The Semantics of Measurement

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The Semantics of Measurement

A dissertation presented
by
Gregory Scontras

to
The Department of Linguistics

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Doctor of Philosophy
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The Semantics of Measurement

Abstract

This thesis examines linguistic phenomena that implicate measurement in the nominal domain. The first is morphological number, as in one book vs. two books. Intuitively, the contrast between singular and plural forms of nouns finds its basis in whether or not some thing measures 1. Chapter 2 develops a formal account of morphological number centered around this measurement. Different classes of words and different languages employ different criteria to determine whether or not something measures 1 for the purpose of morphological singularity.

The second component of the project takes a closer look at the semantics of quantizing nouns, or words that allow for the measurement or counting of individuals. Chapter 3 develops a typology of these quantizing nouns, identifying three classes of words: measure terms (e.g., kilo), container nouns (e.g., glass), and atomizers (e.g., grain), showing that each class yields a distinct interpretation on the basis of diverging structures and semantics.

The third component of the project investigates our representations of measurement, modeled formally by degrees in the semantics. Chapter 4 accesses these representations of measurement through a case study of the word amount, which is shown to inhabit yet another class of quantizing noun: degree nouns. This case study motivates a new semantics for degrees. Formally, degrees are treated as kinds; both are nominalizations of properties. The properties that beget degrees are quantity-uniform, formed via a measure. Treating degrees as kinds ensures that they contain information about the objects that instantiate them.
This new semantics for degrees highlights the four basic elements of the semantics of measurement. First, and perhaps most obviously, we have measure functions in our semantics. These measure functions translate objects onto a scale, allowing for the encoding of gradability. Scales are composed of the second element in our measurement semantics: numbers. Numbers, specifically non-negative real numbers, are taken as semantic primitives. The third element, kinds, often provides the objects of measurement. Kinds are abstract, intensional entities, so the fourth element in our measurement semantics, partitions, delivers maximal instances of the kind (i.e., real-world objects) to be measured. With measures, numbers, kinds, and partitions, we have a semantics of measurement.
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For my parents
Chapter 1

Introduction

Central to theories of natural language are speakers’ mental representations of the words they use, together with the ways these representations are employed in computing the meaning of the sentences that contain them. Identifying the mental representations of our speech and the computational system that manipulates them informs not just the cognitive underpinnings of language, but also how the mind works more generally. The current project focuses on a constellation of linguistic phenomena, those that appeal to or facilitate measurement, as a means to further our understanding of language and the psychological systems that shape it.

Although its scope is ostensibly limited to grammatical measurement, this project deals with fundamental questions that affect the analysis of every noun phrase in every language. By identifying how it is that we use language to count, parcel, and measure the world around us, we stand to uncover how that world is represented in our minds.

1.1 Background

This dissertation is written so that each chapter may stand more or less on its own, which means that the relevant theoretical background is presented as-needed in the context of the relevant discussion. However, certain assumptions about the general framework of compositional semantics underlie the proposals that are developed. Here, we review those assumptions and survey some of the data that motivate the current study.
1.1.1 Measurement

When we use language to describe and interact with the world around us, we reference entities and ascribe to them properties. Measurement stands at the core of this process. Measurement provides the means to answer the question of how much. It affords us the ability to specify and make reference to discrete quantities. It allows us determine properties of individuals, and, by identifying properties, also to ascribe them to individuals. What follows is a brief survey of the ways that natural language makes use of measurement.

Measurement gives rise to gradability, which precipitates properties that are held not wholesale, but rather to a specific extent or point along some scale. Extents are abstract representations of measurement, like three feet or forty pounds or the cardinality ten; they are often referred to as ‘degrees’ in the literature on the semantics of gradability (Kennedy, 1999). A measure function $\mu$ maps an individual (or an event) onto the set of non-negative real numbers, $[0, +\infty]$. Adopting a platonistic approach to numbers in our ontology (more on this in Chapter 2), measure functions receive the semantic type $\langle e, n \rangle$. Minimally, degrees contain information about the measure that determines them and some value of that measure; degrees may be represented as the pair of coordinates $<\mu, n>$, where $n$ is in the range of $\mu$. The degree of length three feet may thus be written $d_{<\mu_{ft},3>}$.  

Our focus here will be on measurement in the nominal domain, where individuals are the object of measurement. This is not to say that measurement does not occur elsewhere in language. In the verbal domain, measurement most often concerns properties of events. For example, the following sentences communicate information about the events that are described. In (1), the speaker describes the extent of time a running event took; (1-b) additionally compares that extent to some contextually determined standard. In (2), the speaker describes intensity. In both cases, degrees are measured.

(1)  
a. John ran the race in fifty minutes.  
b. Mary quickly crossed the finish line.

(2)  
a. Mary loves Bill a lot.  
b. John hit the pole so hard his teeth fell out.
Conceiving of time as a scale, even tense and aspect involve measurement. In (3), event times are compared; the time of an event is an extent along the time scale, which is determined by measurement.

(3)  a. John had finished cooking dinner before Mary came home.
    b. Sue hasn’t finished *Crime and Punishment* yet.

Work on event semantics has motivated the ontological reality of events (Davidson, 1967). Events (or eventualities) are abstract, structured entities. Individuals participate in events, and events determine the existence of individuals. While much work remains to be done on the way that events are named, measured, and ascribed properties, the more we understand events, the more they align with individuals in their structure and behavior (see Schein, 1993; Lasersohn, 1995; Casati and Varzi, 1996; as well as the papers collected in Dölling et al., 2008). For example, Bach (1986) models the domain of events $D_e$ directly after the domain of individuals $D_i$ proposed by Link (1983) (and summarized in the following subsection). Given this similarity, we stand to inform our understanding of measurement as it pertains to events once we make sense of measurement as it pertains to individuals, which are the object of measurement in the nominal domain and the topic of this dissertation.

Within the nominal domain, perhaps the most pervasive use of measurement concerns counting. Cardinal numerals delimit discrete quantities of individuals like three books or fifteen hamburgers. Whenever we use a cardinal numeral, we invoke the cardinality measure $\mu_{\text{card}}$, which maps a (plural) individual to its cardinality. Perlmutter (1970) goes so far as to argue that indefinite *a* is the spell-out of unstressed *one*, so that $\mu_{\text{card}}$ rears its head in most every nominal. This measure also plays a central role in determining morphological number, that is, the choice between singular and plural forms of nouns (the topic of Chapter 2).

Beyond counting, English possesses a wide range of nominals, let us call them ‘quantizing nouns’, that package individuals into discrete quantities. Appearing bare, without a determiner, nouns cannot be used to name specific individuals. For example, bare *carrots* cannot identify a specific quantity of carrots. However, when bare nouns appear with a quantizing noun, suddenly specific reference becomes possible: *one bowl of carrots* identifies a quantity
of carrots, just like *one kilo of carrots* or *one pile of carrots*. However, *bowl* and *kilo* and *pile* vary both in their behavior and in the character of the objects they ultimately reference. A bowl of carrots is a bowl; a kilo of carrots is carrots; a pile of carrots is at once carrots and a pile. It would seem, then, that within the set of quantizing nouns there are distinct subclasses with diverging semantics that yield salient differences in interpretation (the topic of Chapter 3).

Some quantizing nouns, for example *bowl*, invoke measurement only indirectly, through their ability to contain discrete quantities. Others, like *kilo*, directly name a measure in their semantics (e.g., the kilo measure $\mu_{kg}$). Yet others, like *pile* or *grain* or *drop*, serve not to discharge measurement, but to mediate counting by delivering stable minimal parts that may serve as arguments to the cardinality measure $\mu_{\text{CARD}}$. Consider, for example, the contrast between *one oil* and *one drop of oil*. In the former, we have no idea what to count; in the latter, *drop* delivers this information. While their strategies may differ, these quantizing nouns cohere on the basis of their ability to delimit and make reference to discrete quantities of stuff.

In addition to using measurement to carve up the world around us, language affords the ability to reference abstract representations of measurement directly via yet another subclass of quantizing nouns: degree nouns, the paragon example being the word *amount*. In (4), *amount* serves not to name specific carrots, but rather a specific amount thereof, say one kilo or two bags or thirty. Under its most plausible interpretation, the sentence asserts that different carrots were eaten each day, but when measured those carrots evaluated to the same amount. For example, the sentence could assert that each day for a year the speaker ate two carrots.

(4) I ate that amount of carrots every day for a year.

Here we have the means to directly reference degrees, abstract units of measurement, through the use of the degree noun *amount*. In other words, degree nouns reference degrees. By getting clear on the semantics and behavior of degree nouns, we stand to arrive at a better understanding of degrees themselves (the topic of Chapter 4).

Our investigation of measurement within the nominal domain sheds light on the way that
language determines reference to specific quantities. By identifying the means by which measurement transpires via language, together with the representation of measurement itself, we make clear the mechanism that underlies measurement in the semantics of natural language. Ultimately, we arrive at a deeper understanding of the nominal system, as well as of the basic building blocks of semantics.

The remainder of this section gives background on the ontology of things that get measured and parcelled out. We begin with a brief summary of the domain of individuals and its representation of pluralities, then we turn to the representation of kinds and sortal properties, together with the machinery that identifies kinds and properties with the individuals that instantiate them.

1.1.2 Plurality

Here we make explicit the basic assumptions concerning the semantics of plurality. For starters, the term ‘plurality’ is used as a label for collections of entities like the books or Bill and Sue. Work in plural semantics focuses on the inference patterns that result when pluralities are referenced, and on the structure of the domain of individuals that these inference patterns necessitate. The foundational work on this topic is Link (1983), who adopts a mereological account of the logical structure of plurality. Here, we consider a simplified version of Link’s theory.

Link (1983) observes that if a conjunction of individuals serves as the subject to a plural predicate, then each conjunct may serve as the subject to the singular counterpart of the predicate. That is, the sentence in (5) entails (5-a-c), so that each member of the plural subject Alan, Bob, and Charlie is said to be a man. This reading is the DISTRIBUTIVE reading of (5): the property of being a man is distributed among the members of the plural subject. For a predicate such as men to apply distributively to its plural subject, it must be the case that its singular counterpart (i.e., man) is true of each member of the plural subject.
(5) Alan, Bob, and Charlie are men |=
   a. Alan is a man.
   b. Bob is a man.
   c. Charlie is a man.

If we conceive of the plurality consisting of Alan, Bob, and Charlie as a set, then the property of being men applies to each non-singleton subset as well. Conceived of as a mereological sum, the property of being men applies to each part of the plurality. For now it makes little difference whether we model pluralities as sets or sums; either way, the inference in (6) holds.

To allow for the transition into the framework of mereotypology that takes place in Chapter 3, we follow Link in treating pluralities as sums.

(6) Alan, Bob, and Charlie are men |=
   a. Alan and Bob are men.
   b. Alan and Charlie are men.
   c. Bob and Charlie are men.

To derive these patterns of inference, Link makes certain assumptions about the domain of individuals $D_e$. The singular domain consists of atomic individuals. In the case of the singular predicate $man$, its denotation is a subset of this atomic domain; in other words, the denotation of a singular predicate the set of atoms that hold the named property (i.e., the property of being a man), as in (7).

(7) Let $[\text{Alan}] = a,
    \quad [\text{Bob}] = b,
    \quad [\text{Charlie}] = c.$

Assume no other men. Then, $[\text{man}] = \{a, b, c\}$

Given the assumptions in (7), we know that the predicate $man$ denotes the set containing Alan, Bob, and Charlie. Singular predication may be modeled as a process of set inclusion like in (8): for the predication to hold and thus the sentence to be true, the individual referenced by the subject must be a member of the set denoted by the predicate.
To achieve a general theory of predication, Link proposes that the process of set inclusion should hold whether the subject is an atom or a plurality. In other words, predication aligns with set inclusion whether the argument is an atom or a sum or atoms. Now, in (5), we are looking for a single individual that is a member of the predicate $\text{men}$. If the domain of individuals only contains atoms, then there can be no single individual that corresponds to Alan, Bob, and Charlie (and that is a member of $\text{[men]}$). Likewise for (31a-c), there are no individuals in the predicate $\text{men}$ that correspond to any of Alan and Bob, Alan and Charlie, or Bob and Charlie.

To construct a domain of plural individuals out of a domain of atoms, Link introduces the ‘sum’ operator $+$, corresponding to English conjunction. Here a note on terminology is in order: ‘individual’ refers both to atoms (e.g., $\text{John}$) and sums (or ‘pluralities’; e.g., $\text{John and Bill}$). We return to this point presently, once we arrive at the structure of the plural domain.

A predicate $P$ may be closed under $+$, sum formation, by the ‘star’-operator $*$. $*$ composes with a 1-place predicate $P$; $*P$ is the closure of $P$ under $+$. Thus, the denotation of $*P$ is every possible sum of atoms in the denotation of $P$, as in (10).

Once we close the domain of individuals under sum formation, suddenly we have modeled plural individuals in our ontology. Two atoms, say $j$ and $b$, may comprise a sum, $j+b$; the
latter is a plural individual, the former two singular individuals.

The plural domain \(*D*\) is structured on the basis of sum-formation, which yields a complete atomic join semilattice \(S\) on the basis of \(D\). In Fig. 1.1, each of the nodes \(\bullet\) in \(S\) represents a distinct element of the domain of \(S\). \(S\) is built up from the basic elements, or atoms, represented as the bottom layer of the semilattice. Atoms are then combined using \(+\) to form sums, the remaining elements of \(S\). In Fig. 1.1, \(a+b\) represents the sum of the atoms \(a\) and \(b\); \(a+b\) is an individual in \(S\).

Building \(S\) up from atoms via sum-formation introduces a natural order on the domain of \(S\): the individual \(a+b\) has the atoms \(a\) and \(b\) as parts. Link represents this order by the part-of relation \(\leq\); each line in \(S\) indicates this \(\leq\) relation. Thus, the line connecting the atom \(a\) with the sum \(a+b\) represents the fact that \(a\) is part of \(a+b\) \((a \leq a+b)\). If an individual has only one part, itself, that individual is atomic; otherwise, it is non-atomic. For this reason, \(a+b\) is non-atomic.

Within the framework of mereology, we can access the set of atoms that comprise any individual via the atom function \(AT\), modeled in (11).

\[
(11) \quad AT(a+b+c) = \{a, b, c\}
\]

The cardinality function \(\mu_{\text{CARD}}\) counts the number of elements in a set; \(AT\) delivers the atoms to be counted. Observe the behavior of \(\mu_{\text{CARD}}\) in (12).\(^1\)

\[
(12) \quad \begin{align*}
\text{a.} & \quad \mu_{\text{CARD}}(AT(a+b+c)) = 3 \\
\text{b.} & \quad \mu_{\text{CARD}}(a) = 1
\end{align*}
\]

\(^1\)See Schwarzschild (1996) for the motivation behind identifying any atom with the singleton set that contains it, which allows \(\mu_{\text{CARD}}\) to apply directly to the atomic individual \(a\).
Returning to Alan, Bob, Charlie and men, Link continues to assume that the 1-place predicate *man* refers to man atoms. However, the plural predicate *men* includes in its reference man sums, or pluralities. In other words, if the world consists of only three men, Alan, Bob, and Charlie, then the plural predicate *men* denotes the closure of the predicate *man* under +, as in (13).²

(13)  
   a. \([\text{man}] = \{a, b, c\}\)  
   b. \([\text{men}] = \{a, b, c, a+b, b+c, a+c, a+b+c\}\).

Finally, we have a domain in which to find the plurality Alan, Bob, and Charlie. The plural individual is a+b+c, and to ascribe the property of being men to this individual we include it in the set in (13-b).

(14) \([\text{Alan, Bob, and Charlie are men}] = 1 \text{ iff } [\text{men}] (a+b+c), \text{ i.e., iff } a+b+c \in \{a, b, c, a+b, b+c, a+c, a+b+c\}\)

Thus, including in our domain sums of individuals like a+b+c allows predication to always proceed in terms of set inclusion. By building plural predicates out of singular ones via the *-operator, we capture the entailment facts associated with distributivity: if Alan, Bob, and Charlie are men, then the the plural individual they comprise is a member of the plural predicate *men*, which means that atomic parts of the plural individual are members of the singular predicate *man*. We return to the semantics of plurality in Chapter 2.2. For now, the most important takeaway is the terminology: both atoms and sums are labeled ‘individuals’ in our domain.

### 1.1.3 Kinds

Throughout this investigation of measurement, we will see that in most cases, kinds serve as the stuff to get measured. In fact, only the cardinality measure applies directly to the sorts of individuals defined above in the explication of plurality. Otherwise, we use nouns to name

²Here we depart from Link (1983), who has a more conservative view of plural predicates, for reasons that will be made clearer in Chapter 2 once we discuss semantic plurality in the context of number marking.
kinds, whose instances are then measured. This makes sense: a kind is the individual correlate of a property; it is an abstract concept that instantiates as stuff in the world. Measurement provides the means to reference discrete instantiations of kinds. Here, yet another note on terminology is in order, as well as some background about the general theory of kinds that is assumed.

Nouns lead dual lives. Under one guise, they are function-like properties that serve as predicates, which delimit a class of objects that hold the relevant property. For example, in (15-a), the noun bears names the set of bears and the existential construction is used to assert that John likes some members of that set. In (15-b), the set of bears serves as the restrictor to the quantifier every: every member of this set is said to have come up to the speaker and eaten honey.

(15)  
   a. There are bears in the zoo that John like.  
   b. Every bear came up to me and ate some honey.

Under another guise, nouns are argumental: they name individuals directly. Consider the sentences in (16). In (16-a), the sentence ascribes the property of being widespread not to individual bears, or even to collections thereof, but to the bear kind.

(16)  
   a. Bears are widespread.  
   b. John doesn’t like bears.

We use the term ‘kind’ rather liberally. Most transparently, ‘kind’ refers to natural kinds like species of animals or types of plants. The bear kind, written BEAR, is a single entity; individual bears realize the BEAR kind by holding the property of being a bear. One way to conceive of kinds is as saturated properties: the BEAR kind is the nominalization of the (unsaturated) property of being a bear, written bear. A property determines the set of individuals that hold that property; the bear property determines the set of bears. By nominalizing this property, we shift from a set of bears to the concept of being a bear, a single ontological entity (see Krifka, 1995, for discussion). The domain of kinds, $D_k$, is a subset of the domain of individuals, $D_e$.

Natural kinds divide into subkinds, for example subspecies. Brown bears and polar bears
and teddy bears instantiate various subkinds of the bear kind. Crucially, each of these subkinds is itself a kind. The polar bear property determines the set of polar bears, which instantiate the polar bear kind. For our purposes, any concept that relates to real-world objects through a property in this manner will be labeled a generalized kind, or ‘kind’ for short.

Through the modification of kinds, language allows for the construction of new concepts: red wine or cold spring water or bears John likes. While these constructed concepts might not always correspond to natural kinds, they enjoy a similar ontological reality: concepts are the saturated correlates of unsaturated properties. By collapsing over the distinction between ‘kinds’ and ‘concepts’ (Krifka, 1995), or ‘conventional’ and ‘formal’ kinds (Pelletier and Schubert, 1989), both bear and BEARS JOHN LIKES are conceived of formally as generalized kinds. To repeat, they cohere on the basis of their relationship to abstract concepts, properties of individuals, and real-world objects.

The framework of Property Theory (Chierchia and Turner, 1988) makes this relationship explicit and allows for a formal definition of kinds. First, consider the motivation for and role of Montague semantics: Type Theory delivers a general system of semantic categories (Montague, 1973). We start with primitives, individuals and worlds, and then using a constrained mode of composition we construct functions. Individuals, type $e$, model objects in the world; functions, which characterize sets of individuals, model properties. Property Theory adds the idea that propositional functions may be injected into the domain of individuals in a retrievable way. In other words, every function may have an individual correlate.

As predicates, nouns denote functions that characterize sets of individuals. Using lambda notation, the noun bears qua predicate receives the semantics in (17). It denotes the characteristic function of being a bear, which delimits the set of (possibly plural) individuals that hold the bear property. Asserting that Yogi is a bear, we apply the function in (17) to him and attribute to Yogi membership in the set of bears.

3Collapsing over the distinction between what are at times called law-like, conventional, or established kinds and sortal concepts should not be taken as a dismissal of this distinction. Established kinds like BEAR stand apart from sortal concepts like BEARS JOHN LIKES on the basis of two phenomena: 1) established kinds but not sortal concepts may serve as arguments to kind-level predicates, and 2) established kinds but not sortal concepts exhibit scopelessness in episodic sentences. For fuller discussion of this distinction, see Carlson (1977b); Dayal (1992); Chierchia (1998b); as well as Chapter 4 below.
As arguments, nouns denotes kinds. To map the function in (17) into the domain of individuals, we must reimagine it as a semantic whole, not something in need of an argument. Here an example with verbs might be more illustrative. Take the verb runs. It names the predicate runs, which delimits the set of individuals (or events) that runs is true of. Used as an argument, we nominalize the verb: running. Nominalized, verbs may serve as arguments.

(18) a. Running is good for your health.
    b. John loves running.

The same process that nominalizes verbs so that they may serve as arguments transforms a predicative nominal into an individual. From the property of running, something that is true of individuals, we get the concept of running, an entity in its own right. The same holds for bears: from the property of being a bear, we get the BEAR kind. Formally, the nominalization operator \( ^\cap \), also called the ‘down’ operator, turns a function into an individual. The equivalences in (19) hold. For our purposes, any nominalized property will count as a generalized kind.

(19) a. \( \text{BEAR} = ^\cap \lambda x. \ *\text{bear}(x) \)
    b. \( \text{RUNNING} = ^\cap \lambda x. \ *\text{runs}(x) \)

To access the individuals that instantiate a kind, the predication operator \( ^\cup \), also called the ‘up’ operator, turns nominalized properties (back) into their characteristic functions. The equivalences in (20) hold.

(20) a. \( ^\cup \text{BEAR} = ^\cup ^\cap \lambda x. \ *\text{bear}(x) = \lambda x. \ *\text{bear}(x) \)
    b. \( ^\cup \text{RUNNING} = ^\cup ^\cap \lambda x. \ *\text{runs}(x) = \lambda x. \ *\text{runs}(x) \)

Using \( ^\cap \) to nominalize properties into their corresponding individual correlates, i.e., into their corresponding kinds, we map functions into the domain of individuals. To repeat: kinds are individuals, just of a special sort. They exist within the domain of individuals. By predicativizing these individuals with \( ^\cup \), we retrieve a property from its individual correlate.

Two main considerations motivate Property Theory. First, conceived of as functions
from worlds into sets of individuals, two properties may be logically equivalent without being identical. To evidence this fact, Chierchia and Turner (1988) submit the properties named by *being bought* and *being sold*; given the meaning of *buy* and *sell*, these two properties should delimit the same set of individuals: anything sold is necessarily bought, and vice versa. But despite their relationship to the same class of individuals, the concepts of being bought and being sold are distinct (Thomason, 1980; Bealer, 1982). In other words, a concept contains information beyond what a classical set theoretic representation of properties can deliver; Montague semantics mandates a “too-extensional” notion of property (Turner, 1987, p.456). The nominalization process creates individuals, which themselves may hold properties; these individual correlates of predicate functions may stand apart even if the functions share the same extension.

This ability to ascribe properties to concepts (and indirectly to properties) serves as another motivation for Property Theory. Standard typed logics are designed to avoid Russell’s Paradox, which means they preclude self-predication. But Property Theory allows self-predication: functions have individual correlates (i.e., the kinds to which they correspond), so in principle a function may take its own individual correlate as an argument. This is a good result: natural language permits self-predication. Parsons (1979) gives the example in (21), where we ascribe the property of being self-identical to the property of being self-identical.\(^4\)

The logical form for (21-b) appears in (22).

\begin{align*}
(21) \quad & a. \quad \text{Everything has the property of being auto-identical.} \\
& b. \quad \text{The property of being auto-identical has the property of being auto-identical.}
\end{align*}

\begin{align*}
(22) \quad & [\lambda x. \text{*auto-identical}(x)](\\lambda x. \text{*auto-identical}(x))
\end{align*}

Chapter 4.1.1 gives further background on the semantics of nominalization and its treatment of kinds in the context of degree semantics. There we will see that the same process that gets us the BEAR kind from the property of being a bear delivers abstract representations of measurement from the property of measuring specific extents. For now merely note that

\(^4\)Chierchia and Turner (1988) also identify cases of self-predication in mutual belief scenarios (Cresswell, 1985), the semantics of perception (Barwise and Perry, 1983), and other nominalization phenomena (Chierchia, 1982).
the domain of individuals is rich, containing not just real-world objects like books and bears, but also the corresponding concepts. These concepts, formalized as nominalized properties, receive the name ‘kinds’. Given their abstract nature, it should come as no surprise that kinds so often serve as the stuff that gets measured: to reference specific instances of a kind, we must minimally understand what counts as an instance, together with how much/many of those instances are relevant. Quantizing nouns play a central role in the instantiation of kinds.

1.2 Previewing the proposal

Measurement underlies many foundational issues in the study of natural language semantics; this dissertation directly contributes to making sense of three such issues. The first is morphological number, as in one book vs. two books. Intuitively, the contrast between singular and plural forms of nouns finds its basis in whether or not some thing measures 1. Chapter 2 develops a formal account of morphological number centered around this measurement. At the crux of the proposal is a one-ness presupposition attributed to singular morphology. Different classes of words and different languages employ different criteria to determine whether or not something measures 1 for the purpose of morphological singularity.

The second component of the project takes a closer look at the semantics of quantizing nouns, or words that allow for the measurement or counting of individuals. Chapter 3 develops a typology of these quantizing nouns, identifying three classes of words: measure terms (e.g., kilo; Lenning, 1987), container nouns (e.g. glass; Partee and Borschev, 2012), and atomizers (e.g., grain; Chierchia, 1998a), showing that each class yields a distinct interpretation on the basis of diverging structures and semantics. Superficially, each class may compose with a noun denoting some substance (i.e., kind), for example water or rice, and allow for the measuring of discrete quantities of that substance. However, a kilo of water is water, whereas a glass of water is a glass; a grain of rice is at once both rice and a grain. The proposal that results specifies how these differing interpretations arise by attributing categorial and functional differences to the subclasses of quantizing nouns. This proposal successfully predicts a wide range of facts concerning the distribution and behavior of these classes of words: optional
vs. obligatory co-occurrence of substance nouns and numerals, the functional status of the particle/preposition *of*, and constraints on the monotonicity of the measure at play.

The third component of the project investigates the representations of measurement, modeled formally by degrees in the semantics. Chapter 4 accesses these representations of measurement through a case study of the word *amount*, which is shown to inhabit yet another class of quantizing noun: degree nouns. By investigating the exceptional behavior of *amount* and locating this behavior within the landscape of nominal semantics, we motivate a new semantics for degrees. Degrees are shown to behave like kinds in the readings they precipitate and the manner by which they compose with the structures that contain them. Formally, both kinds and degrees are the nominalizations of properties. The properties from which we build degrees are quantity-uniform, formed on the basis of a measure. Treated as nominalized properties, degrees contain information about the objects that instantiate them, which delivers an existential interpretation for degrees, just as for kinds.

Chapter 5 concludes with a brief discussion of how the general program that results may be extended to yet more domains: a general system of degree semantics, mass nouns, and classifier languages.
Chapter 2

The Semantics of Morphological Number

We begin our investigation of measure in natural language with an investigation of number morphology, a phenomenon that ostensibly finds its value on the basis of an evaluation of whether or not some thing measures 1: *one book* contrasts with *two books*, firstly on how many objects are referenced and secondly on the numeral and number morphology expressed. In the first case, we imagine a single book and find a noun in the singular form; with *two books*, we imagine more than one book and the noun appears morphologically plural.

Already we see that the numeral plays a central role in determining the number morphology of the nouns with which it occurs. In English, the contribution of the numeral can be summarized as follows: With *one*, use singular morphology; with all other numerals, use plural. However, describing this pattern and deriving it within a standard framework of compositional semantics prove to be divergently different tasks. First we must understand the means by which we count with numerals, and the effect numerals have on the determination of morphological number. Doing so necessitates not only a semantics for numerals, but also an account of morphological number such that it is sensitive to the numerals present. The task becomes even more difficult once we expand our sights beyond English.

This chapter develops a semantic account of morphological number in the presence of numerals.\(^1\) In addition to accounting for number morphology on basic nouns like *book* in English, the approach extends to cover data from two seemingly disparate domains: 1) number marking on measure terms like *kilo*, which is determined by the numeral co-occurring with

\(^1\)This chapter expands on the proposal put forth in Scontras (2013a,b).
these terms: *one kilo of apples* vs. *two kilos of apples*; and 2) cross-linguistic variation in patterns of number marking: numerals other than ‘one’ obligatorily combining with plural-marked nouns (e.g., English), all numerals obligatorily combining with singular (unmarked) nouns (e.g., Turkish, Hungarian), and numerals optionally combining with either singular or plural nouns (e.g., Western Armenian). Building on the presuppositional approach to morphological number in Sauerland (2003), we see that all of the data considered receive an account once we assume variation in the selection of the measure relevant to the one-ness presupposition of the morphological singular form. Different classes of words and different languages determine whether or not something measures 1 for the purpose of morphological singularity on the basis of diverging criteria.

### 2.1 Number marking and numerals

Speakers of number marking languages decide between singular and plural forms of nouns as they embed them in larger linguistic contexts: In English, *book* is felt to mean something different from *books*, and the choice between these forms is regular and well-defined. If we are talking about a single book, we must use the singular form of the noun; when we are talking about more than one book, we use the plural. While intuitively appealing, this characterization of grammatical number in terms of one vs. more than one faces problems (see the discussion in Sauerland (2003) and Sauerland et al. (2005), as well as in Section 2.2 below). Still, it gives us a point from which to begin investigating the topic at hand: the impact numerals have on the determination of grammatical number.

In English, the numeral *one* requires that the noun it appears with bear singular morphology, thus *one book* and not *one books*. For numerals greater than *one*, plural morphology is required: *two books* and not *two book*. We can describe this pattern using our characterization of grammatical number above: With *one* we are talking about a single thing and so we require the singular form; with greater numerals, we are talking about more than one thing, thus the plural form must be used. The problem lies in explaining how these facts arise: What aspect of the linguistic form is responsible for the choice of grammatical number, and at what level of grammar does it operate?
Suppose that the determination of grammatical number is a wholly syntactic process driven by features of the modificational elements that then agree with features on modified nouns. Such a system would posit a SINGULAR feature on the numeral one and a PLURAL feature on all other numerals, at least in English. When composing with a noun, the number feature of the numeral would value the number feature of the noun and determine its morphological form. Note that this feature distribution, one+SINGULAR and not-one+PLURAL, captures the facts of English, but the system being considered admits a great deal of variation beyond the English pattern. Without ad hoc stipulations concerning the distribution of these features, a numeral could possess any number feature and so we should expect to find languages with unintuitive – and unattested – patterns of number marking. For example, how would we block a language from attributing the PLURAL feature to one and the SINGULAR feature to all other numerals? In other words, how do we rule out languages in which nouns agree with one in the plural and numerals other than one in the singular? The problem with a syntactic, or featural approach is that grammatical number bears only an indirect relationship to the meaning of the elements indexed with it, and so we lack a principled way of constraining the patterns that can be generated.

Here we consider an alternative to the syntactic approach: a semantic account of grammatical number in the presence of numerals that attributes the distinction between singular and plural forms to an interaction between the meaning of numerals and the semantics of the nominal element with which they compose. In developing this system, we consider data from two types of nouns: the basic type, as exemplified by book, and measure terms such as kilo (Lønning, 1987). Measure terms express morphological number, yet their morphology appears to be insensitive to singular vs. plural reference: In a construction such as one kilo of apples, kilo surfaces with singular morphology regardless of the number of individuals referenced (i.e., the number of apples). By augmenting the data to be covered to include measure terms, we highlight the breadth required by the semantic mechanism that modulates grammatical number. We then expand the coverage of the system beyond English, seeing what it takes to account for diverging patterns of number marking such as those found in Turkish.

\footnote{Note that the language described differs from, e.g., English, in that singular and plural morphology behave as expected in the absence of numerals, but with numerals we witness the diverging pattern.}
and Western Armenian (Bale et al., 2011a): in Turkish, all numerals require singular (i.e., unmarked) morphology on the nouns with which they occur; in Western Armenian, numerals optionally compose with singular or plural nouns. The data involving English measure terms and the pattern from Turkish evidence the fact that not every instance of a morphologically singular noun references a single individual. Our task, then, is to allow singular-marked nominals to receive a plural interpretation.

What results is a semantic program centered around a designated functional projection, \#P, from which morphological number features originate (cf. Sauerland, 2003, a.o.). The head of \#P, either \textit{sg} or \textit{pl}, is an operator that establishes conditions on the denotation of the resulting nominal: \textit{sg} checks for singularity of the predicate, and \textit{pl} applies when singularity is not satisfied. In other words, the morphological singular form requires that the nominal indexed with it reference only things that number 1. Variation in the way that singularity is checked captures the cross-linguistic diversity in patterns of number marking that we consider. This variation also accounts for number marking within the second domain of nominals, measure terms. Before we begin to develop this system, however, we must consider in more detail the assumptions at its foundation, together with the data to be explained. This is the topic of the next section.

\section*{2.2 Theoretical background: \#P}

What does it mean for a noun to be semantically singular? Let us assume that the noun must denote a set of atoms. How about semantic plurality? If singularity is tied with atoms, then a plural noun could denote sums of atoms (Link, 1983). We thus carve up our domain of individuals as in Fig. 2.1, where ‘atoms’ is tantamount to singulars, and ‘sums’ is tantamount to plurals.

The singular/plural distinction as realized in Fig. 2.1 makes more precise our intuitions on the contrast between nouns like \textit{book} vs. \textit{books}: the former refers to a set of book atoms (e.g., \{a, b, c\}) while the later refers to a set of book sums (\{a+b, a+c, b+c, a+b+c\}; but see Sauerland et al., 2005, for a finer grained notion of this contrast; we return to this point below). When someone uses singular \textit{book}, he is talking about single individuals; when he
Nominal morphology is in some way sensitive to ten singularities (atoms) and pluralities (sums) in Fig. 2.1; our task is to determine what this way is. If a noun’s denotation contains individuals formed by the sum operation $+$, then that noun appears with plural morphology. Thus, when talking about pluralities we use the plural form of the relevant noun. When referencing atomic individuals, we use the singular form. What follows is a proposal linking semantic number with morphological number.

