of flight to safety in the recursive equilibrium. As public debt is used as input to private debt issuance, the model delivers a safety multiplier. The open economy extension introduces heterogeneity in sovereign risk. Sovereign risk weakens the hedging properties of public debt. The open economy model captures redomestication of sovereign debt by the interaction of risk-shifting motives with endogenous leverage. The empirical discussion points towards the shortage of safe public debt as the key driver of the European debt crisis. Europe is more fragile to the safety multiplier due to the limited participation of investors in the risky technology. The spread between public debt and private debt can be used as a financial stability indicator to reveal the extent of the safety multiplier.

The model has crucial normative implications. The private equilibrium is constrained inefficient because private agents do not internalize the beneficial effects of negative beta securities, which makes them issue too much short-term. Issuing public debt is welfare improving, because it is endogenously endowed with negative beta in the recursive equilibrium. This leads to a higher level of safe assets overall in the economy. Issuance of long-term public debt improves welfare as long as it does not flood the economy up to a point at which its hedging properties are weakened. This provides a well-founded rationale for monetary dominance: the issuance of public liabilities can be used to manipulate prices. Public debt can therefore be seen as an instrument of unconventional monetary policy.

Chapter 2

Leverage and Reputational Fragility in Credit Markets[1]

2.1 Introduction

This paper investigates the determinants of the leverage of financial intermediaries through short-term secured funding markets. The sudden freeze of such markets in 2008 was one of the most surprising and unexpected developments of the financial crisis.[2] Secured debt markets were thought to be immune from traditional bank runs, as each contract is collateralized by a distinct pool of assets. I develop a model where the characteristics of collateral interacts with borrowers' reputation and with the history of the borrower-lender relationship. This model rationalizes the cross-section of secured debt contracts, and provides a narrative for sudden freezes in secured debt markets based on the endogeneity of reputation. Theories of debt capacity are conventionally asset-specific.[3] In Shleifer and Vishny (1992), debt capacity of an asset is its liquidation value. Similarly, in models of endogenous margins such as Geanakoplos (2003) and Simsek (2013), debt capacity only depends on beliefs about the asset which collateralizes the debt contract. However, Figures 2.1 and 2.2 suggest that not only the collateral but also broader attributes of the relationship between the borrower and the lender are priced in. The dataset used in this paper includes all the tri-party repo transactions between 2006 and 2012 involving one of the 145 largest money market funds. Repurchase agreements (repo) contracts are secured debt contracts. Their defining feature is that *two* pricing terms are contracted upon: not only the compensation (the interest rate), but also the protection (the haircut). The rate is defined as the ratio of the ex post promise $\bar{s}$ on the amount of cash raised ex ante D: $1 + r = \frac{\bar{s}}{D}$. The haircut is computed as one minus the ratio of ex ante cash raised D on the ex ante market value of collateral p: $m = 1 - \frac{D}{p}$.

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[1] A previous version of this paper was circulated under the title: "Haircuts and Relationships". I would like to thank Tobias Adrian, Philippe Aghion, Charles Angelucci, Nina Boyarchenko, Adam Copeland, Eduardo Davila, Emmanuel Farhi, Benjamin Friedman, Paul Goldsmith-Pinkham, Oliver Hart, Antoine Martin, Asani Sarkar, Andrei Shleifer, Alp Simsek, Adi Sunderam, Jeremy Stein, Michael Woodford and Eric Zwick for helpful discussions and insightul comments, as well as seminar participants at Harvard, HBS, MIT Sloan, the European Central Bank and the Federal Reserve Bank of New York. JEL numbers: G12, G23, G32, D8, L14. Keywords: leverage cycle, beliefs heterogeneity, secured funding, risk sharing, franchise value, long-term contracts, money market funds.

[2] See for instance Brunnermeier (2009) and Gorton and Metrick (2010).

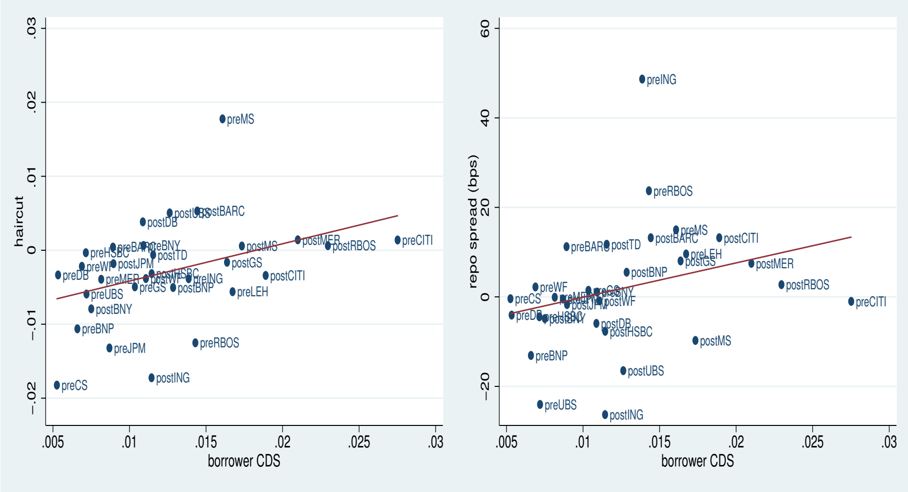

Figure 2.1: *Haircuts and rates in the cross-section of* borrowers*; orthogonalized on time, collateral and lender fixed effects.*

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[3] Debt capacity and loan-to-value are equivalent concepts, i.e. the amount that can be borrowed against an asset or a security. One minus the haircut (or margin) measures debt capacity.

Figure 2.1 plots haircuts and rates in the cross-section of borrowers. It illustrates that borrowers whose perceived creditworthiness by the market - i.e. reputation - is better secure repo contracts that are favorable on the two pricing dimensions: lower rates *and* lower haircuts. Figure 2.2 conducts the same type of exercise in the cross-section of lenders. It illustrates that lenders with more outside options secure repo contracts with more favorable terms: higher rates *and* higher haircuts.

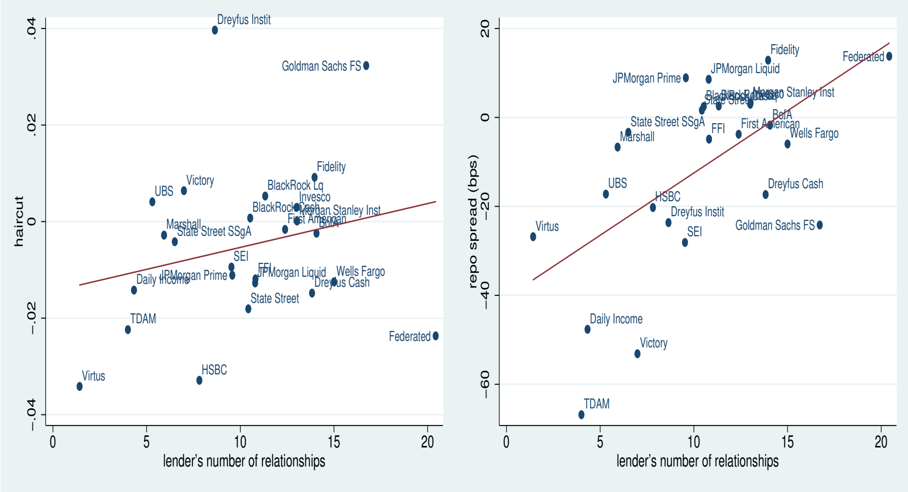

Figure 2.2: *Haircuts and rates in the cross-section of* lenders*; orthogonalized on time, collateral and borrower fixed effects.*

I show in this paper that a positive comovement of haircuts and rates in the cross-section of borrowers can only be rationalized by a dimension of heterogeneity that is not about the collateral. Models using a continuum of beliefs generically predict that a negative correlation between haircuts and rates. On the contrary, in an environment that explicitly takes into account reputation heterogeneity, borrowers with high reputation do enjoy lower haircuts and lower rates. To make this point, I develop a model of multilateral Nash bargaining between I optimists and J pessimists, and I consider two cases: one where optimists only differ according to their beliefs on collateral riskiness, and one where optimists only differ according to their reputation. Only the equilibrium of the second case features a positive comovement of haircuts and rates in the cross-section of borrowers.

I then turn to the dynamic version of the model in order to provide foundations for reputation. I identify reputation as the continuation value of any agent in the infinitely repeated version of the static game. In this environment, agents net worth are the only state variables and reputation can then be solved as a function of net worth. The second result of the paper is that *only with endogenous reputation* leverage can be procyclical with respect to net worth. The intuition is as follows. Reputation is a non-linear function of net worth: it vanishes at a fast pace when net worth gets close to zero. As reputation supports agents debt capacity, equilibrium leverage gets slashed for low levels of net worth. Such dynamics provides an explanation for the ′run on repo′ documented in Gorton and Metrick (2010) without resorting to any coordination failure narrative. The third result is that long-term contracts are valuable in this environment, as they insulate borrowers from margin calls when they have low net worth, precisely the states where they would need to lever up to replenish their capital.

Finally, the fourth and last theoretical result is delivered by an extension of the model featuring *repo chain*. I consider an environment with three types of agents: optimists without franchise value ′Hedge Funds′, optimists with franchise value ′Broker-Dealer′ and pessimists ′Money Markets′. I show that Broker Dealers emerge as financial intermediaries: they rehypothecate collateral, as they are the only ones capable of ′monetizing′ their franchise value. I show that Broker-Dealer leverage is more stable than Hedge-Fund leverage in this equilibrium, consistent with the stylized facts.

The empirical analysis of the repo market confirms the first three theoretical results. I use a hand-collected dataset of repurchase agreement transactions between money market funds as lenders and broker dealers as borrowers. The dataset is constructed, following Krishnamurthy, Nagel and Orlov (2011), by parsing the quarterly SEC filings. The only difference on data source is that I parse for the 145 largest prime institutional money market funds but for no securities lenders. This choice is motivated by the focus of the present paper on the price determinants in the cross-section of borrowers, whereas Krishnamurthy, Nagel and Orlov (2011) focuses on the aggregate time-series of repo funding. In the dataset, borrowers with high reputation, proxied by a low CDS, do secure lower rates and lower haircuts (prediction 1). I also identify the reputation channel by the exposure to the European crisis: controlling for collateral, borrowers that were more exposed to Europe did experience higher haircuts even on US collateral. Moreover, in the time-series, borrowers with low net worth are the ones that face more volatile haircuts (Prediction 2). Finally, borrowers that enjoy long-term relationships with money market funds are more insulated from haircut volatility (Prediction 3). The last two findings can help reconciling Gorton and Metrick (2010), which document dramatic increases in haircuts (with data coming from bilateral repos), with, on the other hand, Copeland, Martin and Walker (2010) which show that haircuts were by and large unresponsive over the crisis (with data coming from tri-party repos).

Related literature. The corporate finance literature traditionally features agency frictions to model limited pledgeability, which is tantamount to a leverage constraint. The agency friction can be moral hazard, such as in Holmstrom and Tirole (1997) and Adrian and Shin (2012), or informational asymmetries about the quality of collateral, such as Dang, Gorton and Holmstrom (2011a). On the contrary, Geanakoplos (2003) and Simsek (2013) rely on beliefs heterogeneity to derive an optimal level of leverage. As I focus in this paper on the debt capacity of financial intermediaries, I choose to cast my model in the latter piece of literature.[4] The key feature of my model is that franchise value cannot be seized by the lenders. It draws from the limited commitment literature, following Hart and Moore (1994) and Albuquerque and Hopenhayn (2004). However, in my model, parties can commit to the debt contract; which is priced taking into account the fact that the franchise value relaxes the default constraint. Due to beliefs heterogeneity, the optimal contracting problem is more akin to an ex ante risk-sharing agreement, in the spirit of Kocherlakota (1996), Lustig (2000) and Alvarez and Jermann (2000). Contrary to the latter, my model features equilibrium default. Beyond beliefs heterogeneity, the only friction in my model is the multilateral Nash bargaining that arises between borrowers and lenders; contractual externalities arise from the collateral constraint, which is shared by all the lenders. Multilateral Nash bargaining is inspired by Stole and Zwiebel (1996),[5] and makes the outside options endogenous. This feature of my model is shared with Krueger and Uhlig (2006), Ghosh and Ray (1996), Phelan (1995), and Rampini and Viswanathan (2012). The optimal long-term contract I derive is a long-term risk-sharing agreement, similar to the labor literature as Harris and Holmstrom (1982), Thomas and Worrall (1988), and not a dynamic contract that helps to mitigate an agency friction as in Biais et al. (2007).

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[4] Given their dispersed lending base, it is hard to think of financial intermediaries as being disciplined by the lenders from a moral hazard friction, or cash stealing friction a la Bolton and Scharfstein (1990) and Bolton and Scharfstein (1990). Similarly, Dang, Gorton and Holmstrom (2011a) microfounds the haircut to prevent information acquisition from a potential lender more sophisticated than the borrower. In the case of repo markets and bank wholesale funding, the ultimate lenders are money market funds and passive money, whose key characteristics seem to be their relative pessimism or risk aversion.

He and Xiong (2012a) also considers a dynamic environment with beliefs heterogeneity and rollover risk to analyze the optimal maturity choice. Differing from their set up, I model rollover risk as being *institution-specific* and not *asset-specific*, and I take into account imperfect competition among lenders. This enables me to capture the idiosyncratic dispersion of repo contracts for a given type of collateral. Geanakoplos and Fostel (2011) also features pricing dispersion of repo contracts, at the cost of introducing a continuum of beliefs types. In my model, two borrowers with the same beliefs but different franchises will have the repo priced differently. Acharya, Gale and Yorulmazer (2011a) is a model of dynamic debt capacity of a long-term asset in presence of small but frequent rollover risk.[6] Contrary to their set up, the risky asset I consider can be short-term. Only institution-specific characteristics (franchise and relationships) are long-term. In my model, impaired debt capacity here arises from the endogeneity of borrower franchise value. Oehmke (2012) analyzes the disorderly liquidation of illiquid collateral. He focuses on the dynamics conditional on the default, whereas I analyze the ex ante choice of leverage under imperfect competition. Brunnermeier and Pedersen (2008) develops a theory of margin spirals based on the feedback loop between market liquidity and funding liquidity, based on asymmetric information. Their mechanism relies on margins set by Value-at-Risk constraints and information extraction for volatility estimates. In my paper, the collateralized debt contract is an optimal contract between heterogeneous agents.[7]

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[5] Such friction is also featured in Ausubel and Deneckere (1993), Acemoglu et al. (2007) and Goldberg and Tille (2012) in the trade context. The alternative would have been a search and matching set up. The latter makes sense between segmented capital markets as in (Duffie and Strulovici, forthcoming) but is harder to justify on liquid wholesale funding markets such as repo.

[6] Other papers analyzing rollover risk are He and Xiong (2012b) and Eisenbach (2011). They usually feature perfect credit markets on banks' liability side. My Industrial Organization set up can be seen as a way to microfound the imperfect credit line, which breaks at a random Poisson random time, in He and Xiong (2012b).

Dynamic models of capital structure traditionally take the size of the balance sheet as exogenous in order to analyze the debt-equity tradeoff.[8] On the contrary, I take the level of net worth (equity) as given and analyze the optimal size of the balance sheet of financial intermediaries. In the dynamic version of the model, the balance sheet size is a control variable and equity is the state variable, i.e. the reverse of traditional models of optimal capital structure. Overall, the present model seems more relevant to analyze the dynamics of leverage of financial institutions whose choice variable is the size of the balance sheet, whereas traditional models of endogenous capital structure seem more relevant for non-financial firms. Finally, the theoretical model developed here implies that high franchise values take on more risk and leverage. This is the reverse of the traditional literature on charter value and reputation building, e.g. Stiglitz and Weiss (1983), Diamond (1989), Keeley (1990), Hellmann et al. (2000), Allen and Gale (2004) and Carletti et al. (2007). In these models, as franchise is destroyed in default, strong franchises take on less risk. Similarly, relationship banking usually hinges on asymmetric information whereas in my model it arises from imperfect competition among lenders.

In the macroeconomics literature, the lender not only cares about the liquidation value he can extract from collateral as in corporate finance, but also about the fact that the borrower cares about the loss of collateral, as in the sovereign debt literature. My model features both aspects of collateral. Gertler and Kiyotaki (2012) introduces rollover risk in a macro model, but features unsecured debt (deposits) and their leverage constraint hinges on an agency friction (cash diversion). The dynamic model developed in this paper delivers procyclical leverage, contrary to the state-independent leverage of Kiyotaki and Moore (1997) and in accordance with empirical evidence of Adrian and Shin (2010).[9] Compared to Adrian and Boyarchenko (2012), I only have one shock that commands both the asset risk and counterparty risk, and the procyclicality of leverage in my model arises from the endogeneity of franchise value, whereas in their model it arises from a risk-sensitive Value-at-risk constraint.

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[7] In Brunnermeier and Oehmke (forthcoming)language, the two liquidity spirals, the loss spiral and the haircut spiral, are jointly determined. This model, in its General Equilibrium version (section 6), provides a framework to analyze their interaction.

[8] Following Merton (1974), Leland (1994) and Leland and Toft (1996).

On the empirical side, Gorton and Metrick (2010), Copeland et al. (2010) and Krishnamurthy et al. (2011) document the behavior of repo markets over the crisis.[10] They focus on the aggregate behavior, whereas I analyze the idiosyncratic component of repo contracts. Duffie and Ashcraft (2007), Afonso et al. (2011), Kuo et al. (2012), Soramaki et al. (2010) and Chernenko and Sunderam (2012) analyze funding conditions on unsecured debt markets. My empirical results complement these studies by showing that idiosyncratic components are priced in secured debt. Kacperczyk and Schnabl (2012) demonstrates that money market funds are heterogeneous in their risk-taking behavior, and that concerns for sponsors' franchise mitigate their risk-taking. Their study taken together with the present one suggests an ambiguous effect of franchise along the intermediation chain: intermediaries closer to the ultimate borrowers use their franchise to lever up more and take on more risk, whereas intermediaries closer to the ultimate lenders (households) use their franchise as a commitment device to take on less risk, and thus mitigating agency frictions.

The rest of the paper is organized as follows. Section 2 develops a static model of endogenous leverage with heterogeneous borrowers. Section 3 extends the model to a dynamic environment by endogeneizing the franchise value. Section 4 analyzes the rehypothecation chain. Section 5 turns to the empirical analysis and tests the three key predictions of the model. Section 6 sketches the normative implications and concludes.

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[9] Acharya and Viswanathan (2011) also delivers procyclical leverage, but with a moral hazard friction.

[10] Compared to Copeland et al. (2010), my dataset has interest rates information, which turn out to be instrumental for the test of the relationships theory. Compared to Krishnamurthy et al. (2011), I have more MMF families (45 against 19) and not exactly the same funds.

2.2 The static model

The goal of this section is to develop a model of repo markets with endogenous prices and quantities of debt contracts, when there are multiple heterogeneous borrowers that can differ whether through their beliefs or through their reputation. There are 3 periods: $t = 0, 1, 2$.

2.2.1 Environment

Agents The economy is populated by two types of agents: there is a discrete number I of type-B agents and a discrete number J of type-L agents. There is one risky asset: investing in one unit of the asset costs p at $t = 0$ and yields a stochastic payoff $s \in \mathbb{S}$ at $t = 1$.[11] The type-B agents are risk-neutral optimists, each endowed with net worth n^B. Their prior distribution of beliefs on the dividend is of density $f_B(s)$ and cumulative distribution function $F_B(s)$. The type-L agents are risk-neutral pessimists, each endowed with net worth n^L. Their prior beliefs are noted $f_L(s)$ for the density and $F_L(s)$ their cumulative distribution function. The type-L agents are pessimists in the sense that $\mathbb{E}_L[s] < \mathbb{E}_B[s]$.[12] I define the subjective expected returns μ^B and μ^L by: $\mathbb{E}_B[s] = p(1 + \mu^B)$ and $\mathbb{E}_L[s] = p(1 + \mu^L)$. Following Simsek (2013), I impose the following parameter restriction on the nature of beliefs heterogeneity.[13]

Assumption 2. *The heterogeneous beliefs of the agents satisfy the hazard rate property:* $\forall s \in \mathbb{S}\ \frac{f_L(s)}{1-F_L(s)} > \frac{f_B(s)}{1-F_B(s)}$

---

[11] p is first taken exogenous (perfectly elastic supply). The asset price is endogenized in the General Equilibrium of section 2.3.

[12] This beliefs heterogeneity creates a rationale for contracting between the two types of agents. It a state-dependent utility heterogeneity, isomorphic to heterogeneity in risk aversion when there is only one asset. All theoretical results therefore transpose to heterogeneity in risk aversion.

[13] Only under this assumption, the debt contract traded in equilibrium is risky. The interest rate on the contract is then non-zero as a credit spread, and this enables to carry out the comparative statics on rates.

Assumption 1 states that optimists are even more optimistic compared to pessimists for high states of nature. Although most results hold under the general property of Assumption 1, I focus on normal beliefs: $f_L \sim N\left(p\left(1+\mu^L\right),\sigma\right)$ and $f_B \sim N\left(p\left(1+\mu^B\right),\sigma\right)$ with $\mu^L < \mu^B$ (Figure 2.3). Such beliefs are tractable and allows for elegant comparative statics with respect to collateral volatility σ.

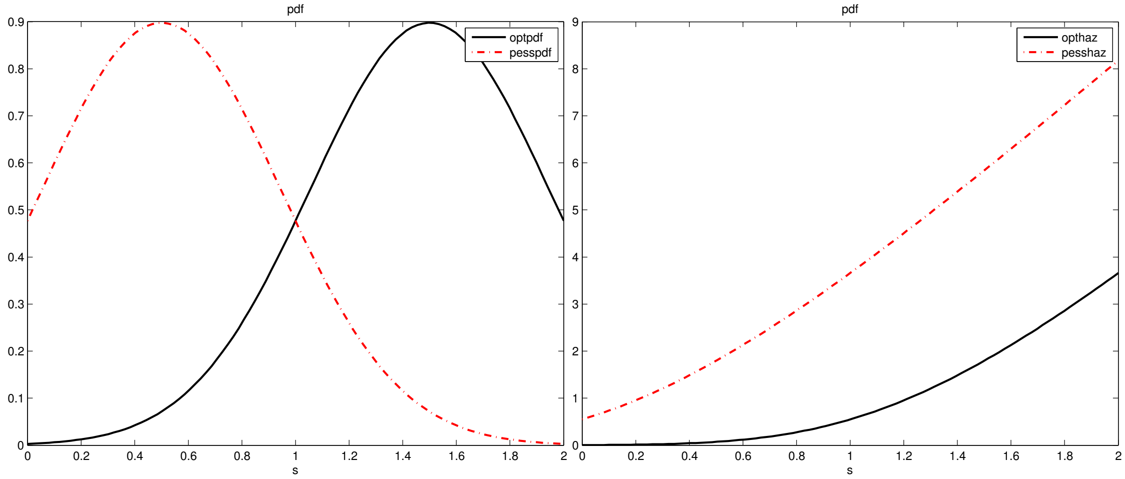

Figure 2.3: *Heterogeneous beliefs satisfying the hazard rate property.*

Financial contracts I restrict the contracting set to collateralized debt: *repo contracts*. A repo contract is a non-state-contingent promise to pay $\bar{s}$ at $t = 2$, made by the seller of the contract (the borrower) to its buyer (the lender). This contract is secured, in the sense that if the borrower does not honor its promise to pay back $\bar{s}$, the lender can push him into default. He then recovers a pre-specified amount of the borrower portfolio in the risky asset. Thus a repo contract has three terms to contract upon: the promise $\bar{s}$, the amount of cash D that is lent at $t = 0$ by the seller to the buyer, and the amount of collateral. By linearity, we can normalize the contracting set to consider that all the repo contracts are collateralized by one unit of the risky asset. The two contract terms $\{\bar{s}, D\}$ are isomorphic to the two observables prices defined below.

Definition 2. The repo spread is: $r = \frac{\bar{s}-D}{D}$ and the haircut on the risky asset is: $m = \frac{p-D}{p}$.

The interest rate r on the contract captures the credit spread. Reciprocally, the value D of the contract can be written: $D = (1-m)\,p$ and the promise: $\bar{s} = (1+r)\,(1-m)\,p$. Given there is only one risky asset, the haircut m also captures leverage and credit supply.

Timeline The key feature of the model relies in the ability of all agents to pledge long-run revenues at anytime, but at a non attractive interest rate. Although agents will never use this possibility at $t = 0$, they will at $t = 1$ in some states in order to avoid default and save the remainder of their long-run revenues. The *reputation* of an agent is encoded in its ability to pledge long-term cash flows.[14] The exact timing is as follows and is illustrated on Figure 2.4.

- At $t = 0$, agents write repo contracts among themselves. Each pair of agents $(i, j) \in I \times J$ enters in x_{ij} units of a repo contract, whose terms are $\left\{r_{ij}, m_{ij}, 1\right\}$: a rate, a haircut and a number of asset units as collateral. These terms can be reframed as a duplet $\left\{\bar{s}_{ij}, D_{ij}\right\}$: a promise and an ex ante value. Prices $\left\{\bar{s}_{ij}, D_{ij}\right\}$ and quantities x_{ij} are determined jointly through multilateral Nash bargaining.

- At $t = 1$, the asset shock s realizes and is observed by all agents. Agents i that face $\sum_{j \in J} x_{ij} s < \sum_{j \in J} x_{ij} \bar{s}_{ij}$ are in situation of distress. In this interim period, any agent can decide to pledge its long-run revenues V^{B_i}, albeit at an exogenous (high) rate r^*. I define $\beta = \frac{1}{1+r^*}$.

- At $t = 2$, contracts are settled, agents receive the share of long-revenues that have not been pledged at $t = 1$, and consume.

---

[14] It can also be thought as the Present Discounted Value of cash-flow generating divisions of the borrower which are not levered but heavily relies on the strength of the franchise value, such as M&A advisory. It can also be thought as the ability to rollover in states of nature.

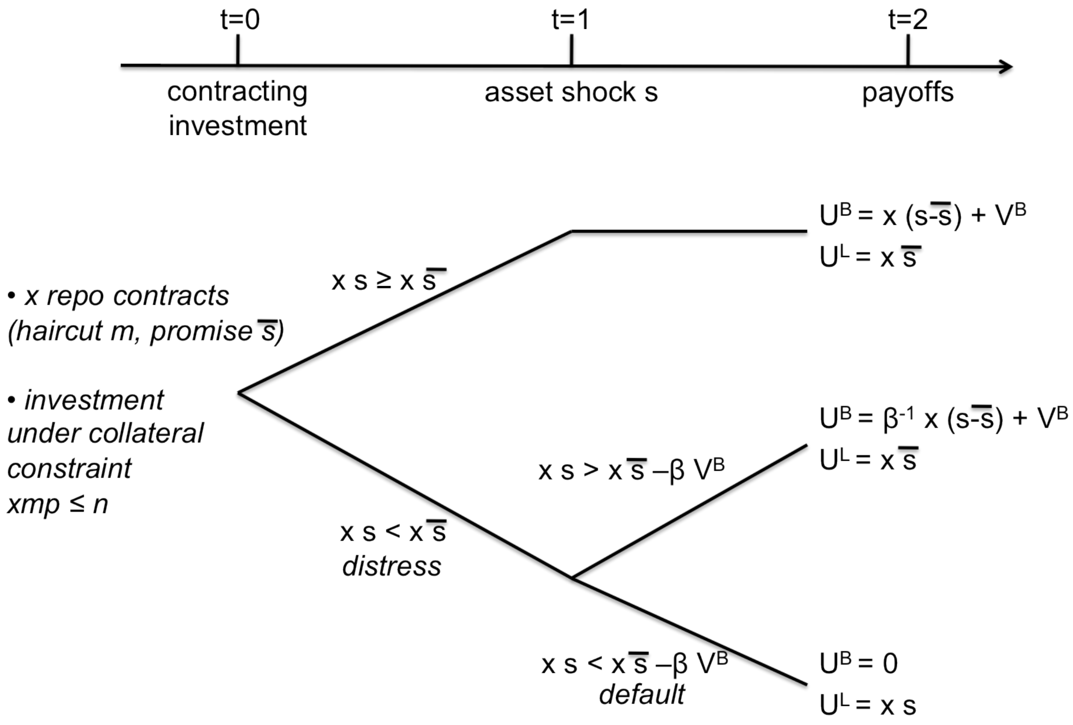
t=0
t=1
t=2
contracting investment
asset shock s
payoffs
• x repo contracts (haircut m, promise s̄)
• investment under collateral constraint xmp ≤ n
x s ≥ x s̄
U^B = x (s-s̄) + V^B
U^L = x s̄
x s < x s̄ distress
x s > x s̄ –β V^B
U^B = β^-1 x (s-s̄) + V^B
U^L = x s̄
x s < x s̄ –β V^B default
U^B = 0
U^L = x s

Figure 2.4: *Timeline of the static model.*

2.2.2 Equilibrium with deep pocket lenders

If the number J of lenders is large and deep pockets ($J\,n^L >> I\,n^B$), they Bertrand-compete away any surplus for them. This is equivalent to granting full bargaining to the borrowers.

Default decision At $t = 1$, borrowers have the option whether to default or to pledge part of their long-run revenues V^{B_i} at the rate r^*. This partitions the states into three regions:

- If $\frac{\sum_{j\in J} x_{ij}\bar{s}_{ij}}{\sum_{j\in J} x_{ij}} < s$, the agent i does not default.
- If $\frac{\sum_{j\in J} x_{ij}\bar{s}_{ij}}{\sum_{j\in J} x_{ij}} - \beta V^{B_i}\frac{1}{\sum_{j\in J} x_{ij}} < s < \frac{\sum_{j\in J} x_{ij}\bar{s}_{ij}}{\sum_{j\in J} x_{ij}}$, the agent pledges a part of its long-revenus and stays afloat.
- If $s < \frac{\sum_{j\in J} x_{ij}\bar{s}_{ij}}{\sum_{j\in J} x_{ij}} - \beta V^{B_i}\frac{1}{\sum_{j\in J} x_{ij}}$, the agent is forced into default.

Let denote $\tilde{s} = s^{def}\left(x_{ij}, \bar{s}_{ij}\right) = \frac{\sum_{j\in J} x_{ij}\bar{s}_{ij}}{\sum_{j\in J} x_{ij}} - \beta V^{B_i}\frac{1}{\sum_{j\in J} x_{ij}}$ the effective default threshold ('riskiness' of the contract), and $\left[\tilde{s}; \frac{\sum_{j\in J} x_{ij}\bar{s}_{ij}}{\sum_{j\in J} x_{ij}}\right]$ the light-distress region. In this region, the borrower is under water but can still survive thanks to the strength of its franchise value V^{B_i}. This allows for a region of the state space where the borrower has negative net worth at $t = 1$. This does not trigger default as the borrower is in effect able to run a simili-Ponzi scheme thanks to its franchise value.[15]

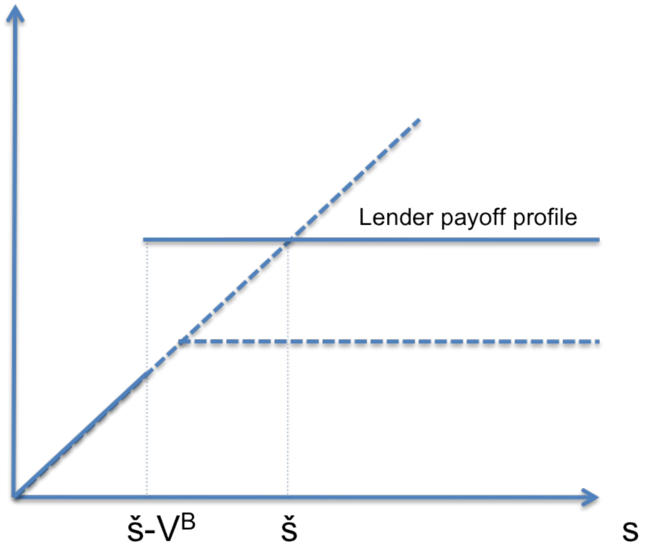

Figure 2.5: *Payoff profile of the lender with borrower franchise. value*

A key assumption is that V^B cannot be seized by the lenders in the states of borrower's default. In practice, repurchase agreement contracts are exempt from the automatic stay thanks to the safe harbor provision,[16] but they usually also are *recourse*: after having seized the collateral, the lender possesses an unsecured claim to the counterparty. I assume that the value of this recourse claim is zero:[17] the recourse feature does not imply any additional recovery for the lender, and V^{B_i} evaporates as soon as the borrower enters bankruptcy. The payoff profile of the lender under a repo contract with respect to the realization of the collateral is illustrated in Figure 2.5.

---

[15] This Ponzi-scheme arises rationally in equilibrium as there is a positive probability of recovery. It can be seen as the symmetric counterpart of rational bubbles of Abreu and Brunnermeier (2003).

[16] See for instance the bankruptcy court decision reported in Schweitzer et al. (2008).

[17] This assumption follows Geanakoplos (1997), making repo contracts non-recourse. The effect of reputation analyzed in this paper are robust to the recourse feature of repo contract (i.e. assuming that lenders have access to the rest of the balance sheet of the borrower beyond the pre-specified collateral in case of default), as long as they cannot seize all of it. See for instance Khan (2010). Such treatment of repo as been applied in the case of Lehman bankruptcy, see Valukas (2012).

The following definition will prove useful in the derivations and for the economic interpretation. The wedge in the valuation of the put underpins the rationale for contracting between borrowers and lenders.

Definition 3. The valuations of the limited liability put with strike $\tilde{s}$, respectively by the type-B agents and by the type-L agents, are noted: $\pi^B(\tilde{s}) = \mathbb{E}_B\left[(\tilde{s}-s)^+\right] = \int_{s^{min}}^{\tilde{s}} F_B(s)ds$ and $\pi^L(\tilde{s}) = \mathbb{E}_L\left[(\tilde{s}-s)^+\right] = \int_{s^{min}}^{\tilde{s}} F_L(s)ds$.

Expected payoffs at $t = 0$ The expected utility of any borrower is the levered gains from the carry trade between investing and its borrowing costs. The franchise value introduces a concern to preserve this franchise value, which works as a continuation term. The expected utility of the borrower in the equilibrium, for a given number of contracts x_{ij}, is then:

$$\begin{aligned} U^B_{I,J} &= \sum_J x_{ij}\,\mathbb{E}_B\left[(s-\bar{s})\,1_{\{no\,distress\}} + \beta^{-1}(s-\bar{s})\,1_{\{light\,distress\}}\right] + \mathbb{E}_B\left[V^B 1_{\{no\,def\}}\right] \quad (2.1)\\ s.t. &\quad \sum_J x_{ij} m_{ij} p \le n^B \end{aligned}$$

If the borrower does not contract at all, it does not lever up and invests only its net worth in the risky asset, yielding $p(1+\mu^B)$ under its beliefs:

$$U^B_0 = n^B\left(1+\mu^B\right) + V^B$$

On the other hand, lenders expected utility are the returns on its portfolio of repo contracts and direct investment in the asset. The expected utility of the lender in the equilibrium, for given contracts x_{ij}, is:

$$U^L_{I,J} = \sum_I x_{ij}\,\mathbb{E}_L\left[1_{\{def\}}s + 1_{\{no\,def\}}(\bar{s}) - D\right] + \left(n^L - \sum_I x_{ij}D\right)(1+\mu^L) \tag{2.2}$$

If a lender does not contract at all, it invests its endowment in the risky asset, valued at

$p(1 + \mu^L)$:

$$U_0^L = n^L \left(1 + \mu^L\right)$$

Under the assumption that lenders are extracting no surplus from the contracting, the following equality holds: $U_{I,J}^L = U_0^L$. This implicitly defines a *contract curve*, plotted on Figure 2.6 in the haircut-rate space.

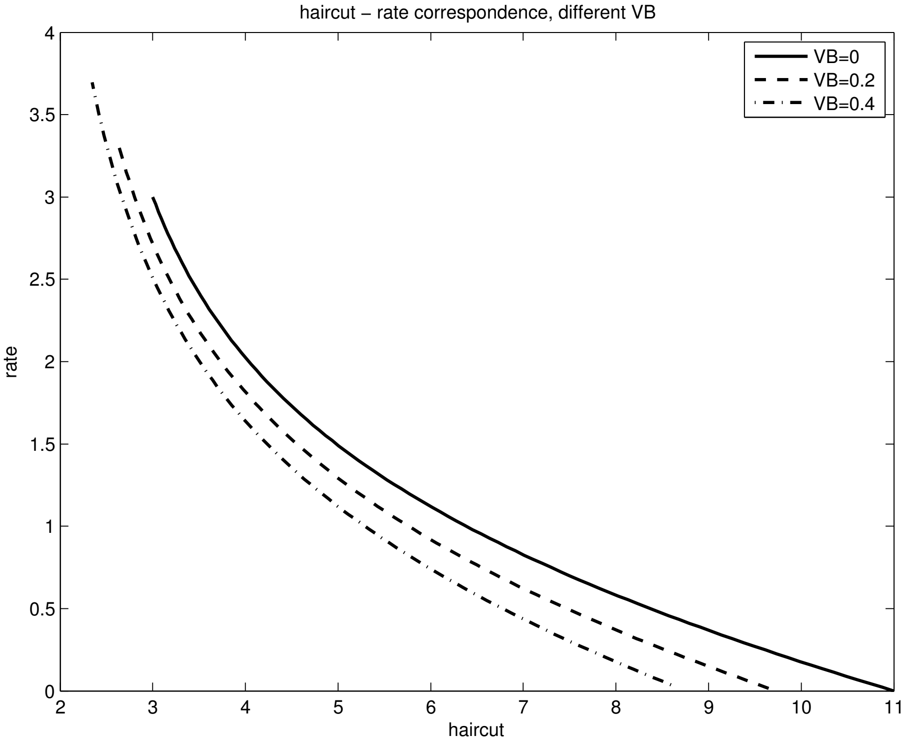

Figure 2.6: *Rate (y axis) - Haircut (x axis) contract curve.*

Equilibrium repo contract The equilibrium is then solved by the program 2.1 of the borrowers, taking the contract curve as a constraint. Its first-order condition leads to the following lemma.

Lemma 10. *For each borrower i, the optimal contract it picks is characterized by its riskiness $\tilde{s}$ that* *satisfies:*

$$\begin{aligned} p = & -\int_{s^{min}}^{\tilde{s}} s f_L ds + \int_{\tilde{s}}^{s^{max}} s f_B ds \frac{1-F_L}{1-F_B} \\ & - V^{B_i} \frac{\left(p - \int_{s^{min}}^{\tilde{s}} s f_L ds - \tilde{s}\left(1-F_L\right)\right)^2}{n^B + \beta V^{B_i}\left(1-F_L\right)} \left(\beta \frac{f_L}{1-F_L} + (1-\beta)\frac{f_B}{1-F_B}\right) \end{aligned} \tag{2.3}$$

The effects of franchise V^{B_i} on the equilibrium are all embedded in one term. The environment features a unique and interior solution for the optimal contract thanks to the continuum of states s, as in Simsek (2013).[18] Proposition 1 then follows easily from the comparative statics.

Proposition 8. ***Franchise Value Collateral.*** *Under Assumption 1, haircuts and rates covary positively in the cross-section of borrowers. A higher franchise secures both lower haircuts and lower rates.*

Proof. Appendix proves that higher franchise V^{B_i} leads to: higher promise $\partial_{V^B}\bar{s} > 0$, lower haircuts $\partial_{V^B} m < 0$, lower riskiness $\partial_{V^B}\tilde{s} < 0$ and lower rates $\partial_{V^B} r < 0$. □

It is easy to see that such property cannot be obtained in any model where borrowers heterogeneity is about collateral, such as beliefs heterogeneity or risk aversion heterogeneity among borrowers. Indeed, in the latter case, all borrowers face the same contract curve 2.2, which is univocally downward sloping. On the contrary, Proposition 1 can be interpreted as Franchise Value being used as intangible collateral in repo borrowing. When endowed with franchise, the borrower is incentivized to lever up more. Despite higher leverage, Proposition 1 shows that refinancing terms are cheaper for the borrower with higher franchise.

An examination of the first-order condition 2.3 convinces that the effect of franchise on haircuts and rates is amplified when beliefs disagreement between borrowers and lenders is higher, and when the collateral is more volatile.

Corollary 5. *Franchise value matters more when collateral is more volatile and when beliefs disagreement is larger.*

---

[18] On the contrary, in Geanakoplos (2009) and He and Xiong (2012a), the borrower program has a knife-edge structure, as their value function is monotonic with respect to the promise $\bar{s}$.

Proof. It follows from , $\frac{\partial}{\partial\sigma}\left(|\frac{\partial m}{\partial V^B}|\right) > 0$, $\frac{\partial}{\partial\sigma}\left(|\frac{\partial r}{\partial V^B}|\right) > 0$, $\frac{\partial|\partial_{V^B} m|}{\partial(\mu^B-\mu^L)} > 0$ and $\frac{\partial|\partial_{V^B} r|}{\partial(\mu^B-\mu^L)} > 0$. $\square$

This corollary informs us that it is precisely in times of distress and on volatile collateral that there is more chance of idiosyncratic dispersion in repo prices. The intangible franchise value collateral depends on the volatility of the underlying asset σ. Recall that the equilibrium haircut comes from the wedge in the perceived values of the put option, and this wedge increase with the volatility of collateral. As a result, haircuts increase with the collateral volatility. The cross-partial is more interesting, as it is a form of *volatility paradox*. As the asset becomes more volatile from the lender's viewpoint, he values tangible collateral less relative to the franchise values of the borrower. As a result, the franchise value collateral channel is magnified, to an extent that can make optimal leverage actually higher with more volatile collateral.

The equilibrium haircut is given by the riskiness of the equilibrium contract:

$$m = \frac{1+\mu^\delta}{1+\mu^{\delta\, barg}} - \frac{1}{p\left(1+\mu^{\delta\, barg}\right)}\left(\tilde{s} + \left(1-F_\delta(\tilde{s})\right)V^B - \pi^\delta(\tilde{s})\right)$$

A Taylor expansion in the franchise obtains a closed-form solution of the haircut as a function of the primitive.

$$m = \left(\frac{1+\mu^\delta}{1+\mu^{\delta\, barg}} - \frac{s^{min}}{p\left(1+\mu^{\delta\, barg}\right)}\right) - \frac{1}{p\left(1+\mu^{\delta\, barg}\right)}\left[\tilde{s} - s^{min} + \pi^\delta(\tilde{s})\right]$$
$$-\left(V^B\right)^2 \frac{\left(1-F_\delta(\tilde{s})\right)}{p\left(1+\mu^{\delta\, barg}\right)} V^B \left(\frac{1}{\mathbb{E}_B\left[s|s>\tilde{s}\right]-\tilde{s}} - \frac{\tilde{s}-\mu^\delta}{\sigma^2} - \frac{\mu^B-\mu^\delta}{\sigma^2}\frac{f_B/\left(1-F_B\right)}{h(0)} + \frac{f_\delta(\tilde{s})}{1-F_\delta(\tilde{s})}\right)$$

This closed-form solution exhibits the different determinants of the equilibrium haircut: the first term s^{min} is the safe component (present in Geanakoplos (2003)), the second is risky debt coming from the wedge in the limited liability put valuation (present in Simsek (2013)). The third new term consists in a direct effect of V^B on m from more promise $\bar{s}$, and an indirect effect from lower riskiness $\tilde{s}$. In the appendix I show that the first direct effect dominates. When $V^B \mapsto \infty$, debt becomes essentially risk-free ($s^{def} = 0$) as in Geanakoplos (2003). When $V^B \mapsto 0$, m tends to the equilibrium margin with no franchise value as in Simsek (2013).

As for interest rate, the closed-form solution is:

$$r = \frac{\tilde{s} + V^B}{p - \frac{1}{\left(1+\mu^{\delta\, barg}\right)}\left(p(1+\mu^\delta) - \tilde{s} - \left(1 - F_\delta(\tilde{s})\right) V^B + \pi^\delta(\tilde{s})\right)} - 1$$

As can be seen from the expression of the rate as a function of $\tilde{s}$, there are two distinct effects from V^B on r. The promise $\bar{s} = \tilde{s} + \beta V^{B_i} \frac{1}{\sum_{j \in J} x_{ij}}$ is higher, but D is also higher. Proposition 1 showed $\frac{\partial \tilde{s}}{\partial V^B} < 0$ and $\frac{\partial \bar{s}}{\partial V^B} > 0$. This means that starting from a situation where $V^B = 0$, increasing V^B de-links $\tilde{s}$ from $\bar{s}$, in such a way that the riskiness $\tilde{s}$ decreases and the promise $\bar{s}$ increases.

The second corollary is concerned with the convexity of the mapping $m(V^B)$, a key element of the procyclical leverage result of the dynamic model below.

Corollary 6. *The haircut is stable for high franchises:* $m(V^B)$ *satisfies* $\frac{\partial^2 m}{\partial (V^B)^2} > 0$.

This proposition implies that the franchise value collateral channel is more sensitive to innovation on franchise when franchise is already low. When franchise value V^B is high, the equilibrium margin is low and stable, as most of the lending is done against V^B and not against the risky collateral. When franchise value is low, the margin abruptly adjusts to high levels, with the limit of the upper bound $m\,(0)$.

2.2.3 Equilibrium with limited lending wealth

I now turn to the general case in which lenders have limited wealth, and show that the main result of positive comovement of haircuts and rates still holds. This relaxation is interesting because Kacperczyk and Schnabl (2012) has empirically shown that the cross-section of money market funds as lenders is highly heterogeneous in their risk attitude. I characterize in this section the equilibrium matching between borrowers with heterogeneous franchise and lenders with heterogeneous pessimism.

In the matching equilibrium, there is not Bertrand competition among the lenders, and the multilateral Nash bargaining is non degenerate. In that case, contractual externalities arise from the co-existence of multiple repo contracts.

Market structure: Multilateral Nash Bargaining at $t = 0$ As observed in the data, repo contracts are highly non-exclusive: in equilibrium, a borrower will contract with several lenders and, reciprocally, a lender will contract with several borrowers.[19] The I optimists and J pessimists are allowed to write repo contracts with any other agent in the economy, and can hold multiple contracts at the same time. The borrower agrees with each lender j on a short-term repo contract, which specifies a value D^j, an interest rate r^j and a haircut m^j. Denote X the set of repo contracts. The set of contracts can be represented as a Cartesian product $X = I \times J \times K$. For $x \in X$, $x(i)$ is the identity of the borrower, $x(j)$ is the identity of the lender and $x(k)$ are the terms of the bilateral contract: $x(k) = \{m, r, colclass, T\}$ where m is the haircut, r is the interest rate, *colclass* is the type of collateral and T is the maturity. I assume for now segmented markets in *colclass* and a unique maturity (overnight), so $x(k) = \{m, r\}$. An *allocation* is a collection of contracts derived as an outcome of the multilateral Nash bargaining. The equilibrium concept is as follows.

Definition 4. An allocation is an equilibrium if it is pairwise stable, i.e. if it is not pairwise blocked by any pair of agents. Pairwise blocking consists in a borrower and a lender that would like to add a new joint contract or replace a previous joint contract while not canceling other contracts.

This equilibrium concept draws from Stole and Zwiebel (1996) and the matching literature.[20] The existence of a non-degenerate equilibrium stems from the contractual externalities imposed by borrowers' aggregate collateral constraints: each lender of a given borrower shares the same collateral constraint. In an equilibrium allocation, the surplus of the pairwise bilateral relationship is equal to its Shapley value in the corresponding cooperative game.

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[19] This model aims at capturing the decentralized nature of money markets such as repo markets. This also relaxes the matching structure of the intermediation market of He and Krishnamurthy (2012b), where matches are identical and exogenously broken at $t + dt$. It can also be seen as microfounding the imperfection of capital markets used by He and Xiong (2012b), which is that borrowers 'have' to rely on a continuum of small creditors. In my static model, borrowers endogenously choose to diversify their creditor structure, and this comes from the endogeneity of the borrowing rate. The multilateral Nash bargaining modelled here captures the pricing of rollover risk in the borrowing rate, a possibility mentioned in their footnote 10.

[20] The repo market is modelled here as a two-sided many-to-many matching market with contracts, which combines matching and contracting (see Roth (1984), Hatfield and Milgrom (2005) and Klaus and Walzl (2009)). With beliefs heterogeneity and contractual externalities from the collateral constraint, there is a breakdown of full substitutability. This way to model the intermediation market differs from He and Krishnamurthy (2012b) where intermediation features a Walrasian equilibrium for risk exposure (equity).

$$S_{i,j} = S_{I,J} = U^B_{I,J} - U^B_{I,J-1} + U^L_{I,J} - U^L_{I-1,J}$$

Following Stole and Zwiebel (1996), I assume Nash bargaining with the bilateral relationship. I denote ω the bargaining power of the lender. The contract $x(i,j)$ must satisfy:

$$\omega \left(U^B_{I,J} - U^B_{I,J-1} \right) = (1-\omega) \left(U^L_{I,J} - U^L_{I-1,J} \right)$$

As benchmark, I first characterize the unique symmetric stable allocation and the optimal contract implementing it. Under this allocation, all possible bilateral relationships (i,j) enter in one contract and each contract feature the same terms $x(k)$. An equilibrium with small J can be referred as concentrated financing, whereas an equilibrium with large J can be referred as dispersed financing.

Lemma 11. *The expected values of the relationship under any contract* (m,r) *for the borrower and for the lender:*

$$\begin{aligned} U^B_{I,J} - U^B_0 &= \frac{n^B}{m} \left[(1-m) \left(\mu^B - r \right) + \frac{1}{p} \pi^B(\tilde{s}) + \left(\frac{1}{p} - \beta m \right) V^B F_B(\tilde{s}) \right] \\ U^L_{I,J} - U^L_0 &= \frac{I}{J} \frac{n^B}{m} \left[(1-m) \left(r - \mu^L \right) - \frac{1}{p} \left(\pi^L(\tilde{s}) + V^B F_L(\tilde{s}) \right) \right] \end{aligned}$$

The expression for the borrowers' expected value tells us that the franchise value has three distinct direct effects. First, it relaxes the default constraint and as such lowers the value of the long put held by the borrower (lower $\pi^B(\tilde{s})$). Second, this is counterbalanced by the states of nature where the borrower does not have to pay back this additional promise V^B: $\frac{1}{mp} V^B F_B(\tilde{s})$. Third, the traditional care about the franchise value also makes expected utility decrease with franchise: $-\beta V^B F_B(\tilde{s})$. These direct effects should make the franchise imply a lower optimal choice of leverage. This is without taking into account the indirect effect of franchise through the price of the loan $r(m)$, i.e. the contract curve.

Under Nash bargaining, the contract curve is slightly modified. I define:

$$\delta = \frac{1}{\frac{1-\omega}{\omega}\frac{I}{J} + 1}$$

δ is a measure of the *effective bargaining power* of the lenders: it is increasing in the bilateral bargaining power ω and it is increasing in the lender intensity J/I. Subsequently, define a weighted average beliefs where each agent is weighted by its effective bargaining power.

Definition 5. The beliefs F_δ are the average beliefs, weighted by δ: $F_\delta \sim (1-\delta)F_L + \delta F_B$

In the normal case, the beliefs F_δ are such that $F_\delta \sim N\left(\mu^\delta, \sigma\right)$ with $\mu^\delta = (1-\delta)\mu^L + \delta\mu^B$. The beliefs δ and B still satisfy the hazard rate order property, in the same order as L and B.

Lemma 12. *The Nash bargaining implies a rate-haircut contract curve:*

$$r(m) = \mu^\delta + \frac{1}{1-m}\left[\frac{1}{p}\left(\pi^\delta(\tilde{s}) + V^B F_\delta(\tilde{s})\right) - \delta\beta m V^B F_B(\tilde{s})\right] - \delta(1-\omega)\frac{m}{1-m}\left[S_{I,J-1} - S_{I-1,J}\right]$$

The contract curve tells that the rate is the sum of two components: the weighted-average of the agents' means, weighted by their respective bargaining powers, and the weighted-average of the agents' perceived values of the limited liability put. The mapping $r(m)$ is decreasing, which is intuitive: as the borrower picks lower haircuts, the loan becomes riskier, and this commands higher interest rates in order to compensate the lender for the credit risk. As in the case with deep picket investors, a higher franchise value shifts the correspondence leftwards.

Equilibrium The equilibrium can be expressed as follows. At $t = 0$, the borrower chooses a haircut m, and thus a size of its balance sheet x (leverage) taking into account the bargaining friction.[21] The maximization program of each borrower takes the form:

$$U^B_{I,J} = \underset{\{x,\bar{s}\}}{Max} \left\{ \sum_J x_{ij} \, \mathbb{E}_B \left[(s-\bar{s}) \, 1_{\{no\,distress\}} + \beta^{-1} (s-\bar{s}) \, 1_{\{light\,distress\}} \right] + \mathbb{E}_B \left[V^B 1_{\{no\,def\}} \right] \right\}$$

$$(collateral\,constraint) \;\; s.t. \sum_J x_{ij} m_{ij} p \leq n^B$$

$$(default\,condition) \;\; s.t. \; default \; i.f.f. \; s < \tilde{s}$$

$$(Nash\,bargaining) \;\; s.t. \; \omega \left(U^B_{I,J} - U^B_{I,J-1} \right) = (1-\omega) \left(U^L_{I,J} - U^L_{I-1,J} \right)$$

The borrower leverage x, counted as units of risky asset purchased, is the sum of the $\tilde{J}$ 'micro-leverages' x^j, which is, the number of units of risky assets that can be purchased on margin through each repo contract j: $x = \sum\limits_{j \in \tilde{J}} x^j$. Rewriting the program with only the contract-haircut m and the contract-rate r, we have:

$$U^B = \underset{\{m,r\}}{Max} \left\{ n^B \left[\left(\frac{1}{m} - 1 \right) \left(r^B - r(m) \right) + \frac{1}{mp} \pi^B(\tilde{s}) + \left(\frac{1}{mp} - \beta \right) V^B F_B(\tilde{s}) \right] \right\}$$

$$\begin{aligned} (Nash\,bargaining) \; s.t. \; r(m) = & \left[(1-\delta)\bar{r} + \delta r^B \right] \\ & + \frac{1}{1-m} \left[(1-\delta)\Pi^L + \delta \Pi^B \right] - \omega(1-\omega) \frac{m}{1-m} \left[S_{I,J-1} - S_{I-1,J} \right] \end{aligned}$$

This program is convex thanks to Assumption 1, and the equilibrium can be solved by induction on the number of borrowers and lenders. The optimal contract is determined as a function of the endogenous outside options $S_{I,J-1}$ and $S_{I-1,J}$. I formulate the induction hypotheses: $U^B_{I,J-1} - U^B_0 = (1-\omega) n^B S_{I,J-1}$ and $U^L_{I-1,J} - U^L_0 = \omega n^B S_{I-1,J}$. Endogenous surplus in bilateral relationships are computed by induction:

$$\begin{aligned} S_{I,J} = n^B \left(1+\mu^\delta\right) \frac{R^{IJ}(\tilde{s})}{R^{\delta U}(\tilde{s})} - \left(1+\mu^B\right) + (1-\omega) S_{I,J-1} \left(\omega \frac{R^{IJ}(\tilde{s})}{R^{\delta U}(\tilde{s})} - 1 \right) \\ - n^B \omega S_{I-1,J} \left((1-\omega) \frac{R^{IJ}(\tilde{s})}{R^{\delta U}(\tilde{s})} + 1 \right) \end{aligned} \tag{2.4}$$

where the ratio $\frac{R^{IJ}(\tilde{s})}{R^{\delta U}(\tilde{s})}$ is greater than 1 and equal to:

---

[21] The equilibrium is given by maximizing over (r, m) the Nash bargaining function $\left(U^B_{I,J} - U^B_{I,J-1} \right)^{\omega} \left(U^L_{I,J} - U^L_{I-1,J} \right)^{(1-\omega)}$.

$$\frac{R^{IJ}(\tilde{s})}{R^{\delta U}(\tilde{s})} = \frac{\left(\left\{p\left(1+\mu^B\right)-\frac{I}{J}p(1+\mu^L)(1-m)\right\}-\left\{\left(1-\frac{I}{J}\right)\left(\tilde{s}+V^B\right)\right\}+\left\{\left(F_B(\tilde{s})-\frac{I}{J}F_L(\tilde{s})\right)V^B\right\}+\left\{\pi^B(\tilde{s})-\frac{I}{J}\pi^L(\tilde{s})\right\}\right)}{p(1+\mu^\delta)-(\tilde{s}+V^B)+F_\delta(\tilde{s})V^B+\pi^\delta(\tilde{s})}$$

The closed form solution for the equilibrium haircut is also derived by induction, and is graphed on Figure 2.7. As in Lemma 1, the characterization of the equilibrium can be characterized by one first-order condition.

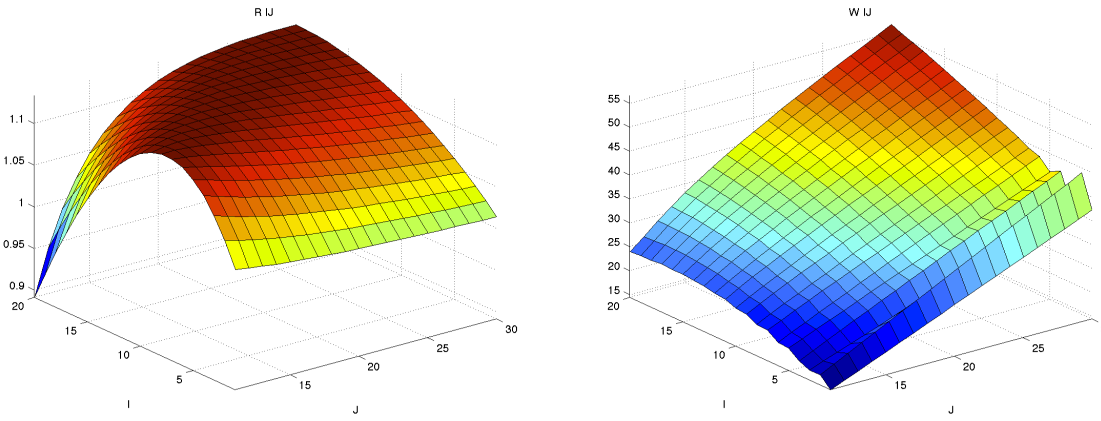

Figure 2.7: *Multilateral Nash bargaining: expected levered returns and welfare, as function of the number of borrowers I and numbers of lenders J.*

Lemma 13. *The optimal contract in the I-J equilibrium is unique and characterized by riskiness $\tilde{s}$:*

$$\begin{aligned} p(1+\mu^\delta) &= \kappa_1 V^B\left(\tilde{s}+V^B\right)+F_\delta(\tilde{s})\,\mathbb{E}_L\left[s|s<\tilde{s}\right]+\left(1-F_\delta(\tilde{s})\right)\,\kappa_2\,\mathbb{E}_B\left[s|s>\tilde{s}\right] \\ &\quad -\frac{\beta V^B}{n^B\left(1+\mu^{\delta\,barg}\right)}\frac{f_B(\tilde{s})}{1-f_B(\tilde{s})V^B-F_B(\tilde{s})}\left(\mathbb{E}_L\left[1_{\{no\,def\}}\left(s-\bar{s}\right)\right]\right)^2 \end{aligned}$$

Lemma 4 characterizes the riskiness of the loans as a function of the primitives of the model: $0 = F\left(\tilde{s};\mu^\delta,\mu^B,\sigma,V^B,p,\mu^{\delta\,barg},\beta\right)$. Intuitively, the borrower picks the optimal promise trading off the gains from levering up with the borrowing costs increasing with the riskiness. Not only the beliefs about the collateral matter, but also the franchise value V^B and the bargaining structure: the lenders' ability to extract surplus (δ) and the outside options $S_{I,J-1}$ and $S_{I-1,J}$. The next corollary shows that the main result, stated in Proposition 1, is robust to the general case of limited lenders net wealth and lenders heterogeneity.

Corollary 7. *Under multiple Nash bargaining, the Proposition 1 result of positive comovement of haircuts and rates in the cross-section of borrowers still holds.*

Furthermore, the more general environment considered here enables to characterize the equilibrium matching between borrowers of heterogeneous reputation and lenders of heterogeneous risk attitude.

Corollary 8. *Lenders reach-for-yield: in equilibrium, less pessimistic lenders pick lower haircuts and higher rates.*

Proof. For $(1-\delta)\frac{\mu^B-\mu^L}{\sigma} < 1$ (mild beliefs heterogeneity and large lender bargaining power): $\frac{\partial \tilde{s}}{\partial \mu^L} > 0$, $\frac{\partial m}{\partial \mu^L} < 0$ and $\frac{\partial r}{\partial \mu^L} > 0$. For $(1-\delta)\frac{\mu^B-\mu^\delta}{\sigma} > 1$: $\frac{\partial \tilde{s}}{\partial \mu^L} < 0$, $\frac{\partial m}{\partial \mu^L} < 0$ and $\frac{\partial r}{\partial \mu^L} < 0$. $\square$

The effect of beliefs disagreement is measured by $\mu^B - \mu^L$. The relative pessimism of the lenders about the asset can be interpreted as their ability to actually seize the collateral in states of nature in which the borrower defaults. A very pessimist lender will ask for higher promise ceteris paribus, and the promise is more dependent on the franchise value in this case. At the same time, a very pessimist lender overestimates the probability of default according to the borrower, which makes the borrower less wary of loosing his continuation value, thereby mitigating the fear of default. This makes the optimal leverage less dependent on the franchise value V^B. This proposition shows that a borrower will be able to lever up even more when the lender does not have the capacity to seize the collateral efficiently. Indeed in this case the lender values much more the promise $\bar{s}$ in states of no default than the actual collateral in states of default (franchise value collateral is more important). In the tri-party repo market $\mu^B - \mu^L$ is higher (more disagreement about the asset) than in the bilateral repo market, so the franchise value collateral channel matters more in the tri-party repo market. This intangible collateral channel, through which the franchise value of the borrower backs the promise of the borrower and enables to achieve lower haircuts (higher leverage), is magnified when beliefs disagreement is high.[22]

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[22] This is equivalent to analyze the pricing of the contract with respect to the respective marginal utilities. When one agent's beliefs is steeper, this agent's marginal utility will be more responsive to the aggregate shock, and as a result will be in less favorable effective bargaining power position against the other agent type.

Finally, the last corollary demonstrates the impact of imperfect competition among lenders on the equilibrium haircuts and rates, and on the franchise value collateral channel.

Corollary 9. *Imperfect competition among lenders temper leverage, but at the same time makes the Franchise Value channel more pivotal.*

Proof. It follows from $\partial_\delta \tilde{s} < 0$, $\partial_\delta m > 0$ and $\partial_\delta r > 0$, as well as $\frac{\partial}{\partial \delta}\left(|\frac{\partial m}{\partial V^B}|\right) > 0$ and $\frac{\partial}{\partial \delta}\left(|\frac{\partial r}{\partial V^B}|\right) < 0$. $\square$

Recall that δ captures the effect of imperfect competition among lenders: the higher δ is, the lesser competition there is among lenders. Intuitively, as the competition among lenders becomes more imperfect (I/J higher) δ increases, so the compound beliefs F^δ become closer to F^B. As a result, beliefs heterogeneity about the tangible collateral matters less than franchise value. From the borrower standpoint, having a dispersed creditor structure enhances its effective bargaining power. A surprising feature of the equilibrium haircut is that it depends on the bargaining structure of the credit market: the haircut equilibrium haircut depends on the number I of borrowers and the number J of lenders. As a result, the total leverage and credit supply in this economy is hindered when the *number* of lenders J is small. This is counterintuive, as one might suppose that the total surplus does not depend on the number of lenders. The wedge arises from the fact that the promise $\bar{s}$ does *not* have the same dependence to m when varying I and J. In economic terms, it is because the difference of valuation in the borrower put option is spread out between the two lenders, enabling the borrower to lever up more. The static model captures a type of diversification benefit from having a dispersed financing. With concentrated financing, the *contractual externalities* between borrowers are exacerbated.[23]

2.2.4 General Equilibrium

I explore here the asset pricing implications of the credit market microstructure. The $t = 0$ price p of the risky asset is endogeneized by relaxing its perfectly elastic supply. Assume now that its supply is fixed, normalized at 1. Combining the optimality conditions arising from the above multilateral Nash bargaining and the risky asset market clearing yields the equilibrium.

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[23] Even more so with capacity constraints on lenders, in accordance with concentration limits faced by money market funds.

Definition 6. The general equilibrium is given by a collection $\left\{x_{ij}, m_{ij}, r_{ij}\right\}_{(i,j)\in I\times J}$ of repo contracts, each specifying a number of units, a haircut and a rate, and a $t = 0$ price p for the risky asset, such that:

i) the outcome of the Nash bargaining among the I borrowers and the J lenders is pairwise stable;

ii) the Walrasian market at $t = 0$ for the risky asset clears: $\sum_i \sum_j x_{ij} = 1$.

In general equilibrium, two countervailing forces are at play for the effect of franchise on haircuts and rates. The partial equilibrium effect of Proposition 1 is still present: higher franchise imply lower haircuts and rates. However, lower haircuts imply higher leverage, hence higher demand in the risky asset market. When the latter is in fixed supply the only margin of adjustment is an inflated asset price p. This in turn tempers leverage, as it makes it deteriorates the expected levered returns of optimists. Finally, Proposition 1 still holds. Only the magnitude of the cross-sectional dispersion is weakened.

Corollary 10. *Proposition 1 is robust to the General Equilibrium.*

In General Equilibrium, the average haircut $\bar{m} = \frac{\sum_I \sum_J x_{ij} m_{ij}}{\sum_I \sum_J x_{ij}}$ satisfy:[24]

$$\bar{m} = I\,\frac{n^B}{p}$$

It implies that the distribution of franchises V^{B_i} is priced in the risky asset through a parsimonious sufficient statistics, the average haircut $\bar{m}$. Its effect is graphed on Figure 2.8 and shows that a distribution skewed towards high franchises props up the asset price p.

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[24] It follows from $\sum_J x_{ij} m_{ij} p = n^B$ and $\sum_i \sum_j x_{ij} = 1$.

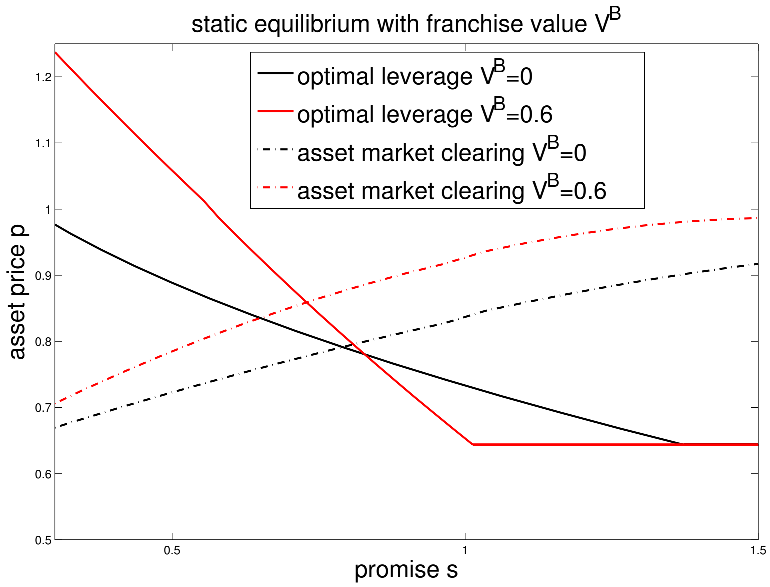

Figure 2.8: *General equilibrium with franchise value.*

In the cross-section of assets, this leads to an endogenous-margin CAPM. With a discrete number of risky assets K, under the same multilateral Nash bargaining , the commonality of borrowers' franchises $\left\{V^{B_i}\right\}$ introduces the correlation between assets. I write m^k and $r^k(m^k)$ as the haircuts and repo rates secured by asset k. There is no risk free asset.

Corollary 11. *Endogenous-margin CAPM: the franchise distribution correlates asset prices.*

The result is not a priori straightforward, as all agents are risk neutrals, hence the traditional CAPM does not hold. The franchises $\left\{V^{B_i}\right\}$ act as pricing kernels, and the more so the larger the franchise is. Precisely the endogenous-margin CAPM formula is:

$$\mathbb{E}_B\left[R^k\right] - r^k(m^k) = \alpha_k + \beta_k V^B$$

with $\beta_k = \frac{m_k F_L(\tilde{s})}{(1-m_k)^2}$ and $\alpha_k = \frac{m_k}{(1-m_k)^2}\int_{s^{min}}^{\tilde{s}} u^k f_L(u)du$. It is a generalization of the Ashcraft, Garleanu and Pedersen (2010) margin-CAPM, by endogeneizing the margins and the risk free rate, and highlight borrowers franchise values as key pricing kernels.[25] The key asset pricing prediction of the model is that asset correlation is higher when franchise values of financial intermediaries are high. This property is consistent with Adrian et al. (2012) which shows the leverage of financial intermediaries, as a single factor, price the cross-section of assets with a R^2 of 0.77. This prediction is contrary to models of fire-sales in which assets are more correlated in bad times.

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[25] Jurek and Stafford (2010) also prices the cross-section of assets in presence of collateralized lending. They feature risk-aversion but leverage and the haircuts are exogenous in their set up.

2.3 The dynamic model

I endogeneize in this section borrowers' franchise values $\left\{V^{B_i}\right\}$, taken as exogenous in the static model. I identify franchise to the continuation value of borrowers in the dynamic version of the static model. This section derives two results. First, haircuts are countercyclical with respect to borrowers' net worth. Although franchise value acts as stabilization in good times, it becomes an amplification force in bad times. The ability of the borrower to lever up today depends on its ability to lever up tomorrow, and this feedback loop creates the high sensitivity of leverage to borrower net worth. Second, this fragility can be mitigated with long-term contracts.

2.3.1 Dynamic environment

The horizon is infinite and time $t, t+1, t+2, ...$ is discrete.

Agents The environment is populated by a number I of borrowers and a number J of lenders, which are all infinitely-lived.[26] The economy is endowed with an infinite supply of Lucas trees, with price p which pay i.i.d dividends at the next period $s(t+1)$. Agents have the same beliefs about this dividend as in the static model.[27] At each period, agents invest and contract through the credit market that has been analyzed in detail in the above section. At each period they consume a fixed fraction c of their total wealth. This artifact implies that agents are simply happy to be rich, and allows to avoid to specify a consumption process for them. Otherwise the borrower would never consume to save its way out of the financing constraint and would eternally postpone consumption. Preferences then are:

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[26] The equilibrium default feature of the model prevents from the need of beliefs switching or of killing optimists at an exogenous Poisson rate of optimists. The ergodic distribution of wealth is not explosive.

