Skill, Job Design, and the Labor Market under Uncertainty

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Skill, Job Design, and the Labor Market under Uncertainty

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Abstract

The labor market matches agents with work, but uncertainty over the type and location of available work reduces the efficiency with which skill can be allocated to its best use. The essays in this dissertation examine the impact of uncertainty on the optimal division of work into jobs and allocation of agents to those jobs using applied economic theory.

The first and second essays focus on the tradeoff between allocating agents in the market (contracting) versus in the firm (employment). Contractors are mobile allowing for a more efficient match between agent skill and task value, but employees are available for tasks that contractors cannot do. In the first essay, two aspects of volatility, urgency and uncertainty, are traded off; urgency requires putting resources in place but at the cost of mobility. The equilibrium divides the labor market into employees and contractors, with the number of employees increasing in urgency and decreasing in uncertainty.

Essay two explores the instability of contract work for agents of different skill levels. It describes an equilibrium, reflecting findings from sociology, in which low skill contractors are excluded from employee positions and face greater instability than high skill contractors. The model shows that such an equilibrium is likely to arise when firms have a low volume of work, tasks require little firm specific knowledge, or labor supply is low relative to the number of possible employee positions.

The third essay discusses the impact of uncertainty on the horizontal division of labor under gains from specialization. It presents a model of team and job design in which the set of tasks required for production is uncertain and the amount of work available is limited.

The assignment of agents to tasks is constrained by the work available, implying naturally arising limits to team size and specialization. Division of labor can be increased by increasing team size, but this is costly as it reduces agent utilization.

All three essays demonstrate that uncertainty increases the value of agent mobility, either between firms or between tasks. Thus, optimal job design and skill allocation are such that specialization is decreasing in uncertainty.

Contents

Preface

Markets and organizations are two ways of allocating resources, including skill, in the economy. An efficient allocation matches each resource to its best use. However, uncertainty makes allocation difficult, because the best use for a resource may not be known in advance. This dissertation explores the impact of uncertainty on the allocation of talent to firms and to jobs in the labor market.

A main theme throughout these essays is that uncertainty increases the value of mobility. Because the best use for an agent's skill is unknown in advance, delaying the commitment of that agent to a particular firm or task has the potential to increase match efficiency. It is clear, then, that as uncertainty increases, specialization of any kind decreases.

The essays presented here discuss several types of specialization: temporal specificity, firm specific knowledge, and area of expertise. Mobility is traded off with preparedness when tasks may be urgent, with performance on firm specific tasks, or with gains from specialization. In all three cases, the results of the model demonstrate that changes in staffing patterns and job design can be attributed to changes in the predictability of workflow.

The first and second essays focus on the tradeoff between allocating agents in the market (contracting) versus in the firm (employment). Contractors are mobile allowing for a more efficient match between agents who differ in skill and tasks that differ in value; however, employees are available for tasks that contractors cannot do.

This argument demonstrates that economy wide trends in staffing practices, such as employment and independent contracting, must be examined from the labor market level. The value of mobility cannot be captured in a transaction level model. Thus, these two

essays, rather than examining a firm's staffing decision as previous papers have done, analyze the entire labor market and identify which agents become employees and which become contractors.

In the first essay, uncertainty is traded off with another aspect of volatility: urgency. Urgency requires putting resources in place, which is at odds with the mobility necessitated by uncertainty. The equilibrium divides the labor market into employees who are allocated to respond to urgency and contractors who remain mobile.

While employees take on any urgent task that needs to be done, contractors' skills are efficiently matched with task value. Thus, an employee's firm assignment is specialized but his job is general, whereas a contractor's firm assignment is general but his job is specialized. The results of the model show that the number of employees is increasing in urgency and decreasing in uncertainty, and job specialization is decreasing in urgency and decreasing in uncertainty.

The second essay explores the emergence of two classes of contract workers, reported by sociologists. It describes an equilibrium, reflecting the findings from sociology, in which low skill contractors are excluded from employee positions and face greater instability than high skill contractors. The model shows that such an equilibrium is likely to arise when firms have a low volume of work, tasks require little firm specific knowledge, or labor supply is low relative to the number of firms.

These two essays reveal a link between the changing nature of production and working patterns of agents. An increase in the use of independent contracting, a decrease in vertical specialization, and an emergence of a class of contingent workers facing low wages and unemployment can all be explained by a decrease in the predictability of available work.

Indeed, work has become less predictable in economies where service and technology sectors are growing while manufacturing is shrinking.

The third essay expands on this idea by examining the impact of uncertainty on the horizontal division of labor. Theorists and practitioners have touted the benefits of division of labor and refinement of jobs in manufacturing for centuries. However, production in manufacturing is uncommon in its consistency and predictability. Instead, many industries can be characterized by workflow that cannot be controlled like an assembly line can. Furthermore, the types of tasks required for production are often uncertain.

The assignment of agents to specific task types is constrained by the work available at a given time. Thus, the final essay shows that team size and specialization are naturally limited when the quantity of work is limited. Furthermore, it demonstrates that the value of moving agents between a variety of task types (i.e. decreasing division of labor) increases when the tasks required for production are not known in advance. Division of labor can be increased by increasing team size, but this is costly as it reduces agent utilization.

The theories presented in this dissertation relate labor market patterns, such as staffing practices, job design, and talent allocation, to characteristics of the labor market and the nature of production and workflows. The results of these models can be used to predict the fraction of agents who are contractors, the degree of job specialization, and job stability in different industries. Future work will take these predictions to data in order to test and refine the models.

In addition, these essays take the labor supply as given. The next step for this research is to extend the models to examine implications for agent skill investment choices, in particular when agents face tournament-like competition.

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Chapter 1

Employees and Contractors: The Role of

Volatility in the Labor Market

1.1 Introduction

Urgency and uncertainty present challenges to matching agents with jobs. Where should
agents be placed when there is uncertainty over the location of important tasks? How can
agents be efficiently placed when there is a limited supply of talent and urgency requires that
tasks be done immediately? This paper argues that employment and contracting can arise
as solutions to allocation frictions. Employees are agents who are allocated locally to tasks,
allowing for responsiveness to urgency. Whereas, contractors are agents allocated globally,
allowing uncertainty to be addressed in the labor market.

In this general equilibrium model, with symmetric information, complete contracts,
and no moral hazard, production occurs when an agent applies his skill to complete a task,
which may take on a number of values. Thus, agents and tasks must be matched, and like

other matching models, social surplus is maximized by allocating the highest skill agents to the highest value tasks, i.e. through assortative matching.

This match cannot generally be achieved because agents are hired by firms, rather than allocated directly to tasks. Uncertainty over which firms will receive high value tasks, together with the urgency of tasks, creates friction in matching agents with tasks. An agent may be allocated after uncertainty is resolved, thereby increasing the efficiency of his match; however, delaying agent allocation is costly because it causes urgent tasks to go undone. The tradeoff between urgency and uncertainty affects the efficiency of production, the nature of employment relationships, and the degree of division of labor in the economy.

The timing of agent allocation reaches a stable equilibrium in which some agents are allocated to firms ex-ante each period while others are allocated ex-post each period. Interpreting agents who are allocated ex-ante as employees and those allocated ex-post as contractors, employment is increasing in urgency and decreasing in uncertainty in the general equilibrium. Employment is also increasing in the supply of high skill labor. The model further predicts that the division of labor is highest when urgency is low and uncertainty is low.

The results of the model support the employment versus contracting interpretation of allocation timing. An agent who is hired before uncertainty is resolved does a subset of the tasks his firm receives. Thus, the job of this agent is a function of the firm to which he is assigned. This result aligns with casual observation, which suggests that the defining characteristic of an employee is that he works for a single firm over an indefinite period. An employee's job is defined by the firm he works for; it depends on the tasks the firm needs to be done.

The model shows that agents who are allocated after information revelation have higher skill utilization because they are able to spread their talent across a variety of firms. An agent who is hired ex-post specializes in a particular type of task, and can work at any one of a number of firms, as long as that firm has the correct task type. This description reflects real world working patterns of contractors who are typically hired for specific projects or periods of time. Contractors work for many firms, or clients, and therefore have the flexibility to specialize in a particular type of task. The firm a contractor works for depends on his job specialization.

The equilibrium also describes three firm staffing policies. In the first policy, the firm hires a high skill agent as an employee. A firm using this policy will have the employee perform any task it receives. In the second policy, the firm hires a low skill agent as an employee. This employee works only on some task types, as the firm will also hire a high skill contractor after it receives a high value, non-urgent task. Lastly, in the third policy, a firm does not hire an employee, but hires a contractor depending on the task type it receives.

The question of when a firm should hire an agent as an employee rather than as an independent contractor is basic to organizational economics, having first been posed by Coase in his 1937 paper that founded the field. The literature on this topic focuses on the role of employees and independent contractors relative to the firm, rather than their role in the economy as a whole. Allocation of authority to the firm has been proposed as the defining characteristic of employment (Coase, 1937; Simon, 1951; Williamson, 1971). Authority is relevant when a choice cannot be specified in the contract in advance, i.e. when there is uncertainty and contracts are incomplete (Williamson, 1971). This paper presents

an alternative to the existing theory of employment that is not based on incomplete contracts and incentive issues.

The existing theory argues that when uncertainty over the state of the world is high, making flexibility important, hiring an employee to execute the decision is better than hiring a contractor because the firm can exercise its authority over the employee (Williamson, 1971, 1975; Tadelis and Williamson, 2013). If this were the case, increased environmental volatility would lead to an increase in traditional employment relationships. In fact, in many industries, the observed phenomenon is the opposite: companies cite flexibility as a reason for hiring contractors and temporary workers. The model in this paper uses a different type of uncertainty, that over task type, to provide an alternative theory for agents as employees and contractors which accounts for these observations.

The issue of whether and why firms have authority over agent actions is under debate. Some argue that relationship specific investments make at least one of the parties better off transacting with its contractual partner than with another party in the market (Klein et al., 1978). When an investment is made in an asset that is used in production, asset ownership determines which party can take the asset if the relationship breaks down, and therefore directly affects the parties' options. When a firm owns an asset, its ability to replace an agent gives it authority over the agent's actions (Grossman and Hart, 1986).

The asset-ownership theory makes strong predictions regarding staffing decisions when physical capital is an essential factor, but does not explain staffing when non-human assets are less important, such as in service industries. For example, the provision of legal services to a single corporate client does not rely on an asset that is owned by the client when the lawyers are in-house employees and owned by the law firm when the lawyers are out-

side counsel. This example indicates that factors other than authority contribute to staffing decisions.

The model in this paper sets aside both incomplete contracting and incentive issues. As such, it is similar in approach to the team theory literature, which focuses on the structure of organizations (Garicano and Van Zandt, 2013). While this literature does discuss trade-offs with respect to delay (Van Zandt, 2003, 1998; Radner, 1993), specialization (Bolton and Dewatripont, 1994; Garicano, 2000), and flexibility through use of local information (Garicano and Van Zandt, 2013), it does not directly address the employment versus contracting problem.

This paper is most closely related to the knowledge hierarchies literature, which argues that a high skill agent's time is leveraged by working with low skill agents so he can focus only on tasks others are unable to do (Garicano, 2000). That result is similar to the specialization result in the current paper; however, specialization here is the job of contractors who can be either low or high skill. The result in this paper that contractors are specialists is in line with the argument elsewhere that specialization is supported by market size (Garicano and Hubbard, 2008); here agents can specialize by offering their skills in the open labor market, which is a larger market than a single firm. In this paper, delay allows for a distinction between agents who are located together and those who are located separately. Thus, this approach is complementary to knowledge hierarchies, in which agents in the same hierarchy may be in the same or different firms.

The paper proceeds as follows. The next section briefly outlines a real world example of the model. Section 1.3 introduces the variables of interest and sets up the model. The equilibrium allocation of agents is derived in Section 1.4. Section 1.5 introduces the results

regarding jobs and how they differ by agent type and by assignment timing. Section 1.6 describes the equilibrium staffing practices of firms and discusses which types of firms use each staffing policy. Section 1.7 explores the impact of uncertainty and urgency on employment and job design. Section 1.8 concludes. All proofs of results are provided in the appendix.

1.2 Example: Corporate Lawyers

As the model is introduced, it will be helpful to refer to a real world example. Corporate law is an industry in which the agents, lawyers, use their skill to work on tasks. Tasks can differ greatly in value. For example, corporate legal transactions and lawsuits can range from tens of thousands of dollars to hundreds of millions of dollars. Furthermore, agents of different skill levels perform differentially on similar tasks. The legal profession comprises a broad range of talent, which is assortatively matched with work; a labor market with lawyers as independent contractors allows for talent to be effectively matched with cases throughout the economy.

In the field of corporate law, some cases are handled by in-house counsel and some by outside lawyers. Legal scholars seeking to explain the rise of in-house counsel have cited firm specific knowledge as an important benefit of using in-house counsel (Lynch, 1979; Machlowitz, 1989; Aibel, 1983; McKinney, 1979), as well as continuity of knowledge (Aibel, 1983; McKinney, 1979) and faster action (Chayes and Chayes, 1985; Machlowitz, 1989; Aibel, 1983; Hackett, 2002). Some have also suggested that in-house counsel has a unique ability to play a preventative role (Lynch, 1979; Chayes and Chayes, 1985; Aibel, 1983; Hackett, 2002). Alternatively, the cited benefits of hiring outside counsel include utilizing specialization

(Chayes and Chayes, 1985; Aibel, 1983) and cost saving of only hiring counsel when needed (Machlowitz, 1989; Aibel, 1983).

The theory presented here encompasses these arguments. Agents who are allocated ex-ante (employees) use the time before tasks arrive to gain knowledge about the firm. Agents who are allocated ex-post (contractors) must catch up, causing a delay that is costly. An ex-post allocated agent, however, is matched with the firm where his skill can be best utilized.

1.3   Model Setup

In an economy where agents use their skill to produce value by completing tasks, suppose there are two types of agent, high skill and low skill, and two values of task that a firm may receive, high value and low value. Frictions arise in the match due to three problems: 1) There is uncertainty over which firms will receive high value tasks; 2) Agents must prepare before working at a firm, and this preparation takes time; and 3) Some tasks are urgent, so delayed matches will cause a loss of value due to the preparation time.

Working on a task requires preparation which is firm, rather than task, specific. As such, the preparation can be done before the task arrives, but preparation for one firm is not helpful for work at a different firm. For example, a lawyer working on a case for a corporation must have an understanding of the corporation's business environment, recent actions it has taken, and the laws and regulations under which it operates. Assume this preparation must be done each time an agent works; agents must keep up with recent firm decisions.
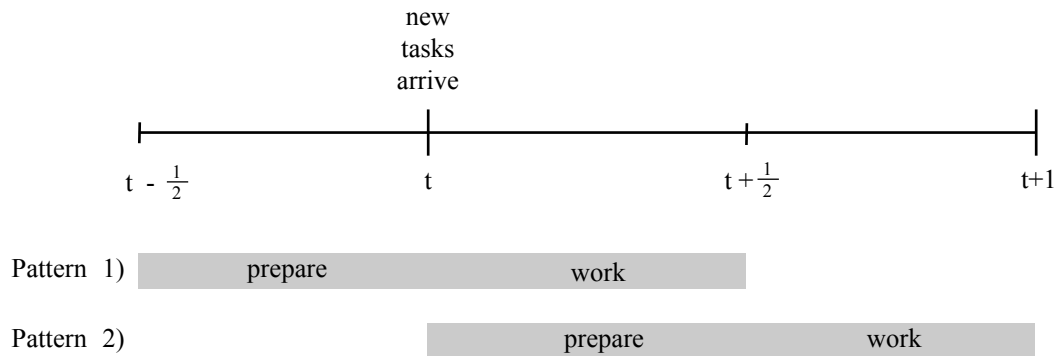
Figure 1.1: Task Arrival and Preparation-Work Patterns

At time t, each firm receives a task, and the type of each firm's task is publicly revealed. Assume that preparation takes half a period, and that while preparing, an agent cannot also be working on a task. Then, in order for an agent to begin working on a task when it arrives, he must be matched with a firm at $t - \frac{1}{2}$. If, on the other hand, an agent is not matched until the tasks arrive, he will not begin working on a task until $t + \frac{1}{2}$. Of all tasks, $\hat{\mu}$ are urgent, expiring before $t + \frac{1}{2}$. The rest of tasks are non-urgent, expiring at the end of one period. Working on a task takes half a period, so the time required for preparation and work is one period.

Each task can be attempted only once; if an agent fails in completing the task, there is no opportunity to attempt the task again. Figure 1.1 shows the timeline, indicating the timing of preparation and of work relative to task arrival. At each firm, the task will either be attempted in the first half of the period by an agent who prepared in the half-period before arrival (Pattern 1), or attempted in the second half of the period by an agent who prepared in the first half of the period (Pattern 2).

Each firm will receive a task of value $v \in \{\underline{v}, \bar{v}\}$, where $\underline{v} < \bar{v}$. This value is independent of task urgency. There are two types of agent, $\underline{\alpha}$ and $\bar{\alpha}$, where α denotes the probability that the agent will successfully complete the task he attempts (otherwise no revenue is generated), and $\underline{\alpha} < \bar{\alpha}$. Each agent's type is public information.

These types can be thought of as lower and higher skill agents, where higher skill agents have an absolute advantage in production. In the example, while all lawyers are highly skilled workers, there is variation within the field; some are elite and some are average. In the model, $\bar{\alpha}$ and $\underline{\alpha}$ will be referred to as 'high' and 'low' skill respectively for simplicity; the relative, not absolute, skill levels of these agent types drive the results of the model.

The fraction of tasks that are high value is $\hat{p} < \frac{1}{2}$, but there is uncertainty over which firms will receive these tasks. All firms are assumed to be ex-ante identical, unless otherwise indicated. For expositional purposes, suppose the fraction of agents who are high skill is equal to the fraction of tasks that are high value; this assumption will be relaxed, and the effect of high skill labor supply analyzed, in Section 1.7. Assume there are as many agents as there are tasks.

1.4 Optimal Agent Allocation

In a multi-period model, there is a tradeoff between ensuring that urgent tasks are done and allowing agents to be efficiently matched with tasks by allocating them ex-post, because ex-post allocated agents cannot work on urgent tasks the subsequent period. This section examines the effect of urgency costs of ex-post assignment. First, a model in which tasks

arrive in each of two periods is presented, allowing for the detailed examination of this tradeoff. Then the implications of extending the model are discussed.

In a finite model, there is no opportunity cost to assigning each agent ex-ante of the first batch arrival of tasks. Each period agents may be reallocated after being allocated ex-ante. Reallocated agents move into the ex-post role in the economy, continuing to work on available efficiently matched non-urgent tasks. The number of agent who are reallocated is limited either by the urgency costs or by the limited supply of non-urgent tasks. Thus, the system will always reach a steady state in which no ex-ante allocated agent is reallocated ex-post.

1.4.1   Three Period Equilibrium Allocation

Suppose the economy lasts for three periods. Specifically, time begins at the start of Period 0. At the beginning of Period 1, each firm receives a task. Some of these tasks are urgent and expire half way through Period 1, and the rest expire at the end of the period. At the beginning of Period 2, each firm receives a second task, which may or may not be urgent, as in Period 1. The timeline is illustrated in Figure 1.2.

Note that ex-ante allocation of an agent for Period 1 does not prevent ex-post reallocation. An agent can prepare to work at a firm before Period 1 begins, and immediately begin preparing to work at a different firm at the start of Period 1. Each agent will be allocated to a firm for preparation in Period 0.

Each agent allocated to a firm ex-ante has some probability of being inefficiently matched with a task. Some high skill agents' firms will receive low value tasks, and some

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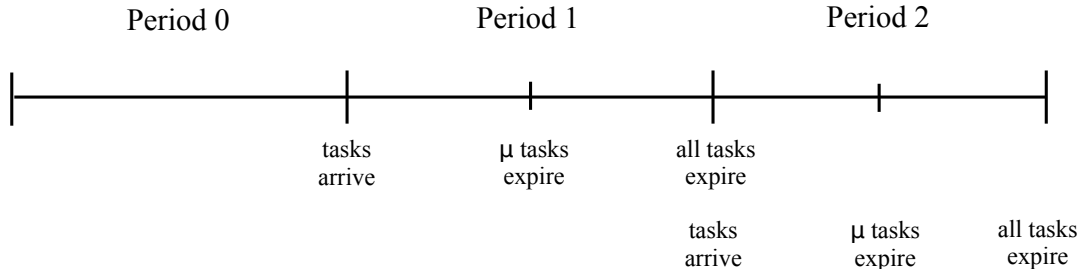
Figure 1.2: Timeline with Tasks Arriving Twice

low skill agents' firms will receive high value tasks. Period 1 surplus may be increased by reassigning some of these agents, but doing so will have an effect on Period 2 surplus. The equilibrium allocation of agents maximizes total surplus, balancing the costs and benefits of ex-post reallocation.

Under the assumption that the fraction of agents who are high skill is equal to the fraction of tasks that are high value, the fraction of agents who are high skill with non-urgent, low value tasks equals the fraction of agents who are low skill with non-urgent, high value tasks.

As such, Period 1 surplus is highest when all non-urgent tasks are efficiently matched with agents: high skill agents do all non-urgent, high value tasks; low skill agents do all non-urgent, low value tasks. Each urgent task is done by the agent allocated to its firm ex-ante, because reallocating these agents will decrease surplus. Figure 1.3 shows the match, including ex-post reassignment, that maximizes Period 1 surplus.

