Competition, Financial Discipline and Growth*

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1 Introduction

The interaction between domestic competition and economic growth of a country or region is a topic of intense policy debate. For example, in a highly-publicized book, Michael Porter (1990) strongly argues that there exists a positive causal relation between competition and growth, since competition at home forces firms to innovate and to be efficient.\(^1\) This "Darwinian" view of competition is supported by recent empirical evidence (e.g. Nickell (1996) or Blundell et al. (1995)) pointing at a positive correlation between competition (measured either by the number of competitors in the same industry or by the inverse of a market share or profitability index) and productivity growth within a firm or industry.

In contrast, the theoretical literature on competition and growth has been pretty much one-sided until recently, stressing mainly the Schumpeterian view that it is the existence of future monopoly rents that induces firms to innovate and thereby the economy to grow. For example, Aghion and Howitt (1992) formalize this very idea in an endogenous growth model. Caballero and Jaffe (1993) obtain a similar effect when competition raises the elasticity of substitution between goods, thereby reducing monopoly rents and also accelerating creative destruction. And Grossman and Helpman (1991) again shows that competition hurts research and development and growth when it facilitates imitation.

How can we reconcile the Darwinian view with the Schumpeterian literature on technological change and growth? A first approach is to modify the technological assumptions generally made in the existing quality-ladder models. For example, Aghion, Harris and Vickers (1995) consider the case where technological progress by leaders and followers in an industry takes place step-by-step rather than through automatic leapfrogging of technological leaders.

\(^1\)In doing so, Porter attacks those who think that promoting national champions is the best way for a country to be competitive. Such a position is of course put forward by firms which are dominant on their domestic market, but is otherwise probably the minority position in policy debates on competition and industrial policies today.
by their rivals, more intense product-market competition between firms with
neck-to-neck technologies will increase each firm's incentive to acquire or
increase its technological lead over the other firms; hence the possibility of
a positive correlation between competition and productivity growth. In an-
other variation, Aghion and Howitt (1996) decompose R&D activities into
research (leading to the discovery of new fundamental paradigms or product
lines) and development (aimed at exploiting the new paradigm and filing up
the new product lines). In this case, more competition between new and
old product lines, parameterized by increased substitutability between them,
will induce developers to switch from old to new lines more rapidly, thereby
inducing a higher level of research and growth.

A second approach, which is the one we emphasize in this paper, is to
introduce agency considerations, and analyze the incentive effects of compe-
tition on technological adoption by non-profit maximizing managers. One
key idea of the paper is that by reducing the amount of slack a manager can
afford while keeping his firm alive, competition, combined with the threat of
liquidation, acts as a disciplinary device which fosters technology adoption
and therefore growth.

These two approaches that extend the basic Schumpeterian paradigm
can be seen as "complementary"; however, they deliver different policy rec-
ommendations. Whilst the former approach would call for selective R&D
subsidies, e.g. favoring the technological "followers" in various industries
and/or inducing higher mobility of developers across industries, the agency
approach developed in this paper will tend to argue against any form of "(un-
monitored) subsidies" to the extent that these might have the perverse effect
of increasing managerial slack and thereby slowing down technological adop-
tion. Our hope is that further empirical work aimed at testing these various
approaches will allow econometricians not only to confront Schumpeterian
growth theory with evidence, but also to sharpen our understanding of how
competition affects growth.²

²Preliminary evidence of the fact that owner control and product market competition
are substitutes as engines to growth has been recently provided by Nickell et al. (1996).
handful of theoretical contributions. This literature focuses on partial equilibrium and looks at a single innovation or technological adoption decision. Hart (1983) and Scharfstein (1988) show that the pressure of competition by profit-maximizing firms may or may not induce other firms to reduce slack depending on managers’ responsiveness to monetary incentives. In a recent paper, Schmidt (1997) extends the analysis and highlights two opposing effects of competition on cost-reduction incentives by a managerial firm. The owner of the firm optimally sets the manager’s financial incentive scheme given the level of competition. Competition induces the manager to engage in cost reduction activities to avoid bankruptcy, whose threat rises with the degree of competition. On the other hand, competition deters the profit-maximizing owner from pushing for cost reduction, the cost of which is fixed whereas its (marginal cost) benefit depends positively on the expected market share of the firm. While the Schumpeterian argument works for the profit-maximizing owner, the Darwinian argument has force for the manager who wants to avoid bankruptcy. Once again, Schmidt finds an ambiguous relation between competition and slack, cost reduction being maximized for an intermediate degree of competition.

The main purpose of this paper is to introduce agency considerations of this type in a growth context and more specifically to analyze the incentive effects of product market competition and financial market discipline on steady-state growth in a dynamic model with non profit-maximizing firms. Section 2 of the paper develops a general equilibrium model of technological adoption, where intermediate producers have to incur a fixed cost to adopt the latest, that is, lowest marginal cost, production technology. In Sections 3 and 4, we show that, for profit-maximizing firms, the Schumpeterian argument works and competition deters adoption and growth. Instead, if the economy is populated by firms controlled by “conservative” entrepreneurs, whose sole desire is to delay adoption of new technologies while keeping the firm alive (i.e. avoiding liquidation), competition acts as a discipline device which fosters technology adoption and growth. “Industrial policy”, that is adoption subsidies, are counterproductive in that case: namely, they allow conservative entrepreneurs to “buy time” since subsidies allow them to break
even with higher slack. On the other hand, competition induces a "virtuous circle" of adoption, since the adoption by one conservative entrepreneur puts pressure on its competitors, in turn forcing them to adopt earlier and more frequently too. This multiplier effect is one among several that arise in our general equilibrium model of repeated innovations and make it richer than the one-shot models described above.³

Sections 2 and 3 present the model, and detail our assumptions about firms' behavior regarding technological adoption. Section 4 analyses the Schumpeterian and Darwinian arguments, highlighting in turn a steady-state with only profit-maximizing firms, then a steady-state with only conservative firms, then a steady-state where both types of firms coexist. Sections 5 and 6 investigate how the existence of financial markets affects the importance of the Darwinian argument. Specifically, we allow for the existence of credit markets, where conservative entrepreneurs can borrow and face the threat of liquidation if they do not repay their debt. Credit markets are here explicit, and function endogenously in contrast with Schmidt (1997), where bankruptcy is an exogenous reduced form. Being saddled with debt forces entrepreneurs to reduce slack, since more frequent adoption becomes necessary for survival. The idea that debt commits managers to pay out cash and thereby reduces slack has been put forward forcefully by Jensen (1986) as the "free cash flow theory of debt" (see also Hart (1995) and Hart and Moore (1995)).

In Section 5, we show that when outstanding debt claims cannot be renegotiated or diluted ex post, and in the absence of uncertainty on the result of adoptions, in the steady-state conservative entrepreneurs will have accumulated just enough debt to force them to behave exactly as profit-maximizers. They first maximize debt accumulation for it allows them to delay adoption in the transition period prior to achieving the steady-state adoption regime. Opening up credit markets thus necessarily has a nonmonotonic effect on growth, being at first negative, while being positive in the steady state. Note

³For the sake of completeness, we also analyze the case of "technology addicted" entrepreneurs, who are biased in favor of innovations beyond what is profit-maximizing. Once again, competition reduces slack here, but this means less frequent innovation and slower growth.
that our approach of the contribution of financial markets to growth differ from the existing literature (e.g. King and Levine (1993)), where financial markets enhance growth simply by offering better screening of investment projects, and thereby reducing the cost of capital. Instead, our paper introduces the concerns of the recent corporate governance literature in the Schumpeterian growth context. In the extreme case of Section 5, all the discipline is performed by credit markets, and product market competition has the Schumpeterian flavor.

In Section 6, we introduce uncertainty on the profitability of adoptions, which limits the power of debt as a discipline device on non-profit maximizing managers. In contrast to Section 5, credit markets do not eliminate all slack, and product market competition has the potential again to reduce slack and thereby speed up technological adoptions. We show however that it crowds out at least partially the positive contribution of financial discipline on adoption and growth. It may even be that this indirect effect of product market competition dominates its direct pro-growth effect, and restores the original Schumpeterian trade-off between competition and growth.

2 Basic Model

2.1 Production and consumption

There are three kinds of tradeable goods in the economy: labor, a consumption (or "final") good, and a continuum of intermediate inputs.

There is a continuum of infinitely-lived individuals, with identical additive preferences over lifetime consumption, and the same rate of time preference \( r > 0 \). Since the marginal utility of consumption is constant (equal to 1), \( r \) is also the rate of interest. Each individual is endowed with a one-unit flow of labor which he/she supplies inelastically at zero disutility cost. Let \( L \) denote the total mass of individuals; \( L \) is then also equal to the total flow of labor supply in the economy. As in Aghion-Howitt (1992), the assumption that utility is linear in consumption makes various specifications of the structure of individuals' portfolios essentially equivalent, by removing any motive
to use capital markets for risk-sharing.\footnote{When it comes to welfare analysis, we shall be more specific about firms’ ownership and assume that firms (in particular in the intermediate inputs sector) are equally owned by all individuals in the economy. This dispersed ownership structure entitles individuals to the whole flow of output (net of the production and innovation costs) and not just to the wage fraction of it.}

The consumption good is produced using a continuum of mass $N$ of intermediate goods, according to the following Dixit-Stiglitz technology:

$$y = \int_0^N A_i \cdot x_i^\alpha di, \quad 0 < \alpha < 1,$$

where $x_i$ denotes the amount of intermediate good $n$ used by the final good sector and $A_i$ is a productivity parameter which measures the quality of intermediate input $i$. In what follows, we shall take the consumption good as the numeraire.

Assuming that each variety of intermediate good is monopolistically produced and that the final good sector is competitive, the inverse demand curve faced by the producer of intermediate input $i$ is simply:

$$p_i(x_i) = A_i \cdot \alpha x_i^{\alpha - 1}.$$

We assume that the set of available intermediate goods ($N$) is exogenously given. In this model growth will arise solely because of the adoption of more efficient technologies by intermediate producers. These producers face two types of decisions: (a) an output decision at each point in time, given their available technology; (b) a decision of when to buy a new technology, i.e. the latest and best available one on the market.