[27] The countercyclicality of haircuts is robust to the introduction of persistent shocks. Beliefs can then be written as $s_{t+1} = a^B + \rho s_t + \sigma\epsilon$ and $s_{t+1} = a^L + \rho s_t + \sigma\epsilon$, $a^L < a^B$ (both perceive the AR(1) for the asset but with different drifts). In the continuous-time representation of the game, the dividend stream follows a random walk (Brownian motion with drift μ^i and volatility σ). I rule out learning. The fact that lenders do not learn about the risky asset could be microfounded by a model of rational inattention. Indeed, Dang et al. (2011b)

$$U_t^i = \mathbb{E}_i \left[\sum_{k=0}^{\infty} \rho^k \, 1_{\{no\,def\}} \, c \, n_{t+k}^i \right]$$

Timeline Each period is broken down in 3 stages, which are the exact same steps as in the static model. The stage timing is as follows and illustrated in Figure 2.9.

- Stage 0 *'evening'*: All the agents enter into multilateral Nash bargaining, and writes the resulting contracts. A bilateral contract (i, j) is the combination of a repo contract $\{x_{ij}, m_{ij}, r_{ij}\}_{(i,j)\in I\times J}$ (specifying a number of units, a haircut and a rate), plus an unsecured long-term promise. The latter is activated only in the equilibrium with long-term contracts.

- Stage 1 *'night'*: The asset shock is realized overnight. Agents i that face $\sum_{j\in J} x_{ij} s < \sum_{j\in J} x_{ij}\bar{s}_{ij}$ are in situation of distress. During the night, any agent can decide to pledge its continuation value V^{B_i}, albeit at an exogenous (high) rate r^*. I define $\beta = \frac{1}{1+r^*}$.

- Stage 2 *'morning'*: contracts are settled. If agents default, they exit the market with outside utility U^{def}. In case of default, debt holders seize the tangible collateral. If agents do not default, they settle both the short-term repo contracts (promises $\bar{s}_{ij}$) and pay back a fraction of the emergency borrowing if contracted at any prior stage 1. This reduces the principal balance on the emergency loan. Next they consume a constant fraction c of the remaining wealth. Before moving on to the next period, agents decide to stay or not in the long-term bilateral relationships if long-term contracts have been written at any prior stage 0.

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show that under debt and high information acquisition costs, the lender does not learn about the underlying asset. Under a flexible technology of information acquisition, Yang (2012) shows the robustness of such result.

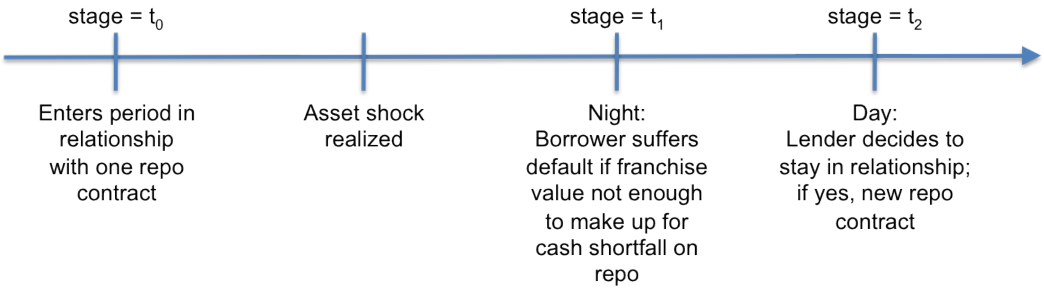
stage = t_0
Enters period in relationship with one repo contract
Asset shock realized
stage = t_1
Night: Borrower suffers default if franchise value not enough to make up for cash shortfall on repo
stage = t_2
Day: Lender decides to stay in relationship; if yes, new repo contract

Figure 2.9: *Timeline of the dynamic model.*

2.3.2 Equilibrium with short-term contracts

In a first step, I rule out long-term contracts. No out-of-the-period promises can be made to lenders; only short-term repo contracts can be written. The policy choices then are the number of contracts x_t^j with each lender j, the haircut m_t^j and the promise $\bar{s}_t^j$ in each of these contracts. The amount of emergency borrowing is also a choice: borrowers decide which fraction ϕ of their continuation value to pledge against immediate liquidity.

The distress region Denote $\bar{s}$ the average level of promises contracted at stage 0: $\bar{s} = \frac{\sum_{j \in J} x_{ij} \bar{s}_{ij}}{\sum_{j \in J} x_{ij}}$. At stage 1 of the dynamic model, any borrower faces the following partition of the state space:

- If $\bar{s} < s$, the agent i does not default.
- If $\tilde{s} < s < \bar{s}$, the agent pledges an endogenous share $\phi(s)$ of its continuation value, and stays afloat.
- If $s < \tilde{s}$, the agent is forced into default.

The state $\tilde{s}$ is an endogenous default barrier that distinguishes the default region from the grace region ('light distress'). In the distress region, borrowers decide how much $\phi \in [0;1]$ to pledge of their continuation value. This share is above what is needed to receive the liquidity that covers exactly the shortfall on short-term promises. Indeed, even the emergency rate

r^* is prohibitive,[28] an agent that stays afloat but with zero net worth enjoys zero utility.[29] Hence by choosing ϕ, the distressed borrower also picks a level of net worth $n_{t+1}^{B\,post-grace} > 0$:

$$n_{t+1}^{B\,post-grace}(s) = \sum_{j\in J} x_{ij}s - \sum_{j\in J} x_{ij}\bar{s}_{ij} + \beta\,\phi(s)U^B\left(n_{t+1}^{B\,post-grace}(s)\right)$$

The policy choice ϕ is determined by this fixed point. Given stage 0 contracting, denote by $\Delta(s)$ the state-contingent cash shortfall on short-term promises (positive in the distress region, and affine decreasing with the state):

$$\Delta_{x,\bar{s}}(s) = \left(\sum_{j\in J} x_{ij}\bar{s}_{ij}\right) - \left(\sum_{j\in J} x_{ij}\right) s = x\bar{s} - xs$$

This yields an expression of the state-contingent $\phi(s)$ as a function of $n_{t+1}^{B\,post-grace}$:

$$\phi(s) = \frac{n_{t+1}^{B\,post-grace}(s) + \Delta(s)}{\beta U^B\left(n_{t+1}^{B\,post-grace}(s)\right)} \tag{2.5}$$

For $s \in (\tilde{s};\bar{s})$:

$$n_{t+1}^{B\,post-grace}(s) = \beta\phi(s)U^B\left(n_{t+1}^{B\,post-grace}(s)\right) - \Delta(s)$$

Taking the value function $U^B(n)$, as well the contracts $\{x, \bar{s}\}$ of stage 0 as given, the fixed point implicitly defines $n_{t+1}^{B\,post-grace}(s)$. For instance, if the value function was linear: $U^B(n) = \theta n$ and $\theta > 1/\beta$ (first-order expansion of the value function), then we obtain:

$$n_{t+1}^{B\,post-grace}(s) = \frac{1}{\beta\phi(s)\theta - 1}\Delta(s)$$

Posit that the policy function ϕ is decreasing and interpolates: $\phi(\tilde{s}) = 1$ and $\phi(\bar{s}) = 0$. The borrower tries to pledge all its continuation value ($\phi = 1$) before entering default.

---

[28] The emergency rate r^* can be thought as exogenous and high, or endogenous and priced by the most pessimistic lender.

[29] Given the consumption process is tied to net worth and agents cannot borrow with zero net worth: as the lenders are all more pessimistic than the optimists, there is no equilibrium in which a pessimist agree to lend to a borrower with zero net worth.

Assume a linear policy function over the grace region:

$$\phi = \frac{\bar{s} - s}{\bar{s} - \tilde{s}}$$

Straightforward algebra on the fixed point yields to:

$$n_{t+1}^{B\,post-grace}(s) = x\frac{(\bar{s} - \tilde{s})}{\beta\theta}\left[1 + \frac{1}{\beta\theta\frac{\bar{s}-s}{\bar{s}-\tilde{s}} - 1}\right]$$

We see that this mapping with respect to s increases from $n_{t+1}^{B\,post-grace}(\tilde{s}) = x\frac{(\bar{s}-\tilde{s})}{\beta\theta-1}$ to the point at which $s = \bar{s} - \frac{1}{\beta\theta}(\bar{s} - \tilde{s})$, before reverting monoticity. By value matching at state $\tilde{s}$ between the default region and the grace region:

$$U^B\left(n_{t+1}^{B\,post-grace}(\tilde{s})\right) = \theta x\frac{(\bar{s} - \tilde{s})}{\beta\theta - 1} = U^{def}$$

This boundary condition characterizes the threshold $\tilde{s}$ between the default region and the grace region:

$$\tilde{s} = \bar{s} - U^{def}\frac{\beta\theta - 1}{\theta x}$$

The default boundary at stage 1 involves not only the value function θ, but also the contracts $\{x, \bar{s}\}$ written at stage 0.

Recursive formulation The environment has a recursive structure with one state variable: the net worth of each borrower $\left\{n_t^{B_i}\right\}_i$. Borrowers' value functions then satisfy the following

Bellman equation:

$$\begin{aligned}
U^B(n_t^B) \;=& \underset{\{x_t, m_t, \bar{s}_t, \phi_t(s)\}}{Max} \quad \mathbb{E}_B\left[1_{\{no\,def\}} c\, n_{t+1} + \rho 1_{\{no\,def\}} U^B\left((1-c)\, n_{t+1}^B\right)\right] \\
& \qquad - \mathbb{E}_B\left[1_{\{grace\}} \rho \phi_t(s)\, U^B\left((1-c)\, n_{t+1}^B\right)\right] \\
(collateral\,constraint) \quad & \sum_J x_{ij} m_{ij} p \leq n^B \\
(default\,condition) \quad & default\, i.i.f\; s < \tilde{s} \\
(contract\,curve) \quad & \omega\left(U_{I,J}^B - U_{I,J-1}^B\right) = (1-\omega)\left(U_{I,J}^L - U_{I-1,J}^L\right) \\
(law\,of\,motion\,of\,wealth) \quad & n_{t+1}^B = \sum_{j \in J} x_{ij}\left(s_{t+1} - \bar{s}_{ij,t}\right) + 1_{\{grace\}} \beta\, \phi_t(s) U^B\left((1-c)\, n_{t+1}^B\right)
\end{aligned}$$

In the stationary Markov equilibrium, the optimal short-term repo contracts picked by borrowers can be characterized by their riskiness $\tilde{s}$ as sufficient statistics. The latter is uniquely determined by the following lemma.

Lemma 14. *The optimal repo contracts picked are characterized by their riskiness $\tilde{s}$, which satisfies:*

$$\begin{aligned}
0 \;=\; & -p(1+\mu^\delta) + \kappa_1 V^B\left(\tilde{s} + V^B\right) + F_\delta(\tilde{s})\, \mathbb{E}_\delta\left[s | s < \tilde{s}\right] + \left(1 - F_\delta(\tilde{s})\right)\, \kappa_2\, \mathbb{E}_B\left[s | s > \tilde{s}\right] \\
& + \beta \frac{1}{1 - \partial_{\tilde{s}} V^B} \frac{\left(R_\delta^{unl}\right)^2}{\partial_{\tilde{s}} R_B^{unl}} \int_{\tilde{s}}^{s^{max}} \partial_{\tilde{s}} R_B^{lev}(s, \tilde{s})\, \partial_{n^B} V^B\left(n^B\left(1 + \mu^{\delta\, barg}\right) R_B^{lev}(s, \tilde{s})\right) f_B(s) ds
\end{aligned}$$

Proof. See Appendix. □

This first order condition implicitly defines the riskiness $\tilde{s}$ of the optimal contract:

$$0 = F^{dyn}\left(\tilde{s}; \mu^\delta, \mu^B, \sigma, V^B, p, \mu^{\delta\, barg}, \beta\right)$$

The first two terms of the first-order condition are identical to the static model. The third new term arises from the endogeneity of the franchise value in the dynamic environment. V^B is not constant anymore, and the borrower takes into account the impact of $\bar{s}$ on the state-contingent $V^B(n_{t+1}^B)$. The term brings in two effects: one is concern about long-term continuation, which tempers leverage. The other one is franchise value collateral, which incentivizes higher leverage. The value function V^B is the fixed point solution of the recursive equilibrium, taking into account two intertemporal interlinkages: the law of motion of wealth, and the franchise value collateral channel.

Compared to the traditional dynamic models of the capital structure,[30] this model features a *grace* region in the dividend space $\mathbb{S}$. When the realization of the dividend s is such as $s \in (\tilde{s}_{\bar{s},V^B};\bar{s})$, the borrower is able not to default. The borrower then has negative net worth, before the credit line cash injection $\beta\,\phi_t(s)U^B\left((1-c)\,n^B_{t+1}\right)$.

The value function is not exactly linear, as the riskiness of the contract depends on the scale of investment x: $\tilde{s} = \bar{s} - U^{def}\frac{\beta\theta-1}{\theta x}$. However this concavity tends asymptotically to linearity as x gets larger and the grace region shrinks to zero measure. The value function is solved on Figure 2.10. The concavity of the franchise value $V^B(n)$ results in the fragility of the franchise value collateral channel exhibited in the static model. At low levels of borrower net worth, franchise value evaporates. The intuition for the concavity of the value function with respect to net worth is as follows. The first-order effect is linear in net worth, as in macro models with financial frictions[31]. In my model, there is an additional role of the value function, which is to relax the default threshold, thus enhancing the debt capacity of the borrower. This feedback loop from rollover ability on the value function is magnified at low levels of net worth and breaks the linearity.

Compounding the concavity of the value function with respect to net worth and the concavity of haircuts with respect to franchise yields the second result of the paper: haircuts are countercyclical with respect to net worth if and only if there is a franchise value channel.

Proposition 9. ***Countercyclical haircuts and rates.***

When $\beta = 0$ (no franchise value channel), haircuts and rates are procyclical.

When $\beta > 0$ (existence of a franchise value channel), haircuts and rates are countercyclical.

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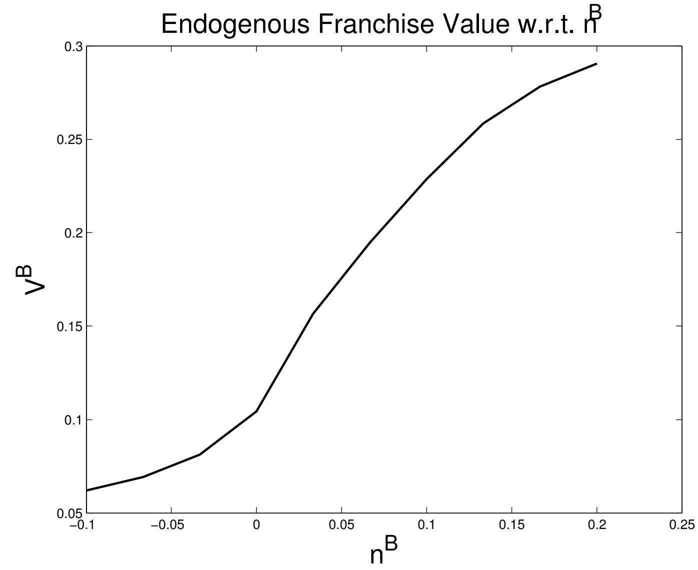
[30] Such as Bolton et al. (forthcoming).

[31] E.g. Brunnermeier and Sannikov (2010) and Gertler and Kiyotaki (2012).



Parameter choice: $\beta = 0.85$, $\sigma = 0.25$, $\mu^B = 1.5$, $\mu^L = 0.5$.

Figure 2.10: *Endogenous franchise value V^B.*

The proposition is illustrated on Figure 2.11. Countercyclical haircuts imply a procyclical leverage.[32] It needs the concavity of the franchise value in order to counteract the direct effect of low net worth: a greater incentive to lever up. The countervailing force is that, at low levels of net worth, the haircut adjust upwards to their no-franchise value levels. For low borrower wealth levels, we have:

$$m(\underset{(-)}{n^B}) = 1 - \frac{1}{p} D\left(\underset{(-)}{n^B}, \underset{(++)}{V^B(n^B)}\right)$$

Furthermore, a consequence of the concavity of the value function with respect to net worth and of the convexity of haircuts to franchise is *fragility of leverage*: haircuts are convex with respect to net worth ($\frac{\partial^2 m}{\partial (n^B)^2} > 0$). This demonstrates that at high levels of net worth,

---

[32] The literature sometimes refers to procyclicality of leverage as with respect to x_t instead of n_t^B. This mechanism of leverage procyclicality is alternative to the 'scary bad news' mechanism of Geanakoplos (2009) and Cao (2011), which relies on an uncertainty shock on the collateral. Their shock is *collateral-specific* whereas my mechanism goes through borrower net worth and is *institution-specific.*

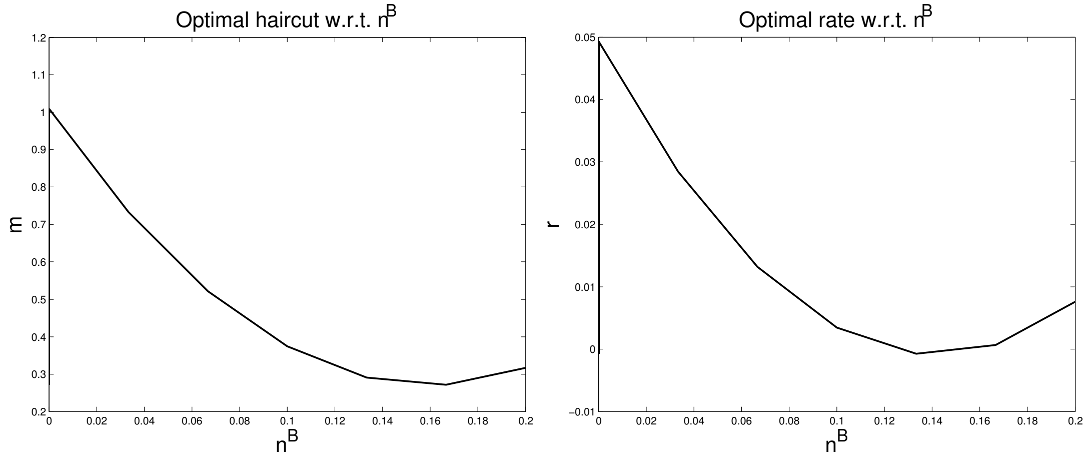


Figure 2.11: *Optimal haircut m and rate r with endogenous franchise value.*

haircuts are low and stable, whereas at low levels of net worth, haircuts are high and unstable. In other words, at low borrower wealth levels the correlation between asset-risk and counterparty-risk $Corr\left(s, V^B(n^B)\right)$ is very high. *It is not a run*, it is a progressive depleting of the borrower debt capacity, which can be very steep when n^B gets closer to zero. It looks like a run on volumes, but it is actually an abrupt adjustment on prices.

2.3.3 Equilibrium with long-term contracts

The existence of a bargaining friction brings a rationale for bilateral long-term contracting.[33] I solve here for the optimal long-term contract, and show how long-term contracting helps mitigating the countercyclicality of haircuts. I also show that dispersed financing (i.e. J/I high) undermines this optimal contract.[34]

I now allow the borrowers to choose between a long-term contract with the lender j (*relationship repo*), or staying out of any long-term relationship (*arm's length repo*). In the latter case, he contracts at each period short-term repo contracts. If the borrower enters a long-term contract, he is able to compensate the lender with promised continuation value V^L_{+1}. This additional instrument enables him to secure lower and more stable haircuts and rates. In the timing of the game, only stage 0 is modified as follows.

---

[33] The bargaining friction breaks the Fudenberg, Holmstrom and Milgrom (1990) irrelevance result of long-term contracts. Without commitment, long-term contracting would still improve over the sequence of short-term contracts in an environment of costly search for counterparties. I leave this set up for future research.

[34] As a consequence, an optimal number of creditors J trades off the benefits of diversification with the costs of dispersed financing due to the inability to promise future continuation value to the lender in this case. It can be seen as a Jacklin (1987) critique: more competition among lenders hurts the optimal contract and leverage.

- Stage 0, *'evening'*: The borrower decides between entering a long-term contract (relationship repo) or staying out. In the former case, the borrower and the lender bargain on the split of the total relationship surplus between the borrower (V^B) and the lender (V^L). This long-term agreement is implemented by a sequence of a short-term (overnight) repo contracts, which specifies a notional D (notional value of debt), an interest rate r and a haircut m, and state-contingent promised continuation values $V^L_{t+1}(s)$. If she decides to stay out, the borrower bargains over short-term contracts as in the static model.[35]

The continuation values of both the borrower and the lender are state-contingent in the long-term contract. As a result, this contracting problem can be seen as an intermediary case between Kocherlakota (1996) (two-sided lack of commitment with autarky as outside options, under complete markets) and Geanakoplos (2003) (one-side lack of commitment with zero as outside option, under incomplete markets).[36] My set up features equilibrium default even under the optimal contract.

---

[35] In the extension without commitment, the lender decides to break up the relationship, it searches for a new borrower match, forming expectations about this franchise value with a mean-field approximation, as franchise value V^B is still concave with respect to net worth n^B, we have by Jensen inequality: $\bar{V^B} = \mathbb{E}_L\left[V^B\left(n^B\right)\right] < V^B\left(\mathbb{E}_L\left[n^B\right]\right)$. This raises the issue of more sophisticated contracts in which the borrower whishes to signal the quality of its balance sheet. In this extension, heterogeneity in lenders information regarding franchise value explains flights to safety as observed in summer 2011 against European banks. With endogenous information acquisition, the optimal long-term relational contract might then want to prevent information acquisition about collateral but foster information acquisition about franchise value (it would be win-win for both parties). A search cost $\theta = \theta(J/I)$, an increasing function of the ratio J/I, would be a reduced-form to capture the bargaining process outside the relationship. J/I is a measure of the tightness of money markets. When $J >> I$, there are many more lenders J than borrowers I, and therefore the search cost of finding a free borrower is very high. Due to the presence of multiple equilibria, the surplus of a new relationship should take into account the probability of sunspot run on the new borrower. This would deteriorate the outside option of the lender, and as a result strengthens the result of lower haircuts thanks to long-term relationships.

[36] The haircut is another price variable for collateralized debt compared to uncollateralized debt. So if this haircut is made state-contingent, it helps completing the markets. The fact that the borrower has *two* choice variables(repo spread r_t and haircut m_t) in effect completes markets and partially overcomes the non-state contingency of overnight short-term debt contracts.

- Expected utility of the borrower:

$$V^B_{I,J,t} - V^B_{I,J-1,t} = \frac{n^B}{m}\left[(1-m)\left(\mu^B - r(m)\right) + \frac{1}{p}\pi^B(\bar{s}) + V^B\frac{F_B(\tilde{s})}{p}\right] + \\ \rho\,\mathbb{E}_B\left[V^B_{I,J,t+1} - V^B_{I,J-1,t+1}\right]$$

- Expected utility of the lender:

$$V^L_{I,J,t} - V^L_{I-1,J,t} = \frac{I}{J}\frac{n^B}{m}\left[(1-m)\left(r(m) - \mu^L\right) - \frac{1}{p}\left(\pi^L(\tilde{s}) + V^B F_L(\tilde{s})\right)\right] + \\ \rho\mathbb{E}_L\left[V^L_{I,J,t+1} - V^L_{I-1,J,t+1}\right]$$

I focus on stationary Markov equilibria.[37] In this case, following Abreu et al. (1990) and Abreu and Pearce (2007),[38] I can use the continuation value of the lender V^L_t as an additional state variable and write the borrowers' maximization program in a recursive formulation:

$$\begin{aligned}
V^B(n^B_t, V^L_t) \; = \; & \underset{\{x_t, m_t, \bar{s}_t, \phi_t(s)\}}{Max} \quad \mathbb{E}_B\left[1_{\{no\,def\}} c\, n_{t+1} + \rho 1_{\{no\,def\}}\left\{V^B\left((1-c)\,n^B_{t+1}\right) - V^L_{t+1}\right\}\right] \\
& \quad - \mathbb{E}_B\left[1_{\{grace\}}\rho\phi_t(s)\,V^B\left((1-c)\,n^B_{t+1}\right)\right] \\
(collateral\,constraint) \quad & \sum_J x_{ij} m_{ij} p \le n^B \\
(default\,condition) \quad & default\,i.i.f\; s < \tilde{s} \\
(contract\,curve) \quad & \omega\left(V^B_{I,J} - V^B_{I,J-1}\right) = (1-\omega)\left(V^L_{I,J} - V^L_{I-1,J}\right) \\
(law\,of\,motion\,of\,wealth) \quad & n^B_{t+1} = \sum_{j\in J} x_{ij}\left(s_{t+1} - \bar{s}_{ij,t}\right) + 1_{\{grace\}}\beta\,\phi_t(s) V^B\left((1-c)\,n^B_{t+1}\right)
\end{aligned}$$

The continuation value is equal to the promised utility $V^L_{I,J,t+1}$ if the borrower does not default, and to $V^L_{I-1,J,t+1}$ if the borrower defaults after seizing collateral. The borrower designs his optimal long-term contract taking the outside option of the lenders as given. The equilibrium optimal contract is then the fixed point on this outside option. The borrower has now a third instrument, beyond the promise $\bar{s}_t$ and the leverage x_t: the state-contingent long-term promise $V^L_{+1}(s)$. The optimization of this policy variable involves the following trade-off. On the one hand, promising more $V^L_{+1}(s)$ enables the borrower to lever up today at no cost. On the other hand, it diminishes the share of the surplus the borrower can enjoy inside the relationship tomorrow, as $\partial_{V^L} V^B < 0$, as shown in the simulation in Figure 2.12, Panel A. As long as:

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[37] Thus I rule out more complicated strategies, where some borrowers might ask the lender what the terms of the contract proposed to him by the other borrowers. I also rule out cooperation among borrowers, which could punish lenders that break up with even lower bargaining power at the start of new relationship.

[38] An alternative would be Marcet and Marimon (2011), where the dynamics of co-state variables give insights on the tightness of the constraint.

$$\rho\, \mathbb{E}_L \left[1_{\{no\, def\}} \left(1_{\{stays\}} V^L_{t+1} + 1_{\{quits\}} V^{L\, out}_{t+1} \right) + 1_{\{def\}} V^{L\, out}_{t+1} \right] > V^L_t$$

then the borrower is able to lever up more than in the case without long-term relationships: $D^{LT} > D^{ST}$.

Lemma 15. *The f.o.c. of the optimal short-term riskiness in presence of long-term contracts is:*

$$\begin{aligned} 0 \;=\;& -p(1+\mu^{\delta}) \\ & +\kappa_1 V^B \left(\tilde{s} + V^B \right) + F_{\delta}(\tilde{s})\, \mathbb{E}_{\delta}\left[s | s < \tilde{s} \right] + \left(1 - F_{\delta}(\tilde{s}) \right)\, \kappa_2\, \mathbb{E}_B \left[s | s > \tilde{s} \right] \\ & + \frac{\beta}{n^B \left(1 + \mu^{\delta\, barg} \right)} \frac{1}{1 - \partial_{\tilde{s}} V^B} \frac{\left(R^{unl}_{\delta} \right)^2}{\partial_{\tilde{s}} R^{unl}_B} \int_{\tilde{s}}^{s^{max}} \left[n^B \left(1 + \mu^{\delta\, barg} \right) \partial_{\tilde{s}} R^{lev}_B \partial_{n^B} V^B + \partial_{\tilde{s}} V^L \partial_{V^L} V^B \right] f_B(s) ds \end{aligned}$$

where

$$\mu^{\delta\, barg} = \mu^{\delta} + \tfrac{\delta}{n^B} \left(U^B_{I,J-1,t} - \beta\, \mathbb{E}_B \left[V^B_{I,J,t+1} - V^B_{I,J-1,t+1} \right] \right) - \tfrac{1-\delta}{n^B} \left(V^L_{I-1,J,t} - \beta \mathbb{E}_L \left[V^L_{I,J,t+1} - V^L_{I-1,J,t+1} \right] \right)$$

The optimal long-term contract substitutes continuation value for haircuts ($\frac{\partial m}{\partial V^L_{I,J,t+1}(s)} < 0$). The interpretation is that both V^B and V^L are supporting the promise $\bar{s}$ (*intangible collateral*). The relationship value V^L helps mitigating the countercyclicality of V^B compared to the equilibrium with only short-term contracts.

Proposition 10. ***Stability of leverage in long-term relationships.***

Leverage under the long-term contract is less volatile than in the sequence of short-term contracts.

Proof. Appendix derives $0 < \partial_{n^B} \tilde{s}^{LT} < \partial_{n^B} \tilde{s}^{ST}$. ☐

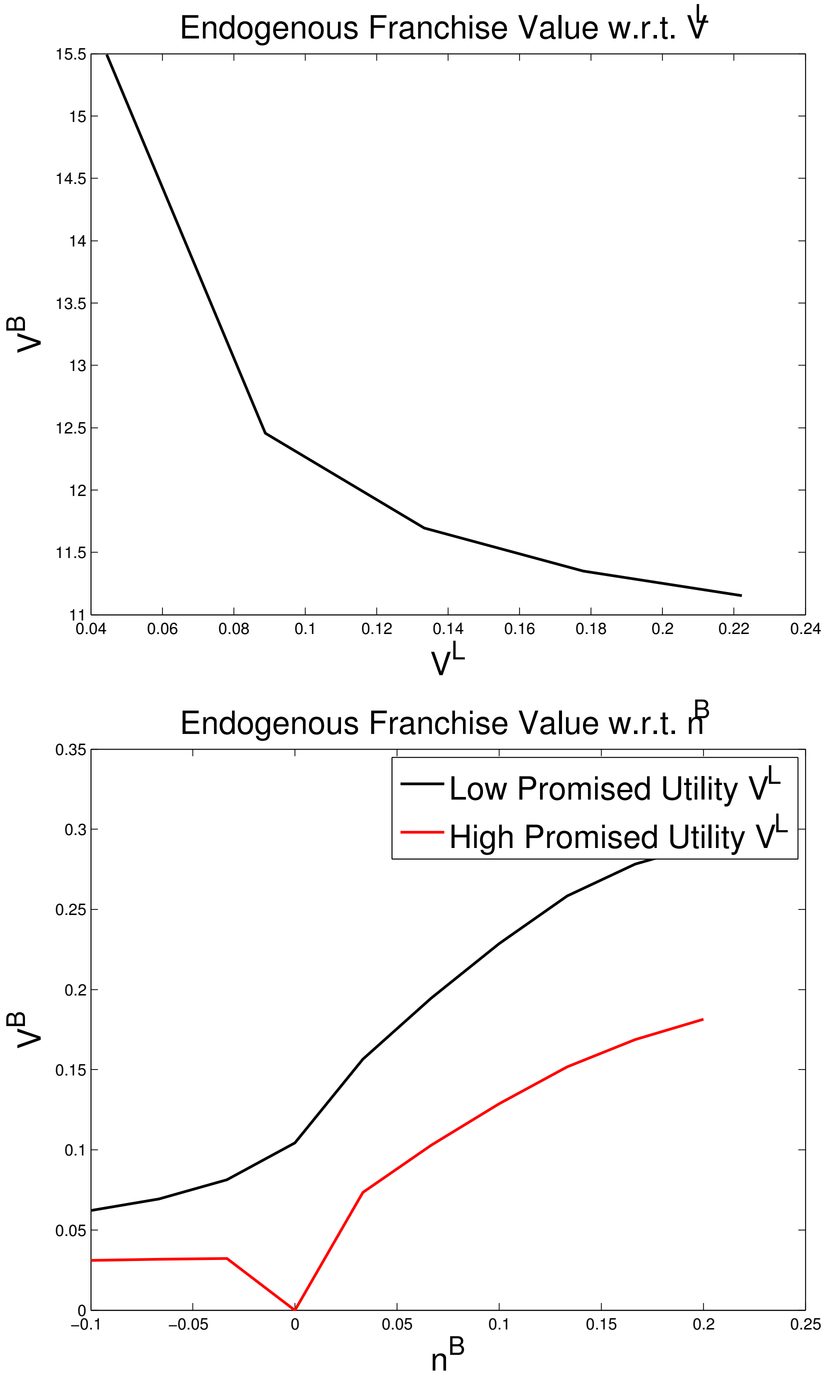

Figure 2.12: *Endogenous franchise value with long-term relationships.*

In long-term relationships, the continuation value is used as a **haircut waiver**, and as a result repo funding is more stable in volumes and in prices (i.e. haircuts and rates). It avoids margin calls exactly in the states where borrower net worth is low, the states where he would like to lever up in order to replenish capital. At low levels of net worth, under short-term contracts, the haircut adjusts abruptly upwards. Under long-term contracts, there is a surplus gain to grant a haircut waiver to the borrower: the lender is ready to maintain a low haircut against the promise of more long-term continuation value.[39] Indeed, the concavity and the fragility of V^B are mitigated for high levels of promised continuation value V_t^L, as illustrated in Panel B of Figure 2.12. The continuation values of the relationship for both parties in excess of their outside option commands not only the current pricing of the repo contract, but also the volatility of the relationship to shocks.

A continuous-time version of the environment and the introduction of persistent shocks enables to derive the following expression for the sensitivity ξ of the lender continuation value to asset innovations:

$$\xi = -\frac{\partial_{nV^L} V^B}{\partial_{V^L V^L} V^B} \frac{n\,\sigma}{mp} > 0$$

When the mapping $V^L \mapsto V^B(V^L)$, we obtain that: $\xi < \sigma$, where σ is the fundamental volatility of the asset. This can be interpreted as long-term relationships mitigating the volatility of asset markets. Borrowers then engage in *volatility transformation*. It is another justification of financial intermediation: insulate the final lender from the shocks on the underlying collateral. The intuition for this insurance result is that it is optimal for both parties to insure the lender against the aggregate shock even in a risk-neutral environment. Long-term relationships enjoy more stable financing. On the contrary, the pricing terms and volume of short-term relationships are more volatile. Therefore, it is optimal for the borrower to concentrate its financing, provided they can commit to long-term contracts.

---

[39] If we add precautionary motive and occasionally binding collateral constraint, this margin waiver is even more valuable in the states of the world in which the collateral constraint binds: it economizes on margin spirals. Moreover, in the framework of Oehmke (2012), waiving a haircut call avoids disorderly liquidation of illiquid collateral, and as such avoid cost of illiquidity and this is translated in repo spread.

Without commitment and costly search, when the lender is far from its participation constraint, the lender is more entranched, and so optimal contract features higher endogenous surplus, and this alters policy functions x_t and $\bar{s}_t$. As such it enhances the endogenous franchise value V^B and achieve lower margins. Moreover, in this case, the margin is less responsive. This a result of the dynamics of the relationship a la Thomas and Worrall (1988): as long as the (IR) do not bind, the optimal contract does not them into account. I conjecture that the lender will not be willing to quit the contract or renegotiate as in the outside option, due to the newness of the relationship, the continuation values are not as high and so the haircuts required to the borrower will be higher than in the current contract. This feature of the optimal contract (higher margins are required in new relationships) endogenously prevent from the borrower from exiting the contract ex-post. This required high margin in a new relationship outside option acts as an endogenous glue (no need of an exogenous cost of breaking up the relationship) to make the optimal contract sustainable.

2.4 Extension: the rehypothecation chain

The goal of this part is to provide an economic rationale to repo chains. I model the pyramiding arrangement consisting of having money market funds lending to broker dealers, which in turn lend to hedge funds. The two lending agreements are secured by the same collateral (rehypothecation). I show how the franchise value of the broker dealer is priced in the optimal contract on both sides of its balance sheet.

I keep only two types of beliefs,[40] but I break down optimists into two sub-groups: the no-franchise optimists ('Hedge Funds': HF) and the optimists that are endowed with franchise value ('Broker Dealers': BD). The pessimists are 'Money Market Funds': MMF. I assume rehypothecation of collateral.[41] I construct an equilibrium in which MMF lends to BD (the *tri-party repo* debt) and in turn BD lends to HF (the *bilateral repo* debt). x^{tri}, m^{tri} and r^{tri} are the number of contracts (each collaterized by one unit of the risky asset), the haircut and the rate of the first transaction ($\bar{s}^{tri}$ is the promise and D^{tri} the value of each contract). x^{bil}, m^{bil}, r^{bil}, $\bar{s}^{bil}$ and D^{bil} are the respective quantities for the second transaction. Only BD enjoys franchise value: $\tilde{s}^{bil} = \bar{s}^{bil}$ and $\tilde{s}^{tri} = \bar{s}^{tri} - V^B$. Figure 2.13 gives the $t = 1$ contractual payoffs.

The balance sheet constraint of HF binds: $pm^{bil}x^{bil} = n^{HF}$. The balance sheet constraint

---

[40] In this environment, the introduction of a 3rd type M with moderate priors and a second state-variable, reputation capital f_t^i, would make the moderates type emerge as Financial Intermediaries, as the one with the relative most acute incentive to build up franchise value.

[41] If the Broker Dealer debt is collateralized by Hedge Fund debt, the analysis is more tedious as the BD debt needs to be priced as a *put on put*, using Geske formula, but the qualitative results of the repo chain are similar.

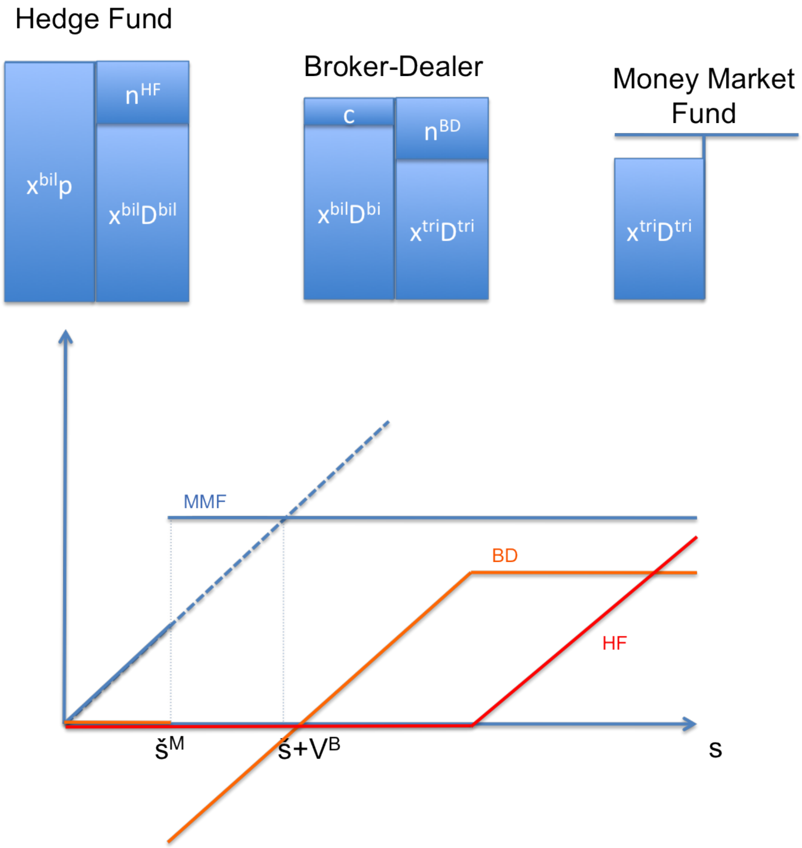
Hedge Fund
$x^{bil}p$
n^{HF}
$x^{bil}D^{bil}$
Broker-Dealer
c
n^{BD}
$x^{bil}D^{bi}$
$x^{tri}D^{tri}$
Money Market Fund
$x^{tri}D^{tri}$
MMF
BD
HF
$\check{s}^{M}$
$\check{s}+V^{B}$
s

Figure 2.13: *Repo chain with rehypothecation of collateral.*

(BS) of the BD is: $x^{bil}D^{bil} \leq n^{HF} + x^{bil}D^{bil}$. Cash raised by the BD non-invested in HF debt is invested in the risky asset. The rehypothecation collateral constraint (CC) imposes: $x^{tri} \leq x^{bil}$. There are two regimes, depending on which of the constraint (CC) or (BS) binds. The $t = 0$ expected payoffs of the agents are as following, denoting the collar $\Delta\pi^B(m^{bil}; m^{tri}) = \pi^B(\tilde{s}^{bil}) - \pi^B(\tilde{s}^{tri})$:

$$\begin{cases} U^{HF} - U_0^{HF} & = \frac{n^{HF}}{m^{bil}p}\left[\left(1-m^{bil}\right)\left(\mu^B - r^{bil}\right) + \pi^B(\tilde{s}^{bil})\right] \\ U^{BD} - U_0^{BD} & = \frac{n^{HF}}{m^{bil}}\left[r^{bil}(m^{bil}; m^{tri}) - r^{tri}(m^{bil}; m^{tri}) - \Delta\pi^B(m^{bil}; m^{tri}) - m^{tri}r^C p\right] \\ U^{MMF} - U_0^{MMF} & = x^{tri}\left[\left(1-m^{tri}\right)\left(r^{tri} - \mu^L\right)p - \pi^L(\tilde{s}^{tri}) - V^B F_B(\tilde{s}^{tri})\right] \end{cases}$$

Solving jointly for the two bargaining processes (HF-BD and BD-MMF) delivers that BD engages in a positive carry trade on repo rates.

Lemma 16. *The Broker-Dealer earns a positive repo rate spread:*

$$r^{bil}(m^{bil};m^{tri}) - r^{tri}(m^{bil};m^{tri}) = \frac{1}{1-\omega_{tri}(1-\omega_{bil})} \left[\omega_{bil}(1-\omega_{tri})\left(\mu^B + \frac{1}{1-m^{bil}}\pi^B(\tilde{s}^{bil}) - \mu^L - \frac{1}{1-m^{tri}}\pi^L(\tilde{s}^{tri})\right)\right] \quad (2.6)$$

From the BD first order conditions in $(m^{bil};m^{tri})$, the rehypothecation chain features even lower haircuts.

Proposition 11. *A high Broker-Dealer franchise value lowers both the bilateral and the tri-party haircuts.*

Proof. Appendix shows $\frac{\partial m^{tri}}{\partial V^B} < 0$ *and* $\frac{\partial m^{bil}}{\partial V^B} < 0$. □

Even if both BD and HF are equally optimistic, they will find a rationale to contract secured debt. The haircut spread $m^{bil} - m^{tri}$ can be negative, and this is sustainable in equilibrium as the BD is compensated through a positive rate spread $r^{bil} - r^{tri}$. It rationalizes bilateral haircuts lower than triparty haircuts in normal times, while observing bilateral haircuts higher than tri party haircuts in stress times.

The introduction of a role for franchise value in the ability to lever up provides a justification for financial intermediation, alternative to the threat of runs as disciplining device as in Diamond and Rajan (2001a) and Diamond and Rajan (2001b), or the returns to scale in monitoring costs as in Diamond (1984) and Holmstrom and Tirole (1997). In my approach, financial intermediaries has a superior ability to develop franchise value, and this helps mitigating the bargaining frictions on both sides of the balance sheet of the broker dealer. It delivers in equilibrium the two-tiered structure: $HF \mapsto Broker-Dealer \mapsto MMF$.[42] Broker-dealers, on their asset side, are more able to seize the collateral than pessimists. At the same time, on their liability side, they are more able to lever up their franchise value than hedge funds. The first array is backed by tangible collateral: so more responsive but also more robust. The second array is backed by intangible franchise value, and therefore less responsive to collateral shocks.

---

[42] This mechanism is reminiscent of the industry practice, especially by universal banks, to *wrap up the collateral* with some of its own credit risk/franchise value.

Moreover, the comparative statics of the haircut spread being positive with respect to volatility, it rationalizes why, with an increase in uncertainty σ, bilateral haircuts are more responsive than tri-party haircuts. This also delivers a more volatile (and procyclical in general equilibrium) leverage for HF than for BD, consistent with Krishnamurthy (2010) evidence. Furthermore, the *haircut spread* can turn negative.

In General equilibrium, when endogeneizing the price p of the asset, p is equal to the sum of its fundamental value and of *two* collateral values: the one enjoyed by HF and the one enjoyed by BD. This happens from rehypothecation practice, a *collateral multiplier* effect. Bringing this repo chain to dynamics requires two state variables: net worth of the BD and of the HF, and derive the value function $V^B(n^B; n^{HF})$ along the lines of section 3. This set up delivers a leverage more procyclical for HF than BD at high levels of BD net worth. On the other hand, at low levels of BD let worth, the concavity of $V^B(n^B; n^{HF})$ makes both HF *and* BD leverage procyclical, a state that can be thought of systemic crisis.

2.5 Empirical analysis

The purpose of this section is to provide support to the 3 key predictions of the model:[43]

1. Proposition 1: in the cross-section, high franchise value borrowers secure lower haircuts *and* rates.
2. Proposition 2: in the time-series, haircuts are more sensitive to borrower's franchise at low net worth.

---

[43] Moreover, haircuts and rates are more sensitive to franchise value when the collateral is illiquid/volatile (Corollary 1), so the effects should be stronger on illiquid collateral. The static model has the additional prediction that haircuts and rates are more sensitive to franchise value with higher effective bargaining power of the lender (Corollary 4). The dynamic model also predicts that haircuts are low and stable at high borrower net worth and high and fragile at low borrower net worth.

3. Proposition 3: haircuts and rates are lower and more stable in long-term relationships.

Taken together, the tests reject the hypothesis that repo markets are perfectly competitive and provide evidence that relationships matter even in secured funding. Repo markets involve four types of prices and volumes determinants: collateral specific (the type of the underlying security of the repo), borrower i specific (its franchise value V^B), lender j specific (its bargaining power ω and its risk attitude), and relationship ij specific (long-term relationship value V^L).

2.5.1 Data

I use a hand-collected dataset of repo transactions (*repurchase agreements)* contracted over the last six years by money market funds.[44] My dataset includes $27,172$ repo transactions extracted from the quarterly SEC filings of the universe of the 145 largest Prime Institutional Money Market Funds. Money Market Funds (MMF) compose the largest volume of repo lending. According to the September 2012 Flow of Fund, US Money Market Funds hold \$508.4 *bn* outstanding in repo contracts for 2012Q2, which represents 65% of total volume of repo lending in the US to banks and broker dealers on this quarter.[45] In turn the repo holdings of the MMFs in the sample account for \$280 *bn*, i.e. 55% of the total MMF holdings. As the sample is composed of the prime institutional money market funds, I argue that the selection bias of the sample works against the tested hypothesis of screening with respect to borrower franchise and relationships. Indeed, smaller funds would have an even more pronounced incentive to trust franchise and relationships over collateral. I merge this dataset with broker-dealers characteristics: CDS from Markit and balance-sheet quantities from Y9-C call reports.

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[44] Most transactions in the dataset are US tri-party (using JP Morgan Chase and BNY Mellon as a clearing bank).

[45] The ratio is computed from Table L207 of Sept. 2012 Flow of Funds. MMF holdings are the liability line 'Money market mutual funds' and total repo lending to banks and broker dealers are the sum of the following asset lines: 'U.S. chartered depository institutions', 'Foreign banking offices in the U.S.', 'Credit unions' and 'Security brokers and dealers'. The aggregate time-series of other source of funding of broker dealers (commercial paper and fed funds) show that these markets do not perfectly substitute to secured funding.

Data from Money Market Fund filings

The identity and CIK numbers of the 145 largest Prime Institutional Money Market Funds is obtained from Peter Crane intelligence. Prime Money Market Funds are a recent financial innovation which allegedly offers higher returns with no risk, and are allowed to invest in non-government securities. Following the procedure of Krishnamurthy et al. (2011), I parse with a Perl script all the quarterly filings of the last 6 years of these 145 MMF (24 quarterly filings for each MMF: forms N-Q, N-CSR and N-CSRS available on SEC Edgar website).[46] MMFs of the same family concatenate their filings in the same html file. I collapse these MMF in one lender identity j, in order to wash out substitution effects from one fund to another within the same family. The haircut can be computed from the collateral fair value and the notional, and the repo rate can be computed from the repurchase amount and the notional.[47] I categorize the collateral described as free-entry text in MMF filings into 9 categories: Treasuries, Agencies, Municipals, Commercial Paper, Corporate Debt, Foreign Debt, Equities, Structured Finance and Mixed Pool. This follows the topography of collateral used in custodian contracts of tri-party agreements. My dataset contains all the repo transactions reported in these SEC filings, and details for each of those: the volume of the transaction, the rate, the haircut, the maturity, the collateral type and the identities of the borrower and the lender. The counterparty identity is manually screened and replaced by the relevant franchise name (e.g. Barclays for Barclay's Capital or any subsidiary of Barclays). I compute the repo spread, as the difference between the repo rate and the Fed fund rate of the same maturity.

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[46] I carry out robustness checks on N-MFP forms, filed monthly by the MMF since November 2010.

[47] For joint repurchase agreements, the haircuts and rates are computed over the entire collateral pool, and assigned to each repo transaction included in the joint repurchase agreement. This type of joint contract usually involves MMF from the same family.

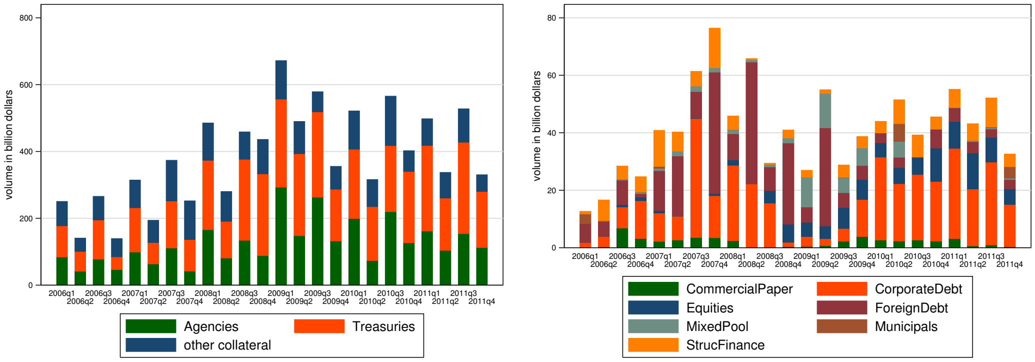

Figure 2.14: *Aggregate repo volume by collateral class.*

Complementary data source

I use specific characteristics of borrowers and of lenders as regressors on repo prices and volumes. For borrowers, I match the broker-dealers included in the repo dataset with Y9-C call reports items: *goodwill* (Y9C item BHCK3163) as proxy for the franchise value V^B and *total equity capital* (Y9C items BHCK3300 and BHCK2948) as proxy for borrower net worth n_t^B. I also fetch their respective exposure to the different funding markets: *commercial paper* (Y9C item BHCK2309) and *fed funds* (Y9C item BHDMB993), in order to control for substitution effects between these markets. I also match the borrowers with market-based measures of their franchise: CDS and CDS lagged 3 months (from Markit).[48] Regarding the lenders, I use CRSP Mutual Fund database to construct a measure of their risk attitude based on Inflows and Yield their experienced in 2008, following the procedure of Kacperczyk and Schnabl (2012) and Chernenko and Sunderam (2012). For collateral, I use volatility index (VIX and TED)) to capture the volatility of the underlying collateral. Finally, to assess the persistence of relationships between Hedge Funds and Broker Dealers, I use prime brokerage information in TASS dataset. For robustness, I also explore the cross-section of haircuts and rates in a sample not from money market funds but from pension funds, which are not regulated by Rule $2a-7$. Data have been obtained under the Freedom of

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[48] CDS of 5 year tenure (the most liquid). I manually take into account franchise mergers (HVB taken over by Unicredit, Wamu by BoA, Wachovia by Wells Fargo).

Information Act.

2.5.2 Summary statistics

Aggregate volumes

Figure 16 plots the time-series of aggregate repo volume from the dataset by collateral class. It follows a pattern analogous to the Flow of Fund repo lending to banks and broker dealers, albeit less dramatic. The discrepancy therefore comes from repo not contracted by money market funds, e.g. the *rest of the world* lending category in Flow of Fund which sharply contracted over the crisis. This is consistent with the hypothesis of the existence of relationships between banks and broker dealers and money market funds that helped sustain a stable level of repo funding over the crisis along these relationships. The right-hand side panel plots the same time-series excluding Treasuries & Agencies. Consistent with Krishnamurthy et al. (2011) and (Martin, 2012), it documents a volatile level of repo funding for structured finance. I use this segment of the repo market to gain power in the test of the franchise value channel. The time-series of aggregate volume has a semestrial spiky shape.[49] This might be due to difference in data reporting between forms N-Q (q2 and q4), and forms N-CSRS/N-CSRS (q1 and q3). This is not of a concern given the cross-sectional analysis.

Pricing terms

Table 2.1 gives the summary statistics of haircuts. For pricing terms (haircut and rate), the dataset is winsorized at the 1% and 99% levels to dismiss reporting mistakes. The majority of repo transactions collateralized by Treasuries command a haircut of 2%. However, some are not, and these are mostly term repos with long-term maturities. Figure 2.18 documents that there is a higher haircut dispersion for more volatile collateral, consistent with Proposition 4 of the model. Summary statistics of repo spreads are given in Table 2.2 and dispersion in Figure 2.19. Even more than haircuts, repo spreads exhibit dispersion around the mean for more volatile collateral. Maturities are the third pricing variable of a repo transaction and its summary statistics are given in Table 2.3. In the specifications presented in the following, I focus on overnight repos (maturity of 1 day) to make sure the results are not driven by maturity risk. Across the three pricing variables, the sample documents an aggregate time-varying funding premium, especially in the Lehman episode and over the European debt crisis. I investigate in the following their idiosyncratic component.

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[49] The relative decline of volume at each end-of-year quarter hints to some window-dressing practices. Although this is not a first-order issue in the present idiosyncratic analysis, I run robustness regressions excluding all quarters q4.

Table 2.1: *Summary statistics of haircuts by collateral class*

col_class	mean	sd	p10	p25	p50	p75	p90
1-Treasuries	.023	.021	.02	.02	.02	.02	.029
2-Agencies	.026	.03	.02	.02	.02	.029	.029
3-CommPaper	.029	.01	.02	.02	.029	.033	.048
4-Municipals	.039	.021	.02	.02	.038	.048	.065
5-CorporateDebt	.048	.033	.02	.029	.048	.056	.074
6-ForeignDebt	.055	.036	.029	.03	.048	.053	.092
7-Equities	.063	.02	.048	.048	.048	.077	.091
8-StrucFinance	.046	.036	.02	.02	.032	.071	.077
9-MixedPool	.027	.013	.02	.02	.02	.035	.048
Total	.03	.03	.02	.02	.02	.029	.048

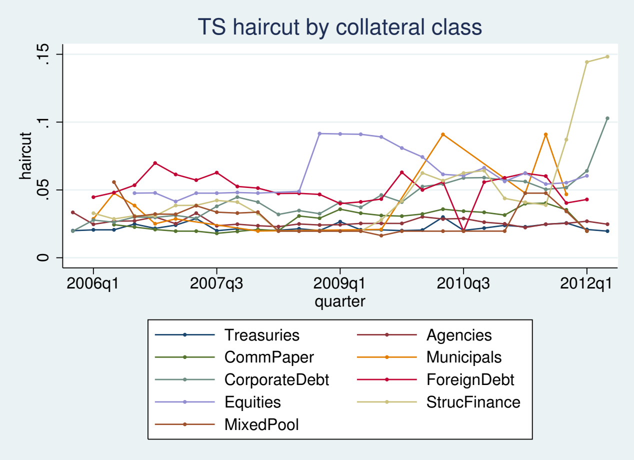

Figure 2.15: *Times-series of haircuts by collateral class.*

Table 2.2: *Summary statistics of repo spreads by collateral class. The repo spread is the spread between the repo rate and the Fed fund rate of the same maturity. It is annualized and given in basis points*

col_class	mean	sd	p10	p25	p50	p75	p90
1-Treasuries	-3.91	43.9	-21	-5.89	1.23	6.25	16.9
2-Agencies	4.14	30	-15.3	-2.7	4.17	14	25
3-CommPaper	21.1	20.3	5.33	11.9	16.4	27.7	49.3
4-Municipals	31.9	31.2	-12.7	16.6	29.7	38.7	81.8
5-CorporateDebt	24.8	42.6	-11.9	12.5	22	40.8	70.1
6-ForeignDebt	55.3	56.9	10.4	20.1	35.9	51.9	171
7-Equities	29.3	29.3	-8.99	18.2	27.7	39.2	53.3
8-StrucFinance	29.5	45	-10.7	2.92	24	44.9	83.4
9-MixedPool	9.47	34.1	-16.8	-3.14	10.5	31.7	36
Total	7.08	42.4	-15.9	-2.9	4.31	17.5	38.9

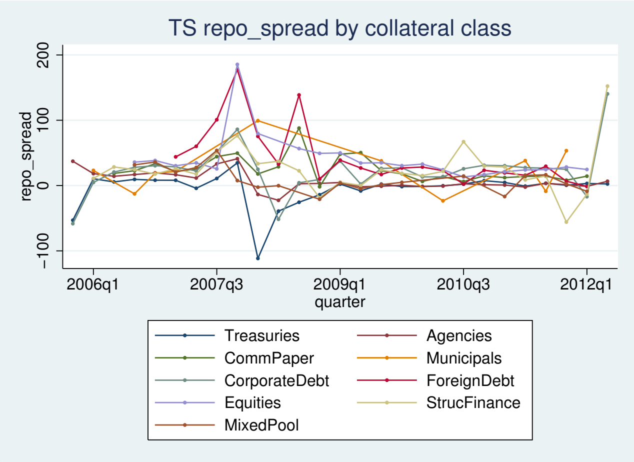

Figure 2.16: *Times-series of repo spreads by collateral class.*

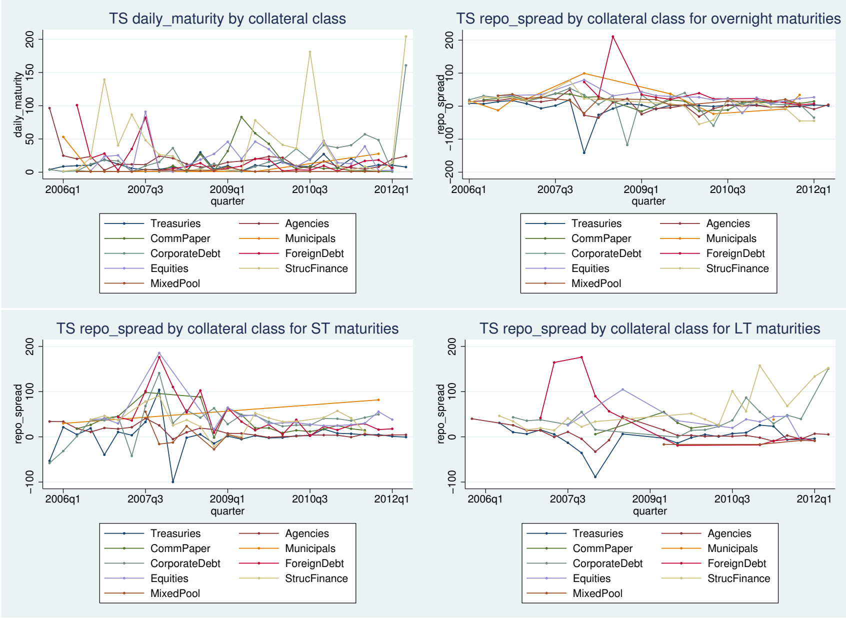

Figure 2.17: *Time-series of repo maturities, by collateral class.*

Table 2.3: *Summary statistics of repo maturities by collateral class.*

col_class	mean	p50	p75	p90
1-Treasuries	9.71	1	3	24
2-Agencies	11.6	3	3	32
3-CommPaper	18.2	1	6	90
4-Municipals	19.9	1	32	90
5-CorporateDebt	24.4	3	31	90
6-ForeignDebt	14.3	2	3	30
7-Equities	23	3	7.5	90
8-StrucFinance	48.1	3	90	182
9-MixedPool	2.36	2	3	4
Total	13.1	2	3	35

Microstructure: network of relationships

In Figure 2.20, I provide a graphical representation of the network of bilateral repo transactions for two key quarters: 2007q3 (start of the crisis) and 2008q3 (midst of the crisis). The dataset features 40 borrowers (broker-dealers) and 45 lenders (the 145 money market funds grouped by family).

One static feature of the network relies in agents' heterogeneity in their *concentration* of financing. On the borrower side, nodes like Merrill, Citi and Goldman Sachs secure repo funding from a variety of sources. On the contrary, Deutsche Bank, Barclays and JP Morgan exhibit concentrated financing. I construct quantitative measures of financing dispersion. For a node $i \in I$ the universe of borrowers, with D_{ij} the repo volume in the relationship between borrower i and lender j over the given quarter, I define the following metrics. The first one uses the extensive margin of relationship existence, the second one is inspired by the Herfindahl index of atomicity:

$$nbrel_i = \frac{\sum_j \#relationships_{ij}}{\sum_i \sum_j \#relationships_{ij}}$$

$$networkscope_i = \frac{1}{1 - \frac{1}{\sum_j \#relationships_{ij}}} \left(1 - \left(\frac{\sum_j D_{ij}}{\sum_i \sum_j D_{ij}}\right)^2\right)$$

$$reposhare_i = \frac{\sum_j D_{ij}}{\sum_i \sum_j D_{ij}}$$

One dynamic feature consists in the *persistence* of bilateral edges, especially the ones involving a borrower exhibiting concentrated financing. These bilateral edges flag the potential existence of a long-term relationship between the two agents. I construct quantitative measures to capture the persistence of bilateral connections:[50]

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[50] Similarly to the static metrics, the first one uses information on the extensive margin (prior existence of the relationship), whereas the second one uses the continuous information of repo flows.

$$persistenceratio_{i,t} = \frac{\sum_j \#relationships_{ij} | existing\,at\,t-1}{\sum_j \#relationships_{ij,t}}$$

And a bilateral-specific measures of long-term relationships:

$$persist\,rel_{ij,t} = 1_{\{link\,ij\,t-1\}}$$

$$history\,rel_{ij,t} = \sum_{t_-} 1_{\{link\,ij\,t_-\}}$$

Table 2.4 presents the summary statistics of these relationship metrics for the 40 borrowers in the sample, along with the balance sheet characteristics from call reports. Figure 2.21 illustrates the time-serie pattern of the heterogeneity of the $persistenceratio_{i,t}$ metrics. Table 2.5 presents the summary statistics of the symmetric metrics for the lenders (grouped in 40 families), along with MMF characteristics obtained from Peter Crane intelligence.

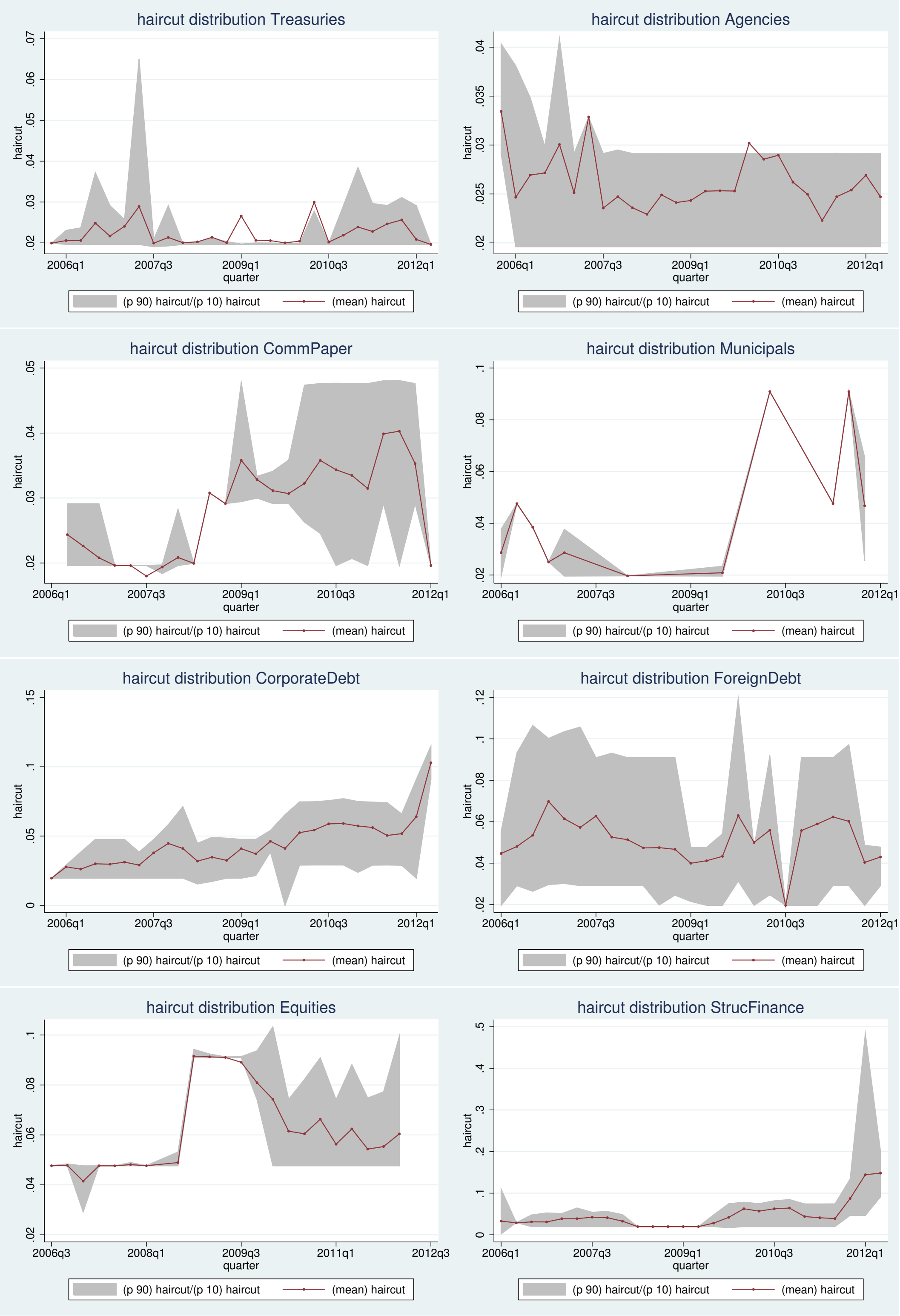


Figure 2.18: *Dispersion in haircuts, by collateral class*

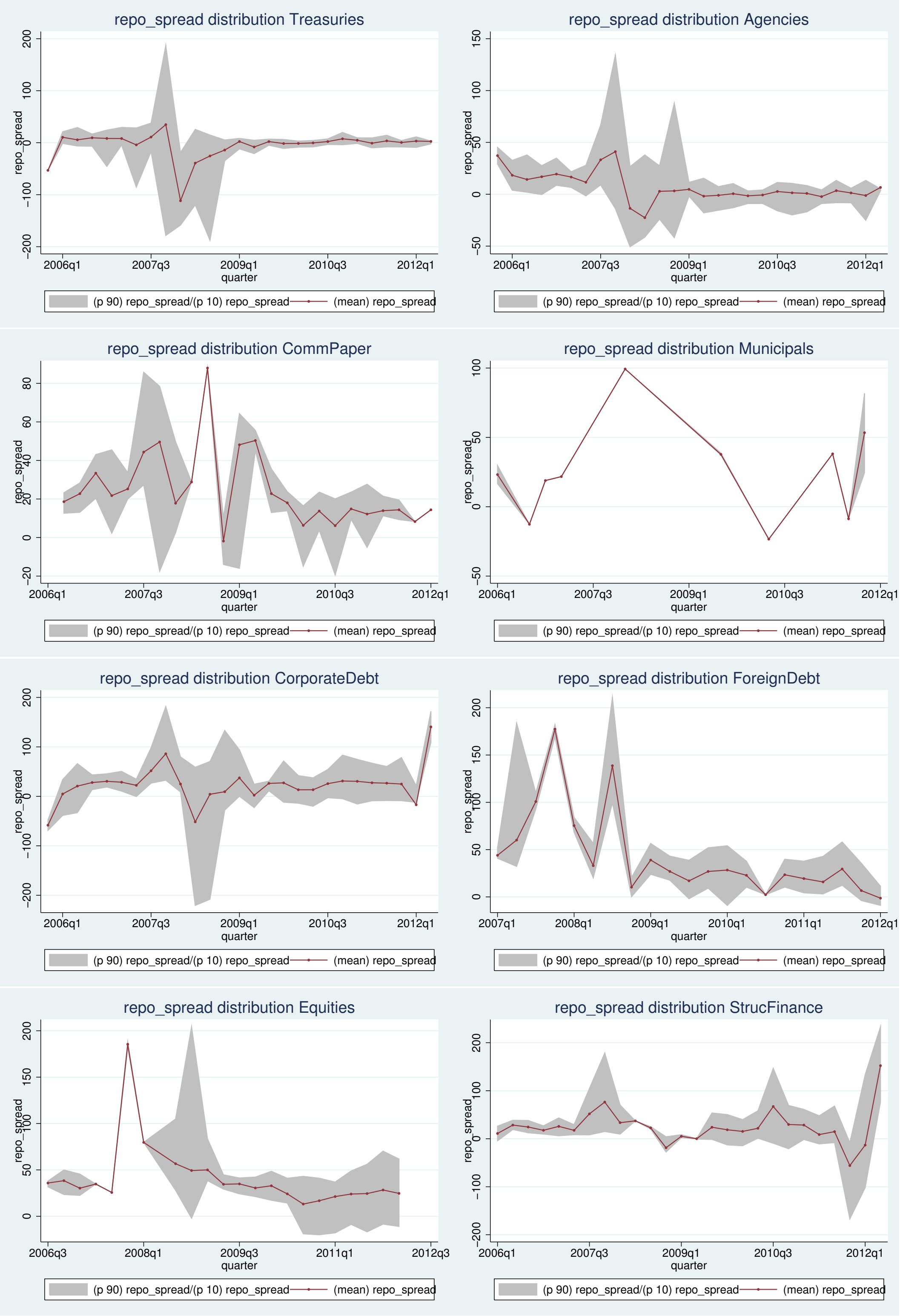
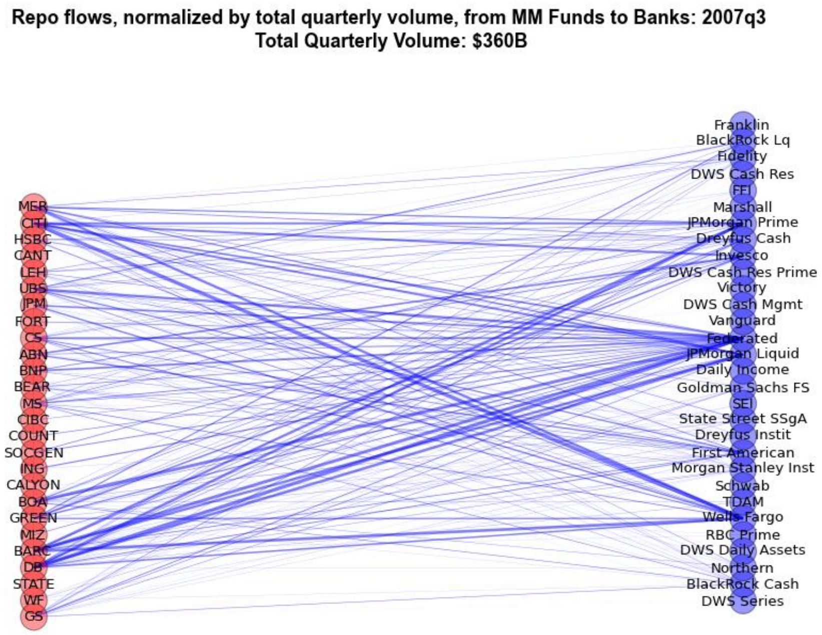


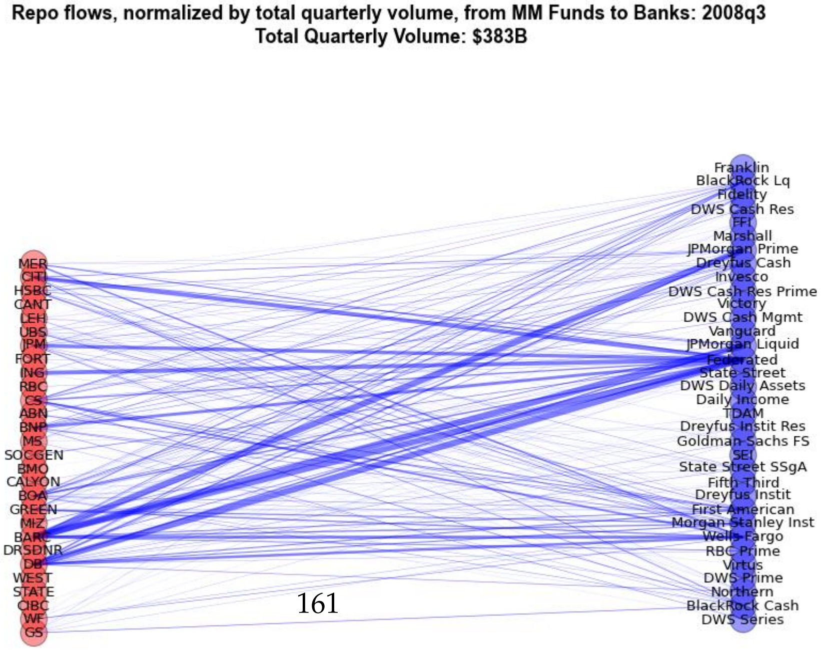
Figure 2.19: *Dispersion in repo spreads, by collateral class*

Figure 2.20: *Network of repo transactions*

Borrowers are vertically aligned on the left of the bipartite graph. Lenders (collapsed by MMF family) are vertically aligned on the right. Each edge connecting one borrower with one lender documents the existence of a bilateral relationship. The thickness of the edge is given by the oustanding repo volume of the bilateral relationship, normalized by the total outstanding volume in the dataset for the quarter 2007q3. The upper panel is 2007q3, the bottom panel is 2008q3.