While reallocation of mismatched agents in a given period increases surplus from work on that period's tasks, it has a negative impact on surplus in the following period. Working during the second half of Period 1 prevents an agent from preparing during that time to

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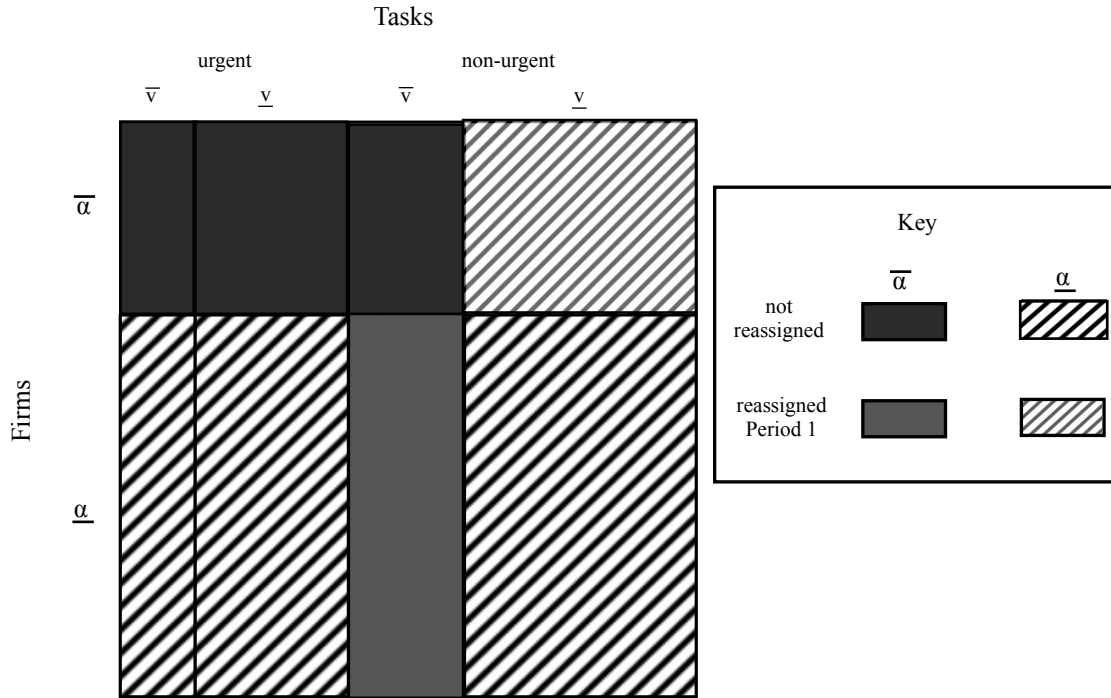
Figure 1.3: Period 1 Match with Ex-post Reallocation

work during the first half of Period 2; therefore, reassigned agents cannot work on urgent tasks in Period 2.

On the other hand, the surplus generated by work on non-urgent tasks in Period 2 cannot decrease as a result of Period 1 reassignment of agents. Reassignment during Period 2, the last period, is not costly; it does not prevent work on future urgent tasks, because there are no future tasks. Therefore, all non-urgent, high value tasks are done by high skill agents in Period 2.

If no agents are reassigned in Period 1, then in Period 2 all non-urgent, low value tasks are done by low skill agents after Period 2 reallocation (see Figure 1.3). However, if any agents are reassigned in Period 1, some non-urgent, low value tasks will be done by high skill agents in Period 2 (as shown in Figure 1.4). Each high skill agent not assigned
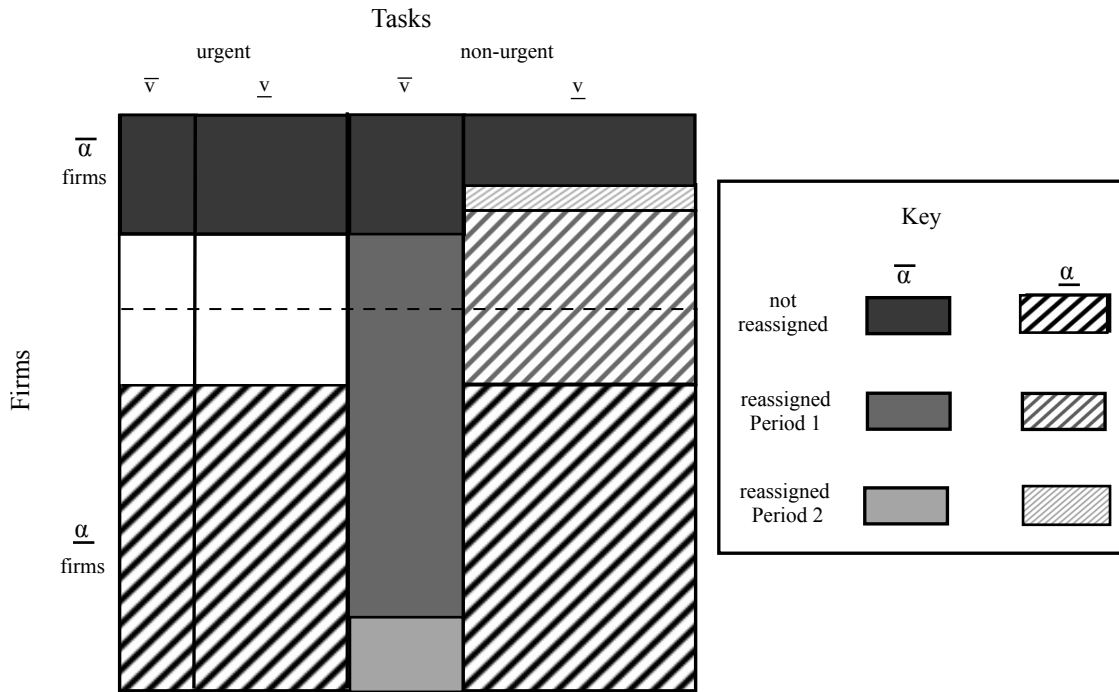
Figure 1.4: Period 2 Match with Ex-ante Unassigned Agents and Ex-post Reallocation

ex-ante for Period 2 (because he was reassigned ex-post in Period 1) will work on a non-urgent, high value task. This allocation decreases the demand for reassignment of Period 2 ex-ante allocated high skill agents from low value to non-urgent, high value tasks. Thus, the surplus generated by work on non-urgent, low value tasks in Period 2 is increasing in Period 1 reassignment—more non-urgent, low value tasks are done by high skill agents.

When the immediate surplus gain and the future gain on non-urgent, low value tasks exceeds the cost from urgent tasks going undone, all agents will be reassigned in Period 1. Note that the future costs are direct, resulting from the unavailability of a Period 1 reassigned agent to work on urgent tasks in Period 2. The potential future benefit comes from a change in the allocation of *other* agents, specifically high skill agents allocated ex-ante for Period 2 to low value tasks who then are not reassigned.

The immediate benefit to reassigning a high skill agent to a non-urgent, high value task is:

$$(\bar{\alpha} - \underline{\alpha})\bar{v} - \bar{\alpha}\underline{v}$$

For each high skill agent that is reassigned, allocating the low skill agent left idle by that reassignment to the non-urgent, low value task left open by that reassignment increases surplus by:

$$\underline{\alpha}\underline{v}$$

Each reassigned agent will not work on urgent tasks in Period 2. Thus conditions[1] for reassignment to be efficient are as follows.

Condition 1.

$$\underline{\alpha}\underline{v} > \underline{\alpha}[\hat{p}\bar{v} + (1 - \hat{p})\underline{v}] \tag{a}$$

$$(\bar{\alpha} - \underline{\alpha})(\bar{v} - \underline{v}) > (\bar{\alpha} + \underline{\alpha})[\hat{p}\bar{v} + (1 - \hat{p})\underline{v}] \tag{b}$$

These conditions hold when $\underline{\alpha}$ is low relative to $\bar{\alpha}$, when $\underline{v}$ is low relative to $\bar{v}$ and when $\hat{\mu}$ is very low. In other words, they hold when the differences between high skill and low skill and high value and low value make allocative efficiency important, and when urgency is very low so that the cost of increasing allocative efficiency is low.

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[1]These conditions are slightly stronger than necessary as the increase in surplus on Period 2 non-urgent tasks would decrease the right hand sides of these inequalities. However, even if the weaker condition fails, the condition that the number of agents and the number of firms is identical is required for no reassignment to occur. For a larger number of firms, some urgent tasks will not be done regardless of ex-ante assignment. Therefore the cost of reassigning an agent ex-post falls. For a larger number of agents than firms, reassignment does not imply that urgent tasks aren't done as the extra agents can be assigned to the firms that are left open.

Proposition 1. *Under Condition 1 all mismatched agents with non-urgent tasks are reassigned in Period 1.*

Agents who are assigned or reassigned ex-post increase allocative efficiency. Their role in the economy is to create a better agent to task match. The role of agents who are assigned ex-ante is to mitigate the costs of urgency by being prepared for newly arriving tasks. In this finite model, agents who are reassigned in a period are moving from the ex-ante role to the ex-post role.

1.4.2   Extended Horizon

Suppose that instead of three periods, the economy lasts n periods, and the future is discounted with discount factor δ. During each period for which surplus can be increased by rematching agents ex-post, agents will be rematched.

Proposition 2. *If the economy lasts for n periods, then for all $\underline{v}$ satisfying Condition 1, there is a δ such that:*

During each period, any ex-ante assigned agents that can be reassigned for an immediate surplus increase will be.

As fewer high skill agents are allocated ex-ante, more non-urgent, high value tasks will be available for ex-post reassignment. Thus, each period more agents are reassigned until the number of high skill agents in the ex-post role equals the number of non-urgent, high value tasks at firms that are not assigned a high skill agent ex-ante (see Figure 1.5). As the time horizon goes to infinity, allocation of agents will approach this steady state.
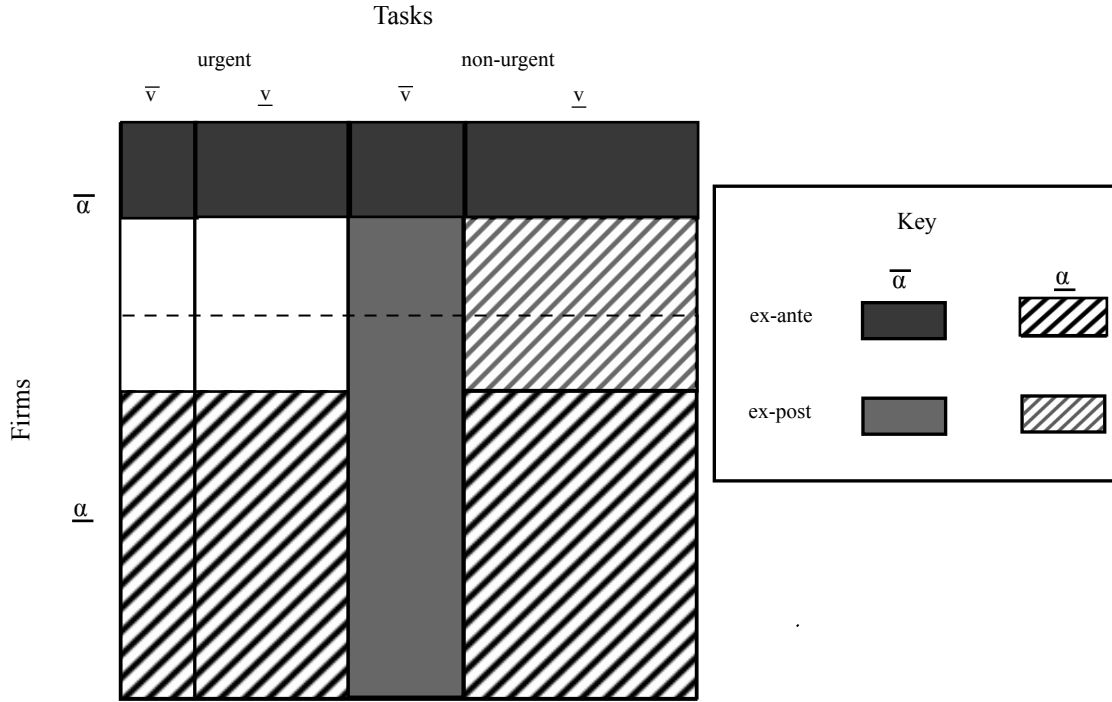
Figure 1.5: Steady State Agent Allocaiton

In this case, after matching unassigned high skill agents with non-urgent tasks ex-post, there are no more non-urgent, high value tasks. Thus, reassigning a high skill agent whose ex-ante assigned firm receives a low value task cannot increase surplus, because there are no high value tasks that he can be reassigned to.

The next two sections describe the behavior of agents and firms in the steady state of this equilibrium allocation. Then the comparative statics of the steady state equilibrium are explored.

1.5 Agent Jobs: Employees and Contractors

An agent's skill level and the timing of his allocation determine the tasks he will attempt. In equilibrium, an agent who is allocated ex-ante will exhibit characteristics typically associated with employees—his productivity will be a function of the firm to which he is assigned. An agent who is allocated ex-post will have characteristics typically associated with contractors—he will always work on a particular type of task, which requires that his skill be spread across many firms.

In the steady state equilibrium, ex-post allocated agents must move between firms, as each firm has a probability less than one of receiving the same task type in the next period. Contractors move throughout the economy, applying their skill to tasks at different firms.

Employees add value because they are available to do tasks as they arrive; they reduce the costs from urgency in the economy. There is no effect on surplus from ex-ante allocated agents moving between firms after they have worked on one task and before they begin preparing for the next. While these agents can move between firms, doing so has no affect on surplus. Movement of employees during a steady state equilibrium cannot, then, be explained by allocative efficiency.

High skill agents are assumed to have an absolute advantage, which makes them more productive. In the equilibrium, high skill agents are also more productive because they are assigned to higher productivity jobs, ones consisting of higher expected value of the tasks. There is an additional differentiation in productivity between contractors and employees of each skill level because of specialization.

1.5.1 High Skill Contractor's Job

A high skill contractor adds value in the economy by working on a high value task received by a firm which was allocated a low skill employee or no employee ex-ante. This allows these agents to maximize their productivity and increases the output of firms that cannot hire a high skill agent as an employee.

High skill agents who are allocated ex-post do only high value tasks. Working on a high value task is more productive than working on a task of uncertain value. Thus, high skill contractors are the most productive agents in the economy[2].

Observation. *High skill agents allocated ex-post work on non-urgent, high value tasks with probability 1. They are the most productive agents in the economy, having the highest expected output.*

Once a high skill agent becomes a contractor, there will always be a high value task for him to work on. However, no individual firm will receive such a task with certainty. Therefore, this agent's skill must be spread across several firms; in the steady state, over time he will necessarily work for multiple firms, even when each firm's probability of receiving a high value task is constant. A high skill contractor's talent is used efficiently because he does not spend time on low value tasks.

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[2]Relaxing the assumption that there are at least as many high skill agents as there are non-urgent, high value tasks, this model shows that the highest skill agents in the economy become contractors, never employees.

1.5.2 High Skill Employee's Job

A high skill agent allocated ex-ante does any task type his firm receives. Because his skill cannot be utilized more efficiently elsewhere, high skill employees spend some of their time working on low value tasks. This makes them less productive than high skill contractors.

Observation. *In the steady state, a high skill agent allocated ex-ante attempts any task that his firm receives.*

The expected performance of a high skill agent allocated ex-ante is

$$\bar{\alpha}[\hat{p}\bar{v} + (1 - \hat{p})\underline{v}]$$

In an economy with firms that differ in p, the assignment of a high skill agent to a particular firm will determine the makeup of that agent's job. A high skill employee works on a high value task with probability p, and a low value task with probability $(1 - p)$, where p may be firm dependent. Thus, the firm an ex-ante allocated high skill agent works for determines his job—the tasks he works on.

1.5.3 Low Skill Employee's Job

Low skill agents allocated ex-ante have the same role in the economy as their high skill counterparts. A low skill employee works on whatever his firm needs done. In the steady-state equilibrium, a low skill agent who is allocated ex-ante will do any task his firm receives except for non-urgent, high value tasks, which are left for high skill agents to do.

Observation. *A low skill agent allocated ex-ante does the low value and urgent tasks that his firm receives.*

The expected value produced each period by a low skill agent allocated ex-ante is:

$$\underline{\alpha}[\hat{p}\hat{\mu}\bar{v} + (1 - \hat{p})\underline{v}]$$

The low skill employee works on a high value task with probability $p\mu$ and a low value task with probability $(1 - p)$, and has a $p(1 - \mu)$ of being idle. Like the high skill employee, his job is a function his firm.

That the low skill agent allocated ex-ante spends some time idle opens the possibility that some low skill agents can be less productive than low skill agents allocated ex-post, who only work on low value tasks.

Condition 2. *More value is generated by an agent who spends all his time on a low value task, than by an agent who works on a firm's urgent and low value tasks, but is idle when the firm receives a non-urgent high value tasks.*

$$\underline{v} \geq \hat{p}\hat{\mu}\bar{v} + (1 - \hat{p})\underline{v}$$

Under this condition, the low skill employee is the least productive agent in this economy.

1.5.4   Low Skill Contractor's Job

Low skill agents allocated ex-post provide value by ensuring no non-urgent tasks remain unassigned. A low skill agent who is allocated ex-post must work on a low value task, because all non-urgent, high value tasks allocated ex-post are matched with high skill agents.

Observation. *Low skill agents allocated ex-post do only non-urgent, low value tasks.*

As a contractor, the agent specializes in a particular type of task; low skill contractors work only on low value tasks. Like the high skill contractor, the low skill contractor must have a large number of potential clients, which collectively have enough tasks to support the agent working full time on this particular type of task.

The results presented thus far reveal the working patterns of agents in this economy. Each agent does a job that depends on his type and on the timing of his allocation. High skill agents always work on tasks with higher expected value than the tasks low skill agents work on. Agents assigned to a firm ex-ante do a greater variety of tasks than their counterparts allocated ex-post, and high and low skill agents allocated ex-ante have over-lapping jobs.

1.6   Firm Staffing: The Use of Employees and Contractors

The staffing practices of firms in this economy balance the cost of uncertainty with the cost of urgency. Firms can choose to staff for flexibility, allowing them to tailor hires to the task at hand. Alternatively, they can staff for preparedness, having an agent ready, even if that agent may not be best-suited for the task.

In the steady-state equilibrium there are three staffing policies, with resulting job design implications, used in this economy.

Observation. *In the steady-state equilibrium, firms have one of three employment policies in this economy:*

1. *Hire high skill agent ex-ante; he does any task.*

2. *Hire low skill agent ex-ante; he does any low value or urgent task. Hire high skill agent ex-post for non-urgent, high value task.*

3. *Hire no agent ex-ante; if task is non-urgent hire high skill agent for high value task, low skill agent for low value task ex-post.*

The tradeoffs of the firm are clear when one considers the market wages that support this equilibrium. The performance of a firm that hires a high skill agent is always higher than that of a firm hiring a low skill or no agent ex-ante. However, the high skill agent ex-ante will have a higher market wage. A low skill worker is paid less, but must be paid even when he does not work. Hiring no agent reduces the wages that must be paid, but also increases the risk that the firm's task will go undone.

1.6.1 Staffing for Flexibility: Firms Differ in Probability of High Value Task

When firms differ in ex-ante probability of receiving a high value task, the ex-ante assignment of an agent to a particular firm has an effect on surplus. High skill agents' advantage in

productivity is higher for a higher expected value task. Thus, for all firms that are allocated agents ex-ante, firms with the highest p must be allocated high skill agents.

Lemma 1. *Under Condition 2, any ex-ante matches must be assortative, with the highest p firm allocated a high skill agent and the lowest p firm allocated a low skill agent.*

As some agents are only allocated ex-post, some firms are not allocated an agent ex-ante. The types of firms that are not allocated an agent ex-ante depend on where assigned agents can be most productive. High skill agents allocated ex-ante will produce

$$\bar{\alpha}[p\bar{v} + (1-p)\underline{v}]$$

This value is increasing in p. Thus, high skill agents must work at the highest p value firms. Similarly, low skill agents allocated ex-ante to a firm of type p produce

$$\underline{\alpha}[p\hat{\mu}\bar{v} + (1-p)\underline{v}]$$

which is decreasing in p due to Assumption 2. Because a low skill employee is idle when his firm receives a non-urgent high value task, his talent is better utilized at firms with a higher probability of a low value task.

Firms that benefit the most from having flexibility in staffing, those whose task type is least certain, are the ones that will hire agents as contractors (ex-post). Firms that have more certainty in their task type will hire employees, low skill for those with a low probability of a high value task and high skill for those with a high probability of a high value task (see Figure 1.6).
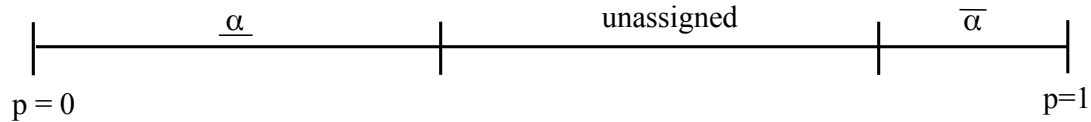
Figure 1.6: Ex-ante Agent Allocation by Firm's Probability of High Value Task

Observation. *Under Condition 2, in the steady state equilibrium, the firm types using each staffing policy are as follows:*

1. *Firms with the highest p hire high skill agents ex-ante, who do any task.*

2. *Firms with mid-range p, hire no agent ex-ante, if task non-urgent hire ex-post: high skill for high value, low skill for low value.*

3. *Firms with low p, hire low skill ex-ante, for low value and urgent tasks. Hire high skill ex-post for non-urgent high value tasks.*

1.7   Comparative Statics: Trends in Staffing

The characteristics of the economy determine the steady state that optimal skill allocation reaches. Factors such as urgency, uncertainty, and high skill labor supply have an impact on the equilibrium size of the labor market for contractors, the division of labor, the firm types that are unassigned ex-ante, and the level and inequality of agent wages.

1.7.1 Size of Contractor Labor Market

The fraction of agents who become contractors is a response to demand for ex-post allocative efficiency. As urgency increases, the quantity of tasks available for reassignment, non-urgent tasks, decreases. Therefore, the ex-post demand for agents decreases and fewer agents become contractors. More workers will be hired as employees (ex-ante), so that fewer tasks are left undone.