\section{Intermediate production decisions}

Intermediate inputs are produced using labor according to a one-to-one technology. An intermediate producer with vintage $\tau$ at date $t \geq \tau$ will choose
output \( x = x_{t, \tau} \) to maximize \( \{ A_t \alpha x^{\alpha-1} x - w_t x \} \),\(^5\) which yields

\[
l_{t, \tau} = x_{t, \tau} = \left( \frac{w_t}{\alpha^2 A_t} \right)^{\frac{1}{\alpha-1}} \tag{2}\]

In steady-state where the leading-edge technology \( A_t \), and consequently the wage rate and also all other costs born by firms grow at the same rate \( g \), the flow demand for labor by an intermediate firm of vintage \( \tau \) at date \( t \) is equal to:\(^6\)

\[
l_{t, \tau} = x_{t, \tau} = \left( \frac{w}{\alpha^2} \right)^{\frac{1}{\alpha-1}} \cdot e^{\frac{g(t-\tau)}{\alpha-1}}\]

where \( w \) is the growth adjusted wage rate and \( (t - \tau) \) is the age of the firm’s vintage. Labor demand thus decreases exponentially with age. Assume that intermediate producers have to bear a fixed operating cost (also in terms of labor):

\[
k_{t, \tau} = w_t k \cdot e^{\rho(t-\tau)} \tag{3}\]

with \( \rho \geq \gamma \) accounting for obsolescence. (Think for example of \( k_{t, \tau} \) as maintenance costs.) Then the net profit flow of a firm of vintage \( \tau \) at date \( t \) is simply equal to:

\[
\pi_{t, \tau} = (\pi(w)) e^{-\frac{g(t-\tau)}{\alpha-1}} - \omega k e^{\rho(t-\tau)} e^{\gamma t} = \psi(w, g, \omega) e^{\gamma t} \tag{4}\]

where \( u = t - \tau \) is the age of the firm’s vintage, and \( \pi_{t, \tau} \) and the profit parameter

\[
\pi(w) = \left( \frac{1 - \alpha}{\alpha} \right) \cdot \left( \alpha^2 \right)^{\frac{1}{\alpha-1}} \cdot \omega^{-\frac{\alpha}{\alpha-1}} \equiv \sigma \cdot \omega^{-\frac{\alpha}{\alpha-1}} \tag{5}\]

are decreasing in the growth adjusted wage rate \( w \). Note that for \( w \) sufficiently low, we have: (i) \( \psi(w, g, 0) > 0 \); (ii) \( \psi_u < 0 \); and (iii) \( \psi(w, g, u) < 0 \) for \( u \) sufficiently large. These properties of \( \psi \) will play a crucial role when analyzing the adoption decisions of intermediate firms.

\(^5\)Since (a) only the adoption of new technologies and not the (static) production decisions may impose a disutility (of effort) to intermediate firms’ managers; and (b) a higher level of static profits allows any kind of intermediate firm to move closer to its most preferred technological adoption decision (see section 2 hereafter), static profit maximization is indeed the relevant objective function on the basis of which instantaneous (flow) production decisions will be made at any instant by all kinds of intermediate firms (no matter their utility or disutility for technological adoptions).

\(^6\)Without loss of generality, we normalize at 1 the productivity parameter at date 0: \( A_0 \equiv 1 \).
2.3  Steady-state growth rate

In order to endogenize growth, we introduce spillovers in technological adoption. Specifically, we follow Caballero-Jaffe (1993) and Aghion-Howitt (1996) and assume that the leading-edge \( A_t \) at date \( t \) grows at a rate proportional to the density of firms that "innovate" at that date, that is adopt the same new vintage \( \tau \). This spillover assumption is the multisector equivalent of the research spillover assumption made in Aghion-Howitt (1992) where, each time a firm innovates, it brings the leading-edge from \( A_{t-1} \) to \( A_t = \gamma A_{t-1}, \gamma > 1 \), i.e. it brings the current state of knowledge from \( A_{t-1} \) to \( \gamma A_{t-1} \). Here also, every (unit of) innovation moves the log of leading-edge technology \( A_t^{\max} \) up by some \( \ln \gamma \). If \( D_t \) denotes the density of firms that innovate at date \( t \), then \( \log A_t^{\max} \) increases by the amount \( D_t \cdot \ln \gamma \).\(^7\) In what follows, we shall assume \( \ln \gamma = 1 \) for notational simplicity and we shall concentrate most of our analysis on steady-states where intermediate firms are uniformly distributed in terms of their respective technological vintages at any date \( t \), and where all intermediate firms adopt the newest technology every \( T \) periods, where \( T > 0 \). In such steady-states, we have:

\[
g = \frac{\dot{A}_t}{A_t} = \frac{N}{T}/N \equiv \frac{1}{T}. \tag{6}
\]

Finally calling \( w_t f \) the fixed (labor) cost of buying vintage \( \tau \), stationarity will require:

\[
w_t f = w f e^{q \tau}. \tag{7}
\]

2.4  Labor market equilibrium

Computing the stationary equilibrium now only requires the computation of \( w \) and of \( T \) since \( g = 1/T \). Given that intermediate firms are uniformly distributed across vintages and that there are thus \( \frac{N}{T} \) firms per vintage in steady-state, the total labor demand at any date \( t \) is given by:

\[
I^d = \int_{t-T}^{t} t \cdot \frac{N}{T} d\tau + \int_{0}^{T} k e^{\rho u} \frac{N}{T} du + \int_{0}^{N} \frac{N}{T}
\]

\(^7\)A natural interpretation is that there is an infinite list of techniques, each embodying a productivity parameter whose log equals that technique’s number on the list, and that innovations result in discovering these techniques one at a time, at a rate proportional to the flow of innovations \( D_t \).
We have moreover that:

\[
\int_{t-T}^{t} \frac{N}{T} d\tau = \left( \frac{\alpha^2}{w} \right)^{\frac{1}{1-\alpha}} \frac{N}{T} \cdot \frac{1 - \alpha}{g} [1 - e^{-\frac{\alpha T}{1-\alpha}}]
\]

Now, using the fact that \( g \cdot T = 1 \) and substituting for \( g \) into \( L^d \), we end up with the following simple labor market clearing equation:

\[
L = \frac{N}{T} \int_0^T ke^{\mu u} du + f \frac{N}{T} + Nsw^{-\frac{1}{1-\alpha}} \tag{L}
\]

where \( s = (1 - \alpha)\alpha^{\frac{2}{1-\alpha}} \cdot (1 - e^{-\frac{1}{1-\alpha}}) > 0 \). Our functional form assumptions imply that \("production labor" requirements are independent of \( T \). Instead, labor requirements for innovation go up when innovation is more prevalent \((N/T \) higher), but this drives down labor requirements for maintenance. As will be seen in section 4, (L) is not monotonic in \((w, T)\) space.

In order to close the model, we now have to specify how firms take their innovation or adoption decisions.

### 3 Entrepreneurial Behavior and the Adoption of New Technologies

Departing from the existing growth literature, we do not restrict our analysis to the case where \"innovating\" firms are pure profit-maximizers. We also consider the case where intermediate producers (i.e. the intermediate firms managers) are \"conservative\" in the sense that they incur a \textit{private cost} from adopting new technologies,\footnote{Such a cost may refer to the required effort by an intermediate firm’s manager to get properly trained for the next technology or to his/her cost of training workers and/or reorganising the firm. In practice, the objective function of intermediate firms’ managers is likely to account for a convex combination of the three \"extreme\" types of behaviour analysed in this and the next sections, namely profit-maximization, conservative management and technological addiction.} and we also briefly refer to the polar case where intermediate entrepreneurs are \"technology addicts\" whose primary objective is to establish a reputation as \textit{pioneers} of new ideas. We shall be mostly interested by the \"conservative entrepreneurs\" case, not least because it generates comparative statics results (in particular regarding the effect of competition and/or debt-financing on technological change and growth) which most depart from existing Schumpeterian models.
There is more than one reason why non-profit maximizing firms (or managers) can survive in a capitalistic environment. A first reason has to do with the competitive environment itself: the less competition among domestic intermediate producers or between them and the “outside world”, the bigger the scope for entrepreneurial slack in (small) firms whose owners are not primarily interested in profit-maximization. 9 A second reason is the existence of an agency problem between intermediate producers and their outside financiers: an important assumption underlying our analysis, in Sections 5 and 6 on debt-financing, is that the adoption dates are non-contractible, that is cannot be included as an enforceable clause in financial contracts between intermediate firms and their outside financiers. (See footnote 11 for an elaboration of this idea.)

We shall restrict ourselves until section 5 to the case of self-financed intermediate firms 10 and analyze how the steady-state equilibria \((w, T)\), respectively when these firms are profit-maximizing or not, vary as we perturb the basic parameters of the model including those reflecting the degree of market competition in the economy.

3.1 Profit-maximizing firm

Given that the sunk cost of adopting new technologies is positive and grows at the economy-wide rate \(g\), a profit-maximizing firm will only innovate at finite intervals of time \(T_1, T_1 + T_2, ..., T_1 + T_2 + ... + T_k, \ldots\) etc. The optimal choice of \(T_j(j \geq 1)\) will maximize the net present value of the firm’s profits flows, equal to:

\[
(W - w_f) + \int_0^{T_1} \psi(w, g, u)e^{-(r-g)u} \cdot du
+ e^{-(r-g)T_1}[-w_f + \int_0^{T_2} \psi(w, g, u)e^{-(r-g)u} \cdot du]
+ e^{-(r-g)(T_1+T_2)}[-w_f + \int_0^{T_3} \psi(w, g, u)e^{-(r-g)u} \cdot du]
+ \ldots \ldots
\]

9 This entrepreneurial view of firms is the one we favor for interpreting the analysis and results in this and the next section. However, our model can accommodate alternative interpretations as we argue below.

