Remembering Raoul Bott (1923–2005)

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Accessibility
Raoul Bott passed away on December 20, 2005. Over a five-decade career he made many profound and fundamental contributions to geometry and topology. This is the second part of a two-part article in the Notices to commemorate his life and work. The first part was an authorized biography, “The life and works of Raoul Bott” [4], which he read and approved a few years before his death. Since then there have been at least three volumes containing remembrances of Raoul Bott by his erstwhile collaborators, colleagues, students, and friends [1], [2], [7]. I have also written elsewhere about my experiences working with him [5]. This second part presents some personal recollections that do not overlap with what has already appeared in print. More reminiscences and appreciations of his work may be found in the upcoming final volume of the Collected Papers of Raoul Bott [6].

Bott had a passion for mathematics, which he kept to the very end, even after his retirement from Harvard. At the same time, he was firmly planted in the real world. As many of his acquaintances would agree, he exemplified the French phrase joie de vivre. His mathematical work speaks for itself, but it is hoped that the following reminiscences can give some idea of his personality, his zest for life, and his humanity.

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Rodolfo Gurdian was one of Raoul Bott’s roommates when they were undergraduates at McGill. The imaginary chicken-stealing incident in this article is a reference to a real chicken leg incident they experienced together at Mont Tremblant, recounted in [4].

What follows is an account of some of the mischief that Raoul Bott and I carried out during our days at McGill.

I met Raoul in 1941, when we were in our first year at McGill University. Both of us lived in Douglas Hall, a student dormitory of the university, but we were in different apartments. He paid attention to me because I got a higher grade in trigonometry. He also noticed that I played the guitar, and I appreciated his piano playing.

The following year we shared an apartment, together with Frazer Farlinger, a student in medicine. Raoul majored in electrical engineering and I in chemical engineering. The difference among us was that Frazer and I had to study very hard, while Raoul didn’t. He used to say that attending the lectures was sufficient for him, since electrical engineering was a very logical subject. His marks were satisfactory but could have been much better if he had only worked harder.

I met Oskar and Celia Pfeffer, his charming stepparents. Realizing that they were not very well off, I called Raoul’s attention to the fact that, being

Rodolfo Gurdian, who has since passed away, wrote this piece around the year 2000 and Raoul Bott read it.

Loring W. Tu, Coordinating Editor
With contributions from Rodolfo Gurdian, Stephen Smale, David Mumford, Arthur Jaffe, Shing-Tung Yau, and Loring W. Tu

The contributions are listed in the order in which the contributors first met Raoul Bott. As the coordinating editor, I have added a short introductory paragraph (in italics) to the beginning of each contribution. —Loring Tu
so talented in mathematics, he could easily obtain a scholarship by studying just a little more. I think I influenced him, because he improved his grades and became one of the best students in the class. In his last years at McGill, I believe he did obtain a scholarship.

Rooming together, we became close friends. We loved to make mischief. On Saturdays we used to go to the movies, often to a theater called The System. Buying just one ticket, we could watch three movies in a row. Although the ticket price was low, both of us being broke, we found a “system” to sneak into The System without paying by taking advantage of the fact that only one person was in charge of the theater’s two entrances. One of the entrances gave access to the upper floor through a wooden staircase, and the other went to the first floor. The trick we devised was for one of us to talk to the ticket clerk, while the other would distract him by running upstairs and making a lot of noise. As the ticket clerk followed the noisemaker, the other one took advantage of the situation to sneak into the theater’s first floor. Of course, once we were in, it was difficult for the ticket clerk to find us, because we sat in the first available seats, feigning to be regular customers.

During the summer months every engineering student was required to work to get some practical experience. Since Douglas Hall was closed for the summer, we rented a room together near the university. Raoul was six foot two, and I just under five foot six. So you can imagine what a strange-looking pair we made! One summer Sunday we decided on a prank. Raoul put on his gray and red bathrobe and a turban and armed himself with a small dagger. I put on my red short pants and a green T-shirt and carried a tambourine. When Raoul went out to the street, I followed, playing the tambourine and dancing in circles around him. People in the street were shocked. Suddenly, Raoul approached an old lady and threatened her with the dagger. When she started screaming, we ran like hell, realizing that the joke and the fun were getting out of hand.

We used to talk about our future careers. I told him that, due to my facility with money, I would dedicate myself to business, which, in fact, I successfully achieved in life. He joked that, since I would become a wealthy man while he, as a professor, would be very poor, eventually he would be forced to come to Costa Rica to seek my help. Finding a thief in my backyard, I would come out with a gun. Upon seeing me, Raoul would shout, “Please, Rodolfo, don’t shoot. It’s me, Raoul, your old friend.” By that time, I would have become an insensitive wealthy man, so I would shout back, “Of course, I recognize you,” and would shoot him dead anyway.

One reason we became good friends might have been that, during our childhood, we both engaged in similar mischief. Moreover, Latin Americans may have more in common with Europeans than with North Americans. So we enjoyed making pranks together.

Stephen Smale
At a conference in 1967 Stephen Smale organized a beach hike in which Raoul Bott nearly drowned. Afterwards, Bott sometimes joked that Smale tried to kill him. When Bott said Smale was his “worst” student, it was not in the mathematical sense but in the moral sense. Here is Smale’s version of the event.

Raoul often introduced me with the words “Steve was my first student” and then added with great emphasis “and my worst!” He described our relationship (often tumultuous) in his talk1 at the conference for my sixtieth birthday. Raoul says there, “Steve tried to drown me,” as he describes one of our excursions.