Repo flows, normalized by total quarterly volume, from MM Funds to Banks: 2007q3
Total Quarterly Volume: $360B



Repo flows, normalized by total quarterly volume, from MM Funds to Banks: 2008q3
Total Quarterly Volume: $383B

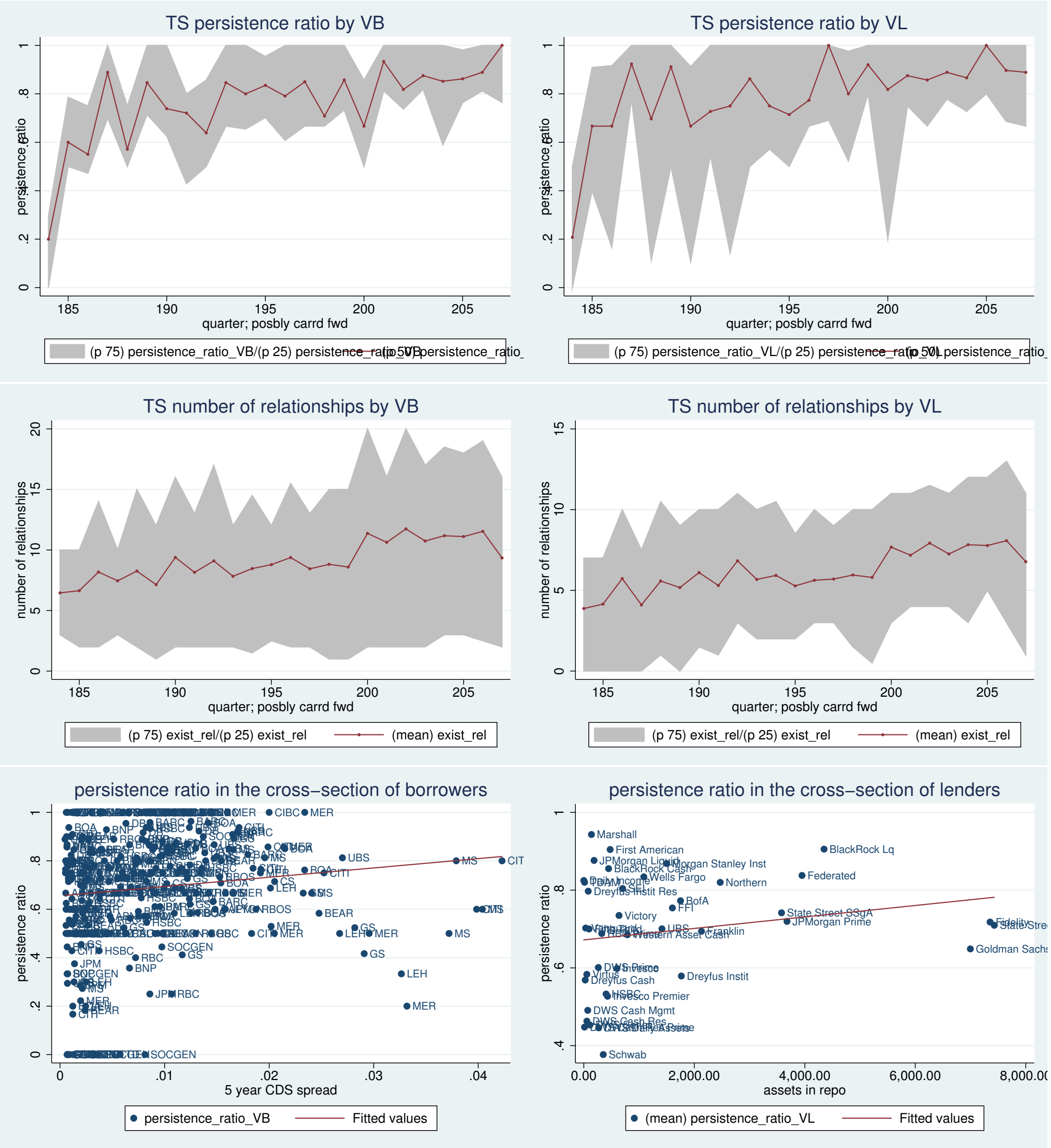


Figure 2.21: *Dynamic metrics of network structure, for borrowers and lenders*

Table 2.4: *Summary statistics of borrowers.*

Name is the identity of the borrower. The following quantities are averaged over the 24 quarters of the sample: Repo is the volume of funding raised by the borrower in the sample (\$$Bil$), Assets are the total assets from Y9-C call reports (\$$Bil$), Net worth is total assets minus total liabilities (\$$Bil$), Cap Ratio is $(totalassets - totalliabilities)\ /totalassets$, CDS is 5 year tenure CDS spread on the borrower name, Scope is the scope of the network of the borrower as defined in the text, PersistNb is the number of relationships of the borrower that already existed in at the previous quarter, NbRel is the number of all relationships of the borrower, NbLTRel is the number of Long-Term relationships, Concentr is equal to one if the borrower has relative concentrated financing (i.e. number of LT relationships superior to the median of number of LT relationships in the sample of borrowers).

Name	Repo	Assets	Networth	CapRatio	CDS	Scope	PersistNb	NbRel	NbLTRel	Concentr
BARC	66	359	21	0.06	0.87	0.89	22	23	25	1
DB	47	21	-7	-0.35	0.62	0.92	27	23	32	1
BNP	31				0.58	0.78	14	17	24	1
BOA	30	991	98	0.07	0.34	0.87	11	20	13	1
RBOS	26	43	25	0.60	1.02	0.91	15	17	16	1
CS	23				0.32	0.86	12	15	12	1
GS	22	702	42	0.06	1.03	0.89	16	20	23	1
CITI	18	1060	-3	-0.02	1.04	0.88	9	13	17	1
MS	17	606	57	0.09	1.29	0.87	11	15	8	1
UBS	17				0.65	0.87	19	16	14	1
MER	16				1.22	0.86	7	14	14	1
SOCGEN	16				0.73	0.80	4	8	8	1
JPM	13	1005	131	0.11	0.71	0.81	9	14	40	1
HSBC	12	192	-62	-0.34	0.55	0.91	8	12	7	1
GREEN	10					0.84	4	10	3	0
LEH	10				3.85	0.75	7	11	0	0
BNY	9	185	87	0.46	0.85	0.47	1	2	1	0
BEAR	9					0.85	7	14	1	0
CALYON	7				0.41	0.92	2	4	6	0
ING	6				0.67	0.66	5	4	6	0
ABN	6	78	-7	-0.08	1.65	0.69	3	6	3	0
RBC	6	69	16	0.23		0.83	4	8	8	1
WF	4	349	73	0.12	0.61	0.74	3	5	7	1
FORT	4				0.70	0.57	2	4	0	0
BMO	3	63	27	0.43	0.56	0.71	1	2	4	0
COUNT	3	110	-25	-0.23		0.51	1	3	2	0
SCOT	3	0	0	0.19	0.45	0.75	0	2	8	1
TD	2	112	71	0.64	0.49	0.88	2	4	8	1
MIZ	2				0.47	0.83	1	2	2	0
DRSDNR	1				0.36	0.47	1	2	0	0
WAMU	1				1.39		0	1	0	0
ABBEY	1				0.34		0	1	0	0
NAT	1				1.56		1	1	2	0
CIBC	0	1	-1	-1.77	0.53	0.74	1	2	3	0
MITSU	0						1	1	0	0
WEST	0						1	1	0	0
NYFED	0						0	1	0	0
CANT	0						1	1	0	0
STATE	0	139	85	0.60		0.37	2	3	0	0

Table 2.5: *Summary statistics of lenders.*

Name is the identity of the lender. The following quantities are averaged over the 24 quarters of the sample: Repo is the volume of funding raised by the borrower in the sample (\$*Bil*), RepoExp is the ratio of repo holdings to Total Net Assets of the MMF, NbRel is the number of all relationships of the borrower, Scope is the scope of the network of the lender as defined in the text, Persist is the ratio of relationships of the borrower that already existed in at the previous quarter. Inflows08 is the cumulative monthly flows over 2008 where $flows_t = \frac{TNA_t - TNA_{t-1}}{TNA_{t-1}}$, InflowsCum is the same quantity cumulated over the 24 quarters of the sample, Yield08 is the cumulative monthly yields over 2008 of the MMF, YieldCum is the same quantity cumulated over the 24 quarters of the sample.

Name	Repo	RepoExp	Inflows08	InflowsCum	Yield08	YieldCum	NbRel	Scope	Persist
Federated	66	0.07	0.22	1.54	-0.00	5.68	19	0.94	0.84
Fidelity	38	0.13	-0.03	0.62	0.07	5.89	12	0.68	0.72
Wells Fargo	34	0.02	1.32	4.58	-0.16	5.15	12	0.95	0.83
JPMorgan Liquid	32	0.00	0.50	0.96	0.04	-12.57	10	0.91	0.88
Invesco	32	0.01		-0.06		-27.43	10	0.89	0.60
JPMorgan Prime	28	0.06	0.44	1.19	-0.01	5.01	9	0.90	0.72
Goldman Sachs FS	20	0.12	-0.20	0.44	-0.31	5.17	8	0.65	0.65
BlackRock Cash	20	0.01	0.53	-0.59	-0.13	-24.51	11	0.93	0.86
First American	20	0.01	1.26	1.36	-0.12	5.20	13	0.97	0.90
BlackRock Lq	16	0.08	0.51	0.26	-0.01	5.61	10	0.90	0.91
Morgan Stanley Inst	14	0.03	-0.51	0.58	-0.09	6.01	12	0.93	0.87
BofA	12	0.03	0.23	1.11	-0.15	5.18	13	0.98	0.77
Dreyfus Cash	11	0.00	1.76	2.66	-0.14	4.37	10	0.95	0.57
Schwab	8	0.01	-0.17	0.20	-0.13	3.90	7	0.86	0.38
Northern	8	0.04	-0.25	0.06	-0.88	3.21	12	0.93	0.82
TDAM	8	0.00	1.01	3.48	4.35	-7.77	3	0.38	0.82
State Street SSgA	8	0.06	-0.12	0.54	-0.18	5.11	11	0.74	0.74
UBS	8	0.02	0.22	0.17	-0.13	5.29	6	0.86	0.70
FFI	6						10	0.91	0.75
State Street	5	0.13	0.25	4.08	4.81	10.66	10	0.84	0.71
HSBC	3	0.01	1.45	2.05	-0.23	5.20	5	0.81	0.53
Dreyfus Instit	3	0.03	0.17	1.32	-0.07	5.73	5	0.90	0.58
Franklin	2	0.04	0.56	1.18	-0.71	3.09	5	0.79	0.69
RBC Prime	2	0.01	0.01	1.27	-0.66	0.74	5	0.87	0.69
Fifth Third	2	0.00	-0.02	0.46	0.14	3.82	6	0.90	0.70
Western Asset Cash	2	0.01	0.86	1.16	-0.00	-12.68	2	0.63	0.69
Dreyfus Instit Res	2	0.00	0.07	-0.75	-1.76	-26.16	6	0.96	0.80
Invesco Premier	2	0.01		0.50		-27.46	8	0.92	0.53
SEI	1	0.01	0.26	0.88	-0.27	4.99	6	0.69	0.80
Marshall	1	0.00	0.23	0.58	0.04	5.75	6	0.82	0.94
Daily Income	1						5	0.94	0.82
DWS Cash Res	1	0.00	-0.09	0.23	-0.21	5.24	3	0.63	0.46
DWS Cash Mgmt	1	0.00	0.07	0.13	-0.27	4.69	3	0.63	0.49
DWS Cash Res Prime	1	0.00	-0.32	-0.76	-0.31	-15.97	3	0.73	0.45
DWS Series	1	0.00	0.31	0.64	-0.16	5.72	3	0.55	0.45
DWS Prime	0	0.00	-0.20	-0.94	-0.28	3.43	4	0.74	0.60
Virtus	0	0.00	-0.15	-1.53	-0.14	5.55	2	0.83	0.58
DWS Daily Assets	0						2	0.80	0.45
Victory	0	0.01	0.15	-0.98	-0.68	3.41	5	0.95	0.73
Vanguard	0	0.00	0.28	1.70	-0.04	5.95	8	0.93	0.70

2.5.3 Test of the model

First, I reject the null hypothesis that repo markets are perfectly competitive. Under this hypothesis, characteristics related to the identity of the borrower and the lender should not matter in repo funding whose price should be solely determined by the nature of collateral. Second, I investigate the power of borrower-specific, lender-specific and bilateral-relationship-specific characteristics to explain repo pricing and volumes.

The first stage is therefore to extract the component in haircuts and repo rates that is not explained by the nature of collateral and aggregate conditions. I carry this out by computing the residual of the OLS of haircuts and repo spreads on quarterly time fixed effects and collateral type fixed effects(specification (0a)):[51]

$$m_{l,ijt} = \mu_t^m + \textstyle\sum_k \beta_k^m 1_{\{col\,k\}} + \epsilon_{l,t}^m$$

$$r_{l,ijt} = \mu_t^r + \textstyle\sum_k \beta_k^r 1_{\{col\,k\}} + \epsilon_{l,t}^r$$

The quarter and collateral class coefficients of the first-stage regression are given in Table 2.6. The coefficients on each collateral class fixed effect are in line with model, which implies that more volatile collateral command higher haircuts and higher rates. The quarterly time fixed-effect coefficients document aggregate change in repo funding over the sample. Interestingly, the OLS does not find an aggregate time-effect on haircuts, but shows a significant positive aggregate coefficient of $44bps$ for 2007q4 at Lehman crisis, immediately followed by significant coefficients from 2008q1 to 2008q4, from $-57bps$ to $-27bps$. Taken together with the Flow of Fund evidence that repo funding volume progressively declined over this period are evidence that the 2007q4 funding stress episode was a negative supply shock (less repo supply), followed by a protracted negative demand shock (less repo demand). The residuals $\hat{m}_{l,t}$ and $\hat{r}_{l,t}$ are the idiosyncratic components I am investigating in the following.

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[51] Arguably, on the US tri-party repo market, for a given collateral type, *the lender does not care about which exact security collateralizes the repo*. Indeed, the lender delegates to the clearing bank the responsibility to check that the collateral posted by the borrower enters in the collateral type agreed upon by the two party, according to the collateral topography of the custodian agreement. Therefore, collateral type fixed effects are sufficient to absorb all the collateral-specific component from the dependent variables.

In order to flag the identity of which borrowers and which lenders get consistent idiosyncratic pricing, I run are the following dummy specification (0b):[52]

$$m_{l,ijt} = \mu_t^m + \sum_k \beta_k^m 1_{\{col\,k\}} + \sum_i \beta_i^m 1_{\{borrower\,i\}} + \sum_j \beta_j^m 1_{\{lender\,j\}} + \epsilon_{l,t}^m$$

$$r_{l,ijt} = \mu_t^r + \sum_k \beta_k^r 1_{\{col\,k\}} + \sum_i \beta_i^r 1_{\{borrower\,i\}} + \sum_j \beta_j^r 1_{\{lender\,j\}} + \epsilon_{l,t}^r$$

Results are reported Tables 2.7 and 2.8, columns 1 and 3. Several dummy variables are omitted due to colinearity. Testing for $\beta_i^m = \beta_j^m = 0$ rejects the null hypothesis of perfect competition. Moreover, dummy coefficient already show that strong franchise borrowers are securing repo with lower spreads and MMF with high bargaining power are getting higher spreads.

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[52] Robustness checks I ran include adding an interaction term $1_{\{col\,k\}} * 1_{\{borrower\,i\}}$ to control for borrowers riskier only because they are more exposed to volatile collateral.

Table 2.6: *First stage OLS (0a): Haircuts and Rates on Collateral Class and Quarters.*

The omitted collateral class is Treasuries, and the omitted quarter is 2006q1. I only report quarterly time-fixed effects that are significant.

	haircut	repo_spread
col_class==2-Agencies	0.00^{***}	7.65^{***}
	(0.00)	(0.63)
col_class==3-CommPaper	0.01^{***}	21.69^{***}
	(0.00)	(2.47)
col_class==4-Municipals	0.02^{***}	34.10^{***}
	(0.00)	(8.19)
col_class==5-CorporateDebt	0.03^{***}	28.17^{***}
	(0.00)	(1.02)
col_class==6-ForeignDebt	0.03^{***}	57.80^{***}
	(0.00)	(1.28)
col_class==7-Equities	0.05^{***}	32.89^{***}
	(0.00)	(2.09)
col_class==8-StrucFinance	0.02^{***}	25.23^{***}
	(0.00)	(1.80)
col_class==9-MixedPool	0.01^{***}	11.66^{**}
	(0.00)	(3.87)
quarter==2007q3	-0.01^{*}	12.35
	(0.00)	(7.42)
quarter==2007q4	-0.00	44.50^{***}
	(0.00)	(7.43)
quarter==2008q1	-0.01^{*}	-57.03^{***}
	(0.00)	(7.40)
quarter==2008q2	-0.01^{*}	-45.07^{***}
	(0.00)	(7.42)
quarter==2008q3	-0.01^{*}	-22.48^{**}
	(0.00)	(7.40)
quarter==2008q4	-0.01^{*}	-27.20^{***}
	(0.00)	(7.40)
N	16,602	16,602
R-squared	0.44	0.30

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2.7: *OLS specification (0b): borrower dummies.*

Borrowers fixed-effects. The specification (0b) is ran on the entire sample (16387 transactions). The smallest borrowers (according to the ranking of Table 2.4) are not displayed. Omitted borrower is ABN Amro.

	haircut	repo_spread
borrower_ticker==BARC	0.01***	4.27
	(0.00)	(3.46)
borrower_ticker==BNP	0.00**	-1.89
	(0.00)	(3.54)
borrower_ticker==BOA	0.00***	2.05
	(0.00)	(3.47)
borrower_ticker==CITI	0.01***	3.96
	(0.00)	(3.56)
borrower_ticker==CS	0.00***	0.87
	(0.00)	(3.49)
borrower_ticker==DB	0.00***	-0.91
	(0.00)	(3.43)
borrower_ticker==GS	0.00***	-4.72
	(0.00)	(3.48)
borrower_ticker==HSBC	0.00**	-3.53
	(0.00)	(3.69)
borrower_ticker==JPM	0.00*	-1.22
	(0.00)	(3.52)
borrower_ticker==LEH	-0.00	-2.74
	(0.00)	(3.93)
borrower_ticker==MER	0.00***	-3.43
	(0.00)	(3.62)
borrower_ticker==MS	0.01***	-1.20
	(0.00)	(3.54)
borrower_ticker==RBOS	0.01***	4.02
	(0.00)	(3.69)
borrower_ticker==SOCGEN	0.00**	1.62
	(0.00)	(3.88)
borrower_ticker==UBS	0.01***	-2.85
	(0.00)	(3.48)
Collateral FE	Y	Y
Quarter FE	Y	Y
N	16,387	16,387
R-squared	0.86	0.38

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2.8: *OLS specification (0b): lender dummies.*

Lenders fixed-effects. The specification (0b) is ran on the entire sample (16387 transactions). The smallest lenders (according to the ranking of Table 2.4) are not displayed. Omitted lender is Blackrock Cash.

	haircut	repo_spread
MMF_name==BlackRock Lq	0.00***	-8.33***
	(0.00)	(1.77)
MMF_name==BofA	-0.00***	-14.48***
	(0.00)	(1.93)
MMF_name==Dreyfus Cash	-0.01***	-13.54***
	(0.00)	(1.98)
MMF_name==Federated	0.00	-12.66***
	(0.00)	(1.40)
MMF_name==Fidelity	0.01***	11.51***
	(0.00)	(1.72)
MMF_name==First American	-0.00***	-15.25***
	(0.00)	(2.03)
MMF_name==Goldman Sachs FS	0.00***	-50.50***
	(0.00)	(2.11)
MMF_name==Invesco	-0.00***	-14.50***
	(0.00)	(2.00)
MMF_name==JPMorgan Liquid	-0.00***	-6.89***
	(0.00)	(1.77)
MMF_name==Morgan Stanley Inst	0.00	-12.07***
	(0.00)	(1.63)
MMF_name==Schwab	0.01***	19.24***
	(0.00)	(5.41)
MMF_name==State Street	-0.00***	-21.60***
	(0.00)	(2.18)
MMF_name==UBS	-0.00**	-14.91***
	(0.00)	(2.30)
MMF_name==Wells Fargo	0.00**	-5.01*
	(0.00)	(2.15)
Collateral FE	Y	Y
Quarter FE	Y	Y
N	16387	16387
R-squared	0.86	0.38

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Test of the effect of borrowers franchise value

Test of Proposition 1: $\frac{\partial m}{\partial V^B} < 0$ and $\frac{\partial r}{\partial V^B} < 0$ To test the franchise value channel, I run a panel regression of haircut and rate on measures of borrower franchise as regressors, still including collateral dummies and time fixed effects (specification (1a)):

$$m_{l,ijt} = \mu_t^m + \beta_B^m V^B + \sum_k \beta_k^m 1_{\{col\,k\}} + \sum_j \beta_j^m 1_{\{lender\,j\}} + \epsilon_{l,t}^m$$

$$r_{l,ijt} = \mu_t^r + \beta_B^r V^B + \sum_k \beta_k^r 1_{\{col\,k\}} + \sum_j \beta_j^r 1_{\{lender\,j\}} + \epsilon_{l,t}^r$$

For measures of V^B I use the following characteristics: CDS, CDS lagged, book value of equity and goodwill.[53] Table 2.9 reports the results. The coefficient β_B^r is statistically and economically significant: $43bps$ has to be compared to the mean gross yield of repo contracts: $20bps$. The coefficient β_B^m is also significant when use CDS lagged one quarter, and net worth computed from Y9-C call reports. I find support that stronger franchise (lower CDS) secure lower haircuts, at the expense of higher rates, consistent with the model.

Compared to Krishnamurthy et al. (2011) which find no effect from borrower identity, two characteristics of the dataset are in order to explain the different results. First, as I parse also smaller money market funds (but not securities lenders), I get more borrowers in the dataset. Moreover, smaller money market funds might be more prone to trust franchise beyond collateral.

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[53] CDS spreads capture risk-neutral default probabilities and a recovery rate from the debt holder standpoint. Therefore it contains a collateral component and a franchise component. The collateral dummies filter out the former component.

Table 2.9: *Specification (1a): continuous OLS test of borrower franchise value.*

This table reports the results of the regression of the pricing variables (haircut and rates) on different proxies of franchise value: $-CDS$, and its determinant in the dynamic model: borrower net worth n^B. CDS is the 5year tenure CDS contract obtained from Markit: *CDS day* is the CDS on the date of the repo transaction, *CDS quarter* is the CDS averaged on the quarter of the repo transaction, and *CDS quarter lagged* is the CDS averaged on the quarter leading the repo transaction. Net worth is computed from the bank holding company Y9-C call report. Time and Collateral Class fixed effects are included in each regression. Repo collateralized by Treasuries or Agencies are excluded from the sample. Standard errors are robust and clustered at the borrower level.

	(1) haircut	(2) haircut	(3) haircut	(4) haircut	(5) haircut	(6) rate	(7) rate	(8) rate	(9) rate	(10) rate
- CDS day	-0.013					-43.240**				
	(0.01)					(15.15)				
- CDS quarter		-0.017					-26.140			
		(0.02)					(20.75)			
- CDS quarter lagged			-0.044*					-46.373*		
			(0.02)					(23.34)		
capital ratio				-0.013					18.391***	
				(0.02)					(5.15)	
net worth					-0.035***					20.506
					(0.01)					(12.66)
Lender FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Collateral FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Quarter FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
N	2670	2998	3001	1686	1686	2670	2998	3001	1686	1686
R-squared	0.87	0.87	0.88	0.86	0.87	0.72	0.70	0.70	0.80	0.80

* p<0.10, ** p<0.05, *** p<0.01

Identification strategy: the European sovereign debt crisis I use the European sovereign debt crisis as an exogenous shock on the franchise value channel. Arguably, this crisis is a shock uncorrelated to the underlying US collateral (US structured finance, US corporate debt), and I take advantage of the fact that European banks act as US financial intermediaries: borrowing from US MMF before lending to US households. The US subsidiaries of European banks are heavily reliant as borrowers on US tri-party repo market, as shown by network graphs (figures 20 and 21) and heavy lenders to US shadow banking sector. The European debt crisis is an exogenous shock to the franchise of European borrowers *i*: borrower ticker equal to DB, CS, UBS, ABN, HSBC, HVB, DRSDNR, SOCGEN, BARC, CALYON, CMZB, BNP, ING, FORT. [54]

I run the following Difference-in-Difference specification (1b):[55]

$$m_{ijt} = \mu_t^m + \beta_1^m * 1_{Eur\,crisis} + \beta_2^m * 1_{Eur\,crisis} + \beta_3^m * 1_{Eur\,crisis} * 1_{Eur\,bank} + \epsilon_{l,t}^m$$

$$r_{ijt} = \mu_t^r + \beta_1^r * 1_{Eur\,crisis} + \beta_2^r * 1_{Eur\,crisis} + \beta_3^r * 1_{Eur\,crisis} * 1_{Eur\,bank} + \epsilon_{l,t}^r$$

The test of the franchise value channel is: $\beta_3^m, \beta_3^r > 0$. Table 2.10 reports the results of this specification. In line with prediction 5, the results are highly significant when using the illiquid collateral sample. The coefficients $\beta_1^m, \beta_1^r < 0$ are also interesting and document a negative demand shock in repo funding over the European crisis.

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[54] If I had data on European lenders investing in US tri-party as hinted by repo volume from Flow of Fund, i.e. *j* belonging to Europe, I could use the European debt crisis as an exogenous shock on the *relationship value* V^L, symmetrically to what I do here on V^B.

[55] Due to the short time period of the sample, robustness checks include to run the specification of $1_{Eur\,bank}$ on symmetric pre- and crisis samples separately, following Bertrand et al. (2004).

Table 2.10: *Specification (1b): Difference-in-Difference on European borrowers.*

This table presents the results of a Difference-in-Difference specification that uses the European sovereign debt crisis as an instrument of European borrowers. Repo transactions contracted on the US tri-party are collateralized by US securities uncorrelated to the European shock. $1_{European\,crisis}$ is a dummy variable equal to one from 2010q1 to 2012q2, and equal to zero from 2006q1 to 2009q4. $1_{European\,bank}$ is a dummy variable equal to one for the following borrowers: DB, CS, UBS, ABN, HSBC, HVB, DRSDNR, SOCGEN, BARC, CALYON, CMZB, BNP, ING, FORT, and equal to zero for the other borrowers. The first two regressions are run on the whole sample, the last two ones are ran on the sample excluding repo transactions collateralized by Treasuries and Agencies.

	(1) haircut	(2) repo_spread	(3) haircut	(4) repo_spread
$1_{Europeancrisis}$	0.00***	0.71	0.01***	-16.16***
	(0.00)	(0.85)	(0.00)	(2.08)
$1_{Europeanbank}$	-0.00	-1.14	-0.01***	-7.70**
	(0.00)	(0.85)	(0.00)	(2.45)
$1_{Europeancrisis}$ * $1_{Europeanbank}$	-0.00	1.98	0.01***	10.01**
	(0.00)	(1.23)	(0.00)	(3.29)
Collateral FE	Y	Y	Y	Y
N	16702	16702	3608	3608
R-squared	0.17	0.14	0.10	0.10

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Test of Corollary 1: $\frac{\partial^2 m}{\partial\sigma\partial V^B} < 0$ and $\frac{\partial^2 r}{\partial\sigma\partial V^B} < 0$ A first piece of evidence is that there is more *dispersion* in haircuts and repo rates for illiquid collateral than for liquid collateral, and during quarters with high volatility than quiet quarters. This is illustrated by the 10th and 90th percentiles on Figures 2.18 and 2.19. Results of the previous specification are much more significant when excluding Treasuries and Agencies from the sample. I also run the dummy specification with interaction terms:

$$m_{l,t} = \mu_t^m + \textstyle\sum_i \sum_k \beta_{ik}^m 1_{\{col\,k\}} * 1_{\{borrower\,i\}} + \epsilon_{l,t}^m$$

$$r_{l,t} = \mu_t^r + \textstyle\sum_i \sum_k \beta_k^r 1_{\{col\,k\}} * 1_{\{borrower\,i\}} + \epsilon_{l,t}^r$$

Test of the effect of lenders risk attitude

Corollary 4 predicts that lenders with more pessimistic beliefs should obtain lower haircuts and lower rates, and also should trust more the franchise value. The first piece of evidence comes from the lenders coefficients in specification (1) (Table 2.8): preeminent money market funds secure higher haircuts and rates. It leads to investigate how the haircut and the repo spread comove in the cross-section of lenders. To this end, I run the following preliminary regression:

$$r_{l,ijt} = \mu_t^r + \alpha\, m_{l,ijt} + \textstyle\sum_k \beta_k^r 1_{\{col\,k\}} + \sum_j \beta_i^r 1_{\{borrower\,i\}} + \epsilon_{l,t}^r$$

The coefficient obtained is positive and significant: $\alpha = 295$ with a standard error (clustered at the lender level) of 28 . This suggests that, *in the market for secured funding*, the MMF risk-return trade off demonstrated by Kacperczyk and Schnabl (2012) is complemented by another type of heterogeneity, the one on the borrower side documented above. I investigate two plausible explanations consistent with this positive coefficient α: lenders heterogeneity in pessimism or in relationship value.

To investigate further the origin of the effect of this heterogeneity on repo lending, I test lender characteristics on both the haircut and the rate (specification (2a)):

$$m_{l,ijt} = \mu_t^m + \beta_L^m\, V^L + \textstyle\sum_k \beta_k^m 1_{\{col\,k\}} + \sum_j \beta_j^m 1_{\{borrower\,i\}} + \epsilon_{l,t}^m$$

$$r_{l,ijt} = \mu_t^r + \beta_L^r V^L + \sum_k \beta_k^r 1_{\{col\, k\}} + \sum_j \beta_j^r 1_{\{borrower\, i\}} + \epsilon_{l,t}^r$$

I use for V^L the following MMF characteristics, proxying pessimism and concentration limits: $RepoExp_j$. Results are reported in Table 2.11. $\beta_L^m > 0$ and $\beta_L^r > 0$ are both economically and statistically significant, showing that MMF with higher bargaining power secure both higher haircuts and higher rates. Moreover, when use for V^L metrics characterizing MMF risk-taking attitude / pessimism (Inflows08 and Yield08), I find $\beta_L^m > 0$ and $\beta_L^r > 0$, consistent with the results on unsecured funding volumes of Kacperczyk and Schnabl (2012) and Chernenko and Sunderam (2012).

Table 2.11: *Specification (2a): continuous OLS test of lender risk-taking attitude.*

This table tests in the cross-section of lenders for the hypothesis of heterogeneity in pessimism (state-dependent utility, isomorphic to risk-aversion). RepoExp is the ratio of repo holding to total asset at the MMF level (proxy for bargaining power). Inflows2008 and Yield08 are respectively the inflows and the yield experienced by the MMF in 2008, used as proxy for MMF risk-taking. Time, Collateral Class and Borrower fixed effects are included. Repo collateralized by Treasuries or Agencies are excluded from the sample. Standard errors are robust and clustered at the Lender level.

	(1) haircut	(2) haircut	(3) haircut	(4) repo_spread	(5) repo_spread	(6) repo_spread
RepoExp	0.016***			22.907***		
	(0.00)			(6.44)		
Inflows2008		-0.048*			50.835	
		(0.03)			(39.87)	
Yield08			0.001			13.380
			(0.00)			(12.38)
Borrower FE	Y	Y	Y	Y	Y	Y
Collateral FE	Y	Y	Y	Y	Y	Y
Quarter FE	Y	Y	Y	Y	Y	Y
N	2982	3411	3411	2982	3411	3411
R-squared	0.89	0.87	0.87	0.50	0.65	0.66

* p<0.10, ** p<0.05, *** p<0.01

Test of the effects of long-term relationships

Proposition 3 states that repo funding is more stable (less countercyclical) for borrowers enjoying long-term relationships. Long-term relationships should enjoy lower and more stable haircuts and rates. I measure the existence of long-term relationships by more *concentrated financing* and *longer history of the relationship*.

Results on pricing variables: haircuts and rates The first exercise is to complement the dummy OLS with relationship dummies. The specification I run are (specification (3a)):

$$m_{l,ijt} = \mu_t^m + \sum_i \beta_i^m 1_{\{borrower\, i\}} + \sum_j \beta_j^m 1_{\{lender\, j\}} + \beta_{ij}^m 1_{\{borrower\, i\}} 1_{\{lender\, j\}} + \sum_k \beta_k^m 1_{\{col\, k\}} + \epsilon_{l,t}^m$$

$$r_{l,ijt} = \mu_t^r + \sum_i \beta_i^r 1_{\{borrower\, i\}} + \sum_j \beta_j^r 1_{\{lender\, j\}} + \beta_{ij}^r 1_{\{borrower\, i\}} 1_{\{lender\, j\}} + \sum_k \beta_k^r 1_{\{col\, k\}} + \epsilon_{l,t}^r$$

Even for the second set of specification, bilateral relationship coefficients are significant, especially on the rates (β_{ij}^r). This dummy regression elicits which (i, j) pair between a broker-dealer borrower and a MMF lender is an actual long-term relationship. The sign of the coefficients β_{ij}^m and β_{ij}^r informs in which direction the bilateral relationship is more in favor. Results of this voluminous dummy regression are available upon demand.

I now investigate how the respective bargaining powers of borrowers and lenders affect the haircut and rate pricing. I run the following specifications with continuous dependent variables (specification (3a) - can be run on first differences):

$$m_{l,ijt} = \mu_t^m + \beta^m Rel_t^B + \gamma^m Rel_t^L + \delta^m history\, rel_{ij} + \sum_k \beta_k^m 1_{\{col\, k\}} + \epsilon_{l,t}^m$$

$$r_{l,ijt} = \mu_t^r + \beta^r Rel_t^B + \gamma^r Rel_t^L + \delta^r history\, rel_{ij} + \sum_k \beta_k^r 1_{\{col\, k\}} + \epsilon_{l,t}^r$$

For Rel_t^B and Rel_t^L, I use different measures of the impact of relationships on respectively the borrowers and the lenders: $RepoShare_i$ and $\#relationships_i$ and $ScopeL$, and the symmetric measures for lenders bargaining powers δ^j.[56] The test of Corollary 4 is $\beta^m, \beta^r > 0$ and

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[56] Even if these regressors are highly endogenous, they capture the bargaining power δ information according to my model (see also Lee and Fong (2012) for a model with endogenous network formation where this property is also true).

$\gamma^m, \gamma^r < 0$. Results on the sample excluding Treasuries and Agencies are reported Table 2.12. The results are more conclusive for repo spreads than for haircuts. This is due to the empirical fact that haircuts are not negotiated on a daily basis, but set in the custodian agreement. We observe a negative significant effect of Rel_t^L both for haircuts and rates. The results of the OLS give some significance for the Rel_t^L . In line with Corollary 1, results on the sample excluding Treasuries and Agencies are even more significant. The results using the history of the relationships are conclusive for *PersistentRel*: persistent relationships are able to achieve lower rates for the borrower.[57]

Result on volume variables First I analyze the stability of the network structure. I use two metrics to capture the stability of bilateral connections. On the intensive margin, $\Delta Volij = |\Delta RepoVolij|$ is the absolute value of the change, from one quarter to another, in the repo volume of bilateral relationship between borrower i and lender j, normalized to the total quarterly repo volume. On the extensive margin, $1_{rel\,ij,t}$ is a dummy variable equal to 1 if the bilateral connection ij at the quarter t. The test is done via probit. The two regressors are $HistoryRel_{ij,t} = \sum_{t_-} 1_{\{link\,ij\,t_-\}}$ and $PersistentRel_{ij,t} = 1_{\{link\,ij\,t_-\}}$. Thus specification (3b) is:

$$|\Delta RepoVolij|_{ijt} = \alpha^{int} + \beta^{int}\, HistoryRel_{ij} + \epsilon_{ij,t}$$

$$E\left[1_{\{link\,ij,t\}}\right] = 1/1 + exp\left(\alpha^{ext} + \beta^{ext} HistoryRel_{ij}\right)$$

The test of the model is $\beta^{int} < 0$ and $\beta^{ext} > 0$. Table 2.13 reports the results and finds significant coefficients consistent with the model. The probit coefficient can be interpreted (dprobit) as: one more quarter of history of the relationship increases by 12% the probability of existence of the relationship in quarter t.

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[57] The existence of previous relationship is still an endogenous variable. One could instrument the prior existence of relationship by lending to first-time borrowers. This would be instruments of an exogenous variation in relationships.

Finally, I test the effect of the existence of long-term relationships on the stability of secured funding volume for borrowers. $\Delta Repo = |\Delta RepoVolB|$ is the absolute value of the change, from one quarter to another, in borrower B repo funding normalized to the total quarterly repo volume. $Repo/ST = Repo/STfunding$ is the ratio, for borrower B, between its repo funding (in \$) and the \$ sum of all its short-term funding sources: fed funds + repo + short-term deposits + commercial paper + short-term liabilities (data from Y9-C call reports). This proxies for the easiness of access to repo for borrower B and captures potential substitution from other funding sources, in case of repo funding difficulties. $VBtotMMFrisk$ is a measure of MMF risk-taking behavior, aggregated at the Borrower level. $NbPersRel$ is the number of bilateral relationships that the borrower already had at the previous quarter. $NbLTRel$ is the number of long-term relationships the borrower enjoys (a long-term relationship is defined when the number of quarter of existence of the relationship is above the median of the universe of relationships). The last two lines are interaction terms to test that long-term relationships help stabilize repo funding.

Borrowers with long-term relationships enjoy more stable funding. It is tested in specification (3c):

$$|\Delta RepoVolB|_{i,t} = \alpha + \beta\, NbLTRel_{i,t} + \epsilon_{i,t}$$

Table 2.14 reports the results of this specification, as well results of specification with potential borrower repo volume stability as regressors. I find that the existence of Long-Term relationships mitigates the sensitivity of repo funding to the quality of the franchise (measured by $-CDSquarter$). The existence of long-term relationships provides an explanation of why repo lending was fairly stable over the crisis. It is consistent with Hrung and Sarkar (2012), which finds an autocorrelation of 0.95 in the volume of borrower repo funding, in the daily tri-party repo data of the Fed.

Table 2.12: *Specification (3a): Regression of haircuts and rates on relationships indicators.*

This table tests for relationship characteristics priced in haircuts and rates. The number of relationships *NbRelB* and *NbRelL* proxy for the concentration of financing for the borrower, and for the concentration of the lending base for the lender. Repo volumes *RepoVolumeB* and *RepoVolumeL* proxy for the respective bargaining powers. *ScopeB* and *ScopeL* is an Herfindhal index measuring the atomicity of the respective counterparty bases: it is high when the counterparties are uniformly dispersed. *PersistentRel* and *HistoryRel* are relationship-specific variables: *PersistentRel* is a dummy variable equal to one if the bilateral connection also exists at the previous quarter, *HistoryRel* counts the number of quarters in which the bilateral connection was existing up to the date of the repo transaction. Time, Collateral Class and Borrower and Lender fixed effects are included. Repo collateralized by Treasuries or Agencies are excluded from the sample. Standard errors are robust and clustered at the relationship level.

	(1) haircut	(2) haircut	(3) haircut	(4) haircut	(5) haircut	(6) rate	(7) rate	(8) rate	(9) rate	(10) rate
NbRelB	0.000					0.395				
	(0.00)					(0.26)				
NbRelL	0.001***					0.295				
	(0.00)					(0.49)				
RepoVolumeB		-0.001					22.369*			
		(0.01)					(11.35)			
RepoVolumeL		-0.019**					-40.176***			
		(0.01)					(15.07)			
ScopeB			0.009					14.301		
			(0.01)					(13.75)		
ScopeL			-0.012**					-0.912		
			(0.01)					(11.33)		
PersistentRel				0.003					-5.762**	
				(0.00)					(2.81)	
HistoryRel					0.000					-0.215
					(0.00)					(0.34)
Borrower FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Lender FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Collateral FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Quarter FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
N	3497	3497	3493	3497	3497	3497	3497	3493	3497	3497
R-squared	0.88	0.88	0.88	0.88	0.88	0.73	0.73	0.73	0.73	0.73

* p<0.10, ** p<0.05, *** p<0.01

Table 2.13: *Specification (3b): Regression of volumes on relationships indicators (long-term relationships and borrower stable network).*

This table tests the effect of long-term relationships on the stability of the network of bilateral connections. $\Delta Volij = |\Delta RepoVolij|$ is the absolute value of the change, from one quarter to another, in the repo volume of bilateral relationship between borrower i and lender j, normalized to the total quarterly repo volume. $1_{rel\,ij,t}$ is a dummy variable equal to 1 if the bilateral connection ij at the quarter t. $NbRelB$ is the number of relationships a borrower has at a given quarter. $ScopeB$ is an Herfindhal index measuring the atomicity of the counterparty base. The two latter regression are run via probit, and the R^2 given for these regressions is McFadden's pseudo R^2.

	(1) ΔVolij	(2) ΔVolij	(3) $1_{relij,t}$	(4) $1_{relij,t}$
NbRelB	-0.000*** (0.00)			
ScopeB		-0.004** (0.00)		
HistoryRel:$\sum_{s<t} 1_{relij,s}$			0.160*** (.0012)	
PersistRel$1_{relij,t-1}$				0.081** (.002)
N	6668	6571	527	2095
R-squared	0.18	0.18	0.47	0.22

* p<0.10, ** p<0.05, *** p<0.01

Table 2.14: *Specification (3c): Regression of volumes on funding stability indicator (long-term relationships and borrower stable funding volume).*

This table tests the effect of long-term relationships on the stability of secured funding *volumes* for one given borrower. $\Delta Repo = |\Delta RepoVolB|$ is the absolute value of the change, from one quarter to another, in borrower B repo funding normalized to the total quarterly repo volume. $Repo/ST = Repo/STfunding$ is the ratio, for borrower B, between its repo funding (in $) and the $ sum of all its short-term funding sources: fed funds + repo + short-term deposits + commercial paper + short-term liabilities (data from Y9-C call reports). This proxies for the easiness of access to repo for borrower B and captures potential substitution from other funding sources, in case of repo funding difficulties. $VBtotMMFrisk$ is a measure of MMF risk-taking behavior, aggregated at the Borrower level. $NbPersRel$ is the number of bilateral relationships that the borrower already had at the previous quarter. $NbLTRel$ is the number of Long-Term relationships the borrower enjoys (a Long-Term relationship is defined when the number of quarter of existence of the relationship is above the median of the universe of relationships). The last two lines are interaction terms to test that Long-Term relationships help stabilize repo funding. Standard errors are robust and clustered at the borrower level.

	(1) ΔRepo	(2) ΔRepo	(3) ΔRepo	(4) ΔRepo	(5) ΔRepo	(6) ΔRepo	(7) ΔRepo	(8) Repo/ST
- CDS quarter	-153.297*					-108.332	-376.478***	
	(80.92)					(78.23)	(102.25)	
CapRatio		-0.013*						
		(0.01)						
Net worth			-0.056					
			(0.03)					
VBtotMMFrisk				-0.000***				0.000***
				(0.00)				(0.00)
NbPersRel					-0.004***			0.018***
					(0.00)			(0.00)
-CDSq*NbLTRel						-32.515***		
						(8.25)		
-CDSq*NbPersRel							25.627***	
							(5.07)	
N	448	238	238	669	669	448	448	246
R-squared	0.01	0.01	0.01	0.01	0.07	0.02	0.10	0.39

* p<0.10, ** p<0.05, *** p<0.01

2.6 Conclusion

This paper develops a model of decentralized markets for secured funding in which the franchise value of borrowers and long-term relationships matter. Continuation values substitute for collateral and sustain low and stable levels for both the haircuts and the rates. Due to the endogeneity of borrower franchise value, haircuts are countercyclical. This can be mitigated by establishing long-term bilateral relationships. This channel is magnified when the collateral is volatile and when lenders competition is imperfect. Franchise value of broker-dealers can also rationalize long intermediation chains as the efficient arrangement to monetize franchise.

The empirical analysis on a hand-collected dataset of repo transactions rejects the null hypothesis that repo markets are perfectly competitive. It demonstrates that relationships are priced even in secured funding, and are instrumental in the stability of refinancing. It shows that this stability is conditional on the existence of franchise and relationships, which can unravel quite abruptly. Even if small compared to ABCP, repo markets can therefore be quite destabilizing for the financial system.

Even though constrained efficient, the present model delivers policy recommendations. It enables to compare the *effectiveness* of different ex post policy instruments to alleviate a credit crunch. Franchise value is concave with respect to borrower net worth. This advocates for *equity injections* (e.g. second Paulson plan) in order to jump start franchise values and restore confidence in secured funding markets. Alternatively, *lending facilities* (LTRO, TALF, PDCF, TSLF, CPFF, AMLF) also enable financial intermediaries leverage by short-circuiting the bargaining friction in private secured funding markets. In the model though, it has a negative side-effect of breaking up welfare-improving long-term relationships in private secured funding markets. On the other hand, the model does not provide any support for policies of *asset purchases* (TARP), as in general equilibrium it would merely have a crowding-out effect of private investment.

Regarding ex-ante policies, the existence of a franchise value channel supports *institution-level* regulation of haircuts and leverage, and not at an asset-level. Finally, the model of the rehypothecation chain advocates for more transparency in prime brokerage, as bargaining outcome on $r^{tri}(m^{tri})$ instead of $r^{tri}(m^{bil}; m^{tri})$ leads to suboptimal choice of haircuts.

An interesting extension of the model is to endow borrowers and lenders with a costly capacity to learn about the collateral, in a rational inattention framework. Under the presence of franchise, the lender cares *less* about the collateral, and as a result learns less about it. This predicts that there is less learning on a specific class of collateral if one strong franchise value is the marginal buyer in this market.

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Chapter 3

Measuring Liquidity Mismatch in the Banking Sector[1]

3.1 Introduction

Liquidity plays an enormous role in financial crises. Fleming (2012) notes that across its many liquidity facilities, the Federal Reserve provided over $1.5 trillion of liquidity support during the crisis. The number is much higher if one includes other forms of government liquidity support such as lending by the Federal Home Loan Bank – lending to banks peaks at $1 trillion in September 2008 – or the Federal Deposit Insurance Corporation guarantees – insurance limits are increased in the crisis, and the guarantees are extended to $336 billion of bonds as of March 2009 (He, Khang and Krishnamurthy (2010)). Recognizing the importance of liquidity in the crisis, the Basel III committee has proposed regulating the liquidity of commercial banks. Yet, despite its importance there is no consensus on how to measure liquidity. Indeed, the only consensus is that liquidity is a slippery concept and is hard to measure.

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[1] Co-authored with Jennie Bai (McDonough School of Business at Georgetown University) and Arvind Krishnamurthy (Kellogg School of Management, Northwestern University and NBER). We thank Viral Acharya, Allen Berger, Christa Bouwman, Markus Brunnermeier, Dong Beom Choi, Adam Copeland, Michael Fleming, Antoine Martin, Klaus Schaeck, and seminar participants at the Federal Reserve Bank of New York, Bank of France, Bank of England, the Department of the Treasury's Office of Financial Research, European Bank Association's 5th Annual Financial Stability Conference at Tilburg, Georgetown University, Copenhagen Business School, University of Rhode Island, for helpful comments. Jonathan Choi provided excellent research assistance. JEL Classification: G21, G28. Keywords: liquidity mismatch; liquidity regulation; market liquidity; funding liquidity.

This paper implements a liquidity measure proposed by Brunnermeier, Gorton and Krishnamurthy (2011, 2012). Their "Liquidity Mismatch Index" (LMI) measures the mismatch between the market liquidity of assets and the funding liquidity of liabilities, at a firm level. There are many empirical challenges that arise in implementing their theoretical measure. We take up these challenges and design a procedure to implement the LMI that relies on balance-sheet as well as off-balance-sheet information of a given bank and market indicators of liquidity and liquidity premia. We construct the LMI for the universe of bank holding companies (BHCs) in the U.S. and describe features of the time-series and cross-sectional properties of the bank-specific and aggregate LMI.

What makes a good liquidity measure? First, we argue that a liquidity measure should be useful for macro-prudential purposes. It should measure liquidity imbalances in the financial system, offering an early indicator of financial crises. It should also quantitatively describe the liquidity condition of the financial sector, and the amount of liquidity the Fed may be called upon to provide in a financial crisis. The LMI performs well on these metrics. An important aspect of the LMI is that it can be aggregated across banks to measure the liquidity mismatch of a group of banks or the entire financial sector. Liquidity measures which are based on ratios, such as Basel's liquidity coverage ratio, do not possess this aggregation property. Our aggregate LMI as shown in Figure 3.3 indicates a growing liquidity mismatch over the period from 2002 to 2007. In 2007 Q1, the LMI is near (negative) \$3 trillion. Thus, had the LMI been computed in 2007, the Fed would have expected that in the event of an aggregate liquidity crisis, it may need to provide \$3 trillion of liquidity to completely mitigate the liquidity-run aspect of the financial crisis.[2] Moreover, the LMI reverses course from 2008 to mid-2009, coincident with the Fed's liquidity injections, returning to its 2004 level. Finally, we show that the LMI methodology can naturally be used to administer a liquidity stress test by a regulator, and present the results of such a stress test at various time points.

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[2] One theoretical rationale to justify liquidity injections by the Fed is an inter-bank market freeze, as modelled in Acharya and Skeie (2011). Another reason is the stigma of borrowing at the discount window. See Ennis and Weinberg (2012) for a signaling model of this stigma. A liquidity injection also prevents coordination failures that can lead to bank insolvency, as in fundamental bank runs models a la Rochet and Vives (2004).

Our second benchmark arises from micro considerations. We argue that an efficient liquidity measure should describe liquidity risk in the cross-section of banks, identifying which banks carry the most liquidity risk. We show that our measure performs well in this dimension. When market-wide liquidity conditions deteriorate, a firm with a worse LMI should be more negatively affected. We examine the cross-section of banks and show that banks with a worse LMI have lower stock returns during the 2008 financial crisis. We also find that across event dates corresponding to a worsening of liquidity conditions, the low LMI banks experience more negative stock returns. Across event dates corresponding to an increase in Fed liquidity provision, the low LMI banks experience more positive stock returns. Our cross-sectional analysis also reveals interesting patterns in the way that firms manage liquidity. We find that banks with high liquidity mismatch (low LMI value) have high stock returns before and after the crisis, when aggregate liquidity conditions were good. Moreover, we find that the banks that have the lowest LMI (i.e. the most liquidity shortfall) are the largest banks, perhaps suggesting a strategy of exploiting the too-big-to-fail backstop. The LMI thus helps to describe the cross-section of liquidity risk in the financial sector. For regulatory purposes, the cross-sectional LMI can help identify systemically important institutions, but here using a liquidity metric.

These two dimensions, the macro and the micro, appear to us to be the most important dimensions on which to evaluate a liquidity measure. One contribution of the paper is to offer these metrics. This is particularly important going forward because there are in principal many ways to measure liquidity and what is needed are benchmarks that can be used to discriminate across measures.

Related Literature

Our paper is related to a small literature on how to measure liquidity. On the practitioner side, there are a number of different metrics that firms use to manage liquidity, ranging from the accounting 'quick' ratio to more sophisticated measures. On the policy side, several central bank studies including Banerjee (2012), de Haan and End (2012) investigate measures for bank liquidity regulation in response to Basel III. The pioneering paper in the academic literature is Berger and Bouwman (2009), which is the first paper to recognize the importance of measuring liquidity and propose a theoretically-motivated liquidity measure. Berger and Bouwman (2009) measure the liquidity mismatch at the bank level and explore the cross-sectional and time-series properties of this liquidity mismatch measure. The principal theoretical difference between the LMI and the Berger-Bouwman measure is that the LMI incorporates information from market measures of liquidity and liquidity premia.[3] In the language of Berger and Bouwman, our liquidity weights are time-varying, while their liquidity weights are only asset and liability specific. Time variation in these liquidity weights is important in capturing liquidity stress during a financial crisis. Another contribution of our paper is in discussing and evaluating the LMI against the benchmarks we have suggested. These benchmarks are important because they help us to calibrate the liquidity weights, which are hard to pin down on purely theoretical grounds.

There is also a banking and corporate finance literature that explores the determinants of liquidity holdings on a firm's balance sheet, for example, Heider, Hoerova and Holhausen (2009), Acharya and Merrouche (2013) and Acharya and Rosa (2013). In most of this literature, liquidity is defined as the cash or liquid assets held on the asset side of the balance sheet. In our approach, liquidity is constructed from both asset and liability side of the balance sheet (as in Berger and Bouwman (2009)), and is furthermore dependent on market-wide liquidity conditions. Each asset and each liability contributes to the liquidity position of the bank. We leave it for future work to revisit this literature using our more comprehensive liquidity measure.

In comparing the stock returns of banks that hold more liquidity on their balance sheet to those holding less liquidity, we find that the former underperform during non-crisis periods. This may be because holding liquidity is on average costly, carrying insurance benefits that are only reaped in crisis periods (as in Holmstrom and Tirole (1998)). It will be interesting to link our findings with a welfare analysis and study the benefits and costs of holding liquidity. It will also be interesting to see if our empirical analysis offers clarity on the optimal regulation of bank liquidity (see e.g., Stein (2013)).

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[3] There is an alternative measure in Berger and Bouwman (2009) that sets the weights on bank loans to vary with the amount of securitization. In our paper, the time-varying feature is generalized to every item on and off balance sheet.

The paper proceeds as follows. The next section builds up a theoretical model for the liquidity mismatch measure and Section 3.3 constructs the empirical measure. Section 3.4 evaluates the LMI in the macro dimension while Section 3.5 evaluates the LMI in the micro dimension. Section 3.6 concludes the paper and discuss future work.

3.2 Liquidity Mismatch Index: Theoretical Framework

The Liquidity Mismatch Index (LMI) of Brunnermeier, Gorton and Krishnamurthy (2011, 2012) provides one approach to measure a bank's liquidity. They define the LMI as the "cash equivalent value" of a firm in a given state assuming that:

i Counterparties act most adversely. That is, parties that have contracts with the firm act to extract as much cash as possible from the firm under the terms of their contracts. This defines the liquidity promised through *liabilities.*

ii The firm computes its best course of action, given the assumed stress event, to raise as much cash against its balance sheet as it can to withstand the cash withdrawals. That is, the firm computes how much cash it can raise from asset sales, pre-existing contracts such as credit lines, and collateralized loans such as repo backed by assets currently held by the firm. The computation assumes that the firm is unable to raise unsecured debt or equity. The total cash raised is the *asset-side liquidity.*

Central to this definition is that liquidity is computed based on a scenario where counterparties act most adversely. To understand why the worst-case is appropriate, consider defining liquidity for a hypothetical Diamond-Dybvig bank that is subject to a bank run. Suppose that the bank owns 100 long-term illiquid assets where early liquidation generates 50. The bank is financed by 75 of short-term demandable deposits and 25 of equity. The liquidity stress that the bank is exposed to is the coordination failure whereby depositors withdraw funds expecting every other depositor to withdraw funds. For this case, the LMI is -25, being the net of 75 and 50. More broadly, the definition of the LMI is based on the idea that liquidity stress always involves coordination failure, which is captured by the scenario that parties with contracts with the bank extract as much cash as possible under the terms of the contract.

The LMI for an entity i at a given time t is the net of the asset and liability liquidity, defined as,

$$LMI_t^i = \sum_k \lambda_{t,a_k} a_{t,k}^i + \sum_{k'} \lambda_{t,l_{k'}} l_{t,k'}^i. \tag{3.1}$$

Assets ($a_{t,k}^i$) and liabilities ($l_{t,k'}^i$) are balance sheet counterparts, varying over time and by asset or liability class (k, k'). The liquidity weights, $\lambda_{t,a_k} > 0$ and $\lambda_{t,l_{k'}} < 0$, are key items to compute. Points (i) and (ii) offer some guidance on these weights, but leave considerable latitude. The main contribution of our paper is to propose liquidity weights λ_{t,a_k} and $\lambda_{t,l_{k'}}$, following a derivation based on banks' optimization through choosing their liquidity mismatch.

3.2.1 Bank Optimization Problem and LMI Derivation for Liabilities

We first focus on computing the liability side LMI, $\sum_{k'} \lambda_{t,l_{k'}} l_{t,k'}^i$. It is easier to explain our methodology by moving to a continuous maturity setting, although we implement the LMI based on a sum of discrete liability classes as in formula (3.1). We use T to denote the maturity of liability class k'. Thus, let $l_{t,T}^i$ be the liability of the bank i due at time T, where the notation $\{l_{t,T}^i\}$ denotes the stream of maturity-dated liabilities.

We are interested in summarizing the stream $\{l_{t,T}^i\}$ as a single number, $LMI(\{l_{t,T}^i\}, t)$, that captures the liquidity features of the liabilities and how it enters into a bank's decision problem. Our measurement system satisfies a recursive principle: given the $LMI(\{l_{t,T}^i\}, t)$ at some date t, the LMI at date $s < t$ is the "discounted value" of $LMI(\{l_{t,T}^i\}, t)$. This structure is natural in this case. Consider a bank which has say 200 of overnight debt and 100 of two-day debt that represents no immediate liquidity stress. Tomorrow, the 100 of debt will become overnight debt and represent immediate liquidity stress. Our LMI measure treats this shrinking of maturity in a smooth recursive fashion.

Denote $V^N(\{l^i_{t,T}\}, t)$ as the value to the bank of choosing liability structure $\{l^i_{t,T}\}$. The bank earns a liquidity premium on its liabilities. In particular, $\pi_{t,T}$ is a liquidity premium the bank earns by issuing a liability of maturity T. The liquidity premium should be thought as the profit on a "carry trade" of issuing liabilities which investor pay a premium $\pi_{t,T}$ and investing the proceeds in long-term assets. Here $\pi_{t,S} > \pi_{t,T}$ for $S < T$, and $\pi_{t,T} = 0$ for large T (i.e. only short-maturity liabilities earn a liquidity premium). Given this liquidity premium structure, the bank is incentivized to issue short-maturity debt. The cost of short-maturity debt is liquidity stress. The bank chooses its liabilities to solve,

$$V^N(\{l^i_{t,T}\}, t) = \max_{\{l^i_{t,T}\}} G(\{l^i_{t,T}\}) + \int_t^{\infty} l^i_{t,T}\pi_{t,T} dT + \psi^i \theta^i LMI(\{l^i_{t,T}\}, t), \tag{3.2}$$

where the function $G(\cdot)$ represents non-liquidity related reasons for choosing a given liability structure, and the liquidity dimension of liabilities is captured in the remaining terms. Note that in writing this expression, and for all of the derivations that follow, we assume for simplicity that the interest rate is effectively zero.

For our purposes, the key term in equation (3.2) is the last one which represents the cost of liquidity stress. We can think of ψ^i as the probability of entering liquidity stress, the LMI (a negative number) as the dollar liquidity need in the stress event, and θ^i as the cost of acquiring the liquidity needed to cover the stress. For example, one way to think about θ^i is that it reflects the implicit and explicit cost for a bank of going to the discount window. This interpretation is natural for a bank risk manager. We will also think about applying our model for regulatory purposes. In this case, θ^i can be interpreted as the regulator's cost of having a bank come to the discount window for access. We pursue this latter angle later in this section when discussing the case where there are many banks with correlated liquidity shocks. Finally, an alternative interpretation of ψ^i is that it is the Lagrange multiplier in a risk management problem for a bank that maximizes value subject to a risk-management or regulatory constraint that $-LMI(\{l^i_{t,T}\},t) < \overline{LMI}$, that is, the risk manager imposes a cap on how negative the liquidity mismatch can become.

As an example, if the relevant stress is the failure to rollover \$100 of overnight debt, which happens with probability 20%, and in which case the bank resorts to the discount window paying an explicit and implicit penalty of 1%, then the numbers are $\psi = 20\%, \theta = 1\%$, and LMI $= -100$.

In a liquidity stress episode, all contractual claimants on the bank act to maximally extract cash from the bank. This means that overnight debt holders refuse to rollover debt and the bank has to cover the cash shortfall from this loss of funding. A liquidity stress episode is defined by a horizon. If the stress lasts for two days, then holders of two-day debt may also refuse to rollover funding, and so on. We assume that at any t, there is a chance μdt that at date $t + dt$ the stress episode ends and firm has access to free liquidity. The liquidity need function, LMI, can be defined recursively,

$$LMI(\{l^i_{t,T}\},t) = -l^i_{t,t}dt + (1-\mu dt)LMI(\{l^i_{t+dt,T}\},t+dt). \tag{3.3}$$

This equation reflects a recursive principle: the LMI at date t is the “discounted value" of LMI at $t + dt$.

We look for an LMI function that is maturity-invariant, that is, a function where the liquidity cost measured at time t of a liability maturing at time T is only a function of $T - t$. Thus consider the function,

$$LMI(\{l^i_{t,T}\},t) = \int_t^\infty l^i_{t,T}\lambda_{T-t}dT \tag{3.4}$$

where λ_{T-t} is a liquidity weight at time t for a liability that matures at time T. It captures the marginal contribution of liability l^i_T to the liquidity pressure on the bank. Substituting the candidate cost function into the recursion equation (3.3) and solving, we find that,

$$\lambda_{T-t} = -e^{-\mu(T-t)}. \tag{3.5}$$

So the liquidity weight is an exponential function of the μ and the liability's time to maturity $T - t$. A high μ implies a low chance of illiquidity, and hence high liquidity. The liquidity weights we have constructed embed the expected duration of liquidity needs. This characterization of liquidity weights is consistent with the description of LMI given in Brunnermeier, Gorton and Krishnamurthy (2012). Their paper considers the following experiment: Suppose that a firm has free access to liquidity (e.g., being able to access equity markets) follows a Poisson process, there is a probability μ that the firm is able to raise equity in any given day. Then, the LMI is based on the expected liquidity outflow going forward. Define the function $f(T, \mu) \in [0, 1]$, where $T = 1$ corresponds to one day and $T = 30$ corresponds to 30 days, as the probability that the firm is unable to access free liquidity by date T. The probability is decreasing in T at a decay rate governed by the parameter μ. Then, the liquidity weight for a given contract $\lambda_{t,L_{k'}}$ with maturity T is proportional to $f(T, \mu)$. Furthermore, there may be times, say during a crisis, when the liquidity stress is likely to last longer so that μ is smaller and $f(T, \mu)$ is higher. In these periods we would expect $\lambda_{t,L_{k'}}$ to be even lower.

3.2.2 Measuring μ

A key variable in the construction of the LMI is μ, which measures the expected duration of the stress event. We aim to map μ into an observable asset price. Consider a hypothetical bank which makes its decisions only based on liquidity considerations (i.e. for this bank, $G(\{l^i_{t,T}\}) = 0$). The first order condition for the bank in choosing $l^i_{t,T}$ is:

$$\pi_{t,T} = \psi^i \theta^i e^{-\mu_t T}. \tag{3.6}$$

The bank earns a liquidity premium on issuing liabilities of maturity T, but at liquidity cost governed by $e^{-\mu_t T}$. The FOC indicates a relation between μ_t and the liquidity premium, which is governed by the market's desire for liquidity.

We propose to measure the liquidity premium using the term structure of OIS-TBill spreads. We assume that $\pi_{t,T}$ is proportional to OIS-TBill spread of the given maturity. This assumption says that when investors have a strong desire to own liquid assets, as reflected in the spread between OIS and T-Bill, any financial intermediary that can issue a liquid liability can earn a premium on this liquidity. There is clear evidence (see Krishnamurthy and Vissing-Jorgensen (2013b) and Nagel (2014)), on the relation between the liquidity premia on bank liabilities and market measures of liquidity premia. The OIS-TBill spread is the pure measure of the liquidity premium, as it is not contaminated by credit risk premia. Under this assumption, μ_t is proportional to $ln(OIS - TBill)/\mathrm{T}$. Thus we use time-series variation in the OIS-TBill spread to pin down μ_t.

The derivation above is carried out with the assumption that μ_t varies over time, but is a constant function of T. However, μ itself has a term structure that reflects an uneven speed of exit from the liquidity event (i.e., μ_t is a function of T). The term structure of μ is reflected in the term structure of the liquidity premia, which is observable. It is straightforward to see that in the general case with T-dependent μ, the liquidity premium at maturity T solves:

$$\pi_{t,T} = \psi^i \theta^i e^{-\int_t^T \mu_{t,s} ds}. \tag{3.7}$$

In our empirical implementation, we assume that this term structure is summarized by two points, a 3-month liquidity spread and a 10 year liquidity spread at every time. Thus we implicitly restrict attention to a two-factor structure for liquidity premia.

3.2.3 LMI Derivation including Assets

Let us next consider the asset side equation, $\sum_k \lambda_{t,a_k} a^i_{t,k}$. In a liquidity stress event, the bank can use its assets to cover liquidity outflows rather than turning to the discount window (or other sources) at cost θ^i per unit liquidity. The asset side LMI measures the benefit from assets in covering the liquidity shortfall.

For each asset, $a_{t,k}$, define its cash-equivalent value as $(1 - m_{t,k})a_{t,k}$. Here m_k is most naturally interpreted as a haircut on a term repurchase contract, so that $(1 - m_{t,k})a_{t,k}$ is the amount of cash the bank can immediately raise using $a_{t,k}$ as collateral. Then the total cash available to the bank is,

$$w_t = \sum_k (1 - m_{t,k}) a^i_{t,k} \tag{3.8}$$

The bank can use these assets to cover the liquidity outflow. Define the LMI including assets as, $LMI(\{l^i_{t,T}\}, w_t, t)$, and note that the LMI satisfies the recursion:

$$LMI(\{l^i_{t,T}\}, w_t, t) = \max_{\Delta_t \geq 0} \Big(-\max(l^i_{t,t} - \Delta_t, 0)dt + (1 - \mu dt) LMI(\{l^i_{t+dt,T}\}, w_t + dw_t, t + dt) \Big) \tag{3.9}$$

where,

$$dw_t = -\Delta_t.$$

At every t, the bank chooses how much of its cash pool, Δ_t, to use towards covering liability at date t, $l_{t,t}$. Given that there is a chance that the liquidity stress episode will end at $t + dt$, and given that the cost of the liquidity shortfall is linear in the shortfall, it is obvious that the solution will call for $\Delta_t = l_{t,t}$ as long as $w_t > 0$, after which $\Delta_t = 0$. We compute the maximum duration that the bank can cover its outflow, T^*, as the solution to,

$$w_t = \int_t^{T^*} l^i_{t,T} dT. \tag{3.10}$$

That is, after T^*, the bank will have run down it's cash pool. By using the assets to cover liquidity outflows until date T^*, the bank avoids costs of,

$$\psi^i \theta^i \int_t^{T^*} l^i_{t,T} \lambda_{T-t} dT,$$

which is therefore also the value to the bank of having assets of w_t.

In implementing our LMI measure, we opt to simplify further. Rather than solving the somewhat complicated equation (3.10) to compute T^* as a function of w_t and then computing, $\int_t^{T^*} l^i_{t,T} \lambda_{T-t} dT$, we instead assume that the cost avoided of having w_t of cash is simply $\psi^i \theta^i w_t$. This approximation is valid as long as T^* is small, so that λ_{T^*-t} is near one, in which case, $\int_t^{T^*} l^i_{t,T} \lambda_{T-t} dT \approx \int_t^{T^*} l^i_{t,T} dT = w_t$. For example, in the case where T^* is one day, the approximation is exact since effectively the cash of w_t is being used to offset today's liquidity outflows one-for-one, saving cost of $\psi^i \theta^i w_t$.