Sauerland (2003) develops an extensional account of the semantics of morphological number: When a DP like *the book* references a single individual, singular morphology surfaces on the nominal and effects singular agreement with other elements in the sentence. When a DP does not reference a single individual, plural morphology and agreement result. The role of checking the numerosity of nominal referents is given to a syntactic head that projects above the determiner; Sauerland terms this element the $\phi$-head. The structure in (1) results.

(1)  
Nominal structure from Sauerland (2003):

```
\phi P \\
  \phi
  \_ \_ DP
  \_ [SG/PL]
  \_ \_ D NP
  \_ \_ the books
```

The $\phi$-heads host number features, which control agreement. The $SG$ head determines the morphological singular form, and the $PL$ head determines the morphological plural.
process establishes agreement within the nominal between nouns, adjectives, determiners, and the \( \phi \)-head. Another process establishes agreement between the \( \phi \)P in subject position and the finite verb.

The crux of Sauerland’s proposal is that only the number features in \( \phi \) are semantically interpreted. Moreover, these features are interpreted as presuppositions. He endows the \( \phi \)-heads with the semantics in (2). They are identity functions that take an individual and return that individual if certain conditions are met.

\[
\text{(2) } \phi\text{-heads from Sauerland (2003):}
\]
\[ a. \ [\text{SG}] = \text{id}_{\{x \in D_e | \neg \exists a (atom(w)(a) \land a < x \land a \neq x)\}} \]
\[ b. \ [\text{PL}] = \text{id}_{D_e} \]

\[
\text{(3) } \phi\text{-heads from Sauerland (2003):}
\]
\[ a. \ [\text{SG}] = \lambda x: \neg \exists a [\text{AT}_w(a) \land a < x \land a \neq x]. \ x \]
\[ b. \ [\text{PL}] = \lambda x. \ x \]

(3) translates Sauerland’s notation into one that matches our conventions here. Singular SG encodes the presupposition that the nominal referent has no atomic proper part, which in effect limits possible referents to atoms or portions of substance (e.g., the water). Plural PL makes no demands beyond requiring that its sister denote a (plural) individual.

There are at least two reasons to doubt the hierarchical placement and referent-checking semantics of Sauerland’s \( \phi \)P. First, nominals express morphological number in the absence of a determiner, as in NP conjunction or compounds. More importantly for our purposes, if the role of the morphological singular, SG, is always to check the atomicity of the referent, we have no hope of allowing singular-marked nominals to refer to a plural individual: singular morphology would mandate atomic reference, which precludes sums. Next, we consider minimal yet significant changes to Sauerland’s general proposal.

Following Sauerland (2003) (see also Sauerland et al., 2005), let us assume that the locus of syntactic number features is a designated functional head. To distinguish the current approach from Sauerland’s, we term this element the \( \# \)-head. Morphological number marking arises as a result of syntactic agreement with \( \# \). In this system, morphological number is
never directly interpreted; the determination of semantic number is a separate but related process. That is, the \#-heads do not contribute directly to the semantics of the nominals with which they compose. Instead, the \#-head that surfaces depends on the semantics of the nominal with which it composes. The details follow.

We find (minimally) two variants of the \#-heads: sg and pl.\(^3\) Here we depart from Sauerland (2003), who assumes that \(\phi\) composes once a nominal references individuals, i.e., at the DP layer. Instead, suppose that \# occurs as the sister to a sub-maximal nominal projection as in (4) and serves as an identity map on the predicate denoted by the nominal with which it composes.

(4) DP

\[
\begin{array}{cc}
D & \#P \\
\hline & \hline
& \# \quad NP
\end{array}
\]

This move allows for the account of measure terms and cross-linguistic patterns developed in Sections 2.4.2 and 2.4.3. Still, a major contribution of Sauerland’s work is the demonstration that sg, and not pl, is semantically marked (see Sauerland et al., 2005, for a discussion of the facts that lead to this conclusion). The \#-head \(\text{sg}\) carries with it a numerical presupposition for one-ness of the property with which it composes, (5-a). To satisfy the presupposition of sg, every member of a predicate denotation must measure 1. Pl carries no such presupposition, (5-b). For now, assume that the measure \(\mu\) relevant to the one-ness presupposition of sg is basic cardinality: \(\mu(x) = \mu_{\text{card}}(x)\). The choice between sg and pl is mediated by Heim’s (1991) principle of Maximize Presupposition, which ensures that sg is used whenever its one-ness presupposition is met.

(5) \#-heads:

a. \([\text{sg}] = \lambda P: \forall x \in P[\mu(x) = 1]. P\)

b. \([\text{pl}] = \lambda P. P\)

Maximize Presupposition requires that when faced with a choice between two forms, for

\(^3\)Additional \#-heads are likely needed to account for dual, trial, paucal, etc., values of grammatical number.
example SG and PL, one must choose the strongest (i.e., most restrictive) form possible. So whenever a predicate may compose with SG, it must. Otherwise, PL surfaces.

At this point we must draw a clear distinction between morphological number, expressed primarily by ø and -s in English and determined by the functional #-heads SG and PL, and semantic number. Assume three books: a, b, and c. In its basic form, semantically singular *book* denotes a set of atoms, (6-a). The star operator * (Link, 1983) closes the semantically singular property, (6-a), under sum formation, +, and produces the plural property, (6-b).\(^4\)

\[(6)\]
\[
a. \; [\text{book}] = \{a, b, c\} \\
b. \; [*\text{book}] = \{a, b, c, a+b, a+c, b+c, a+b+c\}
\]

Suppose that in number marking languages nouns always express grammatical number. In other words, they always appear in the presence of #. The semantically singular property in (6-a) may compose with SG: every member of [book] is atomic and thus has cardinality 1, so the presupposition of SG is satisfied. We thus predict that morphologically singular nouns refer exclusively to atoms. All of the members of the semantically singular predicate in (6-a) are atomic, and to reference a member of this predicate we use the morphologically singular book. The semantically plural property in (6-b) does not satisfy the one-ness presupposition of SG in (5-a) because there are elements of [*book] with cardinality greater than 1, namely all of the sums formed on the basis of + (e.g., a+b, b+c, etc.). By precluding the combination of semantically plural properties with SG, we correctly predict that morphologically singular nouns do not refer to pluralities; if one were to reference pluralities, a semantically plural property would be required, and so the morphologically plural form of the corresponding noun would be used.

Without any presupposition on its use, PL may compose with either of the properties in (6). What blocks PL’s combination with semantically singular properties, (6-a), and thus accounts for why morphologically plural books is not used to refer exclusively to book atoms, is the principle of Maximize Presupposition (Heim, 1991). Compare the strings in (7).

\(^4\) We construe semantic plurality as closure under sum, and not closure under sum less the atoms, in order to account for the behavior of plurals in downward entailing or non-monotonic environments. There, plurals may be used to refer to singularities. For example, if someone asks whether John has children, it would be infelicitous to answer no when he has only one child. For a fuller discussion of these facts, see Sauerland et al. (2005).
In choosing between the use of SG or PL with semantically singular properties, we see that the two options are denotationally equivalent. The one-ness presupposition of SG is met by the semantically singular property: every member has cardinality 1. Without any constraints on its use, PL likewise readily composes with a semantically singular property. Therefore the #-head, either SG or PL, serves as an identity map on the property, returning the same set of individuals it took as an argument.

But Maximize Presupposition necessitates the use of the lexical item with the strongest presuppositions (that are met). Because SG carries stronger presuppositions – PL has none at all – with semantically singular properties we must use SG. It is only when SG’s one-ness presupposition is not satisfied, i.e., when we have a semantically plural property containing individuals with cardinality greater than 1, that PL is used. In this way, morphological number corresponds directly to semantic number: the only licit combinations are SG with semantically singular properties and PL with semantically plural properties.

Next, consider how numerals fit into this program of number marking. Suppose for now that cardinal numerals are restrictive modifiers: they compose with predicates and restrict the predicates’ denotation to those elements with the appropriate cardinality.\(^5\)

\[
\begin{align*}
\text{(8) a. } & \left[\text{one}\right](e,t) = \lambda P \lambda x. P(x) \land \mu_{\text{CARD}}(x) = 1 \\
\text{b. } & \left[\text{two}\right](e,t) = \lambda P \lambda x. P(x) \land \mu_{\text{CARD}}(x) = 2
\end{align*}
\]

Recall the assumptions regarding morphological and semantic number; the semantics for the #-heads are repeated in (9), and the semantics of plurality in (10). Assume further that numerals project between the noun and the # projection: \(# > \text{numeral} > \text{NP}\). We address the motivation behind this structural assumption once we extend the account to measure terms in Section 2.4.

\(^5\)For discussion of numerals as modifiers, see Link (1987), Verkuyl (1993), Carpenter (1995), Landman (2003), among others; we explicate these assumptions regarding numeral semantics in Section 2.4.1.
The numeral *one* may compose with either a semantically singular or a semantically plural property; in either case, the resulting denotation is a set of atoms, each with cardinality 1, (11). This set of atoms satisfies the one-ness presupposition of SG, (12-a): every member has cardinality 1. Because the presupposition of SG is satisfied once *one* composes, Maximize Presupposition rules out the choice of PL, (12-c,d), and thus rules out *one books*. Again, composing restrictive *one* with either a semantically singular or a semantically plural property necessarily returns a set of individuals, each with cardinality 1, a set that allows for the morphological singular form on the basis of SG. Because SG may be used in the presence of *one*, it must be used.

If we want to rule out the composition of a semantically plural property with singular morphology, as in (12-b), we may appeal to a principle of economy, whereby the strings with and without * compete: because (12-a) and (12-b) are denotationally equivalent, and because (12-b) is more complex (it contains the pluralizing *-operator), (12-b) is uneconomical and therefore aberrant.\(^6\)

----

\(^6\)Note that we have no evidence suggesting that the combination of *one* with a semantically plural property should be ruled out by our system: the strings \([\text{SG one book}]\) and \([\text{SG one *book}]\) will both spell out as *one book* and denote the same set of individuals.
The numeral *two* with its restrictive semantics in (8-b) requires that the property with which it composes be semantically plural. When *two* composes with a semantically singular property, it looks among a set of atoms for individuals with the appropriate cardinality and finds none; the result is the empty set. We must say, then, that necessarily denoting the empty set, as in (13-b,d), is deviant and thus ruled out. Such a move should be familiar from recent work on the ungrammaticality that results from logical triviality (e.g., Gajewski, 2002). With semantically plural properties, *two* readily composes and restricts the nominal’s denotation to those individuals with cardinality 2. The one-ness presupposition of SG fails on such a denotation because it is not the case that all members number 1, (13-c). Because the presupposition of SG fails, we must use PL instead, thus *two books* as in (13-a).

At this point we appear to have an account of number marking in the presence of numerals for basic count nouns like *book* in English. Our task now is to extend the coverage of this account. We first consider two different systems of number marking from Turkish and Western Armenian. We then return to English and explore the semantics of measure terms like *kilo*, which, to all intents and purposes, behave as nouns, yet do not appear to refer in the way that a noun like *book* does. Without clear referents to check the atomicity of, we must assess

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7From Sauerland (2003): “Do not use the plural if the resulting meaning is identical to the meaning of the singular in the present context.”
what it means to be semantically singular for these nouns.

2.3 Shortcomings of the present account

Suppose that number marking finds its value (singular vs. plural) on the basis of one-ness, as in the system of morphological number sketched above. Whenever a predicate satisfies the condition of one-ness, singular morphology surfaces. This condition of one-ness remains purposefully vague. We saw that when it is tied to the cardinality of the members of a nominal denotation, the condition correctly captures the pattern of number marking in the presence of numerals on basic nouns like *book* in English. But that cannot be the end of the story. In this section, we consider two additional sets of data that demonstrate the limited ability of cardinality to determine the one-ness of a predicate. We begin with number marking and numerals in Turkish, then we return to English and look at the behavior of measure terms like *kilo*.

2.3.1 Cross-linguistic variation

Languages vary with respect to their patterns of number marking in the presence of numerals. So far, we have considered one type of language, exemplified by English, in which the numeral *one* co-occurs with singular-marked nouns and all other numerals require plural-marked nouns. Here we consider data from two other types of languages. In the first, all numerals obligatorily combine with singular-marked nouns (‘one book’, ‘two book’; e.g., Turkish or Hungarian; Bale et al., 2011a; Farkas and de Swart, 2010); in the second, numerals optionally combine with either singular- or plural-marked nouns (‘one/two book(s)’; e.g., Western Armenian; Bale et al., 2011a). Bale et al. (2011a) offer an account of these facts that treats nominal denotations and numeral semantics as distinct across languages. Farkas and de Swart (2010) derive the patterns within the framework of Optimality Theory. Here we adopt the null hypothesis that the denotations of nouns and numerals remain constant across number marking languages, and assume that a standard compositional semantics determines number morphology. With this in mind, our system for number marking as it stands cannot account for either of these patterns.
Turkish possesses a morphological distinction between singular and plural nouns, as evidenced in (14): the morpheme -lar indexes plurality. The choice of this morphology is regular and well-defined; the singular form is used to reference singular individuals and the plural form references pluralities. So far, tying the one-ness presupposition of sg to basic cardinality can capture these facts, as in the case of English.

(14) a. çocuk  
    boy(sg)  

b. çocuk-lar  
    boy-pl

Unlike with English, however, in Turkish-like languages all numerals, crucially those greater than ‘one’, require singular morphology. Concretely, in the presence of a numeral, -lar is prohibited, (15-b). In other words, nouns in Turkish are obligatorily singular, at least morphologically so, when they occur with numerals.

(15) a. iki çocuk  
    two boy(sg)  
    ‘two boys’

b. *iki çocuk-lar  
    two boy-pl

Despite clear reference to more than one individual, i.e., to two boys, the noun çocuk ‘boy’ in (15-a) expresses singular morphology.

We find a more complex pattern of number marking in Western Armenian. Like Turkish and English, Western Armenian possesses productive plural morphology: the morpheme -ner indexes plurality.

(16) a. degha  
    boy(sg)  

b. degha-ner  
    boy-pl

Western Armenian’s pattern of number marking in the presence of numerals represents a hybrid of the English and Turkish systems: nouns either may appear as morphologically singular in the presence of a numeral greater than ‘one’, as in Turkish, or they may appear.

\*All cross-linguistic data in this subsection come from Bale et al. (2011a).
as morphologically plural, as in English.\(^9\)

\[(17)\]
\[\text{a. yergu degha}
\begin{align*}
\text{two} & \quad \text{boy\textsubscript{sg}} \\
\text{`two boys'}
\end{align*}
\]

\[\text{b. yergu degha-ner}
\begin{align*}
\text{two} & \quad \text{boy-pl} \\
\text{`two boys'}
\end{align*}
\]

Assuming the system of number marking developed in the previous section for English, we predict neither the Turkish facts in (15) nor the Western Armenian facts in (17). The problem is that we have aligned semantic and morphological number so that the morphologically singular nouns are semantically singular, and we have assumed a restrictive semantics for numerals under which numerals greater than ‘one’ require semantic plurality for the predicate with which they compose. Both of the strings in (18) are aberrant. (18-a) fails because ‘two’ applied to a semantically singular predicate returns the empty set. (18-b) fails because the one-ness presupposition of \textit{sg}, at least when it is tied to basic cardinality, cannot be met by a predicate containing plural individuals.

\[(18)\]
\[\text{a. [sg two book]} = \emptyset
\]

\[\text{b. [sg two *book]} = \text{Presupposition failure}
\]

What we need is a way to allow singular-marked nominals to receive a plural interpretation, that is, to be semantically plural. Our approach will be to reevaluate the numerical presupposition we have attributed to the \#-head \textit{sg} so that it may also compose with semantically plural nouns in the presence of a numeral. Before doing so, however, we consider additional data for which our system of number marking must account.

---

\(^9\)This description of morphological number in Western Armenian from Bale et al. (2011a) is likely an idealization, resulting from the confluence of distinct dialects; see Sigler (1996) for a fuller discussion of the facts. Keeping in mind that much more work remains to be done to better understand the nuanced interpretations of these nominals, our goal will be to leave open the option of our approach accounting for the Western Armenian system as it is presented in Bale et al. (2011a).
2.3.2 Measure terms

So far we have been considering number marking on basic nouns like *book* and *boy*. These nouns may be viewed as one-place predicates, denoting sets of individuals holding the relevant property. We defined semantic number for these predicates in terms of the cardinality of the members of their denotations: if a predicate refers to a set of atoms, it is semantically singular; if the predicate is closed under sum-formation, it is semantically plural. But what happens when we have nouns that do not refer to individuals, atomic or otherwise, that still behave regularly with respect to number marking? Of interest are the italicized words in (19).

(19) a. That meat weighs two *kilos*.
    b. I ate two *kilos* of meat.

We must first convince ourselves that measure terms like *kilo* are nouns, or at least nominal to the extent that they should be handled by the same system of number marking that determines the morphology of *book* vs. *books*. To begin, measure terms display regular singular/plural morphology: *kilo* vs. *kilos*. Furthermore, they are free to combine with numerals and when they do they behave as expected, reserving the morphological singular form for the numeral *one*: *one kilo* vs. *two kilos*. Like basic nouns, measure terms constitute an open class: a nonce word may be substituted for a measure term and still we can conclude that the intended meaning involves a quantity or extent identified by the nonce word (but see the discussion in Wellwood, 2014). Finally, measure terms are subject to quantifier restrictions: *many kilos* but not *much kilos*.

Assuming that we take these facts as evidence that measure terms are nouns, what do we make of the semantics of singular vs. plural for them? The current schema relating morphological and semantic number is summarized in (20); it is unclear how this schema could apply to measure terms.

(20) Relationship between nominal referents and morphology:

a. atoms ⇒ SG

b. sums of atoms ⇒ PL
The problem is that measure terms do not appear to refer in the way that boy does. What kind of atoms are kilos, meters, degrees, etc.? What would it mean to close sets of these supposed atoms under sum formation? We thus take as our starting point the idea that measure terms are nouns which do not refer directly to individuals.

Without a referring semantics for measure terms we immediately face a problem in handling these nouns within our system of number marking. Recall the semantics for the #-heads, (21), where the one-ness presupposition of singular morphology depends on the cardinality of the members of the relevant property.

(21)  #’s semantics:

  a. $[\text{sg}] = \lambda P: \forall x \in P[\mu(x) = 1]. P$

  b. $[\text{pl}] = \lambda P. P$

Without atoms to count, the one-ness presupposition of sg is meaningless in the context of measure terms. Perhaps more damningly, even if we have a simple predicate denotation for phrases that contain measure terms such that they denote a set of individuals, what matters to the number morphology of these terms is not cardinality, but rather the measure specified by the term itself: the choice between one kilo and two kilos does not depend on how many atomic individuals weigh the relevant amount. In one kilo of apples, we possibly (and probably) reference more than a single apple, yet singular morphology surfaces on the measure term. What matters in the determination is not the measure in cardinality, but rather the measure in kilograms: when the individuals referenced measure 1 with respect to the kilo measure, singular morphology surfaces on the measure term.

In treating measure terms under our system of number marking, we will need to settle on a semantics for these terms and ensure that the morphological distinction between singular and plural attends to cardinality in the case of basic nouns and specific measures in the case of measure terms.
2.4 Proposed analysis

In what follows, we revise our system of number marking in the presence of numerals from Section 2.2 so that it may handle both measure terms and the observed cross-linguistic variation. We start by adopting a referential semantics for numbers that has cardinal numerals formed on the basis of the functional element \text{CARD} (Zabbal, 2005). Next, we align the semantics of measure terms with \text{CARD} and make clearer the assumptions concerning the measure relevant to the one-ness presupposition of \text{sg}. Finally, we locate the parameter determining cross-linguistic variation in the selection of the measure relevant to the one-ness presupposition of \text{sg}. The resulting proposal attributes measurement – and therefore counting – not to numerals proper, but to a functional projection \text{M(easure)}P. In number marking languages, \text{M}^0 often goes unpronounced, silently relating a numeral with the predicate denoted by a noun.

2.4.1 Numeral semantics

We started with minimal assumptions about numerals: they are property modifiers, type $\langle \langle e, t \rangle, \langle e, t \rangle \rangle$, and they occupy a position intermediate between $\#$ and NP. Now, we fill in the details of these assumptions.

First, concerning their structure, assume that numerals occupy the specifier of a functional projection \text{NumP} (Selkirk 1977; Hurford 1987; Gawron 2002; a.o.), and that \text{NumP} occurs hierarchically between NP and DP (Ritter, 1992).

\begin{equation}
\text{Structure of NumP:}
\end{equation}

```
  DP
     \n  D   NumP
     \n   numeral   Num'
     \n   Num   NP
```

For their semantics, take numerals to be individual-denoting expressions referring to natural
numbers: numerals are of type \( n \). In other words, we adopt a platonistic approach to numbers. The choice of \( \text{Num}^0 \) determines the function of the numeral (e.g., cardinal, ordinal, etc.; Zabbbal 2005). Cardinal numerals are those that serve the purpose of counting; they are formed on the basis of the Num-head \( \text{CARD} \), which takes a predicate and returns a relation between numbers and individuals (in the spirit of Krifka (1989)). In (24), semantically plural \(*\text{boy} \) composes with \( \text{CARD} \) and the numeral \( \text{two} \). The result restricts the denotation of \(*\text{boy} \) to just those (plural) individuals with cardinality 2.

\[
(23) \quad [\text{CARD}] = \lambda P \lambda n \lambda x. \ P(x) \land \mu_{\text{CARD}}(x) = n
\]

\[
(24) \quad [\text{two CARD } \ast\text{boy}] = \lambda x. \ast\text{boy}(x) \land \mu_{\text{CARD}}(x) = 2
\]

Note that \( \text{CARD} \) delivers the restrictive semantics for cardinal numerals that we assumed above: after composing with a predicate and a number \( n \), \( \text{CARD} \) restricts that predicate’s denotation to just those members with cardinality \( n \). This restrictive semantics ensures that cardinals greater than ‘one’ must compose with a semantically plural predicate (formed via \(* \) ), as in (25-b). Were such cardinals to compose with a semantically singular, i.e., atomic predicate, (25-a), the result would be the empty set, (25-c): there are no individuals in the denotation of an atomic predicate with cardinality greater than 1.

\[
(25) \quad \text{Assuming three boys:}
\]

a. \[ [\text{boy}] = \{a, b, c\} \]

b. \[ [*\text{boy}] = \{a, b, c, a+b, a+c, b+c, a+b+c\} \]

c. \[ [\text{two CARD boy}] = \emptyset \]

d. \[ [\text{two CARD } *\text{boy}] = \{a+b, a+c, b+c\} \]

Next, let us preserve the semantics we gave to the \( \# \)-heads, repeated below, and see how this semantics interact with our revised assumptions concerning cardinal numerals. The full nominal structure, including both NumP and \( \#P \), appears in (27).

\[
(26) \quad \#\text{’s semantics:}
\]

a. \[ [\text{SG}] = \lambda P: \forall x \in P[\mu(x) = 1]. \ P \]

b. \[ [\text{PL}] = \lambda P. \ P \]
(27) Nominal structure:

```
DP
   /
  D   #P
     /
    #   NumP
     /
    numeral Num'
      /
   Num   NP
    /
   CARD
```

The #-head takes the nominal, NumP, as an argument. Continue to assume that the measure relevant to the one-ness presupposition of sg is cardinality (note that cardinality is the measure supplied by the closest head to #, CARD; more on this below). Number marking in the presence of numerals proceeds as it did above (cf. (12) and (13)):

(28) Number marking with one:

a. ✓ [sg one CARD book] = \{a, b, c\}

b. ✗ [sg one CARD *book] = \{a, b, c\}, but failure of economy principle

c. ✗ [pl one CARD book] = failure to apply Maximize Presupposition

d. ✗ [pl one CARD *book] = failure to apply Maximize Presupposition

(29) Number marking with two:

a. ✓ [pl two CARD *book] = \{a+b, a+c, b+c\}

b. ✗ [pl two CARD book] = ∅

c. ✗ [sg two CARD *book] = presupposition failure

d. ✗ [sg two CARD book] = ∅

Again, with cardinality determining number marking, one may (and therefore must) compose with sg, which results in singular morphology on the co-occurring nominal, as in (28-a). Concretely, sg (the determinant of singular morphology) checks whether every member of
the denotation of a nominal predicate evaluates to 1 with respect to the measure $\mu_{\text{CARD}}$ in its presupposition. With two and other numerals greater than one, the presupposition of SG fails, so PL must be used instead. The result is plural morphology on the co-occurring nominal, as in (29-a). In other words, we correctly derive one book and two books.

We thus maintain our coverage of basic nouns with numerals. Why, then, have we gone to the trouble of revising our assumptions concerning numerals? As we shall see in what follows, viewing numerals as referring expressions that serve as an argument of the functional counting element CARD allows for a straightforward account of number marking on measure terms.

2.4.2 Accounting for measure terms

Before we can attempt to apply our system of number marking to measure terms, we must settle on a semantics for these nouns. Although we treat measure term semantics much more fully in the following chapter, we begin here with a preliminary semantics. To this end, note that measure terms appear to have two distinct uses. In the first, their ostensibly intransitive use, measure terms compose with a numeral and reference an abstract measurement or extent. They appear intransitive because we lack an overt substance noun. Intransitive measure terms typically occur as the objects of measure verbs (e.g., measure, weigh, etc.), as in (30-a). They also appear in predicative be constructions, (30-b), as well as modifiers of gradable adjectives, (30-c), and in equative constructions, (30-d).

(30) Intransitive measure terms:

a. John weighs 100 kilos.

b. The temperature is 70 degrees.

c. John is two meters tall.

d. Ten degrees Fahrenheit is colder than ten degrees Celsius.

In (30-a), the measure phrase 100 kilos specifies the extent of John’s weight. Similarly, in (30-c), two meters specifies the extent of John’s height.
Intransitive uses of measure terms contrast with their transitive use, where we have an overt nominal argument that provides the material to be measured.\textsuperscript{10} Given its role in the resulting interpretation, here we term this nominal argument the substance noun. The substance noun can be introduced via partitive, (31), or pseudo-partitive constructions, (32). In what follows, we will focus on measure terms in pseudo-partitives, where the connection between the measure term and the substance noun is more direct (Selkirk, 1977).

\begin{quote}
(31) \textit{Partitive}:
\begin{enumerate}
\item I drank two liters of that wine.
\item I ate two kilos of those apples.
\end{enumerate}
\end{quote}

\begin{quote}
(32) \textit{Pseudo-partitive}:
\begin{enumerate}
\item I drank two liters of wine.
\item I ate two kilos of apples.
\end{enumerate}
\end{quote}

In (32), the measure terms serve to quantize the denotations of the substance noun: the measure phrase uses the specified extent familiar from intransitive uses to restrict the denotation of the nominal complement.\textsuperscript{11} For example, in (32-b), \textit{two kilos of apples} denotes a set of apple individuals: those pluralities of apples that measure two kilos. It remains to be shown how transitive measure terms, together with the accompanying numeral, measure and quantize the substance noun. We must also be explicit about how intransitive measure terms interact with a numeral to specify extents along a dimension. Lastly, we must determine the relationship between transitive and intransitive measure terms. Let us work backwards, focusing first on the semantics of measure phrases like \textit{two kilos} and \textit{two kilos of apples}. We can then decide on an appropriate semantics for the measure terms themselves that will yield the desired semantics for measure phrases.

As noted above, measure phrases denote sets of individuals, or predicates. For example, the intransitive measure phrase \textit{100 kilos} in (30-a) names the property of weighing 100 kilos, a property we then ascribe to John. In (32-b), \textit{two kilos of apples} denotes the property of

\textsuperscript{10}Parsons (1970) terms transitive vs. intransitive uses of measure terms 'applied' and 'isolated', respectively.

\textsuperscript{11}Strictly speaking, the measure term restricts the denotation of the predicate counterpart of the kind named by the substance noun.
being a collection of apples that weighs two kilos. Supposing we want our measure phrases to be predicate-denoting, type \langle e, t \rangle, we can conceive of the measure terms as relations between numbers and individuals.

Under this relational conception, in the intransitive use a measure term takes a numeral and returns the set of individuals that satisfy the relevant measure to the extent specified by the numeral. In this way, a measure phrase like 100 kilos will be true of an individual just in case it weighs 100 kilos: supply 100 as the numeral argument to the relation in (33) and the predicate of weighing 100 kilos results, as in (34).

\[(33) \quad \llbracket \text{kilo} \rrbracket_{\langle n, \langle e, t \rangle \rangle} = \lambda n \lambda x. \mu_{kg}(x) = n \]

\[(34) \quad \llbracket 100 \text{ kilos} \rrbracket = \lambda x. \mu_{kg}(x) = 100 \]

In their transitive uses, measure terms take an additional argument: the substance noun. Complements of transitive measure terms used in pseudo-partitive constructions may only be bare plurals or mass nouns, suggesting that they refer at the kind level. We may use the semantics for intransitive measure terms in (33) as the basis for the transitive measure term semantics, where the only difference is that the latter takes an additional kind-denoting internal argument supplied by the substance noun.\(^{12}\)

\[(35) \quad \llbracket \text{kilo} \rrbracket_{\langle k, \langle n, \langle e, t \rangle \rangle \rangle} = \lambda k \lambda n \lambda x. \bigcup k(x) \land \mu_{kg}(x) = n \]

\[(36) \quad \llbracket \text{CARD} \rrbracket = \lambda P \lambda n \lambda x. P(x) \land \mu_{\text{CARD}}(x) = n \]

It bears noting that the semantic type given here for transitive measure terms resembles that given to our Num-head \text{CARD}: \langle k, \langle n, \langle e, t \rangle \rangle \rangle vs. \langle \langle e, t \rangle, \langle n, \langle e, t \rangle \rangle \rangle. The only difference is that where \text{CARD} takes a predicate-denoting argument (it may compose with singular count nouns), measure terms require a kind. The parallels in structure are obvious: \text{CARD} takes a predicate-denoting argument and then a numeral, forming \text{NumP}. A measure term (e.g., kilo) takes a kind-denoting argument and then a numeral, forming \text{M(ensure)P}. The relevant

\(^{12}\)The following chapter provides a detailed discussion of the relationship between transitive and intransitive uses of measure terms. There we will see that so-called ‘intransitive’ uses of measure terms feature an implicit kind argument. In other words, the transitive semantics of measure terms is prior.
structures appear in (37) and (38).  

(37) Cardinal numeral structure:

```
(37) Cardinal numeral structure:

<table>
<thead>
<tr>
<th>DP</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
</tr>
<tr>
<td>#P</td>
</tr>
</tbody>
</table>

| # |
| NumP |

| numeral |
| Num' |

| Num |
| NP |

| CARD |
```

(38) Measure term structure:

```
(38) Measure term structure:

<table>
<thead>
<tr>
<th>DP</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
</tr>
<tr>
<td>#P</td>
</tr>
</tbody>
</table>

| # |
| MP |

| numeral |
| M' |

| M |
| nP |

| kilo |
```

Ostensibly intransitive measure terms lack an overt internal argument. We might therefore conclude that their structure differs from that of a transitive measure term in the absence of a nominal complement, as in (39). So far we have noted both structural and semantic simi-

---

13 For now, the substance noun in the measure term structure in (38) is labeled as nP. The label is meant to indicate only that the substance noun is kind-denoting and expresses morphological number, but likely does not project a full DP. The following chapter provides a fuller discussion of the structure of measure phrases, including the role of the particle of.
larities between CARD and measure terms; we can pursue the parallel between these elements further by observing that, like measure terms, CARD also allows ostensibly intransitive uses. Transitive uses are far more common, and constitute standard cardinal numerals (i.e., *three boys*). Intransitive use of CARD, where a cardinal appears without an overt NP complement, include constructions such as *the boys are three* or *those books number ten*. As with intransitive measure terms, intransitive cardinal numerals serve as predicates. The structure for an intransitive cardinal appears in (40).

(39) **Intransitive measure phrase:**

```
  #P
    # MP
      numeral M
        kilo
```

(40) **Intransitive cardinal numeral:**

```
  #P
    # NumP
      numeral Num
        CARD
```

Given the similarities between CARD and measure terms like *kilo*, the following innovation suggests itself: align CARD with measure terms, such that both instantiate the category M. This move requires us to conceive of MP more generally, taking it to be a measure phrase counting either atoms (CARD) or something more abstract (*kilo*). In both cases, the measure

---

14Chapter 5.1.3 compares CARD with true classifiers in languages like Mandarin Chinese. There, we observe the striking similarity in function and behavior between these two elements. Strengthening the connection between CARD and classifiers, Greenberg (1972) notes that all of the classifier languages he considers allow for intransitive uses of classifiers in parallel to intransitive uses of CARD. He calls these uses ‘anaphoric’, suggesting that the uses are not in fact intransitive but instead presuppose nominal relata. This is the same we will give to the so-called intransitive uses observed here.
is specified by the head of the phrase.

(41) Generalizing MP:

\[
\begin{array}{c}
\text{DP} \\
\text{D} \\
\text{##P} \\
\text{##MP} \\
\text{numeral} \\
\text{M'} \\
\text{M} \\
\text{NP/nP} \\
\end{array}
\]

One advantage of this move is that it allows us to account for number marking on measure terms. First, consider the problem.

As in the case of \text{CARD}, an MP headed by an measure term denotes a nominal predicate, which may then be checked against the one-ness presupposition of \text{sg}. In (42), we have the denotation of \textit{one kilo of apples}.

(42) \([\text{one kilo (of) apples}] = \lambda x. \cup \text{APPLE}(x) \land \mu_{kg}(x) = 1\]

Like \textit{one boy}, \textit{one kilo of apples} denotes a set of individuals. The measure term \textit{kilo} constrains the denotation of \textit{apples} on the basis of the kilo measure, \(\mu_{kg}\). \textit{One kilo of apples} thus denotes the set of apple individuals measuring 1 kilo. However, the average apple weighs approximately 0.2 kilos, so in most scenarios the individuals denoted by \textit{one kilo of apples} will be pluralities, or sums of individuals. In other words, the individuals denoted by \textit{one kilo of apples} will not have cardinality 1. Checking such a set against the one-ness presupposition of \text{sg} relativized to \(\mu_{\text{CARD}}\) therefore fails, and so we incorrectly predict plural morphology on \textit{kilo} in (42): \textit{one kilos of apples}.