Proposition 3. *The quantities of high and low skill agents who become contractors (are hired ex-post) is decreasing in urgency.*

Uncertainty has the opposite effect on the equilibrium quantity of contractors; there are more contractors when uncertainty increases. When uncertainty is high, few firms have a very high probability or a very low probability of receiving a high value task. The firms assigned a high skill agent ex-ante receive fewer non-urgent, high value tasks. Therefore, there are more non-urgent, high value tasks elsewhere for high skill agents to be reassigned to. As the quantity of high skill contractors goes up, the need for low skill contractors increases as well, because there are more firms that are not assigned agents ex-ante with non-urgent, low value tasks that can be done by ex-post assigned low skill agents.

Proposition 4. *The quantities of high and low skill agents who become contractors (are hired ex-post) is increasing in uncertainty.*

Having more contractors in the economy increases the flexibility to efficiently match skill with tasks. When the relationship between assignment to a firm and to a particular type of task decreases, the value of contracting will increase.

Allocative efficiency is particularly difficult when high skill labor is in short supply. In this case, there are fewer agents to work on relatively more high value tasks. As the quantity of high skill agents increases, excess agents will become employees because demand for contractors is already satisfied. Furthermore, when more high skill agents are allocated ex-ante, the demand for high skill contractors goes down. Thus, the fraction of agents who are high skill contractors decreases as the fraction of agents who are high skill increases, which further decreases the fraction of agents who are low skill contractors.

Proposition 5. *The quantity of high skill contractors is decreasing in high skill labor supply. For ex-ante identical firms, the quantity of low skill contractors is decreasing in high skill labor supply.*

Contracting is thus decreasing in urgency, increasing in uncertainty, and decreasing in high skill labor supply.

1.7.2 The Division of Labor

The degree of division of labor in the economy can also be measured. While contractors will always be efficiently matched, uncertainty increases the probability that an employee is mismatched. The degree of job overlap (the probability with which high skill agents do low value tasks and low skill agents do high value tasks) depends on how much division of labor there is for employees and on how many agents become employees.

While contractors are specialists working on only efficiently matched tasks, employees are not. Low skill employees work on urgent, high value tasks, and high skill employees work on urgent and non-urgent, low value tasks. Contractors of both types continue to be

efficiently matched with high skill agents working only on high value tasks and low skill agents working only on low value tasks. Thus, if more agents are employees, the division of labor is lower.

As urgency increases the fraction of agents who become employees is increasing, and therefore, the division of labor is decreasing.

Proposition 6. *As urgency increases, division of labor decreases.*

Similarly, as high skill labor supply is increasing, employment is increasing, causing the division of labor to fall.

Proposition 7. *As high skill labor supply increases, division of labor decreases, when firms are ex-ante identical.*

Finally, uncertainty pushes toward the use of contractors, who are assortatively matched. However, increasing the number of contractors cannot overcome the decrease in division of labor due to employees being mismatched. Therefore, the following result holds.

Proposition 8. *The division of labor is decreasing in uncertainty.*

1.7.3 Wages

In the market equilibrium, each agent and firm will agree to the equilibrium employment policies. Because each type of agent can work as either an employee or a contractor, the wage of each skill level must be constant over allocation timing. If the wage of high skill employees is higher than the wage of high skill contractors, then there will be excess supply of high skill agents in the employment labor market and under supply of high skill agents in

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the contracting labor market. The wage of high skill contractors will then be driven up and the wage of high skill employees driven down until they are equal.

In taking a position at a firm, an agent forgoes the option of working at another firm; thus the agent prepares for the firm and is paid whether he works or not[3]. Therefore, when firms are deciding whether to wait and hire a contractor or to hire a low skill employee, they weigh the cost of not having urgent tasks done with the cost of paying the low skill employee, even if a non-urgent, high value tasks arrives and that agent sits idle.

The market clearing wages in an economy with ex-ante identical firms are such that firms must be indifferent between hiring a high skill agent, low skill agent, and no agent ex-ante. The difference between low skill and high skill wages is due in large part to the differences in skill levels of the agents. However, there is also an allocative effect from work done by contractors. The difference in productivity of contractors exceeds their skill differential because high skill agents are also allocated to high value tasks and low skill agents to low value tasks.

When firms differ in their ex-ante probability of receiving a high value task, the market clearing wages for this economy will sort each ex-ante allocated agent to his highest productivity assignment. Whereas ex-ante identical firms are indifferent between the three staffing policies, in the case of ex-ante heterogeneous firms only the firm type at the boundary between staffing policies will be indifferent.

_____

[3]Contracts are complete, and all agents and firms are risk neutral, so this assumption is equivalent to agents being paid a higher wage only when they work.

Thus, the p value of the firm that determines the wage for high skill agents is larger when firms are ex-ante different than when they are identical. The p value for the firm that determines the wage for low skill agents is smaller when firms are ex-ante different.

1.8   Conclusion

The use of agents in non-employment roles has risen dramatically over the past fifty years. Given that agents in these roles have become a significant fraction of the labor force, understanding the forces driving this change is essential. Contractors provide the flexibility necessary for efficiently allocating talent to tasks in an economy full of uncertainty. The benefit of reserving agents as contractors is limited by urgency, because contractors cannot be in place as tasks arrive.

The model in this paper explores the tradeoff between uncertainty and urgency and produces staffing patterns that closely match the defining characteristics of employees and contractors. Employees are agents who work for a single firm; their jobs are loosely defined and depend on the firm that they work for. Contractors are specialists whose jobs are well defined; whether a contractor will work for a particular firm depends on whether that firm has a task that corresponds with that contractor's job. Furthermore, each contractor is able to sustain this specialization and high level of output relative to employee-agents of the same type because he spreads his skill over many firms.

In many industries, use of outside contractors and temporary workers has increased. Executives explaining these choices cite the importance of workforce flexibility. On the other hand, the industry of corporate law has been moving in-house. To explain this seemingly

opposite trend, legal scholars appeal to urgency and responsiveness. Over the past 40 years, the profession of in-house corporate counsel has risen in prominence. First, the number of lawyers working as in-house counsel has grown (Lynch, 1979; Hackett, 2002; Machlowitz, 1989; Liggio, 2002). Second, while most corporations that employ in-house counsel also use outside counsel, the proportion of a corporation's legal work done in-house has grown (Liggio, 2002; Lynch, 1979)—indeed, for a small number of firms virtually all legal work is performed by in-house counsel (Lynch, 1979; Machlowitz, 1989; Liggio, 2002). Finally, perhaps as a result of these two trends, the reputation of in-house counsel has also risen (Liggio, 2002; Lynch, 1979; Machlowitz, 1989).

The phenomenon of increased reliance on in-house legal counsel is consistent with the results of this model. Corporate law departments range from full service shops to minimally staffed offices that direct work to outside counsel. Firms in between these two poles often retain full responsibility for some areas of legal practice in-house, while delegating all other legal tasks to outside counsel (Liggio, 2002). Liggio's description of the various types of corporate in-house legal practices and his assertion that when outside counsel is hired, it is chosen specifically for a particular task mirror the model's three staffing policy results described in this paper.

The results of the model reflect seemingly opposing hiring trends in the real world. Popular explanations for this dichotomy match the predictions of the model. In many industries, firms have increased their reliance on contractors, consultants, and temporary workers in response to economic uncertainty. On the other hand, the field of corporate in-house counsel has experienced significant growth due to increased urgency and demand for responsiveness.

Previous theories of employment emphasize authority and relationship specific investments. These theories offer important insights into those industries where production is dependent on non-human capital. However, in human capital dominant industries, the model provided here represents the stylized facts in employment and contracting, providing new insight into the value of these roles in the economy.

Chapter 2

Talent Allocation and the Inequality of Contingent Workers

2.1 Introduction

The experiences of agents in their working lives are changing as the nature of employment relationships evolves. Careers are increasingly moving beyond firm boundaries, and as a result, the stability and security of agents' work has become a concern for policy makers and scholars, in addition to the agents themselves. Sociologists, in particular, have noted a relationship between non-standard work relationships and employment instability, especially for low skill agents who are excluded from standard employment. However, economic models of the labor market focus on overall supply side outcomes or demand side concerns rather than the experiences of agents in these changing markets.

This paper presents a theory of the labor market that allows for the examination of agent experience depending on skill level and on type of employment: as an employee or as a contractor. It characterizes the market equilibrium that reflects the instability discussed

by sociologists, and identifies conditions under which such an equilibrium is likely to arise. Specifically, the equilibrium of interest is one in which low skill contractors cycle in and out of unemployment and are unable to move to more stable standard employment positions.

The model is a matching model with no asymmetric information or moral hazard. Agents of different skill levels produce output by working on tasks of different values. Rather than being directly matched with a task, each agent is matched with a firm whose task value is uncertain ex-ante. Each agent can be matched as an employee ex-ante, as a contractor ex-post, or not be matched at all, in which case he is unemployed.

In the leading case, illustrating the evidence from sociology, the optimal match is such that the lowest skill agents in the economy do not become employees because their skill is not competitive for the available positions. Instead, these agents remain unallocated until task types are revealed. In some states they are contractors, while in other states they are unemployed. On the demand side of the labor market, this equilibrium requires that some firms do not hire an employee, so low skill contractors must be more productive than they would be as employees at those firms.

This type of equilibrium is likely to arise when labor supply is relatively limited, so that agents are more productive when spreading their skill across many firms. Similarly, when firms have a small volume of work, contractors can be more productive than employees because they are likely to be matched with a task at one of many firms. Finally, when there are few tasks that can only be done by employees (i.e. in industries where knowledge or skill is general rather than firm specific), the equilibrium is likely to fit this allocation pattern.

Some have noted the role of regulation on the increased use of non-traditional employment relationships (for example, Kalleberg (2000)). Limits to a firm's ability to dismiss

employees (Autor, 2003) and minimum employee benefits requirements (Houseman, 2000) have both been suggested as potentially contributing to the trend. In addition to examining conditions for which the unregulated equilibrium includes low skill contingent workers, the effects of these two types of regulation are explored.

The model reflects several theories regarding firms' choices over staffing arrangements. Sociologists and economists alike have argued that contractors may be used in order to reduce costs (Abraham and Taylor, 1996), to flexibly staff in response to volatility (Abraham and Taylor, 1996; Kalleberg et al., 2003), or to utilize skill not available in-house (Abraham and Taylor, 1996; Matusik and Hill, 1998). All of these forces are at work in the model presented here.

The paper proceeds as follows: The next section gives a very brief overview of the evidence on the contingent workforce from the sociology literature and outlines the facts this theory seeks to explain. Section 2.3 introduces the model, explains how agents are allocated, and gives some general conditions on optimal allocations. In Section 2.4 the equilibrium of interest is characterized, and in Section 2.5 the other two possible equilibrium types are characterized. Section 2.6 gives conditions under which the leading case is likely to arise. Section 2.7 briefly discusses agent allocation over two periods. Section 2.8 considers the affects of employment regulation. Section 2.9 concludes.

2.2 Focus of the Model: The Evidence

This paper is concerned with explaining a particular set of facts established in the sociology literature comparing the experiences of low and high skill workers in non-traditional employ-

ment relationships. The literature concerned with these workers notes that unlike traditional employment relationships, characterized by full-time work over an indefinite period, contingent work lacks security and benefits (Kalleberg, 2000). In addition to these issues, scholars and activists expressed concern that contingent workers were at risk for poverty due to low wages and limited opportunities for training and career advancement (Marler et al., 2002; Vallas, 2012).

Following these arguments, a series of papers examine the experiences of agents working in contingent jobs, and determined that while some workers, particularly those who are low skilled, do encounter this lack of security and of options, others achieve more freedom and power as contingent workers than as traditional employees (Marler et al., 2002; Kunda and Barley, 2006). This line of research used both hard data (i.e. wages, education, etc.), as well as surveys or interviews. For example, Marler et al. (2002) find that, though they have some anxiety regarding the risks, high skill contingent workers choose that arrangement over opportunities for traditional employment and are paid more than their traditional employment counterparts. Low skill contingent workers, on the other hand, hope to find traditional employment positions.

In order to understand and explain the rise of the low skill contingent worker, this paper will focus on finding a labor market equilibrium in which

- Low skill contractors are at risk for unemployment.

- These agents are unable to secure positions as employees.

The papers cited above also make note of a group of high skill agents who choose contracting work over traditional employment. Section 2.5.3 discusses the relationship between this pattern and the equilibrium types characterized.

2.3 A General Model

Suppose the labor market is composed of N agents and M firms. Let a firm be a location where a task may arrive and may be undertaken. Production occurs when an agent completes a task. Agents differ in skill level and tasks differ in value. An agent's skill level, α, is the probability of him successfully completing a task he attempts. Thus, when an agent with skill α works on a task of value, v, the expected output is

$$\alpha v$$

Note than the cross partial of this production function is positive, $\frac{\partial^2}{\partial \alpha \partial v} \alpha v > 0$. Thus, the production maximizing allocation of agents is such that the highest skill agents work on the highest value tasks.

Denote agent i's skill level, α_i. Without loss of generality, let $\alpha_i \geq \alpha_{i'}$ whenever $i > i'$, so that agent 1 has the lowest skill level in the labor market and agent N has the highest. The identity of each agent is public knowledge. Therefore, the productivity of the economy is limited by the information available about the value of the task each firm receives at the time agents are matched with firms. Each firm's task is a random variable that is realized on task arrival. Tasks can take on one of two values, $\bar{v}$ or $\underline{v}$ with $\bar{v} > \underline{v}$.

```
    knowledge                                contractors
    acquired                                  matched

  ├──────────────────────┼──────────────────┼──────────────────┤

  employees                 tasks                               tasks
   matched                  arrive                               done

                            info
                          revealed
```
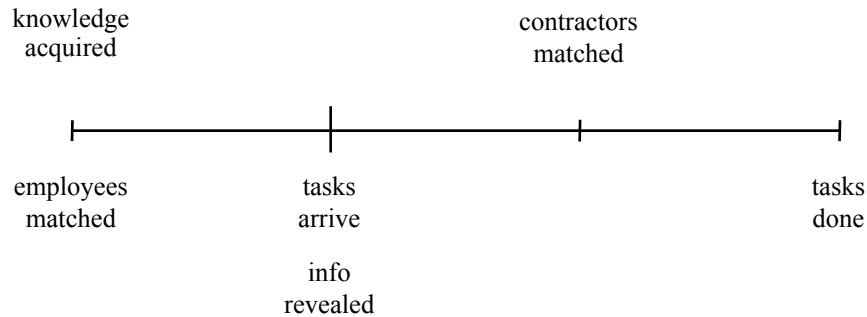
Figure 2.1: Timeline

Each agent must acquire some knowledge before he can work, and this knowledge acquisition must be done before tasks arrive. An agent can either acquire knowledge that is specific to a particular firm or acquire general knowledge. If the agent acquires firm specific knowledge, then he cannot do a task at any other firm. An agent who acquires general knowledge can do a task at any firm; however, some tasks can only be done by an agent with firm specific knowledge. The type of knowledge acquired has no impact on performance other than to determine which tasks an agent can work on.

Call an agent who acquires firm specific knowledge an employee, and the tasks that can only be done by these agents employee specific. All other tasks are referred to as general tasks. Because knowledge must be acquired ex-ante, specific knowledge agents must be matched with firms ex-ante. Call an agent who is matched with a firm ex-post a contractor, and an agent who remains unmatched unemployed. The timeline is shown in Figure 2.1.

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Each firm may receive up to one task. Firm j's probability of receiving a task with value $V \in \{H, L\}$ and specificity[1] $S \in \{E, G\}$ is $p_{j,V,S}$. Then the probability that j does not receive a task is $(1 - \sum_{V,S} p_{j,V,S})$ and the ex-ante expected value of firm j's task is

$$(p_{j,H,E} + p_{j,H,G})\bar{v} + (p_{j,L,E} + p_{j,L,G})\underline{v}$$

Assume $p_{j,V,S} \neq 0$ for all j, V, S, but these may be arbitrarily small.

Assume all $p_{j,V,S}$ and covariances between firms are known so that the economy-wide distribution of the M random variables (tasks) can be calculated. Finally, assume that all agents and firms are risk-neutral. Thus, total surplus is maximized when total production is maximized; the equilibrium is the social planner's allocation of agents to firms maximizing productivity.

2.3.1 Describing an Equilibrium

The equilibrium specifies the allocation agents to firms. Each agent, i, may be matched with a particular firm, j, ex-ante. Agents who are not matched ex-ante may be matched with a firm (including firms already matched ex-ante) ex-post. This ex-post match is a function of the realization of the M random variables in addition to the ex-ante allocation of agents. For example, an agent assigned ex-post may be assigned to firm j in some states and to firm j' in others, depending on the types of tasks received by j and j'.

Given the ex-ante distributions of variables, an agent can be an employee, he can be a contractor in every state (for every possible realization of the random variables), he can

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[1]E denotes employee specific tasks, and G denotes general tasks.

be unemployed in every state, or he can be a contractor in some states and unemployed in other states. For clarity, call an agent who is a contractor in at least one state a contractor and an agent who is unemployed in every state unemployed. Note that it may be impossible for any agent to work as a contractor with certainty, depending on the joint distribution of the random variables.

The task that an agent may work on depends on his firm assignment and the realization of the random variables. Define an agent's job as his firm allocation (specified for each state) and his possible task assignments at the firm(s).

For any allocation of agents to firms, agent i's job consists of him working on high value tasks $q_{i,H}$ of the time and low value tasks $q_{i,L} \leq (1 - q_{i,H})$ of the time. An unemployed agent, for example, will have $q_{i,H} = q_{i,L} = 0$. Thus, the productivity of agent i is under a given allocation can be written

$$\alpha_i [q_{i,H} \bar{v} + q_{i,L} \underline{v}]$$

The total production in the economy, then, is

$$\sum_i \alpha_i [q_{i,H} \bar{v} + q_{i,L} \underline{v}]$$

Condition 1. *A necessary condition for a candidate allocation of agents to be an equilibrium is that for any i and i' with $i' < i$, it must be that $[q_{i,H} \bar{v} + q_{i,L} \underline{v}] \geq [q_{i',H} \bar{v} + q_{i',L} \underline{v}]$. Lower skill agents must have lower productivity jobs.*

If this condition fails, then two agents could switch jobs and total production would be increased; thus, if the condition is violated, the candidate allocation cannot be an equilib-

rium. On the other hand, this condition is not sufficient for an equilibrium, because dividing the set of tasks into different jobs may increase total production, even when the condition is satisfied for a fixed set of jobs.

2.3.2   Ex-Post Agent Allocation

The model as setup so far allows for a few preliminary results regarding ex-post assignment of agents. These results will be useful in the next two sections.

Lemma 1. *For each realization of the random variables, unemployed agents are less skilled than contractors in the following sense:*

If agent i is unemployed, no agent with $i' < i$ can be employed as a contractor.

Proof. See Appendix. $\square$

When agents are allocated to firms ex-post, the highest skilled agents will be allocated first, so any agents left unallocated must be the lowest skill agents of those who have general knowledge.

Lemma 2. *In each state, there are only unemployed agents if each firm with a general task hires an agent.*

Proof. See Appendix. $\square$

An agent can only be unemployed if there are no tasks on which he can work. Unemployed agents are not assigned ex-ante, but they have the potential to be assigned ex-post to a general task. They can only remain unassigned ex-post when there are an insufficient number of tasks for the number of agents available to work on them.

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Lemma 3. *Firm j can only hire agent i as a contractor if the following 2 conditions hold:*

1. *Firm j received a general task.*

2. *No agent with $i' > i$ is an employee of firm j.*

Proof. See Appendix. □

If a firm has not received a general task, then a contractor cannot do any productive work at that firm. Furthermore, if the firm has already hired an employee who has more skill than the potential contractor, hiring the contractor to do the task instead of the employee will decrease the productivity on that task. Because the employee cannot be reassigned, this will decrease overall production.

2.4 Low Skill Contingent Workers: An Equilibrium Characterization

This paper seeks to explain apparent phenomena in the labor market established by sociology scholars. Specifically, the sociology literature discussing contingent workers suggests that there is a class of low skill contractors who cycle in and out of unemployment and are locked out of employee positions, which they would prefer to the unstable positions they hold. This section describes a labor market equilibrium that reflects these assertions. Subsequent sections will identify conditions under which such an equilibrium arises.

First note that the contractors this sociology literature is concerned with must be the lowest skill agents in the economy. This assertion is based on the fact that these agents

would prefer an employee job, but are unable to obtain one. In a market equilibrium, competition drives up the wages of higher skill agents. These agents have more employment options available to them, because many firms want to hire them. Lower skill agents are less desirable to firms, so their employment options are fewer.

If an agent is locked out of a type of job he wants, it must be that the firm offering that job hires a higher skill agent, preventing the lower skill agent from being a competitive candidate for the job. If a contractor was more highly skilled than an agent hired into an employee position, then the firm hiring that agent would be at least as well off hiring that contractor instead of its employee. If the equilibrium places the higher skill agent in a contracting position, this agent must choose that job over the employee position because the compensation is sufficient to attract him[2].

Claim 1. *If i is a contractor who prefers a job as an employee, then all $i' < i$ are contractors.*

Proof. Sketch (full proof in appendix): There cannot be an employee i' who is of lower skill. If there was such an i', i would be competitive for that agent's job and would not be unable to obtain an employee position. If there aren't any lower skill employees, then there must be firms that have not hired an agent as an employee; otherwise, i could not work as a contractor (by Lemma 3). But if there are firms that have not hired an employee, then there cannot be an agent i' who is unemployed (in every state), because surplus would be increased by assigning i' to one of these firms ex-ante. Thus, no lower skill agent can be an employee, and no agent (of higher or lower skill) can be unemployed. All agents $i' < i$ must also be contractors. □

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[2]This argument also explains why high skill and low skill contractors face different levels of stability.

Now suppose that there are n agents who fall into this group of low skill contingent workers. That is all agents $i \in [1, n]$ are contractors and agent $n + 1$ is not a contractor. Following Lemma 1, agent $n+1$ must be an employee; there cannot be an unemployed agent who has higher skill than a contractor. Furthermore, in order for the agents $i \in [1, n]$ to each have a positive probability of being a contractor, each is a contractor in some state, it must be that there is some state in which all of these n agents are contractors.