Furthermore, we categorize the liabilities into maturity buckets rather than computing a continuous maturity structure since in practice we only have data for a coarse categorization of maturity. Putting all of this together, the LMI is,

$$LMI_t^i = \sum_k \lambda_{t,a_k} a_{t,k}^i + \sum_{k'} \lambda_{t,l_{k'}} l_{t,k'}^i.$$

where, the asset-side weights are

$$\lambda_{t,a_k} = 1 - m_{t,k}, \tag{3.11}$$

and the liability-side weights are

$$\lambda_{t,l_{k'}} = -e^{-\mu T_{k'}}. \tag{3.12}$$

3.2.4 Aggregation and Correlated Shocks

Suppose there is a unit measure of banks, indexed by i, each choosing assets and liabilities in a market equilibrium. We can define the aggregate LMI as,

$$\widetilde{LMI}_t = \int_0^1 LMI_t^i di \tag{3.13}$$

We want to capture the idea that running a liquidity mismatch for a given bank at a time when many banks are running a large liquidity mismatch is more costly than when many banks are running a small mismatch. For example, a regulator or a risk manager may want to penalize the LMI more in the case of aggregate shocks than idiosyncratic shocks. To this end, we penalize the liquidity weights in simple way to capture the aggregate dependence. We assume that,

$$\lambda_{t,l_{k'}} = -e^{-\mu T_{k'}} e^{-\gamma \widetilde{LMI}_t} \tag{3.14}$$

This dependence can be thought as follows:

$$\lambda_{t,l_{k'}} = -e^{-\hat{\mu} T_{k'}} \quad \text{with,} \quad \hat{\mu} = \mu + \gamma \frac{\widetilde{LMI}_t}{T}.$$

As $\widetilde{LMI}_t < 0$, the effective duration parameter, $\hat{\mu}$ is lower, which translates to a longer duration of the liquidity event. We can think of our modeling as capturing a risk-adjustment to the probability of a liquidity stress, similar to change of measure common in asset pricing.

The parameter γ (“risk aversion") captures the extent of the risk-adjustment. We likewise assume that,

$$\lambda_{t,a_k} = (1 - m_{t,k})e^{-\gamma \widetilde{LMI}_t}, \tag{3.15}$$

since asset side liquidity is more valuable for a bank facing a more costly liquidity need.

Note that this approach requires us to solve a fixed point problem: $\widetilde{LMI}_t$ is a function of the λs by the definition of the LMI, and the λs are a function of $\widetilde{LMI}_t$. We solve the following equation at every t:

$$\widetilde{LMI}_t = e^{-\gamma \widetilde{LMI}_t} \int_0^1 \left(\sum_k (1 - m_{t,k}) a^i_{t,k} + \sum_{k'} e^{-\mu T_{k'}} l^i_{t,k'} \right) di. \tag{3.16}$$

The linearization of the $e^{-\gamma \widetilde{LMI}_t}$ yields the following expression for the aggregate LMI in equilibrium:

$$\widetilde{LMI}_t = \frac{\int_0^1 \left(\sum_k \widehat{\theta}^i (1 - m_{t,k}) a^i_{t,k} + \sum_{k'} \widehat{\theta}^i e^{-\mu T_{k'}} l^i_{t,k'} \right) di}{1 + \gamma * \int_0^1 \left(\sum_k \widehat{\theta}^i (1 - m_{t,k}) a^i_{t,k} + \sum_{k'} \widehat{\theta}^i e^{-\mu T_{k'}} l^i_{t,k'} \right) di}. \tag{3.17}$$

When $\gamma = 0$, there is no amplification effect. When $\gamma > 0$, the aggregate LMI is higher in absolute value.

3.2.5 Equilibrium

The preceding subsections describe a decision problem that pins down $\widetilde{LMI}$, the aggregate quantity of liquidity supplied by the banking sector. Theoretically, the bank supplied liquidity is part of a general equilibrium in the market for liquidity. Although it is not essential for our measurement exercise, being more explicit in describing this equilibrium may help in providing academic context for the exercise. Following Holmstrom and Tirole (1998), there is a demand for liquidity from the non-financial sector $D(\pi_T)$, which is a function of the liquidity premium. The supply of liquidity is comprised of private supplied liquidity $\int_0^1 LMI^i_t$ and government supplied liquidity S_t. Thus the market clearing condition is,

$$D(\pi_T) = S_t + \widetilde{LMI}_t \tag{3.18}$$

which, along with $\pi_T = \psi^i \theta^i e^{-\mu T}$, pins down the price π_T and the aggregate liquidity $\widetilde{LMI}$. The γ parameter captures the market risk aversion, and the $e^{-\gamma \widetilde{LMI}_t}$ in aggregate LMI can be seen as a change of measure on μ.

3.3 Liquidity Mismatch Index: Empirical Design

Following our theoretical model, we need to collect assets and liabilities for each bank and define their liquidity weights correspondingly. The asset-side liquidity weights are driven by haircuts of underlying securities, while the liability-side weights are determined by liabilities' maturity structure and easiness of rollover ('stickiness'). Both are affected by the expected stress duration, which is pinned down by market liquidity premium.

We construct the LMI for U.S. bank holding companies (BHC). The key source of balance sheet information of BHCs comes from the FRY-9C *Consolidated Report of Condition and Income*, which is completed on a quarterly basis by each BHC with at least \$150 million in total assets before March 2006 or \$500 million since then.[4] The sample period is from 2002:Q1 to 2013:Q1. The dataset includes 2870 BHCs throughout the sample period, starting with 1884 firms in 2002:Q1 and ending with 1176 firms in 2013:Q1. Among 2870 BHCs, there are 41 U.S. subsidiaries of foreign banks, such as Taunus corp (parent company is Deutsche Bank) and Barclays U.S. subsidiary. Table 3.1 lists the summary statistics for these BHCs, including Total Assets (in \$mil), Leverage (the ratio of total liability to total asset), Foreign to Total Deposit ratio, Risk adjusted asset (in \$mil), Tier 1 Risk-based Capital Ratio, Total Capital Ratio, and Tier 1 Leverage Ratio (all three are Basel regulatory measures), as well as return on assets (ROA) and return on equity (ROE). Panel B provides a snapshot of the top 50 BHCs, ranked by their total asset values as of March 31, 2006. The top 50 BHCs together have a total asset of 10.58 trillion US dollars, comprising a large fraction of total

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[4] The Y-9C regulatory reports provide data on the financial condition of a bank holding company, based on the US GAAP consolidation rules, as well as the capital position of the consolidated entity. The balance sheet and income data include items similar to those contained in SEC filings; however, the regulatory reports also contain a rich set of additional information, including data on regulatory capital and risk-weighted assets, off-balance sheet exposures, securitization activities, and so on.

Table 3.1: *Summary statistics of Bank Holding Companies in Y9-C Reports during 2006-2013*

Panel A

	Universe (N=1507)		Public (N=509)		Public US (N=481)		TOP 50 US	
	mean	std	mean	std	mean	std	mean	std
Total Asset ($Mil)	14.89	120.84	35.30	204.39	34.13	205.30	360.85	619.11
Leverage	0.91	0.04	0.91	0.03	0.91	0.03	0.90	0.02
Foreign/Total Deposit	0.01	0.05	0.01	0.07	0.01	0.07	0.11	0.21
Risk-adj. Asset ($Mil)	9.60	74.83	22.65	126.90	22.30	127.90	233.65	380.80
Tier1 Capital Ratio	12.59	5.66	12.30	4.01	12.35	3.96	11.19	2.63
Total Capital Ratio	14.20	5.56	13.89	3.84	13.93	3.79	14.04	2.54
Tier1 Leverage Ratio	9.24	3.50	9.21	2.47	9.26	2.42	8.50	1.94
ROA (annualized)	1.24	4.30	0.96	3.93	0.97	3.95	1.42	3.36
ROE (annualized)	18.57	49.95	16.26	44.89	16.41	45.18	19.71	33.31

Panel B: Top 50 BHCs (rank is based on total asset values as of 2006:Q1)

Rank	Company	Size($Bil)	Leverage	Tier1 Lev. Ratio	Tier1 Risk-based Capital Ratio
1	CITIGROUP	1884.32	0.93	5.16	8.59
2	BANK OF AMER CORP	1463.69	0.91	6.36	8.64
3	JPMORGAN CHASE	1351.52	0.91	6.19	8.66
4	WACHOVIA CORP	707.12	0.90	6.01	7.42
5	METLIFE	527.72	0.93	5.55	9.51
6	WELLS FARGO	482.00	0.90	7.89	8.95
7	HSBC NORTH AMER HOLD	478.16	0.93	5.90	8.13
8	TAUNUS CORP	430.40	0.99	-0.82	-3.97
9	BARCLAYS GROUP US	261.22	0.99	1.08	10.96
10	US BC	219.23	0.90	8.16	8.75
20	BANK OF NY CO	103.46	0.89	6.67	8.19
30	UNIONBANCAL CORP	52.62	0.91	8.44	8.68
40	SYNOVUS FC	31.89	0.88	10.64	10.87
50	WEBSTER FNCL CORP	17.10	0.89	7.68	9.21
Total		10581.94	0.92	6.07	8.04

industry assets.

3.3.1 Asset-side Liquidity Weight

The assets of a bank consist of cash, securities, loans and leases, trading assets, and intangible assets. Under a liquidity shock, a bank can raise cash by borrowing against a given asset or by selling that asset. Asset liquidity weight defines the amount of cash a bank can raise over a short-term horizon for a given asset. A good weight spectrum across assets should comply with two criteria: i) reflecting the market price in real time, ii) capturing the liquidity ranks across assets in a consistent way. For example, assets like cash, and federal funds are ultra liquid and hence should have a fixed sensitivity weight value of one, the highest rank. Assets like fixed and intangible assets are extremely difficult or time-consuming to convert into liquid funds and hence should have a fixed weight value of zero. The challenge is to find a measure which can be applied to various types of in-between assets and also reflect their time-varying market prices.

Implied from our theoretical model, we construct asset liquidity weights from haircut data on repo transactions. (Appendix C.1 shows the details.) One minus the haircut in a repo transaction directly measures how much cash a firm can borrow against an asset, so that the haircut is a natural measure of asset liquidity sensitivity. In addition, the haircuts change over market conditions and hence can reflect the real-time market prices. The haircut is also known to vary with measures of asset price volatility and tail risk for a given asset class, which are commonly associated with market liquidity of the asset. Thus, the haircut is particularly attractive as a single measure of asset liquidity.

We collect haircut data based on repo transactions reported by the Money Market Fund (MMF) sector, which is the largest provider of repo lending to banks and dealers.[5] According to the Flow of Funds data of September 2011, US Money Market Funds have \$458 billion of holdings in repo contracts, representing 46% of the total volume of repo lending in the US. The list of the 145 largest prime institutional Money Market Funds is obtained from Peter Crane intelligence. Our approach follows Krishnamurthy, Nagel and Orlov (2014). For each fund, we further parse forms N-Q, N-CSR and N-CSRS from the SEC Edgar website. We obtain the following details for each repo loan at the date of filing: collateral type, collateral fair value, notional amount, repurchase amount at maturity, and the identities of borrower and lender. Using this information, we compute the haircut from the collateral fair value P and the notional amount D as $m = 1 - P/D$.

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[5] The MMF data measures haircuts in what is known as the tri-party repo market (see Krishnamurthy, Nagel and Orlov (2014) for details). It is apparent that haircuts in the tri-party market were much more stable than in the bilateral repo market (see Copeland, Martin and Walker (2010) and Gorton and Metrick (2010)), which leads to the concern that tri-party haircuts may not accurately capture market liquidity conditions. We conduct a robustness check by calculating the asset-side liquidity sensitivity using the bilateral repo haircuts recovered from our tri-party repo data and the differences of haircut between bilateral and trip-party repo documented in Copeland, Martin and Walker (2011).(We thank the authors for providing the data on the differences of

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bilateral and tri-party repo haircuts.) The resulting liquidity sensitivity weights remain almost unchanged for high-quality assets such as Treasury and agency bonds, but become more variable during the crisis for low-quality assets such as asset-backed securities, corporate debt, and foreign debt. However, when using the bilateral repo haircuts, the impact on the calculation of liquidity mismatch index is not much different from that using the tri-party repo data.

Table 3.2: *Haircuts by Collateral type.*

Collateral	**Mean**	**Std**	**P5**	**P25**	**P50**	**P75**	**P95**
Treasuries	.023	.021	.020	.020	.020	.020	.029
Agencies	.026	.030	.020	.020	.020	.029	.033
CommPaper	.029	.010	.020	.020	.029	.033	.048
Municipals	.039	.021	.020	.020	.038	.048	.091
StrucFinance	.046	.036	.020	.020	.032	.071	.091
CorporateDebt	.048	.033	.020	.029	.048	.056	.085
ForeignDebt	.055	.036	.021	.030	.048	.053	.106
Equities	.063	.020	.048	.048	.048	.077	.094
Average	.030	.030	.020	.020	.020	.029	.074

Between the extreme liquid (cash) and illiquid (intangible) assets, two main categories, securities and trading assets, share the same components. These components resound with the collateral classes in repo transactions: Treasuries, agencies, commercial paper, municipals, corporate debt, foreign debt, structured Finance, and equity. Table 3.2 shows the distribution of haircut rates across collateral types in our sample. It is clear that Treasury bills and bonds have the lowest haircuts when serving as collateral, with an average rate of 2.3%. Agency bonds have the second lowest haircut, on average of 2.6%. Commercial paper and municipal bonds have relatively lower liquidity, and hence slightly higher haircuts, with an average of 2.9% and 3.9% respectively. Structured finance assets, corporate debt and foreign debt have higher haircuts around 5%.

Though sharing similar components, different categories serve as different purposes on the balance sheet. For example, securities categories can be further divided into held-to-maturity securities and available-for-sale securities. Together with securities in trading assets category, these securities should claim different liquidity sensitivity weights depending on their purposes. When facing a liquidity shock, securities in the trading assets category are often the first to be sold, whereas similar securities in the available-for-sale category are the second in the selling consideration, and those in the held-for-maturity category are often untouched unless in an emergency. To accommodate this concern, we assign a constant that scales the liquidity sensitivity weights depending on the purpose a security serves on the balance sheet. The scale coefficient is one for securities in the Trading Assets category, 0.75 for Available-for-Sale Securities, and 0.50 for Held-to-Maturity Securities.

One remaining category undiscussed yet important is bank loans. Ideally, we should use the loan transaction data to compute loan liquidity weights. However, we hesitate to do so for two reasons. First, although our repo market data does not include the collateral class of loans — rather we use the haircut of structural finance repo contract to capture the liquidity of loans secured by real estates, and use the maximum of haircuts across all collateral classes to capture the liquidity of commercial & industry loans — our haircuts for loans match the number as shown in Copeland, Martin and Walker (2011) (category 'Whole Loans' in Table 2). More importantly, even if we derive liquidity sensitivity from loan transaction data, there is a challenge on how to match this weight to the weights from other assets. Overall, we need measures that maintain the relative liquidity rankings across asset categories and it is easier to do this using a single liquidity measure such as haircuts. Clearly, there are subjective judgments that goes into the weights, although this is a general feature of this type of exercise (e.g., the liquidity measures in Basel III are also based on judgment). In particular, the constant scales assigned to different asset categories are ad-hoc. This raises the concern that the sensitivity of the scale may affect the calculation of liquidity mismatch index. We do robustness checks by allowing the scale to change 25 percent lower and higher and recalculated the LMI. These checks result in similar time-series and cross-sectional performance of LMI.

3.3.2 Liability-side Liquidity Weights

Whereas the asset-side is liquidity inflow, with an exposure to liquidity equal to one minus the haircut, the liability-side is liquidity outflow, carrying negative weights. Liability-side liquidity depends on contract maturity and liability's easiness of rollover. The goal of a bank's liquidity risk management is to balance liquidity inflow and outflow in order to achieve an optimal firm value by minimizing the liquidity stress cost.

We implement the theoretical derivation of liquidity weights as follows, with the details shown in Appendix C.1. According to our model, the liability-side liquidity weights (putting aside the γ feedback for now) are determined jointly by $\{\mu, T_{k'}\}$:

$$\lambda_{T-t} = -e^{\mu T_{k'}}.$$

The parameter μ captures the expected stress duration and we estimate it through the combination of short-term and long-term market liquidity premium:

$$\mu T_{k'} = \mu_{ST} min(T_{k'}, 1) + \mu_{LT}(T_{k'} - 1)\mathbb{1}_{T_{k'}>1}. \tag{3.19}$$

The literature has considered many proxies to measure the liquidity premium, so that there is no uniformly accepted candidate to measure μ. The main drawback of proposed measures such as the Libor-OIS spread and the Treasury-Eurodollar spread is that they are contaminated by credit risk (see Smith (2012) for a detailed discussion). We choose to use the spread between OIS swap rates and Treasury bills as our measure, as such a spread is likely to be minimally affected by credit risk. Yet, as Treasury bills are more liquid than overnight federal funds loans, this measure will capture any time variation in the valuation of liquid securities. Furthermore, as opposed to other measures of liquidity premium, say micro-structure measures drawn from stocks or bonds, TOIS is more closely aligned with the funding conditions of financial intermediaries. In equation (3.19), we use the logarithm of 3-month and 10-year OIS-TBill spreads to measure the short-term and long-term liquidity premium, μ_{ST} and μ_{LT}. Figure 3.1 plots the two spreads at daily frequency. We observe that TOIS was volatile and strikingly large since the subprime crisis starting from the summer of 2007, suggesting the deterioration of funding liquidity. It became stable and close to zero since the summer of 2009, reflecting the normalization of liquidity conditions.

The parameter $T_{k'}$ indicates the maturity of liability. For example, overnight financing (federal funds and repo) has a maturity of 0, commercial paper has a maturity of 0.25 year, debt with maturity less or equal than one year has $T = 1$, debt with maturity longer than one year has $T = 5$, subordinated debts have $T = 10$, equity has a maturity of 30 years. For insured deposit, we assign its maturity proxy as $T = 10$ while uninsured deposit is more

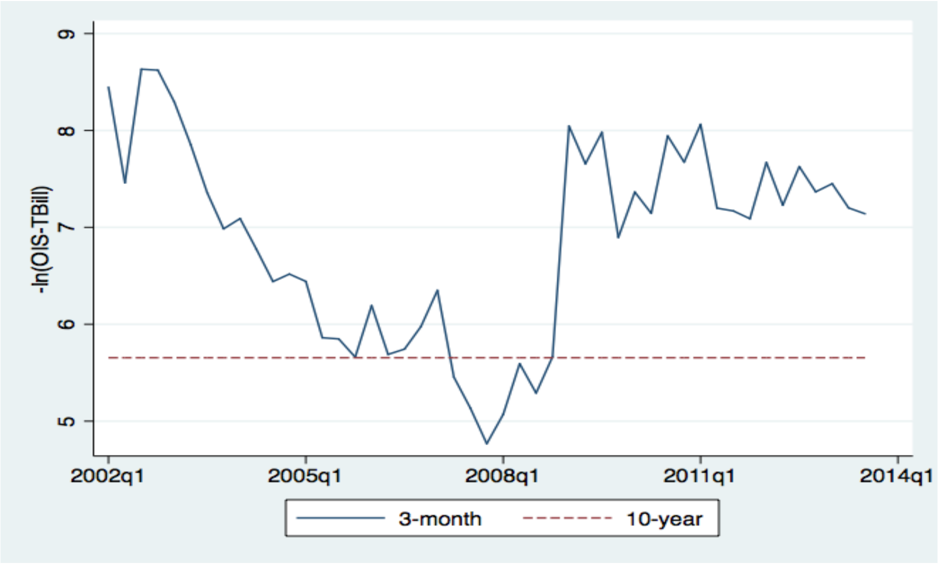

Figure 3.1: *Liquidity factors μ_{ST} and μ_{LT}.*

vulnerable to liquidity withdraw hence has a much shorter maturity proxy, say $T = 1$. For trading liabilities, we follow the rule for trading assets and use the haircut rates to define corresponding liquidity weights. Figure 3.2 shows the liability-side liquidity weight with respect to the maturity parameter $T_{k'}$, conditional on scenarios of market liquidity premium. The left panel shows the case for a longer maturity $T_{k'} \in (0, 15]$ years, and the right panel shows a snapshot for $T_{k'} \in [0, 1]$. In normal times when the OIS-Tbill spread is small (dash blue line, OIS-TBill=0.01), only the very short-term liabilities have high weights. In liquidity crisis (solid black line, OIS-Tbill=0.9), all types of liabilities have significantly larger weights except the very long-duration securities such as equity.

We also examine the liquidity sensitivity of off-balance-sheet securities.[6] We label these off-balance-sheet data as *contingent liabilities*, which include unused commitments, credit lines, securities lend and derivative contracts. Contingent liabilities have played an increasingly important role in determining a bank's liquidity condition, especially during the financial crisis of 2007 - 2009. Given their relative stickiness to rollover in normal times,

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[6] The off-balance-sheet securities are based on Schedule HC-L (Derivatives and Off-Balance-Sheet Items) and HC-S (Servicing, Securitization, and Asset Sales Activities) in Y-9C report.

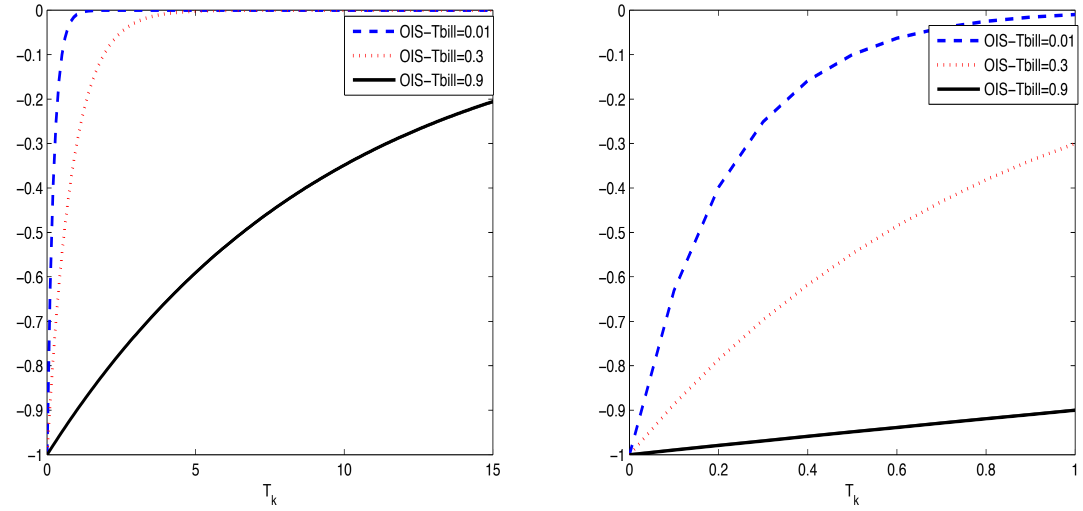

Figure 3.2: *Liability Liquidity Weights as a function of Maturity:* $\lambda_{L_k} = -\exp(-\ln(OIS - Tbill)T_{k'})$.

we assign a maturity proxy of $T = 5$ or 10 years.

Sharing the concern in asset-side liquidity weight, we also do the sensitivity analysis on the maturity parameter $T_{k'}$. The performance of LMI in next two sections remain unchanged for different sets of reasonable maturity setup.

3.4 Macro-Variation in the LMI

3.4.1 LMI as a Macro-Prudential Barometer

The LMI can be aggregated across firms and sectors. This is a property that is not shared by Basel's liquidity coverage measure which is a ratio and hence cannot be meaningfully aggregated. Summed across all BHCs, the aggregate LMI equals the supply of liquidity provided by the banking sector to the non-financial sector. We suggest that this aggregate LMI is a useful barometer for a macro-prudential assessment of systemic risk, which is a principal advantage of our method in measuring liquidity. When the aggregate LMI is low, the banking sector is more susceptible to a liquidity stress (“runs"). Indeed, the macro aspect of the aggregate LMI has already played a role in our construction: in the previous section, we computed the funding liquidity stress via a feedback that depends on the aggregate LMI, relying on the notion that the aggregate LMI is a macro-prudential stress indicator.

Figure 3.3 plots the aggregate liquidity mismatch over the period from 2002 to 2013. Recall that a negative value of LMI at the firm level indicates a balance sheet that is more vulnerable to liquidity stress (i.e. liability illiquidity is greater than the liquidity that can be sourced from assets). Consistent with Diamond and Dybvig (1983) and Gorton and Pennacchi (1990), we find that the banking sector carries a negative liquidity position (that is, the banking sector provides more liquidity than it consumes, hence the banking sector creates liquidity) throughout our sample. The magnitude of the LMI is important as it indicates whether our calibration of the liquidity weights are in the right ballpark. The LMI right before the crisis is about 5.5 trillion dollars which is of the same magnitude as the Fed and other government liquidity provision actions in the crisis.

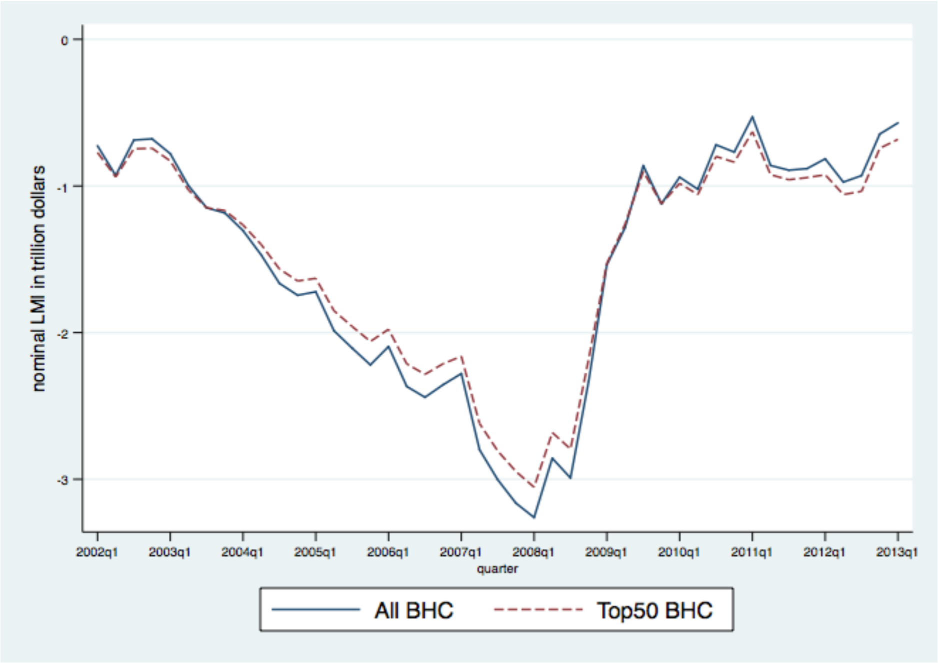

Figure 3.3: *Aggregate Liquidity Mismatch ($trillion) for Top 50 and All BHCs*

The liquidity position evolves markedly over time. At the beginning of our sample, 2002Q1, the total liquidity mismatch was about -0.84 trillion dollars. There was a pronounced increase in the LMI afterwards and an acceleration in 2007. The LMI hit its trough in 2008Q1 when the total mismatch achieved -5.54 trillion dollars, prefacing the financial crisis. The liquidity mismatch reversed with the Fed's liquidity injections and as the crisis faded and recovered to the pre-crisis level by 2009Q1. The trough of the liquidity mismatch occurred two quarters before the Lehman Brothers' bankruptcy and four quarters before the stock market reached its nadir. This suggests that the LMI can serve as a barometer or an early warning signal of a liquidity crisis. The evolution of the LMI is also related to the liquidity intervention by government, which we will discuss further in the next subsection.

Figure 3.3 also plots the time-series of aggregate LMI summed over top 50 BHCs. These BHCs were the primary users of the Fed's liquidity facilities from 2007 to 2009. The aggregate LMI of the top 50 BHCs is very close to that of the universe of BHCs, in terms of both the pattern and the magnitude. This evidence suggests that in dollar amount,

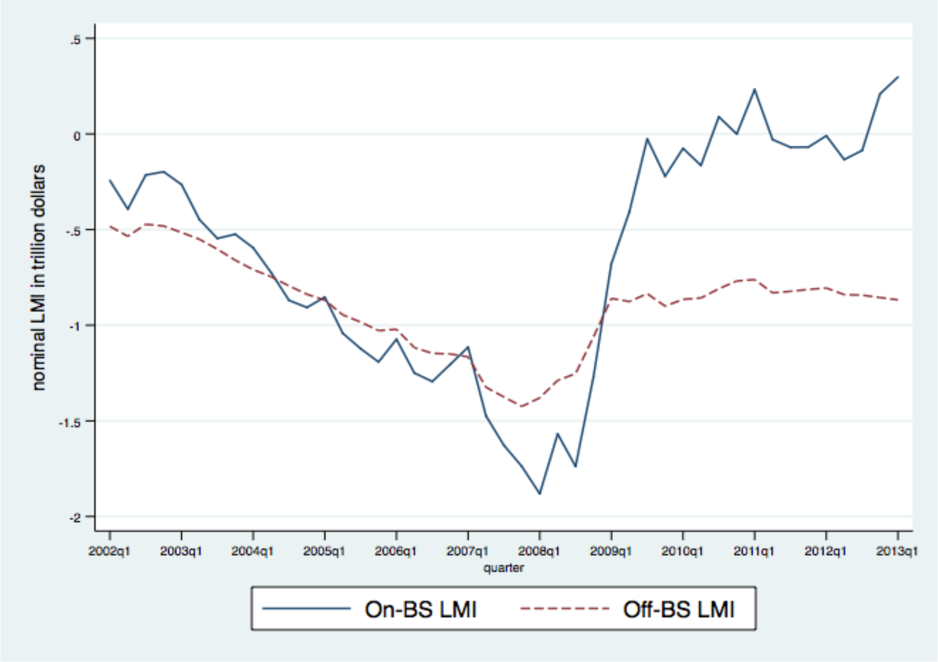

Figure 3.4: *Liquidity Mismatch On and Off Balance Sheet*

the US banking sector's liquidity condition is overwhelmingly determined by large banks represented by the top 50 BHCs. The remaining banks have a small impact totaling about $0 \sim 300$ billion dollars over time.

To understand further the composition of aggregate LMI, we present in Figure 3.4 the liquidity mismatch on and off balance sheet. Clearly, the off-balance-sheet liquidity pressure has been alleviated since the end of 2007. The change seems closely related to regulatory rules such as the Dodd-Frank Act on structured financial products.

3.4.2 LMI Decomposition: Asset, Liability, and Liquidity Weights

The LMI depends on assets, liabilities, and liquidity weights. Panels A and B in Figure 3.5 show the asset- and liability-side liquidity, scaled by total assets, for top 50 BHCs.[7] The scale of the y-axis is in the same order across two panels (asset-side is [0,1] whereas liability-side is [-1,0]), in order to facilitate a comparison of the relative movement in asset and liability liquidity. The red line is the median value while the shade area depicts the 10th to 90th percentiles. Both asset-side and liability-side liquidity contribute to the movement in the LMI, yet the liability side seems to play a bigger role. During 2008–2013, banks slightly increase their asset liquidity while have largely reduced liquidity pressure from the liability side. Panel C in Figure 3.5 plots the ratio of asset liquidity to liability liquidity (in absolute value) for the top 50 BHCs. The movement in the median ratio is consistent with our findings of the time-series pattern in the aggregate LMI.

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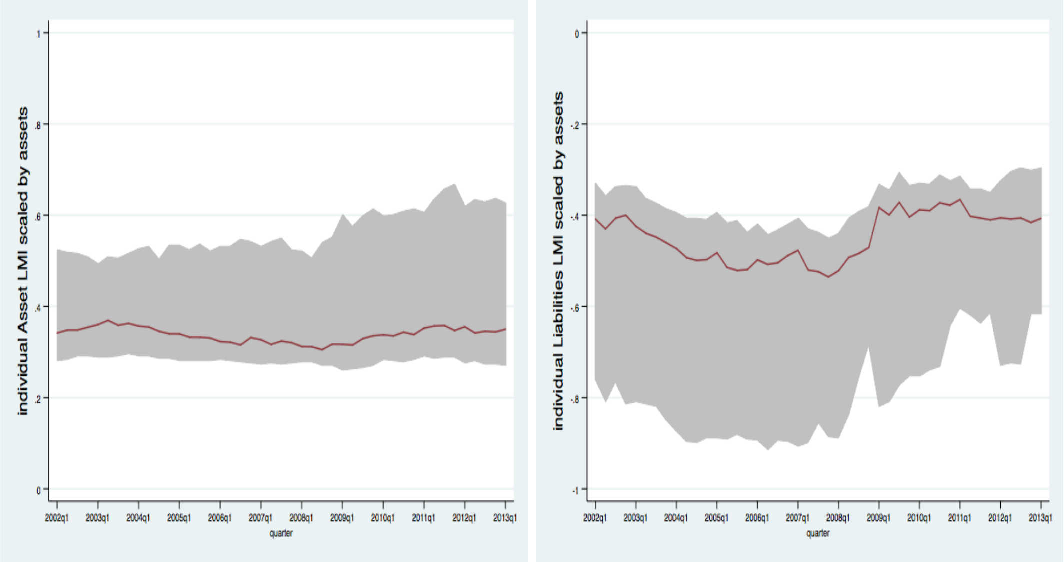
[7] The result remains robust if we extend the analysis to the universe of BHCs. For brevity, we here only report the results for Top 50 banks, given the fact that they dominate the aggregate LMI and hence should be the target of our research.



A. Asset-side Liquidity $(\sum \lambda_{t,A_k} x^i_{t,A_k})$

B. Liability-side Liquidity $(\sum \lambda_{t,L_{k'}} x^i_{t,L_{k'}})$

C. Distribution of the Asset/Liability Liquidity Ratio $(\sum \lambda_{t,A_k} x^i_{t,A_k} / | \sum \lambda_{t,L_{k'}} x^i_{t,L_{k'}} |)$

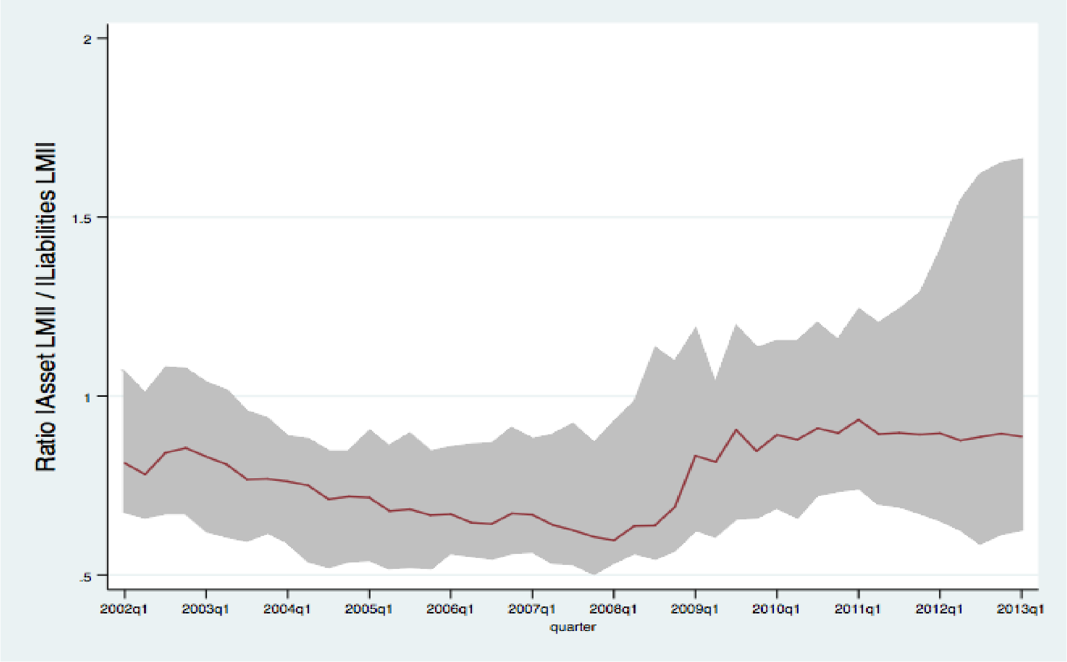


Figure 3.5: *Decomposition of LMI by Assets and Liabilities for Top 50 BHCs. The red solid line is the median value while the shade area depicts the 10th to 90th percentile.*

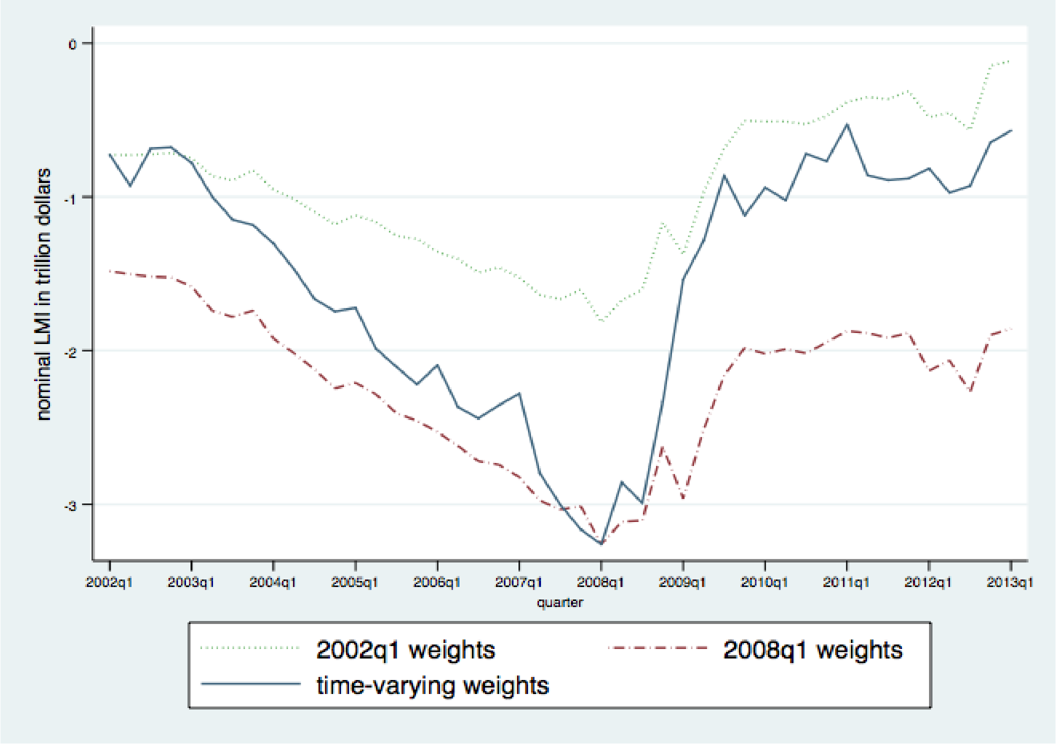


Figure 3.6: *Effect of time-varying liquidity weights on LMI*

We next turn to explaining how the changing liquidity weights contribute to movements in the LMI. Figure 3.6 plots the LMI under three weighting schemes: the blue line is our baseline case with time-varying weights; the green dashed line uses a fixed set of weights as of 2006Q1 (before the financial crisis); and the red dashed line uses weights as of 2008Q1 (the trough of the LMI). All three lines use the same contemporaneous balance sheet information. The three variations show that the time-varying weights contribute to a difference in liquidity of approximately 3 trillion dollars in the trough of 2008Q1, compared with using the pre-crisis weight as of 2006Q1. This figure also highlights the importance of adopting a time-varying weight linked to market conditions in terms of accurately delineating the banking sector liquidity.

3.4.3 Liquidity Stress Test

The Federal Reserve has recently engaged in liquidity stress tests which are designed to examine banks' ability to withstand a given liquidity stress event. The liquidity stress test is an addition to the Supervisory Capital Assessment Program (SCAP), which has become a standard process to test if a bank has sufficient capital to cover a given stress event. The decomposition of Figure 3.6 indicates a simple methodology to run a liquidity stress test within our measurement framework. The only difference across the three lines in Figure 3.6 are the liquidity weights, which in turn are determined by the time-varying repo haircuts m_t and the funding liquidity factor (μ_t). We suggest that a liquidity stress test can be implemented as a set of realizations of the funding liquidity factor or repo haircut, and these realizations can be traced through the liquidity weights to compute the stress effects on the liquidity of a given bank.

T=2008Q1			T=2006Q1			T=2012Q4		
		LMI			LMI			LMI
Benchmark			Benchmark			Benchmark		
	T	-3.26		T	-2.09		T	-0.65
	$[0, T]$	-1.80		$[0, T]$	-1.38		$[0, T]$	-1.55
Stress Scenarios			Stress Scenarios			Stress Scenarios		
μ	$\mu_{[0,T]} - \sigma_{[0,T]}$	-4.03	μ	$\mu_{[0,T]} - \sigma_{[0,T]}$	-2.62	μ	$\mu_{[0,T]} - \sigma_{[0,T]}$	-1.30
	$\mu_{[0,T]} - 2\sigma_{[0,T]}$	-4.91		$\mu_{[0,T]} - 2\sigma_{[0,T]}$	-3.22		$\mu_{[0,T]} - 2\sigma_{[0,T]}$	-2.05
	$\mu_{[0,T]} - 6\sigma_{[0,T]}$	-6.58		$\mu_{[0,T]} - 6\sigma_{[0,T]}$	-5.22		$\mu_{[0,T]} - 6\sigma_{[0,T]}$	-5.92
$\bar{m}$	$\mu_{[0,T]} + \sigma_{[0,T]}$	-3.31	$\bar{m}$	$\mu_{[0,T]} + \sigma_{[0,T]}$	-2.14	$\bar{m}$	$\mu_{[0,T]} + \sigma_{[0,T]}$	-0.67
	$\mu_{[0,T]} + 2\sigma_{[0,T]}$	-3.36		$\mu_{[0,T]} + 2\sigma_{[0,T]}$	-2.18		$\mu_{[0,T]} + 2\sigma_{[0,T]}$	-0.75
	$\mu_{[0,T]} + 6\sigma_{[0,T]}$	-3.56		$\mu_{[0,T]} + 6\sigma_{[0,T]}$	-2.35		$\mu_{[0,T]} + 6\sigma_{[0,T]}$	-1.03

Table 3.3: *Liquidity Stress Test using LMI. The table reports the aggregate LMI (in \$trillion) over all BHCs under stress scenarios when the funding liquidity factor or the cross-collateral average haircut deviate 1-, 2-, 6-σ away from the historical mean up to each time point T. All time-series start on 2002q1.*

We run a liquidity stress test at three time points: 2006Q1 (before the crisis), 2008Q1 (liquidity trough) and 2012Q4 (Fed's first liquidity stress test). Table 3.3 reports the results. Consider the first column corresponding to 2008Q1. The first row in the benchmark, denoted as "T", corresponds to the LMI value as of 2008Q1. The next line, denoted as "[0,T]", reports the historical average LMI up to this time point. We then compute the LMI under stress scenarios. The first line of the funding liquidity scenario reports the LMI based on assets and liabilities as of 2008Q1, but using liquidity weights that are based on one standard deviation (1-sigma) from the historical mean value of the funding liquidity state variable. We similarly report numbers over the next few rows based on weights when funding liquidity factor μ_t or haircut $\bar{m}_t$ is 1, 2, and 6 sigmas away from historical mean values.

The numbers reflect that during the severe liquidity dry-up in 2008Q1 the aggregate liquidity condition was between 2 to 6 sigmas from historical average values. In contrast, in 2012Q4, a six-sigma deviation from its historical average value, either under the funding liquidity or haircut stress scenarios, does not result in an LMI as low as the 2008Q1 scenario. That is, the banking sector currently has a better liquidity profile than when it enters the financial crisis.

3.5 LMI and the Cross-Section of Banks

The previous section presented one benchmark for evaluating the LMI, namely its utility from a macro-prudential viewpoint. We now consider another benchmark for evaluating the LMI. If the LMI contains information regarding the liquidity risk of a given bank, then changes in market liquidity conditions will affect the stock returns of banks differentially depending on their LMI. That is, as market liquidity conditions deteriorate, a firm with a worse liquidity position (lower LMI) should experience a more negative stock return.

We begin this section descriptively. We first show how the LMI of different banks varies over time, and what characteristics of banks correlate with their LMI. Then we examine the stock returns of banks with different LMIs under various liquidity events.

3.5.1 Cross-Sectional LMI

Figure 3.7 plots the cross-sectional distribution of the LMI over the universe of BHCs, with Panel A for the LMI scaled by total assets and Panel B for the LMI in absolute dollar amount. The red solid line is the median value while the shade area depicts the 10th to 90th percentiles.

The median value of the scaled LMI in Panel A follows the aggregate pattern in Figure 3.3. The shaded region (10th and 90th percentiles) is also stable suggesting that bank holding companies tend to have a stable cross-sectional distribution of liquidity. The version in Panel B, where LMI is not scaled rather in dollar amount, tells a different story and indicates a vast heterogeneity across banks' liquidity condition. The figure suggests that bank size (as measured by total assets) plays an important role in differentiating the absolute amount of liquidity mismatch across banks. At the beginning of the sample, the BHCs have a small dispersion in their liquidity conditions. The dispersion widens noticeably after 2007, likely because of the development of structured financial products. After the financial crisis, the dispersion narrowed again as some of these products are unwound, but remains wider than in the early 2000s.

We plot the time-series LMI for twelve representative banks in Figure 3.8, with Panel A for LMI scaled by total assets and Panel B for LMI in dollar amount. The LMI is negative for most of the bank holding companies, illustrating the pervasive liquidity mismatch of the banking sector during the crisis. For banks such as JPMorgan Chase, Bank of America, Wells Fargo, the LMI dramatically deteriorated during the crisis, but improved steadily from 2009 onwards, yet remained negative throughout the sample. For other banks like Goldman Sachs and Morgan Stanley, the LMI was also negative but much smaller in magnitude.[8] For banks like Citibank and Northern Trust, the LMI was negative in the beginning but switched the sign after the crisis, indicating a liquidity-surplus condition.

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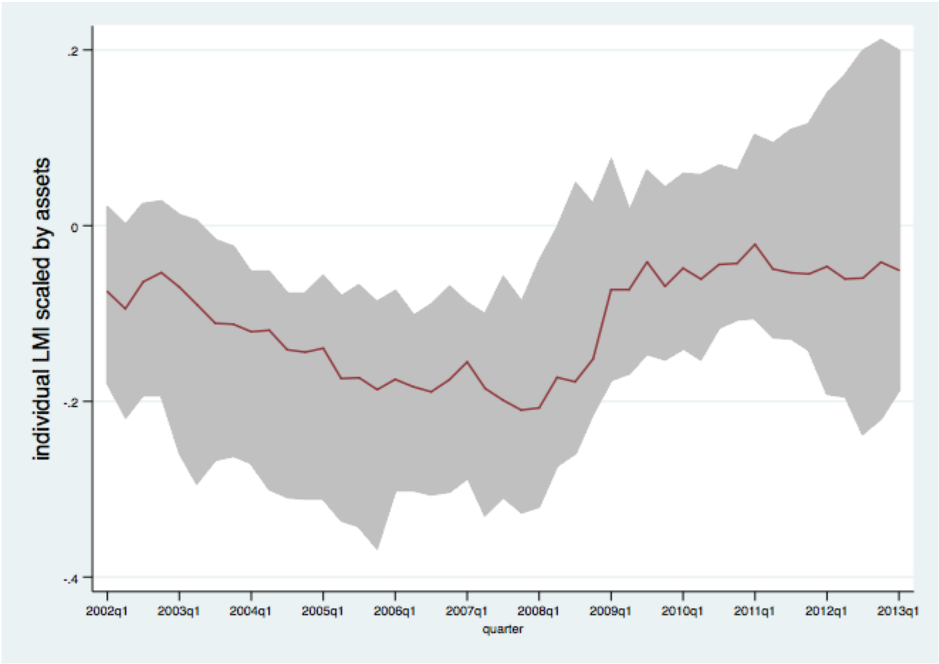
[8] The data for Goldman Sachs and Morgan Stanley begin in 2009Q1 given that these investment banks converted to bank holding companies after the Lehman event in September 2008.

A: LMI Scaled by Total Asset



B: LMI in Absolute Dollar Amount ($Billion)

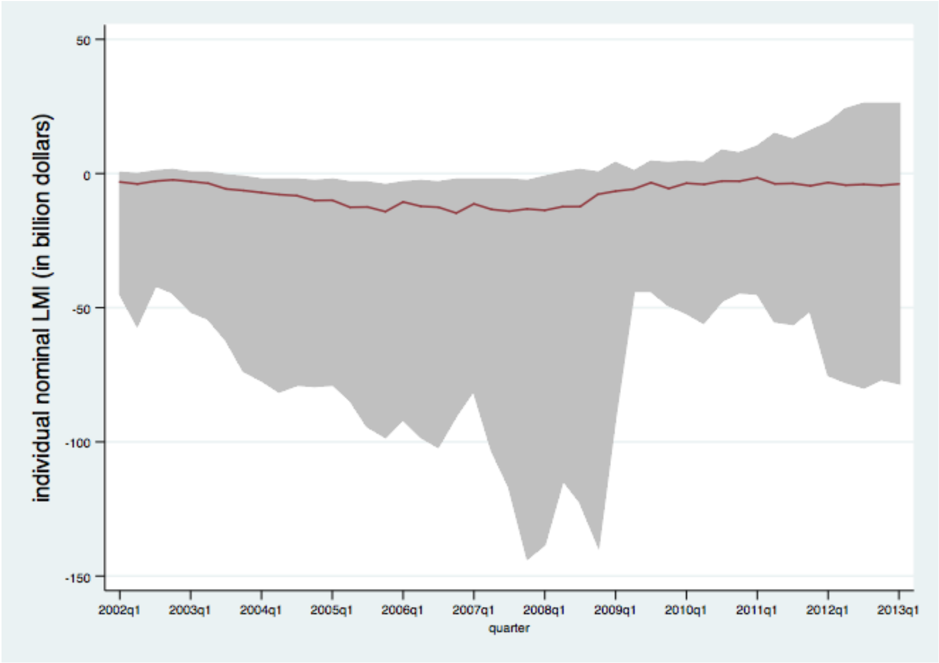


Figure 3.7: *Cross-sectional distribution of Scaled and Dollar LMI for All BHCs. The red solid line is the median value and the shade area depicts the 10th to 90th percentile. The sample is the universe of BHCs filling Y-9C reports.*

Panel A: LMI scaled by Total Assets

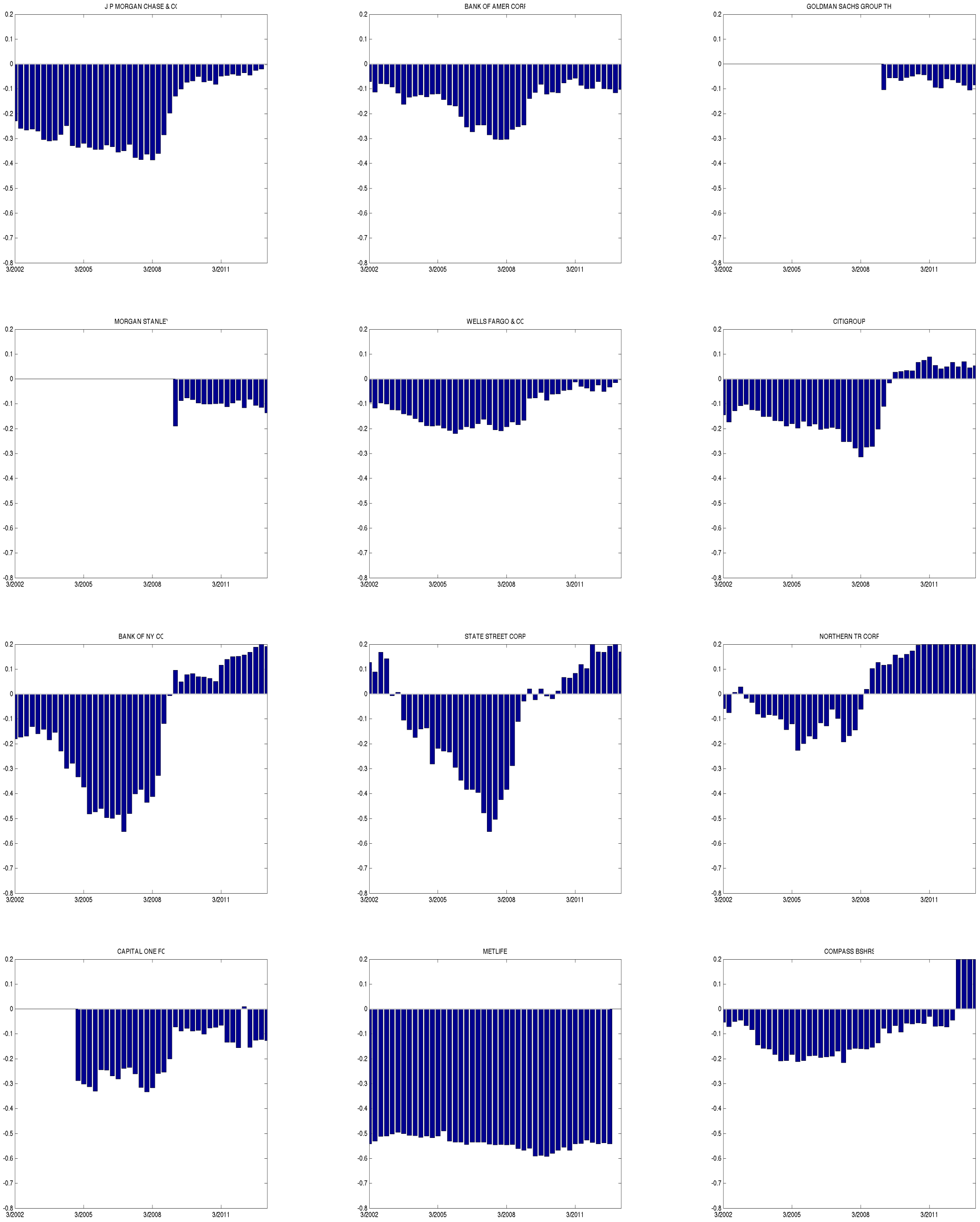


Figure 3.8: *Selected Bank-Level Liquidity Mismatch*

Panel B: LMI in dollar amount ($Thousand)

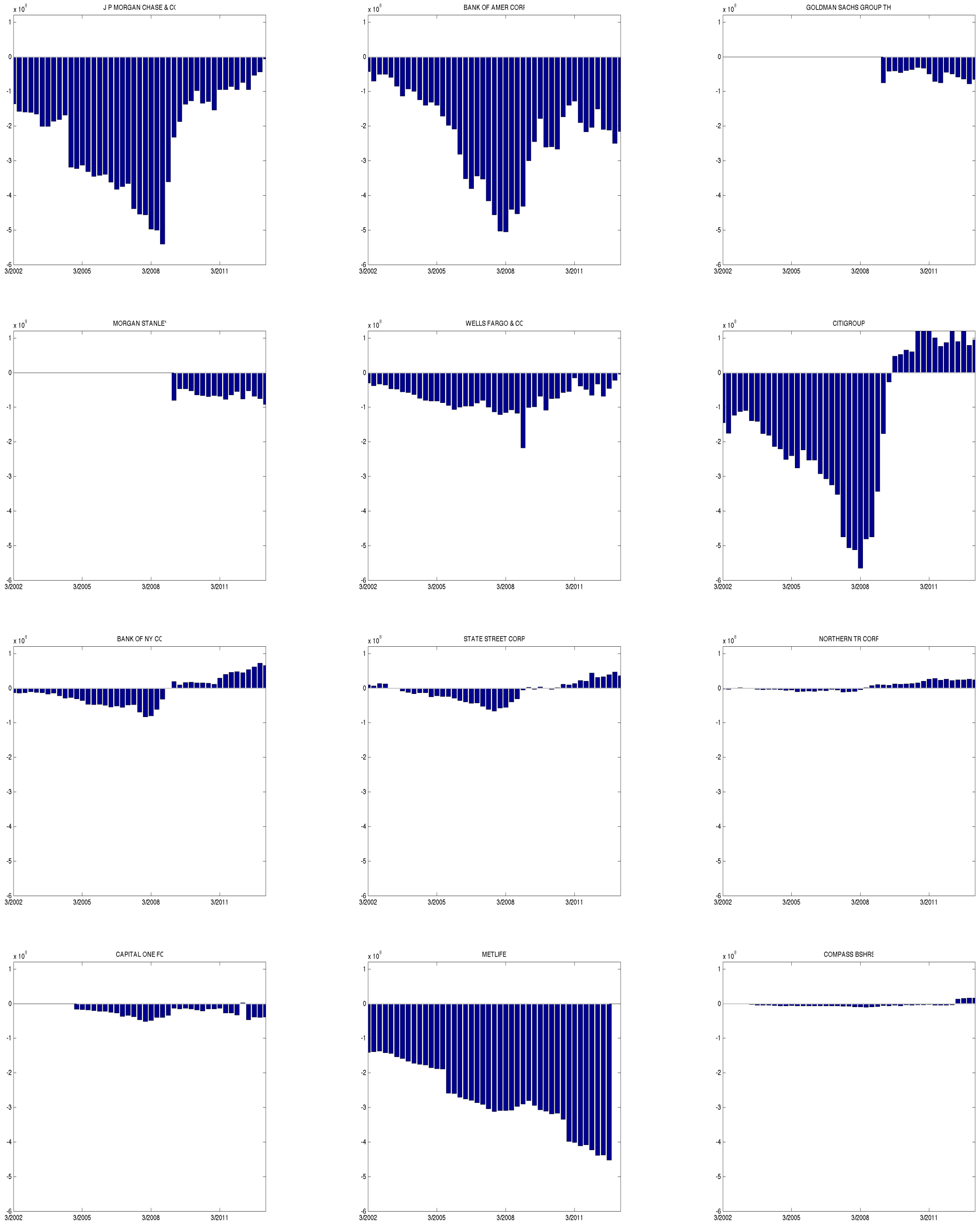


Figure 3.8 (Cont'd) Selected Bank-Level Liquidity Mismatch

The absolute level of the LMI may be useful as an indicator of systemic importance (i.e. "SIFI" status). For this purpose, we plot the bank-level LMI in dollar amount in Panel B, with the same y-axis scale to allow comparison. In the cross-section, banks have strikingly different liquidity levels. Banks like JP Morgan Chase, Bank of America, and Citigroup have large liquidity shortfall during the crisis whereas banks like State Street Corp, Northern Trust have a far smaller liquidity shortfall.

We report the top 15 banks with the most significant liquidity mismatch in Table 3.4, based on the average absolute LMI level over the whole sample. Banks with the most liquidity shortfall also correspond with common notions of the "too-big-to-fail" banks: Bank of America, Citigroup, JP Morgan Chase, Wachovia, and Wells Fargo taking the top positions. These banks experience their most stressed liquidity conditions in 2008Q1. American Express (ranked tenth) had the largest liquidity shortfall in the first quarter of 2009. The mortgage-related financial institution, Countrywide, saw it biggest liquidity mismatch in 2006Q2, one year before the subprime crisis. We also report the top 5 banks with the best average liquidity condition. They are smaller banks, although still among the top 50 BHCs.

We investigate the relationship between the LMI and bank characteristics for the universe of BHCs. Table 3.5 shows the results of regressing scaled-LMI and absolute LMI (in dollar amount), on a set of bank characteristics, which are collected from the Y-9C reports. The univariate specifications suggest that banks tend to have lower scaled LMI (worse liquidity condition) when they have larger risk-weighted assets, or more profitability (measured by return on asset (ROA)), or lower capital ratio. The multivariate specification (4) shows that these results are robust after the inclusion of other BHC characteristics. Specifications (5)-(8) show the similar results using the absolute LMI as dependent variable. Among all bank characteristics, risk-adjusted asset has the most explanatory power on bank liquidity condition.

Table 3.4: *Banks with the most Significant Liquidity Mismatch*

This table shows the LMI summary statistics for selected banks. Panel A presents the summary for banks with the most negative LMI in dollar amount out of the Top 50 banks, ranked by the average LMI values across the sample of 2006Q1 to 2012Q1; Panel B presents the results for banks with the most positive dollar LMI out of the Top 50 banks, that is, with the best liquidity condition. BHC with only one quarterly observation are excluded from the list. **Time** indicates the quarter at which the absolute LMI is the lowest for a given BHC.

Panel A: Banks with the most negative dollar LMI (mean)

Name	Net LMI	LMI (in $bil)				
		Mean	Median	Std	Min	Time
1. Metlife	-0.54	-276	-290	94	-452	2012Q3
2. JPMorgan Chase	-0.22	-235	-186	142	-541	2008Q3
3. Bank of America	-0.15	-229	-209	130	-505	2008Q1
4. Citigroup	-0.10	-147	-175	209	-565	2008Q1
5. Wachovia	-0.14	-78	-82	40	-165	2007Q4
6. Wells Fargo	-0.12	-74	-74	37	-217	2008Q4
7. Morgan Stanley	-0.10	-66	-66	12	-91	2013Q1
8. Goldman Sachs	-0.07	-53	-50	16	-78	2012Q4
9. US BC	-0.15	-38	-41	15	-63	2008Q1
10. Bank of NY	-0.32	-32	-30	17	-55	2006Q4
11. American Express	-0.22	-32	-30	6	-42	2012Q2
12. Countrywide	-0.23	-29	-28	14	-48	2005Q4
13. Capital One	-0.19	-27	-26	13	-51	2007Q4
14. National City	-0.17	-23	-23	6	-34	2008Q1
15. Suntrust	-0.12	-20	-15	10	-41	2008Q1

Panel B: Banks with the most positive dollar LMI (mean)

Name	Net LMI	LMI (in $bil)				
		Mean	Median	Std	Min	Time
1. E Trade	0.13	7	6	1	5	2013Q1
2. IMB	0.19	5	5	0	5	2012Q1
3. Northern Trust	0.03	5	-1	13	-12	2007Q2
4. Commerce	0.06	1	1	2	-4	2007Q4
5. First Niagara	0.00	0	0	1	-1	2011Q3

Table 3.5: *The Relationship of LMI with Bank Characteristics*

This table presents the results of pooled cross-sectional regression for the universe of public bank holding companies during 2006Q1 to 2012Q1. The standard errors are robust and clustered by bank. All variables are adimensional (ratios) except Total Assets, Risk-adjusted Assets and Unscaled LMI are in billion dollars.

	Depend variable: Scaled LMI				Depend variable: Unscaled LMI			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Risk-adj. Assets	-0.19***			-0.14	-0.29***			-0.48
	(0.06)			(0.60)	(0.04)			(0.58)
Tier1 Capital Ratio		1.36***		1.81*		0.14**		-0.45
		(0.35)		(0.93)		(0.06)		(0.33)
ROA (annualized)			-0.53***	-0.60***			-0.04**	-0.02*
			(0.11)	(0.12)			(0.02)	(0.01)
Total Assets				-0.03				0.12
				(0.41)				(0.37)
Total Capital Ratio				0.88				0.46
				(0.87)				(0.33)
Tier1 Leverage Ratio				-2.54***				0.16*
				(0.70)				(0.10)
ROE (annualized)				-0.02**				-0.00
				(0.01)				(0.00)
Constant	-0.08***	-0.25***	-0.08***	-0.18***	-0.00	-0.02**	-0.01**	-0.02
	(0.01)	(0.04)	(0.01)	(0.05)	(0.00)	(0.01)	(0.00)	(0.02)
N	6055	6055	6057	6055	6055	6055	6057	6055
R-squared	0.02	0.11	0.02	0.22	0.53	0.01	0.00	0.54

* p<0.10, ** p<0.05, *** p<0.01

3.5.2 LMI and Market Performance

We first investigate the correlation between LMI and stock market performance.[9] To this end, we sort BHCs and construct two portfolios: LMI High and LMI Low, based on their scaled LMI values averaged over the episode up to 2006Q1. Note that the unscaled LMI is driven almost entirely by bank size, hence it is less suitable when we study the relationship between bank liquidity and stock market performance (although as we have shown, the scaled LMI also correlates positively with size). The High LMI portfolio contains 100 BHCs with the highest LMI (best liquidity condition), and the Low LMI portfolio contains 100 BHCs with the lowest LMI (worst liquidity condition), in the universe of public BHCs during the pre-crisis period. Table 3.6 describes the summary statistics of the banks forming each of the two portfolios.

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[9] Given that the balance sheet information of foreign banks' U.S. subsidiaries cannot match their parent companies' stock market price, we exclude all foreign banks' U.S. subsidiaries in this analysis.

Table 3.6: *Characteristics of LMI High/Low Portfolios*

This table tracks the bank characteristics of two portfolios, LMI High and LMI Low, in the pre-crisis, the crisis, and the post-crisis period. All public BHCs are sorted based on their scaled LMI values averaged over the pre-crisis episode (2006Q1-2007Q2). The LMI High portfolio contains 100 BHCs with the highest LMI (best liquidity condition), and the LMI Low portfolio contains 100 BHCs with the lowest LMI (worst liquidity condition).

All Public US BHCs (Top100/Bottom100)

	Pre-Crisis (2006Q1:2007Q2)				Crisis (2007Q3:2009Q2)				Post-Crisis (2009Q3:2012Q1)			
	Low		High		Low		High		Low		High	
	mean	std	mean	std	mean	std	mean	std	mean	std	mean	std
Total Asset($Mil)	46.36	175.69	2.14	6.00	58.42	240.65	2.64	8.15	74.26	309.68	3.27	10.09
Leverage	0.91	0.03	0.90	0.03	0.91	0.04	0.90	0.03	0.90	0.04	0.90	0.03
Foreign/Total Deposit	0.01	0.04	0.01	0.07	0.01	0.03	0.01	0.07	0.01	0.02	0.01	0.06
Risk-adj. Asset($Mil)	35.32	127.49	1.45	3.99	44.90	177.93	1.80	5.38	51.20	206.42	2.05	6.09
Tier1 Cap Ratio	10.60	2.74	14.16	4.70	10.59	4.36	13.11	3.52	12.77	4.38	14.20	3.81
Total Cap Ratio	12.41	2.45	15.37	4.57	12.51	4.13	14.36	3.41	14.82	4.16	15.62	3.71
Tier1 Leverage	8.97	1.78	9.79	2.26	9.01	2.80	9.61	2.40	9.60	3.09	9.45	2.17
ROA (annualized)	2.41	1.70	2.04	1.76	-0.29	6.33	1.48	3.04	-0.55	4.55	0.96	3.75
ROE (annualized)	31.12	23.76	24.81	22.82	8.72	48.58	19.37	31.20	0.73	55.58	14.26	42.75

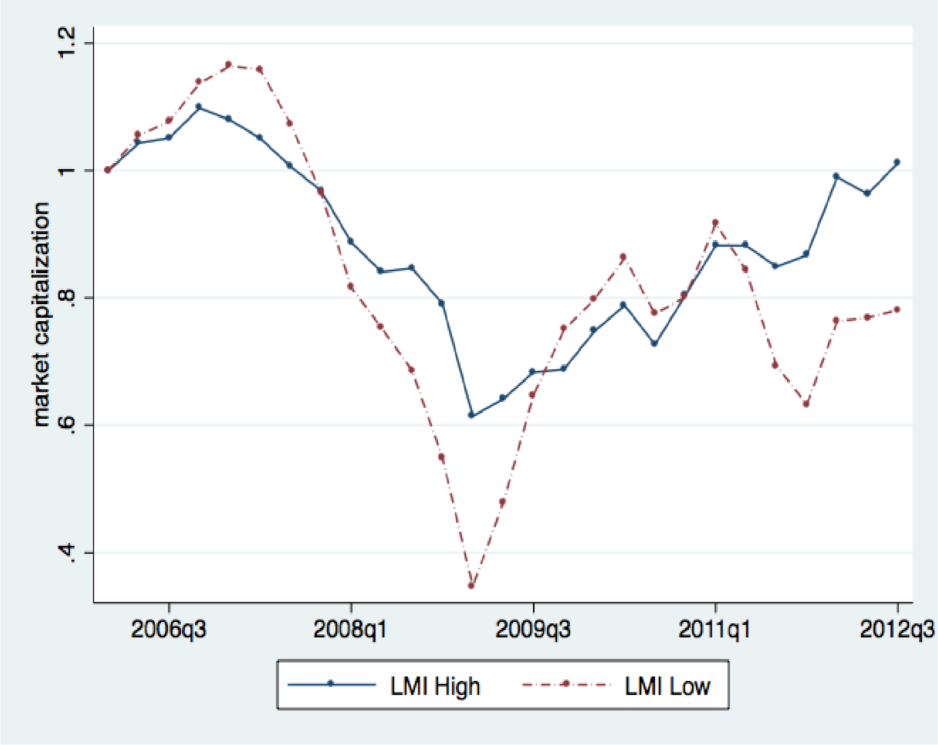


Figure 3.9: *Equity Market Capitalization by ex ante LMI Portfolios. The figure shows the market capitalization scaled by the level as of 2006Q1 for the Low (bottom 100 banks) and High (top 100 banks) portfolios sorted by the average LMI during the pre-crisis period: 2006Q1 - 2007Q2, across all public BHCs. For consistency of balance sheet information and stock market performance, we exclude the U.S. subsidiaries of foreign banks.*

Figure 3.9 presents the cross-section of market performance, where equity market performance is measured by the market capitalization of the portfolio normalized by its level as of 2006Q1. As shown in the figure, the Low LMI portfolio of BHCs outperformed the High LMI portfolio before the crisis. This pattern reversed in the crisis, when banks with a larger liquidity shortfall (the Low LMI portfolio) experienced lower stock returns. The gap between Low and High portfolios continued widening till 2009Q1 (the episode when stock market tumbled to record low point), then narrowed down. The pattern reversed again with the better performance of the Low LMI portfolio in the post-crisis period, though the reverse lasts shortly.

One possible explanation for our finding is that hoarding liquidity is costly, and only generates benefits in crises periods. Thus the low LMI banks are systematically more risky and more exposed to crises than the high LMI banks. Berger and Bouwman (2009) show that banks which are the most active in liquidity creation are rewarded by the stock market. We show that this correlation is dramatically reversed in crisis times. This result provides support to the hypothesis that the banking sector actively creates liquidity in good times (pre-crisis) but at the expense of building fragility, an idea that is tested in the aggregate by Berger and Bouwman (2012). Our cross-sectional approach identifies that the banks that create the most liquidity are the most vulnerable to financial crises.

3.5.3 Event Study: LMI and Liquidity Shock

The LMI is intended to measure the exposure of a bank to a liquidity stress event. If the LMI is informative in this dimension then we should observe differential performance of banks with different LMI across market-wide liquidity events. In particular, we expect that the banks with low LMI (poor liquidity) to perform worse under a negative liquidity shock whereas it performs better under a positive liquidity shock. We follow an event study methodology to test this hypothesis.

We sort the public BHCs and construct two portfolios according to their LMI values at the end of previous quarter, LMI High and LMI Low. Each portfolio contains value-weighted

100 banks with the highest/lowest LMI value. We use the Fama-French three-factor model to compute expected returns within the estimation window of [t-180, t-30], where t denotes the event day of a liquidity shock.

We then choose significant liquidity events in the sample. These events are chosen based on considering a large move in the TOIS spread as well as economic news such as the announcement of Fed liquidity facilities. Note that events cluster in the crisis and hence obscure the effect of liquidity shock. To identify a clean event, we choose the first event over any consecutive 30 days when the TOIS makes a significant negative jump or when the Fed announces the creation of a liquidity-related facility. We end-up with three events on positive liquidity shocks, PDCF (March 17, 2008), CPFF (October 7, 2008), and TALF (November 25, 2008), as well as three events on negative liquidity shocks, $\Delta TOIS$=-59bps (August 20, 2007), $\Delta TOIS$=-30bps (October 10, 2008), and $\Delta TOIS$=-53bps (September 17, 2008).