Note that Sauerland’s (2003) referent-based system does not fare any better: \(\phi\) occurs as the sister to DP, and absolute atomicity of the individual denoted by DP determines number
morphology. However, in *one kilo of apples vs. two+ kilos of apples*, the referent is a quantity of apples and yet the number of apples measuring 1 or 2+ kilos is irrelevant to the number morphology expressed on *kilo*. Number marking on measure terms is determined instead by the value of the numeral present: only with *one* do we have singular morphology.

For our system of number marking to handle both basic nouns and measure terms, the one-ness presupposition of English’s *sg* cannot be invariantly tied to cardinality (and through cardinality to semantic number). However, recall that cardinality does yield the correct pattern of number marking in the case of cardinal numerals: in *one boy*, but not *two boys*, every individual referenced has cardinality 1, so we get singular morphology on the noun. Further note that the cardinality measure, $\mu_{\text{CARD}}$, comes specified by *CARD* in the presence of a cardinal numeral: *CARD* occupies $M^0$, the head closest to $\#$ (cf. the structure in (41)). Here is the claim: in English, the measure specified by the head of $\#’$’s sister determines the measure $\mu$ relevant to the one-ness presupposition of *SG*.

(43)  
$\#’$’s semantics:

   a. $[\text{sg}] = \lambda P : \forall x \in P[ \mu(x) = 1 ]. P$

   b. $[\text{pl}] = \lambda P. P$

With cardinal numerals, *CARD* is the closest head to $\#$ and so the measure relevant to the one-ness presupposition of *SG* is cardinality, $\mu_{\text{CARD}}$. Because *SG* checks for one-ness on the basis of cardinality in the presence of cardinal numerals, the singular/plural distinction on basic nouns like *book* is sensitive to the semantic number of the predicate in question: when the predicate is closed under sum-formation and contains pluralities in its denotation, it no longer satisfies the one-ness presupposition of *SG* and so *pl* must be used. The result is plural morphology with semantically plural predicates (i.e., those that include plural individuals formed via sum). Only when the predicate is semantically singular, and thus atomic, will the one-ness presupposition of *SG* be met on the basis of cardinality. We thus maintain our coverage of number marking on basic nouns, preserving the intuition that singular morphology indexes reference to atoms and plural morphology indexes reference to pluralities (at least in most cases; see Fn. 4). Furthermore, we correctly predict singular morphology on basic nouns only.
with the numeral *one*.

Assuming that the measure in the one-ness presupposition of *sg* is supplied by the closest head, measure terms both specify the relevant measure for which one-ness must be satisfied (e.g., $\mu_{kg}$, $\mu_{degree}$, $\mu_{lb}$, etc.) and have number morphology expressed on them (e.g., *kilo* vs. *kilos*). Here is why: Like CARD, measure terms occupy the head of #’s sister. Also like CARD, measure terms supply a measure: $\mu_{kg}$ in the case of *kilo*, $\mu_{degree}$ in the case of *degree*, etc. With the measure term *kilo* heading MP, $M^0$ comes specified for the kilogram measure, $\mu_{kg}$. In (42), every member of the set denoted by *one kilo of apples* necessarily evaluates to 1 with respect to the kilo measure. With a different numeral, say *two*, no longer does every member measure 1 kilo; in fact, no member does.

Here is our pattern of number marking on English measure terms. To repeat: The measure supplied by the measure term – and not absolute cardinality – determines nominal number morphology. In the presence of numerals, singular morphology is checked against the measure specified by the head closest to #. When those numerals are cardinals, the head is CARD and $\mu_{CARD}$ determines singular morphology. When those numerals are arguments of measure terms, the specific measure named by the term itself determines singular morphology. Crucially, when the numeral is *one*, everything in the denotation of #’s sister will necessarily measure 1 with respect to the measure supplied by $M^0$, allowing for singular morphology. When the numeral is something other than *one*, nothing in the sister of # will measure 1 with respect to the measure supplied by the term, so PL must be used.

In sum, we have seen that the measure relevant to the one-ness presupposition of *sg* is underspecified, and that in English this measure is supplied by the head closest to #. In the case of cardinal numerals, cardinality determines number morphology: CARD is the head of #’s sister and CARD measures cardinality. In the case of measure terms, the specific measure supplied by the given term determines number morphology. With *kilo*, everything in MP must measure 1 kilo in order for the one-ness presupposition of *sg* to be satisfied; only when the numeral *one* appears with *kilo* does this state of affairs holds. In this way, we account for number marking on measure terms in the presence of numerals, which, as we have seen, is sensitive to the numeral present and not to the number of individuals referenced.
The system as it stands yields the desired patterns, but it faces the problem of not being compositional: Some sort of magic looks into the semantics of the measure heads and plugs the relevant measure into the one-ness presupposition of $sg$. We can do better. In order to compositionally attribute the measure internal to $M^0$ to the one-ness presupposition of $sg$, we must consider what all of these measures have in common; we can then hang our system of English number marking on this property of measures.

$MP$ will always denote a property that is quantity-uniform with respect to the measure internal to the semantics of $M^0$. In other words, every individual in the denotation of $MP$ will evaluate to the same extent. (44) provides a formal definition of this notion.

$$\text{(44) Quantity-uniform property:}$$

$$QU_\mu(P) = 1 \text{ iff } \forall x \forall y [ P(x) \land P(y) \rightarrow \mu(x) = \mu(y) ]$$

Take, for example, the $MP$ $\text{one boy}$. Assuming three boys, we get the denotation in (46).

$$\text{(45) } \left[\left[\text{one CARD boy}\right]\right] = \{a, b, c\}$$

In (45), $\text{CARD}$ heads $MP$, and internal to $\text{CARD}$ is the cardinality measure $\mu_{\text{CARD}}$. The predicate $\text{one boy}$ denotes a set of singular boys. When measured by $\mu_{\text{CARD}}$, every member returns the same value, namely 1. With $\text{one kilo of apples}$, $\text{kilo}$ heads $MP$ and supplies the kilo measure $\mu_{\text{kilo}}$; the predicate denotes a set of apple individuals that each return the same value when measured by $\mu_{\text{kilo}}$: 1. The reader can verify that any measure supplied by $M^0$ behaves similarly. Given that the aim is to tie the one-ness presupposition to the measures in $M^0$, all we need do is relativize this presupposition to just those measures that determine quantity-uniform properties, as defined in (60).

$$\text{(46) English $\#$-heads:}$$

a. $\left[\text{SG}\right] = \lambda P: \forall \mu \forall x \in P [ QU_\mu(P) \rightarrow \mu(x) = 1 ]$. $P$

b. $\left[\text{PL}\right] = \lambda P$. $P$

What results is a fully compositional account of English number marking: with cardinal numerals, number marking is sensitive to the quantity-uniform measure $\mu_{\text{CARD}}$, that is, to the semantic number of nominal predicates. With measure terms, number marking is sensitive
to the quantity-uniform measure supplied by the measure term itself, which in effect links number marking to the value of the co-occurring numeral: *one* takes SG regardless of the number of intended referents.

In the next section, we see how our assumptions about the measures relevant to the one-ness presupposition of SG may be extended to provide an account of the cross-linguistic variation in number marking discussed in Section 2.3.1.

### 2.4.3 Relevant measures

In addition to deriving the English pattern of number marking for both basic and measure nouns, we must also introduce sufficient flexibility into our system so that it may account for the patterns in Turkish and Western Armenian. The approach will be to derive the Turkish facts in addition to the English facts, and then assume variation within Western Armenian such that it can employ either the English or the Turkish system.

Recall that in Turkish and languages like it all numerals require singular morphology, which necessitates SG in numeral-noun constructions. With numerals greater than ‘one’, we thus require SG in the presence of a semantically plural property. Consider once again the structure of a nominal predicate modified by a cardinal numeral, as in (47-c).

\[(47)\]

\begin{align*}
\text{a. } [\text{boy}] &= \{a, b, c\} \\
\text{b. } [\star \text{boy}] &= \{a, b, c, a+b, a+c, b+c, a+b+c\} \\
\text{c. } [\text{two \text{CARD} \star \text{boy}}] &= \{a+b, a+c, b+c\}
\end{align*}

The combination of SG with the numeral-modified predicate in (47-c) is problematic because of the way we have aligned semantic and morphological number: we must allow singular-marked nominals to receive a plural interpretation. As was our strategy in accounting for measure terms in the previous subsection, here we will again take advantage of the flexibility allowed for in the selection of the measure \(\mu\) in the one-ness presupposition of SG. In English we said that \(\mu\) is supplied by the head closest to \(\#\), but this need not be the case in all languages.

Given our semantics for \text{CARD}, cardinal numerals serve as restrictive modifiers: they return a subset of a noun’s denotation populated by individuals with the appropriate cardi-
nality. By ensuring that every element has the same cardinality, cardinal numerals quantize the members of the resulting denotation. Crucially, every member of a quantized predicate has no parts that are also members of that predicate; in other words, every member of a quantized predicate is a smallest member (Krifka, 1989).

Take *two boys* in (47-c). This predicate is true of three (plural) individuals: a+b, a+c, and b+c. Each of these individuals has no parts which are also in the denotation of *two boys*. In this way, every member of the predicate *two boys* is a smallest member of the predicate *two boys*: every member is an atom relative to the predicate in question. (We leave it to the reader to check that this situation holds for any cardinal numeral.) In Turkish, then, number marking appears to be sensitive not to absolute atomicity (evaluated by, for example, the cardinality measure $\mu_{\text{CARD}}$) but rather to relative atomicity: quantized predicates bear singular morphology. Here we need a notion of relative atomicity: counting as atomic not with respect to the entire domain, but rather with respect to a specific predicate (Krifka, 1989; Chierchia, 1998b). We term these relative atoms ‘P-atoms’, the smallest elements of P: those elements of P that have no other elements of P as parts.\(^{15}\)

In Turkish, the measure relevant to the one-ness presupposition of SG should count the smallest elements, or relative atoms of nominal predicates. This measure, $\mu_{\text{P-atom}}$, is defined in (48).

\[(48) \quad \mu_{\text{P-atom}}(y) \text{ is defined only if } y \in P; \text{ when defined} \]
\[\mu_{\text{P-atom}}(y) = |\{x \in P: x \leq y \land \neg \exists z \in P[ z < x]\}|\]

\[(49) \quad \text{Turkish } \#\text{-heads:} \]
\[\text{a. } [\text{SG}] = \lambda P. \forall x \in P[ \mu_{\text{P-atom}}(x) = 1]. P \]
\[\text{b. } [\text{PL}] = \lambda P. P \]

\[(50) \quad [\text{two CARD *boy}] = \{a+b, a+c, b+c\}\]

In the presence of cardinal numerals, # composes with a predicate as in (50). Every member of this predicate has no parts which are themselves members of the predicate, therefore

\(^{15}\)This notion of relative atomicity differs from that found in Rothstein (2010a), where atoms are defined relative to a context and not to a predicate.
every member of this predicate measures 1 P-atom. Supposing $\mu_{P\text{-atom}}$ to be the measure relevant to the $\#$-heads, $\text{SG}$ may – and, by Maximize Presupposition, must – be used with $(50)$. In fact, all numeral-noun combinations will have a quantized denotation wherein the elements share a common cardinality, so it will necessarily be the case that every member measures 1 P-atom. In other words, with $\mu_{P\text{-atom}}$ as the measure relevant to $\text{SG}$’s one-ness presupposition, we predict singular morphology with all numerals. This is the pattern in Turkish-like languages.\(^{16}\)

One way to view the distinction between the Turkish and English patterns of number marking in the presence of numerals is as a difference in whether the one-ness presupposition of $\text{SG}$ is relativized to the complement of $\#$ (i.e., $\text{MP}; \mu_{P\text{-atom}}$) or to the head of its complement (i.e., $M^0$). In Turkish, we find the former strategy: because numerals, crucially those greater than ‘one’, quantize the predicates that they modify into sets of relative atoms, the one-ness presupposition of $\text{SG}$ relativized to $\mu_{P\text{-atom}}$ will always be satisfied in the presence of a numeral. In English, we saw that the head of $\#$’s sister supplies the relevant measure: either cardinality in the case of cardinal numerals (supplied by $\text{CARD}$) or the specific measure supplied by measure terms.

The present account makes a prediction about number morphology on measure terms in Turkish. Every member of a predicate like two kilos of apples will measure 1 P-atom. In order to measure more than 1 P-atom, an individual would have to measure two kilos and be a proper part of a different member of the predicate that also measure two kilos. But this is impossible: the monotonicity of the kilogram measure ensures that anything weighing two kilos has no proper parts that weigh two kilos. We therefore expect singular morphology on measure terms like kilo with all numerals in Turkish, which is precisely what we find in $(51)$.

\(^{16}\)Note that the approach correctly predicts singular agreement with all numerals in Turkish even if semantic plurality in such languages is not mere sum-formation, *, but something stronger such as closure under sum less the atoms, * (cf. Link, 1983; for arguments in favor of this stricter approach to plurality in Turkish, see Bale et al., 2011a,b).
Recall that in Western Armenian we find optionality between the English and Turkish systems: numerals greater than ‘one’ optionally combine with either singular- or plural-marked nouns. To account for this optionality, simply assume that each of the two strategies above (phrasal vs. head) is available when selecting the measure relevant to SG’s presupposition. When the phrasal strategy is pursued, one-ness is relativized to P-atoms and so singular-marked nominals appear with numerals greater than ‘one’; when the English-type, head-based strategy is pursued, one-ness is sensitive to cardinality, and so we find plural-marked nominals with these numerals.

We appear to have not only an account of number marking on basic nouns and measure terms in English, but also an account of the cross-linguistic variation observed in patterns of number marking. Crucially, both sets of phenomena receive an account once we assume variation in the measure relevant to the determination of singularity.

2.5 Discussion

In our account of number marking in the presence of numerals, we have considered data from three domains. First, we looked at basic nouns like book in English whose morphological number depends solely on the semantic number of the property denoted by the nominal. We also considered measure terms like kilo, assuming that these measure terms are nouns, at least to the extent that they should be handled by the same system that treats morphological number on basic nouns. Finally, we examined cross-linguistic variation in patterns of number marking, drawing data from Turkish and Western Armenian.

Our account relied on three assumptions: 1) cardinal numerals are formed on the basis of the functional element \( \text{CARD}_{\langle (e,t),(n,(e,t)) \rangle} \), 2) measure terms, like CARD, are relations between
numbers and individuals, and 3) morphological number is determined by the head of the functional projection #P, which serves as an identity map on the predicate denoted by the nominal.

(52) a. \([\text{SG}] = \lambda P: \forall x \in P[ \mu(x) = 1 ]. P \]
b. \([\text{PL}] = \lambda P. P \]

SG carries with it a one-ness presupposition which ensures that every member of the nominal’s denotation measure 1 with respect to some relevant measure \(\mu\). In English, we saw that \(\mu\) is supplied by the head of the complement of #; in the case of cardinal numerals, cardinality determines morphological number. With measure terms, \(\mu\) is supplied by the measure term itself; this accounts for why morphological number on these nouns is sensitive solely to the numeral present. These measures cohere on the basis of determining quantity-uniform predicates, as in (53).

(53) *Quantity-uniform:*

\[ QU_\mu(P) = 1 \iff \forall x \forall y[ P(x) \land P(y) \rightarrow \mu(x) = \mu(y) ] \]

The full semantics for the English #-heads thus checks the one-ness presupposition of SG against quantity-uniform measures:

(54) *English #-heads:*

a. \([\text{SG}] = \lambda P: \forall \mu \forall x \in P[ QU_\mu(P) \rightarrow \mu(x) = 1 ]. P \]
b. \([\text{PL}] = \lambda P. P \]

Cross-linguistic variation in patterns of number marking falls out once we allow variation in the selection of \(\mu\): In English, \(\mu\) is relativized to the head of #’s sister; in Turkish, where all numerals occur with singular-marked nouns, \(\mu\) is relativized to the phrasal complement of # on the basis of relative P-atoms. In other words, Turkish SG evaluates its one-ness presupposition on the basis of relative atomicity via the measure in P-atoms, defined in (55).

(55) \(\mu_{P\text{-atom}}(y)\) is defined only if \(y \in P\); when defined:

\[ \mu_{P\text{-atom}}(y) = |\{ x \in P: x \leq y \land \neg \exists z \in P[ z < x ] \}| \]
(56)  *Turkish #-heads:*

a. \[ [\text{sg}] = \lambda P: \forall x \in P[\mu_{P-\text{atom}}(x) = 1] . P \]
b. \[ [\text{pl}] = \lambda P. P \]

Numeral-modified nominals are quantized such that every member of the predicate is a smallest member, so we correctly predict *sg* with all numerals when one-ness is tied to \( \mu_{P-\text{atom}} \).

In Western Armenian, where the pattern of number marking is intermediate between the English and the Turkish systems, there is optionality in the selection of \( \mu \): either the head or the phrasal approach may apply. Our account of this variation makes do with a uniform syntax and semantics for numerals across these languages (cf. the variation in numeral semantics proposed in Bale et al., 2011a) within a standard semantics framework (cf. the OT account of Farkas and de Swart, 2010).

While the focus of this chapter has been the semantics of morphological number and its interaction with numerals, the claims put forth carry consequences for theories of measurement more broadly. We implicitly took *kilo* to stand in for all measure terms, but distinct subclasses of quantizing nouns have been identified, including container nouns (e.g., *glass*; Partee and Borschev, 2012) and English-style classifiers (e.g., *grain*; Chierchia, 1998a). The next step is to see how these subclasses of measure terms behave within the proposed framework, and whether they in fact possess distinct semantics. This is the topic of the next chapter, which locates measure terms like *kilo* within a typology of quantizing nouns.
Chapter 3

A Typology of Quantizing Nouns

The previous chapter proposed a mechanism by which the number marking on measure terms is determined. The account focused on the word kilo, which was meant to stand in for any measure term. However, we have yet to establish what it means to count as a measure term. Furthermore, we lack a comprehensive description of their distribution, as well as of the interpretations they yield. More importantly, measure terms like kilo inhabit the broader class of quantizing nouns: those nouns that facilitate the counting or measuring of stuff. We therefore begin this chapter by investigating proposed examples of quantizing nouns from the literature. We consider measure terms (e.g., kilo; Lønning, 1987), container nouns (e.g. glass; Partee and Borschev, 2012), and atomizers (e.g., grain; Chierchia, 1998a), examining whether these proposed subclasses in fact possess distinct semantics. We find support for attributing different semantics to each subclass after investigating the different readings that result from their respective uses (e.g., Greenberg, 1972; Selkirk, 1977; Doetjes, 1997; Chierchia, 1998a; Landman, 2004; Rothstein, 2009, 2010b). Three interpretations are considered: the CONTAINER reading yielded by a container noun, the MEASURE reading resulting from a measure term, and the ATOMIZING reading of atomizers.¹

The proposed account of these differences is based on a functional distinction between container nouns, measure terms, and atomizers: with a MEASURE reading, measure terms project M(easure)P and feature the semantics of a measure, as proposed in the preceding

¹To prefigure the case study of the degree noun amount in Chapter 4, each of these three readings is definite in the sense that they reference salient, real-world objects.
chapter, Section 2.4.2; with an container reading, container nouns project NP and have the semantics of basic noun, modified by a prepositional phrase specifying the substance measured. Atomizers serve yet a different function, behaving as transitive nouns and atomizing, or partitioning their nominal relata.

With a better understanding of the readings admitted, together with the conditions that govern their distribution, we revisit the transitive/intransitive distinction from Section 2.4.2, wherein measure terms optionally appear with a complement noun. We then consider the optional co-occurrence of a numeral. The three subclasses of quantizing nouns proposed pattern differently with respect to both phenomena: container nouns optionally appear without a substance noun or numeral, measure terms must appear with a numeral, and atomizers must appear with a substance noun. The restriction on these uses is shown to follow straightforwardly from the semantics given to the different subclasses of measure terms.

3.1 Identifying the object of study

In our examination of number marking on measure terms in the previous chapter, we identified measure terms as words that specify a measure, that is, a relation between individuals and natural numbers: $\mu_{kg}$ in the case of kilo or $\mu_{lb}$ in the case of pound. Another way to conceive of measure terms is as a means by which a substance in quantized – that is, packaged – for the purpose of measuring or counting.\(^2\) In this sense, kilo allows for the measurement of a substance using the standard kilogram unit. A pseudo-partitive like three kilos of apples identifies quantities of apples that, when measured by the kilogram measure, evaluate to 3. With this conception of measure terms as quantizers, what do we make of words like grain, slice, quantity, etc., that, perhaps more directly, enable the counting of a substance (Chierchia, 1998a)? Should these words form a class with measure terms like kilo (Lønning, 1987)? How about nouns like cup or bowl, which appear to implicate measurement in their semantics (Partee and Borschev, 2012)?

As we shall see, delimiting the class of quantizing nouns is not a straightforward endeavor.

\(^2\)Counting is a special case of measuring: individuals are related to natural numbers (which happen to be restricted to the integers). With counting, these numbers correspond to a cardinality.
We begin with uses of the descriptors ‘measure term’, ‘atomizer’, and ‘container noun’. In our attempt to offer concrete ways of understanding these terms, we will find that the proposed classifications are at times ephemeral, allowing for a great deal of overlap. This observed transience of definitions suggests a similar haziness in the distinctions that underlie them. However, an examination of the behavior of these terms supports treating each descriptor as identifying a unique and well-defined class of words, inhabiting the broader class of quantizing nouns. Let us begin with the most unique of these subclasses: atomizers.

Chierchia (1998a) distinguishes between measure terms like *kilo* and what he calls ‘classifiers’ like *grain* or *drop*. Before proceeding, it bears noting that traditional characterizations of classifier languages would have English lack classifiers altogether (e.g., Greenberg, 1972; Allan, 1977a,b; Denny, 1976, 1979; Adams and Conklin, 1973, among many others). Classifiers are taken to be an epiphenomenon of classifier languages: a closed, small, contrasting set of morphemes that designate countable units; classifier languages are those that require these morphemes in the presence of numerals for the purpose of counting the referents of nouns. The surfeit of words in English that (optionally) serve the purpose of enabling counting (i.e., what we are considering as quantizing nouns), together with the language’s ability to directly count with a numeral, suggests a substantial divide between English and classifier languages. However, as we shall see and as the discussion in Greenberg (1972) (see also Lehrer, 1986) identifies, within the candidate set for quantizing nouns that we consider, English does possess a class of expressions that align with the definition of ‘classifier’, at least to the extent that any term in a non-classifier language can. In an attempt to recognize the theoretical distinction between classifier languages and number marking languages like English, we therefore adopt the term ‘atomizer’ for what others have called English classifiers. Once we consider the semantics for atomizers, the motivation for this naming convention will become more clear.

While both measure terms and atomizers inherently relate individuals or quantities with numbers, as evidenced by their free use in the pseudo-partitive frame, Chierchia points out that atomizers impose complex selectional restrictions on the nouns with which they com-
pose.\textsuperscript{3} For example, the atomizer \textit{grain} requires a substance structured with specific dimensional properties (e.g., small, cylindrical, inanimate bodies), hence its inability to compose with amorphous \textit{water} or animate and inappropriately large \textit{men}:

\begin{enumerate}
\item \textit{Four grains of that water/those men} \hfill (Chierchia, 1998a)
\end{enumerate}

Whereas atomizers necessitate certain properties of the nouns and corresponding substances with which they compose, measure terms enjoy a much freer distribution. Still, the use of measure terms is not completely unconstrained. For example, \textit{kilo} requires the substance referenced by its nominal relata to possess the capacity for mass, just as \textit{degree} requires a capacity for temperature. These ontological requirements, however, operate at a fundamentally different level from constraints imposed by classifiers: intuitively, the former constrain the domain of these relations, while the latter constrain their range.

In an attempt to characterize and thus predict the differences between atomizers and measure terms, Chierchia (1998a) attributes different functions to these two classes of words. Atomizers are construed as (partial) functions from pluralities into sets of atoms constituted by members of the pluralities. Thus, \textit{grain} qua function applied to some substance, say rice, returns the set of rice atoms with the appropriate spatial properties. Instead of mapping to or constructing atoms, measure terms receive a semantics under which they are (partial) functions from individuals (plural or atomic) into the set of (non-negative) real numbers. Here it is important to note that Chierchia treats measure terms as measures proper. Applying \textit{kilo} to an apple individual, the result is the kilo measure applied to that individual. As functions into sets of atoms, nominal phrases featuring atomizers, e.g., \textit{two grains of rice}, reference individuals, whereas measure phrases, e.g., \textit{two kilos of apples}, reference something more abstract like number or extent along some scale.

This differentiation of function between atomizers and measure terms stands to explain the differences in selectional restrictions observed above. Because atomizers necessarily access atoms, they may constrain those atoms along certain dimensions. Viewed as a relation between individuals (atomic or otherwise) and abstract numbers, measure terms have no op-

\textsuperscript{3}The label ‘pseudo-partitive’ is used here merely to indicate the string \textit{[numeral] [quantizing noun] [of] [substance noun]}. The discussion that follows describes how this string results from many different structures, only one of which serves as a candidate for the theory-laden label ‘pseudo-partitive’.
portunity to make demands of the individuals at play beyond requiring that they inhabit the relevant function’s domain. On the basis of this functional distinction, two other differences in behavior between atomizers and measure terms fall out.

First, measure terms combine with a restricted set of quantificational determiners. Concretely, quantifiers that operate over individuals like every, most, or no cannot co-occur with measure terms, (2). Such quantifiers readily compose with atomizers, (3). Once we settle on a semantics of these terms, we will see that the quantifier restriction for measure terms falls out from their number-seeking semantics, a feature absent from the denotation of atomizers.

(2) a. ??I bought every/most/no pound of rice from that store.
   b. ??Most liters of wine in this tank are polluted. (Chierchia, 1998a)

(3) I bought every grain of rice in that store.

Second, atomizers allow for adjectival modification as in (4-a), where the property of being beautiful may be attributed to the slices of pizza themselves. Unlike atomizers, measure terms resist such modification; in (4-b), the most plausible interpretation ascribes beauty to the pizza and not to the pounds thereof.

(4) a. I bought two beautiful slices of pizza.
   b. ?I bought two beautiful pounds of pizza. (Chierchia, 1998a)

As mappings into numbers and not sets of individuals, measure terms expectedly resist quantification and modification on the basis of individuals. The term pound does not reference objects in the world like pile or grain does; we do not require the existence of a real-world entity corresponding to a pound, so attempting to characterize or manipulate pounds as one would objects is inappropriate and therefore disallowed. In fact, one might be led by the referencing of individuals in their semantics to align atomizing phrases with basic count nouns like boy or book, to the exclusion of measure terms. As we shall see, however, true atomizers stand apart from both measure terms and count nouns, which instead are aligned with con-

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4The judgments for object-level quantifiers/modifiers and measure terms are more nuanced than Chierchia (1998a) would lead one to believe. At issue is the ability of measure terms to serve as container nouns, a topic we consider in some detail in the following section.
tainer nouns. Furthermore, this contrast between container nouns and measure terms quickly fades once we recognize that each may be used as an instance of the other: container nouns admit uses as measure terms, and measure terms admit uses as container nouns.

Consider the measure term *liter*. As a number-seeking relation formed on the basis of a measure, *liter* resists quantification on the basis of individuals, (2-b), as well as direct modification. In (5), if we are talking about the quantity of wine that was bought, then it is most natural to view *beautiful* as modifying *wine* and not the *liters* thereof. But already by hedging our language and focusing only on a reading under which a quantity of wine is referenced, we have tipped our hand.

(5) I bought two beautiful liters of wine.

One may readily imagine a situation in which objects directly correspond to the amount specified by *liter*, namely wine bottles, and with this in mind it is possible in (5) to view *beautiful* as modifying these bottles and thus *liters* directly. Here is our first encounter with the container interpretation of a measure term, under which the term functions as a container noun in its reference to individuals. This reading of the measure term contrasts with its measure interpretation, where the term references a quantity of some substance instead of its container. In fact, measure and container readings of measure terms may be distinguished overtly in some languages. Doetjes (1997) provides the following examples from Dutch, in which the measure term *kilo* optionally takes plural morphology.

(6) a. Jan heeft twee kilo pruimen gekocht.
    Jan has two kilo(sg) plums bought
    ‘Jan has bought two kilos of plums.’

b. Jan heeft meer kilo-s pruimen gekocht dan Marie
    Jan has more kilo-pl plums bought than Marie
    ‘Jan bought more kilos of plums than Marie did.’

(Doetjes, 1997)

When singular, the measure term favors a measure interpretation: in (6-a), the speaker references the amount of plums Jan has bought. When plural, the term favors a container interpretation; the preferred reading of (6-b) is one under which the number of individual

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5For present purposes, imagine we inhabit a world in which typical wine bottles have a one liter capacity.
units measuring one kilo (say, packages of plums) is being compared. We return to this issue of morphologically distinguishing measure vs. container interpretations cross-linguistically in Section 3.2.4.

Atomizers like *grain* and measure terms like *kilo* are not the only sorts of words that may be used to quantify a substance for the purpose of counting or measuring. Consider the word *glass* or *bowl* or *box*, or any other name for a container. These words lead fruitful lives as basic count nouns, as evidenced by their non-relational, referential uses in the following example.

(7) Mary put the three glasses and a bowl into the box on her table.

It is no coincidence, however, that these words all reference objects whose role is to contain stuff. It is in this role that these so-called ‘container nouns’ quantify a substance for the purpose of counting and therefore constitute yet another candidate class to be included under the title ‘quantizing noun’.

In (8), we witness that just like atomizers and measure terms, container nouns admit a relational use, combining with a noun and a numeral in the pseudo-partitive frame.

(8) Mary put three glasses of water into her soup.

Container nouns also admit both container and measure interpretations: while the measure interpretation under which Mary puts three glasses-worth of water into the soup is much more plausible in (8), one may also read the sentence as stating that Mary put the glasses themselves into the soup, that is, one may get a container interpretation for this use of *glass*. For another example of the truth-conditional distinction between measure and container readings and thus the flexibility required in the semantics of container nouns, consider the following sentence.

(9) John carried three boxes of books into the store.

Imagine a scenario in which a store sells books by the (standard, moderately-sized) box. John delivers the store’s stock, and in doing so he carries the merchandise to fill three of these salable boxes in a single, very large box. In this scenario the sentence in (9) is true under a
reading example thing named referent

measure
three kilos of apples apples
⊆ substance noun

CONTAINER
three glasses of wine glasses
⊆ quantizing noun

ATOMIZING
three grains of rice grains of rice
⊆ substance noun + quantizing noun

Table 3.1: Interpretations available to quantizing nouns summarized in terms of the sort of object that is referenced.

MEASURE reading and false under a CONTAINER reading: John carried only one box into the store, the contents of which measure 3 with respect to the salient box measure.

Here it bears noting a fundamental difference between the CONTAINER and MEASURE reading of quantizing nouns. Consider the CONTAINER reading of three glasses of wine. The referent is three glasses, which happen to contain wine. In other words, the referent belongs to the class of things named by the quantizing noun glass. Contrast this interpretation with the MEASURE interpretation of three liters of wine. The referent in this case is wine, which happens to measure 3 liters in volume. Under the MEASURE reading, the referent belongs to the class of things named by the substance noun wine. Note further that atomizers stand apart in the reading they admit: three grains of rice references something that is at once both grains and rice; the referent of an atomizer belongs in some sense both to the class of things named by the substance noun and to the class of things named by the quantizing noun. Moreover, the function of the atomizer is to partition the denotation of the substance noun into designated, minimal countable units; this reading we term the ATOMIZING interpretation of a quantizing noun. We will return to the semantics that delivers these interpretations, which are summarized in Table 3.1.

Focusing on the ambiguity between MEASURE and CONTAINER interpretations for container nouns, Rothstein (2009) offers four diagnostics which distinguish between the two readings. We consider each diagnostic in turn, seeing also how measure terms and atomizers fare and noting the differences among the three proposed subclasses of words.

Diagnostic 1. Measure suffixes are appropriate only under the MEASURE reading

To the extent that one can force a MEASURE reading for a container noun, Rothstein (2009)
claims that under this reading the noun allows suffixation with -ful, which she calls a ‘measure suffix’. In (10) and the examples that follow for these diagnostics, the (a) examples are meant to be evaluated under a CONTAINER interpretation and the (b) examples are meant to be evaluated under a MEASURE interpretation. Note that under a CONTAINER interpretation, container nouns refuse measure suffixes.

(10) **Container nouns**

a. Three bucket(#ful)s of mud were standing in a row against the wall.

b. We needed three bucket(ful)s of cement to build that wall.  (Rothstein, 2009)

Unlike container nouns, which permit measure suffixes with a MEASURE reading, measure terms appear entirely incompatible with these suffixes under either reading. In fact, the combination of measure terms with measure suffixes is ruled out altogether (e.g., literful, gallonful, tonful; Lehrer, 1986).

(11) **Measure terms**

a. Three liter(*ful)s of mud were standing in a row against the wall.

b. We needed three liter(*ful)s of cement to build that wall.

Like measure terms, atomizers are incompatible with measure suffixes. To keep the examples as similar to Rothstein’s originals as possible, imagine a context in which we are building miniature rice walls:

(12) **Atomizers**

a. Three grain(*ful)s of rice were standing in a row against the wall.

b. We needed three grain(*ful)s of rice to build that wall.

Intuitively, the problem in (12) feels less like one of the morphological composition of an atomizer with a measure suffix, and more like an inability to force the MEASURE reading that would license the measure suffix in the first place. Whereas (11-b) admits a MEASURE interpretation for liter, in (12-b) a parallel reading is inaccessible, with or without the suffix -ful. We return to this issue in our discussion of atomizers in relation to the next diagnostic.
Diagnostic 2. Plural pronouns may not be anteceded under a MEASURE reading

Under a CONTAINER reading, container nouns may serve as antecedents to plural pronouns, (13-a). This situation cannot attain when container nouns receive a MEASURE interpretation, (13-b).

(13) Container nouns
   a. There are two cups of wine on this tray. They are blue.
   b. There are two cups of wine in this soup. #They are blue. (Rothstein, 2009)

Measure terms behave similarly: only under a CONTAINER interpretation may they serve as antecedents to plural pronouns. Crucially, (14-a) succeeds to the extent that we attribute blueness to the liters (i.e., to bottles), and not to the wine itself.

(14) Measure terms
   a. There are two liters of wine on this tray. They are blue.
   b. There are two liters of wine in this soup. #They are blue.