10 The term “self-financed” refers to the case where intermediate firms have enough initial endowments to finance their first innovation and then finance the subsequent innovations out of retained earnings.
where $W$ denotes the intermediate firm's initial wealth endowment at date 0. It is straightforward to show that the optimal adoption decision $T^*_j (j \geq 1)$ satisfies:

$$T^*_1 = T^*_2 = \ldots = T^*$$

where:

$$T^* = \arg \max \left[ -\frac{u f + \int_0^T \psi(w, g, u) e^{-(r-g)u} \cdot du}{1 - e^{-(r-g)T}} \right] \quad (P)$$

### 3.2 Deviation from profit maximization

Whilst it is reasonable to assume that owners-managers of firms with low levels of outside finance will essentially act as pure profit-maximizers (see Jensen-Meckling (1976)), on the other hand in firms with high levels of outside finance (owners)-managers will mostly worry about preserving the private benefits of remaining in business, knowing in advance that the monetary returns from their efforts will mostly accrue to the outside financiers. Then, if the private benefits of remaining solvent are sufficiently large, a deterioration of profit conditions (e.g. as a result of an increase in product market competition) will induce the (owners)-managers to work harder in order to survive such a deterioration. Having looked at the pure profit-maximization case in the previous subsection, we now want to consider the polar case of firms with implicitly high levels of outside finance but also high private benefits of getting financed and remaining solvent, and where therefore the managers are mainly concerned with preserving their private benefits of control over the company while at the same time minimizing innovative effort.\(^\dagger\)

\(^\dagger\)In a companion paper (see Aghion-Dewatripont-Rey (1997b)), we develop a static partial equilibrium model of firms with agency problems and outside finance. In order to get financed, the owner-manager of such a firm can commit to higher effort through making a verifiable (but costly) investment which reduces the marginal cost of effort. We allow for optimal contracting and then show that as the amount of outside finance $I$ increases away from zero, one moves from a region of $I$'s in which a deterioration in profitability conditions reduces effort incentives (we call it the *shirking region*) into a region in which the same kind of deterioration induces the (owner)-manager to commit to higher effort through increasing the amount of verifiable investment in order to get the required outside financing (we call it the *bonding region*). Now, because in the present paper we want to embed firms' decision-making into an infinite-horizon general equilibrium model of innovations and growth, we choose to concentrate on two extreme cases: first, on the pure profit-maximizing case in which initial outside financing is implicitly equal to zero and therefore the owner-manager is fully residual claimant on the monetary revenues generated by his/her innovative activities (see section 3.1 above), and second on the polar case in which outside financing is so high that the (owner)-manager is only concerned about
We thus consider firms in which the owner-manager has the following objective function:

\[ U_0 = \int_0^{\infty} B_t \cdot e^{-\delta t} \cdot dt - \sum_{j \geq 1} C_j \cdot e^{-\delta (T_1 + \ldots + T_j)}, \]

where: \( T_j \) is the time interval between the \((j - 1)^{th}\) and the \(j^{th}\) technological adoption; \( C > 0 \) is the private cost of switching to a new technological vintage (training cost or non monetary cost of reorganizing the firm); \( B_t \) is the current private benefits of control at date \( t \), equal to \( B > 0 \) if the firm has financially survived up to time \( t \) and equal to zero otherwise; \( \delta \) is the subjective rate (not necessarily equal to \( r \)) at which intermediate firms' managers discount future private costs and benefits. Keeping the firm afloat thus means keeping at every point in time a positive net financial wealth. We shall refer to managers of such firms as “non-profit maximizing” or “conservative” managers, although this may not be an ideal terminology.

We shall concentrate our attention on the case where the private benefit \( B \) of remaining solvent is sufficiently large that remaining solvent is the primary objective of conservative managers, and where the rate of time preference \( \delta \) is also sufficiently high the these managers will never find it optimal to speed up the next innovation in order to delay the subsequent ones. More formally, one can establish:

**Proposition 0:** If \( W \) denotes the initial cash endowments of the firm at date zero, the optimal adoption decision of a non profit-maximizing manager with the above objective function is given, for \( B \) and \( \delta \) sufficiently large, by:

\[
T_1 = \tilde{T}(W) \\
T_k = \tilde{T} \equiv \tilde{T}(0) \quad \text{for } k \geq 2,
\]

where \( \tilde{T}(W) \) is the highest solution to the equation:

\[
W + \int_0^T \psi(w, g, u) e^{-(r-g)u} du = w f e^{-(r-g)T}. \tag{F}
\]

preserving the private benefits of remaining solvent but at the lowest possible innovation cost. These two polar cases are, in reduced form, the extreme cases considered in our companion paper, which provides microfoundations for this paper.
Proof: See Appendix A.

In words, if the private benefits of control $B$ are sufficiently large and if conservative managers are sufficiently impatient, then they will always seek to delay as much as possible the next innovation subject to keeping the firm afloat. To see why such behavior is equivalent to the adoption decision $(T_i)_{i \geq 1}$ described in Proposition 0 above, consider an intermediate firm that enters the market with wealth $W_0 = wf$ and innovates at date $t = 0$ (so that its effective initial wealth in period 1 is $W = 0$) but does not innovate thereafter. Such a firm will have accumulated profits at date $t = T$ equal to
$$
\int_0^T \psi(w, g, u)e^{-(r-g)u} \cdot du.
$$
These accumulated profits are first increasing in $T$ (since $\psi(w, g, 0) > 0$, otherwise the market would not exist) and then decreasing (since $\psi(w, g, u) < 0$ for large $u$) and they become eventually negative for $T$ sufficiently large (this follows from the fact that the operating cost $k_{t,r}$ increases with age at a rate $g + \rho > r$). This has two consequences:

- First, it is not optimal for the firm to never innovate again, since by doing so the firm would eventually go bankrupt.

- Second, there exists a maximum date $\bar{T}$ at which the firm’s cumulated profits just cover the adoption cost at that date, $wf \cdot e^{-(r-g)\bar{T}}$; by construction $\bar{T}$ is the highest solution to equation $(F)$ with $W = 0$, and it will be the optimal adoption decision of a conservative firm.

Now, after innovating at time $T = \bar{T}$ the firm starts again with zero accumulated profit and thus faces the same maximization problem as at date $t = 0$. Then, the same argument as above shows that it is optimal for the firm to innovate every $\bar{T}$ periods.

To conclude our typology of patterns of intermediate firm behavior, let us briefly consider the opposite case of a “technology addict” who seeks to maximize the frequency of innovations subject to financial survival. Assume again that the firms enters with wealth $W_0 = wf$ and innovates at date 0 upon entering the market, so that its effective initial wealth in period 1 is $W = 0$. Then, the optimal adoption decision $\tilde{T}$ of a “technology addict” thereafter is the smallest solution to equation $(F)$ with $W = 0$.\(^{12}\)

\(^{12}\)Note that, with $W_0 > wf$ a conservative firm will be able to delay the adoption of its
4 Analysis of the Model

The goal of this section is to analyze the effect of competition policy and industrial policy in this model. In the spirit of our Dixit-Stiglitz specification, competition can be parameterized either by $\alpha$ (which measures the substitutability between intermediate goods) or by $N$, the number of intermediate input varieties (and thus producers). $N$ is exogenous in our model but a slightly less reduced form model would have it depend upon entry conditions: the closer to free entry in the intermediate input market, the smaller the gap between profit-maximizing and non profit-maximizing behavior.

Industrial policy is instead reflected in $f$, which can be lowered by adoption subsidies. While at first glance these should foster faster adoption and growth, in practice they often seem to yield mixed results, generating the fear one subsidizes investment that “would have happened anyway”.

4.1 An intuitive argument

Let us first look at the reaction of a single firm, thus keeping $g$ constant. For the profit-maximizing case, the first-order condition of (P) implies:

$$\int_0^{T^*} \left[ \psi(w, g, u) - \psi(w, g, T^*) \right] e^{-(r-g)u} du = wf.$$

The LHS of this equation is positive, increasing in $T^*$ (since $\psi_T < 0$) and decreasing in $w$. If an increase in $\alpha$ or $N$ drives $\psi(w, g, u)$ down then $T^*$ goes up, which is the usual Schumpeterian result: since $f$ is a fixed cost meant to reduce marginal cost, the incentive to spend it goes up with market share and down with competition. Similarly, an increase in $f$ raises $T^*$, which is the intuitive effect.

What about deviations from profit maximization? Locus (F) consists of the intersections of two curves that be drawn as in Figure 1.

Figure 1 here

first vintage beyond $\overline{T}$, $(T_1 = \overline{T}(W_0 - wf) > \overline{T})$ but it will still be the case that $T_j = \overline{T}$ for $j \geq 2$. Similarly, a “technology addict” with initial wealth $W_0 > wf$ will be able to speed up the first adoption but it will still be the case that $T_j = \overline{T}$ for $j \geq 2$. 

15
A cut in $f$ reduces $\tilde{T}$, but increases $\bar{T}$: the conservative manager can afford a longer period of losses before innovating since the cost of innovation is lower! By contrast, if more competition (e.g. due to an increase in $\alpha$ or $N$) reduces $\psi(w, g, u)$, it will raise $\tilde{T}$ but reduces $\bar{T}$: the conservative manager has to cut slack in order to survive, which means adopting faster.

### 4.2 General analysis

In fact, the insights of the previous subsection remain valid in general equilibrium, i.e. adding to (P) or (F) the condition $gT = 1$ and the labor market equilibrium (L). More precisely, it is shown in Appendix A.2 that for a non-empty set of parameter values there exists a unique equilibrium when all firms are either "conservative" or "technology addicted", and at least one equilibrium when all firms are profit-maximizers.\(^\text{13}\) We shall first consider the effects of increasing the degree of substitutability $\alpha$ between intermediate goods; we shall then move on to analyzing the effects of an increase in the number $N$ of intermediate firms; finally we shall briefly discuss other parametrizations of product market competition.

#### 4.2.1 Increasing the degree of substitutability between intermediate goods.

Here, we shall first concentrate on a simple case: the effect of an increase in $\alpha$ on the adoption policy $\bar{T}$ of a conservative manager, taking the number of intermediate firms as constant, say $N \equiv 1$. Now using the fact that, with Cobb-Douglas production functions in each intermediate sector the gross profit flow in a sector of vintage $\tau$ at date $t$ is equal to $\frac{1-\alpha}{\alpha}$ times the wage bill $w_{t} \cdot x_{t, \tau}$ in that sector; and using the labor market clearing condition (L) and the growth equation $g = \frac{1}{T}$, we can reexpress equation (F) (after elimination of $w$ on both sides of that equation) as:

\(^{13}\)We have not yet, at this point, been able to rule out multiple equilibria in the profit-maximizing case. For the sake of comparative static analysis, we chose to concentrate on the equilibrium with maximum frequency of innovations, to which the results from the above partial equilibrium analysis are shown to carry over.
\[ f e^{-\tau T + 1} = \int_0^T \left\{ \text{"appropriability effect"} \right. \\
\frac{1 - \alpha}{\alpha} \cdot \left[ \frac{1}{1 - e^{1/(1 - \alpha)}} \right] T \left[ L - \frac{f}{T} - \frac{e^{\tau T}}{\rho T} - \frac{1}{k} \right] \left( \text{"productivity effect"} \right) \\
\left. \frac{1}{1 - e^{1/(1 - \alpha)}} \right\} e^{-\tau T} \left( \text{"obsolescence effect"} \right) e^{\tau T} \left( \text{"obsolescence effect"} \right) \frac{e^{-\tau T}}{\rho T} e^{\tau T} \left( \text{"obsolescence effect"} \right) \frac{e^{-\tau T}}{\rho T} e^{\tau T} \left( \text{"obsolescence effect"} \right) \frac{e^{-\tau T}}{\rho T} \\
-ke^{pu} \left\{ e^{-(r-1/T)u} du \right. \\
\left. \frac{e^{-(r-1/T)u}}{\rho T} e^{\tau T} \left( \text{"obsolescence effect"} \right) \frac{e^{-\tau T}}{\rho T} e^{\tau T} \left( \text{"obsolescence effect"} \right) \frac{e^{-\tau T}}{\rho T} \right\} \\
\text{(F)} \]

The two sides of \((F)\), adoption cost and accumulated wealth, are described in Figure 2 below.