It is true that I miscalculated the timing and dangers of an incoming tide at Taylor’s Point on the Olympic Peninsula. It was at a 1967 Battelle conference in Seattle on general relativity that I put together a group of about a dozen people for a three-day hike, camping along the ocean beach. Toward the end of the trip we came to (the notorious) Taylor’s Point and had to make a decision. My wife, Clara; daughter, Laura; and a few others decided on a detour. I convinced the remainder, including Raoul; his wife, Phil; his

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Figure 4. International Symposium on Algebraic Topology, Mexico City, 1956. Front row, from left to right: 1 = William Massey (?), 3 = Friedrich Hirzebruch (?), 4 = Hans Samelson, 5 = Raoul Bott, 6 = J. H. C. Whitehead. Second row: 5 = Witold Hurewicz (who was killed a few days later falling off a pyramid), 7 = Solomon Lefschetz. Third row: 3 = Morris Hirsch, 7 = Leopoldo Nachbin. According to Michael Atiyah, since he was not in the photo, he might have been the lecturer.


daughter, Renee; Mike Shub, his wife (at that time), Beth; and my son, Nat (ten years old), that my tide calculations would justify an attempt to take the sea route. Success was at hand when I looked ahead to see Raoul being battered against the cliffs by the heavy sea. He writes in [3, vol. 2] that he was thinking, “This is how one drowns.” In fact, we all survived (my backpack was lost to the seas). Raoul also writes that, after an excursion with me, he often got on his knees to give thanks: “Back home again and still alive!”

Raoul Bott and I were close friends for over five decades. Already in 1953 we would meet for weekly lunches, as I was taking his class in advanced algebraic topology. I have often spoken of how it was Bott who started me on the road to serious mathematics. He was a great teacher and a great inspiration.

There were sometimes divergences in our approaches to mathematics as well as divergences in which fields we worked in. Very early on, Raoul seemed a bit upset with my working in ordinary differential equations. That subject was for those who found partial differential equations too difficult. Moreover, at that time Raoul was not too crazy about even p.d.e. In Morse theory he preferred using finite-dimensional approximations, in contrast to my using infinite-dimensional manifolds (Palais–Smale).

The last extended time we spent together was on the occasion of his one- to two-month visit to the City University of Hong Kong at my invitation. He and Phil were still drinking martinis with Clara and me after all those years. He was redoing in his own framework my joint result with Mike Shub on the number of real zeros of a system of polynomial equations. Later I saw his writeup with Cliff Taubes.

Living in the world of Raoul Bott was a wonderful part of my life.
David Mumford

David Mumford describes Bott’s “job talk” at Harvard and the transformation in the culture of the Harvard mathematics department upon Bott’s arrival in 1959.

My first encounter with Raoul was most memorable. I was still a lowly undergraduate at Harvard, though taking some advanced courses. One day in 1958, Raoul came to give the colloquium talk, a weekly affair Thursdays at 4:30 in 2 Divinity Avenue (rotating with MIT but not, if I recall correctly, with Brandeis as yet). The stage at 2 Divinity was on a low platform, maybe three feet high with low stairs on each side. After being introduced, Raoul did what no one else had ever done: he marched directly onto the stage with a small jump. His enthusiasm was both physical and mental. He proceeded to mesmerize the assembled crew of senior professors, first explaining Morse theory, a brand new tool for the conservative audience, and then his periodicity theorem. Of course I didn’t know it, but it was a job talk—he was coming to Harvard!

To put this in context, it’s important to know what sort of a place Harvard was at the time. Dave Widdler and Joe Walsh were the most senior and, together with Garrett Birkhoff, represented the tradition from the 1930s. At a dinner party with the Walshes, although a tuxedo was no longer expected, the women still left the dining room after dessert so the men could light their cigars and discuss masculine topics. Lars Ahlfors, Richard Brauer, and Oscar Zariski were the European stars who had put the department on the map. The younger generation was represented by the three American functional analysts: Andy Gleason, George Mackey, and Lynn Loomis. But it was still a very conservative place run by conservative gentlemen. Faculty wives put on white gloves to serve tea before the colloquium. Saunders Mac Lane had left for Chicago, so his fancy ideas in topology, cohomology, sheaves, and so on were unknown areas at Harvard. Semi-simple Lie groups were a distant concept.

Enter Raoul. He was not a breath of fresh air; he was a gale—not merely the new and wonderful mathematical topics that he brought in his tool kit, but his spark, his energy, his fearlessness. You know how universal it is to refrain from asking what you fear may be a stupid question when a seminar speaker begins to lose you? Not Raoul! He regularly raised his hand and asked that “stupid question”, half knowing the answer but wanting to slow down the speaker, hear it again, and bring the rest of us into the circle of mutual appreciation present in the best seminars. Being ashamed not to know something basic was completely alien to his temperament.

And then there were the “colloquium parties”. I think these became a regular fixture in the 1960s: every Thursday the speaker’s host invited a large part of the Boston research community and their spouses to a party. Here were these clusters, typically the guys talking math and their wives talking...
Arthur Jaffe


I first encountered Raoul Bott in 1964. While a student in Princeton I came across the beautiful Bott-Mayberry paper on matrices and graphs and used their representation of a determinant in analyzing a problem in quantum theory. But that experience did little to prepare me for our first face-to-face meeting. Raoul’s personality and spirit struck me with awe; it left an indelible mark in my memory.

Later we became colleagues, and it was then that Raoul evolved into a very special and dear friend. We shared many mathematical discussions together. We also spent hours talking about the world, laughing over an amusing story, listening to music, or sharing a meal with a good wine.

Raoul had always had an interest in physics, but that had some unusual twists. For example, when Raoul learned of normal ordering (a simple form of renormalization), he used to ask me if it could have something to do with the resolution of singularities in algebraic geometry, still an intriguing question.

Raoul described discussions he had with his neighbor Chen Ning Yang during 1955–57 at the time Raoul was a visitor at the Institute for Advanced Study. While they discussed many things, connections were not on the agenda. Only later did one realize their central role in Yang-Mills theory. In the late 1970s we had a long discussion while driving together from Cambridge to an AMS summer meeting in Providence about how important it was to have a good dictionary to translate between gauge theory as physics and differential geometry as mathematics in order for people in the two subjects to communicate.

While I had first met Raoul in a mathematics conference before I came to Harvard in 1967, I really got to know Raoul well during the Les Houches summer school in 1970. Cecile DeWitt had established a famous summer school of theoretical physics after World War II; it was located in a small French village in the Alps, not far from Mont Blanc. The only problem about trying to work in Les Houches was the distraction of a striking view of the Aiguille du Midi. In 1970 the focus of the school was mathematical quantum field theory. Raoul was officially an “observer” at the school, sent by the Battelle Institute, which sponsored the event. Cecile had an interesting philosophy about Les Houches: in order to maximize interaction, the participants at Les Houches should come at the beginning of the meeting and remain there until the end. And this school lasted two full months! Both in the lectures and at the meals in the large dining room, the participants interacted like a large family for sixty days. Raoul brought his wife, Phyllis, and their three young girls, and their son, Tony, also visited on occasion. So, over the course of that summer, I really got to know the Botts. In fact, George and Alice Mackey and their daughter,
Ann were there too, so there was a big contingent from Harvard.