Performing a similar manipulation on atomizers, we find again that they do not permit a MEASURE interpretation. Despite the effort to force a MEASURE interpretation in (15-b), the atomizer provides a suitable antecedent for they; we remain with an ATOMIZING interpretation for the atomizer, a use which, like the CONTAINER interpretation, provides a suitable antecedent for the plural pronoun.

(15) Atomizers
   a. There are two grains of rice on this tray. They are blue.
   b. There are two grains of rice in this soup. They are blue.

It appears, then, that atomizers resist MEASURE readings altogether.

Diagnostic 3. Singular agreement is impossible under a CONTAINER reading

Under a MEASURE interpretation, plural container nouns allow optional singular agreement,
(16-b); container interpretations force plural agreement when the container noun appears in the plural, (16-a) (but see Lehrer, 1986, for a discussion of the limitations of this diagnostic).6

(16) Container nouns acceptability persists with postposed
a. There *is/are two cups of wine on this tray.
   
b. There is/are two cups of wine in this soup.  
      (Rothstein, 2009)

Measure terms behave like container nouns with respect to agreement: in (17-a), if we imagine a CONTAINER interpretation under which we are referencing two bottles of wine, singular agreement is disallowed. Under the MEASURE reading in (17-b), we allow for singular agreement.

(17) Measure terms
a. There *is/are two liters of wine on this tray.
   
b. There is/are two liters of wine in this soup.

Again, atomizers resist the MEASURE reading altogether, so it is unsurprising that they fail to pattern like container nouns or measure terms in allowing singular agreement with this interpretation. In other words, the issue is again not that atomizers pattern differently with respect to the agreement diagnostic, but rather that they lack the range of interpretations presupposed by the diagnostic.

(18) Atomizers
a. There *is/are two grains of rice on this tray.
   
b. There *is/are two grains of rice in this soup.

Attempting to force the CONTAINER interpretation in (18-a), we find as we would expect that singular agreement is unavailable. However, where we attempt to force the MEASURE interpretation in (18-b), we find that singular agreement remains unavailable. Again, it appears

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6Given the permissive nature of agreement in there-existentials, it is important to note that the pattern of acceptability persists with postposed verbs. Consider the following:

(i) Two cups of wine *is/are needed for this tray.
(ii) Two cups of wine is/are needed in this soup.
that the complication with the atomizer \textit{grain} in (18-b) is not that it performs differently with respect to the diagnostic, but rather that it fails to provide the \textsc{measure} interpretation that the diagnostic evaluates. Instead, (18-b) allows only an \textsc{atomizing} interpretation, which, like the \textsc{container} interpretation, precludes agreement in the singular.

\textit{Diagnostic 4. Distributive operators are incompatible with a measure reading}

The last diagnostic concerns the behavior of the distributive operator \textit{each}: under the \textsc{container} reading, such distributive operators may quantify over the individuals in the denotation of a container noun phrase. Under a \textsc{measure} reading, this quantification is disallowed, presumably because use of the quantizing noun references a single entity: a quantity of wine in (19-b).

\begin{enumerate}[label=(19),start=19]
  \item \textit{Container nouns}
    \begin{enumerate}[label=(\alph*),start=a]
      \item The two cups of wine cost 2 euros each.
      \item \#The two cups of wine in this soup cost 2 euros each. \hfill \textit{(Rothstein, 2009)}
    \end{enumerate}

    Like container nouns, measure terms are compatible with distributive operators only under a \textsc{container} interpretation. For \textit{each} to be acceptable in (20-a), we must understand the sentence as discussing the cost of individual vessels of wine, e.g., wine bottles. Once we understand the measure term as referencing a single quantity, as in (20-b), distributive quantification becomes unacceptable.

  \item \textit{Measure terms}
    \begin{enumerate}[label=(\alph*),start=a]
      \item The two liters of wine cost 2 euros each.
      \item \#The two liters of wine in this soup cost 2 euros each.
    \end{enumerate}

    As with the previous diagnostics, with distributive operators atomizers diverge from the patterns of container nouns and measure terms. Accepting that atomizers do not allow \textsc{measure} interpretations, this divergence is expected: in our failed attempt to coerce a \textsc{measure} reading from \textit{grain} in (21-b), we are left with an \textsc{atomizing} interpretation and so
the use of *each* is appropriate.

(21)   **Atomizers**

   a. The two grains of rice cost 2 euros each.
   
   b. The two grains of rice in this soup cost 2 euros each.

It bears repeating that the acceptable use of *each* with an atomizer in (21-b) signals not an inconsistent result for the diagnostic, but rather a divergence between atomizers on the one hand and container nouns and measure terms on the other with respect to their capacity for a **measure** interpretation. The diagnostics we have considered are formulated with respect to readings and not the terms that yield those readings. Table 3.2 summarizes the results of the diagnostics as applied to **measure** and **container** interpretations.

<table>
<thead>
<tr>
<th>Reading</th>
<th>-ful</th>
<th>they</th>
<th>SG</th>
<th>each</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measure</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>Container</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
</tr>
</tbody>
</table>

Table 3.2: Interpretation diagnostics from Rothstein (2009)

As the results demonstrate, these diagnostics are capable of uniquely identifying, if not forcing either the **measure** or the **container** interpretation. But we must remind ourselves of our original focus: identifying readings is useful only inasmuch as it serves our understanding of what a quantizing noun is.

Among the candidate class of words falling under the blanket label ‘quantizing noun’, certain clusterings suggest themselves and have been assumed in the literature: measure terms like *kilo*, container nouns like *glass*, and atomizers like *grain*. Our task has been to evaluate how meaningful these different classifications are, and how deeply they are reflected in the semantics of these terms. To that end, we have repurposed the diagnostics from Rothstein (2009); Table 3.3 presents their results not as applied to specific readings, but as applied to each of the three proposed subclasses of quantizing nouns. A value of ‘Y/N’ signals that the property called for in the diagnostic optionally holds of the subclass depending on

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7The picture becomes more complicated once we include the results of these diagnostics as applied to the **atomizing** interpretation.
Table 3.3: Diagnostics adapted from Rothstein (2009) and applied to subclasses of quantizing nouns

<table>
<thead>
<tr>
<th>TERM</th>
<th>-ful</th>
<th>they</th>
<th>SG</th>
<th>each</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEASURE TERM</td>
<td>NO</td>
<td>Y/N</td>
<td>Y/N</td>
<td>Y/N</td>
</tr>
<tr>
<td>CONTAINER NOUN</td>
<td>Y/N</td>
<td>Y/N</td>
<td>Y/N</td>
<td>Y/N</td>
</tr>
<tr>
<td>ATOMIZER</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
</tr>
</tbody>
</table>

whether it receives a MEASURE or a CONTAINER reading; to interpret Y/N values, refer to Table 3.2.

The most striking feature of the value distribution in Table 3.3 is the clear split between atomizers like grain on the one hand and measure terms and container nouns on the other. As we saw, atomizers resist a MEASURE reading, setting them apart from the measure terms and container nouns. For this reason alone we ought to treat atomizers as a distinct subclass. We return to the properties of this subclass when we adopt a semantics for atomizers in Section 3.3.

Comparing the properties of measure terms and container nouns in Table 3.3, we find considerable overlap. Because both classes allow either MEASURE or CONTAINER interpretations, their behavior as antecedents of plural pronouns, with optional singular agreement, and in allowing quantification by distributive operators is determined solely by the reading they receive. According to Table 3.3, there is only one difference between these two subclasses of words, but it is an important one: measure suffixes are impossible on measure terms.

Consider the role of measure suffixes: -ful affixes to a noun to form a measure term, identifying the quantity that can be held by members of the denotation of that noun. From the Oxford English Dictionary:

(22) -ful: a suffix forming derivatives with the general sense ‘quantity that fills or would fill’ (something); it may be attached at pleasure to any noun denoting an object that can be regarded as holding or containing a more or less definite quantity of anything

(OED Online)

Thus, glassful references the amount a relevant glass can hold, bucketful references the amount a relevant bucket can hold, and so on. By referencing containers, container nouns provide a
ready source for this derivation of a quantity. But what about measure terms?

We have characterized the function of a measure suffix as transforming a property of individuals (e.g., the property of being a glass) into a quantity of substance derived from the volume those individuals may contain (e.g., the amount that would fill a glass). For a measure suffix to be felicitously applied to a measure term, the measure term would have to reference objects with a capacity for containing, but we saw that measures terms reference individuals only indirectly on the basis of world knowledge about associations between the standard unit size relevant from its measure use and the objects that instantiate that unit (e.g., wine bottles in the case of liter). It appears, then, that the measure use of a measure term is somehow prior, and as such measure suffixes cannot apply to these terms. Conversely, the container interpretation of container nouns precedes their measure uses, which are derived via a process similar to that of -ful suffixation (e.g., glass qua quantity corresponds to the capacity of the relevant glass).

Here is the claim: measure terms and container nouns are functionally distinct. One is a measure (i.e., a relation) and the other is a simple predicate; they yield measure and container readings, respectively. When we have a container noun with a measure reading, it is functioning as a measure term. When we have a measure term with a container reading, it is functioning as a container noun. Measure terms shift to simple predicates and yield container interpretations to the extent that there exists a natural correspondence between the measure they specify and the objects that normally instantiate its units. Container nouns shift to relational measures and yield measure interpretations to the extent that there exists a natural correspondence between the objects they reference and the units of a measure.

We have thus identified three subclasses of quantizing nouns: container nouns, measure terms, and atomizers. We have also identified three distinct interpretations: measure, container, and atomizing. Now we must provide a semantics for these quantizing nouns to yield the correct interpretations. We begin with container nouns and measure terms in the next section.
3.2 The semantics of **CONTAINER** vs. **MEASURE** readings

We start with the proposed correspondence between word class and interpretation diagramed in Fig. 3.1. Solid lines indicate a direct relationship between term and interpretation; dashed lines indicate the possibility of deriving one use from the other.

![Diagram](image)

**Figure 3.1:** Relationship between quantizing nouns and the interpretations they yield

Recall that by ‘container noun’ we indicate words that freely admit non-relational, referential uses and denote naturally-occurring containers, or objects with the capacity for holding something inside of them. Examples of container nouns include *glass*, *bowl*, and *box*. Measure terms are words that express a standard unit of measure such as *kilo* or *pound* or *liter* (they are at times also called ‘amount terms’).

A **CONTAINER** interpretation is that under which a quantizing noun is used as a relation between a plurality or substance and objects containing it; the interpretation is therefore referential in the sense that the resulting denotation references concrete objects (that happen to be containers). Under a **CONTAINER** reading, (23) states that Mary carried two objects, each of which was a glass containing water.

(23) Mary carried two glasses of water.

**CONTAINER** interpretations contrast with **MEASURE** interpretations in that the latter reference real-world objects only insomuch as they measure some abstract amount. The quantizing nouns functions as a relation between the specified extent of some measure (i.e., a number) and individuals that evaluate to that extent with respect to the measure. Under a **MEASURE** reading, (24) states that Mary carried a quantity of water whose mass measures two kilos.

(24) Mary carried two kilos of water.

As the diagram in Fig. 3.1 specifies, **CONTAINER** interpretations result from uses of container
nouns and measure interpretations from measure terms. Our first task is to settle on a semantics for container nouns and for measure terms so that the appropriate readings result. However, the story does not end with a semantics yielding container interpretations for container nouns and measure interpretations for measure terms. In addition to identifying default interpretations, Fig. 3.1 illustrates the second component of the proposal, namely the flexibility of these terms’ uses.

Recall that container nouns and measure terms each admit uses as the other, as evidenced by the examples in (5) and (8), repeated in (25).

(25)  a. Mary put three glasses of water into her soup.
    b. I bought two beautiful liters of wine.

The most natural interpretation of (25-a) has Mary putting a quantity of water that measures three glasses into the soup, and not the glasses themselves. (25-a) thus evidences a measure reading for the container noun glass. Similarly, (25-b) evidences a container reading for the measure term liter: under its most natural interpretation, the sentence states that the speaker bought two things, each of which is a liter of wine (i.e., an object independent of the wine it contains) that is beautiful. We therefore must specify the means by which container nouns function as measure terms to yield a measure reading and measure terms function as container nouns to yield a container interpretation. We begin with the semantics for the basic terms, then we turn to the process by which each use is derived from the other.

3.2.1 CONTAINER semantics

CONTAINER readings result from uses of container nouns; this section provides a semantics for container nouns that yields the CONTAINER reading. As we consider the choice points that determine our approach, we must recognize that container nouns are in fact nouns, such that CONTAINER interpretations result from a nominal semantics. But nouns may be characterized by whether or not they are relational, or complement-taking (cf. the distinction between ‘sortal’ and ‘relational’ nouns in Löbner, 1985). In what follows, we consider two approaches that differ in whether or not they treat the semantics of container nouns as relational.
Let us begin by identifying the ingredients of the CONTAINER reading that results from use of a container noun. The phrase *glass of water* includes three elements: the container noun *glass*, the particle *of*, and the substance noun *water*. Notice that we remain agnostic regarding the categorial status of *of* by referring to it as a particle and not a preposition (see Chomsky, 1981, as well as the discussion in Rothstein, 2009); the role that *of* plays will depend on the analysis we give to the first element, the container noun. The last element, *water*, is a noun identifying the substance held within the relevant container.

In analyzing container nouns, the first tack is to treat their semantics as non-relational and derive the CONTAINER use, by which we specify both a container and its contents, via modification by the *of*-phrase (see Rothstein, 2009, for a similar proposal concerning CONTAINER interpretations of the Hebrew free genitive construction). Note that the modification implicated in such an approach cannot (straightforwardly) be intersective: a glass of water is not at once both a glass and water. We therefore require modification of a sort that capitalizes on the fact that these nouns in their basic use denote containers, and therefore attributes to these containers the property of being filled by the relevant substance.

This non-relational semantics ascribes an invariant predicative type to container nouns; they denote a set of containers, as in the case of *glass* in (26).

\[ (26) \quad \text{[glass]} = \lambda x. \text{glass}(x) \]

To derive the CONTAINER interpretation from this predicate semantics for container nouns, we may either call upon a novel process of modification between the container noun and the substance noun, or we may build this novelty into the particle *of*. In order to keep our set of composition rules constrained, we pursue the latter option, attributing to *of* the semantics of containing in (27). As such, we treat *of* as a preposition that takes the substance noun as a complement. No special semantics need be assumed for the substance noun; given the restriction against singular count nouns occurring in this position, substance nouns are

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8Use of the label ‘substance noun’ is not intended to convey that this element must be a mass term; CONTAINER interpretations also result with plural count nouns serving as the substance noun, as in *glass of rocks*. The label simply conveys that the substance noun provides the contents of the container.

9For discussion of the motivation behind keeping our set of composition rules constrained, see Scontras and Nicolae (to appear).
treated semantically as kinds, (28). This restriction to kind-denoting substance nouns is built into the semantics for *of*.

\[(27) \quad \text{[of]} = \lambda k \lambda x. \exists y \left[ y \cap k(y) \land \text{filled-with}(y)(x) \right]\]

\[(28) \quad \text{[water]} = \text{WATER} = \cap \lambda x. \text{water}(x)\]

Composing *of* with the substance noun, we create a property of individuals that are filled with the relevant substance. Here is where we may appeal to intersective modification: the *of*-phrase and the container noun, each a predicate of individuals, compose to yield a new predicate of individuals: those members of the denotation of the container noun that are filled with instances of the substance noun.

\[(29) \quad \text{[glass of water]} = \lambda x. \text{glass}(x) \land \exists y \left[ y \cap \text{WATER}(y) \land \text{filled-with}(y)(x) \right]\]

In this way, *glass of water* is true of an individual just in case it is a glass that is filled with water, and we derive the interpretation without a relational semantics for a container noun.\(^\text{10}\) In favor of this approach is its ability to handle both CONTAINER readings of container nouns, as in (29), as well as basic uses where no substance noun is projected. In each case the semantics of the container noun itself remains unchanged.

Having developed a non-relational approach to container noun semantics, we now consider a different tack, treating the semantics of container nouns as relational and deriving their CONTAINER use via argument saturation by the *of*-phrase. The process is straightforward: rather than attributing the semantics of containing to *of*, this relation must be built into the semantics of the container noun itself. Thus, we lift the predicative type of the basic noun so that it takes the substance noun as an argument. In other words, the container noun is treated as semantically transitive. Under this relational approach, *of* contributes nothing to the resulting semantics.

\[(30) \quad \text{[glass]} = \lambda k \lambda x. \text{glass}(x) \land \exists y \left[ y \cap k(y) \land \text{filled-with}(y)(x) \right]\]

\(^\text{10}\)Partee and Borschev (2012), in response to a similar semantics for container nouns proposed by Rothstein (2009), question whether the semantics in (29) actually captures the individual interpretation (what they call the ‘Container + Contents’ interpretation). At issue is whether we want our semantics to refer to both the container and its contents, or merely to the container, which is in turn characterized by its contents. We refer the reader to Partee and Borschev (2012) for the relevant discussion.
Note that both the relational and non-relational approaches yield the same denotation for the container use of a container noun (compare (29) and (31)).

The relationship between container uses of container nouns (where they denote containers filled with the substance noun) and basic uses (where they simply denote containers) requires spelling out. History tells us that the former derives from the latter, such that basic uses precede container uses (cf. the etymological discussion of cup and gallon in Partee and Borschev, 2012). As we will see, the complexity necessitated by this relationship suggests the first, non-relational approach to container noun semantics over the relational one we now consider.

In her account of the Hebrew Construct State, Rothstein (2009) derives what we are calling container interpretations for container nouns via a type-shifting operation that transforms predicate-denoting container nouns into relations of type $\langle\langle e, t \rangle, \langle e, t \rangle\rangle$, as in (30). The operation Rothstein proposes is CS-SHIFT ('Construct State Shift'), reproduced in (32).

\[\text{CS-SHIFT}([\lambda x. N(x)]) = \lambda P \lambda x. \exists y [N(x) \land P(y) \land R(x, y)]\]

CS-SHIFT applies to a simple noun, type $\langle e, t \rangle$, and transforms the predicate into a relation between predicates and individuals. In (32), $P$ is the predicate that the container noun takes as an argument, and $R$ is a contextually specified relation. For container uses of container nouns, Rothstein takes $R$ to be the contain relation (cf. our filled-with relation). The process of deriving a container interpretation proceeds as in (33).

\[\begin{align*}
\text{a. } & [\text{glass}] = \lambda x. \text{glass}(x) \\
\text{b. } & \text{CS-SHIFT(glass)} = \lambda P \lambda x. \exists y [\text{glass}(x) \land P(y) \land \text{contain}(y)(x)] \\
\text{c. } & \text{CS-SHIFT(glass)(water)} = \lambda x. \exists y [\text{glass}(x) \land \text{water}(y) \land \text{contain}(y)(x)]
\end{align*}\]

The result of applying CS-SHIFT to a container noun with contain as the relevant relation, (33-b), is equivalent to the output of our relational semantics (cf. (30); but see Partee and Borschev, 2012, for discussion of the nuanced difference between the contain and

\[\text{Note that Rothstein treats the substance noun as a predicate, rather than as a kind as we have done.}\]
filled-with relations). We end up with a subset of the individuals in the denotation of the basic use of the container noun, namely those that contain some quantity within the denotation of the substance noun. With an operation like CS-SHIFT, we thus specify the way that container nouns relate to their non-relational uses: the latter are prior, related to the CONTAINER interpretation via a type-shifting operation that transforms a predicate-denoting noun into a relation.

In Hebrew, the language Rothstein uses to motivate CS-SHIFT, the Construct State’s container interpretation does not feature any particle on a par with the of in English container uses. Instead, the Construct State directly joins the container and substance nouns as in (34).

(34) šalosh kosot mayim
    three cup(f.pl.) water
    ‘three cups of water’

(Rothstein, 2009)

Without any prepositional element to tie the semantics of the contain relation to, a non-relational semantics for container nouns appears ill-fated. Thus, Rothstein derives the relational semantics on the basis of CS-SHIFT. But in English we do have evidence for an overt source of the containing relation: of. By attributing this relation to of and keeping the semantics of container nouns unambiguously predicate-denoting, we save ourselves the trouble of stipulating an operation like CS-SHIFT that would yield the desired ambiguity between non-relational, basic uses and relational, CONTAINER uses of container nouns. For this reason, we might want to settle on the non-relational approach to container nouns that we first pursued.

Another consideration in deciding between a PP-modification approach to CONTAINER interpretations and Rothstein’s type-shifting approach concerns the special status of the syntactic frame that yields these interpretations, which at least superficially appears to be the pseudo-partitive. Selkirk (1977) provides a bevy of facts to support her conclusion that pseudo-partitives possess a syntax distinct from true partitives, where the latter involves modification of a noun by a full-fledged PP. Crucially, in pseudo-partitives there is a tighter relationship between the quantizing noun and the substance noun than PP-modification
would allow. To see this distinction, we quickly review Selkirk’s facts.

First, partitives and pseudo-partitives are distinguished on the basis of the ‘Partitive Constraint’, which states that the embedded NP in a true partitive must be specific. Simply put, the substance noun in a partitive must be definite. We thus superficially distinguish the partitive in (35-a) from the pseudo-partitive in (35-b), where the former has a definite substance noun and the latter has instead a mass or plural count noun.

\[(35) \quad \begin{align*}
\text{a. Mary carried three bowls of that soup into the dining room.} & \quad \text{(partitive)} \\
\text{b. Mary carried three bowls of soup/beans into the dining room.} & \quad \text{(pseudo-partitive)}
\end{align*}\]

According to Selkirk, true partitives allow extraposition of the of phrase (i.e., of the substance noun); pseudo-partitives do not. She provides the examples in (36) to illustrate this contrast. Note that the following sentences feature a container noun used with a measure interpretation: the sentences are about eating fudge, not boxes.

\[(36) \quad \begin{align*}
\text{a. They devoured seven boxes of} & \quad \text{\lbrack yourØ\rbrack delicious fudge last night.} \\
\text{b. They devoured seven boxes last night of} & \quad \text{\lbrack yourØ\rbrack delicious fudge.}
\end{align*}\]

When we modify Selkirk’s examples so that they yield a container interpretation, as in (37), suddenly extraposition of the substance noun succeeds.

\[(37) \quad \begin{align*}
\text{a. They bought seven boxes of} & \quad \text{\lbrack yourØ\rbrack delicious fudge last night.} \\
\text{b. They bought seven boxes last night of} & \quad \text{\lbrack yourØ\rbrack delicious fudge.}
\end{align*}\]

In fact, Selkirk herself recognizes the difference between container and measure uses of quantizing nouns. Only the latter, she claims, project pseudo-partitive syntax. For this reason, the remainder of Selkirk’s diagnostics for distinguishing pseudo-partitive from partitive constructions do not apply to container interpretations. In light of the current discussion, we may interpret Selkirk’s claim as stating that container interpretations do not feature a
direct relationship between container and substance nouns, consistent with the extrapolation facts in (36) and (37). In other words, a PP-modification structure for CONTAINER interpretations appears to be justified; these interpretations do not result from true pseudo-partitive structure, but rather from simple adjunction.

To summarize, CONTAINER readings result from the composition of three elements: a container noun, the lexical preposition of, and a substance noun. Example semantics for these elements are repeated in (38).

(38)  
a. \([\text{glass}] = \lambda x. \text{glass}(x)\)

b. \([\text{of}] = \lambda k \lambda x. \exists y[^{\cup}k(y) \land \text{filled-with}(y)(x)]\)

c. \([\text{water}] = \text{WATER}\)

d. \([\text{glass of water}] = \lambda x. \text{glass}(x) \land \exists y[^{\cup}\text{WATER}(y) \land \text{filled-with}(y)(x)]\)

Our semantics, where the container noun, an N head, is modified by the PP headed by of, suggests a structure in which the container noun projects NP, to which a PP adjoins. The syntax for the CONTAINER interpretation of a container noun appears in (39).

(39)  CONTAINER structure:

```
NP
   NP     PP
    |       |
glass  P   nP
    |       |
of    water
```

Here a note is in order on the categorial status of the substance noun. In all of the uses of quantizing nouns that we have an will consider, the substance noun is either bare plural or mass, but never a singular count noun. We encode this restriction by taking the substance noun to be kind-denoting. In English, we cannot tell whether this kind-denoting substance noun projects DP or some functional layer below DP. We do know that the substance noun hosts morphological number, determined by #P, so minimally it must contain more structure
We therefore face a choice: either the substance noun projects DP with a null D responsible for kind-formation, or it projects a sub-maximal functional layer. The assignment of case (or lack thereof) to this sub-maximal projection would then be tied to kind-formation. Given the stable absence of determiners on substance nouns both in English and cross-linguistically (even French, notorious for the obligatory use of determiners, has no D on a substance noun), we label the substance noun nP to signal the sub-maximal nominal the substance noun projects (but keeping in mind the caveats discussed).

Returning to container noun phrases, as an NP the system of number morphology developed in the previous chapter handles a container noun as it would any basic noun (cf. Section 2.4.1). Container noun phrases denote sets of individuals, and their elements are counted by cardinal numerals formed on the basis of the M₀ head card, which takes the container noun phrase as its complement. Number morphology is determined by the # head, which projects above MP; pl must be used because MP will contain in its denotation elements with cardinality greater than 1 (i.e., elements with cardinality 3). The full nominal structure is given in (40).

(40) Counting container nouns:

![Diagram](image)

---

12Note further that the substance noun can be modified, as in three glasses of water from the tap.
We thus have an account of the first aspect of our proposal, namely the correspondence between container nouns and container interpretations. Additionally, we specify the relationship between basic and container uses of container nouns: the nouns’ semantics is invariantly predicative, and whether or not we modify the noun with a prepositional phrase determines the use we observe.

Next, we turn to the correspondence between measure terms and the measure interpretation.

3.2.2 MEASURE semantics

As in the previous subsection, here we begin by identifying the ingredients of the measure reading. We saw that these readings result from uses of measure terms, as in three liters of water. We thus have the measure term liter, the particle of, and the substance noun water. On the surface, all that differs between container and measure readings is whether we have a container noun or a measure term – in other words, whether the quantizing noun names a container or a measure. In what follows, we develop a semantics for measure terms that yields the measure reading. We are guided by the observation that measure terms specify measures (e.g., μkg in the case of kilo, μlb in the case of pound, etc.), so measure interpretations ought to result from measure semantics. In fact, we encountered a semantics for measure terms in the previous chapter (Section 2.4.2), as part of the proposed account of number marking on measure terms. The semantics are restated below, and reevaluated in light of the current discussion.

Throughout our comparison of the container and measure interpretations, we have contrasted them on the basis of whether they are referential: container uses refer to objects (i.e., containers), while measure uses refer to amounts. This contrast is misleading: both glass of water under a container interpretation and liter of water under a measure interpretation refer to objects. What differs is that in the former we refer to an object in the denotation of the quantizing noun (i.e., to a glass) and in the latter we refer to an object in the denotation of the substance noun (i.e., to a quantity of water). In this sense, both container and measure uses are referential, referring to real-world objects; when we say
that MEASURE uses specify amounts, take this as shorthand for specifying the amount that delimits the denotation of the resulting phrase, which contains individuals. This distinction will become crucial in the next chapter when we encounter the degree noun amount.

As we develop a semantics for measure terms that yields their MEASURE interpretation, we must first take into account the distribution of these terms. Of particular importance is the close tie between measure terms and numerals. In every use we have so far encountered, measure terms either explicitly or implicitly call on a numeral to provide a value for the measure that will then constrain the denotation of the substance noun. As we saw in Section 2.4.2, in the phrase three liters of water, we restrict the denotation of the substance noun water to just those quantities of water than measure 3 liters. In a liter of water, no numeral is expressed, but still we understand the phrase as identifying quantities of water that measure a specific value, namely 1, in liters. These facts demonstrate that measure terms operate on numbers in their semantics. In fact, we saw in the previous chapter how this assumption allows us to integrate measure terms into the syntax and semantics of measuring more broadly.

The proposed semantics for measure terms has them take two arguments, the substance noun and the numeral, and yield a set of instances of the substance noun, namely those individuals that return the value of the numeral when measured (cf. the ‘unit of measure’ semantics given in Partee and Borschev, 2012). Consider the semantics of liter in (41).

(41) \[[\text{liter}]\] = \(\lambda k \lambda n \lambda x. \bigcup k(x) \land \mu_{\text{li}}(x) = n\)

When it composes with a substance noun and a numeral, the measure term yields the MEASURE reading: a set containing elements that measure the appropriate amount and instantiate the substance noun. The predicate in (42) denotes a set of water quantities, each measuring three liters in volume.

(42) \[[\text{three liters (of) water}]\] = \(\lambda x. \bigcup \text{water}(x) \land \mu_{\text{li}}(x) = 3\)

This semantics for measure terms differs from the semantics given to container nouns in two important respects. First, measure terms receive a relational semantics, whereas container nouns are treated as simple predicates. This relational semantics for measure terms precipitates the second difference: of in a MEASURE use is introduced syncategorematically,
contributing no semantic content (see Schwarzschild, 2006, for a discussion of this treatment of *of* in constructions with measure terms). Recall that for CONTAINER interpretations we treat *of* as a lexical preposition contributing the semantic **filled-with** relation.

Additionally, we attribute to measure terms a categorial difference: whereas container nouns are in fact nouns, projecting NP, measure terms are at base measures, projecting MP. The trees in (43) illustrate the structural divergence that results from this categorial difference.\(^\text{13}\)

(43) a. *Container noun structure:*

```
(43) a. Container noun structure:
  #P
    #
      MP
        |  
        Numeral  M'
          |  
          3  M  NP
            |  
            CARD  NP  PP
                  |  
                  glass  P  nP
                        |  
                        of  water
```

b. *Measure term structure:*

```
(43) b. Measure term structure:
  #P
    #
      MP
        |  
        Numeral  M'
          |  
          3  M  (of)  nP
            |  
            kilo  water
```

Again, numerals under a CONTAINER interpretation function as cardinals, just as they do with

\(^{13}\)Keep in mind that we label the substance noun as *nP* to signal that it projects a sub-maximal nominal layer.
basic nouns (cf. Section 2.4.1). The cardinal is formed on the basis of \textsc{card}, which heads MP and takes the container noun NP as a complement. Number morphology is determined via the process described in the previous chapter: \textsc{sg} checks for singularity of the elements of the nominal denotation on the basis of cardinality, the measure specified by the M-head \textsc{card}.

By projecting MP rather than serving as its complement (as in the case of container nouns), measure terms preclude the use of \textsc{card} and thus the use of cardinal numerals. But this is as it should be: under a \textsc{measure} reading the numeral is not a cardinal. In \textit{three liters of water}, the numeral \textit{three} does not count individuals. Instead, the numeral specifies the requisite value of the relevant measure, \(\mu_i\) (see Rothstein, 2009; Landman, 2004, for discussion of the same observation).

Recall how number marking works on measure terms (cf. Section 2.4.2). As an instance of \(M^0\), measure terms serve as the head closest to \# and so \textsc{sg} checks for singularity of the elements of the nominal denotation on the basis of the measure specified by the measure term. In \textit{three liters of water}, \(\mu_i\) serves as the measure for which the elements of MP must evaluate to 1; with \textit{three} as the numeral argument of \textit{liter}, the one-ness presupposition of \textsc{sg} fails and so \textsc{pl} must be used, resulting in plural morphology on \textit{liter}. The failure results from the fact that \textit{three} ensures that everything in the denotation of MP evaluates to 3 with respect to the measure in \(M^0\), so there is no hope of these elements satisfying the one-ness presupposition of \textsc{sg}.

We have thus accounted for the second component of the proposal in Fig. 3.1: Measure terms are endowed with a relational semantics that essentially restricts the denotation of the complement substance noun on the basis of the value provided by their second argument, the numeral. The resulting denotation is a set of instances of the substance noun, namely those individuals that evaluate on the basis of the relevant measure to the necessary extent. Here is our \textsc{measure} reading, which derives from the semantics of measure terms. By using the measure term to restrict the denotation of the substance noun, we successfully derive the crucial component of the \textsc{measure} reading: \textit{three liters of water} refers to a quantity of water.
At this point we have a semantics for container nouns and measure terms that delivers container and measure readings, respectively. Next, we need an explanation for the observed variability in the uses of these terms such that each may serve as the other. In other words, we need the means to transform a container noun into a measure term, and vice versa.

3.2.3 Deriving one use from the other

Recall the proposed correspondence between word class and interpretation, repeated in Fig. 3.2. Solid lines indicate an implicational relationship; the dashed line indicates functional versatility such that container nouns enjoy uses as measure terms and measure terms may serve as container nouns. We have developed accounts for the implicational relationships in Fig. 3.2 on the basis of the semantics of these terms, which attributes a categorial distinction to the two classes of words. Now, we must account for their categorial versatility.

![Figure 3.2: Relationship between measure terms and interpretations](image)

So far, our account of container and measure interpretations proceeds via direct mappings from the semantics for the terms involved to the corresponding interpretation. Container nouns, together with the filled-with relation supplied by the preposition of, result in a container interpretation such that objects filled with the appropriate contents are referenced. Measure terms, on the basis of the measure they specify and their relational semantics, deliver a measure interpretation such that objects measuring the appropriate extent are referenced. But this account fails to predict the preferred interpretations of the sentences in (44).

(44) a. Mary poured three glasses of water into her soup.
    b. I dropped two beautiful liters of wine.
In (44-a), we have what appears to be a container noun implicated in a measure reading: Mary is not said to have added three containers filled with water into the soup, but rather to have added a single quantity of water. In (44-b), a measure term yields a container reading: beauty is attributed to two objects, whereas a measure reading would have just a single object accessible, namely a quantity of wine measuring two liters.

Given the semantics we have attributed to container nouns and measure terms, it is incoherent to claim that the unexpected readings result from canonical uses of these terms; we must hold fast to the implications in Fig. 3.2 such that a container noun yields a container interpretation and a measure term yields a measure interpretation. Partee and Borschev (2012, p.447) describe this variation as follows: “the distinction [between container nouns and measure terms] is formally sharp, but the classification of nouns is not.” In (44), what we have are unorthodox uses of the words glass and liter. Instead of serving as a container noun, in (44-a) glass functions as a measure term, resulting in a measure interpretation. In (44-b), liter serves as a container noun and yields a container interpretation. At base, these terms are still container nouns and measure terms, that is, they are born within their prescribed class. We therefore must specify how container nouns acquire measure term semantics and how measure terms acquire container noun semantics. We consider each case in turn.