**Figure 2 here**

Wealth is less than cost for \( T = 0 \) and again as \( T \to \infty \). So the equilibrium, which is the maximal delay possible, occurs at a point like \( E \) where wealth is falling faster than cost. Thus an increase in the competitiveness parameter \( \alpha \) will reduce \( T \), and hence raise growth, if it shifts the wealth curve down.

As indicated in \((F)\), three effects of \( \alpha \) are at work on the steady-state level of accumulated wealth. The appropriability and obsolescence effects are both negative whereas the productivity effect is positive, but the appropriability effect dominates the productivity effect. Thus the overall of increasing \( \alpha \) is to reduce accumulated wealth. Now, contrary to the profit-maximizing case, the effect of a decreased wealth on conservative managers' behavior is to speed up innovations, i.e. to lower \( \bar{T} \), as shown by Figure 2. Hence:

**Proposition 1:** For \( B \) and \( \delta \) sufficiently large, more competition as measured by a higher degree of substitutability between intermediate goods, has the unambiguous effect of increasing growth in the non-profit maximizing case.

### 4.2.2 Increasing the number of intermediate products

To analyze the effects of competition parameters other than the substitutability parameter \( \alpha \), it appears convenient to depict the [general] equilibrium of the model using the following Figure 3 (defining \( w = w^{1-\alpha} \)).

**Figure 3 here**
Deriving the comparative statics results with respect to $N$ turns out to be somewhat tedious and is undertaken in appendix B, where we show that:

i. $(F)$ is convex in $T$, always positive and going to $+\infty$ for $T \to 0$ or $T \to +\infty$. Indeed, in these cases innovation costs and maintenance costs respectively go to infinity, which requires wages going to zero to avoid negative profits.

ii. $(P)$ is decreasing in $T$: as in the previous subsection, higher wages discourage production and thus innovation of a profit maximizing firm.

iii. $(L)$ is concave in $T$ and goes to $-\infty$ when $T$ goes to 0 or $+\infty$, when labor requirements for innovation and maintenance respectively go to infinity. Note that these requirements are independent of $w$, and may exceed $L$, in which case there is no positive $w$ that can ensure labor market equilibrium. Instead, for $L$ high enough, $(L)$ will intersect $(F)$ twice, defining $\tilde{T}$ and $\tilde{T}$. Moreover, when $(P)$ intersects $(L)$ at $T^*$, profits are positive, which means $w$ is lower than the value of $(F)$ at $T^*$. Consequently, $\tilde{T} < T^* < \tilde{T}$. Moreover $(P)$ intersects $(L)$ “from above”.

iv. One can rule out explosive growth, i.e. keep $\tilde{T} > \frac{1}{r}$, by maintaining $f$ high enough (adjusting $L$ if needed to maintain intersections between $(L)$ and $(F)$): this is intuitive, in that a higher $f$ reduces incentives to innovate and thus growth.

v. An increase in $N$ translates $(L)$ downwards and leaves $(F)$ and $(P)$ unchanged; a decrease in $f$ moves $(L)$ upwards and $(F)$ and $(P)$ downwards.

Together, these elements yield:

**Proposition 2:** For $B$ and $\delta$ sufficiently large, while more competition (a higher $N$) reduces growth and higher adoption subsidies (a lower $f$) raise
growth in the profit maximization and technological addiction case, the opposite is true in the non-profit maximizing case.

We thus confirm the simple intuitions of subsection 4.1. Note that Propositions 1 and 2 make no welfare statements: as we know from other endogenous growth models (e.g. Aghion and Howitt (1992)), $T^*$ could be higher or lower than the socially optimal $\tilde{T}$, and policy recommendations should depend on this level $\tilde{T}$ as well as on the assumed pattern of behavior of firm managers.

**Remark:** We chose to emphasize the effects on innovations and growth of increasing the number of and/or the degree of substitutability between intermediate products. However, one can think of other measures of competition.

For example, introducing the possibility of imitations: suppose that with flow probability $q$, a technological vintage can be “imitated” by the outside world which can then produce the same intermediate input at a marginal cost arbitrarily close to zero. Using the same representation of the equilibrium as for the above Proposition 2, namely that depicted in Figure 3, one can show that for $\alpha$ not too small an increase in the rate of imitations $q$ will also have a fostering effect on innovations and growth in the conservative case.\(^{14}\)

Another, related approach to competition is to assume the existence of a competitive fringe imposing a price ceiling ($\tilde{p} \cdot e^{qt}$ at date $t$) to intermediate good producers. The existence of such a ceiling will primarily affect the oldest vintages which are also those that set the highest price (see Section 2.2 above). The lower $\tilde{p}$, the sooner the intermediate firms with older vintages

\(^{14}\)More precisely, both the gross-profit terms embodied in $\psi(w, g, u)$ and the manufacturing demand for labor must be multiplied by $(1 - q)$. We then have:

\[
\frac{dw}{dq} \bigg|_{(L)} = -\frac{w(1 - \alpha)}{1 - q} \quad \text{and} \quad \frac{dw}{dq} \bigg|_{(F)} = -\frac{w}{1 - q}.
\]

In other words, both curves (L) and (F) in Figure 3 will be shifted upwards, although the (F)-curve will shift significantly more than the (L)-curve, thereby leading to a decrease in $\tilde{T}$, i.e. to more growth in equilibrium in the conservative case. Put differently, the fact that the equilibrium wage decreases as a result of more imitations will not prevent the accumulated wealth of intermediate firms from decreasing and therefore will force conservative managers to innovate more frequently in order to survive.
will have to innovate in order to financially survive. Note that contrary to what happens with an increase in the rate of imitations, lowering the price-ceiling \( \bar{p} \) will increase the equilibrium wage \( w \), the reason being that older firms which start being affected by the price ceiling will automatically increase their output and therefore their demand for manufacturing labor.

4.2.3 Coexistence between profit- and non-profit-maximizing firms

We now consider the intermediate case in which the industry consists of both profit-maximizing and conservative firms, in proportions \( \lambda \) and \( 1 - \lambda \) respectively. A steady-state equilibrium \( E_\lambda \) is then a quadruple \((T_p, T_c, g, w)\) such that:

\[
g = \frac{\lambda}{T_p} + \frac{1 - \lambda}{T_c} \tag{8}
\]

\[
T_p = \arg \max \left[ \frac{-w + \int_0^T \psi(w, g, u)e^{-(r-g)u} \cdot du}{1 - e^{-(r-g)T}} \right] \tag{9}
\]

\[
T_c = \min \left\{ T : \int_0^T \psi(w, g, u)e^{-(r-g)u} du = wfe^{-(r-g)T} \right\} \tag{10}
\]

\[
\frac{L}{N} = \frac{\lambda}{T_p} \int_{T_0}^t \left( \frac{w_t}{\alpha^2 A_t} \right)^{\alpha-1} d\tau + \frac{1 - \lambda}{T_c} \int_{T_0}^t \left( \frac{w_t}{\alpha^2 A_t} \right)^{\alpha-1} d\tau + \frac{\lambda}{T_p} \int_0^{T_p} ke^{\rho u} du + \frac{1 - \lambda}{T_c} \int_0^{T_c} ke^{\rho u} du + \left( \frac{\lambda}{T_p} + \frac{1 - \lambda}{T_c} \right) f. \tag{11}
\]

Equation (8) expresses the new average growth rate as equal to the aggregate flow of new innovations generated jointly by profit and non-profit maximizing firms; equations (9) and (10) are nothing but a reexpression of the former equations (P) and (F) determining the periodicity of innovations (T) respectively in the profit-maximizing and in the conservative case; finally equation (11) is the new labor market clearing condition which equates total labor supply to aggregate demand for manufacturing labor plus aggregate demand for maintenance labor plus aggregate research labor flow.

Although it is quite difficult to obtain analytical results for existence or on the sign of the cross-derivatives \( \frac{\partial^2 g}{\partial \delta \delta w} \) or \( \frac{\partial^2 g}{\partial \delta \delta N} \), we have performed a simulation exercise, taking the rate of imitation \( q \) as a measure of product market
competition, and with the following numerical values of the parameters.\footnote{The range for each parameter was derived by fixing the other parameters at their baseline values and then impose the condition that firms would make positive profits so that the product market actually exists. (Typically, \(w,r,k\), and/or \(f\) must not be too high.)}

\[
\begin{align*}
 r &= 0.05 \quad \text{Range: 0.01-0.075} \\
p &= 0.05 \quad \text{Fixed} \\
w &= 0.01 \quad \text{Range: 0.02-0.1} \\
a &= 0.5 \quad \text{Fixed (Equivalent to varying \(q\))} \\
L &= 5 \quad \text{Irrelevant as wage fixe} \\
f &= 3000 \quad \text{Range: 100-10000} \\
k &= 1 \quad k = 0.5 - 4
\end{align*}
\]

Our simulation of the system (8)-(11) yielded the following conclusions:

1. There exists a unique equilibrium for all \(\lambda\) between 0 and 1 over the whole range of numerical parameter values.

2. Increases in the fraction of profit-maximizing firms \(\lambda\) always increase growth.

3. Increases in the rate of imitation \(q\) always increase \(T_p\), i.e. always reduce the frequency of innovations by profit-maximizing firms. (Schumpeterian Effect.)