After returning to Cambridge that fall, I began a mathematical physics seminar at which Raoul became a regular attendee. Several of the students from Les Houches also came to Harvard. Raoul enjoyed them all but became especially fond of Konrad Osterwalder, who eventually spent six years in Cambridge and became one of Raoul’s regular confidants before he left for the E.T.H. Zürich.

In 1976 I helped a friend in Paris organize a summer school in Cargèse, Corsica, where I had been during the summer of 1964. The experience left me with a lasting impression of the beauty and history of this Greek-French village by the Mediterranean Sea. The success of the resulting gathering led to our having five more schools in Cargèse. The second school in 1979 brought together an interesting group of mathematicians and physicists, including Raoul, Michael Atiyah, Jürg Fröhlich, Jim Glimm, Gerard ’t Hooft, Harry Lehmann, Isadore Singer, Kurt Symanzik, Ken Wilson, Edward Witten, and Jean Zinn-Justin. The accompanying photographs (Figures 16 and 17) show that Raoul was in good form in the school: not only did he give beautiful lectures but he animated less formal moments. The combination of the productive and interactive scientific atmosphere, along with an inviting beach, brought Raoul and Phyllis back to Cargèse in 1987 and 1991.

Originally I had been appointed professor of physics at Harvard, although some of my courses were cross-listed in mathematics. But in the spring of 1975 the mathematics department invited me to become a member. Raoul was chairman at the time, and I recall the pleasure with which he described to me that vote. Raoul also enlivened the ensuing faculty meetings following Thursday lunch at the Faculty Club. Until sometime in the 1980s, the department decided on teaching assignments in an old-fashioned way: discuss this at a faculty meeting, which always seemed to have full attendance! The chair wrote on the board a list of necessary courses; the persons present filled in their names in order of seniority in the department. This gave the more senior members of the department an elevated status, in which Raoul amusingly reveled.

In 1978 Raoul and Phyllis became Masters of Dunster House. I had been a happy member of Lowell House ever since some students brought me there during my first year at Harvard. But Raoul asked me to switch and be with him at Dunster, which I eventually did. I brought along a few of my own collaborators, and I have many fond memories of evenings with friends at Dunster: with students and with scientific colleagues in the dining hall, with members of the “Senior Common Room” at their regular meetings, at the Dunster concerts, at the Red Tie dinners, and during many other occasions in the master’s residence with Raoul and Phyllis. Raoul sometimes had his mathematical friends stay in Dunster, such as Fritz Hirzebruch or Michael Atiyah, and he would enjoy letting us know when some mathematician friend of his might be making an unannounced visit to Cambridge.

With our frequent intersection at Dunster House, we often made plans to do things together. We both enjoyed music and often had met at undergraduate concerts. I recall our discussing the concert at Sanders Theater when Yo-Yo Ma’s undergraduate quartet played Brahms. I went with Raoul to the first performance that the Tallis Scholars sang in Boston in a concert at the Church of the Advent. Much after that, I sat with Raoul in his music room in his apartment on Richdale Avenue while he played Bach on his Steinway. Later in that room we discussed the differences between recordings of the Goldberg Variations made by Glenn Gould and András Schiff. (András is another genius from Hungary whom I admire. I have also come to know him as a friend, and I wish that I could have introduced Raoul and András to each other, for they certainly would have hit it off well.)
Raoul did not always remember the seminar schedule, so he enjoyed having his office across from the main seminar room in the department. One felt in the middle of things with people passing by, and with the glass window along the hall corridor, one could always see at a glance what seminars were taking place.

Raoul could turn up at the most unusual time or place. When I married in 1992, Raoul was an usher in the wedding. I recall how proud he was to escort my daughter, Margaret. And I was not surprised in the summer of 2002 to arrive at the airport in Vienna and find Raoul and his granddaughter, Vanessa Scott, there too. They were on their way to make a film about Raoul’s life, a wonderful story I saw in 2006.

Raoul’s life turned upside down when Phyllis had a stroke. He began to spend every day with his Mac computer at Youville Hospital in Cambridge, where Phyllis was recuperating. For a while George Mackey was nearby in the same hospital. During those days Raoul often came to my home for dinner. We sat around the kitchen table over swordfish or bluefish from the broiler. The conversation sometimes turned to music. Raoul admired the virtuosity of my harpist friend, Ursula. Many people regarded Raoul as a father figure, but Ursula was drawn to him for the empathy he expressed for others in need of assistance and for his understanding of the problems of the world. Raoul explained that life would be much easier for him and Phyllis in California rather than in their multistory townhouse. In Cambridge many friends were sad to see them go.

The mathematics department held a dinner in the Faculty Club the night before Raoul and Phyllis left Cambridge. After most other persons went home that evening, I gave Raoul a bottle of excellent Bordeaux. Occasionally I spoke with Raoul by telephone, a nice link between Cambridge and Carlsbad, California. It was wonderful to hear his voice and to get some news. During one of those conversations late in 2005, Raoul let me know that he had left that bottle of Bordeaux with his daughter Candace in Cambridge. She was bringing it to California the next day so he could share it during a small family reunion. At the time I did not realize how Raoul was telling me that his end was upon him. Shortly afterward I cried at the news.

Shing-Tung Yau reflects on Raoul Bott’s influence on him—mathematical, personal, and professional.

I first met Raoul Bott about forty years ago when he briefly visited Shing-Shen Chern and the Berkeley mathematics department. Bott was a great and famous mathematician then, while I was merely a graduate student. My teacher, Chern, was interested in his paper on the localization of Chern numbers for Kähler manifolds and went through it several times during a seminar. I was, of course, very impressed by Bott’s elegant theory. Little did I know that much later this theory would be developed into an extremely valuable tool for computations in geometry. I used this theory myself, along with my coauthors, Bong Lian and Kefeng Liu, in solving the mirror symmetry conjecture (independently solved by Givental), which was part of the broader theory of Calabi–Yau manifolds.