Claim. *If agents $i \in [1, n]$ are contractors, then at least n firms must not have an employee.*

Proof. If agent 1 is a contractor, there is some state in which he is not unemployed ex-post. By Lemma 1, this implies that all of $i \in [2, n]$ are also contractors in this state; thus there must be n tasks for these agents to work on. Each firm receives no more than 1 task, thus these n tasks correspond to n firms. By Lemma 3, a firm hiring agent i ex-post cannot have an employee with higher skill than i. However, none of the agents with less skill than i are employees. Thus, these n firms must not have employees. ◻

Observation. *If agents $i \in [1, n]$ are contractors, no agent in unemployed.*

Proof. Follows directly from Lemma 1. ◻

The equilibrium of interest has the following characteristics:

1. The n lowest skill agents are contractors.

2. Agent $n + 1$ is an employee.

3. No agents are unemployed.

4. At least n firms do not hire an employee.

An equilibrium with these characteristics can be compared to other possible equilibria. Because each agent is either a contractor, an employee, or unemployed, there are three possible types of equilibria based on the job type of the lowest skill agents.

2.5 Other Equilibrium Types

2.5.1 Lowest Skill Agents are Unemployed

Suppose agents $i \in [1, n]$ are unemployed.

Claim. *If the lowest skill agents are unemployed, indeed if there are any unemployed agents in the economy, every firm must have hired an employee.*

Proof. If a firm, j does not hire an employee, then its employee specific tasks must not be done. If agent i is unemployed with probability 1, then $q_{i,H} = q_{i,L} = 0$. Assigning i to firm j increases surplus by at least $\alpha_i[p_{j,H,E}\bar{v} + p_{j,L,E}\underline{v}]$. □

No agent can be unemployed if there are tasks that the agent could be assigned to work on.

Claim. *There are no other unemployed agents in the labor market.*

Suppose agents $i \in [1, n]$ are unemployed. If agent $\hat{i} > n$ is unemployed, then i' must be unemployed for each $i' \in [n, \hat{i}]$.

Proof. By contradiction: First, note that i' cannot be a contractor, by Lemma 3. Next, suppose there is an i' who is an employee. Then Condition 1 is violated because

$$[q_{i',H}\bar{v} + q_{i',L}\underline{v}] > [q_{\hat{i},H}\bar{v} + q_{\hat{i},L}\underline{v}] = 0$$

$\square$

Claim. *If agents $i \in [1, n]$ are unemployed, agent $n + 1$ must be an employee.*

Proof. By the first claim, every firm must have an employee. All employees must have at least as high skill as $n + 1$, because agents 1 through n are unemployed. Then, by Lemma 3, $n + 1$ cannot be a contractor. Thus, $n + 1$ must be an employee. $\square$

An equilibrium in which the least skilled agents are unemployed has the following characteristics:

1. The n lowest skill agents are unemployed.

2. No additional agents are unemployed.

3. Agent $n + 1$ is an employee.

4. Each firm hires an employee.

2.5.2   Lowest Skill Agents are Employees

Suppose agents $i \in [1, n]$ are employees.

Claim. *In an equilibrium in which the lowest skill agents are employees, it must be the case that no agents are unemployed.*

Proof. If i is an employee and $i' > i$ is unemployed, then $[q_{i,H}\bar{v} + q_{i,L}\underline{v}] > [q_{i',H}\bar{v} + q_{i',L}\underline{v}] = 0$ violating Condition 1. $\square$

If an agent i is unemployed, then his productivity is 0 because $q_{i,H} = q_{i,L} = 0$. However, if the lowest skill agent is an employee, then $q_{1,H} > 0$ and $q_{1,L} > 0$. Therefore, such an allocation of agents violates Condition 1, and cannot be an equilibrium.

Observation. *As each agent must be an employee or a contractor, agent $n + 1$ must be a contractor.*

This type of equilibrium will have the following characteristics:

1. The n lowest skill agents are employees.

2. No agents are unemployed.

3. Agent $n + 1$ is a contractor.

2.5.3 High Skill Contractors

In each type of equilibrium, higher skill agents may be contractors, and in some instances the highest skill agents in the economy may be contractors. As noted in the last section, any contractor i for whom there is a lower skill agent $i' < i$ who is a employee must choose his position over standard employment, because in equilibrium all agents and firms's must have individual rationality constraints satisfied.

Agents with general knowledge have the ability to move around the economy to where their skill can be most productive. Thus, depending on the allocation of other agents and on the possible realizations of the random variables a contractor may be more likely to be matched with a task than an employee, and furthermore, can be more likely to be matched with a high value task.

Consider an example following the model presented in Chapter 1. Suppose the ex-post realization of tasks is deterministic, so there is only uncertainty over which firms will receive which tasks. Further suppose that there number of firms is very large, so the ex-post realization of tasks, as well as any number of agents, can be expressed as fractions of the number of firms. Further, assume firms are ex-ante identical. Finally, let there be only two types of agents, high skill with $\bar{\alpha}$ and low skill with $\underline{\alpha}$.

Because the ex-post realization of the random variables is deterministic, the demand for contractors is a deterministic function of the ex-ante assignment of agents. Then, the task assignment of any agent who acquires general knowledge is known ex-ante (only the firm assignment is unknown ex-ante). For any agent that acquires general knowledge, there will be a high value or low value task for him to work on, or no task at all. Because $p_{V,S} > 0$ for all V, S, if there are any contractors, some contractor works on a high value task with certainty.

Then, if any agent is a contractor, a high skill agent must be a contractor. A firm that hires a high skill agent ex-ante cannot increase performance by hiring any agent ex-post. Therefore, a high skill agent who acquires specific knowledge does any task his firm receives. If only low skill agents are contractors, then there is some agent i with skill $\alpha_i = \underline{\alpha}$ and agent $i' > i$ with skill $\alpha_{i'} = \bar{\alpha}$ such that

$$\bar{v} = q_{i,H}\bar{v} + q_{i,L}\underline{v} > q_{i',H}\bar{v} + q_{i',L}\underline{v} = (p_{H,E} + p_{H,G})\bar{v} + (p_{L,E} + p_{L,G})\underline{v}$$

violating Condition 1. If the distribution over tasks is not deterministic, but has a minimum number of realized tasks, the argument continues to hold. In this case, in an equilibrium

where low skill contractors are at risk for unemployment and are excluded from employee positions, there will also be high skill contractors with less instability and who choose those jobs over employee positions.

2.6 When does the Equilibrium Reflect the Evidence?

Now that the three equilibria types have been characterized, it is possible to determine when the equilibrium is likely to be a low skill contractors equilibrium. The information about unemployment in each equilibrium type indicates that the relative number of agents, compared to the number of firms, or task random variables, has an impact on which types of equilibria are possible.

2.6.1 Relative Thickness of Supply and Demand

In any equilibrium, each firm will only hire one agent as an employee. If more than one agent is hired, the higher skill agent will always do the firm's task. Therefore, the lower skill employee will have zero productivity, and surplus can be weakly increased by leaving that agent unallocated ex-ante. Furthermore, the number of contractors is bounded by the number of firms because each firm receives at most one task. Any ex-ante unallocated agents in excess of the maximum number of tasks will be unemployed with probability 1.

Therefore, in an equilibrium in which there are no unemployed agents, the maximum number of agents is $2M$. This is the case in both low skill contractor type of equilibrium and in the low skill employee type of equilibrium. Furthermore, in the low skill contractor
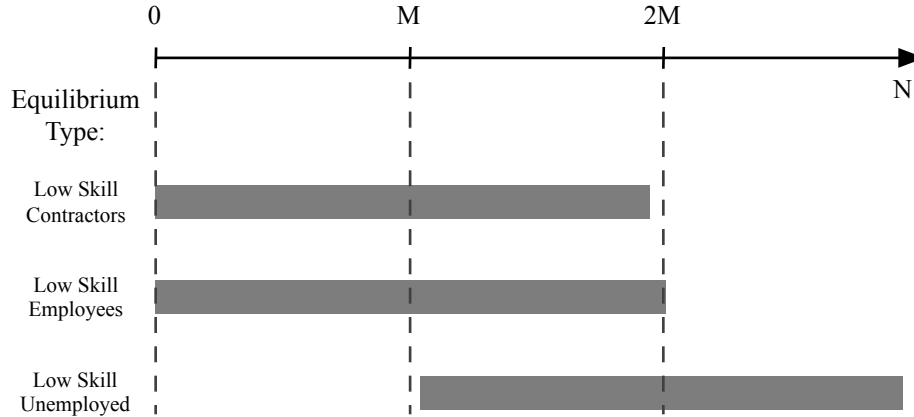
Figure 2.2: Possible Equilibria Types By Relationship Between N and M

type, there must be at least 1 firm without an employee, so the number of agents is bounded above at $2M - 1$.

Finally, in the low skill unemployed type of equilibrium, all firms must hire an employee. Therefore, the number of agents must be at least the number of firms plus one unemployed agent, $M + 1$. These relationships are indicated in Figure 2.2.

As the thickness of the supply side of the market increases in relation to the demand side, the equilibrium is more likely to be a low skill unemployed equilibrium. When the relative supply of labor is small, low skill agents must be employees or contractors. The next subsection will show that when labor supply is very low, low skill agents are likely to be contractors because under these conditions agent mobility becomes important.

2.6.2 A Necessary Condition for Low Skill Contingent Workers

Beyond this basic comparison of all equilibria types, the characteristics of the equilibrium type of interest indicates a condition for its own existence. Recall that in an equilibrium in

which the lowest skill agents are contractors, there are some firms that do not hire employees. This fact can be used to further assess whether an allocation satisfying the characteristics of such an equilibrium is in fact an optimal allocation.

Condition 1 states that for a given set of jobs, agents must be assortatively matched with jobs. A contingent worker allocation calls for an additional necessary condition due to those ex-ante unmatched firms. For such an allocation to be an equilibrium, moving an ex-post assigned agent to one of these firms cannot increase surplus.

Recalling that the n least skilled agent are contractors, call the $m \geq n$ firms without an employee $j \in [1, m]$. The increase from assigning agent i to firm j as an employee is at least:

$$\alpha_i \left[p_{j,H,E} \bar{v} + p_{j,L,E} \underline{v} \right]$$

because employee specific tasks are not otherwise done. Any general tasks previously done by a higher skill agent can still be done by that agent; if a general task had been done by a lower skill agent, it will be done by i after this reassignment, so the increase in surplus may be strictly greater than the expression above.

Agent i's productivity as a contractor is given by

$$q_{i,H} \bar{v} + q_{i,L} \underline{v}$$

Making i an employee would mean that he could not work on the tasks he would work on as a contractor. However, if i is no longer a contractor, some of his tasks may be reassigned to other contractors. Therefore, this is the upper bound of the cost of moving agent i. The

movement of an agent will cause an increase in production when the surplus gain is greater than the surplus loss. Then if

$$\text{surplus loss} \leq \alpha_i \left[q_{i,H} \bar{v} + q_{i,L} \underline{v} \right] < \alpha_i \left[p_{j,H,E} \bar{v} + p_{j,L,E} \underline{v} \right] \leq \text{surplus gain}$$

for some i, j, the allocation cannot be an equilibrium. This inequality is most likely to hold for agent 1 and the firm with $\max_{j}[p_{j,H,E}\bar{v} + p_{j,L,E}\underline{v}]$.

Thus the following condition, then, is necessary for the equilibrium to be a low skill contractor equilibrium.

Condition 2. *A necessary condition for an employment pattern in which firms $j \in [1, m]$ do not hire an employee to be an equilibrium is:*

$$q_{1,H} \bar{v} + q_{1,L} \underline{v} > \max_{j \in [1,m]} p_{j,H,E} \, \bar{v} + p_{j,L,E} \, \underline{v}$$

This condition is likely to hold when $p_{j,H,E}$ and $p_{j,L,E}$ are small relative to $q_{i,H}$ and $q_{i,L}$. Clearly, that is the case when there is little firm specificity. The condition is also likely to hold when each individual firm has a low volume of all tasks, but contractors have a high volume of work because they spread their time among many firms. Thus, as stated in the last subsection, the equilibrium is likely to be characterized by low skill contractors when the number of firms is large relative to the supply of labor.
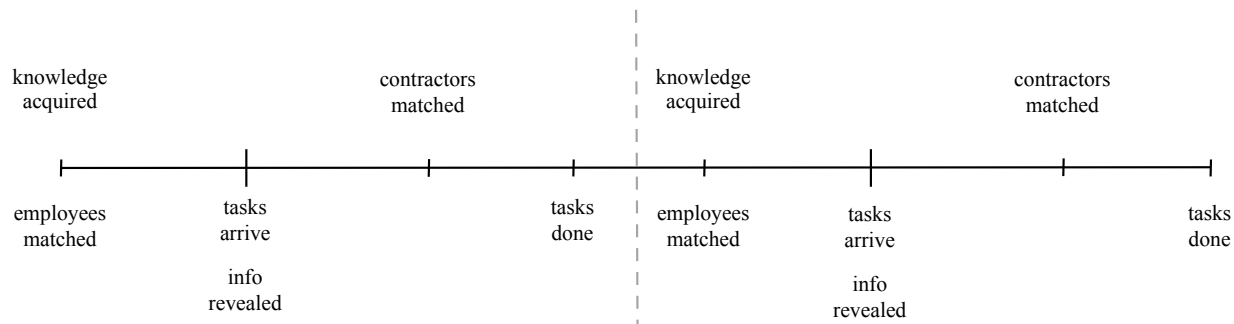
Figure 2.3: Two Period Timeline

2.7 Two Period Model

The literature on contingent workers emphasizes the instability of the jobs of these workers. The model as presented in Section 2.3 captures this instability through a probability of unemployment. Instability could also be an affect of change over time. In order to explore that possibility, this section discusses an extension of the model to two periods. Assuming that agent assignment in the first period puts no restrictions on assignment in the second period, this model is essentially the model presented above repeated twice. The timeline in shown in Figure 2.3.

2.7.1 Stationary Ex-ante Distributions

If in each period the ex-ante distribution of tasks is the same, the allocation strategy will be the same for both periods. That is, if i is assigned to j ex-ante in Period 1, i and j will also be matched ex-ante in Period 2. All agents who are not assigned ex-ante in Period 1 are also not assigned ex-ante in Period 2. Any difference in assignment between Periods 1 and 2 must be a result of a difference in the ex-post realization of tasks. In this way contractors

can cycle in and out of unemployment. However, if the distribution of tasks is not changing, then employed agents will remain employed, and unemployed agents will remain unemployed in both periods.

2.7.2 Changing Distributions

The arguments made by sociologists regarding contingent workers raise an interesting question about low skill workers. When could we expect low skill workers to cycle in and out of unemployment, but into employee positions rather than contractor positions?

Consider an equilibrium to the single period model in which the lowest skill agents are unemployed. Recall that in such an equilibrium, all firms must hire an employee. This result implies that in order for agents to move between unemployment and being employees, there must be a change in the number of firms, or possible tasks. In other words, that pattern can only arise through a pattern of expansion and contraction of the economy.

There may be additional cases where an agent is unemployed in Period 1 but an employee in Period 2. However, in such a circumstance it would have to be the case that the agent had a positive probability of being a contractor in Period 1, but wasn't in the realized state of the world. This may happen when the volume of tasks at some firms is fluctuating.

Agents move between unemployment and contracting because there is uncertainty over the overall demand for labor in the market. Very specific conditions are required for an equilibrium in which some agents move between unemployment and being and employee or being an employee and being a contractor, but no agents move between unemployment and begin a contractor. Namely, the ex-post realization of the variables must be deterministic

in each period, but the size of the market must be fluctuating—total demand for labor is changing but is known ex-ante each period. As this scenario is unlikely, the pattern of instability observed in contingent work may be a feature of most labor market equilibira.

2.8 Employment Policy Considerations

The model presented in this paper captures the forces pushing employers to use contractors rather than employees. Uncertainty and quantity of labor supply are important factors in determining the size of the contracting labor market. In addition, labor regulation may have an impact on the use of contracting.

These policies tend to only apply to employees in an effort to protect these workers. However, they make hiring employees unattractive. Two types of such policies are those affecting minimum compensation (i.e. requiring employers to provide employment insurance, pensions, or health insurance) and limits on dismissals (i.e. an employee cannot be fired before a certain amount of time or under various conditions). This section discusses the impact each of these two types of regulation may have on the equilibrium allocation of agents.

2.8.1 Minimum Compensation Requirements

A policy setting a minimum wage or compensation package (such as health benefits), w^{MC}, may have an impact on the achievable allocation of agents. Consider such a policy that applies only to employees.

Whether and what type of impact a minimum compensation policy will have depends on the level of the policy relative to the unregulated wage of the lowest skill employee, agent i. In an unregulated competitive labor market equilibrium, the wage of agent i must satisfy participation and rationality constraints of both the agent and the firm he works for. Therefore, it must be the case that $w_i \in [0, \alpha_i E[v_i]]$, where $E[v_i]$ is the expected value of the firm's tasks worked on by i. The level of the wage depends on the thickness of each side of the labor market. For example, if the supply side of the market is very thick compared to the demand side (there are many more agents than firms), the wage will be close to the agents' outside option, or 0. On the other hand, if the demand side is thick relative to the supply side (there are few agents compared to firms), the wage will be close to the firm's outside option, $\alpha_i E[v_i]$. Thus, there are three possible cases for the relationship between w^{MC} and w_i.

Case 1: Minimum Compensation is Not Binding

If

$$\alpha_i E_i[v] > w_i > w^{MC}$$

then the lowest skill employee is paid more than the minimum compensation requires. Thus, the policy is not binding and doesn't have an effect on agent allocation or on the compensation of any agents.

Case 2: Minimum Compensation Binding and Raises Wages

If

$$\alpha_i E_i[v] > w^{MC} > w_i$$

then the policy is binding, but the productivity of the lowest skill employee is larger than the minimum compensation. Thus, the firm's participation constraint will continue to be satisfied at the higher wage. The allocation of agents, in this case, will not be impacted by the policy, but wages will be.

It is clear that the wage of agent i will increase, as will any other agents whose wage falls below w^{MC}, because $\alpha_{i'} E_{i'}[v] > \alpha_i E_i[v]$ for all $i' > i$. The wages of all other agents will also increase. The increased wage of the lowest skill worker makes hiring higher skill agents at current wages relatively more attractive, increasing competition for these agents, and thus driving up the wages of those agents.

Because the allocation of agents is not impacted by a policy of this level, this policy does not have an effect on the efficiency of the market. Total production remains the same; however, the policy results in a surplus transfer from firms to agents.

Case 3: Minimum Compensation Binding and Efficiency Decreased

    If

$$w^{MC} > \alpha_i E_i[v] > w_i$$

then the policy is binding, and the higher wage violates the participation constraint of the firm hiring agent i. Thus, this firm will no longer be willing to hire agent i at this level of compensation. This agent will either become a contractor or unemployed.

Beginning with the lowest skill employee, the minimum compensation can be compared with the wage of employees of increasing skill to determine the overall impact on the labor market. Note, however, that if some agents' productivity and wages fall into Case 3, then the outside option of more highly skilled agents may weaken because the pool of

unemployed agents creates competition for their jobs. Thus, the wages used to determine the impact on those agents' jobs must be adjusted[3].

The main ways in which minimum compensation will impact the labor market equilibrium with respect to low skill contractors is in increasing the incidence of such equilibria as well as weakly increasing the numbers of low skill contractors within such equilibria. The regulation only impacts the allocation of agents in Case 3, where some firms no longer hire employees.

These employees may become contractors or they may become unemployed. The displacement of these agents weakly increases the demand for contractors because firms without employees will want to hire contractors, but these firms may have already used contractors for general tasks. The displacement, then, may increase the number of contractors if the there are states in which demand for contractors exceeds supply. Thus, the number of contractors is weakly increasing in with regulation.

Displaced agents are always the lowest skill employees. When these agents become contractors, they are like the low skill contractors in the equilibrium of interest in that there are no employees with lower skill than them. Thus, these regulated equilibria can also reflect sociologists' characterization of the labor market: These agents are at risk of unemployment and they prefer employment jobs that are unavailable to them.

Any labor market with an unregulated equilibrium in which the lowest skill agents are employees or unemployed can, thus, resemble a low skill contractors equilibrium under minimum compensation regulation. The difference between these equilibria under regulation

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[3]An interesting result here is that the minimum wage could potentially reduce the wages of more productive agents. A discussion of this result is outside the scope of this paper.

and the unregulated equilibrium type with lowest skill agents as contractors is that in these equilibria some agents may also be unemployed with probability 1.

2.8.2   Limits on Dismissals

When employees jobs are protected by limits to dismissals, contractors may be used by a firm so as to not commit to employing an agent in the next period. Avoiding commitment is only valuable when a firm's task volume is expected to decrease in the future. However, in an unregulated competitive equilibrium, an agent will move between firms only when the most productive use for his skill changes. If firm j is no longer the best use for agent i's skill, then the wage offered by firm j will be lower than i's outside option, in which case the agent would choose his outside option. So limits on dismissals would not have an affect on allocation.

This argument demonstrates that a policy limiting dismissals can only have an effect on allocation when wages are not allowed to adjust to changes in the market. In this way, limits on dismissals actually function in a similar way to minimum compensation regulation. However, the minimum compensation in this case is agent specific rather than market-wide.

Limits on employee dismissals are argued to have an additional implication for contingent workers. Agents are often hired as contractors before being hired by the same firm as a full time employee. Some suggest that this pattern is a result of limits on employee dismissals combined with asymmetric information about agent skill. A firm is able to learn about an agent's skill only after it has hired the agent. Thus, hiring the agent as a contractor allows the firm to learn about the agent before committing to employing him.

This pattern, however, disproportionately affects low skill contingent workers. The model suggest that because dismissal limiting regulations are equally likely to bind on all skill levels (as the cost is dependent on the employment wage), the value from learning of contracting should apply to any skill level agent. In order for the asymmetric information argument to hold, it must be that firms have better information about the skills of high skill agents. Thus, differences in skill signaling for high and low skill agents may be a promising area of future research.