4. Increases in \(q\) have ambiguous effects on \(T_c\), and therefore on the frequency of innovations by conservative firms. On the one hand there is the direct effect pointed out earlier, namely that more imitation (i.e. more PMC) forces conservative firms to innovate more often \((T_c \downarrow)\). On the other hand, more imitation reduces the frequency of innovations by profit-maximizing firms, which in turn tends to reduce the competitive pressure on conservative firms \((T_c \nearrow)\). Whilst the direct effect always dominates when \(\lambda\) is small, the indirect effect tends to dominate when \(\lambda\) is large (that is, there are few conservative firms) and \(q\) is relatively small. [Figure 4a]

5. Effect of increased imitation on growth is negative when most firms are conservative \((\lambda\ close to 1)\) and strictly positive when most firms are
profit-maximizers (\(\lambda\) close to 0), as one would expect. (Figures 4b and 4c.) Moreover, whilst the relationship between competition and growth is monotonic over a wide range of values for the parameter \(\lambda\), there are values of \(\lambda\) for which the relationship ceases to be monotonic.\(^{16}\)

6. Increases in the fixed cost of innovation, \(f\), always mean higher \(T_p\) and higher \(T_c\), hence lower growth.

7. Increases in the interest rate increase \(T_p\) and \(T_c\), hence lower growth.\(^{17}\)

8. Increased wages always means higher \(T_p\), whilst the effect on \(T_c\) is again ambiguous.

9. Increased \(k\) means lower \(T_p\) and lower \(T_c\), and therefore higher growth.

Figures 4a , 4b, and 4c here

5 The Discipline of Debt-Financing

We now “open” credit markets, assuming that creditors of intermediate firms can have them liquidated in case they fail to meet debt repayments. Instead, in previous sections, we implicitly assumed there was no way creditors of firms could see their property rights enforced, which constrained firms to always have positive net financial wealth. The only entities in the economy which could have negative net financial wealth were consumers, who could borrow the accumulated wealth of firms from them.

In this section, instead, we assume full enforceability of the liquidation threat involved in debt contracting. In particular, we posit:

(a) Irreversible liquidation whenever repayments to current creditors are not honored, without the possibility of renegotiation.

\(^{16}\)For example, when \(\lambda = 0.15\), if \(q\) is low (less than 0.3) then growth falls with \(q\) (because the effect of \(q\) on growth is essentially governed by the effect of \(q\) on \(T_p\)); however, if \(q\) is high then growth increases with \(q\) (because the effect of \(q\) on \(T_c\) becomes the dominating effect).

\(^{17}\)There is a lower limit on \(T_p\) determined by the size of the interest rate. If the interest rate is very low (less than 0.01), the lower bound on \(T_p\) is often binding. If the interest rate is high (greater than 0.075), profit maximizing firms rarely make positive profits.
(b) The existence of covenants that prevent debt-financed firms from contracting any new debt in the future.\textsuperscript{18}

These two assumptions are certainly strong, especially given that liquidation is ex-post inefficient. On the other hand, relaxing either or both of these assumptions and thereby allowing for some dilution of existing debt claims would bring us back towards the "self-financing" setting of the previous sections.\textsuperscript{19} Our main point in this section is precisely that "hard" debt-contracts involving a credible threat of liquidation can be a very powerful instrument to "keep in line" non profit-maximizing managers: that their first concern is to see the firm survive, "commits" them to honor repayment promises \textit{whenever they can}. In turn, this will induce credit markets to lend them money whenever they expect the firm to be able to honor actuarially fair debt contracts. In order to determine equilibrium in this setting, we make two additional assumptions:

(c) The implementation of new technologies is unobservable by potential creditors.

(d) There is perfectly elastic supply of credit at the riskless rate of interest $r$.

Assumption (c) rules out complicated financial contracts, e.g. contingent upon the adoption policy of the borrowing firms. Allowing for such (unrealistic) contracts would only reinforce (but also trivialise) the main result of this section. Assumption (d) means that firms will not suffer from any \textit{liquidity} problem: provided they are \textit{solvent}, firms can always borrow at the riskless rate of interest $r$ which is also equal to the consumers' rate of time preference.

Without any loss of generality, we can then restrict attention to standard debt-contracts involving:

(i) an initial loan "d" to an intermediate good firm.

(ii) a constant growth-adjusted repayment schedule (in steady-state growth);

\textsuperscript{18}This assumption is unnecessarily strong: in particular the following Proposition 3 holds under the weaker assumption that new debt is always \textit{junior} to existing debt.

\textsuperscript{19}Our analysis in this section actually shows that in order to maximize his/her initial amount of sustainable debt, a conservative entrepreneur will find it weakly optimal to include (a) and (b) in the initial debt-contract, were any of these clauses to be enforceable.
that is:
\[ B(d, t) = (r - g)d \cdot e^{rt} \]
at date \( t \) if \( d \) is the initial loan contracted at date 0.

What determines the intermediate firms' solvency, or their debt capacity? The net present value of their profits, which in turn depends on their future adoption decisions. Here, being saddled with debt will force firms to get closer to profit-maximizing adoption decisions, in order to meet repayment promises and avoid liquidation. Existing debt thus plays the same role as a cut in \( \pi \) in reducing (increasing) net cash inflows (outflows) for the firm and thus acting as a discipline device towards profit maximization. As the next Proposition shows, in steady state, the disciplinary effect of debt-financing is extremely powerful. Assume that \( B \) and \( \delta \) are sufficiently large that the optimal adoption decision of conservative managers is described by Proposition 0. We then have:

**Proposition 3:** When intermediate firms can accumulate debt as detailed by assumptions (a-d), the steady state equilibrium amount of debt \( d^* \) leads to profit maximizing adoption decisions even for a conservative entrepreneur: that is, \( \tilde{T}(d) = T^*(d^*) \).

**Proof:** Let \( d^* \) be the net present value of profits under profit maximization just after adoption:

\[
    d^* = \frac{1}{1 - e^{-(r-g)T^*}} \left[ \int_0^{T^*} \psi(w, g, u)e^{-(r-g)u}du - w f e^{-(r-g)T^*} \right]
\]

where \( g = \frac{1}{T^*} \) is the rate of growth associated to the steady state defined by \( T^* \).

A conservative firm which has accumulated debt \( d^* \) will choose the adoption policy \( \tilde{T} \) which is uniquely defined by:

\[
    \int_0^{\tilde{T}} [\psi(w, \pi, g, u) - (r - g)d^*]e^{-(r-g)u}du = wf \cdot e^{-(r-g)\tilde{T}} \quad \text{(F_d)}
\]

and

\[
    \left. \frac{d(LHS)}{dT} \right|_{T=\tilde{T}} \approx \psi(w, g, \tilde{T}) - (r - g)d^* < 0.
\]

\(^{20}\)The same result applies to the case of technological addiction.
That is, the firm will choose the maximum adoption delay \( \bar{T} \) which prevents default on its debt-repayment schedule \( B(d^*, t) \). Since \( T = T^* \) turns out to also be a solution of \( (F_{d^*}) \), we necessarily have: \( \bar{T} = T^* \). In other words, a firm which has accumulated debt \( d^* \) has only one way to service it, that is, by adopting new technologies at the profit-maximizing rate: any deviation from this strategy will unavoidably trigger liquidation at some point, since debt-forgiveness by current creditors is ruled out by assumption (a), and no additional debt can be contracted to "buy time", from assumption (b). While this shows that maximum debt capacity is \( d^* \) and that \( d^* \) induces adoption policy \( T^* \) we can also observe that any debt level below \( d^* \) (and the associated adoption policy) cannot be a steady state debt level: a conservative entrepreneur that would not have reached the maximum debt capacity \( d^* \) could delay the next adoption time by contracting more debt, thereby "committing" to an adoption policy closer to \( T^* \). Consequently, only \( d^* \) and \( T^* \) can emerge as steady state debt level and adoption policy respectively.

\( \square \).

In the steady state, financial discipline completely eliminates the non profit-maximization behavior that arose in previous sections, thanks to the combination of the entrepreneur’s impatience and overriding concern for survival. The discipline force of product market competition is then irrelevant, and we are back in a “Schumpeterian” world! How robust is the above Proposition?

First, as mentioned above, Proposition 2 holds under a weaker assumption than (b), namely that new debt, while being allowed by the existing financial contract, must necessarily be junior to the existing debt. [This weaker assumption (b)', together with (a), corresponds exactly to Hart-Moore (1995).]

To see this, suppose that a conservative entrepreneur decides to contract some new debt \( \Delta \) in addition to \( d^* \). In order to avoid liquidation, this entrepreneur will choose the last adoption period \( T(\Delta) \) consistent with no default
on all forthcoming repayment obligations. That is:\textsuperscript{21}

\[ \Delta + \int_0^{T(\Delta)} \left[ \psi(w, g, u) - (r - g)(d^* + \Delta) \right] e^{-(r - g)u} du - w \int_0^{T(\Delta)} e^{-(r - g)T(\Delta)} \cdot \Delta. \]

After simplifying by \( \Delta \) on both sides of this equation, we get:

\[ \int_0^{T(\Delta)} \left( \psi(w, \pi, g, u) - (r - g)d^* \right) e^{-(r - g)u} \cdot du = w \int_0^{T(\Delta)} e^{-(r - g)T(\Delta)} \]

We thus see that \( T(\Delta) \) is independent of \( \Delta \), in other words it is weakly optimal for a conservative entrepreneur to accumulate the amount of debt \( d^* \).

Second, allowing for renegotiation (i.e. removing assumption (a)) should lead to a multiplicity of subgame perfect equilibria, including one where the maximum contractable debt is equal to the liquidation value of firms, \( L \textsuperscript{22} \) [We have implicitly assumed that \( L = 0 \), but what really matters is that \( L < d^* \), so that debt-financing will not eliminate the role of product market competition as a disciplining device on non-profit maximizing firms].

Third, we have assumed away capital market imperfections that would limit debt-capacity (e.g. due to informational asymmetries and/or imperfect debt-monitoring). Insofar as those imperfections limit total debt capacity to some \( \hat{d} < d^* \), debt-contracting will no longer fully eliminate slack in steady-state which in turn reintroduces a disciplinary role for product market competition. This, however, begs the question of the impact of a change in competition on the debt capacity \( \hat{d} \). The following section starts addressing this question. Although we consider a specific example, we feel that the effects we highlight in that section are robust to alternative specifications of credit market imperfections.

Finally, let us conclude this section by stressing the fact that Proposition 3 only concerns the steady-state. While adoption is faster, and thus growth

\textsuperscript{21}To avoid default, the firm must recover the amount \( \Delta e^{-(r - g)T(\Delta)} \) of cash at the end of the current adoption period.

\textsuperscript{22}This equilibrium would be sustained by the expectation that any amount of debt \( d > l \) will be forgiven in the future and that firms adjust their adoption policies accordingly. The profit-maximizing debt \( d^* \) might still be enforceable as part of a "reputational equilibrium" where creditors would automatically trigger liquidation on any unpaid outstanding obligation in order to prevent further defaults by other firms in the future; however, this can no longer be the unique equilibrium outcome once renegotiation is allowed.
higher, in a steady state with firm debt, the initial effect of an unanticipated opening of credit markets in the conservative firms case is to stop growth altogether! Indeed, short-term conservative managers will use this new opportunity to build up debt instead of adopting new technologies. In fact, firms with relatively new technologies in the steady state without firm debt will even keep building up financial assets. Instead, firms with relatively old technologies will see their net financial position deteriorate. Adoption and growth will resume whenever some firms start hitting their maximum debt capacity, thereby also increasing the pressure to innovate on other firms as in the previous sections. The transition path to the new steady state described in Proposition 3, and the very issue of convergence or non-convergence to this steady state, are natural topics for future research.