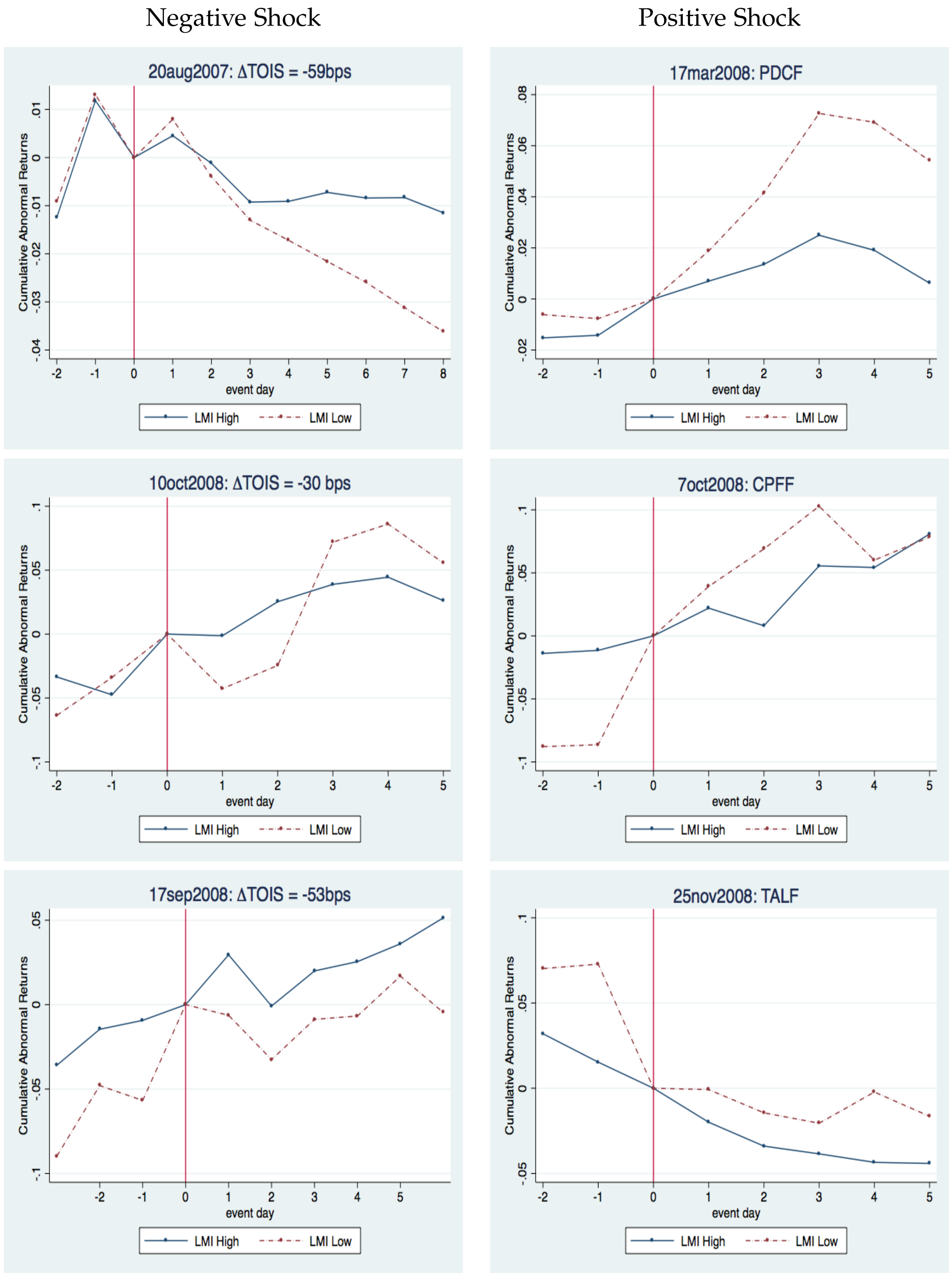


Figure 3.10: *Event Study: LMI and Liquidity Shock. The figure shows the cumulative abnormal return for LMI low (bottom 100 banks) and LMI high (top 100 banks) portfolios sorted by the LMI at the end of previous quarter before an event. Negative events are selected based on the daily change of Tbill-OIS spread, and positive events are selected based on the announcement of Federal Reserve Liquidity facilities.*

Figure 3.10 show the cumulative abnormal returns (CAR) during the [-2, 5] event window, with a normalization on the event date $t = 0$. We observe that the Low LMI portfolio underperforms the High LMI portfolio in days after a negative liquidity shock, whereas it overperforms the High LMI portfolio after a positive liquidity shock, confirming our hypothesis.

3.5.4 LMI and Liquidity Betas

Our findings from the event-study suggest that the LMI may also capture the Beta of the bank to liquidity stress. We pursue this further in this section. Using a similar framework like Flannery and James (1984), we first estimate the sensitivity of common stock returns to the funding liquidity factor, then we examine whether our measure of funding liquidity sensitivity (the beta) is related to the LMI. In detail, we compute a "liquidity beta" by using the funding liquidity factor FL_t as a fourth factor in the standard Fama-French factor analysis:

$$r_{t,k} - r_t^f = \beta^m_{q(t),k} * (r_t^m - r_t^f) + \beta^{HML}_{q(t),k} * r_t^{HML} + \beta^{SMB}_{q(t),k} * r_t^{SMB} + \beta^{FL}_{q(t),k} * FL_t + \epsilon_{t,k} \tag{3.20}$$

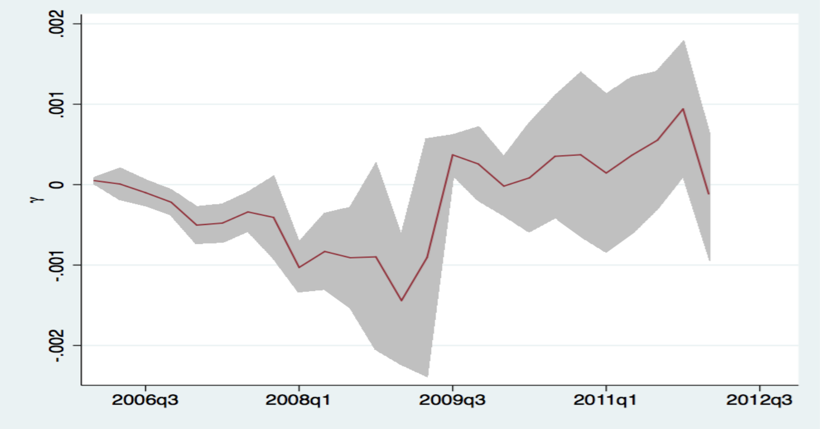
where FL_t is in the form of logarithm. We estimate quarterly time-varying betas $\beta_{q(t),k}$ based on daily returns in a window including the current and previous quarter (six months). Compared to the 'order-flow' liquidity factor used by Pastor and Stambaugh (2003), our funding liquidity factor FL_t does not resort to any stock market price information.

Next we verify that the LMI contains information for the bank's beta. We run a cross-sectional regression of the estimated liquidity betas on the bank's LMI at the current (q) or previous quarters $(q - j)$:

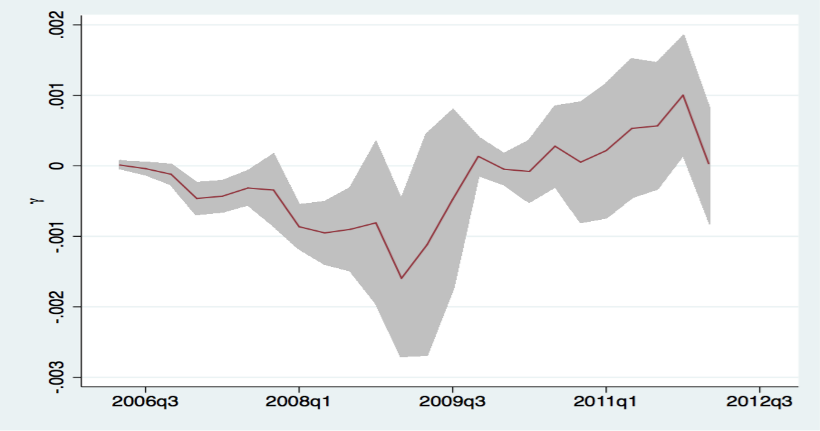
$$\beta^{FL}_{q,k} = \alpha_q + \gamma_q^{LMI} * LMI_{q-j,k} + u_{q,k}, \quad j = 0, 1, 2, \tag{3.21}$$

where we are interested in the coefficient γ^{LMI}.

A: Contemporaneous $(j = 0)$



B: One quarter Lag (j=1)



C: Two quarters Lag (j=2)

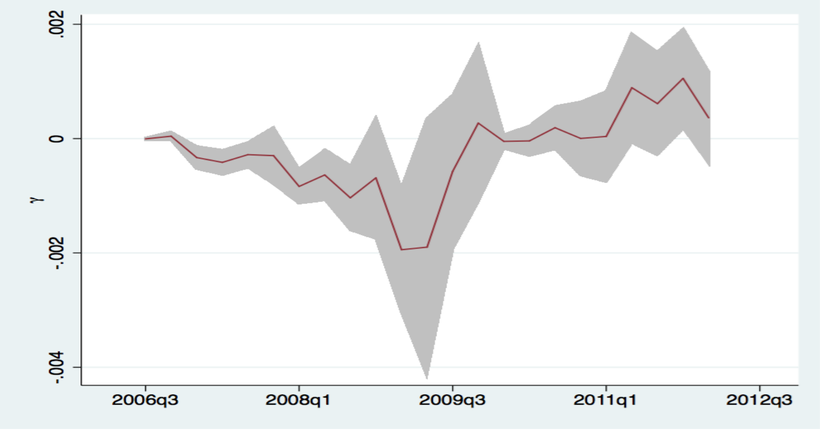


Figure 3.11: *Regression of liquidity betas on LMI: time-series of Liquidity Premium* γ^{LMI}. *The figure shows the coefficients in* $\beta^{FL}_{q(t),k} = \alpha_q + \gamma_q^{LMI} * LMI_{q-j,k} + u_{q,k}$, *where* $LMI_{q-j,k}$ *is bank k's LMI at the quarter* $q - j$, β^{FL} *is the exposure of bank stock return to funding liquidity factor. Shade denotes the +1/-1 standard error.*

Figure 3.11 presents the time-series of the coefficient γ^{LMI}, under different lags of LMI (j=0,1 and 2 quarters). We observe that this coefficient is negative and significant near and during the onset of the crisis, in accordance with the intuition that BHC with more negative LMI have higher positive liquidity betas (these BHCs are more sensitive to the innovation in the liquidity factor). However, the coefficient loses significance after the crisis, suggesting that liquidity risk is priced in stock prices no more, or alternatively, liquidity is no longer a concern in normal times.

3.5.5 LMI and Federal Reserve Liquidity Injection

We next discuss the impact of the government's liquidity injection on the U.S. banking sector's liquidity mismatch during the crisis. The Fed launched a range of new programs to the banking sector in order to support overall market liquidity. Appendix C.2 provides the background on these programs. The liquidity support began in December 2007 with the Term Auction Facility (TAF) and continued with other programs. It is apparent from Figure 3.3 that the improvement in the aggregate liquidity position of the banking sector coincides with the Fed's liquidity injection. While we cannot demonstrate causality, it is likely that the liquidity injection has played a role in the increase of the aggregate LMI.

We study the effect of the Fed injections on the cross-section of LMI. There are 559 financial institutions receiving liquidity from the Fed,[10] among them there are 87 bank holding companies (those submit Y-9C regulatory reports). These BHCs on average borrowed 95.8 billion dollars, with a median value of 0.7 billion dollars. The bank-level borrowing amount ranges from $5 million to $2 trillion. The ten bank holding companies which have received the most liquidity are Citigroup, Morgan Stanley, Bear Sterns, Bank of America, Goldman Sachs, Barclays U.S. subsidiary, JP Morgan Chase, Wells Fargo, Wachovia and Deutsche Bank's US subsidiary, Taunus.

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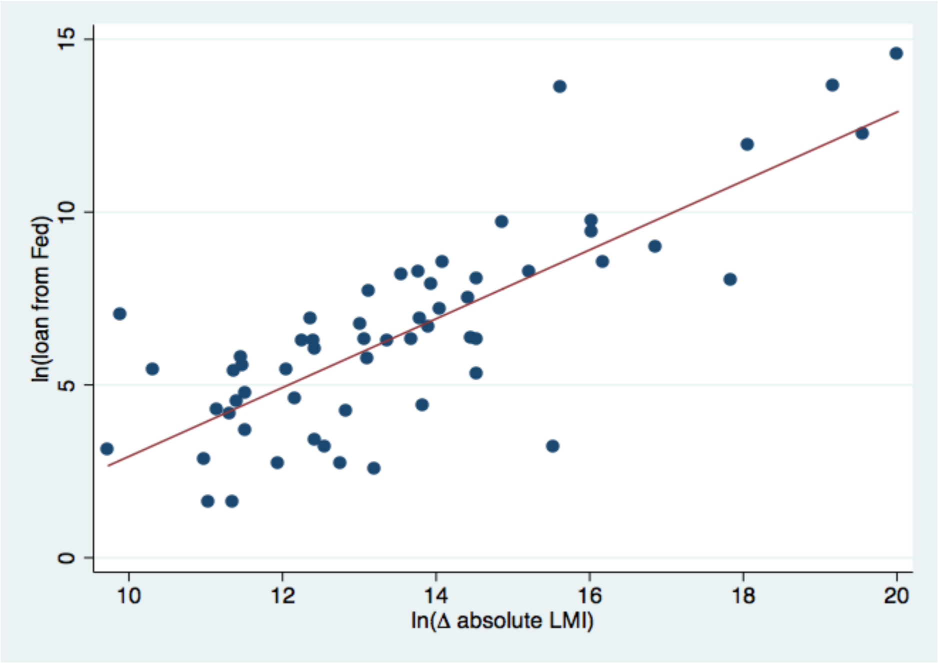
[10] One parent institution may have different subsidiaries receiving the liquidity injection. For example, AllianceBearnStein is an investment asset management company. Under this company, there are seven borrowers listed in the Fed data such as AllianceBearnStein Global Bond Fund, Inc, AllianceBearnStein High Income Fund, Inc, AllianceBearnStein TALF Opportunities Fund, etc.

A: LMI post-crisis minus LMI in the crisis



B: LMI post-crisis minus LMI pre-crisis

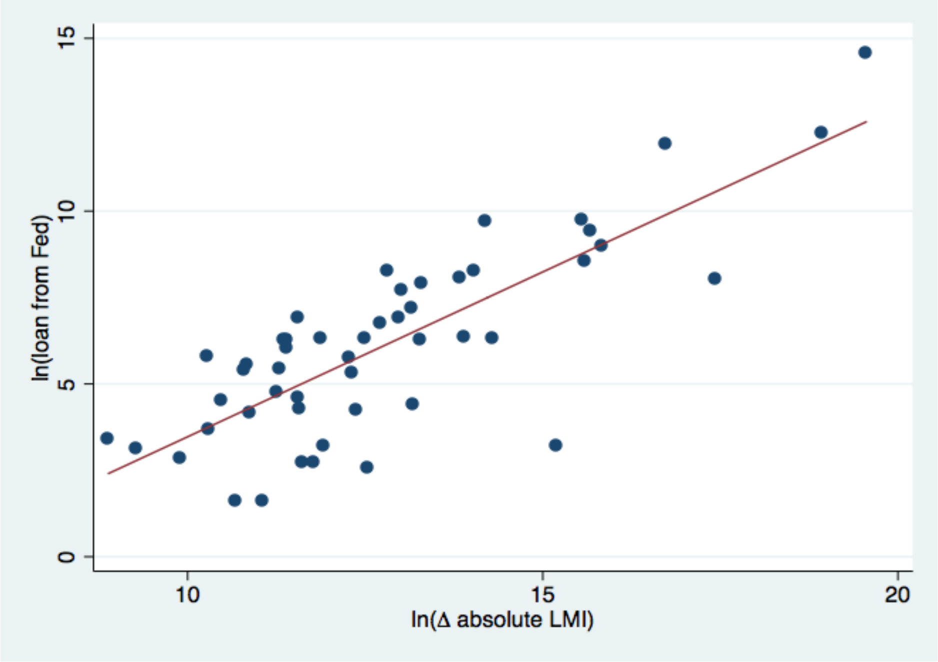


Figure 3.12: *Correlation between Fed liquidity injections and the change of LMI (in dollar amounts)*

Figure 3.12 plots the relation between the Fed liquidity injection and the change in LMI, cross-sectionally. The liquidity injection is measured by the log of the dollar amount of loans received by a given BHC, and the change in LMI is measured by the log of the difference in LMI between the post-crisis and the pre-crisis period (Panel A) and between the post-crisis and the crisis period (Panel B). Both panels document a strong positive correlation between the change in LMI and the level of the Fed liquidity injection. This evidence confirms the effect of the Fed's liquidity facilities on improving the banking sector liquidity.[11]

To formally test the impact of government liquidity injection, we check how the condition that a bank receives a liquidity injection or not has an impact on the relationship of LMI and stock returns. To this end, we run cross-sectional regression of returns $R_{j,T}$ on net LMI and its interaction with Fed injection dummy variables.[12] Table 3.7 presents the results. In Panel A we test the stock market response to *ex ante* LMI, which is the average value during the pre-crisis period (2006Q1 - 2007Q1).

$$R_{j,T} = \alpha_T + \beta_T * netLMI_{j,pre} + \delta_T * netLMI_{j,pre} * 1_{injection} + \epsilon_{j,T}, \quad \text{T = [crisis, post-crisis].} \tag{3.22}$$

where $R_{j,T}$ is annualized stock return compounded over the crisis period (2007Q2 - 2009Q2) and the post-crisis period (2009Q3 - 2012Q1). The Fed injection dummy is equal to 1 if the BHC did receive a Fed liquidity injection, and to 0 otherwise. Panel B replaces *ex ante* LMI by *contemporaneous* LMI, which is the average of LMI over the period on which stock returns are compounded):

$$R_{j,T} = \alpha_T + \beta_T * netLMI_{j,T} + \delta_T * netLMI_{j,T} * 1_{injection} + \epsilon_{j,T}, \quad \text{T = [pre-crisis, crisis, post-crisis].} \tag{3.23}$$

Panel B carries out the same exercise using change in LMI instead of level of LMI.

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[11] Berger, Bouwman, Kick and Schaeck (2013) shows that capital injections and regulatory interventions have a costly persistent effect on reducing liquidity creation. Taken together, their result and our result advocate for liquidity injections in crisis times as a desirable policy intervention.

[12] We also conduct a robustness exercise where the Fed liquidity injection dummy is replaced by the log of the loan amount a bank has received from the Fed. This substitution does not alter the results in terms of signs and significance.

It computes changes in LMI as the difference of net LMI between a pre-period and a post-period.

$$R_{j,T} = \alpha_T + \beta_T * \Delta netLMI_{j,(T-pre)} + \delta_T * \Delta netLMI_{j,(T-pre)} * 1_{injection} + \epsilon_{j,T}, \quad \text{T = [crisis, post-crisis]}. \tag{3.24}$$

Table 3.7 carries two main findings. First, it confirms the previous result that a more negative LMI, regardless of whether it is the pre-crisis LMI or contemporaneous one, predicts underperformance during the crisis and outperformance after the crisis. The second point is related to Fed liquidity injections. The interaction terms always point toward an opposite effect than the direct effect of LMI on returns. This suggests that the banks which have received liquidity injections are more insulated from the fragility a negative LMI has brought to the bank's stock return.

3.6 Conclusion

This paper implements the liquidity measure, LMI, which evaluates the liquidity of a given bank under a liquidity stress event that is parameterized by liquidity weights.

Relative to the Liquidity Coverage Ratio (LCR) of Basel III (which is conceptually closer to our liquidity measurement exercise than the Net Stable Funding Ratio), the LMI has three principal advantages. First, the LMI, unlike the LCR, can be aggregated across banks and thereby provide a macro-prudential liquidity parameter. Second, the LCR uses an arbitrary liquidity horizon of 30 days. Our implementation of the LMI links the liquidity horizon to market based measures of liquidity premia as well as the aggregate LMI. Thus our measurement has the desirable feature that during a financial crisis when liquidity premia are high, the LMI is computed under a longer-lasting liquidity scenario. Likewise, when the aggregate LMI of the financial sector is high, indicating fragility of the banking sector, the LMI is computed under a longer-lasting scenario. Third, the LMI framework provides a natural methodology to implement liquidity stress tests.

Table 3.7: *Case Study: Fed liquidity injection with dummy as Fed injection regressor*

This table illustrates the sensitivity of bank return to Fed liquidity injections depending on their LMI. Over the crisis, banks with lower LMI underperformed the higher LMI banks, but at the same time were more positively sensitive to a Fed liquidity injection.

Panel A: The Impact of Pre-crisis Liquidity Level

	LMI$_{pre}$	LMI$_{pre} * 1_{injection}$	constant	N	Adj-R^2
R_{crisis}	0.125***	-0.150**	-0.021***	356	0.04
	(0.04)	(0.06)	(0.00)		
R_{post}	-0.101**	0.002	0.021***	282	0.03
	(0.04)	(0.09)	(0.00)		

Panel B: The Impact of the Liquidity Change

	ΔLMI$_{crisis-pre}$	ΔLMI$_{crisis-pre} * 1_{injection}$	constant	N	Adj-R^2
R_{crisis}	-0.273***	0.110	-0.027***	356	0.02
	(0.10)	(0.34)	(0.00)		
	ΔLMI$_{post-pre}$	ΔLMI$_{post-pre} * 1_{injection}$	constant	N	Adj-R^2
R_{post}	0.037	-0.026	-0.015*	270	0.03
	(0.05)	(0.12)	(0.01)		

Panel B: The Impact of Contemporaneous Liquidity Level

R_T	LMI$_T$	LMI$_T * 1_{injection}$	constant	N	Adj-R^2
R_{crisis}	0.124***	-0.160**	-0.018***	356	0.03
	(0.04)	(0.08)	(0.00)		
R_{post}	-0.154**	-0.026	-0.044***	270	0.00
	(0.06)	(0.13)	(0.01)		

The LMI has a close precedent, the Berger and Bouwman (2009) liquidity creation measure. The primary change relative to the Berger-Bouwman measure is that the LMI is based on time- and state-dependent liquidity weights. This is an important modification because it naturally links bank liquidity positions to market liquidity conditions, and thus is better suited to serving as a macroprudential barometer (and a stress testing framework). We have shown that the LMI performs well relative to our macroprudential benchmarks. We have also shown that the LMI contains important information regarding the liquidity risks in the cross-section of banks.

We do not view the LMI measures of this paper as a finished product. We have made choices regarding the liquidity weights in computing the LMI. These weights play a central role in the performance of the LMI against our macro and micro benchmarks. It will be interesting to bring in further data to better pin down liquidity weights. Such data may be more detailed measures of security or funding liquidity drawn from financial market measures. Alternatively, such data may be balance sheet information from more banks, such as European banks, which will offer further data on which to calibrate the LMI. In either case, the approach of this paper can serve as template for developing a better liquidity measure.

To conclude we can brush how Monetary Policy would affect LMI. We would argue that: (i) Unconventional monetary policy acts on the liquidity weights: haircuts policy a la Ashcraft, Garleanu and Pedersen (2010) acts on asset side liquidity weight, and liquidity injections by the Fed, as shown in our paper, act on liability side weights (ease TOIS). (ii) Conventional monetary policy acts on the balance sheet quantities: too accommodative monetary policy (low r) would induce banks to reach-for-yield, increase liquidity transformation and lower LMI. As a consequence, taking for granted that we can brush a ballpark for optimal LMI from welfare analysis, monetary policy could be used as a tool to impact LMI and target optimal LMI, whether through balance sheets or liquidity weights. The most effective policy depends on BS quantities / weights sensitivities to policy in current times (in good times, conventional monetary policy should better work, through balance sheet quantities, whereas in bad aggregate times, unconventional monetary policy through market prices of liquidity - liquidity weights - should work better).

Chapter 4

Optimal "Eurobond" Design[1]

4.1 Introduction

This paper studies the optimal design of a joint liability arrangement problem for a group of sovereigns that decide to borrow jointly. Exactly as banks are supplying safe assets to the economy by bundling assets that are negatively correlated on their balance sheet, sovereign authorities should be able to diversify the sovereign risk among themselves.

In the policy debate, one of the often-discussed proposals to alleviate the ongoing European economic crisis has been the issuance of Eurobonds, broadly defined as liabilities shared by several countries. The three most developed policy contributions up to date are: (i) the *Blue Bond* proposal of (Delpla and Von Weizsacker, 2011), (ii) the *Eurobills* proposal of (Hellwig and Philippon, 2011), and (iii) the *European Safe Bonds (ESBies)* proposal of (Brunnermeier and al., 2011). This papers aims at providing theoretical foundations to these policy proposals, by qualifying under which sets of assumptions such arrangement would be beneficial. We also calibrate the optimal levels of pooling and tranching on the Eurozone economy.

Our main result is that Eurobonds is a desirable instrument in order to implement a fiscal union, when explicit transfers are not available. The benefit of pooling is not a priori straightforward, as the Modigliani-Miller theorem makes it generically irrelevant. Worse, joint debt issuance is plagued with a free-rider problem. When given the opportunity to pool a share of its debt with other countries, all countries end up overborrowing due to a moral hazard problem in the issuance of public debt. Nevertheless, in our framework, countries would like to risk-share through a fiscal union, but explicitly transfers are not available, perhaps because of political economy frictions. In this case, Eurobonds prove to be a market-based instrument that enables to implement the risk-sharing agreement. We derive the optimal share of pooling as an interior solution to the trade off between the cost of pooling (free-riding externality) and the benefit of pooling (risk-sharing externality).

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[1] Co-authored with Eduardo Davila (Harvard University). Keywords: sovereign debt, Eurobonds, safe assets.

Eurobonds turn out to lead to a Pareto improvement when there is demand for safety in the economy. We show that pooling sovereign debt into Eurobonds and tranching them into a safe part ('blue bonds' or 'ESBies') provides more safe assets to the economy than the uncoordinated equilibrium, even when countries have access themselves to the tranching technology. The intuition is as follows. Countries that do enjoy a safety premium do not internalize that the safety premium also carries on all the inframarginal units of debt issued. When the source of the safety premium is exogenous to the agents of the zone (i.e. outside demand for safety), countries would be better off by asking the safe countries (which enjoy the safety premium) to issue more individual debt, and risky countries to issue less individual debt. Such Pareto improvement is implemented by the ex-ante arrangement of pooling and tranching the joint liability.

We frame the design of Eurobonds as a second best problem. The social planner wants to redistribute income towards countries with higher need, i.e. higher marginal utility. A ex-ante agreement of joint-issuance will achieve this goal by manipulating asset prices in the sovereign debt market. However, as the agreement only has access to one instrument (the pooled share θ of all individual sovereign debt that is issued), the first-best allocation cannot be achieved. Moreover, we show that the social planner does not want to pool all the debt because of the free-riding externality. As a result the optimal share of pooling to implement the transfer is interior: $\theta \in (0;1)$. In a second step, when we introduce demand for safety, a Pareto improvement can be achieved by joint-liability issuance.

Related Literature

This paper belongs to the vast literature on sovereign default, in the tradition of Eaton and Gersovitz 1981. Given the stochastic nature of our environment though, we capture both strategic and suffered default. Most of the literature is concerned about debt sustainability (Aguiar and Gopinath 2006, Arellano 2008) or maturity choice (Aguiar and Amador 2013). We address here a different question, the design of supranational public debt issuance scheme. This mechanism design approach is inspired by (Davila, 2013), which applies it to the design of bankruptcy policies.

The paper connects a recent literature on the macroeconomic shortage of safe assets: (Caballero and Farhi, 2013), (Gorton and Ordonez, 2013), (Gourinchas and Jeanne, 2012), (Weymuller, 2013a). Whereas the latter addresses the question of optimal public debt issuance to foster private debt creation in a closed economy, our paper is concerned with the mechanism design faced by a supranational authority. Issuing debt to cater to a specific demand for assets is also present in (Greenwood et al., 2010).

4.2 A model of joint sovereign debt issuance

4.2.1 Environment

There are two dates, $t = \{0, 1\}$ and $|I|$ sovereign countries, denoted by $i \in I$, with $I = \{1, \ldots, |I|\}$. For simplicity, we focus most of our discussion in the case of $|I| = 2$. Our model builds upon the canonical model of default in international debt markets.

The only departure is that we allow for an ex-ante institutional arrangement. Before any public debt issuance, sovereigns agree on pooling a share θ of any level of public debt they will issue. The synthetic liability thus created is called a Eurobond. If countries i each decide to issue B_0^i of debt in face value, the Eurobond has a face value of:[2]

$$\theta \sum_i B_0^i$$

This promise is issued *jointly* on international debt markets, at a single price q_0^E. We make the following assumption on seniority: sovereign cannot selectively default on Eurobonds. That is, in the case of default of individual country i, all the debt is written down to zero, both the θB_0^i which is part of the Eurobond, and the residual $(1-\theta) B_0^i$ issued at an individual price. The share θ of pooling is chosen *ex-ante*, that is before sovereigns make issuance decisions $\left\{B_0^i\right\}_i$. Eurobonds consists in this institutional arrangement, which makes bonds fungible. In other words, sovereigns must default on all their debt, they cannot default selectively.[3]

4.2.2 Sovereign choice of public debt issuance

Each country i is populated by a risk averse representative agent which maximizes time separable expected utility. Flow utility $U(\cdot)$ satisfies $U' > 0$ and $U'' < 0$; we allow for a different values of the discount factor β^i, to capture potentially different borrowing needs among countries. We define $J(C_1) = \max_{\xi \in \{0,1\}} \left\{ \mathbb{I}\{\xi = 1\} \cdot U\left(C_{1,D}^i\right) + \mathbb{I}\{\xi = 0\} \cdot U\left(C_{1,ND}^i\right) \right\}$ where $\mathbb{I}\{\cdot\}$ stands for the indicator function for default. Therefore the problem solved by each country is to choose consumption at each date $t = 0, 1$ and state Y_1^i: C_0^i and $\left\{C_1^i Y_1^i\right\}$, the amount of bond issued B_0^i and when to default $\left\{\xi^i\left(Y_1^i\right)\right\}$. Formally, each country solves:

$$\max_{C_0^i, \left\{C_1^i\left(Y_1^i\right)\right\}, B_0^i, \left\{\xi^i\left(Y_1^i\right)\right\}} U\left(C_0^i\right) + \beta^i \mathbb{E}\left[J\left(C_1^i\right)\right]$$

Where C_0^i and C_1^i is the consumption of each country at $t = 0$ and at $t = 1$ respectively. Each country chooses to borrow in the international markets an amount B_0^i. A fraction θ of each dollar borrowed by country i is pooled into a joint liability bond, i.e., a EuroBond (E).

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[2] The face value of any debt contract is how much is promised be repaid tomorrow.

[3] Discuss Russia 98 case, Argentina 2001. Hold outs.

The rest of the amount borrowed remains the individual liability of each country i.

The initial net output of country i at $t = 0$ is given by y_0^i.[4] Output of country i at $t = 1$ is stochastic and denoted by Y_1^i. The joint distribution of Y_1^i is given by a distribution $F\left(Y_1^1, \ldots, Y_1^N\right)$. Define σ^i the volatility of the output i. Following Aguiar and Gopinath 2006 and Arellano 2008, we model default cost as a loss of output. For tractability, this default cost is a proportional fraction δ of GDP at $t = 1$. The budget constraint at $t = 1$ is:[5]

$$C_0^i = Y_0^i + q_{0s}^i B_{0s}^i$$

$$q_{0s}^i = \theta q_0^E + (1-\theta)\,\tilde{q}_0^i$$

where q_0^E is the unit price of a Eurobond of unit face value and $\tilde{q}_0^i$ is the unit price of a unit bond backed only by sovereign i. We define q_0^i as the unit price per dollar owed at $t = 1$ by country i. It is the price that the sovereign faces, taken as given the institutional Eurobond arrangement.

The budget constraints at $t = 1$ when a country i defaults and when it does not are, respectively:

$$C_{1,ND}^i = Y_1^i - B_{0s}^i \tag{4.1}$$

$$C_{1,D}^i = (1-\delta)\,Y_1^i \tag{4.2}$$

We assume that a country that defaults does not pay any of its debt back;[6] neither the individual part $(1-\theta)$ nor the Eurobond part θ.

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[4] The net output can be thought as disposable income.

[5] The parameter δ captures all costs associated with defaulting; international sanctions. We abstract from investment.

[6] The extension to the case in which the sovereign pays back part of its loan is straightforward.

Optimal default decision at $t = 1$

In our setup, a country i decides to default at $t = 1$ when the output realization Y_1^i is sufficiently low.[7] We assume that there is no possibility of renegotiating the contracts,[8] and we rule out the existence of any kind of bailout policy between countries.

A country i decides to default when $C_{1,ND}^i < C_{1,D}^i$. That is when:

$$Y_1^i \leq \frac{B_{0s}^i}{\delta} \tag{4.3}$$

Countries default when their income realization is below an endogenous threshold. Under this formulation, sovereigns may decide to default when they do not have enough resources to repay the debt or also when, even having sufficient resources to pay their debt back, it is more profitable to suffer the penalty than repay. Thus it embeds both strategic and suffered defaults.

Optimal borrowing choice at $t = 0$

Taking into account its optimal default decision at $t = 1$, by backward induction each country i decides at $t = 0$ how much debt to issue:

$$\max_{B_{0s}^i} J\left(B_{0s}^i\right) \equiv U\left(y_0^i + q_{0s}^i B_{0s}^i\right) + \beta^i \left[\int_{\underline{Y_1^i}}^{\frac{B_{0s}^i}{\delta}} U\left((1-\delta)\, Y_1^i\right) dF\left(Y_1^i\right) + \int_{\frac{B_{0s}^i}{\delta}}^{\overline{Y_1^i}} U\left(Y_1^i - B_{0s}^i\right) dF\left(Y_1^i\right)\right]$$

The optimality condition is characterized by the first order condition:[9]

$$U'\left(C_0^i\right)\left[q_{0s}^i + \phi^i \left.\frac{\partial q_{0d}^i}{\partial B_0^i}\right|_i B_{0s}^i\right] = \beta^i \int_{\frac{B_{0s}^i}{\delta}}^{\overline{y_1}} U'\left(Y_1^i - B_{0s}^i\right) dF\left(Y_1^i\right) \tag{4.4}$$

Where $\frac{\partial q_{0d}^i}{\partial B_0^i}$ is the derivative of the demand curve with respect to the total amount of B_0^i. Where ϕ^i denotes whether agents internalize their price impact or not. The general case considers price taking sovereigns, i.e. small open economies. On the other hand, large countries take into account their price impact, i.e. they realize they face a downward sloping demand curve in their own debt.[10] We focus on the latter case.

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[7] Compare to Arellano which has a region of default instead.

[8] This should be the case in equilibrium when debt holders can hold out.

[9] LHS is marginal benefit, RHS is marginal cost of extra unit of borrowing.

The equation 4.4 traces out a supply curve for the sovereign debt of country i. Under mild assumptions on the income distribution, this supply curve is increasing. A key feature of the model is that this supply curve does *not* depend directly on the Eurobond share θ. This policy function will impact debt issuance of country i, but only through its indirect effect on the equilibrium price q_{0s}^i.

In the log-utility case, we derive:

$$\frac{1}{y_0^i + q_{0s}^i B_{0s}^i}\left[q_0^i + \phi^i \left.\frac{\partial q_{0d}^i}{\partial B_0^i}\right|_i B_{0s}^i\right] = \beta^i \int_{\frac{B_{0s}^i}{\delta}}^{\overline{y_1}} \frac{1}{Y_1^i - B_{0s}^i} dF\left(Y_1^i\right)$$

Assumption 3. *The discount factor β^i is sufficiently low.*

Assumption 1 is here to ensure the marginal benefit and the marginal curves for the sovereign do cross out, hence pinning a unique privately optimal level of debt from the sovereign f.o.c. Apart from the log case, non-trivial second order conditions need to be verified to ensure that the supply curve of sovereign debt is upward sloping.

4.2.3 Risk-neutral bond pricing

There are $I+1$ bonds to be priced: the unpooled sovereign bond issued by each individual country, and the Eurobond. International markets price risk neutrally any security, with discount factor $\beta^* = \frac{1}{1+r^*}$. We disregard issues related to the nationality of the debt holders.

Pricing of individual bonds

The bond demand for the fraction of debt not pooled into the Eurobond is priced as such:

$$\tilde{q}_{0d}^i \left(1-\theta\right) B_{0d}^i = \frac{\left(1 - F_i\left(\frac{B_{0d}^i}{\delta}\right)\right)\left(1-\theta\right) B_{0d}^i}{1+r^*} \Rightarrow \tilde{q}_{0d}^i = \frac{1 - F_i\left(\frac{B_{0d}^i}{\delta}\right)}{1+r^*} \tag{4.5}$$

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[10] This is reminiscent of Aguiar and Amador 2013, in which the government acts as a monopsonist: he fully takes into account its price impact.

The prices of individual bonds are generically independent of any correlation structure of the shocks between the countries.

Pricing of Eurobonds

A risk-neutral representation of the assets decomposes linearly the price of the Eurobond. Thus, without any demand for safety from the international lenders, the correlation structure of income shocks does not matter in the pricing of Eurobonds.

For instance, in the case of $|J| = 2$:

$$q_0^E = \frac{\frac{B_{0d}^1}{B_{0d}^1+B_{0d}^2}\mathbb{P}\left[y_1^1 \geq \frac{B_{0d}^1}{\delta}, y_1^2 < \frac{B_{0d}^2}{\delta}\right] + \frac{B_{0d}^2}{B_{0d}^1+B_{0d}^2}\mathbb{P}\left[y_1^1 < \frac{B_{0d}^1}{\delta}, y_1^2 \geq \frac{B_{0d}^2}{\delta}\right] + \mathbb{P}\left[y_1^1 \geq \frac{B_{0d}^1}{\delta}, y_1^2 \geq \frac{B_{0d}^2}{\delta}\right]}{1+r^*}$$

It can be written, with $\chi_i = \frac{B_0^i}{\sum_k B_0^k}$:

$$q_0^E = 1 - \chi_1 F_1\left(\frac{B_{0d}^1}{\delta}\right) - \chi_2 F_2\left(\frac{B_{0d}^2}{\delta}\right) \tag{4.6}$$

Correlations do not matter in the pricing of Eurobonds. Holding a Eurobond is exactly the same as holding B_{0d}^i units of each country i debt. Hence by law of one price, the price of Eurobond cannot differ from the price of the portfolio of individual bonds that exactly replicate the payoffs of the Eurobond.

4.3 Equilibrium: Cournot game in issuance of sovereign debt

We define the uncooperative Cournot game equilibria for this economy. Each country takes into account their price impact when they issue public debt.

Definition 7. A Cournot game is defined as a set of allocations and prices such that:

(i) Sovereigns optimize their borrowing level and default decisions;

(ii) International lenders break even in their public debt holdings.

Effective borrowing price of the sovereign The equilibrium in the debt markets is given by the combination of the first-order conditions of the two borrowing countries and the debt pricing conditions coming from international lenders. We look for an equilibrium in which both the Eurobond and the individual bonds are issued. Lenders pricing conditions act as no arbitrage conditions between these two types of assets. θ does not directly enters in the two bonds prices $\tilde{q}_0^i$ and q_0^E. Hence θ impacts the equilibrium only by controlling the weight between the individual bond and the Eurobond in the effective price faced by the sovereign. For two countries, the effective borrowing prices are:

$$q_0^1 = \theta q_0^E \left(B_0^1, B_0^2\right) + (1-\theta)\, \tilde{q}_{0d}^1 \left(B_0^1\right)$$

$$q_0^2 = \theta q_0^E \left(B_0^1, B_0^2\right) + (1-\theta)\, \tilde{q}_{0d}^2 \left(B_0^2\right)$$

In the general case:

$$q_0^i = \theta q_0^E \left(B_0^i, B_0^{-i}\right) + (1-\theta)\, \tilde{q}_{0d}^i \left(B_0^i\right) \tag{4.7}$$

Using the price equations 4.5 and 4.6, for the individual bond and for the Eurobond, we get:

$$q_0^i = \frac{1 - F_1\left(\frac{B_{0d}^1}{\delta}\right)}{1+r^*} + \frac{\theta\left(1-\chi_1\right)\left(F_1\left(\frac{B_{0d}^1}{\delta}\right) - F_2\left(\frac{B_{0d}^2}{\delta}\right)\right)}{1+r^*}$$

When $\theta = 0$, sovereigns borrow as if they were alone. As long as $\theta > 0$, the effective price faced by the sovereign i depends on the equilibrium amount of debt issued by the other sovereign $-i$.

In the general case with n countries:

$$q_0^i = \frac{1 - \left(\left(1-\theta+\theta\chi_i\right) F_i\left(\frac{B_{0d}^i}{\delta}\right) + \theta \sum_j \chi_j F_j\left(\frac{B_{0d}^j}{\delta}\right)\right)}{1+r^*}$$

Reaction functions $B_0^i = g\left(B_0^{-i}\right)$ The sovereign i decides its public debt issuance by solving:

$$\max_{B_{0s}^i} J\left(B_{0s}^i\right)$$

subject to:

$$q_{0s}^i = \theta q_{0s}^E + (1-\theta)\,\tilde{q}_{0s}^i$$

It explicitly takes into account its price impact:

$$\begin{aligned} J\left(B_{0s}^i\right) \equiv\ & U\left(y_0^i + \theta q_{0s}^E\left(B_0^i, B_0^{-i}\right) + (1-\theta)\,\tilde{q}_{0s}^i\left(B_0^i\right) B_{0s}^i\right) \\ & + \beta^i\left[\int_{\underline{Y_1^i}}^{\frac{B_{0s}^i}{\delta}} U\left((1-\delta)\,Y_1^i\right) dF\left(Y_1^i\right) + \int_{\frac{B_{0s}^i}{\delta}}^{\overline{Y_1^i}} U\left(Y_1^i - B_{0s}^i\right) dF\left(Y_1^i\right)\right] \end{aligned} \tag{4.8}$$

Thus:

$$\frac{dJ}{dB_0} = U'\left(C_0\right)\left[\frac{dq_{0s}^i}{dB_{0s}^i} B_{0s}^i + q_{0s}^i\right] - \beta\int_{\frac{B_{0s}^i}{\delta}}^{\overline{Y_1^i}} U\left(Y_1^i - B_{0s}^i\right) dF\left(Y_1^i\right) = 0$$

By writing the marginal revenue $MR = \left[\frac{dq_{0s}^i}{dB_{0s}^i} B_{0s}^i + q_{0s}^i\right]$ we can write the first-order condition :

$$U'\left(y_0 + q_0 B_0\right) MR = \beta^i\int_{\frac{B_{0s}^i}{\delta}}^{\overline{y_1}} U'\left(Y_1^i - B_{0s}^i\right) dF\left(Y_1^i\right) \tag{4.9}$$

We successively derive:

$$\frac{dq_0^1}{dB_0^1} = -\frac{\left[(1-\theta)+\theta\chi_1\right]\frac{1}{\delta} f_1\left(\frac{B_{0d}^1}{\delta}\right) + \theta\left[\frac{\chi_2}{B_{0d}^1 + B_{0d}^2}\left(F_1\left(\frac{B_{0d}^1}{\delta}\right) - F_2\left(\frac{B_{0d}^2}{\delta}\right)\right)\right]}{1+r^*}$$

$$MR = \frac{1 - F_1\left(\frac{B_{0d}^1}{\delta}\right)}{1+r^*} - \frac{\left[(1-\theta)+\theta\chi_1\right]\frac{B_{0s}^1}{\delta} f_1\left(\frac{B_{0d}^1}{\delta}\right)}{1+r^*} + \frac{\theta\left[\left(1-\chi_1\left(2-\chi_1\right)\right)\left(F_1\left(\frac{B_{0d}^1}{\delta}\right) - F_2\left(\frac{B_{0d}^2}{\delta}\right)\right)\right]}{1+r^*}$$

The first-order condition characterizes the reaction function $B_0^i = g\left(B_0^{-i}\right)$. We are interested in the slope of the reaction function:

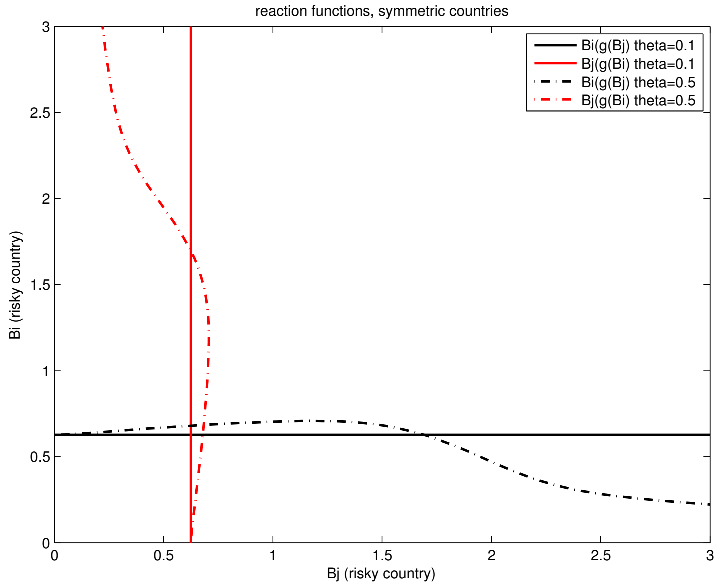

Figure 4.1: *Equilibrium of the issuance Cournot game with symmetric countries. Solid lines are reaction functions under low pooling ($\theta = 0.1$) and dashed lines are reaction functions under high pooling ($\theta = 0.5$).*

$$sign\left(\frac{dB_0^i}{dB_0^{-i}}\right) = U''\left(C_0\right)\underbrace{\frac{dq_0^i}{dB_0^{-i}}}_{<0} B_0^i MR + U'\left(C_0\right)\frac{dMR}{dB_0^{-i}}$$

Lemma 17. *If the pooling share is positive ($\theta > 0$) and the volatility σ^i of output low enough, then the reaction function $B_0^i = g\left(B_0^{-i}\right)$ is upward sloping.*

Lemma 2 formalizes the non-cooperative behavior between the issuance of the two countries. The equilibrium exhibits a free-riding externality which induces countries to overborrow.

4.3.1 Case of symmetric countries

In this case, in equilibrium $B_0^i = B_0^{-i}$, and by inspection of the first-order condition 4.9 we obtain the following proposition.

Proposition 12. *In the case of symmetric countries, individual sovereign borrowing increases with the share θ pooled in the Eurobond.*

Figures 4.1 illustrates the equilibrium with a CARA-normal specification:$U'(C_0) = -e^{-\gamma C_0}$ and $Y_1^i \sim N(\mu^i, \sigma^i)$. It plots the two reaction functions. The equilibrium, which is given by their intersection, moves towards higher levels of borrowing when the Eurobond is introduced.

4.3.2 Case of asymmetric countries

Assume that the output country i is riskier than the output of country $-i$, *ceteris paribus*: the volatilities of output are such that $\sigma^i > \sigma^{-i}$. In this case at the symmetric equilibrium, the marginal revenue for the "safe" country $-i$ than for the "risky" country i. A perturbation argument derives the following proposition.

Proposition 13. *In the case of asymmetric countries, increasing the pooling share θ leads to higher borrowing from the riskier country and to lower borrowing from the safe country.*

Figure 4.2 illustrates the equilibrium with the CARA-normal specification. One reaction function is now downward sloping around the initial symmetric equilibrium. It is explained by the lower marginal revenue for the safe country when the risky country increases the level of its borrowing.

4.4 Optimal level of sovereign debt pooling

We now solve for the social planner's problem. The planner has the ability to choose the pooling share θ and the tranching threshold ψ of the Eurobond. It has as instrument the *institutional* arrangement of how much to pool θ, which is decided ex-ante the actual borrowing choices by sovereigns. It optimizes the values of θ and ψ, subject to the equilibrium described above. We characterize the Pareto frontier in this economy.

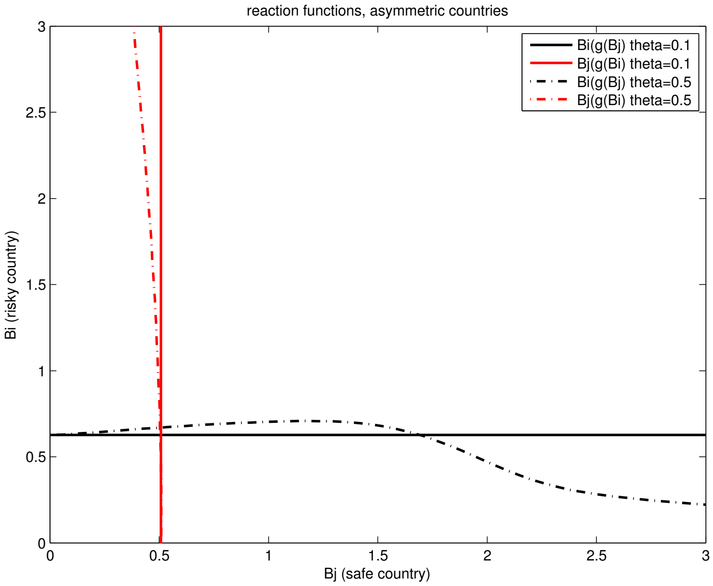

Figure 4.2: *Equilibrium of the issuance Cournot game with asymmetric countries: the output of country i is riskier than the output of country −i. Solid lines are reaction functions under low pooling ($\theta = 0.1$) and dashed lines are reaction functions under high pooling ($\theta = 0.5$).*

4.4.1 Social planner's problem

The planner maximizes a social welfare function with arbitrary weights λ^i. Denote W^i the indirect utility of sovereign i in equilibrium, the social welfare function is:

$$W(\theta) = \max_{\theta} \sum_i \lambda^i W^i$$

The planning problem on θ is:

$$W(\theta) = \max_{\theta} \sum_i \lambda^i \left\{ U\left(C_0^i\right) + \beta \mathbb{E}_0 \left[J\left(C_1^i\right) \right] \right\}$$

This maximization is subject to individual budget constraints (4.1) and (4.2) and to the optimal individual choices for B_0^i, determined by (4.9), and the default thresholds, determined by (4.3).

Substituting budget constraints and individual default decisions, we can write the planning problem as:

$$W\left(\theta\right)=\max_{\theta}\sum_{i}\lambda^{i}\left\{U\left(y_{0}^{i}+q_{0}^{i}B_{0}^{i}\right)+\beta\left[\int_{0}^{\frac{B_{0}^{i}}{\delta}}U\left(\left(1-\delta\right)Y_{1}^{i}\right)f\left(Y_{1}^{i}\right)dY_{1}^{i}+\int_{\frac{B_{0}^{i}}{\delta}}^{\overline{Y_{1}^{i}}}U\left(Y_{1}^{i}-B_{0}^{i}\right)f\left(Y_{1}^{i}\right)dY_{1}^{i}\right]\right\}$$

Subject to

$$q_{0}^{i}=q_{0}^{E}\left(B_{0}^{i},B_{0}^{-i}\right)\ i=\{1,2\}\qquad\text{and}\qquad B_{0}^{i}=g\left(B_{0}^{-i}\right)$$

The planner's optimality condition is given by:

$$\frac{dW}{d\theta}=\sum_{i}\lambda^{i}\left\{\begin{array}{c}U'\left(C_{0}^{i}\right)\left[q_{0}^{i}\frac{dB_{0}^{i}}{d\theta}+\frac{dq_{0}^{i}}{d\theta}B_{0}^{i}\right]\\-\beta\left[\int_{\frac{B_{0}^{i}}{\delta}}^{\overline{Y_{1}^{i}}}U'\left(C_{1}^{i}\right)f\left(Y_{1}^{i}\right)dY_{1}^{i}\right]\frac{dB_{0}^{i}}{d\theta}\end{array}\right\}=0$$

By the envelope theorem, we can substitute the first order condition 4.4 for B_0^i of each agent (which takes the θ as given) in the above equation and find:

$$\begin{aligned}0=\frac{dW}{d\theta}&=\sum_{i}\lambda^{i}\left\{\left[U'\left(C_{0}^{i}\right)q_{0}^{i}-\beta\left[\int_{\frac{B_{0}^{i}}{\delta}}^{\overline{Y_{1}^{i}}}U'\left(C_{1}^{i}\right)f\left(Y_{1}^{i}\right)dY_{1}^{i}\right]\right]\frac{dB_{0}^{i}}{d\theta}+U'\left(C_{0}^{i}\right)\frac{dq_{0}^{i}}{d\theta}B_{0}^{i}\right\}\\&=\sum_{i}\lambda^{i}\left\{\left[-U'\left(C_{0}^{i}\right)\left.\frac{dq_{0}^{i}}{dB_{0}^{i}}\right|_{i}B_{0}^{i}\right]\frac{dB_{0}^{i}}{d\theta}+U'\left(C_{0}^{i}\right)\frac{dq_{0}^{i}}{d\theta}B_{0}^{i}\right\}\\&=\sum_{i}\lambda^{i}U'\left(C_{0}^{i}\right)B_{0}^{i}\left\{\left[-\left.\frac{dq_{0}^{i}}{dB_{0}^{i}}\right|_{i}\right]\frac{dB_{0}^{i}}{d\theta}+\frac{dq_{0}^{i}}{d\theta}\right\}\end{aligned}$$

$\left.\frac{dq_0^i}{dB_0^i}\right|_i$ is the perceived change in borrowing conditions by country i when choosing how much to borrow. $\frac{dB_0^i}{d\theta}$ is the equilibrium response of B_0^i to movements in θ, which is a function in principle of changes in prices too. $\frac{dq_0^i}{d\theta}$ is the equilibrium change in prices of debt issued by a country i; it has the component of the EuroBond and the component of the individual bond. We can thus sign the expression we just found:

$$\frac{dW}{d\theta} = \sum_i \underbrace{\lambda^i U'\left(C_0^i\right) B_0^i}_{>0} \left\{ \underbrace{\left[- \left. \frac{dq_0^i}{dB_0^i} \right|_i \right]}_{<0} \underbrace{\frac{dB_0^i}{d\theta}}_{= \underbrace{\frac{\partial B_0^i}{\partial q_0^i}}_{} \frac{dq_0^i}{d\theta} \gtreqless 0} + \underbrace{\frac{dq_0^i}{d\theta}}_{\gtreqless 0} \right\} \tag{4.10}$$

$$\frac{dW}{d\theta} = \sum_i \underbrace{\lambda^i U'\left(C_0^i\right) B_0^i q_0^i \frac{dq_0^i/q_0^i}{d\theta}}_{weight} \left[1 + \underbrace{\left[- \left. \frac{dq_0^i}{dB_0^i} \right|_i \right]}_{<0} \underbrace{\frac{\partial B_0^i}{\partial q_0^i}}_{>0} \right] = 0$$

The weight term has a leverage term times an elasticity. The planner tries to optimize the price movements. We have shown that there is an interior solution to this problem. In the general case, $\frac{dq_0^i/q_0^i}{d\theta}$ is different for different countries. Increasing θ can make borrowing cheaper for some countries and more expensive for others.

4.4.2 Welfare with symmetric countries

In the case of symmetric countries, as we have $B_0^i = B_0^{-i}$ in equilibrium, the marginal revenue expression is reduced to:

$$MR = \frac{1}{1+r^*} \left[1 - F_1 \left(\frac{B_{0d}^1}{\delta} \right) - \left(1 - \frac{\theta}{2} \right) \frac{B_{0s}^1}{\delta} f_1 \left(\frac{B_{0d}^1}{\delta} \right) \right]$$

For low levels of borrowing, the first term dominates. By increasing the pooling share θ, the level of borrowing increases, due to the negative free-riding externality in debt issuance that makes countries overborrow. The marginal revenue decreases. As a consequence, for each of the symmetric countries, the indirect utility of the country decreases when pooling is introduced: $\frac{dW^i}{d\theta}|_{\theta=0} < 0$. This is due to the It results in sovereign bonds that are riskier, hence a lower bond price and welfare. The following proposition is a direct consequence.

Proposition 14. *With symmetric countries and no exogenous safety demand, introducing Eurobonds* *is welfare destroying. The optimal share of pooling is zero.*

4.4.3 Welfare with asymmetric countries

We analyze the case where the output of one country is riskier the output of the other countries: $\sigma^1 > \sigma^2$ and $\mu^1 < \mu^2$. In this case, from the marginal revenue expression:

$$MR = \frac{1 - F_1\left(\frac{B^1_{0d}}{\delta}\right)}{1+r^*} - \frac{\left[(1-\theta) + \theta\chi_1\right] \frac{B^1_{0s}}{\delta} f_1\left(\frac{B^1_{0d}}{\delta}\right)}{1+r^*} + \frac{\theta\left[\left(1-\chi_1\left(2-\chi_1\right)\right)\left(F_1\left(\frac{B^1_{0d}}{\delta}\right) - F_2\left(\frac{B^2_{0d}}{\delta}\right)\right)\right]}{1+r^*}$$

The last term captures the cross-subsidy from the safer country to the riskier country. As this term increases with the level of pooling, it conveys the intuition of the following proposition.

Proposition 15. *When countries are asymmetric, increasing the share of pooling increases the borrowing and the welfare of the risky country, whereas it decreases the borrowing and the welfare of the safe country.*

In other words, the joint-liability scheme of the Eurobond implements a transfer: $\frac{dW^1}{d\theta}|_{\theta=0} > 0$ and $\frac{dW^2}{d\theta}|_{\theta=0} < 0$. The following figure traces the allocations that can be implemented with different shares θ of pooling. The specification is CARA-normal. As this Pareto frontier is downward sloping, no Pareto improvement can be achieved with the Eurobond instrument.

Case of a utilitarian welfare function We can derive an analytical solution for the optimal pooling threshold as a function of the Pareto weights λ^i. We characterize here the solution in the case of a utilitarian social welfare function. When the terms that sovereigns internalize

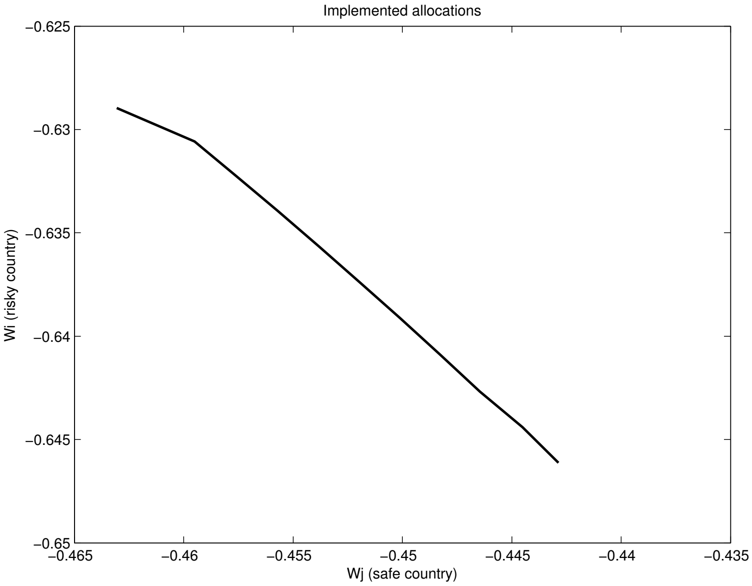

Figure 4.3: *Pareto frontier: allocations that can be implemented by the Eurobond, according to the share of pooling θ.*

are small, we can write, after canceling terms:

$$\begin{aligned}\frac{dW}{d\theta} &= \sum_i \lambda_i U'\left(C_0^i\right) B_0^i \left\{\theta \frac{dq_0^{JL}}{d\theta} + (1-\theta)\frac{dq_0^i}{d\theta} + q_0^E - \tilde{q}_0^i\right\} \\ &= \theta \frac{dq_0^E}{d\theta} \sum_i \lambda_i U'\left(C_0^i\right) B_0^i + (1-\theta) \sum_i \lambda_i U'\left(C_0^i\right) B_0^i \frac{dq_0^i}{d\theta} + (\theta + (1-\theta)) \sum_i \lambda_i U'\left(C_0^i\right) B_0^i \left\{q_0^E - \tilde{q}_0^i\right\} \\ &= \theta \frac{dq_0^E}{d\theta} \sum_i U'\left(C_0^i\right) B_0^i + (1-\theta) \sum_i U'\left(C_0^i\right) B_0^i \frac{dq_0^i}{d\theta}\end{aligned}$$

Where we have assumed that $\lambda_i = 1$ (utilitarian social function), to derive the third line. Setting $\frac{dW}{d\theta} = 0$, we find the following proposition under standard assumptions (Inada conditions).

Proposition 16. *A utilitarian social planner sets an interior pooling share θ^* by satisfying the following equation:*

$$\frac{\theta^*}{1-\theta^*} = \frac{\sum_i U'\left(C_0^i\right) B_0^i \left\{\frac{dq_0^i}{d\theta} + \left(q_0^E - \tilde{q}_0^i\right)\right\}}{\left(-\frac{dq_0^E}{d\theta} - \left(q_0^E - \tilde{q}_0^i\right)\right) \sum_i U'\left(C_0^i\right) B_0^i}$$

This magnitude is between 0 and 1. Note that $\frac{\theta}{1-\theta} \approx \theta$ when θ is small. We will have to show that $\frac{dq_0^E}{d\theta} < 0$ and that $\frac{dq_0^i}{d\theta} > 0$. The denominator captures the aggregate loss and the numerator captures the individual change. Intuitively, the planner wants to tradeoff the gains derived from pooling more (risk-sharing transfer) with the losses from the free-riding externality.

When the utilities are logarithms, we can simplify further the expression for the optimal share of pooling θ^*. The budget constraints is $C_0^i = y_0^i + \left(\theta q_0^{JL} + (1-\theta) q_0^i\right) B_0^i$ (in this formulation y_0 should be seen as disposable income). Working from the equality: $1 = \frac{y_0^i}{C_0^i} + \theta \frac{q_0^{JL} B_0^i}{C_0^i} + (1-\theta) \frac{q_0^i B_0^i}{C_0^i} \Rightarrow 1 = \frac{y_0^i}{C_0^i} + \theta s_i^{JL} + (1-\theta) s_i$, and introducing the elasticity $\varepsilon_\theta^{q_0^{JL}} = \frac{dq_0^{JL}/q_0^{JL}}{d\theta}$, we find that:

$$\frac{\theta^*}{1-\theta^*} = \frac{\sum_i \frac{q_0^i B_0^i}{C_0^i} \frac{dq_0^i/q_0^i}{d\theta}}{\left(-\frac{dq_0^{JL}/q_0^{JL}}{d\theta}\right) \sum_i \frac{q_0^{JL} B_0^i}{C_0^i}} = \frac{\sum_i s_i \varepsilon_\theta^{q_0^i}}{-\varepsilon_\theta^{q_0^{JL}} \sum_i s_i^{JL}}$$

The intuition for the results is as follows. When $\varepsilon_\theta^{q_0^{JL}}$ is very low, that is when markets are willing to accept a large amount of Eurobonds without moving greatly interest rates, this is the optimal policy to be done. When $\theta = 1$ we have too expensive joint liabilities, when $\theta = 0$ we are not doing any risk sharing. Both are suboptimal, so there is a trade-off from the planner's perspective. On the one hand, increasing pooling enhances risk-sharing between the countries. On the other hand, it also exacerbates the pooling externality, i.e. the fact that when an individual sovereign borrows an extra unit, there are states of the joint distribution in which he will not incur the loss, so the Eurobond is 'mispriced', by incentivizing the individual sovereigns to borrow too much: $B^* > B^{SP}$. The latter can be seen as moral hazard. Given the timeline, the pooling level is chosen ex ante the actual debt issuance by individual sovereigns.

4.5 Welfare with exogenous safety demand

We now assume that agents have access to a tranching technology. Any liability promising B can be split in two securities: one part ψB is senior to the second part $(1-\psi) B$. This subordination ensures that the payoff on the senior part is $\psi B \, 1_{\{Y<\psi B\}}$ where Y is the cash-flow collateralizing the liability promise B.

Furthermore we assume that there is an exogenous safety demand in the economy. It can be microfounded by exogenous lenders that are more risk-averse than the international (risk-neutral) lenders currently pricing the public debt. This assumption implies that, in a reduced-form formulation, any liability ψB such that $\mathbb{E}\left[1_{\{Y<\psi B\}}\right] < \alpha$ enjoys a *safety premium* π (see (Stein, 2010) and Weymuller (2013a) for a microfoundation). Compared to the case of risk-neutral lenders pricing the given liability at q, the presence of this exogenous safety demand enhances the price of the liability to $q + \pi$.

Assumption 4. *Each asset that defaults with probability less than α is priced with an extra premium π.*

4.5.1 Bond pricing with safety demand

We analyze the pricing of a two-country Eurobond in this environment. Without safety demand, section 2.3 showed that:

$$q_0^E = 1 - \chi_1 F_1\left(\frac{B_{0d}^1}{\delta}\right) - \chi_2 F_2\left(\frac{B_{0d}^2}{\delta}\right)$$

Now the *correlation structure* of income shocks matters in the pricing of Eurobonds. The state-contingent payoff on the total Eurobond is:

$$EB\left(\left\{y_1^i\right\}_i\right) = \theta \sum_i B_0^i 1_{\left\{y_1^i \leq \frac{B_{0s}^i}{\delta}\right\}}$$

Figure 4.4 illustrates the state-contingent payoffs of Eurobonds. When incomes are too low in both sovereigns, both default. When one does well but not the other, that one defaults. When both sovereigns do fine, there is no default. The correlation structure of the income shocks will determine whether we are more probably in quadrants 1 and 3 (positive correlation) or more in 2 and 4 (negative correlation).

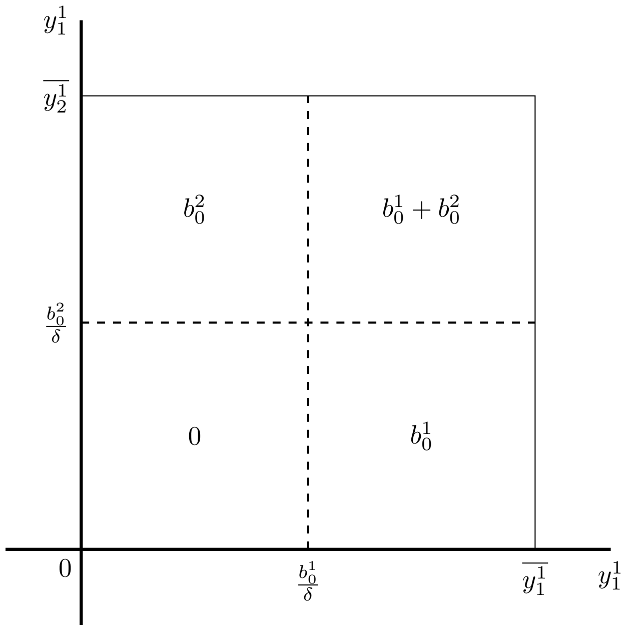

Figure 4.4: *Eurobond payoff: recovery rates as a function of the joint distribution*

Given Assumption 2, now the correlation between the two shocks in the pricing of the Eurobond. Indeed, assume that the parameters are such that the safety premium will be granted to the Eurobond if and only if the probability of ending up in the bottom-left quadrant is low enough:

$$\mathbb{P}\left[y_1^1 < \frac{B_{0d}^1}{\delta}, y_1^2 < \frac{B_{0d}^2}{\delta}\right] < \alpha$$

Thus the pricing of Eurobonds requires to keep track of the exact correlation structure. We consider the case of multivariate normal shocks, which enables us to introduce *copulas* as a useful pricing tool. The multivariate normal copula is:

$$C\left(u, v; \rho\right) = \Phi^2\left(\Phi^{-1}\left(u\right), \Phi^{-1}\left(v\right); \rho\right)$$

The appendix derives the closed-form expression for $\mathbb{P}\left[y_1^1 < \frac{B_{0d}^1}{\delta}, y_1^2 < \frac{B_{0d}^2}{\delta}\right]$ for symmetric countries, denoting $a = \frac{\frac{B^*}{\delta} - \mu}{\sigma}$:

$$\mathbb{P}\left[y_1^1 \geq \frac{B^*}{\delta}, y_1^2 \geq \frac{B^*}{\delta}\right] =$$

$$\frac{1}{2\pi\sigma\sqrt{1-\rho^2}}\left\{\frac{1}{2}\left[1-\Phi\left(a\right)^2\right]+\Delta_\sigma\phi\left(a\right)^2+\left\{\mu\Delta_\sigma-\Delta_\mu\right\}\frac{1}{\sigma\sqrt{2\pi}}\left[1-\Phi\left(\frac{a}{\sqrt{2}}\right)\right]\right\}$$

It leads to the following lemma.

Lemma 18. *The Eurobond price depends on the correlation ρ between the two assets:*

$$q_0^E = g\left(B^*;\rho,\theta\right) = 1-\chi_1 F_1\left(\frac{B_{0d}^1}{\delta}\right)-\chi_2 F_2\left(\frac{B_{0d}^2}{\delta}\right)+\pi\;1_{\left\{\mathbb{P}\left[y_1^1<\frac{B_{0d}^1}{\delta},y_1^2<\frac{B_{0d}^2}{\delta}\right]<\alpha\right\}}$$

Lemma 2 underpins how Eurobonds can create value by overturning Modigliani-Miller: the price of the Eurobond is not equal to the synthetic portfolio of individual bonds replicating the Eurobond payoff. Two benchmarks are given by the following cases: (i) when shocks are independent; (ii) when all countries are ex-ante symmetric. The symmetry of the distribution of shocks implies: $f\left(y_1^1,y_1^2\right)=f\left(y_1^2,y_1^1\right)$. Ex ante debt policy choices are equal: $B_0^1=B_0^2=B^*$.

In the case of symmetric countries, Proposition 3 delivered the negative result that Eurobonds cannot increase welfare. Now the marginal revenue is:

$$\begin{aligned} MR = \frac{1}{1+r^*}&\left[1-F_1\left(\frac{B_{0d}^1}{\delta}\right)-\left(1-\frac{\theta}{2}\right)\frac{B_{0s}^1}{\delta}f_1\left(\frac{B_{0d}^1}{\delta}\right)\right] \\ &+\psi\theta\pi\;1_{\left\{\mathbb{P}\left[y_1^1<\frac{\psi B_{0d}^1}{\delta},y_1^2<\frac{\psi B_{0d}^2}{\delta}\right]<\alpha\right\}}+\left(1-\theta\right)1_{\left\{\mathbb{P}\left[y_1^1<\frac{B_{0d}^1}{\delta}\right]<\alpha\right\}} \end{aligned} \tag{4.11}$$

The sum of the last two terms is strictly increasing with θ as long as:

$$1_{\left\{\mathbb{P}\left[y_1^1<\frac{\psi B_{0d}^1}{\delta},y_1^2<\frac{\psi B_{0d}^2}{\delta}\right]<\alpha\right\}}>1_{\left\{\mathbb{P}\left[y_1^1<\frac{B_{0d}^1}{\delta}\right]<\alpha\right\}}$$

which can be the case only if the country does not enjoy the safety premium individually. It yields to the following proposition.