In order to derive a measure term, and thus a measure interpretation from a container noun, we need to shift the basic predicate semantics of container nouns into the relational type attributed to measure terms. The resulting semantics should hang on the measure derived from the standard volume of the container referenced. In the case of glass, we need to shift a set of glasses into a function that measures individuals with respect to the derived glass measure, $\mu_{\text{glass}}$.

That measure terms may derive from container nouns finds support in the diachronic development of measure terms. Consider the case of gallon, which finds its roots in Gaulish gala ‘vessel’ (Partee and Borschev, 2012, who themselves cite an entry in the Online Etymology Dictionary). A perhaps more obvious example is the measure term foot, which derives

from the length of men’s feet. Thus, we have evidence that the synchronic transformation we are considering operates elsewhere in the grammar: from a concrete, real-world object we derive a standard measure. What we lack is a description of the meaning shift that results in this transformation.

In considering the function of the measure suffix -ful in Section 3.1, we already witnessed the transformation of container nouns into measure terms. We described the role of -ful as operating on the denotation of a noun and identifying the quantity that can be held by individual members of that noun’s denotation. In (45), we see a first attempt at the formal description of the derivation of a measure term via -ful suffixation.

\[(45) \quad [-\text{ful}] = \lambda P \lambda k \lambda n \lambda x. \uplus k(x) \land \mu P(x) = n\]

The suffix -ful takes a predicate as an argument and returns a measure term, type \(\langle k, \langle n, \langle e, t \rangle \rangle \rangle\). This shift is category-changing: a noun is transformed into an M\(^0\)-head. Note that the measure operating in the derived measure term semantics bases itself on the predicate argument of -ful, \(P\). Thus, we expect success in the application of -ful to the extent that deriving the \(P\) measure is possible. In other words, we expect the derivation of a measure term from a noun to the extent that there is a salient correspondence between the objects the noun references and a measure using the potential contents of those objects as units. By referencing containers, container nouns provide natural units of measure for -ful, namely their volume (however abstract; cf. *a bookful of problems*).

In their uses as measure terms, we might say that container nouns undergo a process analogous to -ful suffixation (for a similar proposal, see Rothstein, 2009). Concretely, the (preferred) MEASURE reading of (46) results from a meaning transformation as described in (45) applied silently to *glass*.

\[(46) \quad \text{Mary poured three glasses of water into her soup.}\]

In other words, one should read (46) as stating that Mary poured three glassfuls of water into her soup. Because *glass* provides natural units of measure, namely the volume of a standard glass, the interpretation is transparent: Mary poured a quantity of water equal to the volume of three glasses into her soup. Thus, measure terms derive from container nouns via a lexical
process functionally equivalent to (silent) -ful suffixation, and the measure exploited in the resulting semantics uses the elements of the container noun’s denotation as units. However, this transformation cannot be as simple as the semantics of -ful suffixation in (45) would have one believe.

The problem lies in the creation of a continuous measure from the semantics of a predicate, a measure which crucially maps individuals to non-negative real numbers (and not just integers). In (45), the measure is written as $\mu_P$ – but writing the measure and deriving it compositionally are different tasks. The measure could use instances of P as its standard unit, as in the Concrete Portion reading of container nouns from Partee and Borschev (2012). A schematic indication of the semantics of this meaning shift, again assuming something like -ful suffixation, appears in (47).

(47) Container noun to measure term shift (step 1):

$$[\text{SHIFT}_{C-M}] = \lambda P \lambda k \lambda n \lambda x. \forall k(x) \land \exists y[P(y) \land \text{filled-with}(x)(y) \land \mu_{\text{card}}(y) = n]$$

Take glass. Three glassfuls of water would denote a quantity of water that would fill three glasses. In (47), the variable y ranges over pluralities, so the individual that contains the relevant quantity of water would consist of three glasses. But shifting from counting units that correspond to the volume of an instance of P, say the amount a salient glass can hold, to measuring quantities of stuff along a continuous scale cannot be a process that proceeds compositionally. There is no operator that we could posit that would create a continuous measure for us. Presumably, the shift happens once a standard unit is agreed upon, so that this unit may form the basis of a continuous measure, which itself forms the basis of a measure term (cf. the case of gallon or foot).

Now, consider variation in the opposite direction: container nouns, and thus CONTAINER interpretations, derived from measure terms. Our task is to shift the relational meaning of a measure term to a non-relational, predicate semantics using the measure named by the measure term as its basis. Recall the behavior of measure terms qua container nouns, as evidenced by the CONTAINER interpretation of (48).

(48) I dropped two beautiful liters of soda.
Under the (preferred) container reading, (48) ascribes beauty not to soda, but to the two containers of it that were dropped. The contents of each container is taken to measure one liter. We therefore witness the measure term \textit{liter} functioning as a container noun, referencing containers filled with soda.

Attempting to derive a predicate semantics from an amount term denotation, one might try to delimit a set of objects that evaluate to 1 with respect to the measure called for in the semantics of the amount term. The corresponding meaning shift is defined in (49).

\begin{align*}
\text{(49) Measure term to container noun shift (first attempt):} \\
[\text{SHIFT}_{\text{M-C}}] &= \lambda M \lambda x. \exists k[M(k)(1)(x)]
\end{align*}

\begin{align*}
\text{(50) } \lambda k \lambda n \lambda x. \cup k(x) \land \mu_i(x) = n
\end{align*}

\begin{align*}
\text{(51) } [\text{SHIFT}_{\text{M-C}}(\text{liter})] &= \lambda x. \exists k[\text{liter}(k)(1)(x)]
\end{align*}

In (51), the shift applies to \textit{liter} and returns a set of individuals that each measure one liter, but not the containers thereof. However, the elements in the denotation of a container noun are something above and beyond their contents. Under a container reading, the two beautiful things that the speaker bought in (48) are not merely 1-liter quantities of soda, but the vessels that contain those quantities.

We have not erred in requiring that a derived container noun reference quantities that evaluate to 1 with respect to the relevant measure; in (48), 1-liter quantities of soda are relevant. But we have failed to produce the container aspect of a derived container noun. The result of the meaning shift must denote a class of containers. A revised attempt to do so appears in (52).

\begin{align*}
\text{(52) Measure term to container noun shift (second attempt):} \\
[\text{SHIFT}_{\text{M-C}}] &= \lambda M \lambda x. \exists k \exists y[M(k)(1)(y) \land \text{filled-with}(y)(x)]
\end{align*}

\begin{align*}
\text{(53) } [\text{SHIFT}_{\text{M-C}}(\text{liter})] &= \lambda x. \exists k \exists y[\text{liter}(k)(1)(y) \land \text{filled-with}(y)(x)]
\end{align*}

(53) includes in its denotation any object that would be filled by a liter of some substance. We have at least succeeded in deriving a set of containers from a measure term. With this
candidate semantics for derived container nouns, consider how the proposal for container readings from Section 3.2.1 fares.

Recall that the ingredients to a container reading are the container noun, here derived from a measure term as in (53), the substance noun, and the lexical preposition of, which contributes the filled-with relation.

Composing with the substance noun, of produces the property of being filled with instances of that substance. With soda as the substance noun, of soda denotes a set of objects filled with soda. The derived container noun, (53), itself a predicate, composes with the of phrase via intersective modification. When modified by of soda, \( \text{SHIFT}_{M-C}(\text{liter}) \) identifies any individual that is filled with a liter of soda.

\[
(54) \quad [\text{SHIFT}_{M-C}(\text{liter}) \text{ of soda}] = \lambda x. \exists k \exists y [\text{liter}(k)(1)(y) \land \text{filled-with}(y)(x)] \land \\
\exists y [\text{SODA}(y) \land \text{filled-with}(y)(x)]
\]

We have captured the container aspect of our derived container noun semantics. Still, the semantics for the derived container noun is too liberal. Under the preferred container reading of (48), one imagines a class of objects more specific than any container of one liter of soda, for example those containers that take the form of a plastic bottle found on convenience store shelves. Given the constraints on the resulting denotation that world knowledge imposes, a formal derivation of container noun semantics from a measure term is impossible in the general case. In other words, we cannot pin this transformation to an operator projected in the syntax.

It would appear that we have instead an active lexicon: container nouns are derived from measure terms via categorial reinterpretation of the term as a nominal head (cf. Rothstein, 2009). Because this process takes into account knowledge about the state of the world in the context of the use of the measure term qua container noun, naming it explicitly as in (49) or (52) fails. Instead, the reinterpretation depends on a salient correspondence between specific quantities that evaluate to 1 with respect to the measure term’s measure and a well-defined class of objects with the capacity to contain this quantity.

In summary, we have described the processes by which container nouns function as mea-
sure terms and measure terms function as container nouns. The first transformation occurs when objects referenced by a container noun (i.e., containers) are first used to form units of measure. From these units we extrapolate a continuous measure, and use it as the basis of a measure term. The second transformation, from measure term to container noun, involves reinterpreting a measure term as a nominal head that references a salient class of objects whose potential contents evaluate to 1 with respect to the measure in the semantics of the measure term; this shift occurs in the lexicon. By describing these transformations, we have accounted for the last piece of our proposal relating container nouns and measure terms with CONTAINER and MEASURE interpretations, namely the variable uses of these terms.

3.2.4 Cross-linguistic support for the categorial distinction

Before concluding the discussion of the semantics of container nouns and measure terms, a note on the consistency of the account with cross-linguistic data is in order. Given the proposed categorial distinction between container nouns and measure terms, such that only the former inhabit the syntactic category Noun, we might expect to find syntactic reflexes of this distinction. In what follows, we consider data from Danish and German on two diverging properties of CONTAINER and MEASURE readings. As we shall see, these differences support the proposed categorial distinction between the terms that generate these readings.

By characterizing container nouns as simple predicates in their semantics, we identified the of in CONTAINER readings as a lexical preposition heading an adjoined PP, which contains the substance noun. The proposed structure for the CONTAINER interpretation of a container noun is repeated in (55).

(55) NP
    /   \
   /     \ 
  NP PP
    |     |
   glass P nP
     |     |
    of   water

In contrast to container nouns, measure terms were taken to be relational functions that
compose with the substance noun directly via argument saturation. Thus, the substance noun serves as the complement of the measure term, and the *of* in a MEASURE reading is not a preposition, but merely a marker of (a lack of) case on the measure term’s complement. The proposed structure for the MEASURE interpretation of a measure term is repeated in (56).

\[(56)\]

\[
\text{MP} \quad \\
\text{numeral} \quad \text{M'} \\
\text{M} \quad \text{(of)} \\
\text{kilo} \quad \text{nP} \\
\text{water}
\]

Given its lexical status, we should expect a certain robustness for the preposition in a CONTAINER reading that we do not for the particle in a MEASURE reading. If a language fails to express an intervening particle between a quantizing noun and its substance noun under one of the two possible readings, we therefore expect the particle to be absent under the MEASURE reading. In their discussion of Danish pseudo-partitives, Hankamer and Mikkelsen (2008) identify precisely this pattern. In (57-a), the preposition *med* ‘with’ intervenes between the container noun *pose* ‘bag’ and the substance noun *mel* ‘flour’. In (57-b), a preposition is precluded from intervening between the measure term *liter* and the substance noun *vand* ‘water’.

\[(57)\]

a. pose-r med mel
   bag-PL with flour
   ‘bags of flour’ (*individual*)

b. liter (*af) vand
   liter of water
   ‘liter of water’ (*quantity*)

(Hankamer and Mikkelsen, 2008)

Supposing as we have that only CONTAINER readings project PP, we have an explanation for the unavailability of a preposition in (57-b): the measure term composes directly with the substance noun. Hankamer and Mikkelsen come to a similar conclusion, namely that only
the container noun in (57-a) is a true noun.

Next, consider how the proposed categorial distinction plays out in the domain of number marking. In English, both container nouns and measure terms express morphological number. With one, the relevant quantizing noun appears in the singular form; with greater numerals, plural morphology must be used. We thus observe the following contrasts.

(58) a. Mary carried one cup(*s) of water.
    b. Mary carried two cup*(s) of water.

(59) a. Mary drank one liter(*s) of water.
    b. Mary drank two liter*(s) of water.

Chapter 2 developed a semantic account of nominal number marking; that measure terms express number morphology led us to conclude that they are nominal to the extent that they fall within the purview of our system of grammatical number. In (58), the morphology on the noun cup is determined by the cardinality measure. In (59), number morphology on the measure head liter is determined by the liter measure, i.e., the measure specified by $M^0$. In both cases, if all members of the nominal denotation do not evaluate to 1 with respect to the relevant measure, plural morphology must be used.

But here we pause: Given the proposed categorial distinction between measure terms and container nouns, we might expect a language with a more conservative system of grammatical number to reflect this categorial distinction such that measure terms, as things that are not nouns proper, are not subject to this nominal system of number marking. We would therefore expect measure terms and container nouns to differ on whether they host morphological number. Furthermore, within such a language, derived container nouns would host grammatical number whereas derived measure terms would not. German appears to employ this conservative system of grammatical number and therefore provides cross-linguistic support for the categorial distinction between measure terms and container nouns.15

Grestenberger (2013) notes that plural marking on quantizing nouns in (Viennese) Ger-

---

15 Similar patterns surface in Danish (Hankamer and Mikkelsen, 2008), Swedish (Delsing, 1993, p.204), and Norwegian (Kinn, 2001).
man determines the reading that results from their use. Consider the minimal pair in (60).

In (60-a), the plural-marked container noun Gläser ‘glasses’ expectedly yields a CONTAINER interpretation. In the minimally differing (60-b), Glas appears without plural marking and yields a MEASURE interpretation.

\begin{enumerate}
\item a. Zwei Glä-\text{-}er Wasser
two glass-PL water
‘two glasses of water’ (CONTAINER)
\item b. Zwei Glas Wasser
two glass water
‘two glasses of water’ (MEASURE)
\end{enumerate}

Grestenberger (2013)

Recall the proposal relating container nouns and measure terms to their CONTAINER and MEASURE readings: CONTAINER readings result from the semantics of container nouns, MEASURE readings result from the semantics of measure terms, and both sets of terms enjoy derived uses as the other. Thus, in (60-a), we see the container noun qua noun hosting number morphology, whereas in (60-b) that same word, now used as a derived measure term, appears unmarked for number. That usage as a container noun should explain the absence of number morphology on Glas in (60-b) finds support in the fact that measure terms generally resist plural marking in German. As (61) shows, the measure term kilo cannot appear in the plural.

\begin{enumerate}
\item Zwei Kilo/*Kilos Äpfel
two kilo/kilos apples
‘two kilos of apples’ (MEASURE)
\end{enumerate}

Grestenberger (2013)

We therefore see in German that container nouns but not measure terms express morphological number, and that container nouns qua measure terms appear in the unmarked form.\footnote{This description is a simplification of the German facts. For a much fuller discussion see Grestenberger (2013), who arrives at a conclusion similar to the one arrived at here, namely that measure terms are categorically distinct from container nouns.}

These facts therefore serve as evidence for the categorial distinction proposed between container nouns and measure terms, such that only the former inhabit the syntactic category Noun. In German, then, M\textsuperscript{0} heads are not subject to the system of grammatical number that determines the morphology on nouns. Note that here we make a prediction: container
nouns derived from measure terms should gain a morphological plural form. The example in (6-b) above appears to bear this prediction out, at least for Dutch: a derived container noun expresses morphological number.

### 3.3 The diverging status of atomizers

We began this chapter by examining candidate subclasses of quantizing nouns in English. With the aid of four diagnostics adapted from Rothstein (2009), we distinguished three classes of words: container nouns like *glass*, measure terms like *liter*, and atomizers like *grain*. Recall that the diagnostics we employed were originally meant to discriminate between CONTAINER and MEASURE readings, and not the terms themselves; the results of these tests are reproduced in Table 3.4.

<table>
<thead>
<tr>
<th>READING</th>
<th>-ful</th>
<th>they</th>
<th>SG</th>
<th>each</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEASURE</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>CONTAINER</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
</tr>
</tbody>
</table>

Table 3.4: Interpretation diagnostics from Rothstein (2009)

To summarize: CONTAINER interpretations are impossible with *-ful* suffixed to the quantizing noun, or with singular agreement between the plural quantizing noun and the matrix verb. QUANTITY interpretations prevent the quantizing phrase from serving as the antecedent to a plural pronoun like *they*, as well as quantification by distributive operators such as *each*.

In addition to applying the diagnostics to readings as originally intended, we further applied them directly to the three subclasses of quantizing nouns. Recall the results of these diagnostics, repeated in Table 3.5.

Measure terms and container nouns were shown to vary in their behavior depending on the reading that resulted from their use. The exception was *-ful* suffixation, which necessitates a MEASURE interpretation and is impossible for measure terms. This distinction contributed to the motivation for treating measure terms and container nouns as distinct subclasses, in fact as instantiating distinct syntactic categories. We concluded that measure terms are endowed with a semantics that yields a MEASURE interpretation, that the semantics of container nouns
results in a container interpretation, and that the members of each class enjoy uses as the other to the extent that salient correspondences exist between measures and the objects that may serve as their units.

Contrasting with measure terms and container nouns, in each of our diagnostics it was found that atomizers resist a measure reading. Given that we have taken measure readings to result from the semantics of a measure term, we therefore see that atomizers cannot be used as measure terms. But what is it about the semantics of measure readings, and more fundamentally the semantics of measure terms, that conflicts with the semantics of atomizers? As we saw, measure readings find their basis in the measure specified in the semantics of measure terms. To the extent that the elements of the denotation of a container noun correspond to the units of a measure, these nouns may also serve as measure terms. Atomizers, however, preclude correspondence to a measure. The claim is that this prohibition of measure term usage stems from the fact that atomizers are at once neither referential, identifying objects with the capacity for containing, nor do they appeal directly to measures in their semantics.

With the characteristics discussed above serving as constraints, we now develop a semantics for atomizers in English that appeals not to measurement, but instead to atomization of an amorphous substance, or kind. In developing this semantics, it bears noting that atomizers resist intransitive uses in which a substance noun fails to appear. Chierchia (1998a) observes the aberrance of the following example, in which the atomizer grain is used intransitively, without a substance noun. To the extent that an intransitive use is possible, a substance noun is implicitly assumed.

(62) There were three grains on the floor.  
(Chierchia, 1998a)
In (62), we see that a simple predicate semantics for atomizers is impossible; were these terms predicates, they should unhesitatingly appear without a corresponding substance noun (cf. the behavior of container nouns). From this incompatibility with intransitive uses (i.e., uses without an of phrase), Chierchia concludes that the semantics of atomizers is inherently relational. We should conclude the same. It will not do, then, to propose a predicate semantics for atomizers like that given for container nouns, and stipulate that unlike container nouns the elements of a atomizer’s denotation may never form the basis of a measure (so that atomizers cannot yield a MEASURE interpretation). Again, container nouns are at base non-relational nouns, freely admitting intransitive uses. Therefore, let us submit a semantics for atomizers that is both partitioning (i.e., atomizing) and relational, and that does not appeal to measures. As we do so, keep in mind that the given semantics is meant to cover merely English atomizers; we return to the connection between English atomizers and their counterparts in classifier languages like Mandarin in Chapter 5.

Recall the uses of atomizers we have so far considered, for example grain in (63).

(63) There are two grains of rice in this soup.

(64) *three rice(s)

In (63), the atomizer’s substance noun rice is a mass noun, which when used independently resists counting by cardinal numerals, (64), intuitively because mass nouns are not specified for what counts as a minimal part, or atom (Link, 1983; see also the discussion of mass noun properties in Gillon, 1999). In order to count elements of a mass noun’s denotation, we require information about just what these countable units are. Put plainly, atomizers provide this information: In (63), grain specifies the rice units for counting, namely those small, cylindrical members of rice’s denotation. Thus, atomizers catalyze the counting of the denotation of a noun that otherwise would resist composition with a cardinal numeral. Concretely, atomizer phrases map mass noun denotations into sets of atoms, the units by which cardinality is calculated.

But atomizers do more than simply specify atoms. They also specify physical and spatial properties of these atoms. A grain of sand has a specific shape or physical makeup, just
as a pile of dishes or a stack of laundry or a drop of water. The atoms that atomizers ultimately reference are aggregates of a kind that are assembled in a certain way. The atomizing function of atomizers must therefore track the physical properties of the substance noun and its instances.

To formalize the function of atomizing, that is, mapping into a set of atoms, let us draw on the notion of a partition. Doing so will take us on a considerable detour through the theory of connectedness within the framework of mereotopology (Grimm, 2012; Lima, 2014), but what results is a powerful notion of relative atomicity and, more generally, what it means to be a whole quantity of some stuff. These tools will further prove useful in the discussion of degree semantics in the next chapter.

3.3.1 Partitions

Atomizers compose with a substance noun and designate countable units assembled in a certain way. In (65), *grain* composes with the mass noun *rice*, specifying, or rather constructing atoms in the denotation of *rice*. The rice atoms are minimal instances of rice that are arranged in discrete, small, cylindrical forms. These atoms then get counted by the cardinal numeral *three*.

(65) John picked up three grains of rice from the floor.

This **ATOMIZING** interpretation for atomizers stands apart from **CONTAINER** and **MEASURE** readings on the basis of the referents that result. Recall that under a **CONTAINER** interpretation, *three glasses of wine* refers to an element in the denotation of the quantizing noun; it refers to three glasses. Under a **MEASURE** interpretation, *three liters of wine* refers to an element in the denotation of the substance noun; it refers to a quantity of wine. With the **ATOMIZING** interpretation, the referent of *three grains of rice* seems to be at once both rice and grains. In other words, the entity referred to under an **ATOMIZING** interpretation belongs in some sense both to the denotation of the quantizing noun *grain* and the substance noun *rice*.

Crucial to the understanding of the semantics of atomizers is the facilitation of counting that results from their use. In (65), the atomizer *grain* permits the counting of the mass
noun *rice*. Although the assumptions we make here will ultimately determine the shape of our analysis for the semantics of atomizers, for now we remain agnostic about the structure of a mass noun’s denotation, namely what it is about these nouns that precludes direct counting.\textsuperscript{17} Proposed accounts of mass noun semantics include the double domain approach of Link (1983), who claims that mass nouns find their denotation in a quantificational domain distinct from that of count nouns. Rejecting the double domain approach, Chierchia (1998a) proposes instead that mass nouns differ from count nouns only in that the denotations of mass nouns are inherently plural, that is, closed under sum-formation. In later work, Chierchia (2010) proposes a different account of mass nouns under which their atoms are unstable, or inconstant across worlds. For the present purpose, we need rely only on the fact that mass nouns preclude counting on the basis of atoms, and that atomizers transform mass noun denotations into countable, atomic sets.

Sets are countable when their members are stable units that do not overlap. To create non-overlapping sets, atomizers carve up an otherwise amorphous collection of entities into discrete individuals. In other words, atomizers establish a partition. A formal definition of a partitioning function is adapted from Chierchia (2013, p.9) in (66).

\begin{equation}
\pi \text{ is a function of type } \langle \langle e, t \rangle, \langle e, t \rangle \rangle \\text{ such that for any } P_{\langle e, t \rangle} \text{ and any } x \text{ and } y \text{ in } \pi(P),
\neg \exists z [z \leq x \land z \leq y]
\end{equation}

The partitioning function \(\pi\) in (66) take a set of individuals (a predicate \(P\)) and returns a new set of individuals. Imposed on the new set is the condition that none of its members overlap. Modeled by standard mereology, this no-overlap condition ensures that no two entities share a part. To see the partitioning function at work in the abstract, consider the set in (67-a) and the possible partitioning of this set in (67-b). Note that (67-b) represents just one of many possible partitions. Some of these possible partitions will be natural in that they are suggested or provided by context. For example, a candy bar with pre-formed blocks of chocolate will suggest naturally partitioning the bar into those designated chunks, as opposed

\textsuperscript{17} We return to the issue of mass nouns in Chapter 5.1.2.
to some other arrangement that meets the no-overlap condition of a partition.

\[(67)\]

\[a. \quad P = \{a+b, a+c, a+d, b+c, b+d, c+d, a+b+c, a+b+d, a+c+d, b+c+d, a+b+c+d\}\]

\[b. \quad \pi(P) = \{a+b, c+d\}\]

Now, recall the notion of relative atoms from the previous chapter: evaluating atomicity not with respect to the entire domain, but with respect to a specific predicate. The relative atoms of a predicate \(P\), its \(P\)-atoms, are those elements of \(P\) that have no other elements of \(P\) as parts. On the basis of this notion of relative atomicity, we defined the \(P\)-atom measure, \(\mu_{P-\text{atom}}\), as in (68).

\[(68)\]

\[\mu_{P-\text{atom}}(y) \text{ is defined only if } y \in P; \text{ when defined:} \]

\[\mu_{P-\text{atom}}(y) = |\{x \in P: x \leq y & \neg \exists z \in P[z < x]\}|\]

Once we suppose that counting takes place not over absolute atoms, but over relative atoms via \(\mu_{P-\text{atom}}\), a partitioned predicate becomes amenable to counting: In (67-b), each member of the partitioned predicate measures 1 \(P\)-atom. But here we face a problem: with only the no-overlap condition on partitioning, we allow for some extremely odd counting, at least in the general case.

Imagine the predicate modeled in (67) corresponds to the predicate denotation of the mass noun *water*. When partitioned, we have a set of non-overlapping water atoms relative to a mereological model. Because overlap concerns only the material part-of relation, \(\leq\), these two water atoms may in fact belong to the same portion of water, for example the water contained in a single glass. As long as the atoms do not share any instance of water as parts, we ought to be able to say that the water in the glass when partitioned as in (67) numbers 2. But intuitively the glass contains just a single quantity of water. Something has gone wrong: imposing only no overlap will not suffice to designate countable units. At issue here is the notion of spatial connectedness, the domain of formal topology.

In his investigation of countability in natural language, Grimm (2012) follows recent philosophical work on ontological modeling (Casati and Varzi, 1999; Varzi, 2007) and enriches standard mereological models with the topological notion of connectedness. Lima (2014) extends this mereotopological approach in her study of counting in the Yudja language.
What follows is a brief summary of the relevant aspects of these authors’ works.

First, consider what goes wrong in the water counting scenario above: a single quantity of water divides into two, non-overlapping atoms, yet we would be hard pressed to say that this water numbers 2 in any meaningful sense. Now, imagine that these non-overlapping water atoms were separated spatially, say in two different glasses. Suddenly, counting two quantities of water feels completely natural. Given intuitions like this one, Grimm (2012) and Lima (2014) propose that counting proceeds over maximally self-connected portions of stuff. These maximally self-connected portions are our relative atoms. Now for the formal details.

Mereology is a theory of parthood, described by Leonard and Goodman (1940) as “the calculus of individuals.” Central to mereology is the parthood relation ≤ and the axioms that constrain it. The relation is reflexive, such that every individual is part of itself, (69-a); it is transitive, such that a part of a part of an individual is also a part of that individual, (69-b); and it is antisymmetric, such that if two individuals are part of each other then they are identical, (69-c). Note that variables in these formula and the formula that follow are quantified over universally unless otherwise specified.

(69) The axioms of parthood:

   a. \( x \leq x \) \hspace{1cm} \text{(reflexivity)}
   b. \( x \leq y \land y \leq z \rightarrow x \leq z \) \hspace{1cm} \text{(transitivity)}
   c. \( x \leq y \land y \leq x \rightarrow x = y \) \hspace{1cm} \text{(antisymmetry)}

The proper parthood relation < is defined on the basis of ≤ in (70): a proper part of some individual must be a part of that individual, which must have some other individual as a part.

(70) Proper parthood:

\[ x < y := x \leq y \land \exists z [z \leq y \land \neg(z \leq x)] \]

The overlap relation O, defined in (71), also derives from ≤: two individuals overlap if they share a part.
Overlap:

\[ \text{O}(x)(y) := \exists z \left[ z \leq x \land z \leq y \right] \]

With these relations and their axioms, mereological theory has been used to model the domain of individuals (e.g., Link, 1983). By enriching these models with the formal notion of connectedness, we stand to derive a robust notion minimal wholes and thus relative atoms, the output of atomizers that serve as the basis for counting.

Topology is a theory of shapes in space. As such, topology is concerned with the connectedness relation \( C \), with its axioms in (72). We will encounter many varieties of connectedness; for now suppose that two entities are connected if they touch each other. The relation is reflexive, such that an individual is necessarily connected to itself, (72-a); and it is symmetric, such that if an individual is connected to another individual, then that individual is also connected to it, (72-b). To illustrate: I am connected to myself. I am also connected to my chair, which is furthermore connected to me.

(72) **The axioms of connectedness:**

a. \( C(x)(x) \)  
   \text{(reflexivity)}

b. \( C(x)(y) \rightarrow C(y)(x) \)  
   \text{(symmetry)}

Adopting the axioms in (73) incorporates connectedness into mereological theory. These axioms describe how connectedness interacts with parthood: a part of something is necessarily connected to that individual, (73-a); two individuals that overlap are connected, (73-b); and anything connected to part of an individual is also connected to that individual, (73-c). Here is our mereotopological framework.

(73) **The axioms bridging topology and mereology:**

a. \( x \leq y \rightarrow C(x)(y) \)  
   \text{(integrity)}

b. \( O(x)(y) \rightarrow C(x)(y) \)  
   \text{(unity)}

c. \( x \leq y \rightarrow \forall z[C(x)(z) \rightarrow C(z)(y)] \)  
   \text{(monotonicity)}

Returning once again to the water counting scenario above, our problem was that we identified two parts of what intuitively counted as a single quantity of water. The parts did not
overlap, yet they were connected. In other words, they formed a whole individual. The framework of mereotypology allows us to model this fact formally by incorporating the topological relationship of connectedness within a theory of parthood.

To capture the role of connectedness in countability, Lima (2014) proposes that the individuals we can count are maximally self-connected. In (74), we have the property of self-connectedness:

(74) \textit{Self-connectedness:}
\[
SC(x) := \forall y \forall z [\forall v [O(v)(x) \leftrightarrow (O(v)(y) \lor O(v)(z))] \rightarrow C(y)(z)]
\]

To count as self-connected, an individual's parts must be connected to each other. The water in our counting scenario holds this property: partitioning the water into two non-overlapping parts, these parts are connected to each other (they sit in the same glass). However, each of these water-parts is itself self-connected: splitting an arbitrary part into yet more non-overlapping parts, those parts will still be connected to each other. Self-connectedness will not suffice, then, to satisfy our intuitions about countability; we need a stronger definition of wholes.

Relying only on self-connectedness to determine countability, we confront the problem that an individual can be self-connected and a material part of another individual: the parts of water in a glass are themselves self-connected, and they will have parts that are self-connected, etc. For counting, we must ensure that an individual is not only self-connected (so that it is spatially whole), but also maximal (so that it is not a proper part of any other individual). This property, of being maximally self-connected, appears in (75). Note that the property is relativized to a single kind.\footnote{Alternatively, we could relativize maximal self-connectedness to a predicate (cf. Grimm, 2012).}

(75) \textit{Maximally self-connected:}
\[
MSC(x)(k) := SC(x) \land \cup k(x) \land \neg \exists y [x < y \land SC(y) \land \cup k(y)]
\]

An instance of a kind satisfies the property of maximal self-connectedness when its parts are connected to each other and it is not a proper part of any other self-connected instance of the kind. For the glass of water, only the total quantity of water will meet this requirement: the
water is self connected and not part of any other self-connected quantity of water. Finally, we have a definition of relative atomicity that matches our intuitions about what it means to number 1. In other words, we have the means to model the semantics of atomizers, which partition substances in service of counting. But now back to the connectedness relation itself.

The axioms of topology afford many ways for two individuals to be connected; Grimm (2012) discusses the following five varieties of connectedness. As we shall see, different sorts of connectedness describe different sorts of relative atoms. We begin with the strongest form of connectedness: STRONGLY CONNECTED. Two individuals are strongly connected just in case their interiors overlap. Next is EXTERNALLY CONNECTED, which attains when two individuals are connected but their interiors do not overlap. Then we have BY-CONNECTION, a three-place notion of connectedness: two individuals are by-connected when they are both connected to the same individual. Relatedly, two individuals are MEDIATELY CONNECTED when there is some individual through which they are by connected. Finally, we have the weakest notion of connectedness: PROXIMATELY CONNECTED. Two individuals are proximately connected when they are sufficiently near each other (not necessarily contiguous).

Suppose that a partition results in a set of countable, maximally self-connected individuals. The revised semantics for a partitioning function appears in (76).

\[
\text{(76) Partitioning function } \pi:
\]

\[
\pi \text{ is a function of type } \langle k, \langle e, t \rangle \rangle
\]

such that for any \(k\) and any \(y\) in \(\pi(k)\),

\[
\cup k(y) \& \text{MSC}(y)(k).
\]

The partitioning function \(\pi\) applies to a kind and returns a set of maximally self-connected instances of that kind. In other words, \(\pi\) specifies how the kind instantiates by objects in the world. These individuals are atomic relative to \(\pi(k)\) – for the purpose of counting, each member of \(\pi(k)\) counts as a single P-atom. With the varieties of connectedness just discussed, what it means to be a maximally self-connected individual may vary widely. Now, let us consider how partitioning serve the semantics of an atomizer.

\[19\text{See Grimm (2012) or any of the philosophical work that informs his mereotypological framework for a formal definition of interior. For our purposes, an intuitive notion of interior should suffice.}\]
3.3.2 Atomizer semantics

The atomizer *grain* takes a mass noun like *rice* and returns the set of rice atoms, namely the minimal elements of *rice*’s denotation with the appropriate physical properties. We can start by attributing to *grain* only a partitioning function as in (77).

\[(77) \quad \text{[grain]} = \lambda k \lambda x. \ x \in \pi(k)\]

The output of *grain* applied to a kind-denoting substance noun will be a set of maximally self-connected portions of the relevant substance. Here we make use of the strongest notion of connectedness, which implicates material overlap. Using this strong notion will allow two rice atoms to touch, yet remain discrete (their interiors do not touch). Applied to *rice*, we get the portions of rice that are self-connected and not a proper part of any other portion of rice. Simply put, we get grains of rice. However, the semantics in (77) is incomplete. In addition to identifying atoms, *grain* imposes constraints on those atoms, namely that they hold specific physical or ontological properties. Without identifying these requisite properties, *grain* could be applied to any kind, but as we see in (78), an atomizer is not always indiscriminate in its usage.

\[(78) \quad \text{*Four grains of water/men}\]

The problem with the illicit atomizer phrases in (78) relates to the fact that water units (e.g., drops) and man units (e.g., individual men) are inappropriately configured; they do not count as grains. We therefore must build into the semantics of an atomizer constraints on the set of atoms that results, based on properties of the intended atoms. For *grain*, this means limiting the derived set of atoms to objects that are bounded, small, cylindrical, and inanimate.\(^{20}\) A revised semantics for *grain*, incorporating these constraints, appears in (79).

\[(79) \quad \text{[grain]} = \lambda k \lambda x: \ x \text{ is bounded and small and cylindrical and inanimate.} \ x \in \pi(k)\]

---

\(^{20}\)This listing of properties of grain atoms is not intended to be exhaustive; there are likely many nuanced facets to counting as a grain of something. The list of properties given serves merely to highlight how these properties, whatever they might be, constrain the semantics of an atomizer. Furthermore, we must be careful to not constrain these atomizers too severely, to allow for uses like *grain of truth*. 