In 1971 there was a special program on foliations led by Bott at the Institute for Advanced Study in
Princeton. Since I was graduating from Berkeley, the IAS was an attractive place for me to go. When I applied to several universities, I got a few good offers. Although I could have gotten a higher salary elsewhere, Chern urged me to spend some time at the IAS, partly because of Bott’s program. So I went and enjoyed my year tremendously. At the IAS, I became interested in ways of constructing metrics with special curvature properties and applying them to solve questions in topology. For example, I thought about constructing metrics with positive scalar curvature to create obstructions for a nonabelian group action on a manifold. (I later wrote a paper with Lawson based on this idea.) By studying the wedge product of differential forms under a circle action, I found obstructions to the existence of a topological circle action on a manifold; these obstructions exist in the cohomology ring of the manifold. I showed my work on group actions to Raoul. He was pleased, and his encouragement was really important to a young man like me.

Later on, Raoul had many more interactions with me. After I proved the Calabi conjecture and the positive mass conjecture, the latter with Richard Schoen, he tried hard to convince me to come to Harvard, which offer I did not accept at first. During that process he invited my wife and me to his home for dinner several times. At the time, he was the master of Dunster House at Harvard. It was inspiring to see how much time and energy he invested in college undergraduates. I was truly grateful for his hospitality during my visits to Harvard. In return, I tried to entertain him well when he visited Beijing at the invitation of Chern in 1980. During that visit I proposed the Chinese name Bo Le to him. Bo Le was a famous personage in Chinese history reputed to have the ability to recognize excellent horses, those that can run a thousand miles. Apart from the aptness of its meaning, the name was appropriate phonetically also: “Bo” is the Chinese surname closest to “Bott,” and “Le” is about as close to “Raoul” as a Chinese character can sound. Raoul told me he liked this Chinese name.

The pivotal moment of my life was the time when I was having some trouble in the mathematics department at the University of California in San Diego. I needed help with a decision. Raoul was visiting Berkeley, and I flew to Oakland to have dinner with him. After dinner we had a long discussion about my future. A true statesman, he laid out the pros and cons of what I should do. I felt greatly relieved after talking with him and made the most important decision of my career, which was to come to Harvard, a decision that I have never regretted.

Of course, I learned much more from Raoul during my years at Harvard, no less in statesmanship than in mathematics—he was extraordinarily skilled in handling departmental affairs. I felt truly sad when he passed away. I gave a talk on his life’s work at a Journal of Differential Geometry conference. In preparation for the talk, I researched his contributions to mathematics. I was amazed to learn how much he had accomplished and how much he had done that I did not know about.

Raoul certainly ranks among the most influential mathematicians of the last century. His work was deep, his vision far reaching, and his impact durable. May his spirit always be with us!

Loring W. Tu


Making a Problem Your Own

The first time I met Raoul was at an orientation lunch for incoming graduate students in mathematics at the Harvard Faculty Club. Raoul gave us some advice on how to write a Ph.D. thesis. He said it was like doing a homework problem, but a harder problem. He ended by saying, “Make the problem your own.” It puzzled me what it meant to “make a problem my own,” but I was too intimidated to ask. I thought it was one of those things, like the taste of a certain fruit, that is impossible to explain except to those who have experienced it themselves.

A few years later, when I was an assistant professor at the University of Michigan, my Ph.D. thesis advisor, Phil Griffiths, came to visit. I
picked him up from the airport and drove him to a restaurant. While in the car, we started talking about a mathematical problem. I became so engrossed that I lost all sense of time, place, and orientation. The next thing I knew, a policeman was handing me a ticket for driving the wrong way on a one-way street.

Griffiths advised me helpfully, “Go tell the judge that you were thinking about mathematics.” So I showed up in court to dispute the charge, and I did as Griffiths told me. The judge took a look at my driver’s license and said, “You live only one block away from this street. You have no excuse!” He upheld the fine of seventy-five dollars. At that moment, it dawned on me what Raoul had meant by “making a problem your own.” I think it meant to be so absorbed by the problem that you forget everything else—to be possessed, so to speak.

It has happened to me a few more times, missing a subway stop on my way to the airport or jumping out of bed at night with a solution. Each time I feel that I have finally made a mathematical problem my own.

Bott as a Lecturer

Bott’s lectures were legendary. He had a knack for explaining ideas in simple, easily understood terms, no matter how abstruse, complicated, or abstract the topic. His lectures were always clear and exciting. They were magical in that they gave you the feeling you had understood something, sometimes even when you had not. Not surprisingly, his lectures were popular and his courses heavily enrolled. His courses had impact beyond mathematics students at Harvard, for they were attended also by students and faculty from other departments and other universities. The physicist Cumrun Vafa cited Bott’s courses for changing his perception of modern mathematics and profoundly influencing his later studies [2, p. 277]. Likewise, Edward Witten credited Bott’s lectures with teaching him techniques of geometry and topology, such as Morse theory and equivariant cohomology, which have proven pivotal in his work on supersymmetry.

Bott always seemed glad to be in the classroom. His courses were a lot of fun. In every lecture there were spontaneous moments of laughter. This came about not through preparation and canned jokes but because of his innate sense of humor, unique perspective, colorful phrases, and superb delivery. In his hands, the construction of a spectral sequence could become entertaining. He always focused on the central idea and simple but illuminating examples.

Authority

One year Bott taught the second semester of complex analysis, and the textbook he chose was Lars Ahlfors’s *Complex Analysis*. At some point he departed from the book and gave a different definition. Now students often revere the textbook as the ultimate authority, so a hand shot up and a student blurted out, “But Ahlfors says this, not that!” Bott replied calmly, “Yes, but Bott says that.” As usual, Bott understood things his own way and was not about to faithfully follow any book. In fact, in topology courses he did not even follow his own books, because usually his understanding of the subject had evolved since the book appeared.
A Conscripted Lecture
One day in the early 1980s a poster appeared on the bulletin board of the Harvard mathematics department on the third floor of the Science Center. It looked just like any other announcement, but with a twist. On the top it said, “By popular demand, Professor Raoul Bott will give a lecture on ‘The Atiyah–Singer Index Theorem: What It Really Means’.” The date, time, and place of the lecture were all clearly spelled out. What was unusual about this poster was the presence of an asterisk next to Raoul Bott’s name and a footnote at the bottom: “*Please inform the speaker.”