Proposition 17. *When there is an exogenous safety demand, Eurobond pooling can achieve a Pareto improvement.*

It should be observed that the constructive case that shows the Pareto improvement relies on the two country being risky enough individually in order not to enjoy the safety premium individually. If one country is safe enough to enjoy the safety premium on its individual bond, the financial innovation of Eurobonds cannot increase welfare for this country directly. Nevertheless, in an enriched model with production and open economy consumption, a wealth channel can lead to such Pareto improvement. In this case, the mechanism is as follows. Thanks to Eurobonds, sovereign debt of the risky country gains access to the safe asset status. It leads to a wealth inflow in the risky country, which comes from the exogenous investors. In a second time, the safe country benefits from this exogenous wealth inflow through a terms-of-trade channel on its export goods.

4.6 Conclusion

We analyze the problem of a social planner that has the possibility to force sovereign to pool a share of their debt before borrowing choices. We show that there exists an interior optimal level of pooling to achieve any desired transfer on the Pareto frontier. This optimal level trades-off the risk-sharing benefits of the Eurobond with its cost. The cost of the Eurobond comes from a pooling negative externality that leads to overborrowing.

The optimal pooling threshold can be calibrated on observable market data. The analytical expression only involves elasticities ($\varepsilon_\theta^{q_0^{JL}}$ and $\varepsilon_\theta^{q_0^i}$). The two can be inferred in the European case. Indeed, before the sovereign debt crisis, sovereign debt of the Eurozone was priced jointly, effectively as a Eurobond. Since the crisis, the fragmentation of sovereign debt markets leads to the reversion to a pricing of sovereign bonds on an individual basis.

The Eurobond issuance is a joint arrangement which is tantamount to the two countries entering ex-ante a contract. A natural extension is concerned with the dynamic implications. In a dynamic environment, the contract could be self-enforced, in the same way as in Kocherlakota (1996). The agreement will be self-enforced if and only if the participation constraints for the two countries are satisfied at each period and each state of the world. This introduces additional participation constraints in the program of the social planner. Dynamics would enhance the desirable properties of Eurobonds. Indeed, regarding the effect of voluntary participation on the optimal design of Eurobonds, the safe country will decide to stay in the Eurobonds despite the risky country free-riding on its signature because with some probability this safe country will become the risky country at the next period. A related issue is the time inconsistency in the choice of the optimal pooling share θ. As shown in the above section, θ depends on the conditional correlation structure, which faces innovations as long as shocks are persistent. It incentivizes countries to renegotiate the ex-ante agreement.

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Appendix A

Appendix to Chapter 1

A.1 Theory appendix

A.1.1 4-states example

Consider four equally plausible states. Assume that a $t = 1$ risky payoff for the technology and consider a public security that is imperfectly negatively correlated with the technology:

$$X_1 = s_K = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} \text{ and } X_2 = s_B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 3 \end{bmatrix}$$

Conjecture an equilibrium in which risk-neutral banks sell securities with promise 1 to mean-variance risk-averse investors, and where the bank is pushed in default if and only if one the two lowest states realize. Introduce the conditional payoffs introduced by default:

$$X_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } X_4 = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \tilde{X}_4 = \begin{bmatrix} 4 \\ 3 \\ 0 \\ 0 \end{bmatrix} \text{ and } X_5 = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 3 \end{bmatrix}, \tilde{X}_5 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

In this equilibrium, the bank solves the following program:

$$\begin{aligned} \underset{\{x_K^A, x_B^A, y^A\}}{Max} \; W^A &= \mathbb{E}_0\{x_K^A \tilde{X}_4 + x_B^A \tilde{X}_5 - y^A X_3\} \\ s.t. \quad & x_K^A c_K + x_B^A p_B \leq n^A + y^A q \end{aligned}$$

The three first-order conditions of the bank leads to, denoting λ the Lagrange multiplier on their budget:

$$\lambda = \frac{\mathbb{E}_0\left[\tilde{X}_4\right]}{c_K} = \frac{\mathbb{E}_0\left[\tilde{X}_5\right]}{p_B} = \frac{\mathbb{E}_0\left[X_3\right]}{q}$$

which gives $p_B = \frac{\mathbb{E}_0\left[\tilde{X}_5\right]}{\mathbb{E}_0\left[\tilde{X}_4\right]} c_K$ and $q = \frac{\mathbb{E}_0\left[X_3\right]}{\mathbb{E}_0\left[\tilde{X}_4\right]} c_K$. The mean-variance investor P solves:

$$\begin{aligned} \underset{\{x_K^P, x_B^P, y^P\}}{Max} \; W^P &= L_{\mathbb{E}_0 - \gamma^P \mathbb{V}_0}\left[x_K^P X_1 + x_B^P X_2 + y^P X_3 + x_K^A X_4 + x_B^A X_5\right] \\ s.t. \quad & x_K^P c_K + x_B^P p_B + y^P q \leq n^P \end{aligned}$$

The three first-order conditions of the investor leads to, denoting μ the Lagrange multiplier on their budget and Σ^{ij} the covariance matrix of $\{X_i\}$:

- $c_K \mu = \mathbb{E}_0\left[X_1\right] - 2\gamma^P\left(\Sigma^{11} x_K^P + \Sigma^{12} x_B^P + \Sigma^{13} y^P + \Sigma^{14} x_K^A + \Sigma^{15} x_B^A\right)$
- $p_B \mu = \mathbb{E}_0\left[X_2\right] - 2\gamma^P\left(\Sigma^{21} x_K^P + \Sigma^{22} x_B^P + \Sigma^{23} y^P + \Sigma^{24} x_K^A + \Sigma^{25} x_B^A\right)$
- $q \mu = \mathbb{E}_0\left[X_3\right] - 2\gamma^P\left(\Sigma^{31} x_K^P + \Sigma^{32} x_B^P + \Sigma^{33} y^P + \Sigma^{34} x_K^A + \Sigma^{35} x_B^A\right)$

The market clearings for public debt and private debt are:

$$x_B^A + x_B^A = B \text{ and } y^A = y^P$$

Denote $\Pi = \left[x_K^P; x_B^P; y^P; x_K^A; x_B^A; \mu\right]'$ the vector of equilibrium portfolios and:

$$R = \left[\mathbb{E}_0\left[X_1\right]; \mathbb{E}_0\left[X_2\right]; \mathbb{E}_0\left[X_3\right]; n^P; n^A; B\right]'$$

The equilibrium is characterized by the following linear system:

$$A\Pi = R$$

$$A = \begin{bmatrix} 2\gamma^P\Sigma^{11} & 2\gamma^P\Sigma^{12} & 2\gamma^P\Sigma^{13} & 2\gamma^P\Sigma^{14} & 2\gamma^P\Sigma^{15} & c_K \\ 2\gamma^P\Sigma^{21} & 2\gamma^P\Sigma^{22} & 2\gamma^P\Sigma^{23} & 2\gamma^P\Sigma^{24} & 2\gamma^P\Sigma^{25} & p_B \\ 2\gamma^P\Sigma^{31} & 2\gamma^P\Sigma^{32} & 2\gamma^P\Sigma^{33} & 2\gamma^P\Sigma^{34} & 2\gamma^P\Sigma^{35} & q \\ c_K & p_B & q & 0 & 0 & 0 \\ 0 & 0 & -q & c_K & p_B & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Applying Cramer's rule, developing along the third column, plugging the prices, γ^P and c_K cancel out by multilinearity in the derivative with respect to B. Finally we obtain:

$$\frac{\partial y}{\partial B} = -\frac{\begin{vmatrix} \Sigma^{11} & \Sigma^{12} & \Sigma^{14} & \Sigma^{15} & 1 \\ \Sigma^{21} & \Sigma^{22} & \Sigma^{24} & \Sigma^{25} & \frac{\mathbb{E}_0\left[\tilde{X}_5\right]}{\mathbb{E}_0\left[\tilde{X}_4\right]} \\ \Sigma^{31} & \Sigma^{32} & \Sigma^{34} & \Sigma^{35} & \frac{\mathbb{E}_0\left[\tilde{X}_3\right]}{\mathbb{E}_0\left[\tilde{X}_4\right]} \\ 1 & \frac{\mathbb{E}_0\left[\tilde{X}_5\right]}{\mathbb{E}_0\left[\tilde{X}_4\right]} & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{\mathbb{E}_0\left[\tilde{X}_5\right]}{\mathbb{E}_0\left[\tilde{X}_4\right]} & 0 \end{vmatrix}}{\begin{vmatrix} \Sigma^{11} & \Sigma^{12} & \Sigma^{13} & \Sigma^{14} & \Sigma^{15} & 1 \\ \Sigma^{21} & \Sigma^{22} & \Sigma^{23} & \Sigma^{24} & \Sigma^{25} & \frac{\mathbb{E}_0\left[\tilde{X}_5\right]}{\mathbb{E}_0\left[\tilde{X}_4\right]} \\ \Sigma^{31} & \Sigma^{32} & \Sigma^{33} & \Sigma^{34} & \Sigma^{35} & \frac{\mathbb{E}_0\left[\tilde{X}_3\right]}{\mathbb{E}_0\left[\tilde{X}_4\right]} \\ 1 & \frac{\mathbb{E}_0\left[\tilde{X}_5\right]}{\mathbb{E}_0\left[\tilde{X}_4\right]} & \frac{\mathbb{E}_0\left[\tilde{X}_3\right]}{\mathbb{E}_0\left[\tilde{X}_4\right]} & 0 & 0 & 0 \\ 0 & 0 & -\frac{\mathbb{E}_0\left[\tilde{X}_3\right]}{\mathbb{E}_0\left[\tilde{X}_4\right]} & 1 & \frac{\mathbb{E}_0\left[\tilde{X}_5\right]}{\mathbb{E}_0\left[\tilde{X}_4\right]} & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{vmatrix}}$$

With the given correlation structure and denoting $D = yq$, this example features:

$$\frac{\partial D}{\partial B} > 0$$

A.1.2 Intragenerational equilibrium

Consider the middle-aged generation risk-sharing problem and the generic asset correlation structure as multivariate normal:

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}\right)$$

The portfolio of the borrower is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, the lender's is $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ and the promise of the debt contract is $\bar{s}$. Introduce the following auxiliary variables: $u = X'S - \bar{s}$ and $v = Y'S + \bar{s}$.

$$\begin{bmatrix} u \\ v \end{bmatrix} \sim N\left(\begin{bmatrix} X'\mu - \bar{s} \\ Y'\mu + \bar{s} \end{bmatrix}, \begin{bmatrix} X\Sigma X' \\ Y\Sigma Y' \end{bmatrix}\right)$$

Borrower With the change of variable: $u = X'S - \int x_{\bar{s}}\bar{s}$, we have: $u \sim N\left(\mu_u, \sigma_u^2\right)$ with $\mu_u(x_1, x_2, \bar{s}) = x_1\mu_1 + x_2\mu_2 - \int x_{\bar{s}}\bar{s}$ and $\sigma_u^2(x_1, x_2) = x_1^2\sigma_1^2 + 2\rho x_1 x_2\sigma_1\sigma_2 + x_2^2\sigma_2^2$. Denote Φ and ϕ respectively the density and the cdf of the standard normal distribution. We can write, using the truncated moment generating function for the normal distribution[1,2]:

$$\begin{aligned} W^B &= \mathbb{E}_0\left[\left(X'S - \int x_{\bar{s}}\bar{s}\right) 1_{\{X'S \geq \int x_{\bar{s}}\bar{s}\}}\right] \\ W^B &= \mu_u \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \sigma_u \phi\left(\frac{\mu_u}{\sigma_u}\right) \end{aligned}$$

Lender For the investor, the derivation is more cumbersome in order to take explicitly into account default. With the same change of variable $u = X'S - \int x_{\bar{s}}\bar{s}$ and $v = Y'S + \int y_{\bar{s}}\bar{s}$:

---

[1] We have: $\int_{a_1}^{a_2} y f_{\mu,\sigma^2}(y)dy = \frac{d}{dt}\left(e^{\mu t + \frac{1}{2}\sigma^2 t^2}\left\{\Phi\left(\frac{a_2-\mu}{\sigma} - \sigma t\right) - \Phi\left(\frac{a_1-\mu}{\sigma} - \sigma t\right)\right\}\right)|_{t=0}$

[2] In the general case of a CARA-γ_B utility for borrower is (risk-neutrality is recovered in the neighborhood $\gamma_B \sim 0$):

$$W^B = -\left\{1 - \Phi\left(\frac{\mu_u}{\sigma_u}\right)\right\} - e^{-\gamma_B\mu_u + \frac{1}{2}\gamma_B^2\sigma_u^2}\Phi\left(\frac{\mu_u}{\sigma_u} - \sigma_u\gamma_B\right)$$

$$
\begin{aligned}
W^{L} &= -\mathbb{E}_0\left[e^{-\gamma_L(Y'S+X'S)}1_{\{X'S<\int x_{\bar{s}}\bar{s}\}}+e^{-\gamma_L(Y'S+y_{\bar{s}}\bar{s})}1_{\{X'S\geq\int x_{\bar{s}}\bar{s}\}}\right]\\
&= \int_{-\infty}^{0}\left(\int_{-\infty}^{+\infty}-e^{-\gamma_L v}f_{M_{u,v},\Sigma_{u,v}}(v|u)dv\right)e^{-\gamma_L u}f(u)du\\
&\quad+\int_{0}^{+\infty}\left(\int_{-\infty}^{+\infty}-e^{-\gamma_L v}f_{M_{u,v},\Sigma_{u,v}}(v|u)dv\right)f(u)du\\
&= W^{L\,def}+W^{L\,no\,def}
\end{aligned}
$$

We have $M_{u,v}=\begin{bmatrix} x_1\mu_1+x_2\mu_2-\bar{s} \\ y_1\mu_1+y_2\mu_2+\bar{s}\end{bmatrix}$ and $\Sigma_{u,v}=\begin{bmatrix}\sigma_u^2 & \rho_{uv}\sigma_u\sigma_v \\ \rho_{uv}\sigma_u\sigma_v & \sigma_v^2\end{bmatrix}$ with:

$$
\begin{aligned}
\sigma_u^2 &= x_1^2\sigma_1^2+2\rho x_1x_2\sigma_1\sigma_2+x_2^2\sigma_2^2\\
\sigma_v^2 &= y_1^2\sigma_1^2+2\rho y_1y_2\sigma_1\sigma_2+y_2^2\sigma_2^2\\
\rho_{uv} &= \frac{x_1y_1\sigma_1^2+\rho\left(x_1y_2+x_2y_1\right)\sigma_1\sigma_2+x_2y_2\sigma_2^2}{\sqrt{\left(x_1^2\sigma_1^2+2\rho x_1x_2\sigma_1\sigma_2+x_2^2\sigma_2^2\right)\left(y_1^2\sigma_1^2+2\rho y_1y_2\sigma_1\sigma_2+y_2^2\sigma_2^2\right)}}
\end{aligned}
$$

Using the Moment Generating Function of the normal distribution:

$$
-\int_{-\infty}^{+\infty}e^{-\gamma_L v}f_{M_{u,v},\Sigma_{u,v}}(v|u)dv = -e^{-\gamma_L\left(\mu_v-\frac{\rho_{uv}\sigma_v}{\sigma_u}\mu_u\right)+\frac{1}{2}\gamma_L^2\left(1-\rho_{uv}^2\right)\sigma_v^2-\gamma_L\frac{\rho_{uv}\sigma_v}{\sigma_u}u}
$$

The value function of the lender can be expressed as:

$$
\begin{aligned}
&W^{L}=-e^{-\gamma_L\mu_v+\frac{1}{2}\gamma_L^2\sigma_v^2}\\
&\qquad\left\{e^{-\gamma_L\mu_u+\frac{1}{2}\gamma_L^2\left(\sigma_u^2+2\rho_{uv}\sigma_v\sigma_u\right)}\left\{1-\Phi\left(\frac{\mu_u}{\sigma_u}-\gamma_L\left(\rho_{uv}\sigma_v+\sigma_u\right)\right)\right\}+\Phi\left(\frac{\mu_u}{\sigma_u}-\gamma_L\rho_{uv}\sigma_v\right)\right\}
\end{aligned}
$$

Optimality conditions

Bank maximization We write their Lagrangian:

$$
\underset{\{X,\{x_{\bar{s}}\}\}}{Max}\;L^{B}=\mathbb{E}_0\left[\left(X'S-\int x_{\bar{s}}\bar{s}\right)1_{\{X'S\geq\int x_{\bar{s}}\bar{s}\}}\right]+\lambda\left[n^{B}+\int x_{\bar{s}}q_{\bar{s}}-X'P\right]
$$

Its program takes the price of debt securities $q_{\bar{s}}$. As all securities yield the same interest rate, the rate is the price: $r^{bank} = \frac{\bar{s}}{q_{\bar{s}}}$ and we can write, denoting $D = \int x_{\bar{s}} q_{\bar{s}}$ and $\bar{S} = \int x_{\bar{s}} \bar{s}$: $D = \bar{S} \frac{1}{r^{bank}}$. So the program can be written:

$$\underset{\{X, \{x_{\bar{s}}\}\}}{Max} \; L^B = \mathbb{E}_0 \left[\left(X'S - \bar{S} \right) 1_{\{X'S \geq \bar{S}\}} \right] + \lambda \left[n^B + \bar{S} \frac{1}{r^{bank}} - X'P \right]$$

And the bank f.o.c are: $\frac{dL}{d\bar{S}} = 0 = \frac{\partial W^B}{\partial \bar{s}} + \lambda \frac{1}{r^{bank}}$ and $\frac{dL}{dX} = 0 = \frac{\partial W^B}{\partial x_i} - \lambda p_i$.

Investor maximization Their Lagrangian is:

$$L = -\mathbb{E}_0 \left[e^{-\gamma_L (Y'S + X'S)} 1_{\{X'S < \int x_{\bar{s}} \bar{s}\}} + e^{-\gamma_L (Y'S + y_{\bar{s}} \bar{s})} 1_{\{X'S \geq \int x_{\bar{s}} \bar{s}\}} \right] + \mu \left[n^L - \int y_{\bar{s}} q_{\bar{s}} - Y'P \right]$$

So investor f.o.c are: $\frac{dL}{d\bar{s}} = 0 = \frac{\partial W^L}{\partial \bar{s}} - \mu D'(\bar{s})$ and $\frac{dL}{dY} = 0 = \frac{\partial W^L}{\partial y_i} - \mu p_i$

Introduce the borrower Marginal Rate of Substitution in the portfolio choice from Asset 1 to Asset 2:

$$MRS_B = \frac{\frac{\partial W^B}{\partial x_2}}{\frac{\partial W^B}{\partial x_1}}$$

The Marginal Rate of Transformation from Asset 1 to private contract (promise $\bar{s}$):

$$MRT_B = \frac{\frac{\partial W^B}{\partial \bar{s}}}{\frac{\partial W^B}{\partial x_1}}$$

The lender Marginal Rate of Substitution from Asset 2 to private contract (promise $\bar{s}$):

$$MRS_L = \frac{\frac{\partial W^L}{\partial \bar{s}}}{\frac{\partial W^L}{\partial y_2}}$$

The first-order conditions can be expressed in terms of Marginal Rates of Substitution and of Transformation. In the case of limited participation of risk-averse agents in the risky technology[3], the equilibrium is characterized by the following five equations:

---

[3] This assumption follows Basak and Cuoco (1998) and Cuoco and Kaniel (2011). The general case is solved in the same way and features the same properties as long as the asset correlation is higher than −1. The two cases are compared in section 2.5.4.

- Bank portfolio choice:

$$\frac{p_2}{p_1} = MRS_B = \frac{\frac{\partial W^B}{\partial x_2}}{\frac{\partial W^B}{\partial x_1}} = \frac{\mu_2 \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \frac{\sigma_2(\sigma_2 x_2 + \rho\sigma_1 x_1)}{\sigma_u}\phi\left(\frac{\mu_u}{\sigma_u}\right)}{\mu_1 \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \frac{\sigma_1(\sigma_1 x_1 + \rho\sigma_2 x_2)}{\sigma_u}\phi\left(\frac{\mu_u}{\sigma_u}\right)} \tag{A.1}$$

- Bank leverage choice:

$$\frac{D'(\bar{s})}{p_1} = MRT_B = \frac{\frac{\partial W^B}{\partial \bar{s}}}{\frac{\partial W^B}{\partial x_1}} = -\frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\mu_1 \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \frac{\sigma_1(\sigma_1 x_1 + \rho\sigma_2 x_2)}{\sigma_u}\phi\left(\frac{\mu_u}{\sigma_u}\right)} \tag{A.2}$$

- Investor portfolio choice:

$$\frac{D'(\bar{s})}{p_2} = MRS_L = \frac{\frac{\partial W^L}{\partial \bar{s}}}{\frac{\partial W^L}{\partial y_2}} \tag{A.3}$$

- The two budget constraints:

$$\begin{aligned} y_2 p_2 + D &\leq n^L && \text{(A.4)} \\ x_1 p_1 + x_2 p_2 &\leq n^B + D && \text{(A.5)} \end{aligned}$$

Expressions for marginal rates of substitution and transformation

Borrower Using $\frac{\partial W^B}{\partial \mu_u} = \Phi\left(\frac{\mu_u}{\sigma_u}\right)$ and $\frac{\partial W^B}{\partial \sigma_u} = \phi\left(\frac{\mu_u}{\sigma_u}\right)$, we obtain for the marginal benefits of portfolio investment and the marginal cost of levering:

$$\begin{aligned} \frac{\partial W^B}{\partial x_i} &= \mu_i \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \frac{\sigma_i\left(\sigma_i x_i + \rho\sigma_{-i} x_{-i}\right)}{\sigma_u}\phi\left(\frac{\mu_u}{\sigma_u}\right) \\ \frac{\partial W^B}{\partial \bar{s}} &= -\Phi\left(\frac{\mu_u}{\sigma_u}\right) \end{aligned}$$

Hence closed form expressions for the two B-optimality sufficient statistics MRS_B and MRT_B.

$$MRS_B = \frac{\frac{\partial W^B}{\partial x_2}}{\frac{\partial W^B}{\partial x_1}} = \frac{\mu_2 \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \frac{\sigma_2(\sigma_2 x_2 + \rho\sigma_1 x_1)}{\sigma_u}\phi\left(\frac{\mu_u}{\sigma_u}\right)}{\mu_1 \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \frac{\sigma_1(\sigma_1 x_1 + \rho\sigma_2 x_2)}{\sigma_u}\phi\left(\frac{\mu_u}{\sigma_u}\right)}$$

$$MRT_B = \frac{\frac{\partial W^B}{\partial \bar{s}}}{\frac{\partial W^B}{\partial x_1}} = -\frac{1}{\mu_1 + \frac{\sigma_1(\sigma_1 x_1 + \rho\sigma_2 x_2)}{\sigma_u}\frac{\phi\left(\frac{\mu_u}{\sigma_u}\right)}{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}}$$

Lender The algebraic complexity of investor value function is dramatically simplified under the European feature of limited participation, i.e. investor being restricted access to the risky asset: $y_1 = 0$. The moment of the transformed distribution are now:

$$\begin{aligned} \sigma_v &= y_2\sigma_2 \\ \rho_{uv} &= \frac{(\rho x_1\sigma_1 + x_2\sigma_2)}{\sigma_u} \end{aligned}$$

We have $\frac{\partial\mu_v}{\partial y_2} = \mu_2$, $\frac{\partial\sigma_v}{\partial y_2} = \sigma_2$ and $\frac{\partial\rho_{uv}}{\partial y_2} = 0$. And with respect to the transformed moments:

$$\frac{\partial W^L}{\partial\mu_v} = -\gamma_L W^L$$

Deriving $\frac{\partial W^L}{\partial\sigma_v}$ and $\frac{\partial W^L}{\partial\mu_u}$ delivers closed-form expression for the L-optimality sufficient statistic MRS_L:

$$MRS_L = \frac{num}{dem}$$

$$\begin{aligned} num = {} & (-\gamma_L)\,\Phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L\rho_{uv}\sigma_v\right) \\ & + \frac{1}{\sigma_u}\left\{e^{-\gamma_L\mu_u + \frac{1}{2}\gamma_L^2\left(\sigma_u^2 + 2\rho_{uv}\sigma_v\sigma_u\right)}\phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L\left(\rho_{uv}\sigma_v + \sigma_u\right)\right) - \phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L\rho_{uv}\sigma_v\right)\right\} \end{aligned} \tag{A.6}$$

$$\begin{aligned} den &= \left\{\left(-\mu_2\gamma_L + \gamma_L^2\sigma_2\left(\sigma_v + \rho_{uv}\sigma_u\right)\right)e^{-\gamma_L\mu_u + \frac{1}{2}\gamma_L^2\left(\sigma_u^2 + 2\rho_{uv}\sigma_v\sigma_u\right)}\left\{1 - \Phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L\left(\rho_{uv}\sigma_v + \sigma_u\right)\right)\right\}\right\} \\ & + \left\{\left(-\mu_2\gamma_L + \gamma_L^2\sigma_2\sigma_v\right)\Phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L\rho_{uv}\sigma_v\right)\right\} \\ & + \left(\sigma_2\gamma_L\rho_{uv}\right)\left\{e^{-\gamma_L\mu_u + \frac{1}{2}\gamma_L^2\left(\sigma_u^2 + 2\rho_{uv}\sigma_v\sigma_u\right)}\phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L\left(\rho_{uv}\sigma_v + \sigma_u\right)\right) - \phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L\rho_{uv}\sigma_v\right)\right\} \end{aligned}$$

Bank portfolio choice

From $MRS_B = \frac{p_2}{p_1}$ and using $MRS_B = \frac{\mu_2\Phi\left(\frac{\mu_u}{\sigma_u}\right) + \frac{\sigma_2\left(\sigma_2 x_2 + \rho\sigma_1 x_1\right)}{\sigma_u}\phi\left(\frac{\mu_u}{\sigma_u}\right)}{\mu_1\Phi\left(\frac{\mu_u}{\sigma_u}\right) + \frac{\sigma_1\left(\sigma_1 x_1 + \rho\sigma_2 x_2\right)}{\sigma_u}\phi\left(\frac{\mu_u}{\sigma_u}\right)}$ we obtain:

$$p_1\sigma_2\left(\sigma_2 x_2+\rho\sigma_1 x_1\right)-p_2\sigma_1\left(\sigma_1 x_1+\rho\sigma_2 x_2\right) \quad = \quad \left(p_2\mu_1-p_1\mu_2\right)\sigma_u\frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)} \tag{A.7}$$

Bank leverage choice

From $D'(\bar{s})=-p_1 MRT_B$ and using $MRT_B=-\frac{1}{\mu_1+\frac{\sigma_1(\sigma_1 x_1+\rho\sigma_2 x_2)}{\sigma_u}\frac{\phi\left(\frac{\mu_u}{\sigma_u}\right)}{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}}$ we get:

$$D'(\bar{s})=\frac{p_1}{\mu_1+\frac{\sigma_1(\sigma_1 x_1+\rho\sigma_2 x_2)}{\sigma_u}\frac{\phi\left(\frac{\mu_u}{\sigma_u}\right)}{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}} \tag{A.8}$$

Debt market equilibrium

Combining the two bank optimality conditions (portfolio choice A.7 and leverage choice A.8) and eliminating μ_u we obtain the price of debt at equilibrium:

$$D'(\bar{s})=\frac{1}{\frac{\mu_1}{p_1}+\frac{\frac{\mu_1}{p_1}-\frac{\mu_2}{p_2}}{\frac{p_1}{p_2}\frac{\sigma_2}{\sigma_1}\frac{(\sigma_2 x_2+\rho\sigma_1 x_1)}{(\sigma_1 x_1+\rho\sigma_2 x_2)}-1}}=\frac{p_2}{\mu_2}\frac{1-\frac{p_1}{p_2}\frac{\sigma_2}{\sigma_1}\frac{\rho+\frac{\sigma_2 x_2}{\sigma_1 x_1}}{1+\rho\frac{\sigma_2 x_2}{\sigma_1 x_1}}}{1-\frac{\mu_1}{\mu_2}\frac{\sigma_2}{\sigma_1}\frac{\rho+\frac{\sigma_2 x_2}{\sigma_1 x_1}}{1+\rho\frac{\sigma_2 x_2}{\sigma_1 x_1}}}$$

This shows that the marginal benefit of an extra unit of leverage (of promise $\bar{s}$) is *constant along the contract curve*, i.e. does not depend on the actual promise $\bar{s}$. Using the definition of the price of debt $D'(\bar{s})=\frac{D}{\bar{S}}=\frac{q}{\bar{s}}$ and denote $M\hat{R}S_B$ the second ration on the above right hand side:

$$D(\bar{s})=\frac{p_2}{\mu_2}M\hat{R}S_B\bar{s}=\frac{p_2}{\mu_2}M\hat{R}S_B\left(x_1\mu_1+x_2\mu_2-\mu_u\right)$$

Introduce $r^{bank}=\frac{\bar{s}}{D}=\frac{\mu_2}{p_2}\frac{1}{M\hat{R}S_B}$ and denote $X\left(x_1,x_2;\rho\right)=\frac{\sigma_2 x_2+\rho\sigma_1 x_1}{\sigma_1 x_1+\rho\sigma_2 x_2}=\frac{\rho+\frac{\sigma_2 x_2}{\sigma_1 x_1}}{1+\rho\frac{\sigma_2 x_2}{\sigma_1 x_1}}$, it can be written:

$$M\hat{R}S_B=\frac{1-\frac{p_1}{p_2}\frac{\sigma_2}{\sigma_1}X}{1-\frac{\mu_1}{\mu_2}\frac{\sigma_2}{\sigma_1}X}$$

The ratio $X=\frac{\sigma_2 x_2+\rho\sigma_1 x_1}{\sigma_1 x_1+\rho\sigma_2 x_2}=\frac{\rho+\frac{\sigma_2 x_2}{\sigma_1 x_1}}{1+\rho\frac{\sigma_2 x_2}{\sigma_1 x_1}}$ is a measure of the effective correlation on bank's balance sheet. As $X'(\rho) = \frac{1-\left(\frac{\sigma_2 x_2}{\sigma_1 x_1}\right)^2}{\left(1+\rho\frac{\sigma_2 x_2}{\sigma_1 x_1}\right)^2}$, in the equilibrium that we look for in which $\frac{\sigma_2 x_2}{\sigma_1 x_1} < 1$ (which is feasible under Assumption 1 $\frac{\sigma_2}{\sigma_1} > \frac{p_2}{p_1} > \frac{\mu_2}{\mu_1}$), we obtain that $X(\rho)$ is increasing, between -1 and 1. Furthermore, this functional is concave, with the concavity more marked when $\frac{\sigma_2 x_2}{\sigma_1 x_1}$ high. Besides we directly see that $\frac{p_2}{p_1} > \frac{\mu_2}{\mu_1}$ from Assumption 1 implies, as long as $X < 0$ $M\hat{R}S_B < 1$, but when $X > 0$ we get $M\hat{R}S_B > 1$. So the interest rate on bank debt critically depends on the bank balance sheet correlation measure X. We can ascertain that $r^{bank} > 1$ for sure only in the $X < 0$ case. Define the Safety Mismatch Index as the carry trade on public debt (the opposite is the bank credit spread):

$$\begin{aligned} r^{safe} - r^{bank} &= \frac{\mu_2}{p_2} - \frac{\bar{s}}{D} \\ &= \left(1 - \frac{1}{M\hat{R}S_B}\right)\frac{\mu_2}{p_2} \end{aligned}$$

So the sign of the carry trade *is* the sign of the correlation measure $X(x_1, x_2; \rho)$. Finally:

$$r^{safe} - r^{bank} < 0 \Leftrightarrow \rho < -\frac{\sigma_2 x_2}{\sigma_1 x_1}$$

Furthermore, some algebra delivers the exact dependence:

$$r^{safe} - r^{bank} = \left(r^1 - r^{safe}\right)\frac{\mu_2}{p_1}\left(\frac{1}{1 - \frac{p_1}{p_2}\frac{\sigma_2}{\sigma_1}X} - 1\right)$$

So taking the equilibrium as given (envelope condition), the carry trade SMI increases with $X(x_1, x_2; \rho)$, which itself increases with ρ. The negative carry trade is even more negative when p_2 increases.

Bank Mean Variance Frontier

The bank budget constraint combined with bank portfolio choice is a quadratic system in (x_1, x_2) as $M\hat{R}S_B$ depends on x_1, x_2. Solving it gives (x_1, x_2) as functions of μ_u and $\lambda_u = \sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)}$.

$$\begin{cases} x_1 p_1 + x_2 p_2 & = n^B + D \\ p_1\sigma_2\left(\sigma_2 x_2 + \rho\sigma_1 x_1\right) - p_2\sigma_1\left(\sigma_1 x_1 + \rho\sigma_2 x_2\right) & = \left(p_2\mu_1 - p_1\mu_2\right)\sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)} \end{cases}$$

The portfolio choice directly gives:

$$x_2 = \frac{\left(p_2\mu_1 - p_1\mu_2\right)}{\left(\sigma_2^2 p_1 - \rho\sigma_1\sigma_2 p_2\right)}\sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)} + \frac{\left(\sigma_1^2 p_2 - \rho\sigma_1\sigma_2 p_1\right)}{\left(\sigma_2^2 p_1 - \rho\sigma_1\sigma_2 p_2\right)} x_1 \tag{A.9}$$

Using the value of debt D and the expression of X then plugging x_2 in the budget constraint leads to a polynomial of degree 2 in x_1:

$$T(x_1) = \frac{1}{2}x_1^2 + bx_1 - c = 0$$

There is one and only one positive root x_1 (as long as ρ low enough). The *exact* solution is (x_1 increases with c and decreases with b):

$$x_1 = -b + \sqrt{b^2 + 4c}$$

$$b = \frac{\left[\left(\sigma_2 p_1 - \rho\sigma_1 p_2\right)\frac{\left\{n^B p_2\left(\sigma_1 - \sigma_2\left\{\frac{\mu_1}{\mu_2}(1+\rho) - \frac{p_1}{p_2}\right\}\right) - \frac{\mu_u}{\mu_2}\left(\sigma_1 p_2 - \rho p_1\sigma_2\right)\right\}}{\left(\frac{\mu_1}{\mu_2} - \frac{p_1}{p_2}\right)} + 2\left(1+\rho\right)p_2\left(p_2\mu_1 - p_1\mu_2\right)\sigma_u\frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)}\right]}{2\sigma_1\left[\sigma_2^2 p_1^2 + \left(1 - 2\rho - \rho^2\right)\sigma_1\sigma_2 p_1 p_2 + \sigma_1^2 p_2^2\right]}$$

$$c = \frac{\left[\left(\frac{\sigma_2}{\sigma_1}p_1 - \rho p_2\right)\frac{n^B\left(\frac{\mu_1}{\mu_2}\frac{\sigma_2}{\sigma_1} - \rho\right) - \frac{\mu_u}{\mu_2}\left(\frac{p_1}{p_2}\frac{\sigma_2}{\sigma_1} - \rho\right)}{\frac{\mu_1}{\mu_2} - \frac{p_1}{p_2}} - \frac{1}{\sigma_1^2}\left(p_2\mu_1 - p_1\mu_2\right)\sigma_u\frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)}\right]}{2\sigma_1\left(1-\rho\right)\left[\sigma_2^2 p_1^2 + \left(1 - 2\rho - \rho^2\right)\sigma_1\sigma_2 p_1 p_2 + \sigma_1^2 p_2^2\right]}\left(p_2\mu_1 - p_1\mu_2\right)\sigma_u\frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)}$$

Finally I express $\sigma_u^2 = x_1^2\sigma_1^2 + 2\rho x_1 x_2\sigma_1\sigma_2 + x_2^2\sigma_2^2$ as a function of x_1 and denoting Σ, we obtain:

$$\left(\sigma_2 p_1 - \rho\sigma_1 p_2\right)^2\sigma_u^2 = 2\Sigma c + \delta_u^2 + 2\left\{-b + \sqrt{b^2 + 4c}\right\}\left\{\left(1 - \rho^2\right)\sigma_1^2 p_2\delta_u - \Sigma b\right\}$$

$$\Sigma = \left(1-\rho^2\right)\sigma_1^2\left\{\sigma_2^2 p_1^2 - 2\rho\sigma_1\sigma_2 p_1 p_2 + \sigma_1^2 p_2^2\right\}$$

This equation gives a closed-form expression for the mean-variance frontier. It implicetly defines a functional $\sigma_u \mapsto \mu_u$ which is increasing for low σ_u, before decreasing.

Equilibrium price of debt

The contract curve is defined by $D'(\bar{s}) = \frac{p_2}{\mu_2}\hat{MRS}_L$, i.e.:

$$\hat{MRS}_B = \hat{MRS}_L$$

I develop $\hat{MRS}_L$ in orders of γ_L, using the three following expressions:

$$\Phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L\rho_{uv}\sigma_v\right) = \Phi\left(\frac{\mu_u}{\sigma_u}\right) - \phi\left(\frac{\mu_u}{\sigma_u}\right)\gamma_L\rho_{uv}y_2\sigma_2$$

$$1-\Phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L\left(\rho_{uv}\sigma_v + \sigma_u\right)\right) = 1-\Phi\left(\frac{\mu_u}{\sigma_u}\right) + \phi\left(\frac{\mu_u}{\sigma_u}\right)\gamma_L\left(\rho_{uv}y_2\sigma_2 + \sigma_u\right)$$

$$e^{-\gamma_L\mu_u + \frac{1}{2}\gamma_L^2\left(\sigma_u^2 + 2\rho_{uv}\sigma_v\sigma_u\right)} = 1 - \gamma_L\mu_u + \frac{1}{2}\gamma_L^2\left(\sigma_u^2 + 2\rho_{uv}y_2\sigma_2\sigma_u\right)$$

.

The first order in γ_L gives:

$$\hat{MRS}_L = \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right) - \phi\left(\frac{\mu_u}{\sigma_u}\right)\gamma_L\left\{\left(2-\frac{\mu_u^2}{\sigma_u^2}\right)\rho_{uv}y_2\sigma_2 + \left(\frac{1}{2}-\frac{\mu_u^2}{\sigma_u^2}\right)\sigma_u\right\}}{1-\gamma_L\left\{\left(1-\Phi\left(\frac{\mu_u}{\sigma_u}\right)\right)\mu_u - \Phi\left(\frac{\mu_u}{\sigma_u}\right)\frac{\sigma_2}{\mu_2}\rho_{uv}\sigma_u + \frac{\sigma_2}{\mu_2}\left(y_2\sigma_2 + \rho_{uv}\sigma_u\right) - \phi\left(\frac{\mu_u}{\sigma_u}\right)\right\}}$$

So $\hat{MRS}_L$ *increases in* γ_L for ρ low enough (to make $\rho_{uv} << 1$) and y_2 low enough (i.e. S^b low enough). On the contract curve, we see that implies a higher equilibrium leverage. We also observe that $\hat{MRS}_L$ does not depend on p_2, so easily get the impact of a higher price p_2 here: it increases the slope MRSB: the MB of leverage has to be higher to match the increased price p_2.

We can also get a approximate for the debt-pricing functional in the neighborhood of $\gamma_L \sim 0$, which is an increasing functional $\sigma_u \mapsto \mu_u$:

$$\mu_u = \sigma_u \Phi^{-1}\left(\frac{\mu_2}{p_2}\frac{p_1}{\mu_1}\left(1 - \frac{\sigma_1}{\sigma_2}\frac{\frac{p_2}{p_1} - \frac{\mu_2}{\mu_1}}{X - \frac{\mu_2}{\mu_1}\frac{\sigma_1}{\sigma_2}}\right)\right)$$

Equilibrium price of long-term safe asset

The GE: $x_2 + y_2 = S^b$, and adding the two budget constraints:

$$p_2 S^b + p_1 x_1 \left(\mu_1; p_2\right) = n^B + n^L$$

So p_2 is *depends on the equilibrium only through* x_1 (a key feature of the model):

$$x_1 \left(\mu_1; p_2\right) = n^B \left\{ \frac{(1+\rho)\,\sigma_2 p_2}{\sigma_1 \Sigma}\left(\sigma_2 p_1 - \rho \sigma_1 p_2\right) + \left[\frac{\sigma_1}{\Sigma}\frac{1-\rho}{1+\rho} - 2\frac{(1+\rho)\,p_2^2}{\sigma_1 \Sigma}\right]\mu_1 \sqrt{\delta}\right\}$$

By the implicit function theorem applied to this resource constraint, we can characterize the safe asset price $p_2\left(S^b, \mu_1\right)$.

$$\frac{dp_2}{dS^b} = -\frac{p_2}{S^b + p_1 \partial_{p_2} x_1}$$

So the demand curve is indeed downward sloping ($\frac{dp_2}{dS^b} < 0$) as long as $-\frac{S^b}{p_1} < \partial_{p_2} x_1$.

Second, I investigate the dependence of p_2 on μ_1, as a building block for the dynamic model.

$$\frac{dp_2}{d\mu_1} = -\frac{\partial_{\mu_1} F}{\partial_{p_2} F} = -\frac{p_1 \partial_{\mu_1} x_1}{S^b + p_1 \partial_{p_2} x_1}$$

Third, the dynamic model solution requires $\frac{d^2 p_2}{d(\mu_1)^2}$. I compute it through a double application of implicit function theorem to the resource constraint $F\left(p_2; \mu_1\right) = 0$. Total differentiating twice in μ_1:

$$
\begin{aligned}
\frac{d^2p_2}{d\left(\mu_1\right)^2}\partial_{p_2}F+\frac{dp_2}{d\mu_1}\left(\frac{dp_2}{d\mu_1}\partial_{p_2p_2}F+\partial_{\mu_1p_2}F\right)+\frac{dp_2}{d\mu_1}\partial_{p_2\mu_1}F+\partial_{\mu_1\mu_1}F &= 0\\
\frac{d^2p_2}{d\left(\mu_1\right)^2}\partial_{p_2}F+\left(\frac{dp_2}{d\mu_1}\right)^2\partial_{p_2p_2}F+2\frac{dp_2}{d\mu_1}\partial_{\mu_1p_2}F+\partial_{\mu_1\mu_1}F &= 0
\end{aligned}
$$

$$
\frac{\frac{d^2p_2}{d(\mu_1)^2}}{\frac{dp_2}{d\mu_1}} = \frac{\left(\frac{dp_2}{d\mu_1}\right)^2\partial_{p_2p_2}F+2\frac{dp_2}{d\mu_1}\partial_{\mu_1p_2}F+\partial_{\mu_1\mu_1}F}{\partial_{\mu_1}F}
$$

$\partial_{\mu_1}F$ and $\partial_{\mu_1\mu_1}F$ does not depend on S^b. $\partial_{p_2p_2}F$ by separability. But $\left(\frac{dp_2}{d\mu_1}\right)^2$ *does* depend on S^b. As we have $\partial_{p_2p_2}F = p_1\partial_{p_2p_2}x_1$, which is < 0 (Σ does not dominate - cf. s.o.c.), then this positive effect can counteract the fact that $\partial_{\mu_1p_2}F = p_1\partial_{\mu_1p_2}x_1 < 0$ and $\frac{dp_2}{d\mu_1}$ increasing in S^b.

The second order condition needs to be verified: $\partial_{p_2p_2}x_1 >> 0$, despite $\partial_{\mu_1p_2}x_1 < 0$. Algebra leads to, denoting $u = \frac{p_1\partial_{\mu_1}x_1}{S^b+p_1\partial_{p_2}x_1} = u\left(\underset{(-)}{S^b}\right)$:

$$
-\partial_{\mu_1p_2}x_1 < -\partial_{p_2p_2}x_1u
$$

Beware of the elasticities signs: $\partial_{\mu_1p_2}x_1 < 0$ and $\partial_{p_2p_2}x_1 < 0$ so the above condition is a lower bound on u, i.e. an upper bound on S^b. The elasticities signs are guaranteed with a high enough γ_L.

So finally the second-order Taylor expansion of $p_2\left(\mu_1\right)$ writes:

$$
p_2\left(\mu_1\right) = \alpha - \beta\mu_1 - \gamma\mu_1^2
$$

where $\beta = |\frac{dp_2}{d\mu_1}| = \frac{p_1\partial_{\mu_1}x_1}{S^b+p_1\partial_{p_2}x_1}$
and $\gamma = |\frac{d^2p_2}{d(\mu_1)^2}| = \frac{\left(\frac{dp_2}{d\mu_1}\right)^2\partial_{p_2p_2}F+2\frac{dp_2}{d\mu_1}\partial_{\mu_1p_2}F+\partial_{\mu_1\mu_1}F}{\partial_{p_2}F} = \frac{\left(\frac{dp_2}{d\mu_1}\right)^2p_1\partial_{p_2p_2}x_1+2\frac{dp_2}{d\mu_1}p_1\partial_{\mu_1p_2}x_1+p_1\partial_{\mu_1\mu_1}x_1}{S^b+p_1\partial_{p_2}x_1}$
which means that the price functional $\mu_1 \mapsto p_2$ is decreasing concave.

Equilibrium

Equilibrium is therefore characterized by the MVF and the debt pricing curve:

$$\begin{cases} \left(\sigma_2 p_1 - \rho\sigma_1 p_2\right)^2 \sigma_u^2 & = 2\Sigma c + \delta_u^2 + 2\left\{-b + \sqrt{b^2+4c}\right\}\left\{\left(1-\rho^2\right)\sigma_1^2 p_2 \delta_u - \Sigma b\right\} \\ \frac{\mu_2}{p_2}\frac{p_1}{\mu_1}\left(1 - \frac{\sigma_1}{\sigma_2}\frac{\frac{p_2}{p_1}-\frac{\mu_2}{\mu_1}}{X-\frac{\mu_2}{\mu_1}\frac{\sigma_1}{\sigma_2}}\right) & = \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)-\phi\left(\frac{\mu_u}{\sigma_u}\right)\gamma_L\left\{\left(2-\frac{\mu_u^2}{\sigma_u^2}\right)\rho_{uv}y_2\sigma_2+\left(\frac{1}{2}-\frac{\mu_u^2}{\sigma_u^2}\right)\sigma_u\right\}}{1-\gamma_L\left\{\left(1-\Phi\left(\frac{\mu_u}{\sigma_u}\right)\right)\mu_u-\Phi\left(\frac{\mu_u}{\sigma_u}\right)\frac{\sigma_2}{\mu_2}\rho_{uv}\sigma_u+\frac{\sigma_2}{\mu_2}\left(y_2\sigma_2+\rho_{uv}\sigma_u\right)-\phi\left(\frac{\mu_u}{\sigma_u}\right)\right\}} \end{cases}$$

Denoting $\lambda_u = \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)}$ the MVF delivers:

$$\begin{aligned} \frac{\left(\frac{\rho\sigma_1}{\mu_1}\right)^2}{(1-\rho^2)} \quad = \quad & \lambda_u^2[\left(\frac{\mu_1^2}{\mu_2}\right)^2\left(\frac{1}{n^B}\right)^2(\sigma_u\lambda_u)^2 + 2\left\{1-\left(\frac{\mu_1}{\mu_2}\sigma_1\sigma_2+\sigma_1^2\right)\right\}\frac{\sigma_1\mu_1}{\mu_2}\frac{1}{n^B}\frac{\mu_1}{\sigma_1}(\sigma_u\lambda_u)] \\ & + \left(\sigma_2+\sigma_1\frac{\mu_2}{\mu_1}\right)\frac{\sigma_1\mu_1}{\mu_2}\left\{\left(\frac{\mu_1}{\mu_2}\sigma_1\sigma_2+\sigma_1^2\right)-2\right\}+\frac{1}{(1-\rho^2)} \end{aligned}$$

The degree 4 polynomial in λ_u on the RHS is increasing so it will cross the positive flat line $(\rho\sigma_1)^2$. And the higher ρ^2 is, the higher λ_u needs to be. The lower ρ^2, the lower λ_u so the lower the ellipse mapping is. For n^B low enough:

$$\frac{\left(\frac{\rho\sigma_1}{\mu_1}\right)^2}{(1-\rho^2)} = \lambda_u^2\left[\left(\frac{\mu_1^2}{\mu_2}\right)^2\left(\frac{1}{n^B}\right)^2(\sigma_u\lambda_u)^2 + \left(\sigma_2+\sigma_1\frac{\mu_2}{\mu_1}\right)\frac{\sigma_1\mu_1}{\mu_2}\left\{\left(\frac{\mu_1}{\mu_2}\sigma_1\sigma_2+\sigma_1^2\right)-2\right\}+\frac{1}{(1-\rho^2)}\right]$$

So this a $X\,(aX+b)-c=0$. We then get only one positive root:

$$\lambda_u^2 = \frac{-b+\sqrt{b^2+4c}}{2a}$$

with $a = \left(\frac{\mu_1^2}{\mu_2}\right)^2\left(\frac{1}{n^B}\right)^2\sigma_u^2$, $b = \left(\sigma_2+\sigma_1\frac{\mu_2}{\mu_1}\right)\frac{\sigma_1\mu_1}{\mu_2}\left\{\left(\frac{\mu_1}{\mu_2}\sigma_1\sigma_2+\sigma_1^2\right)-2\right\}+\frac{1}{(1-\rho^2)}$ and $c = \frac{\left(\frac{\rho\sigma_1}{\mu_1}\right)^2}{(1-\rho^2)}$.

So λ_u eq decreases with σ_u, hence μ_u decreases with σ_u. The mapping is:

$$\lambda_u^2 = \left(-b+\sqrt{b^2+4c}\right)\left(\frac{\mu_1^2}{\mu_2}\right)^{-2}\frac{\left(n^B\right)^2}{\sigma_u^2}$$

I write $\lambda_u = \sqrt{\delta}\frac{n^B}{\sigma_u}$, with $\sqrt{\delta}$ increasing in $|\rho|$. δ spikes up when ρ tends to -1. Denoting

ξ^{-1} the inverse function of $\xi : u \mapsto \frac{\Phi(u)}{\phi(u)}$ we have:

$$\mu_u = \sigma_u \xi^{-1}\left(\frac{\sqrt{\delta} n^B}{\sigma_u}\right) \tag{A.10}$$

Now remember the debt pricing curve (in the admissible region $0.5 < \frac{p_2}{\mu_2} M\hat{R}S_B < 1$):

$$\mu_u = \sigma_u \Phi^{-1}\left(\frac{p_2}{\mu_2} M\hat{R}S_B\right) \tag{A.11}$$

By identifying A.10 and A.11:

$$\sigma_u = \frac{\sqrt{\delta} n^B}{\xi\left(\Phi^{-1}\left(\frac{p_2}{\mu_2} M\hat{R}S_B\right)\right)}$$

$$\mu_u = \sqrt{\delta} n^B \frac{\Phi^{-1}\left(\frac{p_2}{\mu_2} M\hat{R}S_B\right)}{\xi\left(\Phi^{-1}\left(\frac{p_2}{\mu_2} M\hat{R}S_B\right)\right)}$$

The function $x/\xi(x)$ is graphed below: it increases in x as long as $x < 1$. We do have $\frac{p_2}{\mu_2} < 1$ and $M\hat{R}S_B < 1$, but Φ^{-1} can take arbitrarily high values. Therefore to get $\mu_u = f\left(\underset{(+)}{p_2}\right)$ we need to make sure that γ_L is large enough, as we have that $\Phi^{-1}\left(\frac{p_2}{\mu_2} M\hat{R}S_B\right)$ is decreasing with γ_L.

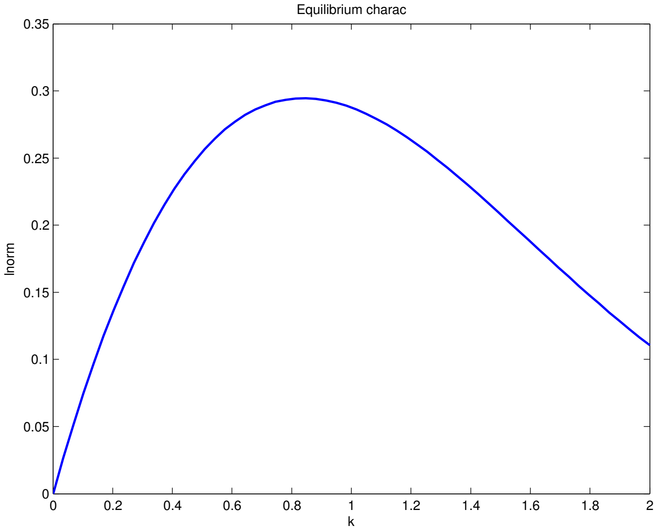

Figure A.1: *Characterization of the equilibrium.*

A.1.3 Intergenerational equilibrium

Endogenous beta

In the Markov equilibria with the one state variable $s^t \equiv \mu_1$, the resource constraint gives:

$$p_2^t\left(s^t\right)S^b + p_1 x_1\left(p_2^t\left(s^t\right); s^t\right) = n^B + n^L$$

Appealing to the static model just solved, the safe asset price functional has the following characteristics[4]:

$$p_2\left(s^{t+1}\right) = \alpha - \beta s^{t+1} - \gamma\left(s^{t+1}\right)^2$$

Generically we have that, for an univariate normal distribution $s \sim N\left(\mu, \sigma\right)$, the correlation between s and $p_2 = \alpha - \beta s - \gamma s^2$, first-order in γ/β[5]:

$$corr\left(s, p_2\right) = -1 + \left(2\mu\frac{\gamma}{\beta}\right)^2$$

Applied to the random walk process of s^t: $s^{t+1} \sim N\left(s^t, \sigma_1\right)$:

- $\hat{\rho} = -1 + \left(2s^t\frac{\gamma}{\beta}\right)^2$
- $\hat{\sigma}_2 = \beta\sigma_1\sqrt{1 + 4s^t\frac{\gamma}{\beta} + 2\left(\frac{\gamma}{\beta}\right)^2\left(s^{t2} + \sigma_1^2\right)}$
- $\hat{\mu}_2 = \alpha - \beta s^t - \gamma\left(s^t\right)^2$

This is a system of 3 equations, with 3 unknowns $(\hat{\rho}, \hat{\sigma}_2, \hat{\mu}_2)$. The recursion implies that β and γ are themselves functions of $(\hat{\rho}, \hat{\sigma}_2, \hat{\mu}_2)$. σ_1 is an exogenous parameter. The state is $s^t \equiv \mu_1$.

---

[4] If I only consider the first-order Taylor expansion $p_2 = \alpha - \beta s^t$, then the endogenous beta is equal to -1. No traction for any comparative statics.

[5] The exact expression in γ/β is:
$\sigma\left(p_2\right) = \beta\sigma\sqrt{1 + 4\mu\frac{\gamma}{\beta} + 2\left(\frac{\gamma}{\beta}\right)^2\left(\mu^2 + \sigma^2\right)}$ and $corr\left(s, p_2\right) = -\frac{1 + 2\mu\frac{\gamma}{\beta}}{\sqrt{1 + 4\mu\frac{\gamma}{\beta} + 2\left(\frac{\gamma}{\beta}\right)^2\left(\mu^2 + \sigma^2\right)}}$

Consider $x_1\left(p_2;\mu_1\right)$. In $p_2=0$: $\Sigma\left(0\right)=\sigma_2^2p_1^2$ and $x_1\left(0\right)=n^B\frac{\sigma_1}{\sigma_2^2p_1^2}\frac{1-\rho}{1+\rho}\mu_1\sqrt{\delta}$ so:

- $\partial_{\mu_1}x_1=n^B\frac{\sigma_1}{\sigma_2^2p_1^2}\frac{1-\rho}{1+\rho}\sqrt{\delta}$
- $\partial_{\mu_1}\partial_{\mu_1}x_1=0$
- $\partial_{p_2}\partial_{\mu_1}x_1=n^B\partial_{p_2}\left(\frac{\sigma_1^2(1-\rho)-2(1+\rho)^2p_2^2}{\sigma_2^2p_1^2+(1-2\rho-\rho^2)\sigma_1\sigma_2p_1p_2+\sigma_1^2p_2^2}\right)\frac{1-\rho}{1+\rho}\sqrt{\delta}$
- $\partial_{p_2}\partial_{p_2}x_1=n^B\frac{1-\rho}{1+\rho}\mu_1\sqrt{\delta}\frac{1}{1+\rho}\partial_{p_2}\partial_{p_2}\left(\frac{\sigma_1^2(1-\rho)-2(1+\rho)^2p_2^2}{\sigma_2^2p_1^2+(1-2\rho-\rho^2)\sigma_1\sigma_2p_1p_2+\sigma_1^2p_2^2}\right)$

By inspection we see that

$$\partial_{p_2}\partial_{p_2}\left(\frac{\sigma_1^2\left(1-\rho\right)-2\left(1+\rho\right)^2p_2^2}{\sigma_2^2p_1^2+\left(1-2\rho-\rho^2\right)\sigma_1\sigma_2p_1p_2+\sigma_1^2p_2^2}\right)<0$$

Introduce:

$$\kappa(p_2)=\frac{\sigma_1^2\left(1-\rho\right)-2\left(1+\rho\right)^2p_2^2}{\sigma_2^2p_1^2+\left(1-2\rho-\rho^2\right)\sigma_1\sigma_2p_1p_2+\sigma_1^2p_2^2}$$

Around $\rho=-1$:

$$\begin{aligned}|\partial_{p_2}\kappa| &= \frac{2\left(1+\rho\right)^2p_2\left(2\sigma_2^2p_1^2+\left(1-2\rho-\rho^2\right)\sigma_1\sigma_2p_1p_2\right)}{\Sigma^2}\\ &\quad+\frac{\sigma_1^2\left(1-\rho\right)\left(\left(1-2\rho-\rho^2\right)\sigma_1\sigma_2p_1+2\sigma_1^2p_2\right)}{\Sigma^2}\\ &= \frac{2\sigma_1^4p_2}{\left[\sigma_2^2p_1^2+\sigma_1^2p_2^2\right]^2}\left(1-\rho\right)\end{aligned}$$

$$\begin{aligned}\partial_{p_2}\partial_{p_2}\kappa &= 2\left(1+\rho\right)^2\left(-2\left(\sigma_2^2p_1^2\right)^2+6\sigma_2^2p_1^2\sigma_1^2p_2^2+2\left(1-2\rho-\rho^2\right)\sigma_1\sigma_2p_1\sigma_1^2p_2^3\right)\\ &\quad+2\sigma_1^2\left(1-\rho\right)\\ &\quad\left\{\left(\left(1-2\rho-\rho^2\right)\sigma_1\sigma_2p_1\right)^2+3\left(\left(1-2\rho-\rho^2\right)\sigma_1\sigma_2p_1\right)\sigma_1^2p_2+\sigma_1^2\left(-\sigma_2^2p_1^2+3\sigma_1^2p_2^2\right)\right\}\end{aligned}$$

As $\hat{\rho}=-1+\left(2s^t\frac{\gamma}{\beta}\right)^2$, only $\frac{\gamma}{\beta}$ is needed. The fact that $\partial_{\mu_1}|\partial_{p_2}x_1|<0$ implies that $\frac{\gamma}{\beta}$ is increasing in S^b.

$$\frac{\gamma}{\beta} = \left(\frac{\partial_{\mu_1} x_1}{\frac{S^b}{p_1} + \partial_{p_2} x_1}\right)^2 \frac{\partial_{p_2 p_2} x_1}{\partial_{\mu_1} x_1} - 2\frac{\partial_{\mu_1 p_2} x_1}{\frac{S^b}{p_1} + \partial_{p_2} x_1}$$

It results in:

$$\rho = -1 + 4\mu^2 \left(\mu_1 \frac{1}{1+\rho} \frac{\kappa \partial_{p_2} \partial_{p_2} \kappa}{\left(\partial_{p_2} \kappa\right)^2} \frac{1}{\left(\frac{S^b}{p_1} \frac{1}{n^B \frac{1-\rho}{1+\rho} \sqrt{\delta} \partial_{p_2} \kappa} + \mu_1\right)^2} - 2 \frac{1}{\frac{S^b}{p_1} \frac{1}{n^B \frac{1-\rho}{1+\rho} \sqrt{\delta} \partial_{p_2} \kappa} + \mu_1}\right)^2$$

As I have shown above, $\partial_{p_2} \kappa < 0$ and $\partial_{p_2} \partial_{p_2} \kappa < 0$. Using the exact expression of κ:

$$\hat{\rho} = -1 + \left(\frac{S^b}{n^B} \frac{\left[\hat{\sigma}_2^2 p_1^2 + \sigma_1^2 \hat{p}_2^2\right]^2}{2\sigma_1^4 p_1^2 \hat{p}_2^2}\right)^2$$

A.1.4 Results: comparative statics

I first show that the two equilibrium variables are as such $\mu_u \left(\underset{(-)}{\rho}, \underset{(+)}{p_2} \right)$ and $\sigma_u \left(\underset{(-)}{\rho}, \underset{(-)}{p_2} \right)$: lower ρ make bank lever up and take more risk, whereas better p_2 make bank lever up more and take less risk. Propositions 1,2 and 3 then follow.

Equilibrium σ_u decreases with p_2. As for dependence in ρ:

$$\frac{\partial \lambda_u^2}{\partial \rho^2} = \frac{1 + \frac{1-\lambda_u^2}{(1-\rho^2)}}{(2a+b)\left(1-\rho^2\right)}$$

which is positive as long as $2 - \rho^2 > \lambda_u^2$.

Equilibrium leverage D

$$D = MRS_B * \bar{s}$$

$$\frac{\partial D}{\partial p_2} = \partial MRS_B * \bar{s} + MRS_B * \partial \bar{s}$$

where $MRS_B = \frac{p_2}{\mu_2} * \frac{1}{1+\frac{\mu_1\sigma_2}{\mu_2\sigma_1}}$. Denote $\theta = \frac{1}{\mu_2+\frac{\mu_1\sigma_2}{\sigma_1}} < 1$.

$$\frac{\partial D}{\partial p_2} < 0 \quad \Leftrightarrow \quad \frac{1}{p_2} < \frac{|\partial \bar{s}|}{\bar{s}}$$

So $\bar{s}$ decreases in p_2 as long this $x/\xi(x)$ increases.

$$\frac{|\partial \bar{s}|}{\bar{s}} \quad = \quad \frac{\frac{\partial}{\partial p_2}\left(\frac{\Phi^{-1}\left(\frac{p_2}{\mu_2}M\hat{R}S_B\right)}{\xi\left(\Phi^{-1}\left(\frac{p_2}{\mu_2}M\hat{R}S_B\right)\right)}\right)}{\frac{\mu_1^2}{\mu_2\sigma_1}\left\{\sqrt{\delta}\frac{\mu_1}{\sigma_1} - \left(\sigma_2 + \sigma_1\frac{\mu_2}{\mu_1}\right)\right\}\left(\mu_1 + \mu_2\frac{\sigma_1}{(-\rho\sigma_2)}\right) + \frac{\mu_1\mu_2}{(-\rho\sigma_1\sigma_2)} - \frac{\Phi^{-1}\left(\frac{p_2}{\mu_2}M\hat{R}S_B\right)}{\xi\left(\Phi^{-1}\left(\frac{p_2}{\mu_2}M\hat{R}S_B\right)\right)}}$$

The equation: $\frac{|\partial \bar{s}|}{\bar{s}} > \frac{1}{p_2}$ is true as long as $p_2 > \bar{p_2} = a/2$. The complementarity (i.e. $D(p_2)$ decreasing) for S^b low enough. Low ρ helps to have the limit a low. Also high γ_L helps: $a\left(\underset{(-)}{\gamma_L}\right)$. And finally we need to make sure that y_2 low. It is the case when S^b low enough in Partial Equilibrium[6]. This proves the safety multiplier in Partial Equilibrium.

Risky asset holdings x_1

Assumption 1 allows to work in the very high μ_1 (relative to μ_2 and μ_u) neighborhood. The analytical expression of $x_1 = -b + \sqrt{b^2 + 4c}$ then gives, after algebra:

$$x_1 = n^B\left\{\frac{(1+\rho)\,\sigma_2 p_2}{\sigma_1\Sigma}\left(\sigma_2 p_1 - \rho\sigma_1 p_2\right) - \left[2\frac{(1+\rho)\,p_2^2}{\sigma_1\Sigma} - \frac{\sigma_1}{\Sigma}\frac{1-\rho}{1+\rho}\right]\mu_1\frac{\lambda_u}{n^B}\right\}$$

The convexity of x_1 with respect to n^B drives the procyclicality of bank leverage in this environment. Using the equilibrium: $\lambda_u = \sqrt{\delta}n^B$:

$$x_1\left(\mu_1; p_2\right) = n^B\left\{\frac{(1+\rho)\,\sigma_2 p_2}{\sigma_1\Sigma}\left(\sigma_2 p_1 - \rho\sigma_1 p_2\right) + \left[\frac{\sigma_1}{\Sigma}\frac{1-\rho}{1+\rho} - 2\frac{(1+\rho)\,p_2^2}{\sigma_1\Sigma}\right]\mu_1\sqrt{\delta}\right\}$$

The elasticities of the partial equilibrium risky asset holding are as following:

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[6] Recall $M\hat{R}S_L = \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)-\phi\left(\frac{\mu_u}{\sigma_u}\right)\gamma_L\left\{\left(2-\frac{\mu_u^2}{\sigma_u^2}\right)\rho_{uv}y_2\sigma_2+\left(\frac{1}{2}-\frac{\mu_u^2}{\sigma_u^2}\right)\sigma_u\right\}}{1-\gamma_L\left\{\left(1-\Phi\left(\frac{\mu_u}{\sigma_u}\right)\right)\mu_u-\Phi\left(\frac{\mu_u}{\sigma_u}\right)\frac{\sigma_2}{\mu_2}\rho_{uv}\sigma_u+\frac{\sigma_2}{\mu_2}\left(y_2\sigma_2+\rho_{uv}\sigma_u\right)-\phi\left(\frac{\mu_u}{\sigma_u}\right)\right\}}$: $\rho_{uv}y_2\sigma_2$ needs to be small enough to make sure the denominator dominates on the numerator.

- $\partial_{\mu_1} x_1 = n^B \left[\frac{\sigma_1}{\Sigma}\frac{1-\rho}{1+\rho} - 2\frac{(1+\rho)p_2^2}{\sigma_1\Sigma}\right]\sqrt{\delta} \; > 0 \, for \, \rho < 0$

$$\partial_{p_2} x_1 = \frac{1}{\Sigma^2}\frac{(1+\rho)\,\sigma_2}{\sigma_1}\left(\sigma_2 p_1 - 2\rho\sigma_1 p_2\right)\left[\sigma_2^2 p_1^2 - \sigma_1^2 p_2^2\right] \\ - \frac{1}{\Sigma^2}\left[\sigma_1\frac{1-\rho}{1+\rho}\partial_{p_2}\Sigma + 2\frac{(1+\rho)}{\sigma_1}p_2\left[2\sigma_2^2 p_1^2 + \left(1-2\rho-\rho^2\right)\sigma_1\sigma_2 p_1 p_2\right]\right]\mu_1\sqrt{\delta}$$

- We already see that $\partial_{p_2} x_1 < 0$ in the neighborhood of $\rho = -1$. Denoting $u = \frac{\sigma_1 p_2}{\sigma_2 p_1}$, this condition writes: $\partial_{p_2} x_1 < 0 \Leftrightarrow \sigma_2\left(\frac{1}{u}+2\right)\left[1-u^2\right] < 4$ which is true as long as $\frac{\sigma_2}{2} < u < 1$. So this refinement of Assumption 1 prevents from imposing *any condition* on ρ to get Proposition 2: $\partial_{p_2} x_1 < 0$.

- Finally the second order condition: $\partial_{p_2}\partial_{\mu_1} x_1 < 0$ to check convexity of the problem:

$$\partial_{p_2}\partial_{\mu_1} x_1 = -\left[\sigma_1\frac{1-\rho}{1+\rho}\partial_{p_2}\Sigma + 2\frac{(1+\rho)}{\sigma_1}p_2\left[2\sigma_2^2 p_1^2 + \left(1-2\rho-\rho^2\right)\sigma_1\sigma_2 p_1 p_2\right]\right]\sqrt{\delta} \; < 0$$

Safe asset holdings x_2

At equilibrium:

$$x_2 = \frac{\mu_1}{(-\rho\sigma_1\sigma_2)}\sqrt{\delta}n^B + \frac{\sigma_1}{(-\rho\sigma_2)}\frac{\mu_1^2}{\mu_2\sigma_1}\left\{\frac{\mu_1}{\sigma_1}\sqrt{\delta} - \left(\sigma_2 + \sigma_1\frac{\mu_2}{\mu_1}\right)\right\}\sqrt{\delta}n^B$$

Inspection of $x_2\left(\mu_u, \sigma_u\right)$ gives $x_2\left(\underset{(+)}{\rho}\,,\,\underset{(+)}{p_2}\right)$. So in PE safe asset holdings x_2 *increase* with high p_2.

A.1.5 Normative analysis

Constrained efficient allocations

Consider the Lagrangian of the social planner under an arbitrary Pareto weights β

$$\begin{aligned}L &= \int -e^{-\gamma_L c_s^L} f(s)ds + \lambda \int c_s^B f(s)ds \\ &+ \int \mu_s \left\{ (x_1+y_1)\, s - c_s^L - c_s^B \right\} f(s)ds \\ &+ \nu_s^L c_s^L + \nu_s^B c_s^B \\ &+ \mu_0 \left\{ n^L + n^B - p_1\,(x_1+y_1) \right\}\end{aligned}$$

The f.o.c. in $i = (x_1+y_1)$ is:

$$\int \mu_s s f(s)ds - \mu_0 p_1 = 0$$

The f.o.c. in c_s^L is:

$$\gamma_L e^{-\gamma_L c_s^L} f(s)ds - \mu_s f(s)ds + \nu_s^L = 0$$

The f.o.c. in c_s^B is:

$$f(s)ds - \mu_s f(s)ds + \nu_s^B = 0$$

Equating μ_s from last two, we get the equality of marginal utility of wealth in state s:

$$\gamma_L e^{-\gamma_L c_s^L} + \frac{\nu_s^L}{f(s)ds} = \lambda + \frac{\nu_s^B}{f(s)ds}$$

We easily derive the constrained first best allocation, denoting $\bar{s} = \frac{p_1}{i_{t-1}}\left[\frac{1}{\gamma} ln\left(\gamma \frac{\beta^P}{\beta^A}\right) - i_t - \sum n^\theta\right]$:

$$\begin{cases} c_s^{L*} = \frac{1}{\gamma} ln\left(\gamma\frac{\beta^P}{\beta^A}\right) \; and \; c_s^{B*} = \frac{n^L+n^B}{p_1} s + i_{t-1}\frac{s_t}{p_K} - i_t - \frac{1}{\gamma} ln\left(\gamma\frac{\beta^P}{\beta^A}\right) & if\ s > \bar{s} \\ c_s^{L*} = \frac{n^L+n^B}{p_1} s + i_{t-1}\frac{s_t}{p_K} - i_t \; and \; c_s^{B*} = 0 & if\ s \leq \bar{s} \end{cases}$$

We recognize the risky debt as in the decentralized equilibrium of the within generation model in the case there is no long-term asset. This is the efficient risk-sharing agreement within generation.