6 The Limits of Debt: Uncertain Payoff from Adoption

Having successively considered the “no-debt” and the “perfect-debt” cases (in the former case product market competition encourages innovations by conservative managers whereas in the latter case the threat of liquidation would suffice to make those managers behave like profit-maximizers), we would like to briefly touch upon the intermediate case of “imperfect debt” in order to get a sense of the robustness of the above results.

In particular, we want to illustrate the following idea: facing the risk of excessive liquidation, firms may decide to limit their debt exposure. On the one hand, this reintroduces product market competition as a disciplinary device complementary to the financial discipline associated with debt. On the other hand, more product market competition will tend to increase the risk of excessive liquidation, thereby making debt-finance less attractive to (conservative) managers. But this in turn implies that product market competition may crowd-out financial discipline. Whether such crowding-out is partial or total, will presumably depend upon circumstances and in particular upon the source of contractual imperfection.

In this section, we shall develop a very simple and therefore stylized ex-
tension of the debt model of the previous section, whereby we can illustrate the crowding-out idea whilst sticking to steady-state analysis. As always in infinite-horizon models, simplicity and tractability comes at the expense of realism, however, at the end of this section, we discuss the robustness of our conclusions to changes in the main assumptions.

We thus extend the analysis by assuming that, upon adopting a new technology, each firm has an infinitely small probability $\varepsilon$ of “being unsuccessful”.23 Specifically, remember that $\psi(w, g, u)$ also depends on $\pi(w)$ (see equation (4)). Redefine $\pi(w)$ as $\bar{\pi}$. “Being unsuccessful” means that the firm is faced with a profit realization $\pi$ lower than $\bar{\pi}$. This example is stylized in the sense that, when currently unsuccessful, firms can get back to the high realization $\bar{\pi}$ upon the next adoption with a probability that is the same as when they are currently successful. We concentrate as in the previous section on the case of conservative entrepreneurs, and we assume:

1. That conservative managers are infinitely risk-averse in the sense that when maximizing their net present expected utility, they put all the emphasis on the worst possible state of nature $\bar{\pi} = (\pi_0, \pi_1, \ldots, \pi_t, \ldots)$, where $\pi_t = \bar{\pi}$ for all $t$.24

2. That neither the implementation of new technologies nor the profit-realization $\pi \in \{\pi, \bar{\pi}\}$ upon adopting new technologies are verifiable.25

Let $d$ denote the maximum debt capacity of a firm that expects to always be unsuccessful given the growth rate $g$. That is:

$$d = \frac{1}{1 - e^{-(r-g)T'^*}} \left[ \int_0^{T'^*} \psi(w, g, u)e^{-(r-g)u}du - w^f \cdot e^{-(r-g)T'^*} \right]$$

(12)

where $T'^*$ is the profit-maximizing adoption policy of such a firm.

One can then establish:

---

23 The reason why we let $\varepsilon \rightarrow 0$ is because we want to stick to steady-state analysis.
24 Assuming infinite risk-aversion will enable us to obtain a crowding-out effect that remains bounded away from zero even when the Poisson probability of being unsuccessful, $\varepsilon$, becomes arbitrarily close to zero.
25 Our analysis would still go through if we assumed that profits can be verified by creditors ex post but only after the firm has been liquidated (i.e. only after the manager has lost his/her private benefits).
**Proposition 4:** When intermediate firms are conservative and their managers are infinitely risk-averse as specified in (1); and when firms can accumulate debt as detailed by assumptions (a)-(d) (with (c) being replaced by (2)), then: \( \forall \delta > 0, \exists B_\delta > 0 \) sufficiently large such that for any \( B \geq B_\delta \) the amount of debt initially acquired by those firms will remain equal to \( d \) no matter how small the probability of failure \( \varepsilon \).

**Proof:** First, an intermediate firm can always avoid default on a debt of size \( d \leq d \), since for any \( d \leq d \) and all \( t \leq T^{**} = T \):

\[
\int_0^t \psi(\overline{\pi}, \overline{w}, g, u) e^{-(r-s)u} du - \overline{w} t e^{-(r-s)u} \geq \int_0^t (r - g) d \cdot e^{-(r-s)u} \cdot du
\]

[This in turn follows from the definition of \( d \) and \( T^{**} \), from \( \psi(\overline{\pi}, \overline{w}, g, u) \geq \psi(\pi, \overline{w}, g, u) \), and from \( \psi(\overline{\pi}, \overline{w}, g, u) \) being decreasing in \( u \).] \( \square \)

In other words, a firm that chooses to adopt new technologies every \( T^{**} \) periods can always service its debt and still finance new adoptions out of its net accumulated profits. And for \( \delta \) sufficiently large, that firm will never choose to speed up innovation this time in order to delay innovation later, nor will it choose to contract a lower level of debt \( d < d \) which also would result in speeding up innovation this time.

Second, choosing any debt level \( d > d \) ex ante for the sole purpose of delaying the first adoption, will expose the firm's manager to the risk of losing his private benefits at some time in the future. Indeed, with positive probability, the firm will remain unsuccessful (\( \overline{\pi} = \pi \)) during \( n \) successive adoptions, where \( n \) is sufficiently large that:

\[
d > \max_T \left[ \int_0^T \psi(\pi, \overline{w}, g, u) e^{-(r-s)u} - \overline{w} t e^{-(r-s)u} | [1 + e^{-(r-s)T} + \ldots + e^{-(r-s)nT}] \right]
\]

where the RHS corresponds to the firm's net accumulated profits if it has been unsuccessful upon \( n \) successive adoptions following the first one.

But for any rate of time preference \( \delta > 0 \), there exists \( B_\delta \) sufficiently large such that the infinitely risk-averse manager will never take the risk of losing his private benefits \( B \geq B_\delta \) any time in the future as the price to pay for delaying the next innovation.
Third, contracting \( d = \bar{d} + \Delta > d \) \emph{ex post} once it has learnt it has been successful upon some technological adoption can only avoid default for sure if the firm puts enough cash aside to service the extra debt \( \Delta \). Letting \( \bar{T} \) denote the firm’s optimal time until the next technological adoption, we have:

\[
\Delta + \bar{T} \int_0^{\bar{T}} [\psi (\bar{\pi}, \bar{w}, g, u) - (r - g)(\bar{d} + \Delta)] e^{-(r-g)u} du - w f e^{-(r-g)\bar{T}} = e^{-(r-g)\bar{T}} \cdot \Delta
\]

that is, after simplifying by \( \Delta \) on both sides of that equation:

\[
\int_0^{\bar{T}} [\psi (\bar{\pi}, \bar{w}, g, u) - (r - g)d] e^{-(r-g)u} du = w f e^{-(r-g)\bar{T}}
\]

In particular \( \bar{T} \) is independent of the extra debt \( \Delta \).

In words, a successful adoption allows the (conservative) firm to buy time (\( \bar{T} > T^{**} \)) while still servicing its debt obligations \( B(d, t) \) and not incurring the risk of future liquidation. However, there is no point contracting more debt than \( \bar{d} \) following a successful adoption. This completes the proof of Proposition 4. \( \square \)

Thus, with uncertainty on post-adoption projects and under the assumptions of Proposition 4, debt-financing will only bring partial discipline on entrepreneurs. \( \bar{T} \) is higher than \( T^{**} \), which itself is higher than the profit-maximizing adoption policy \( T^* \). Competition will then emerge as an additional discipline device, putting pressure on successful firms by reducing their net cash inflows. That is, it will reduce \( \bar{T} \) for a given \( \bar{d} \). However, competition will also tend to reduce cash inflows of unsuccessful firms, and thus the maximum amount of debt \( d \) conservative firms are willing to contract upon! Consequently, product market competition will tend to crowd out financial discipline. Whether such crowding out will be only partial, full or even more than full will in fact depend upon the relative effect of competition on \( \pi \) and \( \bar{\pi} \). This crowding-out effect will vanish if competition involves almost no change on \( \bar{\pi} \). On the other hand, it will grow the higher the effect of competition on \( \pi \) relative to \( \bar{\pi} \). (The difference \( \bar{\pi} - \pi \) parametrises the “cost of debt” in this section).

\textbf{Remark:} For the sake of keeping the (steady-state) analysis tractable and simple, we made several assumptions which may call the robustness of
the whole analysis into question:

i. Unverifiability of profits. As already mentioned in the above footnote 23, what matters for the above analysis to go through is that the managers should care essentially about private benefits, and that if the true profit-realization were to be observed by outside creditors, this would be after the firm has been liquidated and the manager has consequently lost access to future private benefits.

ii. Lack of information gathering by outside creditors. Rather than assume as we did that outside creditors can never observe the true profit realization \( \bar{\pi} \), it would be sufficient to assume that they can observe \( \hat{\pi} \) with probability \( q \) if they incur a monitoring cost \( c(q) \), but that this monitoring cost becomes prohibitively high when \( q \to 1 \). As long as the optimal \( q \) is strictly less than 1, the infinitely risk-averse manager will keep on limiting his debt leverage to \( d = d^* \) when \( \delta \) is sufficiently large and \( B > B^* \) is also sufficiently large.

iii. Lack of debt-renegotiation.\(^{26} \) Although introducing debt-renegotiation in the context of infinite horizon models, turns out to be quite a challenging task, the pioneering work by Gromb (1995) suggests that, if any, allowing for renegotiation would further limit the amount of debt to be contracted upon ex ante by conservative firms, thereby reinforcing our main conclusions in this section.

iv. The absence of severance pay to firms’ managers upon liquidations. Insofar as conservative firms’ managers are mainly concerned with private rather than monetary benefits and private benefits are sufficiently large, debt-financing will remain costly for risk-averse managers and anything (like product market competition) which reduces \( \pi \), will still have a crowding-out effect on debt-financing \( (d) \) and the resulting

---

\(^{26}\) Note that the multiplicity of creditors is often invoked in defense of this assumption.
extent of financial discipline.

A robust conclusion of this section is thus that imperfections in credit markets reestablish the potential role of product market competition as an engine of growth. However, product market competition can be expected to at least partially crowd out financial market discipline.

7 Concluding Remarks

This paper has introduced agency considerations in a model of innovation and growth. It has shown that individual firm behavior can radically affect the impact of competition policy and industry policy: with profit-maximizing firms, product market competition tends to reduce growth in our model, while subsidizing innovation tends to promote it; instead with “conservative firms”, both effects are reversed.\textsuperscript{27}

Financial discipline has also been introduced in a simple way in our dynamic general equilibrium setup. Debt contracts have been shown to be quite powerful under certain assumptions, in particular a credible liquidation threat and no uncertainty. With uncertainty, the fear of liquidation reduces the role of debt, and product market competition can be seen as an alternative instrument to discipline conservative entrepreneurs, even though mutual crowding out can limit this disciplining effect.