A few minutes before the scheduled time on the appointed day, the room was packed. No one had the temerity to inform the speaker about the lecture, so we were all wondering if Raoul Bott was going to show up. At the appointed time, he showed up, made a few jokes, and then proceeded to deliver a wonderful lecture on the Atiyah–Bott fixed point theorem and the Atiyah–Singer index theorem, all in the allotted hour.

Finder’s Fee
Nowhere were Bott’s powers of persuasion more evident than at the seventy-fifth anniversary of the Institute for Advanced Study at Princeton in March 2005. On that occasion he gave a talk reminiscing about how the institute in the fifties changed his life and launched his career (Figure 37). A few days after the conference, Bob MacPherson, a professor at the institute, called him to say that a couple in the audience that day were so moved by Bott’s talk that they donated two million dollars to the institute. Bott recounted the story to me and added, “I should have asked for a finder’s fee.”

Liquors
Coming from a family of teetotalers, I knew nothing about alcohol as a graduate student. At one point I thought it would be good to repair my ignorance in this domain. Raoul had the look of a bon vivant who might be knowledgeable about such things. Just as some students might ask him for good references in topology, when I ran into him in the elevator one day I asked him, “Professor Bott, can you recommend some liquors to me?” He gave me a sly look sideways, and said, “Candy is dandy, but liquor is quicker!” before mentioning a few brands. To this day I remember the aphorism but not the brands of liquor he recommended.

Joint Books
When I first started working on the book Differential Forms in Algebraic Topology with Raoul, I was a graduate student. He thought that we made a great pair working together, because as a graduate student I would know first-hand the difficulties a student would encounter in learning the subject. I think Raoul did not anticipate that it would end up taking up so much of my time. In the end I was glad to have written the book with him. For me it was a form of apprenticeship, and I felt that I had learned a tremendous amount of mathematics from a master.

Raoul was pleased with the resulting book. Once in a lecture I attended, he mentioned some facts—I forget about what, maybe de Rham cohomology or spectral sequences—and told the audience that they could find them all in the “Bible”. There was a momentary perplexity among the audience, and then it transpired that Bott was referring to our joint book. For a devout Catholic like Bott to
compare our book to the Bible must have been the highest form of compliment.

Although we had projected a second volume, Raoul did not mention it after the completion of the first, possibly because he did not want to put me through the experience again. It was many years later that I brought it up. The book would be called *Elements of Equivariant Cohomology*. We worked on it for many years. My chief regret is that we did not finish it while he was alive, but I have hope that it will soon see the light of day.

While working on the books, Raoul often told me to be “generous with credit to others.” Human nature being what it is, we probably all have the tendency to overestimate our own contribution and, conversely, to underestimate that of others. These days, whenever my baser nature threatens to come to the fore, I remember this lesson from Raoul.

One reason we got along so well I think is that with my strict Confucian upbringing, in which every edict is serious, I found Raoul’s wit and irreverence refreshing. As for Raoul, he said that as he got older, he liked more and more the Confucian reverence for the aged.

**Personal Happiness**

Raoul had a playful streak that persisted throughout his life. He liked to tease everyone: his wife, children, friends, colleagues, and even students. His interaction with me was no exception.

His concern for me extended to my personal happiness. My time as a graduate student at Harvard overlapped with that of Nancy Hingston, a good friend of mine and a student of his of whom he thought highly. I remember at a conference, Raoul once put his arm around her shoulder and exclaimed to the public, “My finest student!” On the day that Nancy got married, Raoul said to me, “Loring, you missed your chance.”

**Dust Bunnies**

In my first year as an assistant professor at Michigan, I worked long distance with Raoul on the book *Differential Forms in Algebraic Topology*. That summer I returned to Harvard to facilitate our collaboration. At the time Raoul and his wife, Phyllis, were comasters of Dunster House, a Harvard undergraduate house with three hundred undergraduates. Too cheap to rent a place of my own, I asked Raoul if he had a guestroom for me in the Dunster House master’s residence. Bott readily agreed.

The guestroom was a room attached to the master’s residence but with a separate entrance. This way I had my privacy, but I could go into the master’s residence to use the kitchen and dining room. To afford Raoul and Phyllis their privacy, I normally did not do that except when they were away. The Botts by then had a house on Martha’s
Vineyard and would often spend a large part of the summer there. I worked with Raoul on occasional trips to the Vineyard or when he returned to Cambridge from time to time.

As comasters of Dunster House, Raoul and Phyllis often had to entertain on a large scale, holding receptions for students and parents, for example, and so Harvard provided them with live-in help, who were usually graduate students in fields other than mathematics. The live-in help lived upstairs from the Botts, so that summer I found myself living in the Dunster House master’s residence with three young women, the live-in help of the year.

The first time Raoul came back in the summer, he got very mad at the four of us; apparently we had been living in squalor (though not in sin). Pointing to dust bunnies everywhere, he said, “Look at this!” The three young women were not used to cleaning the house, because during the school year there was a cleaning staff from Harvard. As for me, at that point of my life I was oblivious to dust bunnies; they were simply invisible to me. It was strange that as in mathematics, where, after Raoul showed me his fixed-point theorems, I began to see fixed-point phenomena everywhere, in the same way, after Raoul pointed out those dust bunnies, I began to notice dust bunnies everywhere. After that, each time just before Raoul was to return to Cambridge, my three housemates and I would clean the master’s residence from top to bottom.

Book Contract
The dust-ball incident was one of only two times that I saw Raoul get mad. The other time had to do with the contract for our book. While working on the book, we circulated the manuscript to some colleagues and students for feedback. Possibly because of Raoul’s fame, the book was heavily courted by publishers. Both Walter Kaufmann-Bühler, the mathematics editor at Springer, and Klaus Peters, the editor at Birkhäuser, at the time an independent publisher, came to Harvard to lobby us for their book series. We chose Springer, not only because of its long history and excellent reputation for quality but in part because of the better royalty Springer offered.