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For a given classifier the essential properties will necessarily vary, but the formula for English atomizer semantics is clear: these terms are functions from a kind into the set of minimal instances of that kind, constrained by relevant physical or ontological properties. The crucial ingredient is the partitioning function $\pi$, which may be calibrated on the basis of connectedness type. To repeat, $\pi$ specifies how a kind instantiates. Here it bears noting that English possesses an atomizer whose semantics appears to impose (almost) no restrictions on the atoms that result – it serves merely to partition instances of a kind.\textsuperscript{21} The word is *quantity*, as in the examples in (80).

(80)  
\begin{enumerate}
  \item a. Alan found three quantities of rice on the floor.
  \item b. Bill carried three quantities of water into the other room.
  \item c. Charlie bought three quantities of apples from the farm stand.
\end{enumerate}

In addition to imposing no restriction on the kind with which it composes, *quantity* admits a great deal of flexibility it the arrangement of atoms that result from its use. Quantity thus evidences the strong context sensitivity of partitioning functions. Take *quantity of apples*; the apples need only be proximately connected, that is, sufficiently close to each other. They could also be mediately connected, say sitting together on a table. If there are many apples on the table and context supports specific groupings, say certain apples are touching each other, *quantity* could require that the resulting atoms are externally connected.

In the most general case, then, an English atomizer possesses the semantics in (81), where the partitioning function is sensitive to the variety of connectedness suggested by context.

(81) \[
[\text{quantity}] = \lambda k \lambda x. \ x \in \pi(k)
\]

With (81) serving as the template for atomizer semantics, we have achieved what we set out to: the semantics of atomizers is both relational, taking a nominal argument, and atomizing, returning an atomic set of individuals arranged in a certain way. Our definition of a partition delivers these results.

A final note: The structure called for by there proposed semantics has atomizers taking the substance noun as a complement. In parallel the case of measure terms, the particle

\textsuperscript{21}Some speakers might consider *quantity* illicit with animate substance nouns, as in *three quantities of men.*
of contributes no semantic content. Atomizers themselves are neither container nouns nor measure terms, yet they are nominal (e.g., they express grammatical number, as in one grain of rice vs. two grains of rice). It seems appropriate, then, to treat atomizers as transitive nouns. They create (relatively) atomic predicates, whose members may be counted and quantified over like any basic noun. As nouns, atomizers are counted by cardinal numerals formed by CARD and handled by # in the familiar manner: on the basis of cardinality relative to the nominal predicate, we determine whether the atomizer appears morphologically singular or plural.

(82) Structure of an atomizing nominal:

Throughout our discussion of atomizer semantics we have limited the scope of the account to English; in Chapter 5, we revisit the atomizer semantics proposed here as we compare English atomizers with their counterparts in true classifier languages like Mandarin.

Next, we examine one last aspect of the behavior of quantizing nouns: the distinction between their transitive and intransitive uses. This is the topic of the next section.

3.4 Transitive vs. intransitive uses

In developing a candidate semantics for container nouns, measure terms, and atomizers, we have focused primarily on a single construction, at least superficially so. The relevant string
of words appears in (83).

(83) \[ \text{[numeral] [quantizing noun] [of] [substance noun]} \]

Calling the string in (83) a single construction is not only misleading but wrong. Depending on our choice of quantizing noun, different structures deliver the arrangement of words in (83), not to mention the different readings that result from these different structures. Our choices for the quantizing noun include container nouns, measure terms, and atomizers. On the basis of the measure term used, (83) derives from one of the three structures in (84).\(^\text{22}\)

(84) a. Container noun:

\[
\text{MP} \\
\text{Numeral} \quad \text{M'} \\
| \\
3 \quad \text{M} \quad \text{NP} \\
| \\
\text{CARD} \quad \text{NP} \quad \text{PP} \\
| \\
\text{glass} \quad \text{P} \quad \text{nP} \\
| \\
| \quad | \\
of \quad \text{water}
\]

b. Measure term:

\[
\text{MP} \\
\text{Numeral} \quad \text{M'} \\
| \\
3 \quad \text{M} \quad (\text{of}) \quad \text{nP} \\
| \\
\text{kilo} \quad \text{water}
\]

\(^{22}\)Number morphology on the quantizing nouns in these structures (i.e., \#P) is omitted for simplicity. For a reminder of how number marking works, refer back to Chapter 2.
Measure terms and atomizers share the property of taking the substance noun as a syntactic complement and a semantic argument; the *of* in uses of both is not a lexical preposition. In contrast, container nouns compose with the substance noun indirectly via a PP headed by the preposition *of* and adjoined to NP, modifying the container noun. With syntactic adjunction and semantic modification only in the presence of container nouns, the optional presence of the substance noun in the frame in (83) is predicted only with container nouns serving as the quantizing noun.

Container nouns and atomizers share the property of being counted by cardinal numerals formed on the basis of *card*, a M′-head that takes the maximal projection of the quantizing noun as its complement. However, numerals in the presence of a measure term are not cardinals: *card* does not project. Instead, the measure term heads its own MP and takes the numeral as a semantic argument. In the string in (83), we should find that only container nouns or atomizers permit the absence of the numeral – the numeral must appear with measure terms.

This section investigates the two predictions just sketched. While we ultimately hold to the basic semantics we have given to each subclass of quantizing noun, we shall see that the facts are more complicated than the proposed semantics would have us believe. Considering a broader range of data necessitates a deeper understanding of how our semantics for measurement fits within broader theories of grammar.
3.4.1 The ontological distinction between kilos and cups and grains

We have the opportunity here for an excursion into metaphysics: What sorts of things are kilos and cups and grains? Answering this question calls attention to the fundamental differences we have attributed to the semantics of quantizing nouns.

Cups have an existence independent of their contents. One may talk of cups and not be constantly interrupted by the question “of what?” Cups live in kitchen cabinets or on desks; they come in various shapes and sizes and colors; they are made out of an assortment of materials. Despite their differences, the set of cups coheres on the basis of their common form and purpose, namely that of a bowl-shaped container used for drinking.\textsuperscript{23} Substitute for \textit{cup} any other container noun and a similar state of affairs will hold.

That cups persist independently of their contents is reflected in the predicate semantics we have given to the container noun \textit{cup} in (85).

\begin{equation}
\text{[cup]} = \lambda x. \text{cup}(x)
\end{equation}

This semantics is not relational; solely on the basis of the word \textit{cup} one may identify the set of objects that are cups. They are those objects that hold the property of being a cup.

Unlike cups, kilos do not enjoy an existence independent of the things they measure (or that measure them), at least not in terms of real-world objects. To talk of kilos, one must talk of kilos of \textit{what}. A kilo of apples is a thing that measures one kilo. A kilo of rocks is likewise a thing that measures one kilo. But the first thing is apples, the second rocks. We may therefore delimit the set of things that measure one kilo (or two kilos, or three kilos, etc.), but nowhere in that set will we find a thing that is a kilo. Being a kilo is not a property an object can hold. However, measuring \textit{n} kilos of something is.

\begin{equation}
\text{[kilo]} = \lambda k \lambda n \lambda x. \bigcup k(x) \land \mu_{kg}(x) = n
\end{equation}

We therefore see the need for the relational semantics we have attributed to the measure term \textit{kilo} in (86): only with information about \textit{how many} kilos of \textit{what} may one reference real-world objects. Substitute for \textit{kilo} any other measure term and we find ourselves in a

\textsuperscript{23}Ignore for present purposes the discussion of vagueness: Does a broken cup count as a cup?
similar predicament.

Whereas cups are concrete objects and kilos simpliciter are not, for grains their ontological status is less clear. Grains certainly are objects existing in the world: a grain of rice is something that a person can point to. But a grain of rice is unlike a grain of sand, and each is distinct from a grain of calcium carbonate (the basis of a water hardness measure). One would be hard-pressed to delimit the set of objects that are grains. In fact, even as we use the term ‘grains’, a relatum is assumed. Grains exist only inasmuch as the substance that they are grains of does: the property of being a grain is defined with respect to a substance (i.e., a kind).

However, in contrast to kilos of some stuff, grains of a substance are inherently quantized. Grains of something physically realize in a standardized way that kilos of something do not; with kilos, we need to know how many kilos we are talking about in order to reference a real-world object. With grains, all we need to know is the substance. Consequently, unlike kilos, grains need not reference a specific extent along a scale in order to instantiate.

We see, then, that in order to serve as a property of real-world objects, grains require a concomitant substance, but not a specified amount of that substance, as reflected in the relational semantics in (87).

(87) \[ [\text{grain}] = \lambda k \lambda x: x \text{ is bounded and small and cylindrical and inanimate. } x \in \pi(k) \]

Note that one distinction between an atomizer like grain and a measure term like kilo lies in whether we need a numeral to specify what a given instance of grain or kilo is. Another difference concerns the partition inherent to atomizer semantics, which enables counting (rather than measuring).²⁴ Still, both are relational in that they require a substance to form a property, contrasting with container nouns that readily refer independently of a substance or quantity thereof.

We therefore find conceptual justification for the semantics we have attributed to measure terms. But these considerations serve only to delimit the range of possible analyses, not to determine them. Where our aim is to provide an explanation of natural language phenomena,

²⁴One option, which for now must remain merely a consideration, is that partitions enter the semantics whenever kinds instantiate. Thus, measure terms would feature a partition internal to their semantics, too. We return to this point in the discussion of Chinese classifiers in Chapter 5.1.3.
in the remainder of this section we consider the grammatical underpinnings and implications of the semantics that has been proposed.

3.4.2 Suppressing the substance noun

Although we have focused on uses of measure terms in the string in (83), repeated below in (88-a), we have also encountered apparently intransitive uses of these terms, (88-b), where the measure term appeared without the substance noun (or the word of).

(88) a. [numeral] [measure term] [of] [substance noun]
b. [numeral] [measure term]

Container nouns like *cups* readily admit intransitive uses: *there are three cups on the counter*, *Mary held three cups in her hand*. That container nouns should allow usage without specifying their contents makes sense given the ontological status of their referents: it is possible to talk about cups independently of their contents. Grammatically, these constructions are formed just like any other numeral-noun combination:

(89) MP
    /   /
   /   /
  /   /
 Numeral M'     NP
     |   |   |
    3   M  CARD
     |   |   |
      |   glass

The structure in (89) is licensed by the non-relational (i.e., intransitive), predicate semantics given to container nouns. They are nominal expressions of type \( \langle e, t \rangle \), and they behave as such.

Measure terms like *kilo* resist intransitive uses: neither *there are three kilos on the counter* nor *Mary held three kilos in her hand* sounds acceptable, and to the extent that either of them does they feel strongly elliptical. As we saw, instantiating a kilo is not a property an object may hold, but being a kilo of something is. We therefore require a substance noun in the usage of container nouns, both conceptually and in their semantics. Were we to try
to form a structure like (89) with a measure term, composition would fail at the level of $M'$.

Without its kind-denoting argument, the measure term cannot compose with the rest of the phrase.

(90) $\begin{array}{c}
\text{MP} \\
\text{Numeral} \quad M' \\
\downarrow \\
3 \\
\downarrow \\
kilo
\end{array}$

The prohibition against intransitive uses for measure terms follows from their relational semantics: a measure term, type $\langle\langle e, t \rangle, \langle n, \langle e, t \rangle \rangle \rangle$, requires a nominal internal argument. Without this argument, use of the measure term fails, accounting for its lack of intransitive uses.

While we correctly predict the oddness of intransitive uses of measure terms in expressions such as *there are three kilos on the floor*, as we saw in Section 2.4.2 there are a range of constructions in which measure terms may be used without a substance noun. Recall that superficially intransitive uses of measure terms typically appear as the internal argument of measure verbs (e.g., measure, weigh, etc.), as in (91-a). They also appear in predicative be constructions, (91-b), as well as modifiers of gradable adjectives, (91-c).

(91) a. John weighs 100 kilos.
   b. The temperature is 70 degrees.
   c. John is two meters tall.

In (91-a), the speaker attributes to John the property of measuring 100 with respect to the kilo measure, not the kilo measure of a specific substance. No substance noun appears, nor is one assumed; the construction does not feel elliptical in the way that *there are three kilos on the floor* does (to the extent that this latter sentence is acceptable at all). Thus, our starting point ought to be the observation that measure terms do in fact admit uses without a substance noun.
In the account of number marking on measure terms developed in the previous chapter, we took intransitive measure terms to be relations between numbers and individuals: the intransitive measure term takes a numeral and returns the set of individuals that satisfy the relevant measure to the extent specified by the numeral. The proposed semantics is repeated in (92).

(92) a. \([\text{kilo}]_{\langle n, \langle e, t \rangle \rangle} = \lambda n \lambda x. \mu_{kg}(x) = n\]
b. \([100 \text{ kilos}] = \lambda x. \mu_{kg}(x) = 100\]

An intransitive measure phrase like 100 kilos will be true of an individual just in case it weighs 100 kilos. But what is the relationship between the transitive semantics we have entertained for measure terms and the intransitive semantics in (92-a)? One possibility assumes that there is in fact no relationship between the transitive and intransitive semantics; intransitive kilo in (92-a) and transitive kilo in (93) are two different words.

(93) \([\text{kilo}] = \lambda k \lambda n \lambda x. \cup k(x) \land \mu_{kg}(x) = n\]

In addition to the rampant ambiguity and subsequent explosion of the lexicon such an account necessitates – not to mention the problems posed to a learner – positing no relationship between transitive and intransitive measure terms appears to miss an obvious generalization. The semantics at the core of (92-a) and (93), namely the kilo measure, are the same; all that differs between the two is the presence or absence of a nominal argument. We should therefore consider the relationship between transitive and intransitive semantics of measure terms as one of derivation. It remains to be seen which use is prior, which is derived, and how this derivation proceeds semantically.

Suppose that the intransitive semantics of measure terms is basic. How would we derive a transitive use? We might try increasing the adicity of the measure term via a process of lambda abstraction over the nominal argument, but there is no predicate variable internal to the intransitive semantics to which we could apply such a process (Heim and Kratzer, 1998). Rather than attempting a feat in semantic acrobatics that derives transitive measure terms from intransitive semantics, consider two points. First, we convinced ourselves in Section 3.2.2 that the substance noun plays an essential role in the semantics of measure
terms. Second, by virtue of being able to spell out the closed class of constructions that admit intransitive measure terms in (91), we see that transitive measure terms enjoy a much broader distribution. If one use is derived from the other, it ought to be intransitive measure terms that derive from a transitive semantics.

We may view the process of detransitivization of measure term semantics as existential closure (cf. Heim, 1982), here operating on the substance noun’s argument position. The process would have intransitive measure phrases denote the set of objects that evaluate to the appropriate value with respect to the measure supplied by the measure term, just as in our candidate intransitive semantics in (92). What differs, however, is now we say of those objects denoted that they instantiate some kind. The composition is illustrated in (94).

(94) a. 
   \[
   \begin{array}{c}
   \text{MP} \\
   \text{Numeral} \quad \exists \\
   3 \\
   \mu \\
   \text{kilo}
   \end{array}
   \]

b. \([\text{three kilos}] = \lambda x. \exists k(x) \land \mu_{kg}(x) = 3\]

While \(\exists\)-closure of the substance noun argument position yields the desired semantics for an intransitive measure term, this process of detransitivization must be constrained. Only in a few specified constructions may intransitive measure terms appear. Recall the representative intransitive examples we considered:

(95) a. John weighs 100 kilos.
   b. The temperature is 70 degrees.
   c. John is two meters tall.

What do the measure terms in (95) have in common? At least in (95-b) and (95-c), the measure phrase serves as the predicate of the sentence: (95-b) ascribes to the subject the property of evaluating to 70 with respect to the degree measure; in (95-c),
we say of John that his height is equal to 2 meters.\textsuperscript{25} Perhaps it is not so odd to consider the measure phrase in (95-a) also as a predicate. Doing so would afford us an incisive characterization of intransitive measure terms: they are licensed only when the measure phrases they project are used as predicates.

The trouble with labeling intransitive measure terms as predicates centers around examples like (95-a), where the verb \textit{weigh}, together with the measure phrase, serves as the predicate of the sentence. But consider how this sentence composes. We have given superficially intransitive measure phrases a predicate semantics, type \((e,t)\); this phrase appears bare, without a determiner, so it likely continues to be predicate-denoting at the point at which it composes with \textit{weigh}. The subject, \textit{John}, is individual-denoting, type \(e\). One way to view the contribution of \textit{weigh}, then, is as a function that feeds a predicate its argument, as in (96).

\begin{equation}
(96) \quad \text{[weigh] = } \lambda P \lambda x.\ P(x)
\end{equation}

The semantics for measure verbs cannot be so bleached because the verb imposes selectional restrictions on its complement: John cannot weigh two meters or measure blue. Moreover, the measure phrase complement does not behave as a genuine internal argument because it cannot be passivized (cf. 100 kilos are weighed by John). Assuming the measure verb serves the role of argument-feeder, it does so judiciously, on the basis of presuppositions targeting the dimension of measure called for by the measure term. That we find so few measure verbs (e.g., \textit{weigh, measure}) speaks to the specialized status of this argument-feeding operation.

Before turning to intransitive uses of atomizers, consider one last aspect of the proposed relationship between transitive and intransitive measure terms, where the latter are taken to derive from the semantics of the former. In (95-b), repeated in (97), we see an intransitive use of the measure term \textit{degree}.

\begin{equation}
(97) \quad \text{The temperature is 70 degrees.}
\end{equation}

\begin{equation}
(98) \quad \text{[degree] = } \lambda k \lambda n \lambda x.\ \mathbb{U}k(x) \land \mu_\circ(x) = n
\end{equation}

\textsuperscript{25}The gradable adjective in this construction would act as a simple predicate, specifying the dimension of the measure as one of height. We return to the semantics of gradable adjectives in Chapter 5.1.1.
Supposing as we have that intransitive degree derives from a transitive semantics, the basic entry for degree would be as in (98). We would therefore predict that degree should enjoy transitive uses just like the other measure terms we have encountered. But as Schwarzschild (2006) observes, degree resists transitive uses: substance nouns are precluded from composing with degree. According to Schwarzschild, at issue in (99) is the property of monotonicity, as defined in (100).\footnote{Schwarzschild (2006) defines monotonicity relative to a property, which translates to the corresponding kind for our purposes.}

\begin{align}
(99) & \quad *\text{two degrees of water} \\
(100) & \quad \text{A measure } \mu \text{ is monotonic with respect to a kind } k \text{ iff} \\
& \quad \forall x, y \in \cup k \ [x \leq y \rightarrow \mu(x) \leq \mu(y)]
\end{align}

How does monotonicity account for the ungrammaticality of transitive degree? In (99), the relevant measure is \( \mu_o \), and the kind with respect to which one assesses the measure’s monotonicity is water. If \( \mu_o \) were monotonic with respect to water, any quantity of water measuring two degrees would have no proper parts which also measure two degrees. Our world knowledge tells us this is not the case: any part of a quantity of water that is at two degrees will also be at two degrees, by virtue of the nature of degrees of temperature. It seems, then, that its non–monotonic status as a measure precludes degree from transitive uses. This observation leads to the implicational universal in (101).\footnote{What we have called a ‘transitive use’ Schwarzschild refers to as the ‘pseudo-partitive’ construction (Selkirk, 1977).}

\begin{align}
(101) & \quad \text{Schwarzschild’s generalization:} \\
& \quad \text{Transitive use of measure term } \Rightarrow \text{ monotonic measure}
\end{align}

Schwarzschild’s generalization provides a description of the data from transitive measure terms. As expected, another non-monotonic measure, introduced by the measure term carat, likewise precludes transitive uses: one cannot reference two carats of gold. Now, consider the explanation for this phenomenon.

According to Schwarzschild, ensuring monotonicity in transitive uses is the job of a designated Mon head. \( \text{Mon}^0 \) composes with the substance noun and the resulting constituent...
is modified by the measure term, together with the numeral. The result is Schwarzschild’s structure in (102).

\[
(102) \quad \text{MonP} \quad \text{MP} \quad \text{Mon'} \quad \text{three pounds} \quad \text{Mon}^0 \quad \text{NP}
\]

\[
\text{(of)} \quad \text{meat}
\]

Mon carries with it a presupposition that the measure phrase (MP) is monotonic on the part-whole relation given by the property contributed by the substance noun (NP). This presupposition projects and becomes a condition on the denotation of the maximal projection of Mon, MonP.

\[
(103) \quad [\text{MonP}] = \lambda x. \ \text{NP}(x) \land \text{MP}(x); \text{condition: MP is monotonic on NP}
\]

This account of the lack of transitive uses of non-monotonic measure terms is brute–force engineering; we still lack a principled reason for why Mon should enforce monotonicity and not prohibit it. Worse, we lack evidence of Mon altogether. We therefore consider an account of the ill-formedness of two degrees of water without appeal to a designated functional head that rules it out.

Schwarzschild assumes that non-monotonic degree is precluded from occurring in a specific construction, the pseudo-partitive. But making the restriction against degree construction-specific obscures the fact that only predicative uses of the term are allowed. In other words, it is not the construction, but rather the transitive usage that is incompatible with non-monotonic measures.

The claim is that non-monotonic measure terms resist transitive uses because they are at no point endowed with transitive semantics. Concretely, a non-monotonic measure term

\[\text{28}\] Schwarzschild imagines a different constituency for measure phrases than that which we have assumed. For him, a measure phrase is just the combination of a measure term with a numeral. This measure phrase then modifies the substance noun, rather than taking the noun as a syntactic complement.

\[\text{29}\] There is another way to look at this restriction: non-monotonic measure terms resist uses as arguments. Here is some speculation as to why: A measure term composes with its arguments to form a predicate, modeled
like *degree* is born with the intransitive semantics in (104). With an intransitive semantics, a semantics that allows only predicate uses, we correctly predict that non-monotonic measure terms like *degree* will never find uses in argument position.

(104) \[ \text{[degree]} = \lambda n \lambda x. \mu_o(x) = n \]

Returning once more to the ontological considerations that informed our transitive semantics, it in fact never makes sense when talking of degrees to ask for a clarification of the substance being measured: *of what?* Substitute a different non-monotonic measure term like *carat* and a similar situation holds. Thus, *degree* does not take a substance noun as a complement because it lacks the transitive semantics that would allow it to do so.

Having considered the role of the substance noun in measure term semantics, we turn now to atomizers. The semantics we gave to the atomizer *grain* has it compose with a kind to yield an atomic predicate.

(105) \[ \text{[grain]} = \lambda k \lambda x: x \text{ is bounded and small and cylindrical and inanimate}. x \in \pi(k) \]

Our relational semantics for atomizers, together with our rumination on what sort of thing a grain is (i.e., a grain of *something*), predict that atomizers should resist intransitive uses where no substance noun appears. As Chierchia (1998a) observes, atomizers do in fact resist intransitive uses. To the extent that (106) is permitted, a substance noun is assumed.

(106) ?There were three grains on the floor. (Chierchia, 1998a)

Unlike measure terms, atomizers lack a well-defined class of sanctioned intransitive uses. With atomizers, then, the original prediction, namely that they would disallow optional appearance of the substance noun in the frame in (107), holds.

(107) \[ \text{[numeral [measure term] of] [substance noun]} \]

as a set of individuals. Within the set created by a non-monotonic measure term, there will be rampant overlap among the individuals (every proper part of something measuring two degrees will also measure two degrees). However, the set created by a monotonic measure term will be structured, or non-overlapping (no proper part of something measuring three meters will also measure three meters). When serving an argument, an individual must be extracted from this set. Serving a predicate, we must assess whether some individual is in this set. Conceptually, searching through an overlapping set for an individual is more difficult than verifying whether an individual is in such a set. Hence the ban on monotonic measure terms as arguments: the process of extracting an individual from an overlapping set is too taxing.
Next, we investigate the second prediction of our quantizing noun semantics: that container nouns and atomizers, but not measure terms, allow the absence of the numeral.

### 3.4.3 Suppressing the numeral

The quantizing noun semantics proposed in this chapter has measure terms, but not atomizers or container nouns, obligatorily occurring with the numeral in the string in (108); only the measure term takes this numeral as an argument. The representative lexical entries for quantizing nouns are repeated in (109).

(108) \[ \text{[numeral]} [\text{measure term}] [\text{of}] [\text{substance noun}] \]

(109) a. **Container noun:**

   \[ [\text{cup}] = \lambda x. \text{cup}(x) \]

b. **Measure term:**

   \[ [\text{kilo}] = \lambda k \lambda n \lambda x. \cup k(x) \land \mu_{kg}(x) = n \]

c. **Atomizer:**

   \[ [\text{grain}] = \lambda k \lambda x: x \text{ is bounded and small and cylindrical and inanimate. } x \in \pi(k) \]

As a noun, a term that denotes a set of individuals, we predict uses of *cup* without a numeral in parallel to uses of *book* or *tree* or *table*. This prediction holds: in subject position (*the cup of water fell onto the floor*), in object position (*Mary held the cup of water*), as an oblique (*John hit Bill with the cup of water*), in copular constructions (*that thing is the cup of water*), etc., container nouns freely appear without a numeral.

As a relation between its kind-denoting complement and (appropriately constrained) atomic instances of that kind, atomizers do not take the numeral as an argument. Instead, they are counted via CARD just like nouns. We therefore predict the optional presence of the numeral in (108) when an atomizer serves as the quantizing noun. The prediction holds: *the grain of rice fell onto the floor, Mary held the grain of rice, John hit Bill with the grain of rice, that thing is the grain of rice*; whatever unease speakers may associate with the preceding examples likely stems from the implausible relevance of a single grain of rice. Crucially, the acceptability of these numeral-less instances of atomizers contrasts with numeral-less uses of
measure terms, which follow.

Unlike container nouns and atomizers, measure terms take the numeral in (108) as their (second) argument. We therefore predict that the occurrence of this numeral is obligatory to saturate the denotation of the measure phrase. Consider what happens when the numeral fails to appear with a measure term, as in the examples in (110).

\[(110)\]
\[
\begin{align*}
\text{a. The liter of water fell onto the floor.} \\
\text{b. Mary held the liter of water.} \\
\text{c. John hit Bill with the liter of water.} \\
\text{d. That thing is the liter of water.}
\end{align*}
\]

In (110), the measure term appears without a numeral and the result is perfectly interpretable. One interpretation, the \textit{container} reading, immediately suggests itself for these examples: In (110-a), the container and its contents fell. But as we saw, this \textit{container} interpretation results from uses of derived container nouns. Having already observed the behavior of container nouns with numerals, we focus instead on the second interpretation of the sentences in (110), namely the \textit{measure} interpretation resulting from the use of a measure term.

Under a \textit{measure} interpretation, (110-a) states that some quantity of water measuring one liter fell onto the floor. But the numeral \textit{one} appears nowhere in the sentence. The rest of the examples in (110) behave similarly.\textsuperscript{30} This interpretation supports the conclusion that measure terms may be used without the overt expression of a numeral via a process of \textit{one}-omission (Jiang, 2012; Li, 1997, see also Perlmutter, 1970). To see that there is an implicit \textit{one} in numeral-less uses of measure terms, consider what happens when the measure term appears morphologically plural as in (111).

\[(111)\]
\[
\begin{align*}
\text{a. The liters of water fell onto the floor.} \\
\text{b. Mary held the liters of water.}
\end{align*}
\]

\textsuperscript{30}This \textit{measure} interpretation is perhaps easier to get when the numeral-less measure phrase is an indefinite: \textit{a kilo of water fell onto the floor}. 

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Appearing in the plural without a numeral, measure terms no longer admit a measure reading. In (111-a), we have an individual reading: many things, each of which contains a liter of water, fell to the floor. Thus, in (111-a), we have a derived container noun. So, measure terms may appear without a numeral by suppressing the numeral one, but one is incompatible with the plural marking on the measure terms in (111): one kilos is impossible. Therefore, one-omission, and as a result numeral-less measure terms, is ruled out in (111). Consequently, the prediction that measure terms disallow occurrences without a numeral holds. In the special case where the measure term does appear without a numeral, one is assumed.

3.5 Discussion

We began this chapter intent on identifying what it meant to inhabit the class of words that are quantizing nouns. Building on the description of measure terms from the previous chapter, we characterized quantizing nouns as words whose function is to quantize a substance for the purpose of counting or measuring. We considered three candidate subclasses of measure terms: container nouns like cup, measure terms like kilo, and atomizers like grain. Based on distributional differences, as well as salient distinctions in the meanings that result from uses of members of each candidate subclass, we concluded that container nouns are in fact semantically distinct from measure terms, and that both are distinct from atomizers.

Container nouns were shown to possess a basic predicate semantics, denoting sets of objects with the capacity to contain things. Through modification by a PP headed by the lexical preposition of, which itself composes with a substance noun, container nouns yield a CONTAINER interpretation, denoting objects and their contents. Measure terms compose directly with the substance noun, followed by a numeral, resulting in a MEASURE interpretation, denoting amounts or quantities of the stuff specified by the substance noun. Like measure terms, atomizers compose directly with the substance noun. However, the role they play differs: instead of measuring, atomizers partition, creating a set of atomic instances of the substance noun. The proposed semantics for each subclass of measure terms is given in (112).
(112)  

a. **Container noun:**  
\[ \text{[cup]} = \lambda x. \text{cup}(x) \]

b. **Measure term:**  
\[ \text{[kilo]} = \lambda k \lambda n \lambda x. \cup k(x) \land \mu_{kg}(x) = n \]

c. **Atomizer:**  
\[ \text{[grain]} = \lambda k \lambda x: x \text{ is bounded and small and cylindrical and inanimate. } x \in \pi(k) \]

In addition to direct mappings from container nouns to container readings and from measure terms to measure readings, we also accounted for the processes by which each term enjoys uses as the other. A measure term is derived from a container noun via a process akin to (silent) -ful suffixation, which transforms the noun into an \( M^0 \)-head using the elements of the noun’s denotation as the standard units of the derived measure term’s measure. Container nouns derive from measure terms by reinterpreting a measure term as a nominal head denoting a salient class of objects whose (potential) contents evaluate to 1 with respect to the measure at play in the measure term’s semantics. With a proposal in hand, we then considered cross-linguistic support for the categorial distinction, as well as the variability of uses between container nouns and measure terms. What results is the proposed mapping between quantizing noun and reading specified in Fig. 3.3; solid lines indicate an implicational relationship between term and reading, while the dashed line indicates functional variability between terms.

Given the mapping in Fig. 3.3, we then took a closer look at the behavior and distribution of quantizing nouns. Originally focusing on their uses in the frame in (113), we considered

<table>
<thead>
<tr>
<th>Quantizing Noun</th>
<th>Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONTAINER NOUN</td>
<td>CONTAINER INTERPRETATION</td>
</tr>
<tr>
<td>MEASURE TERM</td>
<td>MEASURE INTERPRETATION</td>
</tr>
<tr>
<td><em>ATOMIZER</em></td>
<td>ATOMIZING INTERPRETATION</td>
</tr>
</tbody>
</table>

Figure 3.3: Relationship between quantizing nouns and the readings that result from their uses
what the proposed semantics predicted about the relationship between a given quantizing noun and the occurrence of the numeral or the substance noun.

(113) \([\text{numeral}] [\text{measure term}] [\text{of}] [\text{substance noun}]\)

The simple predicate semantics for container nouns predicts optional realization of both the numeral and the substance noun in (113). The relational semantics for atomizers, wherein the atomizers takes the substance noun as an argument, predicts only optional realization of the numeral. Both predictions hold: container nouns and atomizers optionally appear with a numeral, but only container nouns allow the suppression of the substance noun.

The semantics given for measure terms, wherein both the substance noun and the numeral are arguments of the term, predicts that both the numeral and the substance noun must appear with a measure term. But as we saw, when a measure phrase is used as a predicate the measure term does not require a substance noun: *John weighs 70 kilos*. We therefore considered how intransitive uses of measure terms (i.e., where the substance noun does not appear) derive from their transitive uses via a process of existential closure targeting the kind-denoting argument position. When not detransitivized by \(\exists\)-closure, measure terms conform to our prediction: the substance noun must appear.

As for the numeral, we saw that it optionally appears with measure terms only when it is assumed to be *one*. An operation *one*-omission (e.g., Jiang, 2012), such that the numeral *one* may go unpronounced in the presence of a measure term, was therefore proposed. Thus, measure terms conform to our prediction: the numeral must appear, although when the numeral is *one* it is optionally pronounced.

In our investigation of quantizing noun behavior, we noted that intransitive uses of non-monotonic measure terms like *degree* cannot plausibly derive from a transitive semantics; these terms with non-monotonic measures do not admit transitive uses: *two degrees of water* is nonsensical. “Non-monotonic” pinpoints the property whereby a quantity of water evaluating to, say, 30 with respect to \(\mu_\circ\) has proper parts that also measure 30 degrees. In contrast, monotonic measures lack this property: a quantity of water measuring 1 kilo will have no proper parts that also measure 1 kilo. We rejected the proposal that non-monotonic measure terms are banned from occurring in a specific construction (the pseudo-partitive, which ne-
cessitates a substance noun) on the basis of functional structure specific to that construction that checks monotonicity (Mon^0; Schwarzschild, 2006). Instead, it was proposed that they simply lack a transitive semantics. Thus, degree and its non-monotonic associates are always functions from numbers to individuals, type \langle n, \langle e, t \rangle \rangle.