This paper is obviously only a first step at tackling the interactions between firm behavior, the nature of competition and growth. In particular, both the simple description of the innovation technology and the assumptions made concerning financial contracts were partly motivated by the need to keep the analysis tractable. But, most importantly, we have taken a reduced-form approach to firm behavior, focusing on the extreme case of “conservative entrepreneurs”. In particular we have assumed that the private benefit of survival, B, and the private cost of adoption, C, of conservative entrepreneurs are high enough to lead to a corner solution, namely the minimization of the frequency of adoptions subject to the survival of

\textsuperscript{27}Aghion et al (1997) discussed in more detail these policy conclusions.
the firm. In subsequent work (see Aghion et al. (1998)), we start to provide microfoundations for this assumption, in a simple static setup. In this paper, we take an explicit agency model of external finance, where an innovator/entrepreneur obtains funds from investors by promising them a share of his financial return (in case of success) but can also "reassure" investors by making verifiable investments that reduce his marginal cost of effort. Such investments can go from the acquisition of research labs to the setting up of monitoring systems such as boards of directors. What we show in this paper is that the equilibrium entrepreneurial effort is non-monotonic in the level of required external finance: a rise in required external finance first lowers effort because the main consequence is a rise in the financial stake of the investor; but gradually at higher and higher levels of external finance, the entrepreneur is forced to commit to a higher probability of success and thus to higher effort, through higher verifiable investment. Interestingly, the analysis leads to two "regimes" in terms of firm behavior: at low levels of external finance, a reduction in the profitability of the firm (due to more competition, say), leads to lower effort, but at high levels of external finance, lower profitability leads to higher effort, in order to convince investors to lend their funds to the firm.

In short this microeconomic model mimics the behavior of our profit-maximizing and conservative firms, for respectively low and high levels of external finance. Moreover, it shows how varying the level of outside finance continuously fills the gap between these two extreme patterns of behavior. These microfoundations are however derived in a static and partial equilibrium context. Extending them to the dynamic context of this paper would further strengthen our results.
APPENDIX A: Proof of Proposition 0

Consider an intermediate firm with initial wealth $W$ at date 0. For $B$ sufficiently large, it is optimal for the conservative manager to never go bankrupt. The optimal adoption plan thus solves the following program:

$$\min_{(t_i, W_i)} C \sum_{i \geq 1} e^{-\delta(t_1 + \ldots + t_i)}$$

s.t. \[
\begin{aligned}
W_{i+1} &= W_i + \int_{t_i + \ldots + t_{i-1}}^{t_i + \ldots + t_{i-1}} \psi(w, g, t - (t_1 + \ldots + t_{i-1})) e^{-(r-g)\mu t} dt - f \omega e^{-(r-g)(t_1 + \ldots + t_i)} \\
W_i &\geq 0, \quad t_i \geq 0 \quad \text{for all } i \geq 1
\end{aligned}
\]

Defining $w_i \equiv W_i e^{(r-g)(t_1 + \ldots + t_i)}$ and $\varphi(t) \equiv \int_0^t \psi(w, g, u) e^{(r-g)(t-u)} du - f \omega$ this program is equivalent to (setting $w_0 \equiv W$)

$$\min_{(t_i, W_i)} \sum_{i \geq 1} e^{-\delta(t_1 + \ldots + t_i)}$$

\[(P)\]

s.t. \[
\begin{aligned}
w_{i+1} &= \varphi(t_i) + w_{i-1} e^{(r-g)t_i} \quad \text{for } i \geq 1, \\
w_i &\geq 0, \quad t_i \geq 0 \quad \text{for } i \geq 1.
\end{aligned}
\] \[(A1 \text{ and } A2)\]

It is straightforward to check that for any $w \geq 0$, there exists $t^*(w) > 0$ and $\tilde{t}(w), t^*(w)$ such that $\varphi(t) + we^{(r-g)t}$ increases for $t < t^*(w)$ and decreases for $t > t^*(w)$, and becomes negative $t > \tilde{t}(w)$. Moreover, starting from a plan $(t_i, w_i)_{i \geq 1}$ satisfying (A1) and (A2) and such that $t_{i_0} < t^*(w_{i_0})$ for some $i_0$, a slight increase in $t_{i_0}$ would enhance the objective and only relax the constraints in (P). Hence, without loss of generality, (A2) can be replaced in (P) with:

$$t^*(w_{i-1}) \leq t_i \leq \tilde{t}(w_{i-1}) \quad \text{for } i \geq 1 \quad (A2')$$

Now, fix $\varepsilon > 0$ and consider the set of plans satisfying (A1) and (A2') and such that:

$$\rho_i \equiv -\frac{d}{dt} \left[ \varphi(t) + w_{i-1} e^{(r-g)t} \right]_{t=t_i} \leq \varepsilon \quad \text{for } i \geq 2. \quad (A3)$$
Then for $\varepsilon$ positive but small, each $t_i$ (for $i \geq 2$) must be close to $t^*(w_{i-1})$ - and thus far from $\overline{t}(w_{i-1})$, implying that no constraint in (A2) is binding for $i \geq 2$. But then, a slight increase, in say, $t_2$ would still would not violate those constraints and would enhance the objective, a contradiction. Hence, attention can be restricted to the set of adoption plans such that at least one condition in (A3) is violated. For each such plan, let $k \geq 2$ refer to the first occurrence of this violation, and, assuming $t_1 < \overline{t}(w_0)$, consider the following change:

$$
\begin{align*}
&\begin{cases}
  dt_1 > 0 \\
  dt_i = 0 & \text{for } i \neq 1, k \\
  dt_k < 0 & \text{such that } w_k \text{ remains unchanged}
\end{cases}
\end{align*}
$$

The sequence $(w_i)_{i \geq 1}$, defined by (A1) is thus only affected for $i = 1, \ldots, k - 1$:

$$
\begin{align*}
  dw_i &= -\rho_1 e^{(r-g)(t_2+\ldots+t_i)} & \text{for } i = 1, \ldots, k - 1, \\
  dw_i &= 0 & \text{for } i \geq k
\end{align*}
$$

whereas $dt_k$ is defined by

$$
-\rho_1 e^{(r-g)(t_2+\ldots+t_k)} dt_1 - \rho_k dt_k = 0
$$

or equivalently

$$
dt_k = -\rho_1 e^{(r-g)(t_2+\ldots+t_k)} \rho_k
$$

(Noting that this is indeed a feasible change: since $t_1 < \overline{t}(w_0)$ and $t_i$ is close to $t^*(w_{i-1})$ for $i = 2, \ldots, k - 1$, $w_i$ is positive for $i = 2, \ldots, k - 1$.)

The impact of this change on the objective is thus given by:

$$
\left[ -\delta \sum_{t=1}^{k-1} e^{-\delta(t_1+\ldots+t_t)} \right] dt_1 - \delta e^{-\delta(t_1+\ldots+t_k)} \left[ 1 + \sum_{t=k+1}^{k} e^{-\delta(t_{k+1}+\ldots+t_t)} \right] (dt_1 + dt_k)
$$

$$
\leq -\delta e^{-\delta t_1} dt_1 - \delta e^{-\delta(t_1+\ldots+t_k)} \left[ 1 + \sum_{t=k+1}^{k} e^{-\delta(t_{k+1}+\ldots+t_t)} \right] dt_k
$$

$$
= -\delta e^{-\delta t_1} dt_1 \left[ 1 - \left( 1 + \sum_{t=k+1}^{k} e^{-\delta(t_{k+1}+\ldots+t_t)} \frac{\rho_1 e^{-|\delta - (r-g)|((t_2+\ldots+t_k)}}{\rho_k} \right) \right]
$$

$$
\leq -\delta e^{-\delta t_1} dt_1 \left[ 1 - \frac{\rho_1 e^{-|\delta - (r-g)|t^*}}{\varepsilon} \frac{1}{1 - e^{-\delta t^*}} \right].
$$

(using $\delta > r - g, t_i \geq t^* \equiv t^*(0), \rho_k > \varepsilon$ and $k \geq 2$)
But for any $w_0$ in a range $[0, w]$, $t_1$ cannot exceed $\bar{t}(w)$ and thus $\rho_1$ bounded above by $\varphi'(\bar{t}(w))$. Hence for any $w_0 \leq w$, there exists $\delta^*(w)$ such that the above expressions is negative for $\delta \geq \delta^*(w)$. Hence, for $w_0 \leq w$ and $\delta \geq \delta^*(w)$, any optimal adoption plan is such that $t_1 = \bar{t}(w)$ (and thus $w_i = 0$ for $i \geq 1$ and $t_i = \bar{T} \equiv \bar{t}(0)$ for $i \geq 2$). \qed
APPENDIX B: Sketch of proof of subsection 4.2.2

We proceed by proving a number of lemmata. Let us start with $T^*$. We have:

$$T^* = \arg \max_T \pi(T) = \frac{\int_0^T \psi(w, g, u)e^{-(r-g)u}du - wf}{1 - e^{-(r-g)T}}$$

with moreover $g = \frac{1}{T}$. Let us compute $\pi'(T)$ for a given firm, i.e. with $g$ constant. This gives:

$$\pi'(T) = \frac{(r - g)e^{-(r-g)T}}{(1 - e^{-(r-g)T})^2} \left[ \int_0^T \psi(w, g, T) - \psi(w, g, u) \right] e^{-(r-g)u}du + wf. \tag{B1}$$

The solution $T^*$ thus satisfies:

$$\int_0^T \left[ \psi(w, \frac{1}{T^*}, T^*) - \psi(w, \frac{1}{T^*}, u) \right] e^{-(r-g)u}du + wf = 0. \tag{B2}$$

At $T = T^*$, using (B1), the sign of $\pi''(T)$ is the sign of:

$$\psi(w, g, T)e^{-(r-g)T} + \int_0^T \psi_T(w, g, T)e^{-(r-g)u}du - \psi(w, g, T)e^{-(r-g)T}$$

which is negative since $\psi_T(w, g, T)$ is negative. At $T^*$, firms thus maximize profits.

Equation (B2) defines the profit-maximizing locus of points $w(T^*)$. Call (P) its inverse ($1/w(T^*)$).