2.9 Conclusion

As the use of non-standard employment relationships has increased, the reported growth of an underclass of contingent workers has caused concern among sociologists and policy makers. Understanding the conditions that lead to these labor market patterns is essential both from a policy perspective and for agents who are experiencing instability.

While other papers focus on trends in over the entire labor market or on the firm's decision of which staffing practices to use, this paper specifically explores the experiences of individual agents. Using a basic model with no contracting problems (i.e. moral hazard or asymmetric information), it demonstrates that particular patterns identified by sociologists can be part of the equilibrium in a competitive, unregulated labor market.

The conditions under which this type of equilibrium is likely to arise reflect the uncertainty which firms have been increasingly confronting in the modern economy. In particular, limited and uncertain staffing needs can both contribute to increased contracting. Further-

more, regulations that have been implemented to protect employees impact the equilibrium allocation of agents in the labor market, and make this type of equilibrium more likely.

The general model outlined here can be used as a basis for understanding how agents move through the labor market as conditions change. An important simplifying assumption made here is that agents skill levels are observable, which allows for an efficient match. Future work can examine employment patterns when there is asymmetric information about skill. This type of extension would allow for the identification of the informational value of different employment practices.

Chapter 3

Division of Labor, Specialization, and Uncertainty

3.1   Introduction

The division of labor is a fundamental characteristic of society; production is shared among groups, and no member of a group would be able to produce his share of consumed output individually. The nature of humans as social animals implies production will be shared as no individual can survive alone, but the particular way in which work is divided has an impact on output. The division of labor facilitates specialization and, thus, increases productivity. Yet in many instances, labor is not completely divided; some types of tasks are shared among multiple agents rather being divided to take full advantage of gains from specialization.

For example, in an academic department, research (and to a lesser extent teaching) is often very specialized with relatively little overlap between two member's areas of expertise. Dividing labor allows each individual to develop a deeper expertise on a narrower field. However, the members of a department will share other duties, taking turns on recruitment

committees, for example. Within a household, some tasks may be specialized to one spouse or the other, but certain tasks are done by both spouses, and different households differ in the amount of overlap.

The term division of labor has been used to describe the number of agents working together to produce a particular good (Becker and Murphy, 1992; Wadeson, 2013). An assumption implicit in this definition is that agents working together each work on different tasks[1]. However, the above examples illustrate that division of labor can vary even with fixed team size. This paper presents a model of team production that allows the division of labor and team size to vary independently in order to identify conditions under which job overlap outperforms division of labor. Output is produced by agents working on projects, which consist of a variety of tasks. Team size and job design (the assignment of tasks to the agents in the team) are optimized given the distribution of over the types of tasks in projects, as well as the quantity of projects.

The results of the model encompass established results in the literature, where increasing returns to specialization drive the division of labor (Becker and Murphy, 1992; Stigler, 1951). The model demonstrates that job overlap is at a minimum when the tasks required for production are predictable and unlimited. The literature discussing division of labor and specialization has assumed that the distribution of task types required for production is constant and that the quantity produced is limited only by the productivity of the team (Becker and Murphy, 1992; Bolton and Dewatripont, 1994; Rosen, 1983; Wadeson,

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[1]Indeed the results of these papers, which specifically discuss division of labor, and others that focus on specialization (i.e. Bolton and Dewatripont (1994) and Rosen (1983)) result in non-overlapping specializaitons

2013)[2]. Both of these assumptions reflect a focus on manufacturing industries embedded in the discussion of division of labor since Adam Smith.

While the manufacture of a good requires a set of tasks that is identical for each good produced, production in other industries is not so predictable. In agriculture it may be unclear when the harvest will take place and which crop will be ripe for harvesting first; a customer service department may deal with more sales one day and more technical support a different day; a salon may not know if there will be more demand for haircuts or for manicures. These examples illustrate that job overlap, as opposed to division of labor, provides value under uncertainty by allowing agents to move to the tasks where their labor most needed at any given time.

These non-manufacturing examples share another feature: the quantity of work available at a particular time is typically limited. Whereas an academic can always write another paper and a pin factory can alway produce another pin, the customer service agent can only increase the practice of his expertise when an additional client requests his services. Thus, for some types of tasks, unlike manufacturing, decreasing the breadth of specialization does not imply an increase in the depth of specialization. Therefore, it is not always the case that division of labor leads to gains from specialization.

The model shows that when the distribution of task types is uncertain but the total amount of work is fixed, the optimal job design will assign the same set of tasks to multiple agents, who can then field a fluctuating volume of tasks in their area of expertise. If the distributions of task types and quantify of work both vary, then there is a tradeoff between

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[2]Note that Wadeson (2013) only assumes unlimited production potential, not uncertain workflow. However, the current paper shows that both assumptions must be relaxed for overlapping jobs

specialization, requiring increased team size, and agent utilization. Employing more agents allows labor to be divided more, but the potential gains from specialization are offset by the cost of reducing utilization of agent time.

Limits on productive tasks are descriptive of service industries, but also reflect an inability to smooth production over time. While, the total amount of work on a particular task type may be enough to support an agent specializing, when tasks cannot be moved between periods, constraints on the agent's time can prevent him from gaining expertise. Thus, the model can be used to analyze an economy that is increasingly service based as well as increasingly volatile.

The paper proceeds as follows: The next section sets up the model. Section 3.3 replicates previous results, then examines the effects of deterministic and stochastic limits to project quantity on specialization. Section 3.4 discusses the effect of task distribution on job overlap for unlimited projects and deterministic project limits. The tradeoff between specialization and agent utilization is discussed in Section 3.5. Section 3.6 concludes.

3.2   Model Setup

An agent is able to specialize, increasing his productivity on a particular type of task by spending more time working on that type of task. Examples of this phenomenon abound, but this fact is particularly salient to academics who specialize in narrow subfields. Specialization is facilitated by the division of labor. When an agent does not need to produce each good and service himself, he can use time that would otherwise be devoted to producing a wider variety of goods to specialize.

Suppose that tasks have equal productive potential, but are horizontally differentiated. Let Ω be the set of task types, s. A project P is a set of tasks, with distribution over Ω, $F_P(s)$. Suppose that all projects are the same size, and normalize $|P| = 1$. Assume that tasks require a fixed amount of time to complete, which is identical for all tasks; then in some sense the size of P measures both the number of tasks in it and the number of man-hours required to complete the project. Therefore, say a project takes one unit of agent-time to complete. For simplicity, assume each project is continuously divisible so a project can be divided in any way between any number of agents.

Let $\tau_{i,s}$ be the frequency with which an agent, i, works on task type $s \in \Omega$. His productivity from working on any task of type s is $E(\tau_{i,s})$. There are decreasing returns to practice, so $E' > 0$ and $E'' < 0$. Let $A_{P,s}$ be the set of agents who work on type s tasks in project P. Project production is a function of the productivity on each task in the project. Following Becker and Murphy (1992), assume production is determined by the minimum performance on a task in the project. Production on project P is given by

$$Y(P) = \min_{s \in P} \left[\min_{i \in A_{P,s}} E(\tau_{i,s}) \right]$$

Becker and Murphy (1992) argue that for a productive activity that *requires* all tasks to complete, the production function will be the minimum function. Their argument focuses on on goods and interprets Y as quantity produced; however, this production function also applies to quality, which is a more natural interpretation when considering productive value on a project of fixed size, as modeled here.

The output quality interpretation can be applied to goods; a product is sometimes only as good as its lowest quality part. Moreover, quality may be a better measure of productive value than quantity for the provision of services. The value of customer service is in customer retention which depends on quality. One bad experience can cause a customer to switch brands, so the minimum quality of service is essential in determining productive value.

3.3 Limits to Specialization

3.3.1 Specialization Limited by the Extent of the Market

This model can be used to replicate the result from Becker and Murphy (1992), based on Smith's (1776) argument, that the division of labor is limited by the extent of the market. Let $\Omega = [0, 1]$ so there is no limit to how narrow an agent's area of expertise can be. Assume all projects are identical. Let F_P be uniform on Ω. Further, suppose that the availability of projects is unlimited, so that as soon as one project is completed another can begin. This assumption applies well to industries like manufacturing of consumer goods or research production; as noted in the introduction, it is always possible to produce an additional pin or an additional paper.

An agent who works on a project alone divides his time evenly among all tasks, which in this case is the same as dividing time evenly among task types due to the uniform distribution. Thus, the agent will do one task of each type ($f_P(s) = 1 \ \forall s$) during each

period, $\tau_s = 1$ for all s. The production of this agent will be

$$E(1)$$

Suppose an agent works on m projects in a unit of time. Without loss of generality[3] denote the set of tasks he completes for project j, $[\underline{s}_j, \bar{s}_j]$. The time constraint for this agent is

$$\sum_{j=1}^{m}(\bar{s}_j - \underline{s}_j) = 1$$

An agent's proficiency on a type of task, s, is a function of the number of these m projects for which this type of task is in the agent's task set. Define a binary variable $\delta_{j,s} = 1$ when $s \in [\underline{s}_j, \bar{s}_j]$ and 0 otherwise. Then

$$\tau_s = \sum_{j=1}^{m}\delta_{j,s}$$

A team of n agents can complete up to n projects each period. The maximum possible output on a singe project is

$$E(n)$$

because if $\tau_{i,s} > n$ for some agent i and task type s, then $\tau_{i,s'} < n$ for some $s' \neq s$, otherwise i's time constraint is violated. This maximum is achievable on all n projects by setting $[\underline{s}_{i,j}, \bar{s}_{i,j}] = [\frac{i-1}{n}, \frac{i}{n}]$ for agents $i \in [1, n]$. The optimal task assignment for a team of n agents divides the set of task types into n non-overlapping jobs of equal size. Then the per unit

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[3]All tasks have equal value and are equally likely; therefore tasks can be relabeled so that each agent's job is a continuous set.

time output of n agents is

$$nE(n)$$

The productivity of each agent in a team increases with the team size. Therefore, the optimal team includes all agents in the market, denoted N.

Becker and Murphy (1992) argue that this result shows that the division of labor is limited by the extent of the market. While it is true that specialization here is increasing in the number of agents and is only limited by the total number of agents in the market, in a sense labor is equally divided for all n because no two agents work on the same task type for any size team.

3.3.2 Specialization Limited by the Availability of Projects

The assumption that the number of projects is unlimited allows any size specialization to fill an agent's time, which gives the agent the practice required to improve proficiency. Assume instead that the number of tasks available in a given period is limited. Limits to the amount of work available are typical in service industries. Customer service representatives, hair stylists, and dry cleaners (to name a few) can only provide their services when a customer requests them.

Suppose that the number of projects available in a unit of time is $M < N$. Then M agents can complete the available work during that time. Maximum production is achieved by partitioning Ω into M subsets, $[\underline{s}_i, \bar{s}_i]$, with $M|[\underline{s}_i, \bar{s}_i]| \leq 1$. The depth of specialization is clearly limited by the number of tasks available. Increasing the size of the team beyond M agents, while supported by market size, cannot increase production. Further, if there is

any other activity with value $\epsilon > 0$ that agents can participate in, including leisure, the size of the team, as well as the narrowness of specialization is strictly limited by the volume of projects available.

3.3.3 Uncertainty over Project Limits

When project availability is limited, it is usually also uncertain. Demand for services may be cyclical or fluctuate unpredictably. Rather than assisting twenty customers each day, a customer service department may help five one day and thirty the next.

Note that under uncertainty an agent's proficiency will depend on the realization of the random variable over time. The following analysis uses average frequency to determine an agent's proficiency. This notion of proficiency is advantageous for two reasons. First, for a long horizon, any realization of the random variables will have the same average. Furthermore, as opposed to a discounting function, with a long horizon, the average frequency does not fluctuate with fluctuations in work volume[4].

Assume that in a unit of time, there are $m \in \mathbb{R}$ projects available. (A continuous approximation will simplify the notation and calculations.) Let $G(m)$ be the distribution over this number of projects. Suppose a team consists of n agents, then for all $m \geq n$, the job design problem is like that when the availability of projects is unlimited. In these cases, the optimal division would be n jobs of equal size. For $m < n$, the limit of task availability

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[4]This analysis, then, focuses on long-term job and team design. The results also hold for a two period model in which proficiency is gained in the first period and used for production in the second. A two period example is given in the Appendix to illustrate that the same forces drive optimal job design.

is binding; however, in these cases, dividing projects into n equal size jobs for $n \geq m$ does not decrease productivity.

Therefore, the optimal division will be to assign each of the n agents an equal size subset of task types. For example

$$[\underline{s}_i, \bar{s}_i] = \left[\frac{i-1}{n}, \frac{i}{n} \right]$$

for $1 \leq i \leq n$. For all m, each agent does his subset of each of the m projects available. For each agent i in the team, the team works on an ith task whenever $m \geq i$. Therefore,

$$\tau_s = \int_0^n 1 - G(x)dx$$

and total output is given by

$$\left(\int_0^n 1 - G(x)dx \right) E \left(\int_0^n 1 - G(x)dx \right)$$

Team Size and Uncertainty

The previous two subsections have identified the optimal team size in two extreme cases: When there is certainty over the number of projects and this number is limited below the total number of agents, the optimal team size is equal to the number of projects available. On the other hand, when the probability that the number of projects available exceeds the number of agents goes to 1, the optimal team size is the total number of agents.

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These two cases suggest that optimal team size is a function of the distribution of task availability, $G(m)$. Indeed, the optimal team size is increasing in mean of this distribution. Some distributions can be ranked in terms of uncertainty, for example, when one is a mean preserving spread of another. In addition to increasing in the mean of $G(m)$, team size is increasing in uncertainty for a refinement of mean preserving spreads.

The output function in terms of n derived in early sections is weakly increasing in n. Suppose now that there is a cost to adding agents to a team $C(n)$, with $C' > 0$ and $C'' \geq 0$. A brief discussion of possible cost functions and interpretations is given below, but for now assume that this function is such that there is an internal solution to the optimal team size problem.

$$\max_n \left(\int_0^n 1 - G(x)dx \right) E \left(\int_0^n 1 - G(x)dx \right) - C(n)$$

It will help to pick a specific proficiency function E; following Becker and Murphy (1992) again, let $E(x) = x^\theta$ with $\theta \in (0, 1)$. Then the first order condition of the optimization problem is

$$(1 + \theta) \left(\int_0^n 1 - G(x)dx \right)^\theta (1 - G(n)) - C'(n) = 0$$

For an internal solution to exist, the first term must be decreasing in n. This fact combined with the fact that $C'' \geq 0$ indicates a condition for the optimal team size to be increasing. Denoting the optimal team size n_k when the distribution is G_k, it must be that $n_2 > n_1$ when

$$(1 + \theta) \left(\int_0^n 1 - G_2(x)dx \right)^\theta (1 - G_2(n)) > (1 + \theta) \left(\int_0^n 1 - G_1(x)dx \right)^\theta (1 - G_1(n)) \quad (1)$$

at $n = n_1$.

It is plain, then, that $n_2 > n_1$ when G_2 first order stochastic dominates G_1, because inequality (1) holds for all n. When there is a higher probability of more projects being available, the optimal team has more members. More interestingly, a mean preserving spread can also lead to an increase in the optimal team size. In this way, optimal team size is increasing in uncertainty, for a given expected number of projects.

Let $G_r(m)$ be a distribution on the number of projects available, and let G_{r+1} be a mean-preserving spread of G_r.

Claim 1. *There is a probability* $(1 - G_r^*) \in \left((1 - G_r(n_r)), (1 - G_r(n_r))^{\frac{1}{1+\theta}} \right)$ *such that* $n_{r+1} \geq n_r$ *if and only if*

$$(1 - G_{r+1}(n_r)) \geq (1 - G_r^*) \tag{2}$$

Proof. See Appendix.

A mean preserving spread satisfying inequality (1) places more weight on higher project limit values. Then a mean preserving spread of G_r with a sufficiently thicker right tail above the optimal team size for G_r has a larger optimal team size than G_r. When uncertainty over the number of projects available increases in a way that puts sufficiently more weight on the number of projects exceeding the optimal team size, the team size increases. even when the mean number of projects remains the same.

Observation. Any *mean preserving spread of a distribution with no uncertainty increases optimal team size.*

This is because $(1 - G_r(n_r^*)) = 0$ for a distribution in which all weight is on n_r^*, and $(1 - G_{r+1}(n_r^*)) > 0$ for any mean preserving spread of G_r.

Elsewhere in the literature regarding division of labor, specialization, and team size, coordination and communication costs are suggested as limits to team size (Becker and Murphy, 1992; Bolton and Dewatripont, 1994). The functional form of these costs is considered to be convex. Another possibility is that there are outside activities, like leisure, that provide a private value but cannot be traded. In this case, the cost function would be linear in team size.

This type of cost function has not been a focus in the past because an internal solution to the team size problem with linear cost is only possible if the production function is concave in team size, which is at odds with increasing returns to scale. When uncertainty over the number of available projects is introduced to the model of specialization, the production function may be concave in team size, depending on the shape of the distribution. Thus, it is possible for team size to be limited by uncertainty, in a setting where communication or coordination costs are minimal.

3.4   Limits to the Division of Labor

3.4.1   Task Sharing under Predictable Production

The result that the set of task types is divided into non-overlapping jobs rests on the assumption that all projects are uniformly distributed over all possible task types, Ω. It may not be the case that the optimal specialization requires a partition of the set of task types into n subsets for n agents. Joint production is not always maximized with a division of

labor and trade. Even with unlimited divisibility of task types, some agents may share a specialization because proportionally more of some tasks are required for project completion.

Assume again that all projects are identical and that there is unlimited availability of projects. Let the density of task types in a project be the piecewise function:

$$f(s) = \begin{cases} \frac{2}{3} & \text{if } s \in [0, \frac{1}{2}] \\ \\ \frac{4}{3} & \text{if } s \in (\frac{1}{2}, 1] \end{cases}$$

Then a team of n agents can complete n projects in one period. The maximum proficiency that can decide project output is $E(\tau_s) = (E\frac{2}{3}n)$; that is the maximum output on a project is the maximum proficiency with which an agent can do the least frequent type of task. For every one task of type $s \in [0, \frac{1}{2}]$, there are two of type $s' = s + \frac{1}{2}$. Thus, if an agent completes all tasks of type $s' \in (\frac{1}{2}, 1]$, the proficiency on this task type is $E(\frac{4}{3}n)$, while total production will remain

$$nE\left(\frac{2}{3}n\right)$$

because performance on less frequent tasks cannot be increased. Therefore, the following two task allocations, both of which divide the project into n equally sized jobs, achieve maximum output[5]. Task types $s \in [0, \frac{1}{2}]$ are divided into equal segments, with no two agents working on the same type in this region. One allocation would divide the task types $s \in (\frac{1}{2}, 1]$ in a similar way, with no over overlapping jobs, but with smaller segments for each agent than

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[5]In this example n may be the maximum number of agents or the maximum number to tasks, depending which is the limiting factor.

in the range $[0, \frac{1}{2})$. That is, for each agent, $i \in [1, n]$

$$[\underline{s}_i, \bar{s}_i] = \left[\frac{1}{2}\frac{i-1}{n}, \frac{1}{2}\frac{i}{n}\right] + \left[\frac{1}{2} + \frac{1}{2}\frac{i-1}{n}, \frac{1}{2} + \frac{1}{2}\frac{i}{n}\right]$$

The other optimal allocation divides task types in $s \in (\frac{1}{2}, 1]$ into larger jobs that are then shared by two agents:

$$[\underline{s}_i, \bar{s}_i] = \left[\frac{1}{2}\frac{i-1}{n}, \frac{1}{2}\frac{i}{n}\right] + \left[\frac{1}{2} + \frac{j-1}{n}, \frac{1}{2} + \frac{j}{n}\right]$$

with

$$j = \begin{cases} \frac{i+1}{2} & \text{if } i \text{ odd} \\\\ \frac{i}{2} & \text{if } i \text{ even} \end{cases}$$

for $i \in [1, n]$.

The example shows that for high volume tasks, it can be equally productive to have multiple agents share the same specialty as it would be to have them divide that specially, becoming even more proficient in a smaller set of task types. In this case, the result is due to the fact that lower volume tasks cannot support the same level of specialization, so the gains from further dividing labor on high value tasks cannot be realized. However, this is an important case to consider because it has implications for when project distributions over task types vary. In fact, it may be that overlapping jobs are strictly better than a complete division of labor because there is uncertainty over which task types will be high frequency.

3.4.2   Task Sharing under Project Uncertainty

To illustrate this point, consider a simple example of the model. Suppose that there are two task types, $\Omega = \{A, B\}$. Further suppose that the number of projects is limited, but there are two projects with certainty. Finally, suppose that each project consists entirely of one of the task types. There is a probability, p_A of there being two projects of type A tasks, a probability p_B of there being two projects of type B tasks and a $(1 - p_A - p_B)$ of there being one project of each type.