We have now in hand an understanding of the means by which measuring and counting proceed in the nominal domain. The following chapter expands the typology proposed here with a case study of the word amount.
Chapter 4

Amount Semantics

The previous chapter provides a typology of quantizing nouns. Here we probe and extend that typology with an analysis of the word *amount*. Despite its superficial similarities with the atomizer *quantity*, we will see that the peculiar behavior of *amount* necessitates an expansion of our inventory of quantizing nouns with yet another distinct class: degree nouns. Unlike atomizers, which partition the overlapping denotations of substances and refer directly to real-world objects, degree nouns like *amount* appeal to measurement in their semantics and through a measure reference abstract entities; namely, amounts of stuff.

This case study of *amount* accomplishes three things: First, we expand and refine the semantics of measurement, endowing it with the ability to reference abstract representations of measurement (i.e., degrees). Second, we develop a new semantics for degrees under which they are semantically complex (i.e., not semantic primitives or simple points on a scale; cf. Kennedy, 1999): degrees are aligned with kinds and treated as nominalized quantity-uniform properties. While the idea that degrees should align with kinds has existed in one form or another for some time (see Anderson and Morzycki, 2012, for discussion and further motivation), we find here the first systematic implementation of this idea within the broader framework of compositional semantics, an implementation that situates degrees within a comprehensive theory of measurement. Third, we show how this new notion of degrees sheds light on the analysis of so-called “amount” relatives (Carlson, 1977a; Heim, 1987; Grosu and Landman, 1998). The approach developed here eschews the ad-hoc nature of previous attempts and more readily accounts for the intuition that amount relatives reference objects,
not degrees. We begin with a look at the behavior of amount.

4.1 A new kind of degree

Throughout our investigation of measurement in natural language, our strategy has been to identify cases where measures enter into the semantic calculation. A particularly perspicuous and interesting case is the word amount, as in (1). The sentence is ambiguous; not all of the readings implicate measurement.

(1) John brought that amount of apples with him to the party last week.

To see measurement at work in the semantics of amount, we must first identify the readings this word admits. Under the first reading, amount behaves analogously to the atomizer quantity. As we saw in the previous chapter, quantity makes no use of measures in its semantics. Rather, it establishes a partition over the overlapping denotation of the substance noun with which it composes. Under this quantity-like reading of (1), amount partitions the denotation of apples and, crucially, references specific apples. In fact, under this quantity-like reading, (1) may be directly paraphrased as in (2), where quantity stands in for amount.

(2) John brought that quantity of apples with him to the party last week.

Imagine that two bowls of apples sit on a table. The speaker points to one of them and utters (1) (or (2)), intending the quantity-like reading. He thus conveys that those specific apples there, in the bowl on the table, were brought by John last week. We already have the means to deliver this reading: amount receives the same denotation as quantity. It composes with the substance noun apples, establishing a partition on apples; that picks out the contextually relevant quantity of apples. Of these apples we assert that John brought them to the party last week. The sentences in (3) provide more examples to highlight this reading: in each case, that amount most likely refers to a contextually salient quantity (i.e., set) of apples. So far, no measurement.
(3)  
  a. That amount of apples is rotten.
  b. That amount of apples fell out of the bag and onto the floor, where it is now.
  c. I put away every amount of apples that you brought in.

Under its second reading, *amount* does not reference concrete objects. In the same two-bowl scenario just described, suppose that the bowl the speaker points at was just filled by the speaker’s housemate, who recently returned from the supermarket. Here the speaker does not intend to communicate with (1) that his housemate bought the same apples that John brought to the party the previous week. Instead, he conveys that the apples filling the bowl measure the same (say, in weight or volume or number) as the apples John brought to the party – the apples are different, but the abstract amount is the same. In other words, he bought an amount of apples equal to the amount that John brought.

The sentences in (4) more transparently demonstrate this second reading of *amount*. In (4-a), it is highly unlikely that the speaker eats the same apples every day. Similarly, in (4-b), one hazards to assume that the speaker and the addressee are eating the same apples. Instead, both sentences appeal to abstract amounts determined on the basis of a measure; these abstract amounts are instantiated at various points in time by different objects, which are acted on accordingly.\(^1\) For example, suppose a dietary regimen mandates the eating of two kilos of apples each day; *that amount of apples* in (4-a) could then refer to that abstract amount, two kilos, which was differently instantiated by apples each day (i.e., the speaker ate different apples each day, but each day the apples that the speaker ate measured two kilos).

(4)  
  a. I ate that amount of apples every day for a year.
  b. I ate the amount of apples that you ate.
  c. I want the amount of apples that Bill received.

*Amount* is not alone in its status as a degree noun; other nouns that behave like *amount* and thus fall within the class of degree nouns include *size* and *length* and *weight* (a subset of what Partee 1987 calls “attribute” nouns). Compare (4) with the sentences in (5). These sentences share the ability to reference objects indirectly via an abstract measurement.

\(^1\)See Cartwright (1970) for a similar observation, which she attributes to Russell (1938).
(5)  a. I sold that length of rope every day for a year.

b. I cut the length of rope that you cut.

c. I bought that size (of) shirt for my entire life.²

d. I wore the size (of) shirt that you wore.

Here we see measurement at work in the semantics of *amount* and other degree nouns, but this semantics appears highly context-sensitive. To evaluate *amount*, first we need to fix the measure by which we determine amounts; in the verifying scenario given for (4-a), context fixed that measure to weight. Additionally, the amounts referenced appear to be substance-bound, applying only to individuals named by the substance noun (*apples* in (4) and *rope* or *shirt* in (5)). Overtly specifying a different substance to which *amount* applies results in a cumbersome utterance, interpretable only under a metalinguistic guise. Consider (6).

(6) I ate that amount of apples every day for a year – in bananas.

It would appear that the substance noun is an argument of *amount*; in this way, *amount* functions as a transitive quantizing noun. The amounts to which we refer are restricted such that they apply only to objects named by the substance noun. We return to this issue in our discussion of the status of the substance noun for other quantizing nouns below, as well as in Section 4.1.1.

Let us settle on some terminology that differentiates these two readings of *amount*. Under the first, object-level interpretation, *amount* behaves like *quantity* and the resulting expression receives a simple DEFINITE interpretation: *that amount of apples* references the maximal relevant apple individual. Under the second, abstract amount interpretation, *that amount of apples* variously instantiates with different apple individuals; this we term the EXISTENTIAL definite interpretation (cf. the EXISTENTIAL reading for kinds from Carlson 1977b, which we discuss in detail below).

²Size may compose directly with the substance noun, without an intervening *of*, supporting the claim that degree nouns take the substance noun as an argument. Why the other degree nouns preserve the particle *of* remains an open question. See Zamparelli (1998) for further discussion of this particle.
(7) I want that amount of apples.
   a. DEFINITE interpretation: I want those apples there
   b. EXISTENTIAL interpretation: I want some apples that measure the relevant amount

In addition to DEFINITE and EXISTENTIAL interpretations, *amount* admits another, more abstract reading: the DIRECT interpretation, which directly references an abstract amount and stands out from the EXISTENTIAL interpretation by its lack of instantiation as real-world objects. This reading is particularly apparent when *amount* appears bare, without a substance noun. Imagine ordering wine from a menu, which lists prices for three different amounts (i.e., measurements) of wine. One may utter either of the sentences in (8), implicating only abstract measurements, not the wine that could instantiate them.

(8) a. I want the largest amount.
   b. That amount is too much.

We also see the DIRECT interpretation in specificational sentences like in (9).

(9) That amount is five kilos.

Finally, bare *amount* receives a DIRECT interpretation when it is indirectly modified by a substance noun, as in (10). Imagine cashing out at a (disreputable) casino, where winnings may be instantiated by various commodities. Uttering the sentence in (10), the speaker uses *that amount* to reference an abstract measurement (say, a sum of money).

(10) I want that amount in diamonds.

Recall the behavior of the EXISTENTIAL interpretation: an abstract amount is instantiated by the objects holding the appropriate property. It would appear that the DIRECT interpretation is somehow prior to this reading: first we settle on the abstract amount (the DIRECT interpretation), then we instantiate it (the EXISTENTIAL interpretation). We have thus identified three distinct, though related interpretations for *amount*. They appear in (11) (keep in mind that not all readings are always equally salient, hence the use of various tricks to highlight these interpretations in the sentences considered above).
(11) I want that amount of apples.

✓ DEFINITE interpretation: I want those apples there
✓ DIRECT interpretation: I want that abstract amount/measurement
✓ EXISTENTIAL interpretation: I want some apples that measure the relevant amount

Given our focus on the semantics of measurement, the DIRECT and EXISTENTIAL readings of amount in (11) will be our primary concern; measurement is not implicated in the DEFINITE interpretation. As was our strategy in the previous chapter, we will here endow amount with a semantics that delivers these readings, while taking note of the flexibility necessary to allow amount to function as an atomizer like quantity and yield a DEFINITE interpretation. In fact, we must also allow quantity to function like amount.

We saw in examples (1) and (2) that amount and quantity are interchangeable under a DEFINITE interpretation: both words may be used to specify discrete quantities of a substance. In (12), we see that quantity also admits an EXISTENTIAL interpretation (compare (4-a)).

(12) I ate that quantity of apples every day for a year.

It is unlikely that the speaker means to convey that he ate the same apples every day when uttering (12). Instead, as was the case with amount, here quantity is used to specify an abstract amount that variously instantiates as apples. Under this reading, quantity receives an EXISTENTIAL interpretation. Here we make a prediction: we hypothesized above that underlying the EXISTENTIAL reading is the DIRECT interpretation, whereby abstract amounts are referenced but not instantiated by objects. Put differently, we have hypothesized that the EXISTENTIAL interpretation derives from a DIRECT use of the noun: first a measurement is referenced, and then it is instantiated. Given that we observe EXISTENTIAL uses of quantity (cf. (12)), we predict DIRECT uses for the noun as well. This prediction appears to hold: compare (13) with the examples featuring amount in (8).

(13) a. I want the largest quantity.
    b. That quantity is too much.
While the most salient reading for (13) might be a definite one (where a substance is assumed and a salient subset of it is referenced), it also admits the *direct* interpretation like *amount* in (8), as we would expect if the *direct* interpretation precedes the *existential* interpretation. Under the *direct* reading, *quantity* is used to make claims about abstract amounts.

What about the other sub-classes of quantizing nouns? In (14), we test the availability of the *definite* interpretation for the container noun *glass*. Both sentences allow this *definite* interpretation: specific quantities of water are referenced by *glass of water* (likely through a reinterpretation of *glass* as a measure term).

(14) a. That glass of water smells like chlorine.
   b. I drank every glass of water that you brought.

In (15), we test the availability of the *direct* interpretation for *glass*. Neither sentence delivers this interpretation. (15-a) fails completely. In (15-b), *glass* functions as a basic noun and the speaker makes claims about glasses, not about measurements.

(15) a. #I want that glass, but of milk.
   b. I want the largest glass.

Finally, in (16), we test the availability of the *existential* interpretation. Here we see that no such reading arises. Both sentences express a *definite* interpretation of *glass*, resulting in the unlikely assertion that the same water was consumed at various points in time.

(16) a. I drank that glass of water every day for a year.
   b. I drank the glass of water that you drank.

The resulting pattern is summarized in (17). Unlike the atomizer *quantity*, here we see that *glass* may not function like *amount* and reference abstract measurements (the reader can verify that no container nouns may).
I want that glass of water.

✓ DEFINITE interpretation: I want that water there
✗ DIRECT interpretation: I want that abstract amount/measurement
✗ EXISTENTIAL interpretation: I want some water that measures the relevant amount

This finding should come as no surprise: in the previous chapter we saw that container nouns are simple predicates with no measurement in their semantics. But if measurement is a sufficient quality to behave as amount, we might expect measure terms like liter to admit direct and existential interpretations. We begin with the definite interpretation in (18), which liter admits via its measure term semantics.

(18) a. That liter of water smells like chlorine.
   b. I drank every liter of water that you brought.

We next test the direct interpretation for liter. Assuming it is available, the direct interpretation would reference an abstract measurement. However, to the extent that they succeed at all, the sentences in (19) receive only a definite interpretation whereby a container is referenced.

(19) a. I want that liter, but of milk.
   b. That liter is too much.

Turning to the existential interpretation, in (20) we see that no such reading arises for liter. The sentences simply cannot convey that different quantities of water, each measuring one liter, were consumed at various times. We summarize the results for the measure term liter in (21).

(20) a. I drank that liter of water every day for a year.
   b. I drank the liter of water that you drank.

---

3We did, however, see that container nouns may function as measure terms when a natural correspondence exists between the container referenced and a measure that uses the container as units. In fact, in (14), glass likely serves as a measure term.
(21) I want that (one) liter of water.

✓ DEFINITE interpretation: I want that water there
✗ DIRECT interpretation: I want that abstract amount/measurement
✗ EXISTENTIAL interpretation: I want some water that measures the relevant amount

The measure term *liter*, in spite of naming a measure, may not be used to reference abstract amounts determined by that measure, or variable instantiations of such amounts. We see, then, that *amount* and the other degree nouns truly stand out among the quantizing nouns. Only they and *quantity*, when used under a similar guise, make reference to abstract measurements.

Despite its unique status among the quantizing nouns, once we broaden our investigation to include words without any hint of measurement in their semantics, we find a noun that behaves similarly to *amount*: *kind*. In fact, any name for a kind admits analogues to the DIRECT and EXISTENTIAL interpretations we have identified. Consider the behavior of *kind* in (22).

(22) a. I ate that kind of apple every day for a year.
   b. I ate the kind of apple that you ate.

The sentence in (22-a) does not assert that the same apple was eaten each day for a year; instead, different instantiations of the same kind of apple (say, McIntosh) were eaten each day. A similar situation obtains in (22-b), which crucially does not assert that the speaker and the addressee ate the same apple. In parallel to the behavior of *amount* in (4-a), here we have an EXISTENTIAL reading of *kind*: the noun is used to name an abstract property, being a specific kind of apple, and this property is differently instantiated by real-world objects, which are acted on accordingly.

If this EXISTENTIAL reading for *kind* is the same beast as the reading we identified for *amount*, and if we are right in supposing that the EXISTENTIAL reading derives from a more abstract, DIRECT reading, then we should observe this DIRECT reading for *kind*. In fact, we do. First, recall what is meant by the label “DIRECT” for *amount*: the abstract measurement
referenced by *amount* is named and predicated of directly (e.g., by asserting that an amount of wine is too much, as in (8-b); or by specifying how the measurement instantiates, as in (10)). With *kind*, we are not referencing measurements, but rather kinds. Thus, with *kind*, a **direct** interpretation has the kind named entering into a direct predication relationship. Consider the sentences in (23), where properties are ascribed directly to kinds.

(23) a. That kind of apple is widespread.
    b. That kind of whale is extinct.

The **direct** interpretation for *kind* also arises when the noun enters into generic constructions; here, the named kind serves as the restrictor to a modal and behaves like an indefinite. For example, a paraphrase for (24-b) could be, “it is generally the case that apples of that kind have worms” (Chierchia, 1995).

(24) a. That kind of apple goes down easy.
    b. That kind of apple has worms.

We see, then, that *kind* behaves like *amount* in its ability to yield both **direct** (i.e., kind-referencing) and **existential** (i.e., kind-instantiating) interpretations. Recall that in addition to these two interpretations, *amount* also allows a **definite** interpretation whereby real-word objects are referenced, as in (25).

(25) I want that amount of apples.

We have supposed that this **definite** interpretation of *amount* arises when *amount* receives a partitioning *quantity* semantics (by a process to be made clear below in Section 4.1.2). Returning to *kind*, no such **definite** interpretation is possible. Simply put, *that kind of apple* can never refer to a specific, salient apple. It may only reference an apple kind, and through this apple kind instantiations thereof. It would seem, then, that unlike *amount*, *kind* cannot receive a partitioning semantics. In fact, the **definite** interpretation for *amount* will arise through a reinterpretation of the word as an atomizer like *quantity*, a strategy not available to *kind*. However, *kind* does allow abstract reference to kinds (the **direct**
interpretation), and through such reference differential instantiation of the relevant kind (the existential interpretation). The interpretations available to kind are summarized in (26).

(26) I want that kind of apple.
   ✗ DEFINITE interpretation: I want that apple there
   ✓ DIRECT interpretation: I want that abstract kind
   ✓ EXISTENTIAL interpretation: I want some apple of the appropriate kind

We finally have a noun with behavior similar to amount. To see more clearly the similarity in behavior between amount and kind (and kinds), we next consider a broader range of examples. As will become apparent, the parallels between kind and amount are not accidental. As such, our semantics for amount will be modeled on the semantics of kinds.

In his discussion of bare plurals and their role as names of kinds, Carlson (1977b) examines the behavior of the noun kind. We use his observations as a point of comparison with the behavior of amount. First, Carlson identifies the peculiar relationship between kind and its substance noun. In (27) and (28) (Carlson’s examples (11) and (12), p.341), Carlson notes the contrast between the behavior of kind’s substance noun and run-of-the-mill DPs in their ability to relativize, be questioned, and pronominalize (cf. the restrictions on pseudo-partitive syntax observed in Selkirk, 1977, and the anti-anaphora property reported in Zamparelli, 1998).

(27) a. ??Those are the beans that Bob ate three kinds of ___.
   b. ??What did Bob see two kinds of?
   c. ??Bob saw three kinds of \{ \begin{align*} \text{them} \\ \text{it} \end{align*} \} yesterday.

(28) a. Those are the beans that Bob ate three pounds of ___.
   b. What did Bob eat two pounds of?
   c. Bob ate three pounds of \{ \begin{align*} \text{them} \\ \text{it} \end{align*} \} yesterday.

\[With \text{kind}, \text{the direct interpretation whereby kinds are referenced is perhaps more apparent with the verb like, as in I like that kind of apple. Kind-level predicates illustrate this reading even more clearly, as in that kind of animal is extinct and the sentences in (23).\]
Whereas the substance nouns in (28) freely relativize, get questioned, and pronominalize, in (27), *kind*'s substance noun resists participation in all three phenomena. Crucially, each of the targeted nominals in (28) is a full DP. In (27), it is more natural to assume a kind-denoting substance noun, rather than a definite one – an assumption that reduces perceived acceptability. Compare this behavior with *amount* in (29); with all three phenomena, *amount* aligns with *kind* – the sentences are possible, but they contrast with (28) in acceptability. Like with *kind* in (27), the discomfort associated with the sentences in (29) likely stems from a clash between assuming a kind-denoting substance noun and manipulating a full DP.

(29)  
\[
\begin{align*}
&\text{a. ??Those are the beans that Bob ate three amounts of } \underline{\text{ }}. \\
&\text{b. ??What did Bob } \begin{cases} \text{see} \\
\text{eat} \end{cases} \text{ two amounts of?} \\
&\text{c. ??Bob saw three amounts of } \begin{cases} \text{them} \\
\text{it} \end{cases} \text{ yesterday.}
\end{align*}
\]

Next, Carlson notes that there are uses of *kind* that attribute properties to kinds of things, and crucially not to denumerable objects; here we have the *direct* interpretation of *kind*. For example, (30-a) (Carlson’s example (16a), p.343) and (30-b) make claims about specific kinds of animal, not specific instantiations of those kinds. Any kind-level predicate will deliver this *direct* interpretation for *kind*.

(30)  
\[
\begin{align*}
&\text{a. Some kind of animal is common.} \\
&\text{b. That kind of dog is widespread.}
\end{align*}
\]

But like *amount*, *kind* may also be used to make claims about instantiations of the kinds that are named; here we have the *existential* interpretation. In (31) (Carlson’s example (20), p.344), the speaker conveys that two kinds of dogs (say, pit bulls and collies) are *instantiated* by dogs in the next room (say, by Bruiser and Rex).

(31) Two kinds of dogs are in the next room.

Carlson furthermore shows that the dimension by which we evaluate *kind* must be fixed, so that their realizations are disjoint. He gives the example of Fido, the watch-dog collie.
Watch-dogs are a kind of dog; collies are, too. However, if Fido and no other dog is in the next room, (31) cannot (easily) describe this situation. The problem is that kind here would have been used to reference kinds that share realizations. To further illustrate this point, Carlson considers possible responses to a request to enumerate all the kinds of cars. “Fords, convertibles, road-racers, sedans, Chevrolets, . . . ” would be out as a response, but a list of brand names would be fine. Only in the latter case are the sets of objects that realize the kinds disjoint.

Amount exhibits the same behavior. (32) may not be used to assert that two different amounts of apples (say, apples weighing three kilos and apples numbering five) are instantiated by the same apples in the next room.

(32) Two amounts of apples are in the next room.

Even when the amounts are differently instantiated, (32) must assume a single, fixed measure by which we evaluate amount. As with kind, amount requires a fixed dimension by which it is evaluated; with amount, this dimension is determined by the contextually-specified measure. So, two quantities of apples, one weighing three kilos and another weighing four, could verify (32). Likewise, a pit bull and a collie, or a watch dog and a lap dog, verify (31).

In fact, any name for a kind aligns with amount in its ability to yield both direct (i.e., kind-naming) and existential (i.e., kind-instantiating) interpretations. Crucially, the existential interpretation that evaded container nouns and measure terms freely arises from uses of kind names. We illustrate the availability of these readings with common nouns (e.g., apples; see Carlson 1977b for a description of the common noun class, and motivation for treating common nouns as names for kinds). Again, when we make the definite interpretation highly unlikely, the existential interpretation becomes particularly salient. In (33), the same apples were probably not eaten each day, but instantiations of the same kind of apple likely were (cf. (4-a)). Given the current hypothesis that existential interpretations derive from direct ones, we expect those apples to exhibit this direct interpretation. In (34), we privilege this interpretation by combining common nouns with a kind-level predicates.
(33) I ate those apples every day for a year.
    ← every day for a year I ate some apples of that kind

(34) a. Those apples are common.
    b. That animal is extinct.

Building on Carlson’s investigation of kind and kinds, Wilkinson (1995) discusses the facts about kind mentioned above (i.e., its ability to yield both direct and existential interpretations), and to them adds three more observations that are relevant to our discussion of amount. First, Wilkinson notes that kind optionally appears without a common/substance noun. Compare the sentences in (35) (Wilkinson’s examples (17) and (26), pp.386–7).

(35) a. That kind of animal is sitting on my lawn.
    b. An animal of that kind is sitting on my lawn.

Both sentences receive an existential interpretation: an instantiation of the appropriate animal kind is sitting on the speaker’s lawn. In (35-a), kind appears superficially transitive; Carlson treats it as a modifier of the common noun animal. In (35-b), kind appears superficially intransitive. For now we leave aside the specifics of the transitive/intransitive semantics for kind (but see below in Section 4.1.1 for discussion); what is relevant is the parallel in behavior with amount. In (36), we replicate the transitive/intransitive ambiguity with amount.

(36) a. That amount of water is too much.
    b. Water in that amount is too much.

Next, Wilkinson observes that definite kind may serve as the pivot of a be-existential, ostensibly flouting the Definiteness Restriction, which precludes definite NPs from occurring in this position (Milsark, 1974; Heim, 1987). Compare (37-a), featuring kind, with (37-b), featuring the basic definite those books (Wilkinson’s examples (12) and (13), p.384); whereas those books may not serve as the pivot of an existential, those kinds does so freely (see also Zamparelli 1998).
definite direct existential

amount ✓ ✓ ✓
quantity ✓ ✓ ✓
glass ✓
kilo ✓
kind ✓ ✓ ✓

Table 4.1: Available nominal interpretations: a comparison with amount

(37)  a. There are those kinds of books in the library.
     b. *There are those books in the library.
     c. There are books in the library.

(38)  a. There is that amount of apples in our kitchen.
     b. There are those amounts of wine on the menu.

Comparing the sentences in (38) with (37-a), we see that amount yet again patterns with kind: both may serve as the pivot of an existential. We return to amounts in existential sentences and the Definiteness Restriction in Section 4.4.

Finally, Wilkinson observes that definite kind may be bound by adverbs of quantification like indefinites (see also Zamparelli, 1998). The sentences in (39) (Wilkinson’s examples (14) and (15), p.384) make the same assertion: it is usually the case that an equation of the specified kind has two different solutions.

(39)  a. That kind of equation usually has two different solutions.
     b. An equation of that kind usually has two different solutions.

(40)  a. That amount of apples usually busts the bags.
     b. Apples in that amount usually bust the bags.

Once again, amount patterns with kind: it, too, may be bound by quantificational adverbs, such that the sentences in (40) are synonymous. Like kind, amount behaves as an indefinite in certain contexts.

To review, amount may be used to reference objects directly (the definite interpretation). Under this guise, amount behaves like quantity and establishes a partition over the overlapping substance noun denotation. Amount may also be used to reference abstract
measurements (the direct interpretation) or make claims about objects that can instantiate those measurements (the existential interpretation). Our focus is on the semantics of measurement, so these latter interpretations of amount are our primary concern. Amount stands out among the quantizing nouns in its ability to deliver these interpretations – container nouns and measure terms do not admit direct or existential readings. However, names for kinds and kind specifically do exhibit both direct and existential interpretations. We review the possible interpretations for the nouns that we have considered in Table 4.1. Using kind semantics as our guide, we turn now to the denotation of amount.

4.1.1 Amount semantics

In characterizing the existential interpretation of amount, we noted how objects are referenced indirectly via their correspondence to abstract amounts. Moreover, we identified the nature of this correspondence as one of measurement. Consider the existential reading of (41):

(41) I ate that amount of apples every day for a year.

\[ \rightarrow \text{ every day for a year I ate some apples that measured the relevant amount} \]

In uttering (41), the speaker references an amount and assert that the amount was instantiated each day by apples, which he ate. But what are amounts, and, crucially, how are they instantiated?

It would appear that amounts are degrees, e.g., three kilos or four feet. Degrees are instantiated by the individuals that hold them: apples weighing three kilos, trees reaching four feet in height, etc. In other words, the abstract degree indicated by that amount of apples instantiates as any set of apples that evaluates to the appropriate value with respect to the relevant measure, i.e., that reaches the specified amount. These individuals are specified in (42), where \( \mu_f \) is the contextually-specified measure and \( n_i \) is the relevant value in the range of that measure. Setting \( \mu_f \) to the measure in kilograms, \( \mu_{kg} \), and \( n_i \) to 3, we get a set of apple individuals, each weighing three kilos. (42) would thus serve as the characteristic function for the property of holding the degree three kilos of apples.
However, in using *amount* to reference abstract representations of measurement, that is, degrees, one crucially does not reference individuals. Uttering *I want that amount of apples* with an EXISTENTIAL interpretation, the speaker makes no claims to the apples he wants. Instead, the speaker indicates a desired amount, which will be instantiated by apples. Thus, the denotation in (42) is a poor candidate for *amount of apples*, given that (42) denotes a set of apples whereas *amount of apples* denotes a set of amounts.

The language we are using to describe the referents of *amount* and their behavior in the sentences that embed it reveals our strategy in formalizing *amount*’s semantics: *amount* references abstract representations of measurement which may be instantiated by objects in the world. Thus, the set to which *amount* refers should contain abstract entities, which must be instantiated by objects. Here we find a parallel with the semantics of kinds: abstract entities corresponding to properties, which are defined in terms of the objects that instantiate them. Moreover, we noted in the previous section the striking similarity in behavior between *amount* and *kind*: both admit direct and existential interpretations whereby abstract entities or real-world objects that instantiate them are indicated by the use of these terms. *Amount* also enjoys the exceptional distribution pattern that characterizes *kind*. Here is the explanation for this overlap in behavior: *amount* refers to a set of degrees; *kind* to a set of kinds. Degrees, like kinds, are the individual correlates of properties of individuals; kinds and degrees are the same sort of entity.

Anderson and Morzycki (2012) arrive at a similar conclusion, namely that degrees are kinds, using a different set of data. Their primary concern is modification as it relates to degrees, manners, and kinds. They show a broad range of functional elements that appear to apply to all three sorts of entities, for example English *how*, *as*, and *such*. They also note the behavior of the Polish anaphoric expression *tak*, which refers to kinds, manners, and degrees; for our purposes, the flexibility between kind and degree reference is relevant, as in (43) (from their example (1)).
Their ultimate conclusion is couched in a Neo-Davidsonian framework: degrees are kinds of states. While the proposal developed here is in principle compatible with Anderson and Morzycki’s approach, we make do without appeal to events or states. In other words, our kinds are of a different sort.

First, we review the behavior of kinds. With kind-level predicates (*widespread, extinct, common, etc.*), kinds enter directly into predication relations and yield a direct interpretation. The sentences in (44) ascribe properties directly to kinds.

(44)  
a. That kind of dog is extinct.  
b. Dogs are extinct.

In generic constructions, kinds restrict the generic modal (a universal quantifier) and behave as simple indefinites (Chierchia, 1998b). The sentences in (45) make claims about what usually transpires with (specific types of) dogs.

(45)  
a. That kind of dog barks.  
b. Dogs bark.

In episodic contexts, there is no modal to bind the kind and there is no kind-level predicate for which it may serve as an argument. To compose a kind with a non-kind-selecting predicate, the kind must undergo a type adjustment similar to noun-incorporation (cf. van Geenhoven, 1998). The result is the existential interpretation, as in (46).

(46)  
a. That kind of dog is barking outside my window.  
b. Dogs are barking outside my window.

What follows is a brief introduction to the semantics of kinds, building on the primer given in Chapter 1.1.3, which delivers these three distinct uses.
Take the dog kind. It corresponds to the property of being a dog. Dogs instantiate the dog property. Formally, kinds are built from properties via a process of nominalization via the ‘down’ operator $\cap$ (see Chierchia 1998b for discussion). (Kinds may also be taken as primitives; more on this below.) Simply put, the dog kind is the individual correlate of the property of being a dog. Kinds behave as individuals because they are individuals. Crucially, they may be referred to and serve as arguments to predicates. Chierchia gives the following semantics for the $\cap$-operator.\footnote{Where we have generalized the notion of kinds to include any sortal concept (cf. Chapter 1.1.3), K, the set of kinds, should include any kind formed on the basis of a +closed predicate. This move precludes bare singular nouns from enjoying kind reference.}

$$\cap P = \begin{cases} \lambda s. \iota P_s, & \text{if } \lambda s. \iota P_s \text{ is in } K \text{ (the set of kinds)} \\ \text{undefined, otherwise} & \end{cases}$$

where $P_s$ is the extension of $P$ in $s$.

Just as kinds may be constructed from properties via nominalization, properties may be retrieved from kinds via predicativization. The ‘up’ operator $\cup$ applies to a kind and returns the property from which the kind was built. Taking kinds as primitives, the ‘up’ operator $\cup$ applies to a kind and returns the function that characterizes it. Applied to the dog kind, $\cup$ returns the property of being a dog, that is, a set of possible dogs. Chierchia (1998b, p.349) schematizes the relationship between properties and kinds in Fig. 4.1.

When viewed extensionally, the correspondence between kinds and properties may be thought of in terms of sets and the functions that characterize them; the distinction between kinds and properties thus boils down to one of saturation: kinds are saturated, properties are not.

Figure 4.1: Diagram of property–kind correspondence from Chierchia (1998b)
It bears repeating that the set of kinds is a subset of the domain of individuals. Fido is a dog; he is also an individual. The dog kind is the individual correlate of the property of being a dog; it, too, is an individual. When the dog kind serves as the argument to kind-level predicates, as in (44) and (48), we reference the kind directly (and attribute properties to it); here we have the direct interpretation.

(48) Dogs are extinct.

\[ \leftrightarrow \text{extinct}(\cap_*\text{dog}) \text{ or } \text{extinct(DOG)} \]

(where dog is the property of being a dog and DOG is the corresponding kind)

Generic contexts arise from uses of the generic operator, GN, a modalized universal quantifier licensed by a functional aspect head (Chierchia, 1995, 1998b, also see the other papers in Carlson and Pelletier, 1995). This generic operator behaves like a quantificational adverb (Lewis, 1975). It quantifies over appropriate individuals in situations, contextually restricted by the variable C. What results is an assertion about those individuals in the appropriate situations (e.g., dogs barking when they are awake and enervated, etc.). With kinds, the generic operator shifts the kind to a predicative type and quantifies (universally) over the members of the kind. Thus, in generic contexts kinds yield a universal reading. The formalization of (49) may be paraphrased as, “every situation s of the appropriate type containing appropriate instances x of the dog kind is a situation in which x barks.”

(49) Dogs bark.

\[ \leftrightarrow \text{GN } x,s[1^*\text{dog}(x) \land C(x, s)][^*\text{bark}(x, s)] \]

When the dog kind serves as an argument to a non-kind-selecting predicate in an episodic context, there is no GN operator to bind it. Moreover, the predicate attributes properties not to kinds, but to objects – that is, to instances of kinds. Instead of ascribing a property to the entire dog kind, the sentences in (50) assert that there is an instantiation of the dog kind (i.e., some dogs) that is barking. In other words, the sentences assert that there exist individuals belonging to the dog kind that hold the property of barking.
(50)  
   a. That kind of dog is barking outside my window.  
   b. Dogs are barking outside my window.

To compose with non-kind-selecting predicates, kinds in episode sentences are bound by an existential quantifier. Chierchia (1998b) terms this process, whereby kinds compose with non-kind-selecting predicates to yield an existential interpretation, Derived Kind Predication (DKP).

(51) Derived Kind Predication:
   If P applies to objects and k denotes a kind, then P(k) = ∃x[∪k(x) ∧ P(x)]

(52) [dogs are barking outside my window]
   = barking-outside-my-window(DOG)
   via DKP
   = ∃x[∪DOG(x) ∧ barking-outside-my-window(x)]

In (50-b), DKP applies at the level of the predicate barking outside my window, which does not select for kinds. The result is existential quantification over members of the dog kind, as in (52). Here we have the existential interpretation. By having DKP apply at the level of the predicate, we derive the scopelessness (i.e., obligatory narrow scope) of kinds in episode sentences (Carlson, 1977b; Chierchia, 1998b). Compare the (a) examples, featuring overt indefinites, with the (b) examples, featuring bare plurals (i.e., kinds). Only the former allow scope ambiguity such that the nominal takes scope over the relevant scope-bearing element.

(53)  
   a. John wants to see an apple.         (√ want > ∃; √ ∃ > want)
   b. John wants to see apples.          (√ want > ∃; X ∃ > want)
   c. John wants to see that kind of apple. (√ want > ∃; X ∃ > want)

(54)  
   a. John ate an apple repeatedly.      (√ repeat. > ∃; √ ∃ > repeat.)
   b. John ate apples repeatedly.        (√ repeat. > ∃; X ∃ > repeat.)
   c. John ate that kind of apple repeatedly. (√ repeat. > ∃; X ∃ > repeat.)
With DKP formulated as in (51), the scopelessness of kinds in the (b) examples falls out straightforwardly. DKP applies at the level of the predicate; the scope of the existential quantification contributed by DKP is thus bound to this predicate. In other words, the existential quantifier contributed by DKP is scopally inert. Note that in the (c) examples of (53) and (54), the noun kind patterns with bare plurals in its scopeless behavior. We expect this behavior, given that both bare plurals and kind of x name kinds and require DKP to compose in episodic contexts. Let us consider how by giving a denotation for kind.