**Lemma 1:** (P) slopes downward for $T^* > \frac{1}{r}$.

**Proof:** Define $w = \frac{1}{\omega^{1-\alpha}}$. Using the definition of $\psi(w, g, u)$, (B2) implies:

$$\frac{\sigma}{w} \int_0^T \left( e^{-(r-g)(1-\alpha)} - e^{-1-\alpha} \right) e^{-(r-g)u}du$$

$$= f - k \int_0^T \left( e^{\alpha T^* - e^{\alpha u}} \right) e^{-(r-g)u}du,$$

or equivalently

37
\[
\frac{\sigma}{w} = \frac{b}{T^*} \int f - k \left( \frac{e^{\rho T^*} - e^{1 + (\rho - r)T^*}}{rT^* - 1} - \frac{e^{1 + (\rho - r)T^* - 1}}{1 + (\rho - r)T^*} \right)
\]

(B3)

Simple algebra shows that:

i. the term that multiplies \( k \) in the numerator is positive and finite at \( T^* = \frac{1}{r} \), and is monotonically increasing to \( +\infty \) when \( T^* \) tends to \( +\infty \). (I.e. its derivative is positive and bounded away from zero\(^{28}\)).

ii. the denominator is positive and monotonically decreasing for \( T^* \) going from \( \frac{1}{r} \) to \( +\infty \). Indeed, defining \( y = \frac{1}{1 - \alpha} \geq 1 \) and \( x = rT^* - 1 \geq 0 \), one can redefine the denominator as:

\[
\int_0^y h(x, z)dz
\]

where \( h(x, z) = \frac{dx}{dz} \left[ e^x - e^{-z} \right] \). One can verify that \( h(x, z) > 0 \) and that \( h_x(x, z) < 0 \) for \( z \geq 1 \) and \( x \geq 0 \).\(^{29}\)

iii. the denominator times \( T^* \) is positive and monotonically increasing for \( T^* \) going from \( \frac{1}{r} \) to \( +\infty \). Indeed, multiply this expression by \( r \), which gives:

\[
(1 + x) \int_0^y h(x, z)dz
\]

Taking this derivative w.r.t. \( x \), one can show that it is positive\(^{30}\).

Taken together, these three statements prove the lemma, since \( \sigma \) is positive and \( \frac{\sigma}{w} \) is the sum of a positive decreasing term in \( T^* \) and a negative increasing term in \( T^* \). \( \square \)

Let us now turn to locus (F), as defined in section 3. Using earlier definitions, it is equivalent to:

\(^{28}\)This can be shown by expressing this term as a single fraction and taking its derivative, whose numerator is composed of a set of positive terms, some of which are exponential while the numerator is only polynomial.

\(^{29}\)\( h_x(x, z) < 0 \) can be shown with the following steps: (i) \( h_x(0, 1) < 0 \); (ii) \( h_{xx}(x, 1) < 0 \); (iii) \( h_{xy}(x, y) < 0 \).

\(^{30}\)This can be done in two steps. First for \( 0 \leq x < y \); then for \( x > 1 \).
\[
\frac{\sigma}{w} = \left[ k \cdot \frac{e^{1+(\rho-r)T} - 1}{1+(\rho-r)T} \cdot \frac{1}{1-e^{-(\frac{\alpha}{1-\alpha})+rT}} + f \cdot \frac{e^{1-rT}}{T} \cdot \frac{1}{1-e^{-(\frac{\alpha}{1-\alpha})+rT}} \right]
\]

**Lemma 2:** \( \frac{\sigma}{w} \) defined by \((F)\) is strictly positive and convex for \( T \in \left[ \frac{1}{\rho}, +\infty \right] \), and goes to \(+\infty\) with \( T \).

**Proof:** Relabeling \((B4)\) as \( \frac{\sigma}{w} = kA(T) \cdot C(T) + fB(T) \cdot C(T) \), one can show that:

i. \( A(T) \) is positive increasing and convex for \( 1 + (\rho - r)T \geq 1 \).

ii. \( C(T) \) is positive, increasing and convex for \( \frac{\alpha}{1-\alpha} + rT \geq 0 \).

iii. \( B(T)C(T) \) is positive since \( \frac{e^{\frac{\alpha}{1-\alpha}+rT}}{T} \) and \( \frac{e^r}{e^{rT}-e^{-\alpha/1-\alpha}} \) are both positive, decreasing and convex in \( T \).

iv. \( A(T) \cdot C(T) \) goes to \(+\infty\) with \( T \). \( \square \)

Finally, consider locus \((L)\), as defined in section 3. Using earlier definitions, it is equivalent to:

\[
\frac{s}{w} = \frac{L}{N} - k \frac{e^{\rho T} - 1}{\rho T} - f \cdot \frac{1}{T}
\]

**Locus \((L)\)** defines \( \frac{s}{w} \) as a concave function of \( T \) which goes to \(-\infty\) when \( T \) goes to 0 or to \(+\infty\).

What can we say about the intersections of these curves?

i. Only \((L)\) depends on \( \frac{L}{N} \). Keep in mind that \( \sigma \) and \( s \) are positive and depend only on \( \alpha \). Keep \( N \) constant. Then, there is a unique level \( \bar{L} \) of \( L \) such that, for \( L > \bar{L} \), \((F)\) and \((L)\) will have two intersections; for \( L = \bar{L} \), \((F)\) and \((L)\) will have one intersection; and for \( L < \bar{L} \), \((F)\) and \((L)\) will have no intersection (see figures A1 and A2). Economically, \( L \geq \bar{L} \) means that production is viable, while \( L > \bar{L} \) means firms can make strictly positive profits. The two intersections are \( \bar{\sigma} \) and \( \bar{T} \) with \( \bar{\sigma} < \bar{T} \).

ii. When \( L = \bar{L} \), \((P)\) cuts \((L)\) at the same point as \((F)\). When \( L > \bar{L} \), at \( T^* \) profits are strictly positive for \((P)\), but zero for \((F)\), which means that \( \frac{1}{w} \) has to be higher for \((P)\) when it
intersects (L) at \(T^*\). Consequently, we must have \(\tilde{T} = T^* < \hat{T}\).
(See Figures A1 and A2).

Finally, we want growth to be finite, i.e. \(\tilde{T} > \frac{1}{r}\). What is required for that in terms of parameter values? Let us start for example with \(\hat{T} = \frac{1}{r}\), which will first require (L) and (F) to intersect at \(T = \frac{1}{r}\). Combining (B4) and (B5) yields, for \(T = \frac{1}{r}\):

\[
\frac{1}{s} \left[ \frac{\hat{L}}{N} - \left( rf + k \frac{e^{\rho/r} - 1}{\rho/r} \right) \right] = \frac{1}{\sigma} \left[ rf + k \frac{e^{\rho/r} - 1}{e/r} \right] \frac{1 - \alpha}{1 - e^{\frac{1}{1 - \alpha}}} \tag{B6}
\]

which defines a positive \(\hat{L}\) (higher than \(\hat{L}\) in fact). In order to make sure that \(\hat{L}\) defines \(T = \frac{1}{r}\) and not \(\hat{T} = \frac{1}{r}\), sufficient conditions are that the slope of (F) be negative and the slope of (L) be positive. What does this imply in terms of parameter values, given that \(\hat{L}\) will change with the parameters that appear in (A.6)?

**Lemma 3:** For \(f\) (and thus \(\hat{L}\)) large enough, the slope of (F) and (L) are respectively negative and positive at \(T = \frac{1}{r}\).

**Proof:** The derivation of (B4) w.r.t \(T\) is finite at \(T = \frac{1}{r}\). For \(f\) large enough, its sign will be that of:

\[
\frac{\partial}{\partial T} \left[ \frac{e^{1-rT}}{T} \cdot \frac{\alpha}{1-\alpha} + rT \right]
\]

which, at \(T = \frac{1}{r}\), equals the sign of:

\[
(1 - \alpha) - (e - \alpha)e^{\frac{1}{1-\alpha}}
\]

which is negative.

The derivative of (B5) w.r.t \(T\) is also finite at \(T = \frac{1}{r}\). For \(f\) large enough, its sign will be positive. \(\Box\).

Finally we have:
Lemma 4: for $f$ (and thus $\hat{L}$) large enough, (B3) is above (B4) and (B5) in the $(1/w, T)$ space at $T = \frac{1}{r}$.

Proof: At $T = \frac{1}{r}$, the coefficient of $f$ in (B3) is:

$$\frac{r}{1-\alpha} / \left[ 1 - c^{\frac{-1}{1-\alpha}} \right]$$

which is positive, as we know from Lemma 1, point (ii). This is bigger than the coefficient of $f$ in (B4), which is:

$$\frac{r}{1-\alpha} / \left[ 1 - c^{\frac{-1}{1-\alpha}} \right].$$

For $f$ large enough, (B3) is thus above (B4) (and (B5), which intersects (B4) since $L = \hat{L}$) at $T = \frac{1}{r}$. □

For $f$ large enough and $L \in [\hat{L}; \tilde{L}]$, we thus have $\frac{1}{r} < \tilde{T} < T^* < \tilde{T}$, and (P) cuts (L) “from above” at the lowest possible $T^*$. Let us end this appendix with comparative statics results in space $(1/w, T)$:

Lemma 5: an increase in $\frac{k}{N}$ translates (L) upwards and leaves (F) and (P) unchanged; an increase in $k$ moves (F) upwards and (L) and (P) downwards; an increase in $f$ moves (L) downwards and (F) and (P) upwards.

Proof: Obvious from (B3), (B4) and (B5). □
References


Figure 1

\[ \int_0^\tau \varphi(w, g, u)^{-\tau s^u} du \]

\[ wfe^{-(r-s)T} \]
Figure 2

Effect of an increase in $\alpha$
Figure 4a

Effect of Increased Competition on Time between Innovations for Conservative Firms

When lambda is small, direct effect dominates. As lambda increases, this effect is offset by the indirect effect.
Figure 4b

Effect of Competition on Growth
As a function of the proportion, $\lambda$ of profit maximizing firms

The overall effect of imitation in growth is positive when most firms are conservative and it is negative when a significant number of firms are profit maximizing. Although small in absolute terms, the increase in growth for $\lambda=0.1$ is significant in percentage terms.
Figure 4c

Effect of Competition on Growth
As a function of the proportion, $\lambda$ of profit maximizing firms

Note: Ideally, we would have like to see the positive effect of increased competition on growth occurring with only a small proportion of conservative firms. This does not happen here, mainly because conservative firms only contribute a very small proportion of the innovations that cause growth.
Figure A2: $L = \hat{L}$

\[ \frac{1}{w} \]

\[ (P) \]

\[ (L) \]

\[ \frac{\bar{T}}{T} = T^* = \bar{T} \]