First suppose the team consists of two agents. Then when both projects are the same, each agent will work on one of them. The only question is how to divide the work on two different projects. Let $\gamma_{i,s}$ be agent i's share of the project containing only s type tasks. Then, agent i's frequency of working on s is

$$\tau_{i,s} = p_s + (1 - p_s - p_{s'})\gamma_{i,s}$$

Note that, $\sum_i \gamma_{i,s} = 1$ and $\sum_s \gamma_{i,s} = 1$, so $\gamma_{i,s} = \gamma_{i',s'} = \gamma$. Thus, there is only one choice parameter to optimize output. Because the production function is the minimum function, output is discontinuous at $\gamma = 0$. Further, because the two agents have the same potential

for productivity, the domain of output can be limited to $\gamma \in [0, \frac{1}{2}]$.

$$E[Y(\gamma = 0)] =$$

$$p_A[E(p_A) + E(1 - p_B)]$$

$$+ (1 - p_A - p_B)[E(1 - p_B) + E(1 - p_A)]$$

$$+ p_B[E(p_B) + E(1 - p_A)]$$

$$E[Y(\gamma \neq 0)] =$$

$$p_A[E(p_A + (1 - p_A - p_B)\gamma) + E(p_A + (1 - p_A - p_B)(1 - \gamma))]$$

$$+ (1 - p_A - p_B)[E(p_A + (1 - p_A - p_B)\gamma) + E(p_B + (1 - p_A - p_B)\gamma)]$$

$$+ p_B[E(p_B + (1 - p_a - p_b)(1 - \gamma)) + E(p_B + (1 - p_a - p_b)\gamma)]$$

For $\gamma \neq 0$, $E[Y(\gamma)]$ is increasing in γ. Thus, optimal task sharing is either $\gamma = 0$ or $\gamma = \frac{1}{2}$. In either state where both projects are the same, $\gamma = \frac{1}{2}$ does better for all p_A and p_B because $E(\tau)$ is concave. When $p_A = p_B = 0$, the optimum is at $\gamma = 0$; when there is always one task of each type, each agent specializes in one type of task. However, as p_A and p_B increase, the performance of specialization decreases in the state where the two projects are different, because each agent must spend *some* time on the other type of task. Thus, the division of labor is decreasing in p_A and in p_B.

A larger team would weaken the constraints in this optimization problem, allowing for the sharing of type A tasks to vary independently of the sharing of type B tasks; $\gamma_{i,A} \neq \gamma_{i',B}$. Although each task type may support the work of two agents, a three agent team always

dominates a team of four agents. When two agents share work on a task type, they must share it equally. Therefore, if it is optimal for each task type to be shared, the same two agents can share both task types.

Three agents perform better than two only when $\gamma_s = 0$ is optimal for one of the task types but not the other. In this case, the time constraints cannot be satisfied with only two agents. One agent is required to work on a project of type A (for example) whenever such a project is available, but two agents should share the single B type project when there is one of each type. With only two agents, one agent would be required to work on one and a half projects with probability $(1 - p_A - p_B)$.

While the distribution over the number of projects for each task may be such that the optimal team size is two agents, combining these task types so one team works on both types is at least as efficient because an agent on team A can work on a type B task when he is not working on a type A task. Because the quantity of different task types do not fluctuate together, job sharing is possible, and job sharing allows for the same output to be achieved with a smaller number of agents.

This argument elucidates the role of a constant project limit in demonstrating the optimality of job sharing in this example. The next section discusses optimal job sharing when the volume of different task types can vary freely. First, a brief illustration will show that the job sharing with constant project limit result does not depend on a finite number of task types.

Consider a team of n agents, and assume that the number of projects in a unit of time is also n. Suppose all n projects in a given period are identical, but projects may be distributed over the set of task types, $\Omega = [0, 1]$, in a number of ways. Specifically, assume

that there is a probability p that the tasks in each project are distributed uniformly, and there is a $(1-p)$ probability that half of the task types are twice as likely as the rest (as in Section 3.4.1) and that every possible subset $S \subset \Omega$ with $|S| = \frac{1}{2}$ can be weighted this way. (The particular probability distribution over potential projects is not essential to this illustration.)

As outlined in Section 3.3.1, the optimal division of labor when the project's distribution over tasks is uniform is a partition of $\Omega = [0,1]$ into n equal sized subsets. In some states this division of labor is not feasible because if agents are limited to working only on tasks inside this specialty, the time constraint of some agents will be violated—they will not be able to complete all tasks in their specialty.

In particular, there will be some state in which all of agent i's tasks are given higher weight. Recalling the distribution

$$
f(s) = \begin{cases} \frac{2}{3} & \text{if } s \notin S \\[2ex] \frac{4}{3} & \text{if } s \in S \end{cases}
$$

Agent i's segment of task types will require

$$
n\frac{4}{3}\frac{1}{n} > 1
$$

units of time to complete. Thus, no partition of Ω into n equal subsets is a feasible division of labor in this case. In fact, no partition of Ω into n sets is feasible. Instead, Ω can be partitioned into $\frac{n}{2}$ subsets, with each subset being shared by two agents. Then for

any realization of the task type distribution, there will be a sufficient number of agents to complete each task type when agents only work on tasks within their specialization. In this case, agents are less specialized, and labor is less divided. As the volatility of project distributions increases (i.e. the maximum volume of each task type increases), the division of labor decreases—more agents share a specialization.

When the volume of a particular task type fluctuates, more agents will be required to work on that type of task in some states than are required in other states. Then, there is a positive probability that an additional agent will spend some time working on that task type. Maximizing gains from practice for one agent comes at the cost of less proficient agents sometimes working on that type of task. Reducing the division of labor, by creating overlapping jobs, allows all agents who work on a particular type of task to do so with a moderate level of proficiency. When there is a sufficiently high probability of there being excess tasks, a team of moderate proficiency agents will perform better than a team with some experts and some novices.

3.5   Job and Team Design: Varying Task Type and Project Quantity

The assumptions used in the previous sections have required that task types be related to each other in particular ways, either always co-occurring because there is a fixed task type distribution (Section 3.3) or being negatively correlated because there is a fixed project limit (Section 3.4). In some industries the availability of different task types may be related in

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other ways (or may even be independent). This section uses a simple example of the model to discuss the impact of weakening the assumptions used above on optimal job design and team size.

Suppose that there are four task types ($\Omega = \{A, B, C, D\}$), and that each project consists entirely of a single task type. Further suppose that for each task type, in a unit of time, up to one project may be available; let p_s be the probability that a project of type s is available. Finally, suppose there is a value to leisure that is ω. Then if the assignment of agents to tasks is done for each type individually, an agent would be assigned to task type s when $p_s E(p_s) \geq \omega$.

The optimal overall job design and team size depend on the joint distribution of project availability. To illustrate the relationship between distribution and optimal team design, the fixed task type and the fixed project limit distribution examples of this setup will be considered before discussing more general distributions.

3.5.1   Fixed Task Type Distribution

Suppose that $Pr[\, s' \,|\, \hat{s} \,] = 1$ for all s' and $\hat{s} \in \Omega$; task types are co-occurring. There is a fixed distribution over task types; namely, in each state, each task type represents a quarter of all projects. It must be that $p_s = p$ for all $s \in \Omega$ and that there are always either four projects or none.

In this case, the team size is the sum of the individually optimal teams. If and only if $p_{\hat{s}} E(p_{\hat{s}}) \geq \omega$ for some $\hat{s} \in \Omega$, $p_s E(p_s) \geq \omega$ for all $s \in \Omega$. Optimally, there is no job overlap in this case because if two agents share two task types, the performance on those task type's

projects decreases. Thus, the optimal combined team for all task types has the same size and job design as the optimal separate teams for each task type.

3.5.2 Fixed Project Limit with Negative Correlation

At the other extreme, assume that there is always one project, so that $Pr[\,s'\,|\,\hat{s}\,] = 0$ for each $s', \hat{s} \in \Omega$. Then one agent always does at least as well as a larger team, because maximum production of a task type s is $p_s E(p_s)$, and one agent can achieve this level of production for all task types. Thus, rather than separating different task types, it is always better to combine them into a single team; division of labor is at a minimum.

Note that if it is optimal for an individual task type to be done separately, then that task type will be done in the combined team: If $p_{s'} E(p_{s'}) \geq \omega$ for some s', then $\sum_s p_s E(p_s) \geq \omega$. In addition, some task types that would not be done individually may be done in the combined team because spare time can be used for tasks that are not frequent enough to command their own team. Thus, the combined team never does worse than individual teams, and it performs strictly better than individual teams whenever $\sum_s p_s E(p_s) \geq \omega$, because more projects are done, sometimes with fewer agents.

3.5.3 Illustration of Other Distributions

In general the number of projects available could be anywhere between zero and four. Suppose the probability of two projects being available is near one, and that each combination of two projects is equally likely.

Consider a team of two agents. This team must have some job overlap because for any two mutually exclusive subsets of Ω, there is some probability that the two available task types are in one of those subsets. Thus, there is at least one task type for which two agents share responsibility, and the performance of this task type could be increased by assigning all projects of this type to a single agent. Increasing proficiency in this way can only be achieved by increasing the team size. With a team of four agents, for example, each project could be done with maximum productivity, $p_s E(p_s)$.

Suppose that there are never more than two projects. Then increasing the team size above two does not increase the number of projects that are done. Thus, when the probability of three or four projects is sufficiently low, increasing the team size increases the productivity with which projects are done, but it decreases the utilization of agent time. That is, each task type can become more specialized because an agent is able to spend more of his time on only that task type, but the agent also spends more time idle, as he does not use his spare time to work on other types of tasks.

Thus there is a tradeoff between specialization and efficiency of agent time. Division of labor and specialization are increased by increasing the number of agents. However, under limited workflow and uncertainty over task types, the gains from adding agents to the team are constrained because the probability that an agent is idle increases and because specialization is limited by the availability of tasks within a narrow field.

3.6 Conclusion

The division of labor is a concept fundamental to the study of economics. It has important implications across the economy, but especially for job and team design. Intuitively, the particular way in which labor is divided should depend on the nature of the work being done. However, the existing literature hasn't addressed the relationship between workflow and the division of labor.

During the last century, production in post-industrial economies has shifted away from manufacturing industries towards service and technology industries. The workflow in these industries differs greatly from that in manufacturing, implying observed patterns in the division of labor should be expected to change. In order to better understand these changes, our theories must be adapted to reflect non-manufacturing industries.

The model presented in this paper illustrates the impact of uncertain workflows on job and team design. It shows that proficiency from specialization may be limited by the availability of tasks. In addition, as uncertainty over the distribution of task types increases, specialization and the division of labor both decrease, because the ability to move agents between different types of tasks becomes more important. Finally, uncertain workload can lead to an increase in team size; this increase in size may allow for a greater division of labor, but can also result in decreased agent utilization.

In addition to elucidating the relationship between shifts in the economy and changes in division of labor, job design, and team design, this model has cross-sectional implications. It suggests that differences in division of labor and specialization across industries may be attributable to differences in the nature and uncertainty of the work in those industries.

Furthermore, it indicates that within an industry, some tasks are more likely to be shared among groups than others; therefore, some tasks will have a lower degree of specialization than others. In this way it connects differences in job design and specialization (for example between faculty and administrators) with differences in the nature of work (research generation versus service provision).

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Appendix to Chapter 1

Equilibrium Proofs

Proof of Proposition 1: Two Period Equilibruim

Proposition 1. *Under Condition 1 all mismatched agents with non-urgent tasks are reassigned in Period 1.*

Let $\bar{\eta}$ be the fraction of agents who are high skill and reassigned in Period 1, and let $\underline{\eta}$ be the fraction of agents who are low skill and reassigned in Period 1.

Lemma 1. *The surplus from work on non-urgent tasks in Period 2 is weakly increasing in the fraction of reassigned agents.*

Proof. If no agents are reassigned in Period 1, $\hat{p}$ high skill and $(1 - \hat{p})$ low skill agent are allocated ex-ante to firms for work in Period 2. After tasks arrive in Period 2, $\hat{p}(1 - \hat{\mu})(1 - \hat{p})$ high skill agents will have received non-urgent, high value tasks. Similarly, $(1 - \hat{p})(1 - \hat{\mu})\hat{p}$ low skill agents will have received non-urgent low value tasks. All other non-urgent tasks are efficiently matched with agents. For each non-urgent, high value task without a high skill agent, there is a high skill agent with a non-urgent low value task, and by rematching these

two agents, surplus can be increased by

$$(\bar{\alpha} - \underline{\alpha})(\bar{v} - \underline{v}) > 0$$

If a high skill agent is reassigned during Period 1, then he is unassigned when Period 2 starts, as he worked instead of preparing during the second half of Period 1. Assigning these agents to non-urgent high value tasks generates more surplus than reassigning a high skill agent from a low value task.

$$(\bar{\alpha} - \underline{\alpha})\bar{v} > (\bar{\alpha} - \underline{\alpha})\bar{v} - \bar{\alpha}\underline{v}$$

Thus, for each reassigned high skill agent, the fraction of tasks that are non-urgent and high value at firms without high skill agents is increasing at the marginal rate of

$$(1 - \hat{\mu})\hat{p}$$

but the fraction of agents who are high skill and unassigned increases at the marginal rate of 1, thus, the marginal change in non-urgent, high value tasks available to rematch with assigned high skill agents is

$$(1 - \hat{\mu})\hat{p} - 1 < 0$$

Thus, all non-urgent, high value tasks will be done by high skill agents regardless of the fraction of agents who are reassigned.

Furthermore, the fraction of agents who are high skill with non-urgent, low value tasks and who are not reassigned in Period 2 is

$$\hat{p} - (\hat{p} - \bar{\eta})\hat{\mu} - (1 - \hat{\mu})\hat{p} = \bar{\eta}\hat{\mu}$$

For an increase in reassigned high skill agents, there is a corresponding marginal increase of $\hat{\mu}$ in the fraction of tasks that are non-urgent, low value and done by high skill agents.

Corollary 1. *Following from Lemma 1, reassigning an agent of skill level α during Period 1 decreases surplus in Period 2 by a maximum of*

$$\alpha\hat{\mu}[\hat{p}\bar{v} + (1 - \hat{p})\underline{v}]$$

Each reassigned agent corresponds to a firm that is unmatched ex-ante of Period 2. This firm's urgent task will not be done, but would have generated $\alpha\hat{\mu}[\hat{p}\bar{v} + (1 - \hat{p})\underline{v}]$ if the reassigned agent had been available.

Proof of Proposition. For each reassigned high skill agent, reassigning a low skill agent to the non-urgent task at his firm leads to a change in surplus of

$$\underline{\alpha}\underline{v} - \underline{\alpha}\hat{\mu}[\hat{p}\bar{v} + (1 - \hat{p})\underline{v}]$$

which is positive when Condition 1 a) holds. Thus, each low value agent at a firm with a non-urgent, high value tasks is reallocated to a firm with a non-urgent, low value task.

Thus, for each reassigned high skill agent, a low skill will also be reassigned. This generates a marginal surplus in Period 1 of

$$(\bar{\alpha} - \underline{\alpha})(\bar{v} - \underline{v})$$

Then the net total surplus change from reassigning this agent is

$$(\bar{\alpha} - \underline{\alpha})(\bar{v} - \underline{v}) - (\bar{\alpha} + \underline{\alpha})\hat{\mu}[\hat{p}\bar{v} + (1 - \hat{p})\underline{v}]$$

which is positive when Condition 1 b) holds. Thus, each high skill agent at a firm with a non-urgent, low value task is reassigned to a non-urgent, high value task. □

Convergence

Lemma 2. *Once a high skill agent is reassigned, there will be a non-urgent, high value task for him to work on during each subsequent period.*

Proof. Suppose $\bar{\eta}_t$ high skill agents have been reassigned before period t begins. Further, suppose that $\bar{\eta}_{t-1}$ high skill agents were not allocated ex-ante of $t - 1$. It must be that $\bar{\eta}_t$, the number of agents reassigned before t, is less than the number of non-urgent, high value tasks without an ex-ante assigned high skill agent in period $t - 1$

$$\bar{\eta}_t \leq [1 - (\hat{p} - \bar{\eta}_{t-1})](1 - \hat{\mu})\hat{p} \tag{1}$$

Otherwise, an agent would have no task to do in Period $t-1$ and could be allocated ex-ante for Period t.

The fraction of tasks that are non-urgent, high value and not matched with a high skill ex-ante assigned agent in Period t is

$$[1 - (\hat{p} - \bar{\eta}_t)](1 - \hat{\mu})\hat{p}$$

So the condition that there are non-urgent high value tasks for all unassigned high skill agents to do in Period t is

$$\bar{\eta}_t \leq [1 - (\hat{p} - \bar{\eta}_t)](1 - \hat{\mu})\hat{p}$$

$$\bar{\eta}_t[1 - (1 - \hat{\mu})\hat{p}] \leq (1 - \hat{p})(1 - \hat{\mu})\hat{p} \tag{2}$$

From Equation 1 we have

$$\bar{\eta}_t[1 - (1 - \hat{\mu})\hat{p}] \leq [1 - (\hat{p} - \bar{\eta}_{t-1})](1 - \hat{\mu})\hat{p}[1 - (1 - \hat{\mu})\hat{p}]$$

$$\leq (1 - \hat{p})(1 - \hat{\mu})\hat{p}[1 - (1 - \hat{\mu})\hat{p}]$$

$$< (1 - \hat{p})(1 - \hat{\mu})\hat{p}$$

Therefore, Inequality 2 must hold when Equation 1 holds, and there must be a sufficient number of tasks for all reassigned high skill agents to work on.

$\square$

Observation. *Each period for which the number of high skill agents not assigned ex-ante, $\bar{\eta}$, satisfies the following condition, there will be an opportunity for surplus increase from ex-post reassignment.*

$$\bar{\eta}[1 - (1 - \hat{\mu})\hat{p}] < (1 - \hat{p})(1 - \hat{\mu})\hat{p}$$

If this inequality is satisfied, then in this period, there are strictly more non-urgent, high value tasks not assigned a high skill agent ex-ante than there are unassigned high skill agents.

Proposition 2. *If the economy lasts for n periods, then for all $\underline{v}$ satisfying Condition 1, there is a δ such that:*

During each period, any ex-ante assigned agents that can be reassigned for an immediate surplus increase will be.

Proof. Following from Proposition 1, it must be that in the second to the last period $n - 1$, any agents that can be reassigned will be. Then, in $n - 2$, any reassignment will have no effect on surplus from work on non-urgent tasks in the following two periods. However, it will introduce a cost of urgent tasks that go undone *each* of the last two periods. Given the discount rate, δ, the net change in surplus from reallocating a high skill agent in period $n - 2$ is

$$\underline{\alpha v} - (\delta - \delta^2)\underline{\alpha}\hat{\mu}[\hat{p}\bar{v} + (1 - \hat{p})\underline{v}]$$

$$\geq \underline{\alpha v} - \underline{\alpha}\hat{\mu}[\hat{p}\bar{v} + (1 - \hat{p})\underline{v}] > 0$$

Therefore, if there is a non-urgent, high value task without a high skill agent, reassigning a high skill agent from a low value task will increase surplus.

Using backward induction, the net surplus from reassignment of a high skill agent in Period $n - t$ is

$$\underline{\alpha}\underline{v} - \sum_{1}^{t} \delta^t \underline{\alpha}\hat{\mu}[\hat{p}\bar{v} + (1 - \hat{p})\underline{v}]$$

$$= \underline{\alpha}\underline{v} - \frac{\delta}{1 - \delta}\underline{\alpha}\hat{\mu}[\hat{p}\bar{v} + (1 - \hat{p})\underline{v}] \tag{3}$$

$$> \underline{\alpha}\underline{v} - \underline{\alpha}\hat{\mu}[\hat{p}\bar{v} + (1 - \hat{p})\underline{v}] > 0 \tag{4}$$

The inequality holds between lines (3) and (4) for any $\delta < \frac{1}{2}$.

For high skill agents, then the net increase in surplus from reassignment in $n - 2$ is

$$(\bar{\alpha} - \underline{\alpha})(\bar{v} - \underline{v}) - (\delta - \delta^2)\bar{\alpha}\hat{\mu}[\hat{p}\bar{v} + (1 - \hat{p})\underline{v}]$$

$$> (\bar{\alpha} - \underline{\alpha})(\bar{v} - \underline{v}) - \bar{\alpha}\hat{\mu}[\hat{p}\bar{v} + (1 - \hat{p})\underline{v}] > 0$$

Therefore, if there is a non-urgent, high value task without a high skill agent, reassigning a high skill agent from a low value task will increase surplus.

Using backward induction, the net surplus from reassignment of a high skill agent in Period $n - t$ is

$$(\bar{\alpha} - \underline{\alpha})(\bar{v} - \underline{v}) - \sum_{1}^{t} \delta^t \bar{\alpha} \hat{\mu} [\hat{p} \bar{v} + (1 - \hat{p}) \underline{v}]$$

$$= (\bar{\alpha} - \underline{\alpha})(\bar{v} - \underline{v}) - \frac{\delta}{1 - \delta} \bar{\alpha} \hat{\mu} [\hat{p} \bar{v} + (1 - \hat{p}) \underline{v}] > 0$$

Thus, in each period for which there is an opportunity to reassign agents in a (same period) surplus increasing way, those agents will be reassigned. $\qquad \square$

Comparative Statics Proofs

Proposition. *Under Condition 1, as the number of periods approaches infinity, the assignment of agents converges to the following:*

- *The number of high skill agents are allocated ex-post is*

$$\bar{\eta}^* = \frac{(1 - \hat{p})(1 - \hat{\mu})\hat{p}}{[1 - (1 - \hat{\mu})\hat{p}]}$$

- $\hat{p} - \bar{\eta}^*$ *high skill agents are allocated ex-ante*

- *The number of low skill agents allocated ex-post is*

$$\underline{\eta}^* = \frac{(1 - \hat{p})^2 (1 - \hat{\mu})^2 \hat{p}}{[1 - (1 - \hat{\mu})\hat{p}][1 - (1 - \hat{\mu})(1 - \hat{p})]}$$

- $(1 - \hat{p} - \underline{\eta}^*)$ *low skill agents are allocated ex-ante*

Proof. For each period in which $\bar{\eta}_t < (1 - \bar{\eta}_t)(1 - \hat{\mu})\hat{p}$, it must be that

$$\bar{\eta}_{t+1} = (1 - \bar{\eta}_t)(1 - \hat{\mu})\hat{p} > \bar{\eta}_t$$

Thus, $\bar{\eta}_t$ is increasing but bounded above by

$$\bar{\eta}^* = \frac{(1 - \hat{p})(1 - \hat{\mu})\hat{p}}{[1 - (1 - \hat{\mu})\hat{p}]}$$

So the number of high skill contractors approaches a steady state in which it is equal to $\bar{\eta}^*$. The number of low skill contractors is the number of non-urgent, low value tasks at firms without an employee, giving the expression for $\underline{\eta}^*$. $\qquad\square$

Varying High Skill Labor Supply

Let η be the total number of high skill agents, and suppose that $\eta < \hat{p}$.