After the book was published, Kaufmann-Bühler was quite happy, because as he told me, “The book was selling like hotcakes.” He passed away a few years later and was replaced by a succession of editors at Springer. At one point, one of the new editors sent me a letter, pleading difficult financial circumstances at Springer and asking Raoul and me to sign a new contract with a lower royalty rate.

For Raoul, I think the royalty was not an issue at all, but for me, a low-paid assistant professor at the time, it was much more significant. With the letter in hand, I walked into Raoul’s office, looking frantic. When Raoul saw me and read the letter, he got quite mad. He said, “They signed a contract. Tough luck.” He then picked up the phone and called the editor. In his usual authoritative voice, he told the editor firmly that we had no intention of renegotiating the contract. That was the end of it. Springer backed off and seems to have flourished.

Style
At a conference in Montreal in 2008, Michael Atiyah said that someday historians of mathematics may want to decipher joint papers to figure out who wrote what. In some cases this may be quite easy. Raoul was a consummate stylist. His writings were pithy. He had a colorful, inimitable way of expressing himself. People have often come up
to me to tell me how much they like our book. Sometimes, as if to prove that they have read it, they cite specific passages that they like best. Much to my chagrin, these are usually not the ones I wrote.

Sleeping in Another Woman’s Bed
Jane Kister was a young logician at the University of Oxford in the seventies. In the fall of 1978, just after marrying the topologist Jim Kister, Jane spent a sabbatical semester at MIT. At a reception at Harvard, Raoul put his arm around her and announced, “I’ve slept in this woman’s bed.” Jane’s face turned beet red. What happened was that Jane was also on sabbatical in the spring of 1977 and had rented her house in Oxford to the Botts. It was indeed true that Raoul had slept in Jane’s bed, though not simultaneously with her.

While visiting England in the early eighties, Raoul thought that he had also slept in Queen Elizabeth’s bed, but of course without the queen in it. In his Collected Papers he credited this experience with his sudden joint insight with Michael Atiyah into the relation between equivariant cohomology and the moment map [3, Volume 4, p. xiii]: “Possibly the night I had spent in the erstwhile bed of Queen Elizabeth had something to do with it!” According to a recent message from Atiyah, the queen was Victoria, not Elizabeth. Raoul had stayed with the Atiyahs in the Master’s Lodge at Trinity College, Cambridge, where Atiyah was then the master. In her time, Queen Victoria and her consort, Prince Albert, did in fact stay as guests at Trinity College, and the four-poster bed that they used became a guest bed.

Lecture Preparation
One year when I was at the University of Michigan, Raoul was invited to give a lecture in a prestigious series. During his visit to Ann Arbor, Raoul stayed with me in my one-bedroom apartment. The morning of the lecture, he was writing his lecture notes. After writing seven pages, he said, “That’s enough. I will not be able to cover more than five pages in an hour.” I have found this to be a useful rule of thumb: five to seven pages of handwritten notes are about right for an hour lecture on the blackboard. I learned more from Raoul’s leisurely but well-timed pace of five handwritten pages in an hour than from other people’s fifty slides, each densely packed with information.

Another Narrow Escape
Raoul’s life seemed to be blessed. He left his native Hungary/Slovakia before the Nazi invasion, survived near-drowning in an expedition organized by Stephen Smale, and visited India without a visa at a time when visas were required. In Ann Arbor he also had a narrow escape.

At the end of his visit to Ann Arbor, I drove him to the Detroit International Airport, twenty miles away, in my Ford Maverick. It was a used car that I had bought from a departing postdoc at the University of Michigan. Soon after I purchased the car, I noticed that it was leaking transmission fluid, but the rate of the leak was so slow—just one or two drops a day—that it did not seem worthwhile to replace the entire transmission. On the highway as we were heading towards the airport, the car started smoking under the hood. We were alarmed, but Raoul had a plane to catch and the airport was not so far away, so I continued driving at full speed.

Just as we arrived at the airport, dense white smoke billowed from under the hood and the car went dead. It looked like it could explode. Raoul hurriedly ran to his flight, and I jumped out of the car. After his return to Boston he called me to make sure that I was still alive.
The Toaster Incident at Dunster House
Raoul navigated the perils of academic politics with consummate skill. He and Phyllis were comasters of Dunster House for six years. After they stepped down, another professor was appointed as the master. To distinguish him from Raoul, I will call him the new master. The new master was a very nice man, but his term was marked by controversy. I will give one example. It stemmed from a toaster oven.

Some Jewish students did not want to eat the food in the dining hall for reasons of keeping kosher. They asked the new master for a toaster oven so that they could heat up their own kosher food. The new master bought a toaster oven for them. One of the tutors (academic advisors) at Dunster House, an activist with strong principles, wrote a letter to the student paper, the Harvard Crimson, criticizing the use of house funds to buy the toaster oven, because in his view this was an act of favoritism towards one particular religion, akin to a violation of the separation of church and state, a founding principle of our republic.

The new master fired this tutor. More letters followed in the Crimson. It was no longer about the toaster oven, but about the new master’s leadership. Other tutors wrote letters, accusing the master of autocracy and partiality, of favoring some tutors over others. There were calls for the master’s ouster. Students organized demonstrations in Harvard Yard supporting the fired tutor. Professor Edmund Lin, a former chair of the Department of Molecular Genetics at Harvard Medical School and a member of the Senior Common Room of Dunster House, wrote a letter to President Rudenstein of Harvard, calling for the master’s resignation. Only at Harvard could there be a raging debate about constitutional principles arising from a toaster oven. This was when the new master’s five-year term was up for renewal. President Rudenstein asked to meet with Raoul, evidently because he valued Raoul’s judgment. Knowing that I was a close friend of Edmund Lin, Raoul asked me if I knew what was going on. I did, not only because of my friendship with Edmund Lin but also because I read the Crimson every day. Raoul did not read the Crimson.