The f.o.c. on investment gives:

$$c_s^{L*}\left[1-\beta\int_{\bar{s}} sf(s)ds\right] = \beta\gamma e^{-\gamma\sum n^{\theta}} e^{\gamma i_t}\int^{\bar{s}} e^{-\gamma i_{t-1}} sf(s)ds$$

Jointly with the definition of $\bar{s}$ and the efficient consumptions, the history $\{i_t\}$ of efficient levels of investment is well defined. As the equilibrium is recursive, we can write $i_t = a\, s_t$ and solve for a.

The indirect utilities derived with this allocations parametric in λ traces the Pareto frontier:

$$\begin{aligned} V^L(\lambda) &= W^L\left(\left\{c_s^{L*}\right\}\right) \\ &= \mathbb{E}_0\left[\left(-e^{-\gamma_L\left(\frac{n^L+n^B}{p_1}s\right)}\right)1_{\left\{s\leq\frac{1}{\gamma_L}ln\left(\frac{\gamma_L}{\lambda}\right)\frac{p_1}{n^L+n^B}\right\}} + \left(-e^{-\gamma_L\left(\frac{1}{\gamma_L}ln\left(\frac{\gamma_L}{\lambda}\right)\right)}\right)1_{\left\{s>\frac{1}{\gamma_L}ln\left(\frac{\gamma_L}{\lambda}\right)\frac{p_1}{n^L+n^B}\right\}}\right] \\ &= \int_{-\infty}^{\frac{1}{\gamma_L}ln\left(\frac{\gamma_L}{\lambda}\right)\frac{p_1}{n^L+n^B}}\left(-e^{-\gamma_L\left(\frac{n^L+n^B}{p_1}s\right)}\right)f(s)ds + \int_{\frac{1}{\gamma_L}ln\left(\frac{\gamma_L}{\lambda}\right)\frac{p_1}{n^L+n^B}}^{+\infty}\left(-e^{-\gamma_L\left(\frac{1}{\gamma_L}ln\left(\frac{\gamma_L}{\lambda}\right)\right)}\right)f(s)ds \end{aligned}$$

and

$$\begin{aligned} V^B(\lambda) &= W^B\left(\left\{c_s^{B*}\right\}\right) \\ &= \mathbb{E}_0\left[(0)\,1_{\left\{s\leq\frac{1}{\gamma_L}ln\left(\frac{\gamma_L}{\lambda}\right)\frac{p_1}{n^L+n^B}\right\}} + \left(\frac{n^L+n^B}{p_1}s - \frac{1}{\gamma_L}ln\left(\frac{\gamma_L}{\lambda}\right)\right)1_{\left\{s>\frac{1}{\gamma_L}ln\left(\frac{\gamma_L}{\lambda}\right)\frac{p_1}{n^L+n^B}\right\}}\right] \\ &= \int_{-\infty}^{\frac{1}{\gamma_L}ln\left(\frac{\gamma_L}{\lambda}\right)\frac{p_1}{n^L+n^B}}(0)\,f(s)ds + \int_{\frac{1}{\gamma_L}ln\left(\frac{\gamma_L}{\lambda}\right)\frac{p_1}{n^L+n^B}}^{+\infty}\left(\frac{n^L+n^B}{p_1}s - \frac{1}{\gamma_L}ln\left(\frac{\gamma_L}{\lambda}\right)\right)f(s)ds \end{aligned}$$

Public debt issuance

The Social Planner problem: which financial policy $\left(S^k, \tau^B, \tau^L\right)$ maximizes the welfare of the investors under the competitive equilibrium.

Computation of the indirect utility of risk averse investors

$$\begin{aligned} W^L = &-e^{-\gamma_L\mu_v+\frac{1}{2}\gamma_L^2\sigma_v^2} \\ &\left\{e^{-\gamma_L\mu_u+\frac{1}{2}\gamma_L^2\left(\sigma_u^2+2\rho_{uv}\sigma_v\sigma_u\right)}\left\{1-\Phi\left(\frac{\mu_u}{\sigma_u}-\gamma_L\left(\rho_{uv}\sigma_v+\sigma_u\right)\right)\right\}+\Phi\left(\frac{\mu_u}{\sigma_u}-\gamma_L\rho_{uv}\sigma_v\right)\right\} \end{aligned}$$

Developing in orders of γ_L, we can write:

$$W^L = -\left\{1-\gamma_L\left(y_2\mu_2+\bar{s}+\frac{1}{2}\mu_u-\frac{1}{2}\sqrt{\frac{2}{\pi}}\sigma_u\right)\right\}$$

Using lender budget:

$$W^L \propto \mu_2\frac{n^L-D}{p_2}+\bar{s}+\frac{1}{2}\mu_u-\frac{1}{2}\sqrt{\frac{2}{\pi}}\sigma_u$$

Recall $D=\frac{p_2}{\mu_2}M\hat{R}S_B\bar{s}$:

$$W^L \propto r^{safe}n^L+\left(r^{bank}-r^{safe}\right)D+\frac{1}{2}\mu_u-\frac{1}{2}\sqrt{\frac{2}{\pi}}\sigma_u$$

We sign as follows:

$$W^L = \underset{(-)}{r^{safe}}n^L+\underset{(-)}{SMI}\,\underset{(-)}{D}+0.5\underset{(+)}{\mu_u}-0.4\underset{(+)}{\sigma_u}$$

The trade-off is as follows: increasing p_2 worsens the wealth effect and the safety multiplier, but as bank takes less risk, it is also beneficial. There is a direct effect on n^L (a 'wealth effect': the first term $\underset{(-)}{r^{safe}}n^L$), and then there is the indirect SMI/safety creation effect (and additionally there is the cost of default: the two last terms). Both tends to call for more S^b, as long as ρ is low enough. Given that ρ is endogenous in the dynamic model, higher S^b leads to a higher ρ. As a result, there is an interior solution for S^b. Contrary to Lorenzoni and Werning (2013) and Calvo (1988), and in line with common practice, the government picks S^b and not p_2S^b. Under this last assumption, there is no Laffer curve, hence no multiple equilibria.

A.1.6 Extension with sovereign risk

Closed economy

Following the assumption made in the text the public debt market value is: $\tilde{p}_2^t = \kappa s^t \; 1_{\{s^t < \bar{\bar{s}}\}} + p_2^t \; 1_{\{s^t \geq \bar{\bar{s}}\}}$. We focus on Markov equilibria, defined exactly in the same way as in the main model. The covariance of the *post public default* price $\tilde{p}_2^t$ with s^t is now:

$$cov\left(\tilde{p}_2^t, s^t\right) \;=\; \kappa\, cov\left(s^t \; 1_{\{s^t < \bar{\bar{s}}\}}, s^t\right) + cov\left(p_2^t \; 1_{\{s^t \geq \bar{\bar{s}}\}}, s^t\right)$$

We derive, denoting $s_1^{t=1} = \mu_1$, $h = \frac{\bar{\bar{s}} - \mu_1}{\sigma_1}$:

$$\begin{aligned} cov\left(s^t \; 1_{\{s^t < \bar{\bar{s}}\}}, s^t\right) &= \sigma_1^2 \Phi\left(h\right) - \left\{\sigma_1 h + \mu_1\right\} \sigma_1 \phi\left(h\right) \\ cov\left(p_2^t \; 1_{\{s^t \geq \bar{\bar{s}}\}}, s^t\right) &= -\beta \sigma_1^2 \left(1 - \Phi\left(h\right)\right) + \left\{\alpha - \beta\mu_1 - \beta h \sigma_1\right\} \sigma_1 \phi\left(h\right) \end{aligned}$$

Therefore the total covariance is, using the linear approximation $p_2^t\left(s^t\right) = \alpha - \beta s^t$:

$$cov\left(\tilde{p}_2^t, s^t\right) \;=\; -\beta\sigma_1^2 + \left(\beta + \kappa\right)\sigma_1^2 \Phi\left(h\right) + \left\{\alpha - \left(\beta + \kappa\right)\mu_1 - \left(\beta + \kappa\right)\sigma_1 h\right\} \sigma_1 \phi\left(h\right)$$

We also have $\tilde{\mu_2} = \alpha\left(1 - \Phi\left(h\right)\right) - \beta\mu_1 + \left(\beta + \kappa\right)\left\{\mu\Phi\left(h\right) - \sigma\phi\left(h\right)\right\}$ and $\tilde{\sigma_2} = \beta\sigma_1$ in a low sovereign risk approximation.

Now, to solve for the fixed point, use the explicit expression of α and β:

$$\beta = -\frac{p_1}{S^b} \frac{\left(\mu_1 \tilde{\sigma_2} - \tilde{\rho}\tilde{\mu_2}\sigma_1\right)}{\tilde{\mu_2}\sigma_1^3} \sqrt{\delta} n^B$$

and $\alpha = \frac{n^B + n^L}{S^b}$ and $cov\left(\tilde{p}_2^t, s^t\right) = \tilde{\rho}\sigma_1\tilde{\sigma_2}$.

As in the dynamic model with no sovereign risk, we eliminate $\tilde{\mu_2}$ and $\tilde{\sigma_2}$, and collate in $\tilde{\rho}$. In the low sovereign risk approximation to get rid of the ϕ terms:

$$\tilde{\rho} = -1 + \left(1 + \frac{\kappa}{\beta}\right)\Phi\left(h\right) \tag{A.12}$$

The right strategy is to add the sovereign term through the linear term and the quadratic term from the dynamic model, and only then solve for the fixed point. This gives:

$$\tilde{\rho} = -1 + \left(1 + \frac{\bar{\kappa}}{\beta}\right)\Phi\left(h\right) + \frac{16\mu^2}{\left(\mu - \frac{S^b}{p_1}\frac{1}{n^B\frac{1-\rho}{1+\rho}\sqrt{\delta}|\partial_{p_2}\kappa|}\right)^2}$$

with $\beta = |\frac{dp_2}{d\mu_1}| = \frac{p_1\partial_{\mu_1}x_1}{S^b+p_1\partial_{p_2}x_1}$ and $\partial_{\mu_1}x_1 = n^B\frac{\sigma_1}{\sigma_2^2p_1^2}\frac{1-\rho}{1+\rho}\sqrt{\delta}$. So we write the new beta:

$$\tilde{\rho} + 1 = \left(1 + \frac{\bar{\kappa}}{\beta}\right)\Phi\left(h\right) + \frac{16\mu^2}{\left(\mu - \frac{S^b}{p_1}\frac{1}{n^B\frac{1-\rho}{1+\rho}\sqrt{\delta}|\partial_{p_2}\kappa|}\right)^2}$$

Doing the same manipulation as in the no sovereign environment, denoting $u = \tilde{\rho} + 1$:

$$u = \left(1 + \bar{\kappa}\frac{\partial_{p_2}\kappa}{\kappa}\left(\mu_1 - \frac{S^b}{p_1}\frac{1}{n^B\frac{1-\rho}{1+\rho}\sqrt{\delta}|\partial_{p_2}\kappa|}\right)\right)\Phi\left(h\right) + \frac{16\mu^2}{\left(\mu - \frac{S^b}{p_1}\frac{1}{n^B\frac{1-\rho}{1+\rho}\sqrt{\delta}|\partial_{p_2}\kappa|}\right)^2}$$

Of the same manner as in the no sovereign risk model:

$$\tilde{\rho} = -1 + \frac{\left(\frac{S^b}{n^B}\frac{\left[\hat{\sigma}_2^2p_1^2+\sigma_1^2\hat{p}_2^2\right]^2}{2\sigma_1^4p_1^2\hat{p}_2^2}\right)^2 + \left(1 + \kappa\frac{1}{\hat{p}_2}\left(s^t - \frac{S^b}{n^B}\frac{\left[\hat{\sigma}_2^2p_1^2+\sigma_1^2\hat{p}_2^2\right]^2}{2\sigma_1^4p_1^2\hat{p}_2^2}\right)\right)\Phi\left(\frac{\bar{\bar{s}}-s^t}{\sigma_1}\right)}{1 + \kappa\frac{1}{\hat{p}_2}\frac{S^b}{n^B}\frac{\left[\hat{\sigma}_2^2p_1^2+\sigma_1^2\hat{p}_2^2\right]^2}{2\sigma_1^4p_1^2\hat{p}_2^2}\Phi\left(\frac{\bar{\bar{s}}-s^t}{\sigma_1}\right)}$$

We directly observe that increasing $\bar{s}$ (sovereign risk through h) increases $\tilde{\rho}$ and therefore destroys the endogenous hedging properties of public debt.

Open economy

In the set up of the open economy model, we have:

$$\sigma_u^2 = x_1^2\sigma_1^2 + 2\rho x_1x_2\sigma_1\sigma_2 + x_2^2\sigma_2^2 + x_3^2\sigma_3^2 + 2\rho x_1x_3\sigma_1\sigma_3$$

$$\mu_u = x_1\mu_1 + x_2\mu_2 + x_3\mu_3 - \bar{s}$$

It adds a third f.o.c fro each bank:

$$MRS_B^S = \frac{\frac{\partial W^B}{\partial x_2}}{\frac{\partial W^B}{\partial x_1}} = \frac{\mu_2 \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \sigma_2 \frac{x_2\sigma_2 + \rho_2 x_1 \sigma_1}{\sigma_u} \phi\left(\frac{\mu_u}{\sigma_u}\right)}{\mu_1 \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \sigma_1 \frac{(x_1\sigma_1 + \rho_2 x_2 \sigma_2 + \rho_3 x_3 \sigma_3)}{\sigma_u} \phi\left(\frac{\mu_u}{\sigma_u}\right)}$$

$$MRS_B^N = \frac{\frac{\partial W^B}{\partial x_3}}{\frac{\partial W^B}{\partial x_1}} = \frac{\mu_3 \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \sigma_3 \frac{x_3\sigma_3 + \rho_3 x_1 \sigma_1}{\sigma_u} \phi\left(\frac{\mu_u}{\sigma_u}\right)}{\mu_1 \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \sigma_1 \frac{(x_1\sigma_1 + \rho_2 x_2 \sigma_2 + \rho_3 x_3 \sigma_3)}{\sigma_u} \phi\left(\frac{\mu_u}{\sigma_u}\right)}$$

In the open economy we derive:

$$(p_1\sigma_2\rho_2 - p_2\sigma_1)\, x_1\sigma_1 + (p_1\sigma_2 - p_2\sigma_1\rho_2)\, x_2\sigma_2 + (0 - p_2\sigma_1\rho_3)\, x_3\sigma_3 = (p_2\mu_1 - p_1\mu_2)\, \sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)}$$

Similarly, on the third asset:

$$(p_1\sigma_3\rho_3 - p_3\sigma_1)\, x_1\sigma_1 + (0 - p_3\sigma_1\rho_2)\, x_2\sigma_2 + (p_1\sigma_3 - p_3\sigma_1\rho_3)\, x_3\sigma_3 = (p_3\mu_1 - p_1\mu_3)\, \sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)}$$

The second object is the Marginal Rate of Transformation between "Asset 1" and "leverage '"(promise $\bar{s}$):

$$D'(\bar{s}) = \frac{p_1}{\mu_1 + \sigma_1 \frac{(x_1\sigma_1 + \rho_2 x_2\sigma_2 + \rho_3 x_3 \sigma_3)}{\sigma_u} \frac{\phi\left(\frac{\mu_u}{\sigma_u}\right)}{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}}$$

So the asset side correlation metrics is now:

$$X = \frac{(x_2\sigma_2 + \rho_2 x_1 \sigma_1)}{(x_1\sigma_1 + \rho_2 x_2 \sigma_2 + \rho_3 x_3 \sigma_3)}$$

and we still have:

$$D'(\bar{s}) \quad = \quad \frac{p_2}{\mu_2} \frac{1 - \frac{p_1}{p_2}\frac{\sigma_2}{\sigma_1} X}{1 - \frac{\mu_1}{\mu_2}\frac{\sigma_2}{\sigma_1} X}$$

So in the open economy model, all the results go through as long as we use the new X. Consider the South bank:

$$\begin{cases} x_1 p_1 + x_2 p_2 + x_3 p_3 & = n^B + \alpha \\ \left(p_1\sigma_2\rho_2 - p_2\sigma_1\right) x_1\sigma_1 + \left(p_1\sigma_2 - p_2\sigma_1\rho_2\right) x_2\sigma_2 + \left(0 - p_2\sigma_1\rho_3\right) x_3\sigma_3 & = \left(p_2\mu_1 - p_1\mu_2\right)\sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)} \\ \left(p_1\sigma_3\rho_3 - p_3\sigma_1\right) x_1\sigma_1 + \left(0 - p_3\sigma_1\rho_2\right) x_2\sigma_2 + \left(p_1\sigma_3 - p_3\sigma_1\rho_3\right) x_3\sigma_3 & = \left(p_3\mu_1 - p_1\mu_3\right)\sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)} \end{cases}$$

Where we denote:

$$\alpha = \frac{p_2}{\mu_2}\hat{MRS}_B\left(x_1\mu_1 + x_2\mu_2 + x_3\mu_3 - \mu_u\right)$$

By inverting the 2x2 system in $(x_2\sigma_2, x_3\sigma_3)$:

$$\begin{cases} \left(p_1\sigma_2 - p_2\sigma_1\rho_2\right) x_2\sigma_2 + \left(0 - p_2\sigma_1\rho_3\right) x_3\sigma_3 & = \left(p_2\mu_1 - p_1\mu_2\right)\sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)} - \left(p_1\sigma_2\rho_2 - p_2\sigma_1\right) x_1\sigma_1 \\ \left(0 - p_3\sigma_1\rho_2\right) x_2\sigma_2 + \left(p_1\sigma_3 - p_3\sigma_1\rho_3\right) x_3\sigma_3 & = \left(p_3\mu_1 - p_1\mu_3\right)\sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)} - \left(p_1\sigma_3\rho_3 - p_3\sigma_1\right) x_1\sigma_1 \end{cases}$$

Linear algebra (matrix notation for the general case of N sovereign bonds) leads to:

$$x_2\sigma_2 = \Sigma_2^u \sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)} + \Sigma_2^1 x_1\sigma_1$$

with $\Sigma_2^u = p_2 p_1\left(\sigma_3\mu_1 - \sigma_1\rho_3\right) - p_1^2\sigma_3 + p_1 p_3\sigma_1\rho_3$ and $\Sigma_2^1 = p_1 p_2\sigma_1\sigma_3\left(1 - \rho_3^2\right) - p_1^2\sigma_3\sigma_2\rho_2 + p_1 p_3\sigma_1\sigma_2\rho_2\rho_3$.

By symmetry

$$x_3\sigma_3 = \Sigma_3^u \sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)} + \Sigma_3^1 x_1\sigma_1$$

Finally, using the budget constraint of the bank:

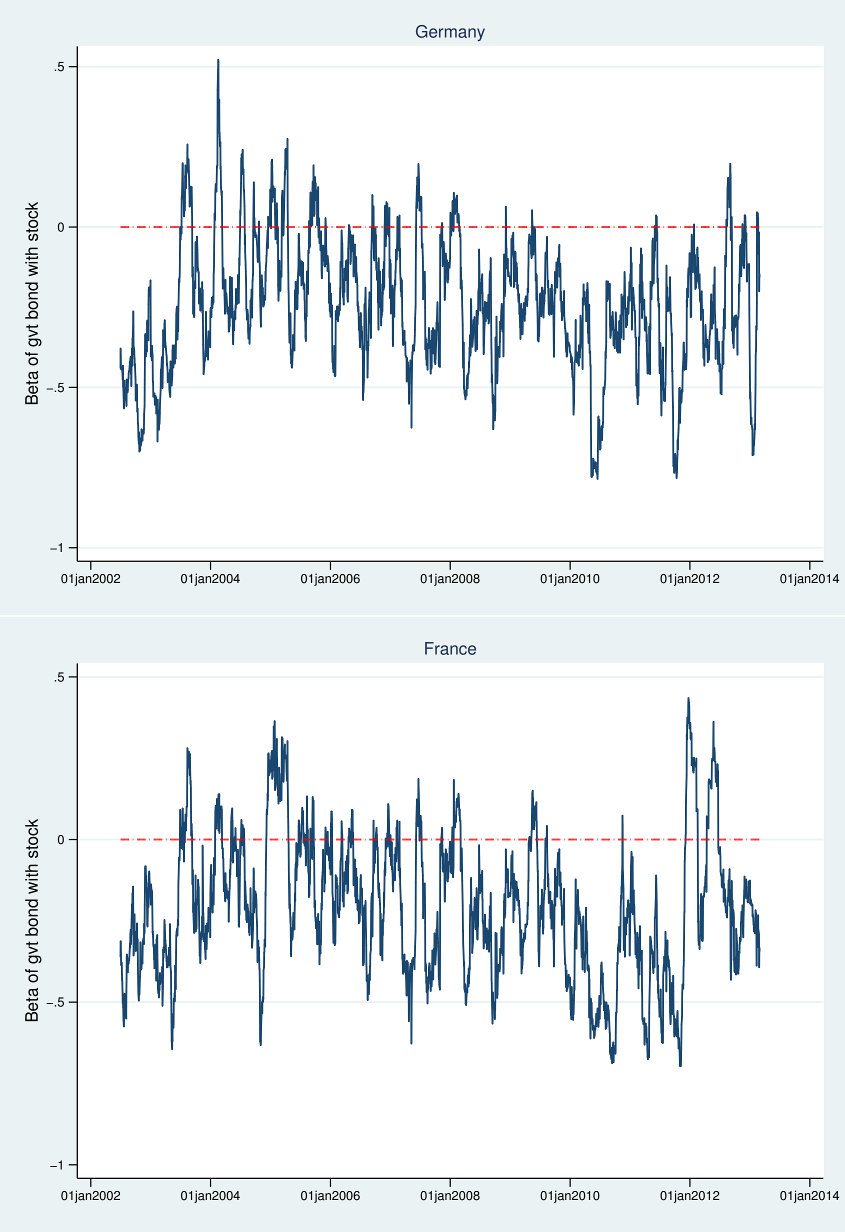
$$\begin{aligned}
\left(p_1 - \frac{p_2}{\mu_2}M\hat{R}S_B\mu_1\right)x_1 + \left(p_2 - \frac{p_2}{\mu_2}M\hat{R}S_B\mu_2\right)x_2 + \left(p_3 - \frac{p_2}{\mu_2}M\hat{R}S_B\mu_3\right)x_3 &= n^B - \frac{p_2}{\mu_2}M\hat{R}S_B\mu_u \\
\left[p_1 + p_2\frac{\sigma_1}{\sigma_2}\Sigma_2^1 + p_3\frac{\sigma_1}{\sigma_3}\Sigma_3^1 - M\hat{R}S_B\left(\frac{p_2}{\mu_2}\mu_1 + \frac{p_2}{\mu_2}\mu_2\frac{\sigma_1}{\sigma_2}\Sigma_2^1 + \frac{p_2}{\mu_2}\mu_3\frac{\sigma_1}{\sigma_3}\Sigma_3^1\right)\right]x_1 & \\
+\left[p_2\frac{1}{\sigma_2}\Sigma_2^u + p_3\frac{1}{\sigma_3}\Sigma_3^u - \frac{p_2}{\mu_2}M\hat{R}S_B\left(\frac{\mu_2}{\sigma_2}\Sigma_2^u + \frac{\mu_3}{\sigma_3}\Sigma_3^u\right)\right]\sigma_u\frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)} &= n^B - \frac{p_2}{\mu_2}M\hat{R}S_B\mu_u
\end{aligned}$$

Now using $M\hat{R}S_B = \frac{1-\frac{p_1}{p_2}\frac{\sigma_2}{\sigma_1}X}{1-\frac{\mu_1}{\mu_2}\frac{\sigma_2}{\sigma_1}X}$ and $X = \frac{(x_2\sigma_2+\rho_2 x_1\sigma_1)}{(x_1\sigma_1+\rho_2 x_2\sigma_2+\rho_3 x_3\sigma_3)}$, this defines the equilibrium. The equilibrium $\mu_u\sigma_u$ is the same as in the closed economy (the debt-pricing mapping, so far the two banks are symmetric.). This pins down a $x_1(\mu_u, \sigma_u)$.

We show the "redomestication" result as follows, through a perturbation argument. Consider South bond loosing its hedging property: $\rho_3 < \rho_2 = 0$ *and* higher volatility $\sigma_2 > \sigma_3$. This amounts to a higher γ_L so at the margin, lower μ_u and higher σ_u. It is less expensive to lever up for a South bank. From the eq $x_1(\mu_u, \sigma_u)$, more investment x_1. From the x_2 and x_3 expressions (as $\sigma_u\frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)}$ is pinned down by the closed economy equilibrium), an increasing x_1 *crowds in* x_2 *and* x_3 (to hedge against default). The sensitivity of which one increases more is commanded by $\frac{\Sigma_2^1}{\sigma_2}$ and $\frac{\Sigma_3^1}{\sigma_3}$. To make x_2 increase more, despite σ_2 increasing, it suffices to make $\Sigma_2^1 > \Sigma_3^1$. This is the case as ρ_3^2 decreases, making Σ_2^1 increasing more than Σ_3^1. As result, x_2^{South} increases more at the margin than x_2^{North}. Starting from the symmetric equilibrium, we obtain redomestication of public debt.

A.2 Empirical Appendix

A.2.1 Betas of Eurozone government bonds


Germany

France

Figure A.2: *Betas of government bonds with DJ EUROSTOXX 50 . Core countries: Germany (top) and France (bottom). Source: Financial Database.*

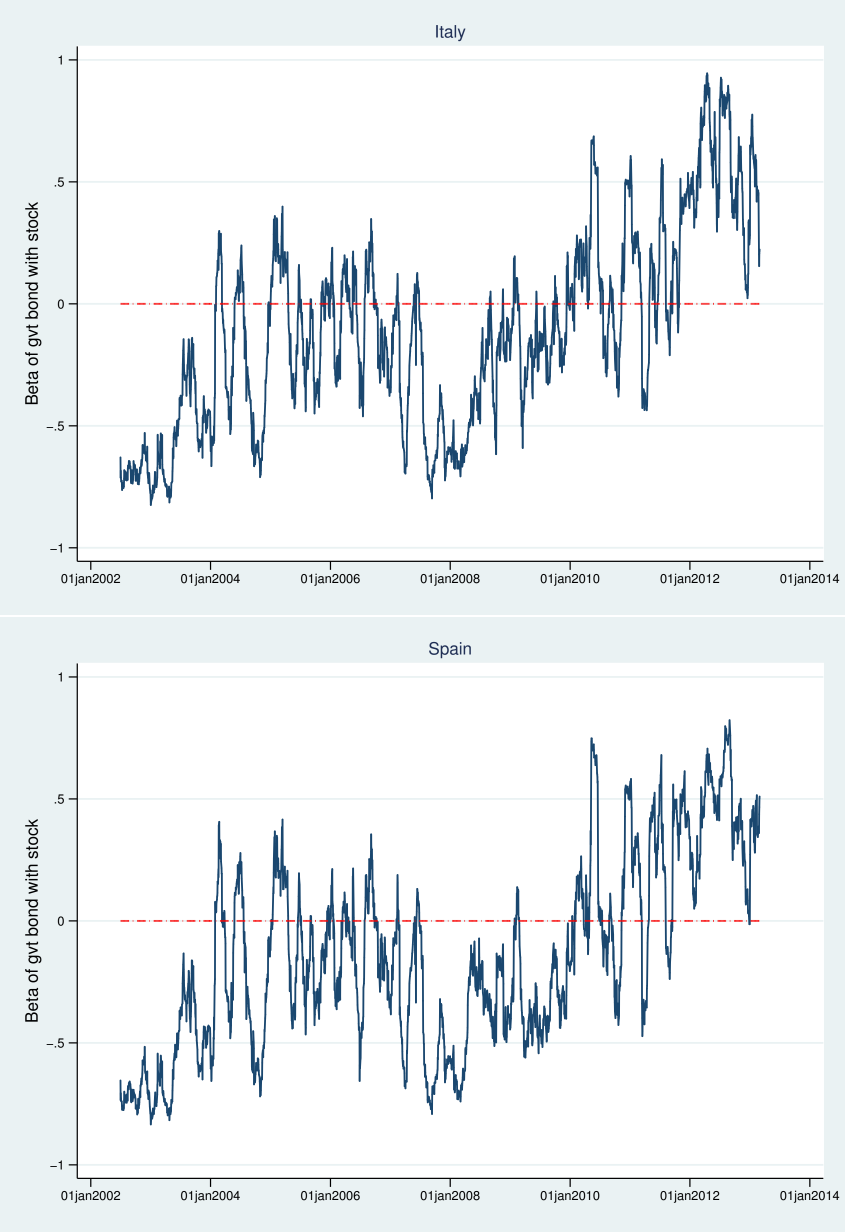


Figure A.3: *Betas of government bonds with DJ EUROSTOXX 50 . Periphery countries: Italy (top) and Spain (bottom). Source: Financial Database.*

A.2.2 Concentration of ownership for Eurozone public debt

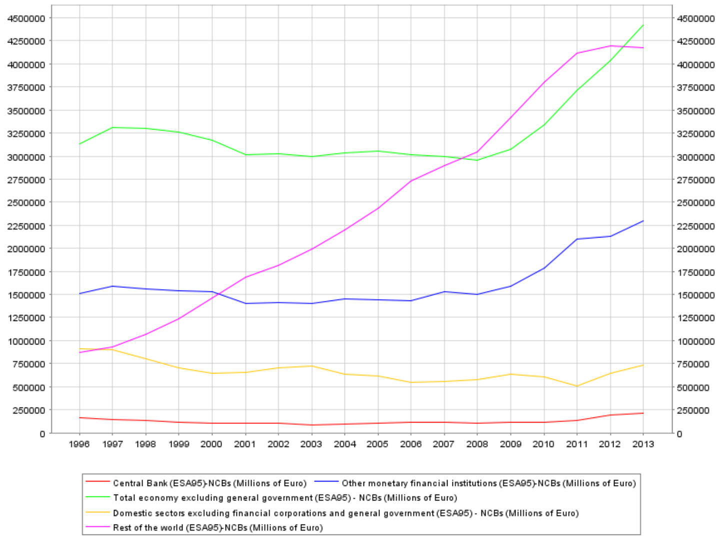


Figure A.4: *Breakdown of Eurozone public debt by type of holders.*

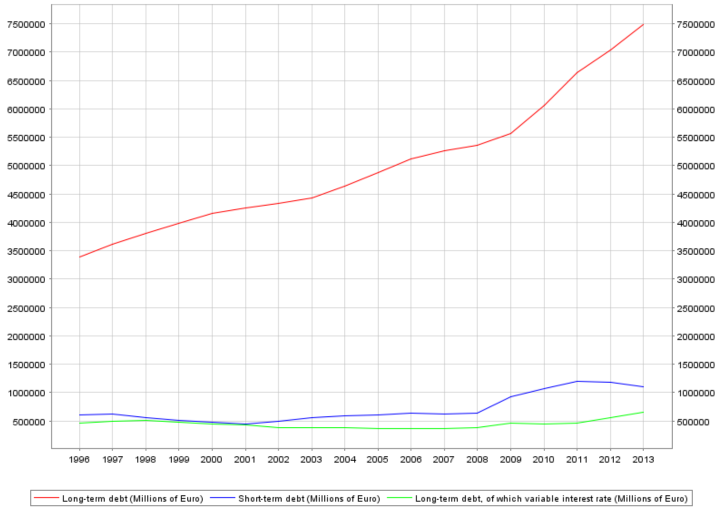


Figure A.5: *Breakdown of Eurozone public debt by maturity: long-term in red, short-term in blue.*

A.2.3 Predictive regression of $SMI = r^{safe} - r^{bank}$ on bank profits Π^B

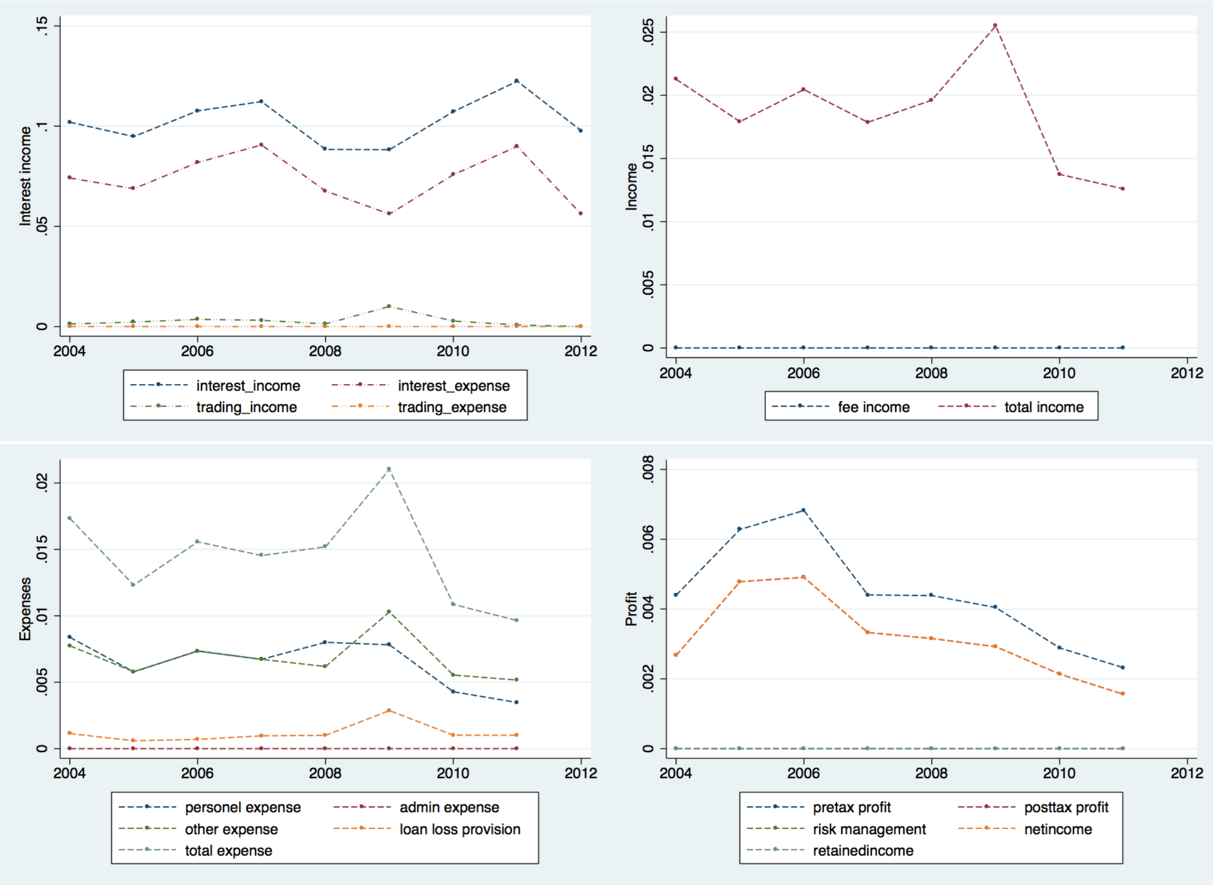


Figure A.6: *Decomposition of European banks profits.*

A.2.4 Open economy

Redomestication of government debt

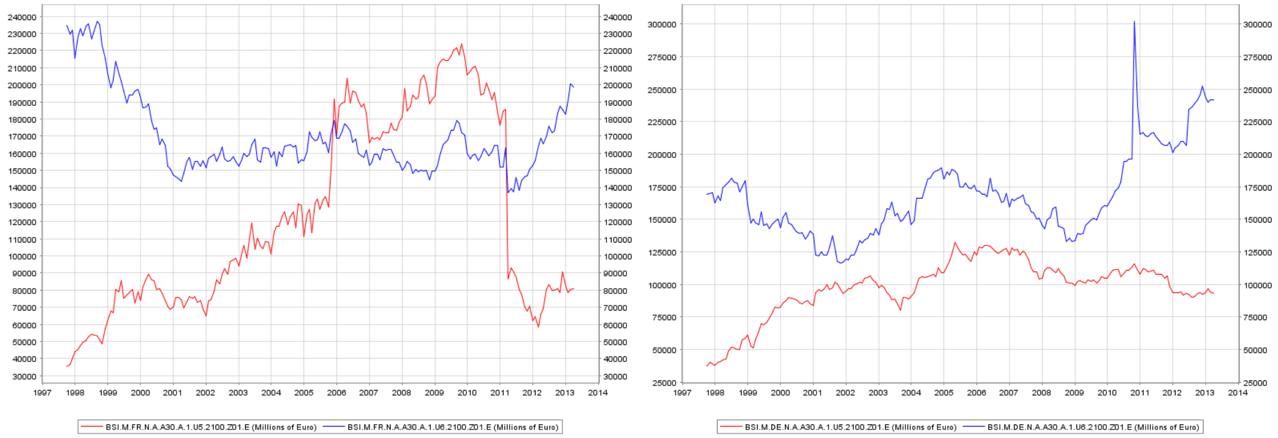


Figure A.7: *France (left) and Germany (right) government bond holdings by banks. Foreign debt in red, domestic debt in blue.*

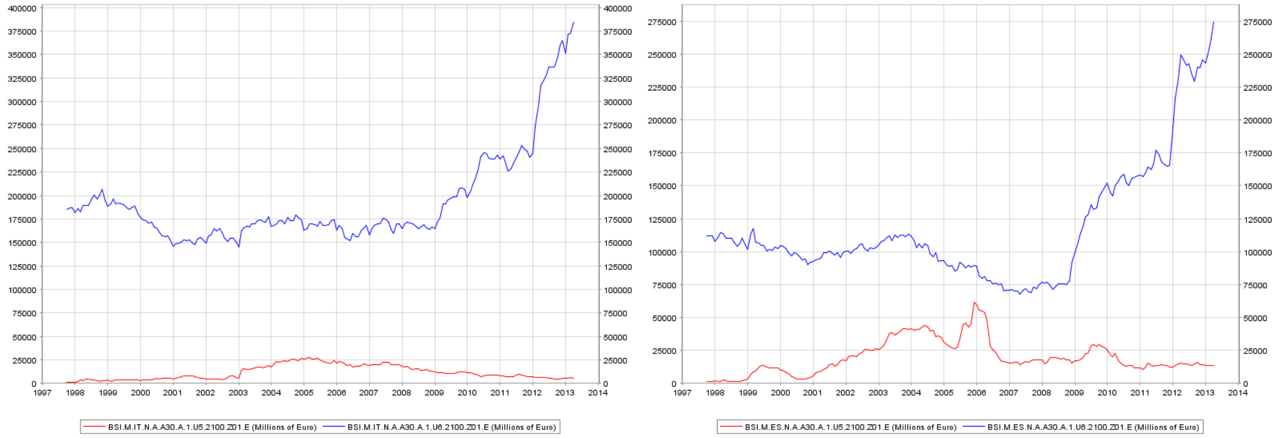


Figure A.8: *Italy (left) and Spain (right) government bond holdings by banks. Foreign debt in red, domestic debt in blue.*

Bank leverage by country

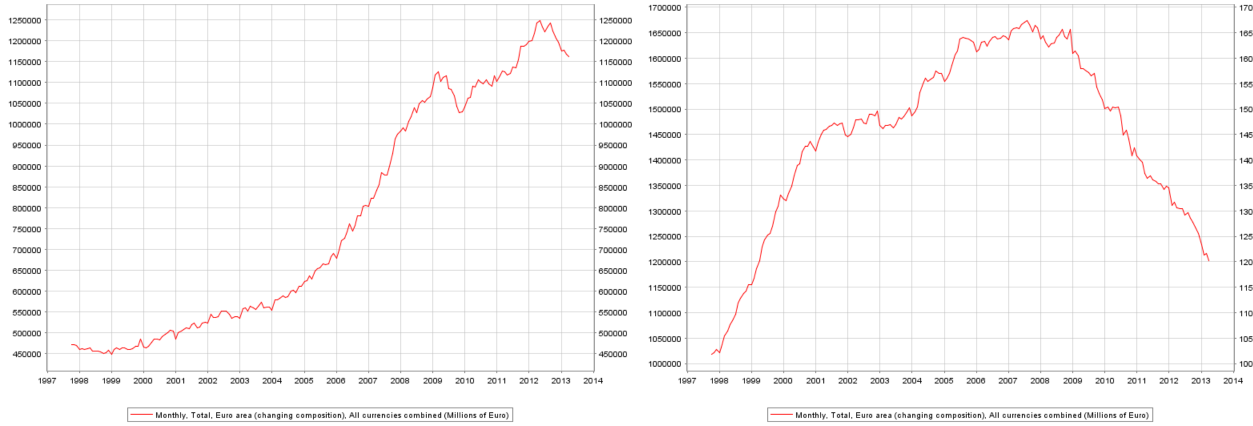


Figure A.9: *France (left) and Germany (right) bank short-term debt: decrease of leverage. Source: ECB.*

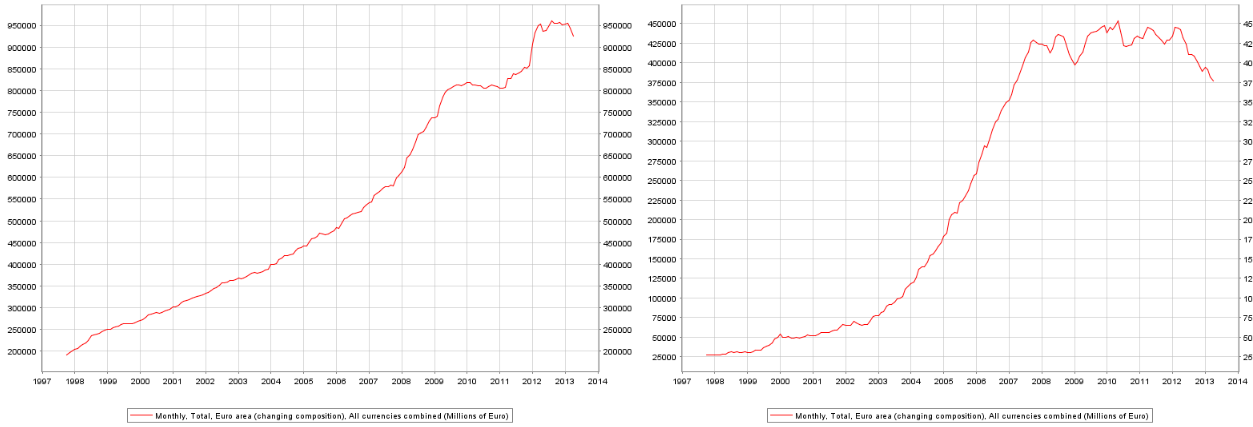


Figure A.10: *Italy (left) and Spain (right) bank short-term debt: stable leverage. Source: ECB.*

Appendix B

Appendix to Chapter 2

B.1 Static model

B.1.1 Optimal contract

There are I borrowers and J lenders. We assume there are no inactive borrower or lender in equilibrium. We focus on symmetric pairwise stable equilibria. The collateral constraint of each borrower is: $\sum_J x_{ij} m_{ij} p \leq n^B$ so $x_{ij} = \frac{1}{J} \frac{n^B}{m_{ij} p}$. We note $m_{ij} = m$.

Expected utility of each borrower The expected utility of the borrower, for given contracts x_{ij}, is:

$$U^B_{I,J} = \textstyle\sum_J x_{ij}\, \mathbb{E}_B \left[1_{\{no\,def\}} \left(s - \bar{s} \right) \right] + \beta \mathbb{E}_B \left[1_{\{no\,def\}} V^B \right] \text{ with } \textstyle\sum_J x_{ij} m_{ij} p \leq n^B$$

Using $\tilde{s} = \bar{s} - V^B$ the default threshold, recalling $mp = p - D$ and $\pi^B(\tilde{s}) = \int_{s^{min}}^{\tilde{s}} \left(\tilde{s} - s \right) f_B(s) ds$ (with s^{min} and s^{max} potentially equal to $\pm\infty$):

$$\mathbb{E}_B \left[1_{\{no\,def\}} \left(s - \bar{s} \right) \right] = \int_{\bar{s} - V^B}^{s^{max}} \left(s - \bar{s} \right) ds = p \left(1 + \mu^B \right) - \tilde{s} - \left(1 - F_B(\tilde{s}) \right) V^B + \pi^B(\tilde{s})$$

$$\left(U^B_{I,J} - U^B_0 \right) (\tilde{s}, D) \quad = \quad \frac{n^B}{p - D} \left(p \left(1 + \mu^B \right) - \tilde{s} - \left(1 - F_B(\tilde{s}) \right) V^B + \pi^B(\tilde{s}) \right) + \beta V^B \left(1 - F_B(\tilde{s}) \right) - n^B \left(1 + \mu^B \right) + \beta V^B$$

The expected utility of the borrower can also be written as a function of r and m only:

$$\left(U^B_{I,J} - U^B_0\right)(r,m) \quad = \quad \frac{n^B}{m}\left[(1-m)\left(\mu^B - r\right) + \frac{1}{p}\pi^B(\tilde{s}) + \left(1 - \frac{\beta}{n^B}mp\right)V^B\frac{1}{p}F_B(\tilde{s})\right]$$

Expected utility of each lender The expected utility of each lender, for given contracts x_{ij}, is[1]:

$$\left(U^L_{I,J} - U^L_0\right)(\tilde{s}, D) = \textstyle\sum_i x_{ij}\mathbb{E}_L\left[1_{\{def\}}s + 1_{\{no\,def\}}\bar{s}\right] + \left(n^L - \sum_i x_{ij}D\right)(1+\mu^L) - n^L(1+\mu^L)$$

With $\mathbb{E}_L\left[1_{\{def\}}s + 1_{\{no\,def\}}\bar{s}\right] = \int_{s^{min}}^{\tilde{s}} s\, f_L(s)ds + \left(\tilde{s} + V^B\right)\left(1 - F_L(\tilde{s})\right) = \tilde{s} + \left(1 - F_L(\tilde{s})\right)V^B - \pi^L(\tilde{s})$:

$$\left(U^L_{I,J} - U^L_0\right)(\tilde{s}, D) \quad = \quad \frac{I}{J}\frac{n^B}{p-D}\left(\tilde{s} + \left(1 - F_L(\tilde{s})\right)V^B - \pi^L(\tilde{s}) - (1+\mu^L)D\right)$$

We can also write it as a function of r and m only:

$$\left(U^L_{I,J} - U^L_0\right)(r,m) \quad = \quad \frac{I}{J}\frac{n^B}{m}\left[(1-m)\left(r - \mu^L\right) - \left(\frac{1}{p}\pi^L(\tilde{s}) + V^B\frac{1}{p}F_L(\tilde{s})\right)\right]$$

Multilateral Nash bargaining Denoting ω the bargaining power of the lender, the surplus of each bilateral relationship is shared according to:

$$\omega\left(U^B_{I,J} - U^B_{I,J-1}\right) = (1-\omega)\left(U^L_{I,J} - U^L_{I-1,J}\right)$$

I formulate the induction hypothesis: $U^B_{I,J-1} - U^B_0 = (1-\omega)n^B S_{I,J-1}$ and $U^L_{I-1,J} - U^L_0 = \omega n^B S_{I-1,J}$. The bargaining does not into account he continuation term of the borrower (static bargaining).

---

[1] The extension where lenders have mean-variance preferences adds a penalty term quadratic in x^2_{ij}. This delivers extra diversification benefits with respect to J. Lender heterogeneity can also be thought along the γ dimension.

$$\left(U^B_{I,J} - U^B_0\right)(r,m) \quad = \quad \frac{I}{J}\frac{n^B}{m}\left[(1-m)\left(r - \mu^L\right) - \Pi^L(\tilde{s})\right] - \frac{I}{J^2}\gamma\left(\frac{n^B}{mp}\right)^2$$

$$\omega \left\{ \frac{n^B}{p-D} \mathbb{E}_B \left[1_{\{no\,def\}} \left(s - \bar{s} \right) \right] - n^B \left(1 + \mu^B \right) \right\} =$$
$$(1-\omega) \left\{ \frac{I}{J} \frac{n^B}{p-D} \left(\mathbb{E}_L \left[1_{\{def\}} s + 1_{\{no\,def\}} \bar{s} \right] - (1+\mu^L) D \right) \right\}$$

I define:

$$\delta = \frac{\omega}{(1-\omega)\frac{I}{J}+\omega} \text{ and } 1-\delta = \frac{(1-\omega)\frac{I}{J}}{(1-\omega)\frac{I}{J}+\omega} \text{ and } \delta^{IJ} = \frac{(1-\omega)\omega}{(1-\omega)\frac{I}{J}+\omega} = \delta(1-\omega)$$

$\delta \in [0;1]$ is a measure of the *lenders effective bargaining power*. Rearranging terms yields the value D of the contract obtained from the bargaining:

$$D = \frac{(1-\delta)\mathbb{E}_L\left[1_{\{def\}}s+1_{\{no\,def\}}\bar{s}\right]+\delta\left(\left(1+\mu^B\right)p-\mathbb{E}_B\left[1_{\{no\,def\}}(s-\bar{s})\right]\right)+\delta^{IJ}\left(S_{I,J-1}-S_{I-1,J}\right)p}{(1-\delta)(1+\mu^L)+\delta(1+\mu^B)+\delta^{IJ}\left(S_{I,J-1}-S_{I-1,J}\right)}$$

I define the levered return (per unit of net worth) perceived by the borrower:

$$R_B^{lev} = \frac{\mathbb{E}_B\left[1_{\{no\,def\}}(s-\bar{s})\right]}{p-D}$$

I note $\mu^\delta = (1-\delta)\mu^L + \delta\mu^B$ and $\mu^{\delta\,barg} = \mu^\delta + \delta^{IJ}\left(S_{I,J-1} - S_{I-1,J}\right)$. Given the value of the contract D derived from the bargaining:

$$R_B^{lev} = \left(1+\mu^{\delta\,barg}\right) \frac{\mathbb{E}_B\left[1_{\{no\,def\}}(s-\bar{s})\right]}{(1-\delta)(1+\mu^L)p-(1-\delta)\mathbb{E}_L\left[1_{\{def\}}s+1_{\{no\,def\}}\bar{s}\right]+\delta\mathbb{E}_B\left[1_{\{no\,def\}}(s-\bar{s})\right]}$$

I now introduce the compound distribution: $F_\delta(s) = (1-\delta)F_L(s) + \delta F_B(s)$. It is a linear combination of normal distributions, thus: $F_\delta \sim N\left(\mu^\delta; \sigma\right)$. The beliefs δ and B still satisfy the hazard rate order property:

$$\forall \tilde{s}, \frac{f_\delta(\tilde{s})}{1-F_\delta(\tilde{s})} > \frac{f_B(\tilde{s})}{1-F_B(\tilde{s})}$$

Using the expressions of $\mathbb{E}_L\left[1_{\{def\}}s + 1_{\{no\,def\}}\bar{s}\right]$ and $\mathbb{E}_B\left[1_{\{no\,def\}}\left(s-\bar{s}\right)\right]$ as a function of the put perceived valuation $\pi^B(\tilde{s})$ and $\pi^L(\tilde{s})$, and denoting $\pi^\delta(\tilde{s})$ the put valuation under the compound beliefs F_δ, I obtain the following expression for R_B^{lev}, introducing $R_\delta^{unl}(\tilde{s})$ and $R_B^{unl}(\tilde{s})$:

$$R_B^{lev}(\tilde{s}) = \left(1+\mu^{\delta\, barg}\right) \frac{p\left(1+\mu^B\right)-\tilde{s}-(1-F_B(\tilde{s}))V^B+\pi^B(\tilde{s})}{p(1+\mu^\delta)-\tilde{s}-(1-F_\delta(\tilde{s}))V^B+\pi^\delta(\tilde{s})} = \left(1+\mu^{\delta\, barg}\right) \frac{R_B^{unl}(\tilde{s})}{R_\delta^{unl}(\tilde{s})}$$

As a result I have expressed the borrower expected utility as a function of the contract riskiness $\tilde{s}$ only:

$$\begin{aligned}\left(U_{I,J}^B - U_0^B\right)(\tilde{s}) = n^B \left(1+\mu^{\delta\, barg}\right) \\ \left(\frac{p\left(1+\mu^B\right)-\tilde{s}-(1-F_B(\tilde{s}))\,V^B+\pi^B(\tilde{s})}{p(1+\mu^\delta)-\tilde{s}-(1-F_\delta(\tilde{s}))\,V^B+\pi^\delta(\tilde{s})} - \frac{\beta V^B}{n^B\left(1+\mu^{\delta\, barg}\right)}F_B(\tilde{s})\right) \\ - n^B\left(1+\mu^B\right)\end{aligned}$$

Borrower maximization program The borrower solves for the optimal riskiness $\tilde{s}$:

$$\underset{\{\tilde{s}\}}{Max}\ \left(U_{I,J}^B - U_0^B\right)(\tilde{s}) \propto \underset{\{\tilde{s}\}}{Max}\left(\frac{R_B^{unl}(\tilde{s})}{R_\delta^{unl}(\tilde{s})} - \frac{\beta V^B}{n^B\left(1+\mu^{\delta\, barg}\right)}F_B(\tilde{s})\right)$$

The first order condition delivers:

$$0 = -R_\delta^{unl} + R_B^{unl}\frac{\partial_{\tilde{s}} R_\delta^{unl}}{\partial_{\tilde{s}} R_B^{unl}} - \frac{\beta V^B}{n^B\left(1+\mu^{\delta\, barg}\right)}\frac{\left(R_\delta^{unl}\right)^2}{\partial_{\tilde{s}} R_B^{unl}} f_B$$

Rearranging terms:

$$\begin{aligned}\left(1+\mu^\delta\right)p \quad = \quad & \left[(1-F_\delta(\tilde{s})) - \frac{1-f_\delta(\tilde{s})V^B - F_\delta(\tilde{s})}{1-f_B(\tilde{s})V^B - F_B(\tilde{s})}\left(1-F_B(\tilde{s})\right)\right]V^B \\ & + \left[\tilde{s}-\pi^\delta(\tilde{s})\right] + \frac{1-f_\delta(\tilde{s})V^B - F_\delta(\tilde{s})}{1-f_B(\tilde{s})V^B - F_B(\tilde{s})}\left[p\left(1+\mu^B\right)-\tilde{s}+\pi^B(\tilde{s})\right] \\ & - \frac{\beta V^B}{n^B\left(1+\mu^{\delta\, barg}\right)}\frac{f_B(\tilde{s})}{1-f_B(\tilde{s})V^B - F_B(\tilde{s})}\left(\mathbb{E}_\delta\left[1_{\{no\, def\}}\left(s-\bar{s}\right)\right]\right)^2\end{aligned}$$

Introducing the auxiliary functions measuring the distortion of beliefs with V^B:

$$\kappa_1(\tilde{s};V^B) = \frac{f_\delta(\tilde{s})(1-F_B(\tilde{s}))-f_B(\tilde{s})(1-F_\delta(\tilde{s}))}{1-F_B(\tilde{s})-f_B(\tilde{s})\,V^B} \text{ and } \kappa_2(\tilde{s};V^B) = \frac{1-V^B\frac{f_\delta(\tilde{s})}{1-F_\delta(\tilde{s})}}{1-V^B\frac{f_B(\tilde{s})}{1-F_B(\tilde{s})}}$$

$$\kappa_3(\tilde{s};V^B) = \frac{1-F_\delta(\tilde{s})-f_\delta(\tilde{s})\,V^B}{1-F_B(\tilde{s})-f_B(\tilde{s})\,V^B} = \frac{1-F_\delta(\tilde{s})}{1-F_B(\tilde{s})}\kappa_2(\tilde{s};V^B)$$

By the hazard rate property we have $0 \leq \kappa^1 \leq 1$ and $0 \leq \kappa^3 \leq 1$. κ_1 and κ_3 are related by: $(1 - F_\delta(\tilde{s})) - \kappa_3 (1 - F_B(\tilde{s})) = V^B \kappa_1$. I then write the f.o.c:

$$\begin{aligned} p(1+\mu^\delta) &= \kappa_1 V^B \left(\tilde{s} + V^B\right) + F_\delta(\tilde{s})\, \mathbb{E}_\delta\left[s | s < \tilde{s}\right] + (1 - F_\delta(\tilde{s}))\ \kappa_2\, \mathbb{E}_B\left[s | s > \tilde{s}\right] \\ &\quad - \frac{\beta V^B}{n^B \left(1 + \mu^{\delta\, barg}\right)} \frac{f_B(\tilde{s})}{1 - f_B(\tilde{s}) V^B - F_B(\tilde{s})} \left(\mathbb{E}_\delta\left[1_{\{no\, def\}}\left(s - \bar{s}\right)\right]\right)^2 \end{aligned}$$

This first order condition implicitly defines the riskiness $\tilde{s}$ of the optimal contract:

$$0 = F\left(\tilde{s}; \mu^\delta, \mu^B, \sigma, V^B, p, \mu^{\delta\, barg}, \beta\right)$$

B.1.2 Comparative statics

For any given parameter $a \in \left\{\mu^\delta, \mu^B, \sigma, V^B, p, \mu^{\delta\, barg}, \beta\right\}$, the implicit function theorem gives: $\partial_a \tilde{s} = -\frac{\partial_a F}{\partial_{\tilde{s}} F}$.

I first compute $\partial_{\tilde{s}} F$ (for $\beta \sim 0$: weak care for continuation):

$$\partial_{\tilde{s}} F = \partial_{\tilde{s}} \kappa^1 V^B \left(\tilde{s} + V^B\right) + \kappa^1 V^B + \tilde{s} f_\delta(\tilde{s}) + \partial_{\tilde{s}} \kappa_3 \left(1 - F_B(\tilde{s})\right) \mathbb{E}_B\left[s | s > \tilde{s}\right] - \kappa_3 \tilde{s} f_B(\tilde{s})$$

We can verify that:

$$\partial_{\tilde{s}} \kappa_3 = -\kappa_3 \left[\frac{\left(f_\delta + V^B f'_\delta\right)}{(1 - F_\delta(\tilde{s}) - V^B f_\delta(\tilde{s}))} - \frac{\left(f_B + V^B f'_B\right)}{(1 - F_B(\tilde{s}) - V^B f_B(\tilde{s}))}\right]$$

Let $h : V^B \mapsto h(V^B) = \frac{\left(f_\delta(\tilde{s}) + V^B f'_\delta(\tilde{s})\right)}{(1 - F_\delta(\tilde{s}) - V^B f_\delta(\tilde{s}))} - \frac{\left(f_B(\tilde{s}) + V^B f'_B(\tilde{s})\right)}{(1 - F_B(\tilde{s}) - V^B f_B(\tilde{s}))}$. With normal beliefs:

$$h(V^B) = \frac{\left(f_\delta(\tilde{s}) - \frac{\tilde{s} - \mu^L}{\sigma} V^B f_\delta(\tilde{s})\right)}{(1 - F_\delta(\tilde{s}) - V^B f_\delta(\tilde{s}))} - \frac{\left(f_B(\tilde{s}) - \frac{\tilde{s} - \mu^B}{\sigma} V^B f_B(\tilde{s})\right)}{(1 - F_B(\tilde{s}) - V^B f_B(\tilde{s}))}$$

We show that $\forall V^B, h(V^B) > 0$. We have $h(0) = \frac{f_\delta(\tilde{s})}{1 - F_\delta(\tilde{s})} - \frac{f_B(\tilde{s})}{1 - F_B(\tilde{s})} > 0$ by the hazard rate order property. We also have $h(+\infty) \sim -\frac{f'_\delta(\tilde{s})}{f_\delta(\tilde{s})} + \frac{f'_B(\tilde{s})}{f_B(\tilde{s})} = \frac{\tilde{s} - \mu^\delta}{\sigma} - \frac{\tilde{s} - \mu^B}{\sigma} = \frac{\mu^B - \mu^\delta}{\sigma} > 0$. After rearranging terms, we get:

$$
\begin{gathered}
\left(1-\tfrac{f_B}{(1-F_B)}\,V^B\right)\left(1-\tfrac{f_\delta}{(1-F_\delta)}\,V^B\right)h(V^B) = \\
h(0)-\tfrac{\tilde{s}-\mu^\delta}{\sigma^2}h(0)V^B-\tfrac{\mu^B-\mu^\delta}{\sigma^2}\tfrac{f_B}{(1-F_B)}V^B\left(1-\tfrac{f_\delta}{(1-F_\delta)}\,V^B\right)
\end{gathered}
$$

The result obtains by monoticity of function h: $\partial_{\tilde{s}}\kappa_3 = -\kappa_3 h(\tilde{s};V^B) < 0$.

Similarly, with $(1-F_\delta(\tilde{s}))-\kappa_3\,(1-F_B(\tilde{s})) = V^B\kappa_1$ we get:

$$
\begin{aligned}
\partial_{\tilde{s}}\left(V^B\kappa_1\right) &= -f_\delta(\tilde{s})+\kappa_3 f_B(\tilde{s})+(1-F_B(\tilde{s}))\,\kappa_3 h(\tilde{s};V^B) \\
\\
\partial_{\tilde{s}}F &= -\left(1-F_B\right)\kappa_3 h(V^B)\left[\mathbb{E}_B\left[s|s>\tilde{s}\right]-\tilde{s}-V^B\right]
\end{aligned}
$$

As $\mathbb{E}_B\left[s|s>\tilde{s}\right]>\tilde{s}$, we obtain: $\partial_{\tilde{s}}F<0$ for small V^B.

With respect to franchise value V^B (borrower heterogeneity)

$$
\partial_{V^B}F = \partial_{V^B}\kappa_1 V^B\left(\tilde{s}+V^B\right)+\kappa_1\left(\tilde{s}+2V^B\right)+\partial_{V^B}\kappa_3\left(1-F_B(\tilde{s})\right)\mathbb{E}_B\left[s|s>\tilde{s}\right]
$$

Using $\partial_{V^B}\kappa_1 = \frac{f_B(\tilde{s})}{1-F_B(\tilde{s})-f_B(\tilde{s})V^B}\kappa_1$ and $\partial_{V^B}\kappa_3 = -\frac{1}{1-F_B(\tilde{s})-f_B(\tilde{s})V^B}\kappa_1$, it implies:

$$
\partial_{V^B}F = -\tfrac{\kappa_1(1-F_B)}{1-F_B-f_BV^B}\left[\mathbb{E}_B\left[s|s>\tilde{s}\right]-\tilde{s}-2V^B+\tfrac{f_B}{(1-F_B)}\left(V^B\right)^2\right]
$$

I obtain the following equation for the comparative statics $\partial_{V^B}\tilde{s}$, using:

$$
\frac{h(0)}{\left(1-\frac{f_B}{(1-F_B)}\,V^B\right)\left(1-\frac{f_\delta}{(1-F_\delta)}\,V^B\right)h(V^B)} = \frac{1}{1-\frac{\tilde{s}-\mu^\delta}{\sigma^2}V^B-\frac{\mu^B-\mu^\delta}{\sigma^2}\frac{f_B/(1-F_B)}{h(0)}V^B\left(1-\frac{f_\delta}{(1-F_\delta)}\,V^B\right)}
$$

$$
\partial_{V^B}\tilde{s} = \frac{1}{1-\frac{\tilde{s}-\mu^\delta}{\sigma^2}V^B-\frac{\mu^B-\mu^\delta}{\sigma^2}\frac{f_B/(1-F_B)}{h(0)}V^B\left(1-\frac{f_\delta}{(1-F_\delta)}\,V^B\right)}\left(-1+\frac{V^B\left(1-\frac{f_B}{(1-F_B)}V^B\right)}{\mathbb{E}_B\left[s|s>\tilde{s}\right]-\tilde{s}-V^B}\right)
$$

By Taylor expanding first-order in V^B:

$$
\partial_{V^B}\tilde{s} = -1+\left(\tfrac{1}{\mathbb{E}_B[s|s>\tilde{s}]-\tilde{s}}-\tfrac{\tilde{s}-\mu^\delta}{\sigma^2}-\tfrac{\mu^B-\mu^\delta}{\sigma^2}\tfrac{f_B/(1-F_B)}{h(0)}\right)V^B
$$

At this order $\partial_{V^B}\tilde{s}<0$. The comparative statics of contract characteristics all derive from this expression for $\partial_{V^B}\tilde{s}$. The value of the contract D can be expressed as a function of the riskiness $\tilde{s}$:

$$D = \frac{1}{1+\mu^{\delta\, barg}} \left(p\left(\mu^{\delta\, barg} - \mu^{\delta}\right) + \tilde{s} + \left(1 - F_{\delta}(\tilde{s})\right) V^{B} - \pi^{\delta}(\tilde{s})\right)$$

- Promise $\bar{s} = s^{def} + V^{B}$. The first-order Taylor expansion in V^{B} gives:

$$\partial_{V^B}\bar{s} = \partial_{V^B}\tilde{s} + 1 = \left(\frac{1}{\mathbb{E}_B[s|s>\tilde{s}] - \tilde{s}} - \frac{\tilde{s}-\mu^{\delta}}{\sigma^2} - \frac{\mu^{B}-\mu^{\delta}}{\sigma^2}\frac{f_B/(1-F_B)}{h(0)}\right) V^{B}$$

Thus starting from $V^{B} = 0$ the promise increases. It also shows $-1 \leq \partial_{V^B}\tilde{s} < 0$. We also notice the convexity and the null derivative at $V^{B} = 0$.

- Haircut: $m = 1 - \frac{D}{p}$:

$$m = \frac{1}{p\left(1+\mu^{\delta\, barg}\right)}\left(p(1+\mu^{\delta}) - \tilde{s} - \left(1 - F_{\delta}(\tilde{s})\right) V^{B} + \pi^{\delta}(\tilde{s})\right)$$

We obtain, as $-1 \leq \partial_{V^B}\tilde{s} < 0$ and $0 \leq 1 - V^{B}\frac{f_{\delta}(\tilde{s})}{1-F_{\delta}(\tilde{s})} < 1$:

$$\partial_{V^B} m = -\frac{1-F_{\delta}}{p\left(1+\mu^{\delta\, barg}\right)}\left[1 + \partial_{V^B}\tilde{s}\left(1 - V^{B}\frac{f_{\delta}}{1-F_{\delta}}\right)\right]$$

First-order Taylor expansion in V^{B}:

$$\partial_{V^B} m = -\frac{\left(1-F_{\delta}(\tilde{s})\right)}{p\left(1+\mu^{\delta\, barg}\right)} V^{B}\left(\frac{1}{\mathbb{E}_B\left[s|s>\tilde{s}\right] - \tilde{s}} - \frac{\tilde{s}-\mu^{\delta}}{\sigma^2} - \frac{\mu^{B}-\mu^{\delta}}{\sigma^2}\frac{f_B/\left(1-F_B\right)}{h(0)} + \frac{f_{\delta}(\tilde{s})}{1-F_{\delta}(\tilde{s})}\right)$$

Thus $\partial_{V^B} m < 0$. We also notice the concavity of $m(V^{B})$ in the neighborhood of $V^{B} = 0$:

$$\partial_{V^B V^B} m_{\{V^B=0\}} = -\frac{1-F_{\delta}}{p\left(1+\mu^{\delta\, barg}\right)}\frac{1}{\mathbb{E}_B[s|s>\tilde{s}] - \tilde{s}} < 0$$

- Rate: $r = \frac{\tilde{s}+V^{B}}{D} - 1$

$$\partial_{V^B} r = \frac{1}{D^2}\left[\left(1 + \partial_{V^B}\tilde{s}\right) D - \left(\tilde{s} + V^{B}\right)\partial_{V^B} D\right]$$

Using the expression of contract value D:

$$\partial_{V^B} r = \frac{1}{D^2\left(1+\mu^{\delta\, barg}\right)}\left[\left(1 + \partial_{V^B}\tilde{s}\right)\left(p\left(\mu^{\delta\, barg} - \mu^{\delta}\right) - \pi^{\delta}(\tilde{s}) + F_{\delta}(\tilde{s})\tilde{s}\right) + \partial_{V^B}\tilde{s}\left(\tilde{s} + V^{B}\right) V^{B} f_{\delta}(\tilde{s})\right]$$

From above, we have $-1 \leq \partial_{V^B}\tilde{s} < 0$, denoting $\mu^{\delta\, barg} - \mu^{\delta} = barg$ and by integration by parts, $F_{\delta}(\tilde{s})\tilde{s} - \pi^{\delta}(\tilde{s}) = F_{\delta}(\tilde{s})\,\mathbb{E}_{\delta}\left[s|s<\tilde{s}\right]$:

$$\partial_{V^B} r =$$

$$\frac{V^B}{D^2\left(1+\mu^{\delta\, barg}\right)}\left[-\left(\tilde{s}+V^B\right) f_\delta(\tilde{s})+\partial_{V^B}\bar{s}\left(\left(\tilde{s}+V^B\right) V^B f_\delta(\tilde{s})+p\, barg+F_\delta(\tilde{s})\,\mathbb{E}_L\left[s|s<\tilde{s}\right]\right)\right]$$

As a result we have $\partial_{V^B} r_{\{V^B=0\}}=0$. By Taylor expanding first-order in V^B:

$$\partial_{V^B} r =$$

$$\frac{V^B}{D^2\left(1+\mu^{\delta\, barg}\right)}\left[-\tilde{s} f_\delta(\tilde{s})+\left(\frac{1}{\mathbb{E}_B[s|s>\tilde{s}]-\tilde{s}}-\frac{\tilde{s}-\mu^\delta}{\sigma^2}-\frac{\mu^B-\mu^\delta}{\sigma^2}\frac{f_B/(1-F_B)}{h(0)}\right) V^B\left(p\, barg+F_\delta(\tilde{s})\,\mathbb{E}_L\left[s|s<\tilde{s}\right]\right)\right]$$

Thus for $V^B=0$ we have $\partial_{V^B} r=0$. For V^B small, $\partial_{V^B} r<0$. However the comparative statics is reversed as V^Bgrows larger (but still $V^B<\mu^B$).