Observation. *When $\eta < \hat{p}$, the number of high skill agents allocated ex-post is*

$$\bar{\eta}^* = \frac{(1 - \eta)(1 - \hat{\mu})\hat{p}}{[1 - (1 - \hat{\mu})\hat{p}]}$$

and the number of low skill agents allocated ex-post is

$$\underline{\eta}^* = \frac{\hat{p}(1 - \hat{p})(1 - \hat{\mu})^2(1 - \eta)}{[1 - (1 - \hat{\mu})\hat{p}][1 - (1 - \hat{\mu})(1 - \hat{p})]}$$

Ex-Ante Identical Firms

Numbers of Contractors

Proposition 3. *The fractions of agents who are high and low skill contractors (are hired ex-post) is decreasing in urgency.*

$$\frac{\partial \bar{\eta}^*}{\partial \hat{\mu}} < 0$$

and

$$\frac{\partial \underline{\eta}^*}{\partial \hat{\mu}} < 0$$

Proof. The fraction of agents who are high skill contractors is

$$\bar{\eta}^* = \frac{(1 - \eta)(1 - \hat{\mu})\hat{p}}{[1 - (1 - \hat{\mu})\hat{p}]}$$

$$\frac{\partial \bar{\eta}^*}{\partial \hat{\mu}} = \frac{-[1 - (1 - \hat{\mu})\hat{p}](1 - \hat{\mu})\hat{p}(1 - \eta) - (1 - \eta)(1 - \hat{\mu})\hat{p}^2}{[1 - (1 - \hat{\mu})\hat{p}]^2} < 0$$

As the number of high skill agents is constant, the number of high skill employees is $\eta - \bar{\eta}^*$, it must be that the number of high skill employees is increasing in urgency, $\hat{\mu}$.

The fraction of agents who are low skill contractors is

$$\underline{\eta}^* = \frac{\hat{p}(1 - \hat{p})(1 - \hat{\mu})^2(1 - \eta)}{[1 - (1 - \hat{\mu})\hat{p}][1 - (1 - \hat{\mu})(1 - \hat{p})]}$$

$$\frac{\partial \underline{\eta}^*}{\partial \hat{\mu}} = \frac{-[1 - (1 - \hat{\mu})(1 - \hat{p})][\bar{\eta}^*(1 - \hat{p}) - (1 - \hat{p})(1 - \hat{\mu})\frac{\partial \bar{\eta}^*}{\partial \hat{\mu}}] - \bar{\eta}^*(1 - \hat{\mu})(1 - \hat{p})^2}{[1 - (1 - \hat{\mu})(1 - \hat{p})]^2} < 0$$

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As the number of low skill agents is constant, the number of low skill employees is $(1-\eta-\underline{\eta}^*)$, we have that the number of high skill employees is increasing in urgency, $\hat{\mu}$.

□

Proposition 5. *The quantity of high and low skill contractors are decreasing in high skill labor supply.*

$$\frac{\partial \bar{\eta}^*}{\partial \eta} < 0$$

and

$$\frac{\partial \underline{\eta}^*}{\partial \eta} < 0$$

Proof. The fraction of agents who are high skill contractors is

$$\bar{\eta}^* = \frac{(1-\eta)(1-\hat{\mu})\hat{p}}{[1-(1-\hat{\mu})\hat{p}]}$$

$$\frac{\partial \bar{\eta}^*}{\partial \eta} = \frac{-(1-\hat{\mu})\hat{p}}{1-(1-\hat{\mu}\hat{p}]} < 0$$

So the quantity of high skill contractors is decreasing as the quantity of high skill agents increases.

The fraction of agents who are low skill contractors is

$$\underline{\eta}^* = \frac{\bar{\eta}^*(1-\hat{p})(1-\hat{\mu})}{[1-(1-\hat{\mu})(1-\hat{p})]}$$

$$\frac{\partial \underline{\eta}^*}{\partial \eta} = \frac{\partial \bar{\eta}^*}{\partial \eta} \frac{(1-\hat{p})(1-\hat{\mu})}{[1-(1-\hat{\mu})(1-\hat{p})]} < 0$$

So the quantity of low skill contractors is also decreasing in high skill labor supply.

□

Division of Labor

Define the division of labor as the fraction of tasks that are matched assortatively (high value with high skill agents and low value with low skill agents). Recalling that ex-post allocated agents are always matched assortatively, this fraction can be written:

$$\bar{\eta}^* + (\eta - \bar{\eta}^*)\hat{p} + \underline{\eta}^* + (1 - \eta - \underline{\eta}^*)(1 - \hat{p})$$

Proposition 6. *The division of labor is decreasing in urgency.*

Proof. The division of labor is

$$\bar{\eta}^* + (\eta - \bar{\eta}^*)\hat{p} + \underline{\eta}^* + (1 - \eta - \underline{\eta}^*)(1 - \hat{p})$$

The derivative with respect to urgency is

$$\frac{\partial \bar{\eta}^*}{\partial \hat{\mu}}(1 - \hat{p}) + \frac{\partial \underline{\eta}^*}{\partial \hat{\mu}}\hat{p} < 0$$

The inequality holds by Proposition 4 ($\frac{\partial \bar{\eta}^*}{\partial \hat{\mu}} < 0$ and $\frac{\partial \underline{\eta}^*}{\partial \hat{\mu}} < 0$). □

Proposition 7. *The division of labor is decreasing in the supply of high skill labor.*

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Proof. The division of labor is

$$\bar{\eta}^* + (\eta - \bar{\eta}^*)\hat{p} + \underline{\eta}^* + (1 - \eta - \underline{\eta}^*)(1 - \hat{p})$$

which can be written

$$\bar{\eta}^*(1 - \hat{p}) + \underline{\eta}^*\hat{p} + \eta\hat{p} + (1 - \eta)(1 - \hat{p})$$

The derivative with respect to η is

$$\frac{\partial \bar{\eta}^*}{\partial \eta}(1 - \hat{p}) + \frac{\partial \underline{\eta}^*}{\partial \eta}\hat{p} + (2\hat{p} - 1) < 0$$

The inequality holds by Proposition 5 ($\frac{\partial \bar{\eta}^*}{\partial \eta} < 0$ and $\frac{\partial \underline{\eta}^*}{\partial \eta} < 0$) and $\hat{p} < \frac{1}{2}$.

$\square$

Wages: Levels and Inequality

Let $w(\alpha)$ be the wage of agent type α. The profits of firms following each of the employment policies are as follows:

$$\pi_{\bar{\alpha}} = \bar{\alpha}[\hat{p}\bar{v} + (1 - \hat{p})\underline{v}] - w(\bar{\alpha})$$

$$\pi_{\underline{\alpha}} = \underline{\alpha}[\hat{p}\hat{\mu}\bar{v} + (1 - \hat{p})\underline{v}] - w(\underline{\alpha}) + \hat{p}(1 - \hat{\mu})[\bar{\alpha}\bar{v} - w(\bar{\alpha})]$$

$$\pi_{wait} = (1 - \hat{\mu})[\hat{p}(\bar{\alpha}\bar{v} - w(\bar{\alpha})) + (1 - \hat{p})(\underline{\alpha}\underline{v} - w(\underline{\alpha}))]$$

The indifference of firms between hiring a low skill agent and waiting to hire an agent, $\pi_{wait} = \pi_{\underline{\alpha}}$, determines the wage of the low skill agent.

$$w(\underline{\alpha}) = \underline{\alpha}\,v\,\frac{\mu}{r_v}\frac{[\hat{p} + (1-\hat{p})r_v]}{[\hat{p} + (1-\hat{p})\mu]}$$

The indifference of firms between hiring a high skill agent and waiting to hire an agent, $\pi_{\bar{\alpha}} = \pi_{wait}$ determines the wage of the high skill agent.

$$w(\bar{\alpha}) = \bar{\alpha}\bar{v}\left[\frac{\mu\hat{p} + (1-\hat{p})[1-(1-\mu)r_v]}{[1-\hat{p}(1-\mu)]}\right] + \frac{(1-\mu)(1-\hat{p})}{[1-\hat{p}(1-\mu)]}w(\underline{\alpha})$$

A firm of type p_*, the highest type firm to hire a low skill agent ex-ante, will be indifferent between this policy and waiting to hire ex-post. This indifference will determine the wage for the low skill agent.

$$w(\underline{\alpha}) = \underline{\alpha}\,v\,\frac{\mu}{r_v}\frac{[p_* + (1-p_*)r_v]}{[p_* + (1-p_*)\mu]}$$

A firm of type p^*, the lowest type firm to hire a high skill agent ex-ante, will be indifferent between hiring a high skill agent and waiting to hire ex-post.

$$w(\bar{\alpha}) = \bar{\alpha}\bar{v}\left[\frac{\mu p^* + (1-p^*)[1-(1-\mu)r_v]}{[1-p^*(1-\mu)]}\right] + \frac{(1-\mu)(1-p^*)}{[1-p^*(1-\mu)]}w(\underline{\alpha})$$

In an economy with ex-ante different firms, high skill agents are allocated to higher productivity firms. This effect increases inequality in the wages of high and low skill agents.

A firm of type p_*, the highest type firm to hire a low skill agent ex-ante, will be indifferent between this policy and waiting to hire ex-post. This indifference will determine the wage for the low skill agent.

$$w(\underline{\alpha}) = \underline{\alpha}\,\underline{v}\,\frac{\mu}{r_v}\frac{[p_* + (1 - p_*)r_v]}{[p_* + (1 - p_*)\mu]}$$

A firm of type p^*, the lowest type firm to hire a high skill agent ex-ante, will be indifferent between hiring a high skill agent and waiting to hire ex-post.

$$w(\bar{\alpha}) = \bar{\alpha}\bar{v}\left[\frac{\mu p^* + (1 - p^*)[1 - (1 - \mu)\alpha]r_v}{[1 - p^*(1 - \mu)]}\right] + \frac{(1 - \mu)(1 - p^*)}{[1 - p^*(1 - \mu)]}w(\underline{\alpha})$$

In an economy with ex-ante different firms, high skill agents are allocated to higher productivity firms. This effect increases inequality in the wages of high and low skill agents.

Two Types of Firms

In this subsection, the assumption that all firms are ex-ante identical is relaxed. Let there be two types of firms, one type has an ex-ante $p > \frac{1}{2}$ probability of receiving a high value task, and the other has an ex-ante $(1 - p)$ probability of receiving a high value task. Of all firms, γ are the high type, with $p > \frac{1}{2}$ probability of a high value task. The fraction of tasks that are high value is $\hat{p} = p\gamma + (1 - p)(1 - \gamma)$.

In addition to the variables discussed in the previous subsection, this section will examine the types of firms are not matched with agents ex-ante as well as how uncertainty impacts the number of contractors, the division of labor, and wages.

Holding the number of high value tasks, $\hat{p}$, constant, uncertainty is increasing as p decreases. When p is close to 1, firm type is a good indicator for task type, making it easier to assign high skill agents to high value tasks. However, as p decreases, there is less certainty over the location of high value tasks ex-ante.

Note that as p changes, γ also changes. Rewriting $\hat{p} = p\gamma + (1-p)(1-\gamma)$ gives:

$$\gamma = \frac{\hat{p} - (1-p)}{2p - 1}$$

Then, γ is decreasing as p decreases (uncertainty increases), and $\gamma \in [0, \hat{p})$.

Ex-ante assigned agents will be assertively matched with firms. High skill agents assigned ex-ante will first be assigned to high probability (p) firms. If there are more such agents than such firms, some ex-ante assigned, high skill agents will be allocated to low probability $(1-p)$ firms. Similarly, low skill agents assigned ex-ante will first be assigned to low probability firms. Low skill agents assigned ex-ante will only be allocated to high probability firms if there is an excess of these agents.

There are three cases for the ex-ante assignment of agents:

Case 1: $\gamma > \eta + \underline{\eta}^* > \eta - \bar{\eta}^*$

There are fewer high skill agents assigned ex-ante than there are high probability firms, and there are more low skill agents assigned ex-ante than there are low probability firms. High skill, ex-ante assigned agents work for high probability firms. Low skill, ex-ante agents work for low and high probability firms. All ex-post assigned agents work for high probability firms.

$$\bar{\eta}^* = \frac{\hat{p}(1 - \hat{\mu}) - \eta p(1 - \hat{\mu})}{1 - p(1 - \hat{\mu})}$$

$$\underline{\eta}^* = \frac{\bar{\eta}^*(1 - p)(1 - \hat{\mu})}{1 - (1 - p)(1 - \hat{\mu})}$$

Case 2: $\eta + \underline{\eta}^* > \gamma > \eta - \bar{\eta}^*$

There are fewer high skill agents than there are high probability firms, and there are fewer low skill agents than there are low probability firms. High skill, ex-ante assigned agents work for high probability firms. Low skill, ex-ante agents work for low probability firms. Ex-post assigned agents work for high and low probability firms.

$$\bar{\eta}^* = \frac{\hat{p}(1 - \hat{\mu}) - \eta p(1 - \hat{\mu})}{1 - p(1 - \hat{\mu})}$$

$$\underline{\eta}^* = \frac{(1 - \hat{\mu})[(1 - \hat{p}) - (\eta - \bar{\eta}^*)(1 - p) - (1 - \eta)p]}{1 - (1 - \hat{\mu})p}$$

Case 3: $\eta + \underline{\eta}^* > \eta - \bar{\eta}^* > \gamma$

There are more high skill agents than high probability firms. High skill, ex-ante assigned agents work for high probability and low probability firms. All ex-post assigned and low skill, ex-ante assigned agents work for low probability firms.

$$\bar{\eta}^* = \frac{(1 - \eta)(1 - \hat{\mu})(1 - p)}{1 - (1 - \hat{\mu})(1 - p)}$$

$$\underline{\eta}^* = \frac{\bar{\eta}^*(1 - \hat{\mu})p}{1 - p(1 - \hat{\mu})}$$

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Number of Contractors

Proposition 3. *The quantities of contractors are decreasing in urgency.*

Proof. First consider $\bar{\eta}^*$. In Cases 1 and 2,

$$\bar{\eta}^* = \frac{\hat{p}(1 - \hat{\mu}) - \eta p(1 - \hat{\mu})}{1 - p(1 - \hat{\mu})}$$

and

$$\frac{\partial \bar{\eta}^*}{\partial \hat{\mu}} = \frac{-[1 - p(1 - \hat{\mu})](\hat{p} - \eta p) - p[\hat{p}(1 - \hat{\mu}) - \eta p(1 - \hat{\mu})]}{[1 - p(1 - \hat{\mu})]^2} < 0$$

In Case 3

$$\bar{\eta}^* = \frac{(1 - \eta)(1 - \hat{\mu})(1 - p)}{1 - (1 - \hat{\mu})(1 - p)}$$

and

$$\frac{\partial \bar{\eta}^*}{\partial \hat{\mu}} = \frac{-[1 - (1 - \hat{\mu})(1 - p)][(1 - \eta)(1 - p)] - [(1 - \eta)(1 - \hat{\mu})(1 - p)](1 - p)}{[1 - (1 - \hat{\mu})(1 - p)]^2} < 0$$

Next consider $\underline{\eta}^*$. In Case 1,

$$\underline{\eta}^* = \frac{\bar{\eta}^*(1 - p)(1 - \hat{\mu})}{1 - (1 - p)(1 - \hat{\mu})}$$

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and

$$\frac{\partial \underline{\eta}^*}{\partial \hat{\mu}}$$
$$= \frac{\partial \bar{\eta}^*}{\partial \hat{\mu}} \frac{(1-p)(1-\hat{\mu})}{1-(1-p)(1-\hat{\mu})} - \frac{[1-(1-p)(1-\hat{\mu})]\bar{\eta}^*(1-p) - [\bar{\eta}^*(1-p)(1-\hat{\mu})](1-p)}{[1-(1-p)(1-\hat{\mu})]^2}$$
$$< 0$$

In Case 2,

$$\underline{\eta}^* = \frac{(1-\hat{\mu})[(1-\hat{p}) - (\eta - \bar{\eta}^*)(1-p) - (1-\eta)p]}{1-(1-\hat{\mu})p}$$

and

$$\frac{\partial \underline{\eta}^*}{\partial \hat{\mu}} =$$
$$\frac{\partial \bar{\eta}^*}{\partial \hat{\mu}} \frac{(1-p)(1-\hat{\mu})[1-(1-\hat{\mu})p]}{[1-(1-\hat{\mu}p]^2} - \frac{(1-\hat{p}) + (\eta - \bar{\eta}^*)(2p-1) + \bar{\eta}^*p}{[1-(1-\hat{\mu})p]}$$
$$- \frac{(1-\hat{\mu})[(1-\hat{p}) + (\eta - \bar{\eta}^*)(2p-1) + \bar{\eta}^*p]}{[1-(1-\hat{\mu}p]^2}$$
$$< 0$$

Finally, in Case 3,

$$\underline{\eta}^* = \frac{\bar{\eta}^*(1-\hat{\mu})p}{1-p(1-\hat{\mu})}$$

and

$$\frac{\partial \underline{\eta}^*}{\partial \hat{\mu}} = \frac{\partial \bar{\eta}^*}{\partial \hat{\mu}} \frac{p(1-\hat{\mu})}{1-p(1-\hat{\mu})} - \frac{[1-p(1-\hat{\mu})]\bar{\eta}^*p - [\bar{\eta}^*p(1-\hat{\mu})]p}{[1-p(1-\hat{\mu})]^2} < 0$$

□

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Proposition 4. *The fractions of high and low skill agents who become contractors (are hired ex-post) is increasing in uncertainty.*

Proof. First consider $\bar{\eta}^*$. In Cases 1 and 2,

$$\bar{\eta}^* = \frac{\hat{p}(1 - \hat{\mu}) - \eta p(1 - \hat{\mu})}{1 - p(1 - \hat{\mu})}$$

and

$$\begin{aligned}
\frac{\partial \bar{\eta}^*}{\partial p} &= \frac{-[1 - p(1 - \hat{\mu})]\eta(1 - \hat{\mu}) + [\hat{p}(1 - \hat{\mu}) - \eta p(1 - \hat{\mu})](1 - \hat{\mu})}{[1 - p(1 - \hat{\mu})]^2} \\
&= \frac{-\eta(1 - \hat{\mu}) + \hat{p}(1 - \hat{\mu})^2}{[1 - p(1 - \hat{\mu})]^2} \\
&\leq 0
\end{aligned}$$

The inequality holds because $\eta \in ((1 - \hat{\mu})\hat{p}, \hat{p})$.

In Case 3,

$$\bar{\eta}^* = \frac{(1 - \eta)(1 - \hat{\mu})(1 - p)}{1 - (1 - \hat{\mu})(1 - p)}$$

and

$$\frac{\partial \bar{\eta}^*}{\partial p} = \frac{-[1 - (1 - \hat{\mu})(1 - p)](1 - \eta)(1 - \hat{\mu}) - [(1 - \eta)(1 - \hat{\mu})(1 - p)](1 - \hat{\mu})}{[1 - (1 - \hat{\mu})(1 - p)]^2} < 0$$

Next, consider $\underline{\eta}^*$. In Case 1,

$$\underline{\eta}^* = \frac{\bar{\eta}^*(1 - p)(1 - \hat{\mu})}{1 - (1 - p)(1 - \hat{\mu})}$$

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and

$$\frac{\partial \underline{\eta}^*}{\partial p}$$

$$= -\frac{\bar{\eta}^* \left[[1 - (1-p)(1-\hat{\mu})](1-\hat{\mu}) + [(1-p)(1-\hat{\mu})](1-\hat{\mu})\right]}{[1 - (1-p)(1-\hat{\mu})]^2} + \frac{\partial \bar{\eta}^*}{\partial p} \frac{(1-p)(1-\hat{\mu})}{1 - (1-p)(1-\hat{\mu})}$$

$$< 0$$

In Case 2,

$$\underline{\eta}^* = \frac{(1-\hat{\mu})[(1-\hat{p}) - (\eta - \bar{\eta}^*)(1-p) - (1-\eta)p]}{1 - (1-\hat{\mu})p}$$

and

$$\frac{\partial \underline{\eta}^*}{\partial p} =$$

$$\frac{[1 - (1-\hat{\mu})p](1-\hat{\mu})[2\eta - 1 - \bar{\eta}^* + \frac{\partial \bar{\eta}^*}{\partial p}(1-p)]}{[1 - (1-\hat{\mu})p]^2}$$

$$+ \frac{(1-\hat{\mu})^2[(1-\hat{p}) - (\eta - \bar{\eta}^*)(1-p) - (1-\eta)p]}{[1 - (1-\hat{\mu})p]^2}$$

$$= \frac{[1 - (1-\hat{\mu})p](1-\hat{\mu})(2\eta - 1) - 2(1-\hat{\mu})^2(1-p)\bar{\eta}^* - (1-\hat{\mu})^2 p(1 - \bar{\eta}^*)}{[1 - (1-\hat{\mu})p]^2}$$

$$+ \frac{(1-\hat{\mu})^2[(\eta - \bar{\eta}^*) - [\hat{p} - (1-p)]]}{[1 - (1-\hat{\mu})p]^2}$$

$$< 0$$

In Case 3,

$$\underline{\eta}^* = \frac{\bar{\eta}^*(1-\hat{\mu})p}{1 - p(1-\hat{\mu})}$$

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and

$$\frac{\partial \underline{\eta}^*}{\partial p} = \frac{\bar{\eta}^* \left[[1 - p(1 - \hat{\mu})](1 - \hat{\mu}) + (1 - \hat{\mu})^2 p\right]}{[1 - p(1 - \hat{\mu})]^2} + \frac{\partial \bar{\eta}^*}{\partial p} \frac{\bar{\eta}^*(1 - \hat{\mu})p}{1 - p(1 - \hat{\mu})}$$

$$= \bar{\eta}^* \left[\frac{(1 - \hat{\mu})}{[1 - p(1 - \hat{\mu})]^2} - \frac{(1 - \hat{\mu})p}{(1 - p)[1 - p(1 - \hat{\mu})][1 - (1 - \hat{\mu})(1 - p)]}\right]$$

$$= \bar{\eta}^* \left[\frac{(1 - \hat{\mu})(1 - p)[1 - (1 - \hat{\mu})(1 - p)] - (1 - \hat{\mu})p[1 - p(1 - \hat{\mu})]}{[1 - p(1 - \hat{\mu})]^2(1 - p)[1 - (1 - \hat{\mu})(1 - p)]}\right]$$

$$= \bar{\eta}^* \left[\frac{(1 - \hat{\mu})\hat{\mu}(1 - 2p)}{[1 - p(1 - \hat{\mu})]^2(1 - p)[1 - (1 - \hat{\mu})(1 - p)]}\right]$$

$$\leq 0$$

The inequality holds because $p > \frac{1}{2}$. □

Proposition 5. *The quantity of high skill contractors is decreasing in high skill labor supply.*