When I explained the incident to Raoul, his immediate reaction was “An activist troublemaker? You should never fire someone like that. If you do, there is no end to the trouble. You should give him tenure!” Raoul had a very good nose for staying out of trouble. Of course, this did not mean that he would give every activist tenure. It just meant that in this case the stakes were not high enough to fire the tutor. Raoul then said pensively, “Ed Lin was always so quiet when I was the master. He must have thought that I was doing a good job.”

It so happened that the new master was an ethnic Chinese from Indonesia, a resident tutor whom he particularly liked and was accused of being partial to was a Chinese-American, and the professor calling for the master’s ouster was a Chinese from China. Raoul turned to me and asked, “Is this one of those Chinese battles so inscrutable to us Westerners?”

I do not know what he said to President Rudenstein. Rudenstein renewed the contract of the new master. The controversy died down after the students graduated. Edmund Lin told me afterwards, “I am sure it was Raoul who saved the new master’s skin.”

Foreign Languages
Raoul had a wonderful self-deprecating sense of humor. He was a talented linguist. He spoke German, Hungarian, and Slovak fluently, not to mention English, of which he was a master. But there is a limit to the number of languages one can learn or need to learn. I like his experience with Italian. Before a conference in Italy, he bought a cassette course on Italian. Repeating the sentences on the cassette tape, he studied Italian for two weeks. When he got to Italy, he found that he had forgotten all the sentences except for one. He told me that the one sentence he could say in Italian was “Ascolti e ripeta,” which means “Listen and repeat.”
Nonmathematical Activities

In spite of his prodigious output in mathematics, Raoul found time to do other things. As comasters of Dunster House, Raoul and Phyllis actively participated in the life of the undergraduates, sharing meals with them, meeting with their parents, and Organizing and attending cultural activities in the house. Raoul played the piano well enough to give public performances. Ever the good sport, he took part in an undergraduate theater production, playing a Hungarian linguist in My Fair Lady. At one Halloween party, Raoul and Phyllis dressed up as a pirate king and a young maiden, but two students upstaged them by dressing up as Raoul and Phyllis Bott! The male student sported a big beard and was chock-full of gray hair, and to top it off, he was carrying Raoul’s signature briefcase (Figure 15).

An avid swimmer and a regular on the clothing-optional beach of Martha’s Vineyard, Raoul earned himself the sobriquet “The Mayor of Lucy Vincent Beach”. He played tennis and bicycled to work. Once when I visited his home, he showed me with great pride some kitchen renovation, saying that he did it all with a router.

Material Enjoyment

From Raoul, I learned that a lifelong dedication to intellectual pursuits is not incompatible with enjoyment of material things.

Raoul bought a beautiful house on Martha’s Vineyard. Although the house was not right on the water, it was surrounded by an expanse of wild vegetation and had an unobstructed view of the ocean. There was even a brook on the property. Since most of the houses there were hidden in dense foliage, Raoul’s house had a view of nature with no other sign of human habitation. One day another house rose up, towering above the canopy of trees in full view from Raoul’s window, the only house visible in otherwise pristine nature. Raoul said it stuck out like a sore thumb, but he was philosophical about it. After all, his own house might be a sore thumb to the other owner.

While we were working on the book Differential Forms in Algebraic Topology, he teased me about the enormous amount of time I was spending on it, asking me if I thought that with the expected royalty it would come out to minimum wage. Then he said, “I want to buy a boat with it.” I thought he was joking, but years later he did buy a boat.

Raoul had a fascination with cars, and on one visit he proudly showed me his collection, a single 2-inch exact replica of a Jaguar that he said a student of his gave him. Finally, at the age of seventy-four, he bought a BMW, exemplifying another piece of advice he gave me: “Live it up!”

Mineral Collection

One of the pleasures of talking to Raoul was the unexpected insight that he often offered. Sometime in the early nineties, Raoul received in the mail a calendar of Steve and Clara Smale’s priceless collection of natural crystals, lovingly and beautifully photographed by Steve Smale himself. Raoul showed me the calendar in his office, and while admiring the breathtaking beauty of the...
minerals, he said, “What a way to avoid inheritance tax! You just have to slip a few of these to your children.” Of course, he did not mean it as an estate-planning tip; besides, I had neither a fortune nor children to benefit from this advice, but it was so characteristic of Raoul to have a unique perspective on everything.

Practical Advice
Fresh out of graduate school, I once visited Raoul on Martha’s Vineyard to work on our joint book. Sitting on a bench surveying his beautiful estate, he said to me, “Loring, buy land.” At the time I was too poor to buy anything, but time has borne out the wisdom of his advice, especially when the land is in a well-chosen location like Martha’s Vineyard.

One of Raoul’s observations on life has played a crucial role in my mental equilibrium. When he was at the Institute for Advanced Study at Princeton in 1949–51, he once had a conversation with John von Neumann, a fellow Hungarian who was at the time a professor at the institute. Von Neumann told Raoul that he had known only one great mathematician, David Hilbert, and that having been a prodigy in his youth, he never felt that he had lived up to his promise. Raoul wrote in [3, Volume 4, p. 270], “So you see, it is not difficult to be found wanting—one just needs an appropriate measuring rod.” If even von Neumann felt inadequate in his achievement in comparison with Hilbert’s, what chance for professional satisfaction do we ordinary mortals have? After Raoul recounted this incident to me, I resolved never to compare myself with anyone else, especially not with my friends and classmates who have achieved greatness.

I was fortunate to be in the job market during a brief window of opportunity when there were many jobs available, and so I actually had a few choices. Tufts had a fine reputation and excellent colleagues, but what clinched the deal was what Raoul said to me, “It will be nice to have you in the backyard.” The physical proximity made collaboration easier, and after moving to Tufts, I worked on a few more joint projects with him and had the pleasure of attending more of his courses.

Favorite Theorems
When I was writing “The life and works of Raoul Bott” in 2001, I interviewed Raoul and asked him to list three of his own theorems that he liked the best. He had trouble doing it, saying that it was like asking him which of his children he liked best. Eventually he came up with a list of the top five. The Atiyah–Bott fixed point theorem for elliptic complexes was not one of them.