With respect to drifts μ^L (lender heterogeneity) and μ^B, and asset price p

Using $\partial_{\mu^L} F_L(\tilde{s})=-\frac{1}{\sigma} F_L(\tilde{s})$ and $\partial_{\mu^L}\pi_L(\tilde{s})=-\frac{1}{\sigma}\pi_L(\tilde{s})$, we derive (first-order in V^B):

$$\partial_{\mu^L} F \quad = \quad -(1-\delta)\left[p-\frac{1}{\sigma} F_\delta(\tilde{s})\left(\mathbb{E}_B\left[s|s>\tilde{s}\right]-\mathbb{E}_\delta\left[s|s<\tilde{s}\right]\right)\right]$$

We have $\mathbb{E}_B\left[s|s>\tilde{s}\right]>p(1+\mu^B)$ and $\mathbb{E}_\delta\left[s|s<\tilde{s}\right]<p(1+\mu^\delta)$. Using the first-order condition again: $F_\delta(\tilde{s})\left(\mathbb{E}_B\left[s|s>\tilde{s}\right]-\mathbb{E}_\delta\left[s|s<\tilde{s}\right]\right)=\mathbb{E}_B\left[s|s>\tilde{s}\right]-p(1+\mu^\delta)-\kappa_1 V^B\left(\tilde{s}+V^B\right)$

$$\partial_{\mu^L} F \quad = \quad -(1-\delta)\left[p-\frac{1}{\sigma}\left(\mathbb{E}_B\left[s|s>\tilde{s}\right]-p(1+\mu^\delta)-\kappa_1 V^B\left(\tilde{s}+V^B\right)\right)\right]$$

As we have $\mathbb{E}_B\left[s|s>\tilde{s}\right]>\mu^B$:

$$\left(\mathbb{E}_B\left[s|s>\tilde{s}\right]-p(1+\mu^\delta)-\kappa_1 V^B\left(\tilde{s}+V^B\right)\right)>p(\mu^B-\mu^\delta)-\kappa_1 V^B\left(\tilde{s}+V^B\right)$$

so for $\frac{\mu^B-\mu^\delta}{\sigma}<1$ we have $\partial_{\mu^L}\tilde{s}>0$: a less pessimist lender implies a higher optimal riskiness.

Similarly, $\partial_{\mu^B} F=(1-\delta)\left(\frac{1-F_L(\tilde{s})}{1-F_B(\tilde{s})}\right) p>0$ so $\partial_{\mu^B}\tilde{s}>0$.

On the haircut:

$$\begin{aligned}\partial_{\mu^B} m &= (1-\delta)\,\partial_{\mu^\delta}\left(\frac{1}{p\left(1+\mu^{\delta\, barg}\right)}\right)\left(p(1+\mu^\delta)-\tilde{s}-(1-F_\delta(\tilde{s}))\,V^B+\pi^\delta(\tilde{s})\right)\\ &\quad +\frac{1}{p\left(1+\mu^{\delta\, barg}\right)}\left(p\,(1-\delta)-\partial_{\mu^B}\tilde{s}(1-F_\delta(\tilde{s})-V^B f_\delta(\tilde{s}))\right)\end{aligned}$$

So $\partial_{\mu^L} m < 0$ as soon as $\frac{\mu^B-\mu^\delta}{\sigma} < 1$ (mild beliefs heterogeneity) and similarly $\partial_{\mu^B} m < 0$.

On the rate:

$$\partial_{\mu^L} r = -\left(\tilde{s}+V^B\right)(1-\delta)\,p+\partial_{\mu^B}\tilde{s}\left(p\, barg-\pi^\delta+2\left(\tilde{s}+V^B\right)\left(1-F_\delta-V^B f_\delta\right)\right)$$

So for δ large enough and mild beliefs heterogeneity ($\frac{\mu^B-\mu^\delta}{\sigma} < 1$): $\partial_{\mu^L} r > 0$.

As for the comparative statics w.r.t the asset price p: $\partial_p F = -(1+\mu^\delta)$ so $\partial_p \tilde{s} < 0$, the lower the asset price the riskier the loan is chosen.

$$\partial_p m = \frac{\left(\tilde{s}+(1-F_\delta(\tilde{s}))\,V^B-\pi^\delta(\tilde{s})\right)}{p^2\left(1+\mu^{\delta\, barg}\right)}-\partial_p\tilde{s}\left(1-F_\delta(\tilde{s})-V^B f_\delta(\tilde{s})\right) > 0$$

$$\partial_p r \propto \left(\frac{barg}{1+\mu^{\delta\, barg}}\right)\left(\partial_p\tilde{s}\, p-1\right)+\partial_p\tilde{s}\left[\mathbb{E}_\delta\left[s|s<\tilde{s}\right]F_\delta(\tilde{s})+\left(\tilde{s}+V^B\right)V^B f_\delta(\tilde{s}))\right] < 0$$

Interactions: with respect to asset volatility σ

We analyze the sensitivity of the franchise value collateral channel with respect the volatility of the risky asset σ. There is a non-zero effect, despite risk-neutrality. Using standard properties of the Gaussian distribution: $\partial_\sigma F_\delta(\tilde{s}) = \frac{\mu^\delta-\tilde{s}}{\sigma^2} f_\delta(\tilde{s})$, which is negative as long as $\tilde{s} > \mu^\delta$, and $\partial_\sigma f_\delta(\tilde{s}) = \frac{\left(\mu^\delta-\tilde{s}\right)^2}{\sigma^3} f_\delta(\tilde{s}) > 0$. At small V^B, $\partial_{V^B}\tilde{s} = -1$ so the volatility σ has no effect on the franchise value channel. The expression for $\partial_{V^B}\tilde{s}$ first-order in V^B:

$$\partial_{V^B}\tilde{s} = -1+\left(\frac{1}{\mathbb{E}_B[s|s>\tilde{s}]-\tilde{s}}-\frac{\tilde{s}-\mu^\delta}{\sigma^2}-\frac{\mu^B-\mu^\delta}{\sigma^2}\frac{f_B/(1-F_B)}{h(0)}\right)V^B$$

As a result, using $\partial_\sigma\mathbb{E}_B\left[s|s>\tilde{s}\right] < 0$ as $\tilde{s} < \mu^B$: $\partial_\sigma|\partial_{V^B}\tilde{s}| > 0$. Subsequently, in first-order in V^B:

$$|\partial_{V^B} m| = \frac{(1-F_\delta(\tilde{s}))}{p\left(1+\mu^{\delta\, barg}\right)}V^B\left(\frac{1}{\mathbb{E}_B[s|s>\tilde{s}]-\tilde{s}}-\frac{\tilde{s}-\mu^\delta}{\sigma^2}-\frac{\mu^B-\mu^\delta}{\sigma^2}\frac{f_B/(1-F_B)}{h(0)}+\frac{f_\delta(\tilde{s})}{1-F_\delta(\tilde{s})}\right)$$

The three dependences $\mathbb{E}_B\left[s|s>\tilde{s}\right]^{-1}$, $-F_\delta(\tilde{s})$ and σ^{-2} are all positive so we obtain: $\partial_\sigma|\partial_{V^B}m|>0$.

$$|\partial_{V^B}r| =$$
$$\frac{V^B}{D^2\left(1+\mu^{\delta\, barg}\right)}\left[\tilde{s}f_\delta(\tilde{s})-\left(\frac{1}{\mathbb{E}_B[s|s>\tilde{s}]-\tilde{s}}-\frac{\tilde{s}-\mu^\delta}{\sigma^2}-\frac{\mu^B-\mu^\delta}{\sigma^2}\frac{f_B/(1-F_B)}{h(0)}\right)V^B\left(p\, barg+F_\delta(\tilde{s})\,\mathbb{E}_L\left[s|s<\tilde{s}\right]\right)\right]$$

Given that $\partial_\sigma f_\delta>0$, we obtain in the neighborhood of $V^B=0$: $\partial_\sigma|\partial_{V^B}r|>0$.

Interactions: with respect to bargaining parameter δ

δ captures all the effect of imperfect competition among lenders.

$$\partial_\delta F =$$
$$-\left(\mu^B-\mu^L\right)p+\partial_\delta\kappa_1 V^B(\tilde{s}+V^B)+\partial_\delta\left(F_\delta(\tilde{s})\,\mathbb{E}_\delta\left[s|s<\tilde{s}\right]\right)+\partial_\delta\kappa_3\,\left(1-F_B(\tilde{s})\right)\mathbb{E}_B\left[s|s>\tilde{s}\right]$$

We have $\partial_\delta\kappa_1=-\frac{f_L-f_B}{1-V^B\frac{f_B}{1-F_B}}$ and $\partial_\delta\kappa_3=\frac{\left(F_L-F_B+V^B(f_L-f_B)\right)}{1-F_B-V^Bf_B}$. First-order in V^B: $\partial_\delta F=-F^L(\tilde{s}^\delta)$.

Where the last F^L is the characterization function of optimal promise $\tilde{s}^\delta$ (under parameter $\delta=0$ but evaluated at δ). As we have $\partial_{\tilde{s}}F<0$ and $F^L(\tilde{s}^0)=0$, I can conclude: $\partial_\delta F<0$ and therefore $\partial_\delta\tilde{s}<0$.

$$\begin{aligned}\partial_\delta m &= \partial_\delta\left(\frac{1}{1+\frac{barg}{1+\mu^\delta}}\right)+\partial_\delta\left(-\frac{1}{p\left(1+\mu^\delta+barg\right)}\right)\left(\tilde{s}+\left(1-F_\delta(\tilde{s})\right)V^B-\pi^\delta(\tilde{s})\right)\\&\quad+\frac{1}{p\left(1+\mu^{\delta\, barg}\right)}\left(-\partial_\delta\tilde{s}\right)\left(1-F_\delta(\tilde{s})-V^Bf_\delta(\tilde{s})\right)\end{aligned}$$

$$\partial_\delta r\propto\left(\mu^B-\mu^L\right)\left(\frac{barg}{\left(1+\mu^{\delta\, barg}\right)^2}-\frac{1}{(1-F_\delta)h(0)\mu^B}\left[p\left(\frac{barg}{1+\mu^{\delta\, barg}}\right)+\mu^LF_\delta(\tilde{s})+\left(\tilde{s}+V^B\right)V^Bf_\delta(\tilde{s}))\right]\right)$$

as the second bracket is an increasing function of beliefs disagreement $\mu^B-\mu^L$. So we obtain:

$$\partial_\delta m>0 \text{ and } \partial_\delta r>0$$

- Haircut:

From $|\partial_{V^B} m| = \frac{(1-F_\delta(\tilde{s}))}{p\left(1+\mu^{\delta\, barg}\right)} V^B \left(\frac{1}{\mathbb{E}_B[s|s>\tilde{s}]-\tilde{s}} - \frac{\tilde{s}-\mu^\delta}{\sigma^2} - \frac{\mu^B-\mu^\delta}{\sigma^2} \frac{f_B/(1-F_B)}{h(0)} + \frac{f_\delta(\tilde{s})}{1-F_\delta(\tilde{s})} \right)$, for σ high enough and mild beliefs disagreement:

$$\partial_\delta |\partial_{V^B} m| \propto \frac{barg\,(F_L-F_B)+(1-F_B)(1+\mu^L)-(1-F_L)(1+\mu^B)}{\left(1+\mu^{\delta\, barg}\right)^2} + \frac{(1-F_\delta(\tilde{s}))}{\left(1+\mu^{\delta\, barg}\right)} \left(-\partial_\delta \tilde{s}\right) \left(\frac{\tilde{s} f_B(\tilde{s})}{\left(F_B(\tilde{s})\pi^B(\tilde{s})\right)^2} \right) > 0$$

- Rate:

From

$$|\partial_{V^B} r| = \frac{V^B}{D^2\left(1+\mu^{\delta\, barg}\right)}$$
$$\left[\tilde{s} f_\delta(\tilde{s}) - \left(\frac{1}{\mathbb{E}_B\left[s|s>\tilde{s}\right]-\tilde{s}} - \frac{\tilde{s}-\mu^\delta}{\sigma^2} - \frac{\mu^B-\mu^\delta}{\sigma^2} \frac{f_B/\left(1-F_B\right)}{h(0)} \right) V^B \left(p\, barg + F_\delta(\tilde{s})\, \mathbb{E}_L\left[s|s<\tilde{s}\right] \right) \right]$$

and first-order in V^B:

$$\partial_\delta |\partial_{V^B} r| \quad \propto \quad \partial_\delta \left(\frac{V^B}{D^2\left(1+\mu^{\delta\, barg}\right)} \tilde{s} f_\delta(\tilde{s}) \right)$$

The three dependences w.r.t. δ are decreasing ($\tilde{s}$, f_δ and $\left(\mu^{\delta\, barg}\right)^{-1}$) therefore we derive the surprising result that when competition is more imperfect among lenders there is *less* screening with respect the franchise value: $\partial_\delta |\partial_{V^B} r| < 0$.

- With respect to endogenous outside options *barg*. From the f.o.c: $\partial_{barg} F = 0$ so $\partial_{barg} \tilde{s} = 0$. As we have:

$$m \quad = \quad \frac{1}{p\left(1+\mu^\delta+barg\right)} \left(p(1+\mu^\delta) - \tilde{s} - \left(1-F_\delta(\tilde{s})\right) V^B + \pi^\delta(\tilde{s}) \right)$$

$$\begin{aligned} \partial_{barg} r \quad &\propto \quad -\partial_{barg} \left(\frac{barg}{1+\mu^\delta+barg} \right) \\ &= \quad -\frac{1}{\left(1+\mu^\delta+barg\right)^2} \left(1+\mu^\delta\right) \end{aligned}$$

As a result $\partial_{barg} m < 0$ and $\partial_{\mu^{\delta\, barg}} r < 0$.

B.1.3 Endogenous bilateral surplus

The two last comparative statics shows that the optimal riskiness $\tilde{s}$ does depend on δ (and thus of I and J), but not on *barg* (and thus not on the outside options $barg = \omega(1-\omega)\left(S_{I,J-1} - S_{I-1,J}\right)$). However the optimal haircut does depend on both δ and *barg*. We can compute the surplus by induction.

$$\begin{aligned} S_{I,J} &= \left(U^B_{I,J} - U^B_{I,J-1}\right) + \left(U^L_{I,J} - U^L_{I-1,J}\right) \\ &= \left(U^B_{I,J} - U^B_0\right) - \left(U^B_{I,J} - U^B_0\right) + \left(U^L_{I,J} - U^L_0\right) - \left(U^L_{I,J} - U^L_0\right) \end{aligned}$$

We formulate the induction hypothesis: $U^B_{I,J-1} - U^B_0 = (1-\omega)n^B S_{I,J-1}$ and $U^L_{I-1,J} - U^L_0 = \omega n^B S_{I-1,J}$.

$$R^{unl}_{IJ}(\tilde{s}) = p\left(1+\mu^B\right) - \frac{I}{J}p(1+\mu^L)(1-m) - \left(1-\frac{I}{J}\right)\left(\tilde{s}+V^B\right) + \left(F_B(\tilde{s}) - \frac{I}{J}F_L(\tilde{s})\right)V^B + \pi^B(\tilde{s}) - \frac{I}{J}\pi^L(\tilde{s})$$

$$\frac{R^{unl}_{IJ}(\tilde{s})}{R^{unl}_{\delta}(\tilde{s})} = \\ \frac{\left(\left\{p\left(1+\mu^B\right) - \tfrac{I}{J}p(1+\mu^L)(1-m)\right\} - \left\{\left(1-\tfrac{I}{J}\right)\left(\tilde{s}+V^B\right)\right\} + \left\{\left(F_B(\tilde{s}) - \tfrac{I}{J}F_L(\tilde{s})\right)V^B\right\} + \left\{\pi^B(\tilde{s}) - \tfrac{I}{J}\pi^L(\tilde{s})\right\}\right)}{p\left(1+\mu^\delta\right) - \left(\tilde{s}+V^B\right) + F_\delta(\tilde{s})V^B + \pi^\delta(\tilde{s})}$$

$$S_{I,J} = \frac{n^B}{m_{IJ}p}R^{IJ}(\tilde{s}) - n^B\left(1+\mu^B\right) - (1-\omega)n^B S_{I,J-1} - \omega n^B S_{I-1,J}$$

Plugging $m_{IJ}p = \frac{1}{\left(1+\mu^\delta+\omega(1-\omega)\left(S_{I,J-1}-S_{I-1,J}\right)\right)}R^{\delta\,U}(\tilde{s})$ in the definition of the surplus:

$$\begin{aligned} S_{I,J} = n^B\left(1+\mu^\delta\right)\frac{R^{IJ}(\tilde{s})}{R^{\delta\,U}(\tilde{s})} - n^B\left(1+\mu^B\right) \\ + n^B(1-\omega)S_{I,J-1}\left(\omega\frac{R^{IJ}(\tilde{s})}{R^{\delta\,U}(\tilde{s})} - 1\right) - n^B\omega S_{I-1,J}\left((1-\omega)\frac{R^{IJ}(\tilde{s})}{R^{\delta\,U}(\tilde{s})} + 1\right) \end{aligned}$$

This equation recursively characterizes the endogenous surplus of the bilateral relationship (I, J). The surplus depends on how it is shared (ω), and is equal to zero if beliefs are identical.

B.2 Dynamic model in discrete time

B.2.1 Equilibrium with short-term contracts

We verify Blackwell sufficiency conditions of the Bellman equation show existence and uniqueness of function value V^B. There are no multiplicity of equilibria where each lender lends to all borrowers. However there are other asymmetric equilibria, for instance in which no lender lends to a given borrower, and as such the franchise of the latter is low, reinforcing the equilibrium property of not lending to this given borrower.

The only difference is that agents now takes into account the endogeneity of the franchise value V^B with respect to net worth n^B_{t+1}, and as a result the impact of the choice of $\tilde{s}$ on the value of its franchise tomorrow.

$$\begin{aligned}\left(U^B_{I,J} - U^B_0\right)(\tilde{s}) \quad &= \quad n^B\left[\left(1+\mu^{\delta\, barg}\right)R^{lev}_B(\tilde{s}) - \left(1+\mu^B\right)\right] \\ &\quad +\beta\mathbb{E}_B\left[1_{\{no\, def\}}V^B\left(n^B\left(1+\mu^{\delta\, barg}\right)R^{lev}_B(s,\tilde{s})\right) - V^B\left(n^B s\right)\right]\end{aligned}$$

The optimal riskiness comes from the following maximization:

$$\underset{\{\tilde{s}\}}{Max}\left(n^B\left[\left(1+\mu^{\delta\, barg}\right)R^{lev}_B(\tilde{s}) - \left(1+\mu^B\right)\right] + \beta\mathbb{E}_B\left[1_{\{no\, def\}}V^B\left(n^B\left(1+\mu^{\delta\, barg}\right)R^L_B(s,\tilde{s})\right) - V^B\left(n^B s\right)\right]\right)$$

I note the continuation component:

$$w(\tilde{s}) = \beta\frac{1}{n^B\left(1+\mu^{\delta\, barg}\right)}\mathbb{E}_B\left[1_{\{no\, def\}}V^B\left(n^B\left(1+\mu^{\delta\, barg}\right)R^{lev}_B(s,\tilde{s})\right) - V^B\left(n^B s\right)\right]$$

The f.o.c can be written, using $\partial_{V^B}R^{lev}_B(\tilde{s}) = -\frac{(1-F_\delta)R^{\delta\, unl} - (1-F_B)R^{B\, unl}}{\left(R^{\delta\, unl}\right)^2}$ (in the static model, $\partial_{\tilde{s}}R^{lev}_B(\tilde{s}) = 0$):

$$0 =$$
$$-R_{\delta}^{unl} + R_{B}^{unl}\frac{\partial_{\tilde{s}}R_{\delta}^{unl}}{\partial_{\tilde{s}}R_{B}^{unl}} + \frac{\beta}{n^{B}\left(1+\mu^{\delta\, barg}\right)}\frac{1}{1-\partial_{\tilde{s}}V^{B}}\frac{\left(R_{\delta}^{unl}\right)^{2}}{\partial_{\tilde{s}}R_{B}^{unl}}\partial_{\tilde{s}}\mathbb{E}_{B}\left[1_{\{no\, def\}}V^{B}\left(n^{B}\left(1+\mu^{\delta\, barg}\right)R_{B}^{lev}(s,\tilde{s})\right)\right]$$

I compute $\partial_{\tilde{s}}\mathbb{E}_{B}\left[1_{\{no\, def\}}V^{B}\left(n^{B}\left(1+\mu^{\delta\, barg}\right)R_{B}^{lev}(s,\tilde{s})\right)\right]$ with static bargaining:

$$\begin{aligned}\partial_{\tilde{s}}\mathbb{E}_{B}\left[1_{\{no\, def\}}V^{B}\left(n^{B}\left(1+\mu^{\delta\, barg}\right)R_{B}^{lev}(s,\tilde{s})\right)\right] &= -f_{B}(\tilde{s})V^{B}\left(n^{B}\left(1+\mu^{\delta\, barg}\right)R_{B}^{lev}(s^{def},\tilde{s})\right)\\ &\quad+\int_{\tilde{s}}^{s^{max}}\partial_{\tilde{s}}V^{B}\left(n^{B}\left(1+\mu^{\delta\, barg}\right)R_{B}^{lev}(s,\tilde{s})\right)f_{B}(s)ds\end{aligned}$$

As we have $V^{B}\left(n^{B}\left(1+\mu^{\delta\, barg}\right)R_{B}^{L}(s^{def},\tilde{s})\right)=0$ in the dynamic model:

$$\begin{aligned}&\partial_{\tilde{s}}\mathbb{E}_{B}\left[1_{\{no\, def\}}V^{B}\left(n^{B}\left(1+\mu^{\delta\, barg}\right)R_{B}^{lev}(s,\tilde{s})\right)\right]\\ &\qquad= n^{B}\left(1+\mu^{\delta\, barg}\right)\int_{\tilde{s}}^{s^{max}}\partial_{\tilde{s}}R_{B}^{lev}(s,\tilde{s})\,\partial_{n^{B}}V^{B}\left(n^{B}\left(1+\mu^{\delta\, barg}\right)R_{B}^{lev}(s,\tilde{s})\right)dF_{B}\end{aligned}$$

So the f.o.c can be written, rearranging terms:

$$\begin{aligned}0 &= -p(1+\mu^{\delta}) + \kappa_{1}V^{B}\left(\tilde{s}+V^{B}\right) + F_{\delta}(\tilde{s})\,\mathbb{E}_{\delta}\left[s|s<\tilde{s}\right] + \left(1-F_{\delta}(\tilde{s})\right)\,\kappa_{2}\,\mathbb{E}_{B}\left[s|s>\tilde{s}\right]\\ &\quad+\beta\frac{1}{1-\partial_{\tilde{s}}V^{B}}\frac{\left(R_{\delta}^{unl}\right)^{2}}{\partial_{\tilde{s}}R_{B}^{unl}}\int_{\tilde{s}}^{s^{max}}\partial_{\tilde{s}}R_{B}^{lev}(s,\tilde{s})\,\partial_{n^{B}}V^{B}\left(n^{B}\left(1+\mu^{\delta\, barg}\right)R_{B}^{lev}(s,\tilde{s})\right)f_{B}(s)ds\end{aligned}$$

Concavity of the value function $V^{B}(n_{t})$ Differentiating this f.o.c w.r.t n^{B} (optimal riskiness does not move first-order by the envelope condition), using

$$\partial_{\tilde{s}}V^{B} = \partial_{\tilde{s}}R_{B}^{lev}(\tilde{s})\,n^{B}\left(1+\mu^{\delta\, barg}\right)\,\partial_{n^{B}}V^{B}$$

$$\begin{aligned}0 &= \left(1-\partial_{\tilde{s}}V^{B}\right)\kappa_{1}\partial_{n^{B}}V^{B}\left(\tilde{s}+2V^{B}\right)\\ &\quad+\left(-\partial_{\tilde{s}}R_{B}^{lev}(\tilde{s})\left(1+\mu^{\delta\, barg}\right)\left(n^{B}\,\partial_{n^{B}n^{B}}V^{B}+\partial_{n^{B}}V^{B}\right)\right)\kappa_{1}V^{B}\left(\tilde{s}+V^{B}\right)\\ &\quad+\beta\frac{\left(R_{\delta}^{unl}\right)^{2}}{\partial_{\tilde{s}}R_{B}^{unl}}\int_{\tilde{s}}^{s^{max}}\partial_{\tilde{s}}R_{B}^{lev}(s,\tilde{s})\,\partial_{n^{B}n^{B}}V^{B}\left(n_{t+1}^{B}\right)f_{B}(s)ds\end{aligned}$$

Developing first-order in n^{B}:

$$-\left(1-\partial_{\tilde{s}}V^{B}\right)\kappa_{1}\partial_{n^{B}}V^{B}\left(\tilde{s}+2V^{B}\right) = \beta\frac{\left(R_{\delta}^{unl}\right)^{2}}{\partial_{\tilde{s}}R_{B}^{unl}}\int_{\tilde{s}}^{s^{max}}\partial_{\tilde{s}}R_{B}^{lev}(s,\tilde{s})\,\partial_{n^{B}n^{B}}V^{B}\left(n_{t+1}^{B}\right)f_{B}(s)ds$$

Using $\partial_{n^B} V^B > 0$, this equation implies: $\partial_{n^B n^B} V^B < 0$.

Comparative statics of m w.r.t n_t^B The first-order condition delivers:

$$\begin{aligned}
\partial_{n^B} F^{dyn} &= \kappa_1 \partial_{n^B} V^B \left(\tilde{s} + 2V^B\right) \\
&\quad + \left(-\partial_{\tilde{s}} R_B^{lev}(\tilde{s}) \left(1 + \mu^{\delta\, barg}\right) \left(n^B\, \partial_{n^B n^B} V^B + \partial_{n^B} V^B\right)\right) \frac{1}{1 - \partial_{\tilde{s}} V^B} \kappa_1 V^B \left(\tilde{s} + V^B\right) \\
&\quad + \beta \frac{1}{1 - \partial_{\tilde{s}} V^B} \frac{\left(R_\delta^{unl}\right)^2}{\partial_{\tilde{s}} R_B^{unl}} \int_{\tilde{s}}^{s^{max}} \partial_{\tilde{s}} R_B^{lev}(s, \tilde{s})\, \partial_{n^B n^B} V^B \left(n_{t+1}^B\right) f_B(s) ds
\end{aligned}$$

At low n^B: $\partial_{n^B n^B} V^B \left(n_{t+1}^B\right) < 0$ and $\left(n^B\, \partial_{n^B n^B} V^B + \partial_{n^B} V^B\right) > 0$. Therefore $\partial_{n^B} F^{dyn} < 0$, which, along with $\partial_{\tilde{s}} F^{dyn} > 0$, results in $\partial_{n^B} \tilde{s} > 0$.

- Haircut m: $m = \frac{1}{p\left(1+\mu^{\delta\, barg}\right)} \left(p(1 + \mu^\delta) - \tilde{s} - (1 - F_\delta(\tilde{s}))\, V^B + \pi^\delta(\tilde{s})\right)$

So $\partial_{n^B} m = -\frac{1}{p\left(1+\mu^{\delta\, barg}\right)} \partial_{n^B} \tilde{s} \left[1 - F^\delta(\tilde{s}) - V^B f^\delta(\tilde{s})\right]$ implies $\partial_{n^B} m < 0$

Fragility:

$$\partial_{n^B n^B} m = -\frac{1}{p\left(1 + \mu^{\delta\, barg}\right)} \left(\partial_{n^B n^B} \tilde{s} \left[1 - F^\delta(\tilde{s}) - V^B f^\delta(\tilde{s})\right] - \left(\partial_{n^B} \tilde{s}\right)^2 \left[f^\delta(\tilde{s}) \left(1 - V^B \frac{\tilde{s} - \mu^\delta}{\sigma}\right)\right]\right)$$

At low net worth levels n^B: $\partial_{n^B n^B} m > 0$

$$\begin{aligned}
\partial_{n^B} \tilde{s} &= -A \left(\partial_{n^B} V^B + n^B\, \partial_{n^B n^B} V^B\right) \\
\partial_{n^B n^B} \tilde{s} &= -A \left(2 \partial_{n^B n^B} V^B + n^B\, \partial_{n^B n^B n^B} V^B\right) \\
\partial_{n^B n^B} \tilde{s} &\propto -2A \partial_{n^B n^B} V^B
\end{aligned}$$

- Rate r: $r = \frac{\tilde{s} + V^B}{p\left(\frac{barg}{1+\mu^{\delta\, barg}}\right) + \tilde{s} + (1 - F_\delta(\tilde{s}))\, V^B - \pi^\delta(\tilde{s})} - 1$

$$\partial_{\tilde{s}} r = \frac{1}{\left(p\left(\frac{barg}{1+\mu^{\delta\, barg}}\right) + \tilde{s} + (1 - F_\delta(\tilde{s}))\, V^B - \pi^\delta(\tilde{s})\right)^2} \left[p\left(\frac{barg}{1 + \mu^{\delta\, barg}}\right) + E_\delta\left[s | s < \tilde{s}\right] F_\delta(\tilde{s}) + \left(\tilde{s} + V^B\right) V^B f_\delta(\tilde{s})\right)\right]$$

So $\partial_{n^B} r = \partial_{n^B} \tilde{s} \times \partial_{\tilde{s}} r + \partial_{n^B} V^B \times \partial_{V^B} r$ with $\partial_{V^B} r < 0$.

At low levels of n_t^B at which $\partial_{n^B} V^B$ high enough: $\partial_{n^B} r < 0$.

B.2.2 Equilibrium with long-term contracts

Following Abreu-Pearce-Stacchetti (1980), we can use the promised utility to the agent (who is the lender here) as state variable. We now have two state variables: the borrower net worth n_t^B and the lender continuation value V^L.

- Expected utility of the borrower:

$$\begin{aligned} U^B_{I,J,t} - U^B_{I,J-1,t} &= \frac{n^B}{m}\left[(1-m)\left(\mu^B - r(m)\right) + \frac{1}{p}\pi^B(\bar{s}) + V^B\frac{F_B(\tilde{s})}{p}\right] \\ &\quad +\beta\,\mathbb{E}_B\left[V^B_{I,J,t+1} - V^B_{I,J-1,t+1}\right] \end{aligned}$$

- Expected utility of the lender:

$$\begin{aligned} V^L_{I,J,t} - V^L_{I-1,J,t} &= \frac{I}{J}\frac{n^B}{m}\left[(1-m)\left(r(m) - \mu^L\right) - \frac{1}{p}\left(\pi^L(\tilde{s}) + V^B F_L(\tilde{s})\right)\right] \\ &\quad +\beta\mathbb{E}_L\left[V^L_{I,J,t+1} - V^L_{I-1,J,t+1}\right] \end{aligned}$$

- The bilateral Nash bargaining delivers a contract value $D = \frac{num}{den}$:

$$num = (1-\delta)\mathbb{E}_L\left[1_{\{def\}}s + 1_{\{no\,def\}}\bar{s}\right] + \delta\left(\left(1+\mu^B\right)p - \mathbb{E}_B\left[1_{\{no\,def\}}\left(s-\bar{s}\right)\right]\right) +$$
$$\tfrac{1}{n^B}\left(\delta\left(U^B_{I,J-1,t} - \beta\,\mathbb{E}_B\left[V^B_{I,J,t+1} - V^B_{I,J-1,t+1}\right]\right) - (1-\delta)\left(V^L_{I-1,J,t} - \beta\mathbb{E}_L\left[V^L_{I,J,t+1} - V^L_{I-1,J,t+1}\right]\right)\right)p$$
$$den = (1-\delta)(1+\mu^L) + \delta\left(1+\mu^B\right) +$$
$$\tfrac{1}{n^B}\left(\delta\left(U^B_{I,J-1,t} - \beta\,\mathbb{E}_B\left[V^B_{I,J,t+1} - V^B_{I,J-1,t+1}\right]\right) - (1-\delta)\left(V^L_{I-1,J,t} - \beta\mathbb{E}_L\left[V^L_{I,J,t+1} - V^L_{I-1,J,t+1}\right]\right)\right)$$

The only change of the dynamics compared with static bargaining is the value of $\mu^{\delta\,barg}$:

$$\mu^{\delta\,barg} = \mu^\delta +$$
$$\tfrac{1}{n^B}\left(\delta\left(U^B_{I,J-1,t} - \beta\,\mathbb{E}_B\left[V^B_{I,J,t+1} - V^B_{I,J-1,t+1}\right]\right) - (1-\delta)\left(V^L_{I-1,J,t} - \beta\mathbb{E}_L\left[V^L_{I,J,t+1} - V^L_{I-1,J,t+1}\right]\right)\right)$$

- The borrower maximization program is now:

$$\begin{aligned} \underset{\{\tilde{s}, V^L_{I,J,t+1}\}}{Max} \quad & n^B\left[\left(1+\mu^{\delta\,barg}(V^L_{I,J,t+1})\right)R_B^{lev}(\tilde{s}) - \left(1+\mu^B\right)\right] \\ & +\beta\mathbb{E}_B\left[1_{\{no\,def\}}V^B\left(n^B\left(1+\mu^{\delta\,barg}\right)R_B^L(s,\tilde{s}), V^L_{I,J,t+1}\right) - V^B\left(n^B s\right)\right] \end{aligned}$$

- And the dynamic f.o.c. still holds:

$$
\begin{aligned}
0 = & -R_{\delta}^{unl} + R_{B}^{unl}\frac{\partial_{\tilde{s}}R_{\delta}^{unl}}{\partial_{\tilde{s}}R_{B}^{unl}} \\
& + \frac{\beta}{n^{B}\left(1+\mu^{\delta\, barg}\right)}\frac{1}{1-\partial_{\tilde{s}}V^{B}}\frac{\left(R_{\delta}^{unl}\right)^{2}}{\partial_{\tilde{s}}R_{B}^{unl}}\partial_{\tilde{s}}\mathbb{E}_{B}\left[1_{\{no\, def\}}V^{B}\left(n^{B}\left(1+\mu^{\delta\, barg}\right)R_{B}^{lev}(s,\tilde{s})\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
& \partial_{\tilde{s}}\mathbb{E}_{B}\left[1_{\{no\, def\}}V^{B}\left(n^{B}\left(1+\mu^{\delta\, barg}\right)R_{B}^{lev}(s,\tilde{s})\right)\right] \\
& \qquad = \int_{\tilde{s}}^{s^{max}}\left[n^{B}\left(1+\mu^{\delta\, barg}\right)\partial_{\tilde{s}}R_{B}^{lev}\partial_{n^{B}}V^{B}+\partial_{\tilde{s}}V^{L}\partial_{V^{L}}V^{B}\right]f_{B}(s)ds
\end{aligned}
$$

So the f.o.c can be written, rearranging terms:

$$
\begin{aligned}
0 \; = \; & -p(1+\mu^{\delta}) \\
& +\kappa_{1}V^{B}\left(\tilde{s}+V^{B}\right)+F_{\delta}(\tilde{s})\,\mathbb{E}_{\delta}\left[s|s<\tilde{s}\right]+\left(1-F_{\delta}(\tilde{s})\right)\,\kappa_{2}\,\mathbb{E}_{B}\left[s|s>\tilde{s}\right] \\
& +\beta\frac{1}{1-\partial_{\tilde{s}}V^{B}}\frac{\left(R_{\delta}^{unl}\right)^{2}}{\partial_{\tilde{s}}R_{B}^{unl}}\int_{\tilde{s}}^{s^{max}}\left[\partial_{\tilde{s}}R_{B}^{lev}\partial_{n^{B}}V^{B}+\frac{1}{n^{B}\left(1+\mu^{\delta\, barg}\right)}\partial_{\tilde{s}}V^{L}\partial_{V^{L}}V^{B}\right]f_{B}(s)ds
\end{aligned}
$$

I show now how the optimal riskiness $\tilde{s}$ depends on the continuation value $V_{I,J,t+1}^{L}$ promised to the lender. The countercyclicality of the haircut comes from $\partial_{\tilde{s}}R_{B}^{lev}\partial_{n^{B}}V^{B}$ in the continuation term, which by variational argument as a positive impact on optimal riskiness $\tilde{s}$ (same effect as of minus price $-p$). The optimal long-term contract counteracts this countercycality by the term $\frac{1}{n^{B}\left(1+\mu^{\delta\, barg}\right)}\partial_{\tilde{s}}V^{L}\partial_{V^{L}}V^{B}$: as n^{B} decreases, this term increases as long as $\frac{\partial_{\tilde{s}}V^{L}\partial_{V^{L}}V^{B}}{\left(1+\mu^{\delta\, barg}\right)}$ does not decrease too fast. The impact of the continuation term is dampened, making the variation of optimal riskiness $\tilde{s}$ less sensitive to the state variable n^{B}.

- The f.o.c in promised continuation value delivers $V_{I,J,t+1}^{L}$:

$$
\begin{aligned}
\underset{\{\tilde{s},V_{I,J,t+1}^{L}\}}{Max} \quad & n^{B}\left[\left(1+\mu^{\delta\, barg}(V_{I,J,t+1}^{L})\right)R_{B}^{lev}(\tilde{s})-\left(1+\mu^{B}\right)\right] \\
& +\beta\mathbb{E}_{B}\left[1_{\{no\, def\}}V^{B}\left(n^{B}\left(1+\mu^{\delta\, barg}\right)R_{B}^{lev}(s,\tilde{s}),V_{I,J,t+1}^{L}\right)-V^{B}\left(n^{B}s\right)\right]
\end{aligned}
$$

$$
0 = n^{B}\partial_{V_{I,J,t+1}^{L}}\mu^{\delta\, barg}R_{B}^{lev}(\tilde{s})+\beta\mathbb{E}_{B}\left[1_{\{no\, def\}}\partial_{V_{I,J,t+1}^{L}}V^{B}\right]
$$

Using $\partial_{V^L_{I,J,t+1}} \mu^{\delta\, barg} = \frac{\beta(1-\delta)}{n^B}$: $0 = (1-\delta) R^{lev}_B(\tilde{s}) + \mathbb{E}_B \left[1_{\{no\, def\}} \partial_{V^L_{I,J,t+1}} V^B \right]$

The intuition is that increasing the long-term promise $V^L_{I,J,t+1}$ to the lender by one unit increase the short-term gain for the borrower by $(1-\delta) R^{lev}_B(\tilde{s})$ but decreases its long-term expectation by $\mathbb{E}_B \left[1_{\{no\, def\}} |\partial_{V^L_{I,J,t+1}} V^B| \right]$. Under the optimal long-term contract, the two legs are equalized.

$$\partial_{V^L_{I,J}} V^B < 0 \text{ and } |\partial_{V^L_{I,J}} V^B| = (1-\delta) R^{lev}_B(\tilde{s})$$

- Finally I show formally that $0 < \partial_{n^B} \tilde{s}^{LT} < \partial_{n^B} \tilde{s}^{ST}$.

$$\begin{aligned} \partial_{\tilde{s}} F^{dyn} \quad = \quad & -\left(1 - F_B\right) \kappa_3 h(V^B) \left[\mathbb{E}_B \left[s | s > \tilde{s} \right] - \tilde{s} - V^B \right] \\ & + \beta \frac{1}{1 - \partial_{\tilde{s}} V^B} \frac{\left(R^{unl}_{\delta}\right)^2}{\partial_{\tilde{s}} R^{unl}_B} \int_{\tilde{s}}^{s^{max}} [\partial_{\tilde{s}\tilde{s}} R^{lev}_B(s, \tilde{s})\, \partial_{n^B} V^B \left(n^B_{t+1}\right) \\ & + \left(\partial_{\tilde{s}} R^{lev}_B(s, \tilde{s})\right)^2 n^B \left(1 + \mu^{\delta\, barg}\right) \partial_{n^B n^B} V^B \left(n^B_{t+1}\right) \end{aligned}$$

$$+ \frac{1}{n^B \left(1 + \mu^{\delta\, barg}\right)} \partial_{\tilde{s}} V^L \partial_{V^L} V^B] f_B(s) ds$$

$$\begin{aligned} \partial_{n^B} F^{dyn} \quad = \quad & \kappa_1 \partial_{n^B} V^B \left(\tilde{s} + 2V^B \right) + \left(-\partial_{\tilde{s}} R^{lev}_B(\tilde{s}) \left(1 + \mu^{\delta\, barg}\right) \left(n^B\, \partial_{n^B n^B} V^B + \partial_{n^B} V^B \right) \right) \\ & \frac{1}{1 - \partial_{\tilde{s}} V^B} \kappa_1 V^B \left(\tilde{s} + V^B \right) \\ & + \beta \frac{1}{1 - \partial_{\tilde{s}} V^B} \frac{\left(R^{unl}_{\delta}\right)^2}{\partial_{\tilde{s}} R^{unl}_B} \\ & \int_{\tilde{s}}^{s^{max}} \left[\partial_{\tilde{s}} R^{lev}_B(s, \tilde{s})\, \partial_{n^B n^B} V^B \left(n^B_{t+1}\right) - \frac{1}{\left(n^B\right)^2 \left(1 + \mu^{\delta\, barg}\right)} \partial_{\tilde{s}} V^L \partial_{V^L} V^B \right] f_B(s) ds \end{aligned}$$

As $\partial_{V^L} V^B < 0$ the term $-\frac{1}{\left(n^B\right)^2 \left(1 + \mu^{\delta\, barg}\right)} \partial_{\tilde{s}} V^L \partial_{V^L} V^B$ is positive and therefore counteracts the negativity of $\partial_{n^B n^B} V^B \left(n^B_{t+1}\right) < 0$. Using $|\partial_{V^L_{I,J}} V^B| = (1-\delta) R^{lev}_B(\tilde{s})$ (f.o.c in $V^L_{I,J,t+1}$) this counteract effect on $\partial_{n^B} F^{dyn}$ is larger than the effect of $\frac{1}{n^B \left(1 + \mu^{\delta\, barg}\right)} \partial_{\tilde{s}} V^L \partial_{V^L} V^B$ on $\partial_{\tilde{s}} F^{dyn}$.

$$\begin{aligned}
\partial_{n^B}\tilde{s}^{LT} &= \frac{\left(-\partial_{n^B}F^{dyn\,LT}\right)}{\partial_{\tilde{s}}F^{dyn\,LT}} \\
&= \frac{\left(-\partial_{n^B}F^{dyn\,ST}\right) - \beta\frac{1}{1-\partial_{\tilde{s}}V^B}\frac{\left(R_\delta^{unl}\right)^2}{\partial_{\tilde{s}}R_B^{unl}}\int_{\tilde{s}}^{s^{max}}\frac{1}{\left(n^B\right)^2\left(1+\mu^{\delta\,barg}\right)}\partial_{\tilde{s}}V^L\left(-\partial_{V^L}V^B\right)f_B(s)ds}{\partial_{\tilde{s}}F^{dyn\,ST} + \beta\frac{1}{1-\partial_{\tilde{s}}V^B}\frac{\left(R_\delta^{unl}\right)^2}{\partial_{\tilde{s}}R_B^{unl}}\int_{\tilde{s}}^{s^{max}}\frac{1}{n^B\left(1+\mu^{\delta\,barg}\right)}\partial_{\tilde{s}}V^L\partial_{V^L}V^B f_B(s)ds} \\
&< -\frac{\partial_{n^B}F^{dyn\,ST}}{\partial_{\tilde{s}}F^{dyn\,ST}} \\
&= \partial_{n^B}\tilde{s}^{ST}
\end{aligned}$$

Optimal riskiness is less senstive to net worth under long-term contracts than under short-term contracts.

B.3 The rehypothecation chain

We stack two I-J set ups described above. There is one multilateral Nash bargaining between the HF and the BD and one multilateral Nash bargaining between the BD and the MMF. This implies two $r(m)$ mappings, which are inter-related.

Expected utilities of each agent

- Hedge Fund expected value (its balance sheet constraint always binds - no cash hoarding - so $pm^{bil}x^{bil} = n^{HF}$):

$$U^{HF} - U_0^{HF} = \frac{n^{HF}}{m^{bil}p}\left[\left(1-m^{bil}\right)\left(\mu^B - r^{bil}\right) + \pi^B(\tilde{s}^{bil})\right]$$

- Broker Dealer expected value (its balance sheet constraint does not always bind: $p\left(1-m^{bil}\right)x^{bil} \leq n^{BD} + p\left(1-m^{tri}\right)x^{tri}$):

$$\begin{aligned}
U^{BD} - U_0^{BD} = \; & x^{bil}\left[\left(1-m^{bil}\right)\left(1+r^{bil}\right)p - \pi^B(\tilde{s}^{bil})\right] \\
& - x^{tri}\left[\left(1-m^{tri}\right)\left(1+r^{tri}\right)p + \pi^B(\tilde{s}^{tri}) + \left(1-\beta n^B/x^{tri}\right)V^B F_B(\tilde{s}^{tri})\right] \\
& - n^{BD}\left[\left(1-m^{bil}\right)\left(1+r^{bil}\right)p - \pi^B(\tilde{s}^{bil})\right]
\end{aligned}$$

- Money Market Fund expected value:

$$U^{MMF} - U_0^{MMF} = x^{tri}\left[\left(1-m^{tri}\right)\left(r^{tri}-\mu^L\right)p - \pi^L(\tilde{s}^{tri}) - V^B F_B(\tilde{s}^{tri})\right]$$

1/ Scarce collateral regime Assume we are in a scarce collateral regime. In this case:

- The BD collateral constraint binds: $x^{tri} = x^{bil}$
- The BD balance sheet constraint is slack: $p\left(1-m^{bil}\right)x^{bil} < n^{BD} + p\left(1-m^{tri}\right)x^{tri}$

Combining with $pm^{bil}x^{bil} = n^{HF}$ we get: $x^C = n^B + n^{HF}\left(1-\frac{m^{tri}}{m^{bil}}\right)$

In this regime, we can write the BD expected utility as:

$$U^{BD} - U_0^{BD} = \frac{n^{HF}}{m^{bil}}\left[r^{bil}(m^{bil};m^{tri}) - r^{tri}(m^{bil};m^{tri}) - \Delta\pi^B(m^{bil};m^{tri}) - m^{tri}r^C p\right]$$

where we introduce the value of the collar: $\Delta\pi^B(m^{bil};m^{tri}) = \pi^B(\tilde{s}^{bil}) - \pi^B(\tilde{s}^{tri}) = \int_{\tilde{s}^{tri}-V^B}^{\tilde{s}^{bil}} F_B(s)ds$. The first term can be called the *repo spread* (carry trade from lending at a higher rate than borrowing). The second term (the collar) arises from the composition of the two put options the borrower bears (long with MMF, short with HF) and would be traced to a *haircut spread.* The third is cash gains arising from collateral management (high haircuts secured against HF, low haircuts against MMF).

Derivation of the two mappings $r^{bil}(m^{bil};m^{tri})$ and $r^{tri}(m^{bil};m^{tri})$ Denote ω_{bil} the bargaining power of the lender BD in the HF-BD bilateral repo and ω_{tri} the bargaining power of the lender MMF in the BD-MMF triparty repo. These two Nash bargainings imply:

$$(1-\omega_{bil})\left(U^{BD} - U_0^{BD}\right) = \omega_{bil}\left(U^{HF} - U_0^{HF}\right)$$

$$(1-\omega_{tri})\left(U^{MMF} - U_0^{MMF}\right) = \omega_{tri}\left(U^{BD} - U_0^{BD}\right)$$

Following the static bargaining from the I-J model, we get:

$$r^{bil}\left(m^{bil};m^{tri}\right) = \omega_{bil}\mu^B + (1-\omega_{bil})r^{tri}\left(m^{bil};m^{tri}\right) + \omega_{bil}\frac{1}{1-m^{bil}}\pi^B(\tilde{s}^{bil}) + (1-\omega_{bil})\frac{1}{1-m^{bil}}\Delta\pi^B(\tilde{s}^{bil};\tilde{s}^{tri})$$

$$\begin{aligned} r^{tri}\left(m^{bil};m^{tri}\right) = {} & \omega_{tri} r^{bil}\left(m^{bil};m^{tri}\right) + (1-\omega_{tri})\mu^L \\ & - \omega_{tri}\frac{1}{1-m^{tri}}\Delta\pi^B(\tilde{s}^{bil};\tilde{s}^{tri}) + (1-\omega_{tri})\frac{1}{1-m^{tri}}\pi^L(\tilde{s}^{tri}) \end{aligned}$$

The solution of the linear system, by Cramer's rule, is:

$$\begin{cases} r^{bil}\left(m^{bil};m^{tri}\right) & = \frac{1}{1-\omega_{tri}(1-\omega_{bil})} \\ & \left[\omega_{bil}\left(\mu^B + \frac{1}{1-m^{bil}}\pi^B(\tilde{s}^{bil})\right) + (1-\omega_{bil})(1-\omega_{tri})\left(\mu^L + \frac{1}{1-m^{tri}}\pi^L(\tilde{s}^{tri}) + \frac{1}{1-m^{tri}}\Delta\pi^B(\tilde{s}^{bil};\tilde{s}^{tri})\right)\right] \\ r^{tri}\left(m^{bil};m^{tri}\right) & = \frac{1}{1-\omega_{tri}(1-\omega_{bil})} \\ & \left[\omega_{bil}\omega_{tri}\left(\mu^B + \frac{1}{1-m^{bil}}\pi^B(\tilde{s}^{bil}) - \frac{1}{1-m^{tri}}\Delta\pi^B(\tilde{s}^{bil};\tilde{s}^{tri})\right) + (1-\omega_{tri})\left(\mu^L + \frac{1}{1-m^{tri}}\pi^L(\tilde{s}^{tri})\right)\right] \end{cases}$$

Maximization program of the Broker Dealer The equilibrium is given by the BD program:

$$\begin{aligned} & \underset{\{x^{BD},x^{MMF},\tilde{s}^{BD},\tilde{s}^{MMF}\}}{Max} \{x^{BD}\left(\tilde{s}^{BD} - \pi^B(\tilde{s}^{BD})\right) \\ & \qquad - x^{MMF}\left(\tilde{s}^{MMF} - \pi^B(\tilde{s}^{MMF}) + V^B(1-F_B(\tilde{s}^{MMF}))\right) \\ & \qquad + x^c\bar{r} + \beta n\, V^B(1-F_B(\tilde{s}^{MMF}))\} \\ (BD\,balance\,sheet\,constraint) \quad & s.t.\ x^c + x^{BD}D^{HF} \leq n^{BD} + x^{MMF}D^{BD}(\tilde{s}^{MMF}) \\ (BD\,collateral\,constraint) \quad & s.t.\ x^{MMF} \leq x^{BD} \\ (BD\,default\,condition) \quad & s.t.\ default\ i.f.f.\ s < \tilde{s}^{MMF} \\ (Nash\,bargainings) \quad & s.t.\ r^{bil}(m^{bil};m^{tri})\ and\ r^{tri}(m^{bil};m^{tri}) \end{aligned}$$

This can be written as only functions of m^{bil} and m^{tri}, using the solution of the joint Nash bargainings.

$$\underset{\{m^{bil};m^{tri}\}}{Max} U^{BD} - U^{BD}_0 = \frac{n^{HF}}{m^{bil}}\left[r^{bil}(m^{bil};m^{tri}) - r^{tri}(m^{bil};m^{tri}) - \Delta\Pi^B(m^{bil};m^{tri}) - m^{tri}r^C p\right]$$

where:

$$\begin{aligned} & r^{bil}(m^{bil};m^{tri}) - r^{tri}(m^{bil};m^{tri}) = \\ & \quad \frac{1}{1-\omega_{tri}(1-\omega_{bil})}\left[\omega_{bil}(1-\omega_{tri})\left(\mu^B + \frac{1}{1-m^{bil}}\pi^B(\tilde{s}^{bil}) - \mu^L - \frac{1}{1-m^{tri}}\pi^L(\tilde{s}^{tri})\right)\right] \end{aligned}$$

As a result the BD maximizes the following functional $U^{BD} - U^{BD}_0$:

$$\underset{\{m^{bil};m^{tri}\}}{Max} \frac{n^{HF}}{m^{bil}} \left[\frac{\omega_{bil}(1-\omega_{tri})}{1-\omega_{tri}(1-\omega_{bil})}\left(\mu^B + \frac{1}{1-m^{bil}}\pi^B(\tilde{s}^{bil}) - \mu^L - \frac{1}{1-m^{tri}}\pi^L(\tilde{s}^{tri})\right) - \Delta\Pi^B(m^{bil};m^{tri}) - m^{tri}r^C p\right]$$

- Fo.c. with respect to m^{tri} (it uniquely defines m^{tri}, independently from m^{bil}):

$$\begin{aligned} 0 &= H^{tri}(m^{tri};\omega_{bil},\omega_{tri},V^B,\mu^L) \\ &= \frac{\omega_{bil}(1-\omega_{tri})}{1-\omega_{tri}(1-\omega_{bil})}\left(\frac{1}{1-m^{tri}}\right)^2 \left[\mathbb{E}_L\left[s|s<\tilde{s}^{tri}\right]+V^B\right]F_L(\tilde{s}^{tri}) - (1+r^{bil})pF_L(\tilde{s}^{tri}) - \mu^L p \end{aligned}$$

Comparative statics:

$$\begin{aligned} \partial_{m^{tri}}H^{tri} &= 2\left(\frac{1}{1-m^{tri}}\right)^3 \frac{\omega_{bil}(1-\omega_{tri})}{1-\omega_{tri}(1-\omega_{bil})}\left[\mathbb{E}_L\left[s|s<\tilde{s}^{tri}\right]+V^B\right]F_L(\tilde{s}^{tri}) \\ &\quad -\left(\tilde{s}^{tri}f_L(\tilde{s}^{tri})F_L(\tilde{s}^{tri}) + \left[\mathbb{E}_L\left[s|s<\tilde{s}^{tri}\right]+V^B\right]f_L(\tilde{s}^{tri}) - (1+r^{bil})pf_L(\tilde{s}^{tri})\right) \end{aligned}$$

So $\partial_{m^{tri}}H^{tri} > 0$. Similarly $\partial_{\omega_{bil}}H^{tri} > 0$ and $\partial_{\omega_{tri}}H^{tri} < 0$ and $\partial_{V^B}H^{tri} > 0$ and $\partial_{\mu^L}H^{tri} < 0$, so $\partial_{\omega_{bil}}m^{tri} < 0$ and $\partial_{\omega_{tri}}m^{tri} > 0$ and $\partial_{\mu^L}m^{tri} > 0$ and $\partial_{V^B}m^{tri} < 0$

- Fo.c. with respect to m^{bil}:

$$\begin{aligned} 0 &= H^{bil}(m^{tri};\omega_{bil},\omega_{tri},V^B,\mu^L) \\ &= \frac{1}{n^{HF}}\left[U^{BD}-U_0^{BD}\right] + \frac{\omega_{bil}(1-\omega_{tri})}{1-\omega_{tri}(1-\omega_{bil})}\left(\frac{1}{1-m^{bil}}\right)^2\left[\int_{s^{min}}^{\tilde{s}^{bil}} uf_B(u)du\right] - (1+r^{bil})pF_B(\tilde{s}^{bil}) \end{aligned}$$

$$\partial_{V^B}F = \left[\frac{\omega_{bil}(1-\omega_{tri})}{1-\omega_{tri}(1-\omega_{bil})}F_L(\tilde{s}^{tri}) - F_B(\tilde{s}^{bil})\right]$$

For high ω_{tri} and low ω_{bil}: $\partial_{V^B}H^{bil} < 0$. We also have: $\partial_{m^{bil}}H^{bil} < 0$, $\partial_{\omega_{bil}}H^{bil} > 0$ and $\partial_{\omega_{tri}}H^{bil} < 0$ and $\partial_{V^B}H^{bil} < 0$. As a result: $\partial_{\omega_{bil}}m^{bil} > 0$, $\partial_{\omega_{tri}}m^{bil} < 0$ and $\partial_{V^B}m^{bil} < 0$.

Appendix C

Appendix to Chapter 3

C.1 Computing the LMI

We propose an integrated measure of the Liquidity Mismatch undertaken by individual banks. We factor in *all* the balance sheet information and account for the effect of each balance sheet entry on the liquidity pressure faced by a bank. The implementation of the LMI is parsimonious yet captures the key degrees of liquidity variability (balance-sheet item heterogeneity and time-series variation). Moreover, the methodology developed in this paper is flexible and can be improved on in future research. For instance, the $\beta_{L'_k}$ sensitivity parameters can be refined and could be estimated, for example, by regressing the corresponding-maturity bond yield on the funding liquidity factor.

ASSETS

Category	A_k	λ_{A_k}
Cash	cash and balances due from depository institutions	1
	federal funds sold	1
	securities purchased under agreements to resell	1
Trading Assets	Treasury securities	$(1 - m_{Trsy})$
	agency securities, excluding MBS	$(1 - m_{Agency})$
	residential MBS guaranteed by FNMA, FHLMC, or GNMA	$(1 - m_{Agency})$
	securities Issued by States and U.S. Pol. Subdivisions	$(1 - m_{Muni})$

	non-agency MBS	$(1 - m_{StrucFin})$
	structured product	$(1 - m_{StrucFin})$
	corporate debt	$(1 - m_{CorporateDebt})$
Available for Sale	Treasury securities	$0.75(1 - m_{Trsy})$
	agency securities, excluding MBS	$0.75(1 - m_{Agency})$
	residential MBS guaranteed by FNMA, FHLMC, or GNMA	$0.75(1 - m_{Agency})$
	securities Issued by States and U.S. Pol. Subdivisions	$0.75(1 - m_{Muni})$
	non-agency MBS	$0.75(1 - m_{StrucFin})$
	structured product	$0.75(1 - m_{StrucFin})$
	corporate debt	$0.75(1 - m_{CorporateDebt})$
	foreign debt	$0.75(1 - m_{ForeignDebt})$
	equity securities	$0.75(1 - m_{Equity})$
Held for Maturity	Treasury securities	$0.50(1 - m_{Trsy})$
	agency securities, excluding MBS	$0.50(1 - m_{Agency})$
	residential MBS guaranteed by FNMA, FHLMC, or GNMA	$0.50(1 - m_{Agency})$
	securities Issued by States and U.S. Pol. Subdivisions	$0.50(1 - m_{Muni})$
	non-agency MBS	$0.50(1 - m_{StrucFin})$
	structured product	$0.50(1 - m_{StrucFin})$
	corporate debt	$0.50(1 - m_{CorporateDebt})$
	foreign debt	$0.50(1 - m_{ForeignDebt})$
Loans	loans secured by real estates	$0.25(1 - m_{StrucFin})$
	commercial & Industry Loans	$0.50(1 - Max\{m_k\})$
	other Loans	$0.50(1 - Max\{m_k\})$
	lease financing receivables	$0.50(1 - Max\{m_k\})$
Fixed Assets	premises and fixed assets	0
	other real estate owned	0
	investment in unconsolidated subsidiaries	0
Intangible Assets	goodwill and other intangible assets	0
Other Assets		0

LIABILITIES: $\lambda_L = -\exp(-\mu T_{k'})$

Category	$L_{k'}$	$T_{k'}$
Fed Funds	overnight federal funds purchased	0

Repo	securities sold under repo	0
Deposits[1]	insured	10
	uninsured	1
Trading Liabilities	trading liabilities	$(1-m_{k'})$
Other Borrowed Money	commercial paper	0.25
	with maturity <= 1 year	1
	with maturity > 1 year	5
Other Liabilities	subordinated notes and debenture	10
	other liabilities	10
Total Equity Capital	equity	30
Contingent Liabilities[2]	unused commitments	5
	Credit Lines	10
	Securities Lent	5
	Collateral Values	10

Notes: 1. A bank's deposit can be decomposed into multiple categories: insured and uninsured deposits, interest-bearing and noninterest-bearing deposits, domestic and foreign deposits, time deposits and broker deposits, and so on. Among them, insured and uninsured category directly relates to a bank's liquidity condition. The Federal Deposit Insurance Corporation (FDIC) provides deposit insurance in order to guarantee the safety of deposits in member banks. Such deposits, since fully guaranteed by the FDIC, should have little influence on a bank's liquidity. However, the insured and uninsured category are not clearly broken down in the Y-9C report. We collect such data instead from the Call Report FFIEC 031 Schedule RC-O – Other Data for Deposit Insurance and FICO. The Call Report data are for banks that are subsidiaries of the BHCs which file the Y9C. Therefore we manually merge the call reports data back to their highest holding company. The deposits at the BHC level is thus the sum of deposits of all its subsidiary commercial banks.

Based on the FDIC insurance limits and the call report decomposition data, we calculate the insured deposit as the combination of i) all deposit lower than the FDIC limit K and ii) the first K dollar amount in the accounts above the limit multiplying the number of such deposit accounts. There are two insurance coverage changes in our sample period. First, the FDIC increased insurance limits from \$100,000 to \$250,000 per depositor on October 3, 2008. Yet this change is not reflected in the Call Report RC-O until 2009:Q3. We follow the data availability and change our definition for insured/uninsured deposit beginning in 2009:Q3. Second, the FDIC increased the insurance for retirement accounts from \$100,000 to \$250,000 on March 14, 2006. This change is reflected in the 2006:Q2 call reports and our definition reflects this change beginning in 2006:Q2.

2. We study four types of contingent liabilities that may exert a pressure on bank's liquidity. Many banks carry *unused commitments*, including revolving loans secured by residential properties, unused credit card lines, commitments to fund commercial real estate, construction, and land development loans, securities underwriting, commitments to commercial and industrial loans, and commitments to provide liquidity to asset-backed commercial paper conduits and other securitization structures. The second type are *credit lines*, including financial standby letters of credit and foreign office guarantees, performance standby letters of credit and foreign office guarantees, commercial and similar letters of credit.[1] A third type of contingent liability is *securities lent*. The last type of contingent liability in our study is the *derivative* contract. Item 7 in Schedule HC-L lists the gross notional amount of credit derivative contracts, including credit default swaps, total return swaps, credit options and other credit derivatives. However, such gross notional amount does not reflect the contracts' liquidity. What matters in a credit derivative contract in terms of liquidity impact is the additional collateral or margin required in a stress event. We therefore use Item 15 to collect the fair value of collateral posted for over-the-counter derivatives.[2]

C.2 Background on Federal Liquidity Injection

The Federal Reserve System (Fed) undertook numerous measures to restore economic stability from the financial crisis of 2007 - 2009. Beyond its conventional monetary policy tools, the central bank, citing "unusual and exigent circumstances," launched a range of new programs to the banking sector in order to support overall market liquidity.

Conventionally, the Fed uses open market operations and the discount window as its principal tools to manage reserves in the banking sector. During the crisis, however, the effectiveness of the discount window was limited because of a sigma effect. Banks were reluctant to approach the discount window since such action could cause market participants to draw adverse inference about the bank's financial condition (see, for example, Peristiani (1998), Furfine (2003), Armantier, Ghysels, Sarkar and Shrader (2011)).

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[1] Berger and Bouwman (2009) consider unused commitments and standby letters of credit as asset-side liquidity whereas we treat them as liability-side liquidity. It's true that unused commitments and credit lines are similar to loans and hence can be treated as assets, yet they become assets only when used. In terms of liquidity, they belong to potential liquidity outflow as other liability classes. Therefore we treat them in line with liabilities given the common feature that they all exert liquidity pressure.

[2] The collateral type contains U.S. Treasury securities, U.S. government and government-sponsored agency debt securities, corporate bonds, equity securities, and other collateral. The collateral value is further divided into groups by the counterparty, for example, a) bank and security firms, b) Monoline financial guarantors, c) hedge funds, d) sovereign governments, and e) corporations and all other counterparties.

Data under item 15 is required to be completed only by the bank holding companies with total assets of 10$billion or more, and such requirement starts only since the second quarter of 2009. Not surprisingly, we only find such data for large BHCs such as J.P. Morgan Chase, Bank of America, etc.

Given the borrowing stigma and inflexibility of open market operations, the Fed proceeded to introduce additional facilities increase liquidity, including the Term Auction Facility (TAF), Term Securities Lending Facility (TSLF), Primary Dealer Credit Facility (PDCF), Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility (AMLF), Commercial Paper Funding Facility (CPFF), Money Market Investor Funding Facility (MMIFF), and Term Asset-Backed Securities (TALF). Fleming (2012) provides a summary on these lending facilities. We summarize their key features in the following table.

Facility	Announcement	Expiration	Participants	Term
TAF	Dec12, 2007	Mar08, 2010	Depository Inst.	28 or 84 days
TSLF	Mar11, 2008	Feb01, 2010	Primary dealers	28 days
PDCF	Mar17, 2008	Feb01, 2010	Primary dealers	overnight
AMLF	Sep19, 2008	Feb01, 2010	BHCs and branches of foreign banks	<120 days for D* <270 days for non-D
CPFF	Oct07, 2008	Feb01, 2010	U.S. CP issuers	3 months
MMIFF	Oct21, 2008	Oct30, 2009	Money Mkt Funds	90 days or less
TALF	Nov25, 2008	Jun30, 2010	U.S. eligible banks	<5 years

*: D denotes depository institutions; non-D is non-depository institutions.

The Fed announced the first facility, Term Auction Facility (TAF) on December 12, 2007 to address the funding pressure in short-term lending markets. Through the TAF, the Fed auctioned loans to depository institutions, typically for terms of 28 or 84 days. Later, to address liquidity pressures in the term funding markets, the Feds introduced the Term Securities Lending Facility (TSLF) on March 11, 2008. Through TSLF, the Fed auctioned loans of Treasury securities to primary dealers for terms of 28 days. Another related facility, the Primary Dealer Credit Facility (PDCF), was announced on March 16, through which the Fed made overnight loans to primary dealers. The bankruptcy of Lehman Brothers on September 15, 2008 led to unparalleled disruptions of the money market. On September 19, the Fed announced created the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility (AMLF). It provided loans to U.S. bank holding companies, and U.S. branches and agencies of foreign banks to purchase eligible asset-backed commercial paper from money market mutual funds. On October 7, the Fed further announced the creation of the Commercial Paper Funding Facility (CPFF), through which the Fed provided credit to a special-purpose vehicle (SPV) that, in turn, bought newly issued three-month commercial paper. Two weeks later on October 21, the Fed established the Money Market Investor Funding Facility (MMIFF). All three money market-related facilities expired on February 1, 2010. Lastly, the Fed introduced the Term Asset-Backed Securities (TALF) on November 25, 2008, through which the Fed made loans to borrowers with eligible asset-backed securities as collateral.

Appendix D

Appendix to Chapter 4

D.1 Pricing of Eurobonds with exogenous safety demand

It can be derived closed-form, assuming a multivariate normal distribution:

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}\right)$$

The symmetry assumption is: $\mu_1 = \mu_2$ and $\sigma_1 = \sigma_2$. The computation of $\mathbb{P}\left[y_1^1 \geq \frac{B^*}{\delta}, y_1^2 \geq \frac{B^*}{\delta}\right]$, using conditional pdf:

$$\mathbb{P}\left[y_1^1 \geq \frac{B^*}{\delta}, y_1^2 \geq \frac{B^*}{\delta}\right] = \int_{\frac{B^*}{\delta}} \left(\int_{\frac{B^*}{\delta}} f\left(u|v\right) du\right) f(v) dv$$

where: $f\left(s_1|s_2\right) \sim N\left(\mu_1 + \frac{\sigma_1}{\sigma_2}\rho(s_2 - \mu_2), \sqrt{(1-\rho^2)\sigma_1^2}\right)$

so in the symmetric case: $f\left(s_1|s_2\right) \sim N\left(\mu + \rho(s_2 - \mu), (1-\rho^2)\sigma^2\right)$.

$$\int_{\frac{B^*}{\delta}} \left(\int_{\frac{B^*}{\delta}} f\left(u|v\right) du\right) f(v) dv = \int_{\frac{B^*}{\delta}} \left(1 - \Phi\left(\frac{\frac{B^*}{\delta} - \mu - \rho(v-\mu)}{\sqrt{1-\rho^2}\sigma}\right)\right) f(v) dv$$

Using:

$$\int_x^\infty \Phi\left(z\right)\phi(z) dz = \frac{1}{2}\left[1 - \left(\Phi\left(x\right)\right)^2\right]$$

$$\tilde{\mu} = \frac{1}{\rho}\frac{B^*}{\delta} - \left(\frac{1}{\rho} - 1\right)\mu = \mu + \frac{1}{\rho}\frac{B^*}{\delta} - \frac{1}{\rho}\mu$$

$$\tilde{\sigma} = \frac{\sqrt{1-\rho^2}}{\rho}\sigma$$

We obtain:

$$\mathbb{P}\left[y_1^1 \geq \frac{B^*}{\delta}, y_1^2 \geq \frac{B^*}{\delta}\right] = \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}} \int_{\frac{B^*}{\delta}} \Phi\left(\frac{v - \tilde{\mu}}{\tilde{\sigma}}\right) \phi\left(\frac{v-\mu}{\sigma}\right) dv$$

Introduce:

$$\Delta\sigma = \left(\frac{1}{\tilde{\sigma}} - \frac{1}{\sigma}\right) = \left(\frac{1}{\frac{\sqrt{1-\rho^2}}{\rho}\sigma} - \frac{1}{\sigma}\right) = \frac{1}{\sigma}\left(\frac{\rho}{\sqrt{1-\rho^2}} - 1\right)$$

$$\Delta\mu = \left(\frac{\tilde{\mu}}{\tilde{\sigma}} - \frac{\mu}{\sigma}\right) = \left(\frac{\mu + \frac{1}{\rho}\frac{B^*}{\delta} - \frac{1}{\rho}\mu}{\frac{\sqrt{1-\rho^2}}{\rho}\sigma} - \frac{\mu}{\sigma}\right) = \frac{\mu}{\sigma}\left(\frac{\rho + \left[\frac{1}{\mu}\frac{B^*}{\delta} - 1\right]}{\sqrt{1-\rho^2}} - 1\right)$$

In the copula:

$$\mathbb{P}\left[y_1^1 \geq \frac{B^*}{\delta}, y_1^2 \geq \frac{B^*}{\delta}\right] = \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}} \int_{\frac{B^*}{\delta}} \left\{\Phi\left(\frac{v-\mu}{\sigma}\right) + \frac{1}{\sigma}\phi\left(\frac{v-\mu}{\sigma}\right)\left[v\Delta\sigma - \Delta\mu\right]\right\}\phi\left(\frac{v-\mu}{\sigma}\right) dv$$

By change of variable, using:

$$\int_a \phi^2 z dz = -\int_a \phi\phi' dz = \phi\left(a\right)^2$$

$$\int_a \phi^2 dz = \frac{1}{2\pi}\int_a e^{-\frac{1}{2}\frac{z^2}{1/2}} dz = \frac{1}{\sqrt{2\pi}}\int_a e^{-\frac{1}{2}\frac{z^2}{1/2}} dz = \frac{1}{\sqrt{2\pi}}\left[1 - \Phi\left(\frac{a}{\sqrt{2}}\right)\right]$$

$$\int_a \Phi\phi dz = \frac{1}{2}\left[1 - \Phi\left(a\right)^2\right]$$

and denoting:

$$a = \frac{\frac{B^*}{\delta} - \mu}{\sigma}$$

We obtain:

$$\mathbb{P}\left[y_1^1 \geq \frac{B^*}{\delta}, y_1^2 \geq \frac{B^*}{\delta}\right] = \frac{1}{2\pi\sigma\sqrt{1-\rho^2}}\left\{\frac{1}{2}\left[1 - \Phi\left(a\right)^2\right] + \Delta_\sigma\phi\left(a\right)^2 + \left\{\mu\Delta_\sigma - \Delta_\mu\right\}\frac{1}{\sigma\sqrt{2\pi}}\left[1 - \Phi\left(\frac{a}{\sqrt{2}}\right)\right]\right\}$$