Proof. First consider $\bar{\eta}^*$ In Cases 1 and 2

$$\bar{\eta}^* = \frac{\hat{p}(1 - \hat{\mu}) - \eta p(1 - \hat{\mu})}{1 - p(1 - \hat{\mu})}$$

and

$$\frac{\partial \bar{\eta}^*}{\partial \eta} = -\frac{p(1 - \hat{\mu})}{1 - p(1 - \hat{\mu})} < 0$$

In Case 3

$$\bar{\eta}^* = \frac{(1 - \eta)(1 - \hat{\mu})(1 - p)}{1 - (1 - \hat{\mu})(1 - p)}$$

and

$$\frac{\partial \bar{\eta}^*}{\partial \eta} = -\frac{(1 - \hat{\mu})(1 - p)}{1 - (1 - \hat{\mu})(1 - p)} < 0$$

□

Division of Labor

Recall that for ex-ante identical firms, the division of labor is

$$(\eta - \bar{\eta}^*)\hat{p} + \bar{\eta}^* + (1 - \eta - \underline{\eta}^*)(1 - \hat{p}) + \underline{\eta}^*$$

The function for the division of labor under the assumption that there are two types of firms depends on the value of γ, the fraction of firms with a high probability of a high value task.

$$\min\{(\eta - \bar{\eta}^*), \gamma\}p + \max\{0, (\eta - \bar{\eta}^*) - \gamma\}(1 - p) + \bar{\eta}^*$$

$$+ \min\{(1 - \eta - \underline{\eta}^*), (1 - \gamma)\}p + \max\{0, (1 - \eta - \underline{\eta}^*) - (1 - \gamma)\}(1 - p) + \underline{\eta}^*$$

Case 1: $\gamma > \eta + \underline{\eta}^* > \eta - \bar{\eta}^*$

$$(\eta - \bar{\eta}^*)p + \bar{\eta}^* + (1 - \gamma)p + [(1 - \eta - \underline{\eta}^*) - (1 - \gamma)](1 - p) + \underline{\eta}^*$$

$$(\eta - \gamma)(2p - 1) + \bar{\eta}^*(1 - p) + (1 + \underline{\eta}^*)p$$

Case 2: $\eta + \underline{\eta}^* > \gamma > \eta - \bar{\eta}^*$

$$(\eta - \bar{\eta}^*)p + \bar{\eta}^* + (1 - \eta - \underline{\eta}^*)p + \underline{\eta}^*$$

$$(1 - \bar{\eta}^* - \underline{\eta}^*)p + \underline{\eta}^* + \bar{\eta}^*$$

Case 3: $\eta + \underline{\eta}^* > \eta - \bar{\eta}^* > \gamma$

$$\gamma p + (\eta - \bar{\eta}^* - \gamma)(1 - p) + \bar{\eta}^* + [(1 - \eta - \underline{\eta}^*)]p + \underline{\eta}^*$$

$$\gamma(2p - 1) + (\eta - \bar{\eta}^*)(1 - p) + \bar{\eta}^* + (1 - \eta - \underline{\eta}^*)p + \underline{\eta}^*$$

Proposition 6. *The division of labor is decreasing in urgency.*

Proof. Case 1: $\gamma > \eta + \underline{\eta}^* > \eta - \bar{\eta}^*$

$$(1 - \bar{\eta}^* - \underline{\eta}^*)p + \underline{\eta}^* + \bar{\eta}^*$$

The derivative is

$$\left(\frac{\partial \bar{\eta}^*}{\partial \hat{\mu}} + \frac{\partial \underline{\eta}^*}{\partial \hat{\mu}} \right)(1 - p) < 0$$

Case 2: $\eta + \underline{\eta}^* > \gamma > \eta - \bar{\eta}^*$

$$(\eta - \gamma)(2p - 1) + \bar{\eta}^*(1 - p) + (1 + \underline{\eta}^*)p$$

The derivative is

$$\frac{\partial \bar{\eta}^*}{\partial \hat{\mu}}(1 - p) + \frac{\partial \underline{\eta}^*}{\partial \hat{\mu}} p < 0$$

Case 3: $\eta + \underline{\eta}^* > \eta - \bar{\eta}^* > \gamma$

$$\gamma(2p - 1) + (\eta - \bar{\eta}^*)(1 - p) + \bar{\eta}^* + (1 - \eta - \underline{\eta}^*)p + \underline{\eta}^*$$

The derivative is

$$\frac{\partial \bar{\eta}^*}{\partial \hat{\mu}} p + \frac{\partial \underline{\eta}^*}{\partial \hat{\mu}} (1 - p) < 0$$

□

Lemma 3. *If*

$$\bar{\eta}^* = \frac{\hat{p}(1 - \hat{\mu}) - \eta p(1 - \hat{\mu})}{1 - p(1 - \hat{\mu})}$$

(as in Cases 1 and 2) then

$$(\eta - \bar{\eta}^*) + \frac{\partial \bar{\eta}^*}{\partial p}(1 - p) > 0$$

Proof.

$$(\eta - \bar{\eta}^*) + \frac{\partial \bar{\eta}^*}{\partial p}(1 - p)$$

$$= \eta - \frac{\hat{p}(1 - \hat{\mu}) - \eta p(1 - \hat{\mu})}{[1 - p(1 - \hat{\mu})]} - (1 - p)\frac{[1 - p(1 - \hat{\mu})]\eta(1 - \hat{\mu}) - (1 - \hat{\mu})[\hat{p}(1 - \hat{\mu}) - \eta p(1 - \hat{\mu})]}{[1 - p(1 - \hat{\mu})]^2}$$

$$= \frac{\eta - \eta p(1 - \hat{\mu}) - \hat{p}(1 - \hat{\mu}) + p\hat{p}(1 - \hat{\mu})^2 - (1 - p)\eta(1 - \hat{\mu}) + \hat{p}(1 - p)(1 - \hat{\mu})^2}{[1 - p(1 - \hat{\mu})]^2}$$

$$= \frac{\hat{\mu}[\eta - \hat{p}(1 - \hat{\mu})]}{[1 - p(1 - \hat{\mu})]^2}$$

$$> 0$$

□

Proposition 8. *The division of labor is decreasing in uncertainty.*

Proof. Recall that as p increases, uncertainty decreases. Thus, we need to show that the derivatives are positive for the proof. *Case 1: $\gamma > \eta + \underline{\eta}^* > \eta - \bar{\eta}^*$*

$$(1 - \bar{\eta}^* - \underline{\eta}^*)p + \underline{\eta}^* + \bar{\eta}^*$$

The derivative is

$$\left[(\eta - \bar{\eta}^*) + \frac{\partial \bar{\eta}^*}{\partial p}(1 - p)\right] + \left[\eta + \underline{\eta}^* + \frac{\partial \underline{\eta}^*}{\partial p}p\right] > 0$$

Both terms are positive: the first by Lemma 3, the second as follows:

$$
\begin{aligned}
&\eta + \underline{\eta}^* + \frac{\partial \underline{\eta}^*}{\partial p}p \\
&= \frac{\eta\hat{\mu} + \hat{p}(1-p)(1-\hat{\mu})^2}{[1 - p(1-\hat{\mu})][1 - (1-p)(1-\hat{\mu})]} - \frac{(1-p)(1-\hat{\mu})[\hat{p}(1-\hat{m}u) - \eta p(1-\hat{\mu})]}{[1 - (1-p)(1-\hat{\mu})]^2} \\
&\quad - \frac{(1-p)^2(1-\hat{\mu})^2[\eta - \hat{p}(1-\hat{\mu})]}{[1 - p(1-\hat{\mu})]^2[1 - (1-p)(1-\hat{\mu})]} \\
&= \frac{\eta\hat{\mu}[1 - (1-\hat{\mu})[p + (1-p)^2]] + \eta\hat{\mu}^2(1-p)}{[1 - p(1-\hat{\mu})]^2[1 - (1-p)(1-\hat{\mu})]} - \frac{(1-p)^2(1-\hat{\mu})^2[\eta(1-\hat{\mu}) - \hat{p}(1-\hat{\mu})]}{[1 - p(1-\hat{\mu})]^2[1 - (1-p)(1-\hat{\mu})]} \\
&\quad + \frac{(1-p)(1-\hat{\mu})[\eta p(1-\hat{\mu})]}{[1 - (1-p)(1-\hat{\mu})]^2} + \frac{\hat{p}(1-p)(1-\hat{\mu})^3(2p-1)}{[1 - p(1-\hat{\mu})][1 - (1-p)(1-\hat{\mu})]^2} \\
&> 0
\end{aligned}
$$

Case 2: $\eta + \underline{\eta}^* > \gamma > \eta - \bar{\eta}^*$

$$(\eta - \gamma)(2p - 1) + \bar{\eta}^*(1 - p) + (1 + \underline{\eta}^*)p$$

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The derivative is

$$\left[(\eta - \bar{\eta}^*) + \frac{\partial \bar{\eta}^*}{\partial p}(1 - p)\right] + \left[(1 - \eta - \underline{\eta}^*) + \frac{\partial \underline{\eta}^*}{\partial p}(1 - p)\right] > 0$$

Both terms are positive: the first by Lemma 3, the second as follows:

$$(1 - \eta - \underline{\eta}^*) + \frac{\partial \underline{\eta}^*}{\partial p}(1 - p)$$

$$= (1 - \eta) - \frac{(1 - \hat{\mu})(1 - p) - (1 - \eta)p(1 - \hat{\mu})}{[1 - p(1 - \hat{\mu})]}$$

$$+ (1 - p)\frac{\partial \bar{\eta}^*}{\partial p}\left[\frac{(1 - \hat{\mu})(1 - p) - (1 - \eta)p(1 - \hat{\mu})}{[1 - p(1 - \hat{\mu})]}\right]$$

$$+ \left[(\eta - \bar{\eta}^*) + \frac{\partial \bar{\eta}^*}{\partial p}(1 - p)\right]\frac{(1 - p)(1 - \hat{\mu})}{[1 - p(1 - \hat{\mu})]}$$

$$+ (\eta - \bar{\eta}^*)\frac{1 - (1 - \hat{\mu})^2}{[1 - p(1 - \hat{\mu})]^2}$$

$$> 0$$

Case 3: $\eta + \underline{\eta}^* > \eta - \bar{\eta}^* > \gamma$

$$\gamma(2p - 1) + (\eta - \bar{\eta}^*)(1 - p) + \bar{\eta}^* + (1 - \eta - \underline{\eta}^*)p + \underline{\eta}^*$$

The derivative is

$$\left[1 - (\eta - \bar{\eta}^*) + \frac{\partial \bar{\eta}^*}{\partial p}p\right] + \left[(1 - \eta - \underline{\eta}^*) + \frac{\partial \underline{\eta}^*}{\partial p}(1 - p)\right] > 0$$

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The first term is positive:

$$1 - (\eta - \bar{\eta}^*) + \frac{\partial \bar{\eta}^*}{\partial p} p$$

$$= \frac{(1 - \eta)}{[1 - (1 - p)(1 - \hat{\mu})]} - \frac{p(1 - \eta)(1 - \hat{\mu})}{[1 - (1 - p)(1 - \hat{\mu})]^2}$$

$$= \frac{(1 - \eta)\hat{\mu}}{[1 - (1 - p)(1 - \hat{\mu})]^2}$$

The second term is also positive:

$$(1 - \eta - \underline{\eta}^*) + \frac{\partial \underline{\eta}^*}{\partial p}(1 - p)$$

$$= \frac{\hat{\mu}(1 - \eta)}{[1 - p(1 - \hat{\mu})][1 - (1 - p)(1 - \hat{\mu})]} + \frac{(1 - \eta)(1 - \hat{\mu})^2(1 - p)^2}{[1 - p(1 - \hat{\mu})]^2[1 - (1 - p)(1 - \hat{\mu})]}$$

$$- \frac{(1 - \eta)(1 - \hat{\mu})^2 p(1 - p)}{[1 - p(1 - \hat{\mu})][1 - (1 - p)(1 - \hat{\mu})]^2}$$

$$= \frac{(1 - \eta)\hat{\mu}\left[[1 - (1 - \hat{\mu})[(1 - p) + p(1 - p)(1 - \hat{\mu})]\right]}{[1 - p(1 - \hat{\mu})][1 - (1 - p)(1 - \hat{\mu})]^2}$$

$$+ \frac{(1 - \eta)(1 - \hat{\mu})^2(1 - p)\left[[1 - (1 - \hat{\mu})[p + (1 - p)^2]] + (1 - \hat{\mu})^2 p^2\right]}{[1 - p(1 - \hat{\mu})]^2[1 - (1 - p)(1 - \hat{\mu})]^2}$$

$$> 0$$

$\square$

Appendix to Chapter 2

Condition 1. *A necessary condition for a candidate allocation of agents to be an equilibrium is that for any i and i' with $i' < i$, it must be that $[q_{i,H}\bar{v} + q_{i,L}\underline{v}] \geq [q_{i',H}\bar{v} + q_{i',L}\underline{v}]$.*

Proof. Holding all jobs constant, suppose there is an $i' < i$ with $[q_{i,H}\bar{v} + q_{i,L}\underline{v}] < [q_{i',H}\bar{v} + q_{i',L}\underline{v}]$. Then without changing any other assignment, switching the jobs of these two agents changes output by

$$(\alpha_{i'} - \alpha_i)[q_{i,H}\bar{v} + q_{i,L}\underline{v}] + (\alpha_i - \alpha_{i'})[q_{i',H}\bar{v} + q_{i',L}\underline{v}]$$

$$= (\alpha_i - \alpha_{i'})([q_{i',H}\bar{v} + q_{i',L}\underline{v}] - [q_{i,H}\bar{v} + q_{i,L}\underline{v}]) > 0$$

$\square$

Lemma 1. *For each realization of the random variables, unemployed agents are less skilled than contractors in the following sense:*

> *If agent i is unemployed, no agent with $i' < i$ can be employed as a contractor.*

Proof. Follows directly from Condition 1: Suppose agent $i' < i$ is employed as a contractor. Then $[q_{i',H}\bar{v} + q_{i',L}\underline{v}] \geq [q_{i,H}\bar{v} + q_{i,L}\underline{v}] = 0$, violating Condition 1. $\square$

Lemma 2. *There are only unemployed agents if each firm with a general task hires an agent.*

Proof. If agent i is unemployed and firm j receives a general task with value v_j but has not hired an agent, surplus is increased by $\alpha_i v_j$ when j hires i ex-post. $\square$

Lemma 3. *Firm j can only agent i as an independent contractor if the following 2 conditions hold:*

1. *Firm j received a general task.*

2. *No agent with $i' < i$ is an employee of firm j.*

Proof. Part 1) If firm j does not have a general tasks, then i with general knowledge cannot produce any value at that firm.

Part 2) If firm j has hired $i' > i$ as an employee, then i' can only produce value at firm j (that value is $\alpha_{i'} v_j$). If j hires i ex-post to do this task instead then the change in surplus is $(\alpha_i - \alpha_{i'}) v_j < 0$. $\square$

Claim 1. *If i is a contractor who prefers a job as an employee, then all $i' < i$ are contractors.*

Proof. There cannot be an employee who is of lower skill (that firm would be willing to hire i and get higher productivity...). If there aren't any lower skill employees, then there must be firms that have not hired an agent as an employee (otherwise i could not work as a contractor). But if there are firms that have not hired an employee, then there cannot be any agents (of lower or of high skill) who are unemployed (in every state). Thus, no lower skill agent can be an employee, and no lower skill agent can be unemployed. All less skilled agents must also be contractors.

If there is an $i' < i$ who is an employee, then $[q_{i,H} \bar{v} + q_{i,L} \underline{v}] > [q_{i',H} \bar{v} + q_{i',L} \underline{v}]$, with $q_{i,V}, q_{i',V} > 0$. If there is an $i' < i$ who is unemployed, then $\square$

Appendix to Chapter 3

Proof of Condition for Increasing Team Size

Let G be a distribution over the number of projects available. Let G_{r+1} be a mean preserving spread of G_r and let n_r and n_{r+1} be the optimal team sizes for distributions G_r and G_{r+1} respectively.

Claim 1. *There is a probability $(1-G_r^*) \in \left((1-G_r(n_r)), (1-G_r(n_r))^{\frac{1}{1+\theta}} \right)$ such that $n_{r+1} \geq n_r$ if and only if*

$$(1 - G_{r+1}(n_r)) \geq (1 - G_r^*) \tag{1}$$

Recall that if an interior solution to the optimization problem exists, $n_{r+1} > n_r$ when

$$(1+\theta) \left(\int_0^{n_r} 1 - G_{r+1}(x)dx \right)^\theta (1 - G_{r+1}(n_r)) > (1+\theta) \left(\int_0^{n_r} 1 - G_r(x)dx \right)^\theta (1 - G_r(n_r)) \tag{2}$$

By definition, if G_2 is a mean preserving spread of G_1, then

$$\int_0^n G_2(x)dx \geq \int_0^n G_1(x)dx$$

for all n and

$$\int_0^\infty 1 - G_2(x)dx = \int_0^\infty 1 - G_1(x)dx = E[x]$$

Then inequality (2) can only be satisfied when $(1 - G_{r+1}(n_r)) > (1 - G_r(n_r))$.

Because $(1 - G(x)) \in (0, 1]$ is decreasing,

$$(1 - G(n))n > \left(\int_0^n 1 - G(x)dx\right)^\theta (1 - G(n)) > (1 - G(n))^{1+\theta}n$$

for all n and all G. Thus, if $(1 - G_{r+1}(n_r))^{1+\theta} = (1 - G_r(n_r))$,

$$\left(\int_0^{n_r} 1 - G_{r+1}(x)dx\right)^\theta (1 - G_{r+1}(n_r)) >$$

$$(1 - G_{r+1}(n_r))^{1+\theta}n_r = (1 - G_r(n_r))n_r$$

$$> \left(\int_0^{n_r} 1 - G_r(x)dx\right)^\theta (1 - G_r(n_r))$$

Thus, Inequality 2 holds at $(1 - G_{r+1}(n_r)) = (1 - G_r(n_r))^{\frac{1}{1+\theta}}$. Then there is a $(1 - G_r^*) <$ $(1 - G_r(n_r))^{\frac{1}{1+\theta}}$ such that if $(1 - G_{r+1}(n_r)) = (1 - G_r^*)$ the inequality holds with equality. Then whenever $(1 - G_{r+1}(n_r)) \geq (1 - G_r^*)$ the inequality must hold. Because the inequality cannot hold for $(1 - G_r(n_r)) = (1 - G_r(n_r))$, it must be that $(1 - G_r^*) \in$ $\left((1 - G_r(n_r)), (1 - G_r(n_r))^{\frac{1}{1+\theta}}\right)$.

$\square$

Two Period Example

This section considers job design under uncertainty over task type distribution in the short-run. Consider a two period model in which agents accrue proficiency only after completing work. Then agents work to accrue skill in the first period, and work to produce output in the second period.

Following the example given in Section 3.4.2: Suppose that there are two task types, $\Omega = \{A, B\}$. Further suppose that the number of projects is limited, but there are two projects with certainty. Finally, suppose that each project consists entirely of one of the task types. There is a probability, p_A of there being two projects of type A tasks, a probability p_B of there being two projects of type B tasks and a $(1 - p_A - p_B)$ of there being one project of each type.

Job design is only relevant when there is one project of each type in the first period. Each project could be assigned to one agent, in which case Period 2 output would be:

$$p_A[E(1) + E(0)]$$

$$+ (1 - p_A - p_B)[E(1) + E(1)]$$

$$+ p_B[E(0) + E(1)]$$

Alternatively, each project could be split between the two agents. Let $\gamma \in (0, \frac{1}{2}]$ be the smaller share of a project, noting that γ when the team consists of two agents. In this case, output is given by:

$$p_A[E(1 - \gamma) + E(\gamma)]$$

$$+ (1 - p_A - p_B)[E(1 - \gamma) + E(1 - \gamma)]$$

$$+ p_B[E(\gamma) + E(1 - \gamma)]$$

The derivative of this expression with respect to γ is

$$(p_A + p_B)E'(\gamma) - (2 - p_A - p_B)E'(1 - \gamma)$$

Because E is concave, $E'(\gamma) \geq E'(1 - \gamma)$ on the domain of γ. Then, for a sufficiently large $(p_A + p_B)$, the optimal γ is positive, and the optimum goes to $\frac{1}{2}$ as $(p_A + p_B)$ goes to 1. When the probability of there being excess tasks of a single type increases, it is more important for both agents to have some proficiency in each type of task.

The difference between this example and the one given in the text is that here in Period 2 when there is one project of each type, they are not shared between the agents. Sharing tasks in the last period of a finite model decreases output because the minimum proficiency applied to each project decreases. There is also no benefit to sharing the projects because there are no future projects.

In a finite model with more periods, project sharing in intermediate periods will contribute to agent's performance in subsequent periods, even though it decreases current performance. These intermediate periods are similar, then, to the analysis in the text. When there are more than two periods, project sharing in the first period becomes even

more valuable because it increases the performance on future shared projects. Taking this added benefit to its extreme, a very long time horizon, optimal task sharing must be $\gamma = \frac{1}{2}$, as shown in the text.