After the memorial service for Raoul in January 2006, Michael Atiyah gave a compelling lecture on why the Atiyah–Bott fixed point theorem should have been one of Raoul’s top five favorite theorems. I think Raoul would have agreed. The list of five was a rather artificial framework and should probably not be taken too literally. It was what came to Raoul’s mind on the spur of the moment, but he simply could not fit all of his favorite theorems in there. In the end, my article included another thirteen in addition to the top-five list.

The Wolf Prize
Raoul used to say that there were two kinds of mathematicians, smart ones and dumb ones. The smart ones were people like Michael Atiyah and Jean-Pierre Serre, who understood new ideas quickly. He classified himself as a dumb mathematician, because understanding came to him slowly. This may be so, but his understanding was profound, as his corpus of many beautiful and deep theorems attests. If he did not understand something, he had no hesitation in saying so. When he was awarded the Wolf Prize, he told me that he...
was in very good company, because he was sharing the prize with Serre.

One of them had to give a speech in the Knesset, the Israeli parliament. According to Raoul, Serre wanted him to give the speech, because Serre thought that Raoul “had a better stage presence” and that Raoul “looked more like a mathematician.” But how to explain to the Israeli lawmakers the research for which they were being awarded the prize? This is the usual conundrum of pure mathematicians called upon to explain their work. Serre came up with a gem that Bott incorporated into his speech:

> Mr. President of the State, Mr. Speaker of the Knesset, Mr. Minister of Education, Members of the Diplomatic Corps, Dear Colleagues and Guests:

> It is a great honor for me to rise in this beautiful chamber and in so distinguished a company to accept the Wolf Prize in Mathematics on behalf of Jean-Pierre Serre and myself.

> Thank you.

> In our field alone the previous winners of this Prize include both heroes of our youth and cherished friends. And if we look beyond, well, who would not be delighted—as well as humbled—to join a list that, so to speak, starts with Marc Chagall!

> My first words of thanks here are in tribute to Ricardo and Francisca Wolf for setting up a foundation so much in tune with the most essential need of our ever-shrinking planet. The universality of their purpose speaks for itself:

> “To promote science and art for the benefit of mankind.”

> And how inspired of them to see the commonality of art and science, and to include mathematics, where these two spheres of endeavor are well nigh indistinguishable, in their generous bequest.

> But we feel doubly honored that a small and relatively new country, with so many pressing and highly nontrivial—as we say in our mathematical jargon—problems on its agenda, nevertheless finds time to bestow this award at its highest level. This act alone is a moving tribute to the life of the spirit in a world mostly concerned with more mundane things.

> Unfortunately, the very term “Mathematics” strikes terror in most mortal hearts, and so it is possibly appropriate here to put our subject into some sort of perspective. And I can think of no better way of doing this than to divulge to you just how my junior, but much wiser, colleague Jean-Pierre Serre cajoled me into being the one to deliver this acceptance speech. “For if I were to give the speech,” he argued, “then all I would say is that while the other sciences search for the rules that God has chosen for this Universe, we mathematicians search for the rules that even God has to obey.” And I certainly couldn’t let him get away with that!

> But, after this little tongue in cheek, my time is definitely up!

> Still, please permit me two more words of thanks. The first is to the committee that had a long enough memory to settle on us from amongst so large an array of worthy and younger candidates. And our final thank you is to our families and especially our wives, who for a lifetime have put up with our absent-minded ways and have been our anchors in the real world.

**Final Years**

After Phyllis became partially disabled following an operation, the Botts moved to California in the fall of 2004, where the year-round good weather permitted Phyllis more opportunities for outdoor mobility in a wheelchair. In [5] I mentioned some of the coincidences in Raoul’s life and my own in terms of the places where we ended up—McGill, Princeton, Harvard, Michigan—wherever he went, I followed a few decades later, if only in the vicinity sometimes. The final coincidence was that the town the Botts moved to, Carlsbad, California, was
only twenty-five miles from my parents’ house! So it was easy for me to continue to visit the Botts.

Soon after their move, Raoul was diagnosed with lung cancer. In spite of the poor prognosis, he was his usual cheerful self. He explained the principle of chemotherapy to me this way: “It tries to kill the cancer faster than it kills you.” He faced the prospect of death with equanimity. When I asked him if he would be returning to Massachusetts at some point, he pointed to the ground and said, “I am going in here.”

It has often been said that mathematics is a young person’s game. Raoul’s life is a particularly inspiring counterexample. I saw him three weeks before he passed away. I had been working on a problem with him on the volume of a symplectic quotient. He was in top form mentally. He explained to me a new way of looking at the problem that greatly simplified it. I cried, “This is so simple!” He said, “That’s the way I like it.”

At the age of eighty-two, battling cancer, he was still trying to understand integration on a symplectic quotient. There was a paper of Victor Guillemin and Jaap Kalkman on the subject, but he wanted to understand it in his own way. Clearly, his motivation was not any external reward, like an NSF grant or more honors. He simply wanted to understand. He was a true mathematician.

His life showed us what is humanly possible. He continued to make beautiful discoveries and publish important papers to the very end.

Royal Society

In the final year of his life, Bott was inducted into the Royal Society. The Royal Society dates back to 1660 and is a roster of luminaries in the history of science. Each new fellow signs in a book that has the signatures of all former and current fellows. For health reasons, Bott was not able to travel to London for the signing, but Michael Atiyah, a former president of the Royal Society, brought to California the actual page from the book Bott was to sign. For good measure, Atiyah also brought Raoul a scanned and bound copy of the preceding pages. An induction ceremony was held at the Kavli Institute for Theoretical Physics, University of California, Santa Barbara, in October 2005.

When I visited Raoul in California a month later, he excitedly showed me pages from his copy of the Royal Society book, exclaiming, “Look at this! Christopher Wren! Isaac Newton! George Stokes! Lord Kelvin!” For a man of science, this may be the ultimate good company.

3Raoul was buried in the Chilmark cemetery on his beloved Martha’s Vineyard, so he did return to Massachusetts after all.

References

Figure 37. Speaking at the 75th Anniversary of the Institute at Princeton, March 2005.

Figure 38. Induction into the Royal Society, ceremony at UC Santa Barbara, October 2005.


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