First, consider a phrase like kind of dog. It denotes a set of subkinds of the DOG kind, containing, say, collies, beagles, poodles, etc. The noun kind thus extracts from its substance noun (e.g., dog) all of its subkinds. We attribute this subkind extraction to the operator SUBKIND, which takes a kind and returns a subkind of it. But recall the discussion of Carlson (1977b) above, specifically the disjointness requirement on kind: when extracting subkinds, kind must be restricted to a certain dimension of evaluation so that kind of dog cannot denote the set consisting of, say, collies, long-haired dogs, beagles, and big dogs. Long-haired dogs are a subkind of DOG, as are collies. But these subkinds share instantiations; they are not disjoint. Hence, the SUBKIND function must be restricted to a certain dimension of evaluation, notated as SUBKIND$_f$, to enforce disjointness.

Next, consider the behavior of kind: in the general case, it composes with a kind-denoting noun and returns its subkinds.$^6$ In this use, kind is transitive, taking the substance noun as an argument. Discussing the inherently relational semantics of kind, Zamparelli (1998) conceives of kind as an unsaturated function, writing, “a kind is always a kind of something.” We should encode this fact in the semantics of kind. The resulting denotation is transitive, or relational as in (55).

\[ [\text{kind}] = \lambda g \lambda k. \text{SUBKIND}_f(g)(k) \]

Kind takes a kind-denoting argument, the substance noun, and returns a set of kinds. This set contains subkinds of the substance noun. The subkinds are extracted by the SUBKIND$_f$ function. These subkinds are individual correlates of the properties of being a subkind of

---

$^6$For now we leave aside ostensibly intransitive uses of kind, as in a dog of that kind. Zamparelli (1998) provides arguments for treating these uses as underlyingly transitive, with kind taking dog as an argument.
dog. A candidate denotation for *kind of dog* appears in (56).

\[(56) \quad \begin{align*}
\text{[kind of dog]} &= \lambda k. \text{SUBL}_\text{k}(\text{DOG})(k) \\
\text{[kind of dog]} &= \left\{ \begin{array}{l}
\cap \lambda x. \ast \text{collie}(x) \\
\cap \lambda x. \ast \text{beagle}(x) \\
\cap \lambda x. \ast \text{poodle}(x) \\
\vdots
\end{array} \right\}
\end{align*}\]

With *kind* returning a set of (sub)kinds, its scopelessness is predicted in episodic sentences. In (50-a), DKP applies at the level of the predicate to allow *that kind of dog* to serve as an argument. What results is the **existential** interpretation, whose scope is tied to the predicate itself.

Now we return to degrees and the word that names them: *amount*. As we have seen, degrees are abstract representations of measurement. These representations behave as individuals: speakers may reference degrees and provide them as arguments to predicates. When the predicate may apply directly to degrees, we get a **direct** reading. Furthermore, these degree individuals correspond to properties: sets of individuals that hold the relevant degree. When a predicate applies to objects, we make claims about individuals that instantiate the relevant degrees, getting an **existential** reading. We thus have an association between kinds and degrees: both are nominalizations of properties that are instantiated by objects in the world. *Amount* behaves like *kind* because both terms denotes entities of the same sort: nominalized properties. An **existential** reading results when a degree serves as the argument to an object-level predicate and something like DKP mediates their composition.

But what sort of property begets a degree? Because they are abstract representations of measurement, degrees must be built from properties whose semantics appeals to a measure. In its simplest form, a degree is the nominalization of a property defined on the basis of a measure, as in (57). By determining how the a kind instantiates for the purpose of measurement, the partitioning function $\pi$ internal to the semantics of degrees ensures that they apply to contextually-supported maximal instances of stuff. Degrees are thus conceived of as information bundles with four coordinates: $<\mu, n, \pi, k>$. Setting $\mu_f$ to the kilogram measure, $\mu_{kg}$, and its value $n$ to 3, we get the *three kilo* degree as in (59). This degree is the
individual correlate of the property of weighing three kilos; predicativising the degree via the \( \cup \) operator returns the set of things that weigh three kilos.

\[
\text{DEGREE} := \cap \lambda x. \exists k [ \mu_f(x) = n \land \pi(k)(x)]
\]

where \( \mu_f \) is a contextually-specified measure,
\( n \) is some number in the range of the measure \( \mu_f \),
and \( \pi \) is a contextually-supplied partition

\[
\text{(58) Partitioning function } \pi:\]

\( \pi \) is a function of type \( \langle k, \langle e, t \rangle \rangle \)
such that for any \( k \) and any \( y \) in \( \pi(k) \),
\( \cup k(y) \& \text{MSC}(y)(k) \).

\[
\text{(59) } \cap \lambda x. \exists k [ \mu_{kg}(x) = 3 \land \pi(k)(x)]
\]

Note that the property from which a degree is built is quantity-uniform with respect to the measure \( \mu_f \) specified in the property’s semantics: everything holding this property evaluates to the same \( n \) with respect to \( \mu_f \). In (59), every object in the de-nominalized property weighs the same: three kilos. This notion, being a quantity-uniform property, is defined in (60) (cf. Chapter 2).

\[
\text{(60) Quantity-uniform property: }
\]

\[
\text{QU}_{\mu}(P) = 1 \iff \forall x \forall y [ P(x) \land P(y) \rightarrow \mu(x) = \mu(y) ]
\]

Degrees are thus nominalizations of quantity-uniform properties. \textit{Three kilos qua degree} is the individual correlate of the property something holds when it weighs three kilos. Similarly, \textit{that amount} is the individual correlate of the property something holds when it measures the appropriate amount. As individuals, degrees enter into semantic computation as arguments. Composing with a predicate that may apply directly to degrees, \textit{that amount} yields a \textit{direct} interpretation. By predicativizing them via \( \cup \), as in the process of DKP, degrees grant us access to the individuals that instantiate them. Hence, degrees also admit an \textit{existential} interpretation.
Finally we have the means to provide a denotation for the noun *amount*: it denotes a set of degrees, nominalized quantity-uniform properties formed on the basis of a contextually-specified measure. But as with *kind*, *amount* behaves like a transitive noun, relating a substance with amounts thereof. Echoing Zamparelli (1998), an amount is always an amount of *something*. Rarely does one find bare *amount*, that is, an instance of the degree noun without a substance noun like *apples* specifying what the amounts are of. In (61), we attempt to construct examples of bare *amount*, but the result feels inherently relational: a substance is assumed.

(61)  

\begin{enumerate}
\item John brought that amount with him to the party last week.
\item I would like the amount that Bill had.
\end{enumerate}

In both of the sentences in (61), the tendency is to assume elision of the substance noun and attribute to *amount* a specific substance it is measuring, similar to the process of ∃-closure that yields apparently intransitive measure terms. We therefore encode the substance noun, a bare plural or mass term, as an argument of *amount*. Doing so allows the degrees to which *amount* refers to be both quantity- and quality-uniform. In other words, the degrees referenced by *amount* are tied to the kind supplied by the substance noun. To make available salient concrete portions of the substance for measurement, the kind denoted by the substance noun gets instantiated and partitioned by π. In (62), we illustrate a quantity- and quality-uniform degree: every member of the denominalized property is an instance of the *apple* kind.

(62)  \quad \cap \lambda x. \mu_f(x) = n \land \pi(\text{APPLE})(x)

Taking *amount* to be inherently transitive, relating instances of the substance noun with amounts thereof, we arrive at the following semantics for *amount*:

(63)  \quad [\text{amount}] = \lambda k \lambda d. \exists n[d = \cap \lambda x. \mu_f(x) = n \land \pi(k)(x)]

where \( \mu_f \) is a contextually-specified measure,

\( n \) is some number in the range of the measure \( \mu_f \),

and \( \pi \) is a contextually-supplied partition.
Transitive *amount* first takes the kind-denoting substance noun as an argument, partitions it, then relates this partitioned set to a set of quantity- and quality-uniform degrees. In this way, amounts are always of *something*. Simply put, *amount* encodes directly the kind variable that gets existentially quantified in our degree template in (64).

(64)  
\[
\text{DEGREE} := \cap \lambda x. \exists k[\mu_f(x) = n \land \pi(k)(x)]
\]

where $\mu_f$ is a contextually-specified measure,

$n$ is some number in the range of the measure $\mu_f$,

and $\pi$ is a contextually-supplied partition

Note, however, that we have encountered ostensibly intransitive uses of *amount*, repeated below. Recall the scenario in which the sentences in (65) are considered: looking over a wine menu, the speaker makes claims about the amounts on offer. Here, as in (61), a substance is assumed, namely wine. Similarly in the specificational sentence in (66), a substance is assumed. The problematic case for quality-uniform degrees is (67). For now we set aside this intransitive use of *amount*, merely noting its circumlocutory paraphrase.

(65)  
\begin{itemize}
  \item a. I want the largest amount.
  \item b. That amount is too much.
\end{itemize}

(66)  
That amount is five kilos.

(67)  
I want that amount in diamonds.

\[\mapsto \text{I want that amount to be realized in diamonds}\]

As with *kind*, *amount* receives a relational semantics under which it takes a kind-denoting substance noun as an argument and relates the kind with a set of nominalized properties, that is, with a set of degrees. By building degrees from properties, we may access the members of those properties just as we access the instantiations of a kind. Thus, (41), repeated in (68), references an amount (i.e., a degree) and asserts that this degree was variously instantiated by apples, which were eaten each day over the course of a year. This instantiation process proceeds with degrees just as it did with kinds: via existential quantification over the members of denominalized degrees. The mechanism remains DKP, which we generalize in (69) to apply
to both kinds and degrees, that is, to any nominalized property (cf. (51))).

\[(68) \quad \text{I ate that amount of apples every day for a year} \]
\[\quad \leftrightarrow \quad \text{every day for a year I ate some apples that measured the relevant amount}\]

\[(69) \quad \text{Generalized DKP:} \]
\[\text{If P apples to objects and y denotes a nominalized property, then} \]
\[P(y) = \exists x[\cup y(x) \land P(x)]\]

In (70), we see a simplified derivation for the sentence in (68). Two features are crucial: first, *that amount of apples* denotes a degree; second, this degree composes with the object-level predicate *eat* via Generalized DKP. The result is an existential interpretation; the speaker asserts that there was instantiation of the *amount-of-apples* degree that he ate.

\[(70) \quad [\text{I ate that amount of apples...}] \]
\[= \text{ate(that-amount-of-apples)}(I) \]
\[\quad \text{via Generalized DKP} \]
\[= \exists x[\cup \text{that-amount-of-apples}(x) \land \text{ate}(x)(I)]\]

By taking seriously the similarities in behavior between *amount* and *kind*, we arrived at a kind semantics for degrees. Degrees, like kinds, are the individual correlates of properties, for example the property of attaining a certain degree (e.g., weighing 3 kilos) or belonging to a specific kind (e.g., being a poodle). Our aim has been the existential interpretation whereby measurements are variously instantiated by objects that are ascribed properties. Associating degrees with properties grants us access to the objects that instantiate them, just as associating kinds with properties grants us access to their members. Taking degrees as semantic primitives that merely indicate points on a scale (e.g., Kennedy, 1999), we would have no hope of deriving the existential interpretation that characterizes *amount*. We would also miss the generalization that captures the striking similarity in behavior between *amount* and *kind*: both nouns reference nominalized properties.

Our next step is to specify how the internal composition of *that amount of apples* proceeds such that the result is a degree. However, before investigating the semantics of degree
composition, we first compare the degree noun semantics for *amount* in (63) with the other quantizing nouns we encountered in the previous chapter.

### 4.1.2 Degree nouns in our typology

We began this section by comparing the behavior of *amount* and *quantity*. Both words admit *definite, direct*, and *existential* interpretations. Both require a substance noun. Despite these similarities, we have posited two distinct subclasses of quantizing nouns: degree nouns and atomizers. Here we explore this hypothesis in more detail to ensure that degree nouns in fact inhabit a diverging subclass. We will see that the nouns *amount* and *quantity* are more or less interchangeable, but the classes to which they belong pull apart in predicted ways once we attribute measurement to the semantics of degree nouns and partitioning to the semantics of atomizers.

In the previous chapter, we identified the semantics of *quantity* as one of atomizing; it and the other atomizers (e.g., *pile, grain, heap*) establish a partition over the substances with which they compose. The partition enforces no overlap on the basis of maximally self-connected individuals, which creates a (relatively) atomic predicate denotation whose elements are susceptible to counting. *Quantity* is the most general atomizer, presupposing nothing about the substance with which it composes and making no claims about the (relative) atoms that result. We repeat the semantics for *quantity* in (71); recall that the crucial ingredient is the partitioning function $\pi$.

$$
(71) \quad \text{Atomizers:} \\
[\text{quantity}] = \lambda k \lambda x. \ x \in \pi(k) \\
\text{where } \pi \text{ is a variable of type } \langle k, \langle e, t \rangle \rangle \\
\text{such that for any } k \text{ and any } y \text{ in } \pi(k), \\
\cup k(y) \ & MSC(y)(k)
$$

With the semantics in (71), *quantity* yields a *definite, atomic* interpretation when embedded in larger linguistic contexts. Its use references specific objects that instantiate the substance noun.
(72)  
\[ \text{a. I brought that quantity of apples.} \]
\[ \quad \rightarrow \quad \text{I brought those apples there that I am pointing at} \]
\[ \text{b. I want that quantity of apples.} \]
\[ \quad \rightarrow \quad \text{I want those apples there that I am pointing at} \]

For *amount*, we provided a measuring semantics whereby the substance noun is related to amounts thereof. These amounts are degrees, which are built as nominalized quantity-uniform properties. By building degrees from properties, we enable access to the objects that instantiate degrees, a move that ultimately delivers the existential interpretation that characterizes *amount* and degree nouns. We repeat the semantics for *amount* in (73); here the crucial ingredient is the contextually-supplied measure $\mu_f$.

(73)  
\[ \text{Degree nouns:} \]
\[ [\text{amount}] = \lambda k \lambda d. \exists n[d = \cap \lambda x. \mu_f(x) = n \land \pi(k)(x)] \]
where $\mu_f$ is a contextually-specified measure,
$n$ is some number in the range of the measure $\mu_f$,
and $\pi$ is a contextually-supplied partition

(74)  
\[ \text{a. I ate that amount of apples every day.} \]
\[ \quad \rightarrow \quad \text{every day I ate some apples that measured the relevant amount} \]
\[ \text{b. I want that amount of apples.} \]
\[ \quad \rightarrow \quad \text{I want some apples that measure the relevant amount.} \]

Despite the diverging semantics these nouns receive, *quantity* may yield an existential interpretation and *amount* may yield a definite interpretation. In (75) and (76), we revisit the relevant facts.

(75)  
I brought that quantity/amount of apples.
\[ \quad \rightarrow \quad \text{I brought those apples there that I am pointing at (definite)} \]

(76)  
I ate that amount/quantity of apples every day.
\[ \quad \rightarrow \quad \text{every day I ate some apples that measured the relevant amount (existential)} \]
Have we erred in ascribing to these two nouns diverging semantics and positing distinct subclasses of quantizing nouns to which they belong? To see that the answer to this question is no, we must adopt a broader prospective: *quantity* evidences the subclass of atomizers, and *amount* evidences the subclass of degree nouns. These subclasses contain other members; *quantity* and *amount* are provided merely as the most general instances of their respective subclass. Looking at other atomizers and degree nouns, we see that *quantity* and *amount* are privileged by their similarities in behavior.

Consider the atomizers *grain* and *pile*. Like *quantity*, these nouns compose with a substance noun and yield a partitioned, atomic denotation. Their use thus yields an ATOMIZING, DEFINITE interpretation that references specific objects.

(77) a. I dropped that grain of rice on the floor.
   ⇔ I dropped that rice there that I am pointing at
b. I want to knock that pile of books over.
   ⇔ I want to knock over those books there that I am pointing at

However, unlike *quantity*, these other, more specific atomizers resist the EXISTENTIAL usage that characterizes *amount*. In (78), no measurement enters into the semantic computation; only a DEFINITE, ATOMIZING interpretation is possible.7

(78) a. I will eat that grain of rice again tomorrow.

b. Bring me the pile of books that you brought yesterday.

It would appear that *quantity* alone enjoys EXISTENTIAL uses.

Next, consider the more specific degree nouns *size* and *length*. Like *amount*, these nouns compose with a substance noun and yield a set of degrees, namely, measurements of the substance noun. In episodic sentences, their use yields an EXISTENTIAL interpretation implicating instances of these measurements.

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7Note that (78-a) also admits a subkind reading, paraphrased roughly as, “I will eat that kind of rice again tomorrow.” Crucially, this reading does not involve measurement.
(79)  

a. I dropped the size (of) rock that you dropped.
   \[\rightarrow I \text{ dropped a rock of the same size as the rock that you dropped}\]

b. I used that length of rope on all of my climbs.
   \[\rightarrow \text{ on all of my climbs a used a rope that measured the relevant length}\]

When we try to use \textit{size} and \textit{length} to yield a \textsc{definite} interpretation, we find \textit{length} but not \textit{size} permits it. In (80-a), \textit{that size (of) rock} cannot be used to refer to a specific rock. However, \textit{that length of rope} freely refers to a length segment. Recall that \textit{amount}, like \textit{length}, admits the \textsc{definite} interpretation.

(80)  

a. I dropped that size (of) rock on the floor.

b. I dropped that length of rope on the floor.

To summarize, atomizers yield a \textsc{definite} interpretation, and \textit{quantity} stands out among the atomizers in its ability to also yield an \textsc{existential} interpretation. Degree nouns yield an \textsc{existential} interpretation, and some of them also yield a \textsc{definite} interpretation (e.g., \textit{amount}, \textit{length}). Now for the explanation: the measuring semantics of degree nouns delivers the \textsc{existential} interpretation; the partitioning semantics of atomizers delivers the \textsc{definite} interpretation. The process that shifts degree noun semantics to atomizer semantics is compositional; we thus expect this process to apply broadly, and not just to \textit{amount}, such that degree nouns enjoy uses as atomizers and yield a \textsc{definite} interpretation. There is no corresponding semantic shift that yields degree noun semantics from atomizer semantics; when \textit{quantity} serves as an degree noun to yield an \textsc{existential} interpretation, it derives from a genuine lexical ambiguity. Let us consider each case in turn.

As with the other classes of quantizing nouns, here we see flexibility in usage such that degree nouns may serve as atomizers, giving the \textsc{definite} reading of \textit{that amount of apples} in (75) and \textit{that length of rope} in (80-b). This shift, from degree noun to atomizer, proceeds compositionally as in (81): given the partition function \(\pi\) internal to the semantics of \textit{amount}, all we need to do is generate the set of all individuals to which the degrees denoted by \textit{amount} apply. The set contains non-overlapping instances of the relevant kind, just like the output of \textit{quantity}.  

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Degree noun (AMT) to atomizer (ATM) shift:

\[ [\text{SHIFT}_{\text{AMT} \rightarrow \text{ATM}}] = \lambda \tau(k,(d,t)) \lambda k \lambda x \; \exists d \in A(k)[\langle k \rangle] \]

\[ \lambda d(x) \]

a. \[ \text{[amount]} = \lambda k \lambda d \; \exists n[d = \cap \lambda x. \mu_f(x) = n \land \pi(k)(x)] \]

b. \[ [\text{SHIFT}_{\text{AMT} \rightarrow \text{ATM}}(\text{amount})](k) = \lambda x. \exists d \in A(k)[\langle k \rangle] \]

Given the compositional nature of this semantic shift, it should come as no surprise that it applies to more than just \textit{amount}. As we saw, the more specific degree noun \textit{length} also undergoes the shift in (81) to yield a \textit{definite} interpretation.

The process that builds degree nouns from atomizers is less straightforward. The problem lies in the measure inherent to a degree noun’s semantics. We faced a similar problem in formalizing the shift from container nouns to measure terms in the previous chapter. Shifting \textit{glass} qua container, (82-a), to a name for a measure, (82-b), requires a correspondence between containers and standard units of measure.

\[(82) \]

a. I broke three glasses of water.

b. I drank three glasses of water.

From these standard units we extrapolate a standard measure, whether in the case of container nouns like \textit{glass} or atomizers like \textit{quantity}. But for this process of extrapolation, we simply do not possess enough information within the semantics of these nouns to straightforwardly build a continuous measure. Instead, we saw that the process is indirect, starting with the association between containers and standard units of measure, and from these units deriving a measure. Given its non-compositional nature, we therefore expect the shift from partitioning, atomizer semantics to measuring, degree noun semantics to apply less liberally than the compositional shift in the opposite direction (cf. (81)).

In fact, we have seen that \textit{quantity} stands out among the atomizers in its ability to possess degree noun semantics; none of the other atomizers admit the \textit{existential} interpretation that characterizes degree nouns. It would appear, then, that \textit{quantity} alone enjoys uses as a degree noun because of a genuine lexical ambiguity: it has both partitioning and measuring variants (atomizing: \textit{quantity}_{\text{ATM}}; and \textit{amount}-like: \textit{quantity}_{\text{AMT}}). There is no compositional process that builds measuring \textit{quantity}_{\text{AMT}} from partitioning \textit{quantity}_{\text{ATM}}. Without such a
process, it should come as no surprise that the other atomizers fail to exhibit this ambiguity.

By comparing not just quantity and amount, but a broader class of atomizers against a broader class of degree nouns, we find justification for treating these two nouns as belonging to distinct subclasses of quantizing nouns. Atomizers deliver an ATOMIZING, DEFINITE interpretation via their partitioning semantics. Degree nouns have a measuring semantics and yield an EXISTENTIAL interpretation. Compositionally, the measuring degree noun semantics may be shifted to a partitioning, atomizer semantics; when degree nouns yield a DEFINITE interpretation they have undergone this shift. There is no complementary process that shifts atomizers to degree nouns, delivering an EXISTENTIAL interpretation for atomizers. Quantity alone yields EXISTENTIAL uses because the word is ambiguous.

We thus have evidence for treating the class of atomizers as distinct from the class of degree nouns. But before adopting degree nouns as a subclass of quantizing noun, let us check the behavior of degree nouns against the behavior of the other subclasses we identified in the previous chapter. In addition to atomizers, we have container nouns like glass and measure terms like kilo. Container nouns are simple predicates (they reference objects) with no measurement in their semantics, so amount stands apart if only on the basis of its appeal to measurement. However, as their name suggests, measure terms like liter do appeal to measures; in fact, they name them. But as we saw above, neither container nouns nor measure terms may receive the EXISTENTIAL reading that characterizes degree nouns. The relevant examples are repeated in (83) and (84); these sentences receive only the (highly implausible) DEFINITE interpretation whereby a specific instance of water is consumed repeatedly. They contrast with the degree nouns in (85), which readily yield the EXISTENTIAL interpretation.

(83) Container noun:
   a. I drank that glass of water every day for a year.
   b. I drank the glass of water that you drank.

(84) Measure term:
   a. I drank that liter of water every day for a year
   b. I drank the liter of water that you drank.
Degree noun:

a. I drank that amount of water every day for a year.

b. I drank that size (of) glass every day for a year.

It would appear, then, that degree nouns do stand apart from container nouns and measure terms. Only degree nouns reference degrees, which may be instantiated by objects in episodic contexts via the process of Generalized DKP. As we saw in the previous subsection, it is the application of Generalized DKP to the degree ultimately denoted by degree nouns that delivers the existential interpretation. Container nouns and measure terms reference objects, so Generalized DKP has no opportunity to apply.

What we have is a distinct subclass of quantizing noun. In addition to container nouns and measure terms and atomizers, we also have degree nouns, as evidenced by amount. Degree nouns reference abstract representations of measurement (i.e., degrees) by appealing to measurement directly in their semantics. The subclasses that result, together with representative examples and denotations, appear in (86).

Subclasses of quantizing nouns

a. **Container nouns:**
   
   \[ \text{[glass]} = \lambda x. \text{glass}(x) \]

b. **Measure terms:**
   
   \[ \text{[kilo]} = \lambda k \lambda n \lambda x. \mu_{kg}(x) = n \land \lor k(x) \]

c. **Atomizers:**
   
   \[ \text{[quantity]} = \lambda k \lambda x. x \in \pi(k) \]
   
   where \( \pi \) is a variable of type \( \langle k, \langle e, t \rangle \rangle \)
   
   such that for any \( k \) and any \( y \) in \( \pi(k) \),
   
   \[ \lor k(y) \land \text{MSC}(y)(k) \]

d. **Degree nouns:**
   
   \[ \text{[amount]} = \lambda k \lambda d. \exists n[d = \cap \lambda x. \mu_f(x) = n \land \pi(k)(x)] \]
   
   where \( \mu_f \) is a contextually-specified measure, and
   
   \( n \) is some number in the range of the measure \( \mu_f \)
To summarize: Container nouns (e.g., glass) are simple predicates, denoting a set of objects. Measure terms (e.g., kilo) are number-seeking relations, composing with a substance noun and a numeral; the result is a set of instances of the substance noun that, when measured by the measure named, evaluate to the extent specified by the numeral. Atomizers (e.g., quantity) are partitioning functions; they compose with a substance noun and return an atomic (i.e., non-overlapping) set of objects, susceptible to counting. Degree nouns (amount) are context sensitive relations between a substance noun and amounts thereof; they yield a set of degrees, which are nominalized quantity-uniform properties formed on the basis of a contextually-supplied measure. Table 4.2 summarizes the sorts of entities each subclass references, together with the feature that characterizes their semantics (e.g., measuring vs. partitioning). We include the noun kind as a reminder of the similarity between it and amount – both reference nominalized properties; amount is endowed with a measuring semantics.

Our next task is to investigate how semantic composition proceeds in the structures that contain amount. Doing so allows us to evaluate the conception of degrees as nominalized quantity-uniform properties.

### 4.2 Referencing amounts

To review: amount relates a kind-denoting substance noun with a set of amounts of that substance. This set is a set of degrees; degrees are conceived of as nominalized quantity-uniform properties formed on the basis of a measure. Amount is highly context-sensitive, such that this measure $\mu_f$ and its value $n$ are contextually determined. Additionally, the partitioning function $\pi$ that returns instances of the substance noun receives its specification.
from context. The resulting denotation for the phrase *amount of apples* appears in (87).

(87) a. \[ \text{[amount]} = \lambda k \lambda d. \exists n[d = \bigcap \lambda x. \mu_f(x) = n \land \pi(k)(x)] \]

b. \[ \text{[amount of apples]} = \lambda d. \exists n[d = \bigcap \lambda x. \mu_f(x) = n \land \pi(\text{APPLE})(x)] \]

*Amount* inhabits the subclass of degree nouns. The class stands apart from container nouns (e.g., *glass*), measure terms (e.g., *kilo*), and atomizers (e.g., *grain*) in its ability to yield the existential interpretation in (88).

(88) I ate that amount of apples every day for a year.

→ every day for a year I ate some apples that measured the relevant amount

*Kind* and other kind-denoting nominals pattern with *amount* and deliver the existential interpretation in episodic contexts. Hence the conception of degrees, like kinds, as nominalized properties.

Having settled on a semantics for *amount*, our task now is to determine how this semantics interacts with the structures in which *amount* participates. We start modestly, taking notice of the complex degrees that result from the composition of *amount* with the substance noun. Our focus will be the existential interpretation that characterizes degree nouns. Consider *amount of apples*. We saw that the substance noun is an argument of *amount*. We therefore treat *amount* as a transitive noun; the particle *of* makes no semantic contribution (as in the treatment of *of* for measure terms or atomizers in the previous chapter). The structure in (89) results.

(89) \[
\begin{array}{c}
\text{NP} \\
\text{N} \quad \text{of} \\
\text{amount} \\
\text{apples}
\end{array}
\]

For our purposes, treating the bare plural/mass substance noun as referring to a number-neutral property or to a kind makes little difference; as with the other quantizing nouns, we proceed under the assumption that it refers to a kind in order to exclude singular count
nouns from this position. By composing with its substance noun argument and contextually determining the measure in its semantics, *amount* returns a set of nominalized quantity- and quality-uniform properties. This set is a set of degrees, ordered on the basis of the contextually-determined measure. In (90), we have context set this measure to $\mu_{kg}$, the measure in kilograms. The result is a set of kilograms-of-apples degrees.

\[(90) \quad [\text{amount of apples}] = \lambda d. \exists n[d = \bigcap \lambda x. \mu_{kg}(x) = n \land \pi(\text{APPLE})(x)]\]

\[= \left\{ \begin{array}{l}
\bigcap \lambda x. \mu_{kg}(x) = 1 \land \pi(\text{APPLE})(x) \\
\bigcap \lambda x. \mu_{kg}(x) = 2 \land \pi(\text{APPLE})(x) \\
\bigcap \lambda x. \mu_{kg}(x) = 3 \land \pi(\text{APPLE})(x) \\
\ldots
\end{array} \right\} \]

How do we get from a set of degrees to the relevant degree? In other words, how do we arrive at a single degree from the NP denotation in (90)? Consider the behavior of *amount of apples* when it serves as the argument of the demonstrative *that*.

\[(91) \quad \text{John bought that amount of apples.} \]

Here is a situation in which the sentence in (91) may be uttered felicitously: a quantity of apples sits on a table; the speaker points to these apples, and intends an existential interpretation. The speaker conveys that John bought some apples equal in amount to the apples to which the speaker points. Suppose *amount of apples* denotes a set of kilograms-of-apples degrees as in (90). The demonstrative *that* takes this set of degrees and returns the maximal degree that applies to those apples on the table. In other words, we access this abstract degree through the objects that instantiate it. This process obtains for *that* when
it composes with nominalized properties elsewhere (cf. Partee, 1987): through the indicated object that instantiates it, we access the property.

(92)  a. I love that color of shirt!
     b. That style of art never took off.
     c. I wish that kind of animal would stay out of my garden.

Inherent to the semantics of demonstrative *that* is the individual THAT, that is, the salient object that is indicated. To access the kind/degree-level entity the indicated object instantiates, demonstrative *that* receives the semantics in (93).

\[(93) \quad [\text{that}] = \lambda A. \iota y[A(y) \land \cup y(\text{THAT})]\]

where A is a set of individuals, either nominalized properties or objects, and THAT is the salient object indicated in the use of the demonstrative

The \(\cup\) operator in the semantics of *that* predicativizes the individuals its argument denotes, which allows them to apply to the specified object THAT. When *that* composes with a set of nominalized properties, that is, kinds or degrees, \(\cup\) predicativizes these entities to return the properties from which they are built. We now have the means by which to compositionally reference specific, complex degrees.

In (91), we access the abstract amount of apples by first identifying the relevant apples (i.e., by establishing a pointer to them with *that*) and then picking out the degree that applies to these apples. Suppose the relevant apples comprise the object \(a+b+c\), and that \(\mu_{kg}(a+b+c) = n_{a+b+c}\). What results is (94), where *that amount of apples* references the kilograms-of-apples degree that \(a+b+c\) holds.

\[(94) \quad [\text{that}](\text{[amount of apples]})
\begin{align*}
= & \quad [\text{that}](\lambda d. \exists n[d = \cap x. \mu_{kg}(x) = n \land \pi(\text{APPLE})(x)]) \\
= & \quad \iota y[\lambda d. \exists n[d = \cap x. \mu_{kg}(x) = n \land \pi(\text{APPLE})(x)](y) \land \cup y(\text{THAT})] \\
= & \quad \iota y[\lambda d. \exists n[d = \cap x. \mu_{kg}(x) = n \land \pi(\text{APPLE})(x)](y) \land \cup y(a+b+c)] \\
= & \quad \cap x. \mu_{kg}(x) = n_{a+b+c} \land \pi(\text{APPLE})(x)
\end{align*}\]
The result references a degree: a nominalized quantity-uniform set of apples; everything in that set evaluates to $n_{a+b+c}$ with respect to the kilogram measure. For this degree to compose with the structure that embeds it, we apply Generalized DKP to type-shift the nominalized property for object-level argument slots.

(95) \[[\text{John bought that amount of apples}]\]

$$= \text{bought}(\forall x. \mu_{kg}(x) = n_{a+b+c} \land \pi(\text{APPLE})(x))(\text{John})$$

via Generalized DKP

$$= \exists y[\forall x. \mu_{kg}(x) = n_{a+b+c} \land \pi(\text{APPLE})(x))(y) \land \text{bought}(y)(\text{John})]$$

Here we have the existential interpretation: (95) asserts that John bought some apples equal in weight to the salient apples indicated by that.

To see that the semantics for that in (93) applies in the same fashion for kinds, consider the derivation in (96). Assume the indicated dog, b, is a beagle.

(96) John bought that kind of dog

a. \[[\text{that kind of dog}]\]

$$= \exists y[\forall x. \text{beagle}(x)](y) \land \text{bought}(y)(\text{John})]$$

b. \[[\text{John bought that kind of dog}]\]

$$= \exists y[\forall x. \text{beagle}(x))(y) \land \text{bought}(y)(\text{John})]$$

The sentence in (96) asserts that John bought some dog that belongs to the BEAGLE kind, that is, that John bought a beagle. We have successfully derived the existential interpretation for both degrees and kinds on the basis of the semantics for demonstrative that in (93).

Verifying that we have not lost anything in our new semantics for that, let us consider its more basic uses: when that takes a simple predicate as an argument, as in that boy, it returns the individual in the denotation of the predicate that is identical to the specified individual
that. In other words, when *that* takes a set of objects as an argument, it returns the unique, salient object from this set. To see how this proceeds on the basis of the semantics for *that* in (93), we must understand how the predicativizing \( \cup \) operator works when it apples to an object-level individual.

Applied an object \( a \), the \( \cup \) operator shifts that object into a property. What results in the property of being identical to \( a \). Here we make use of the IDENT operator from Partee (1987), defined as in (97).

\[
(97) \quad \text{Object predicativization:}
\]

\[
\cup a := \text{IDENT}(a) = \lambda x. x = a
\]

Suppose we have the boy \( a \) (i.e., Alan). Predicativizing \( a \), \( \cup a \), yields the property of being identical to Alan. In other words, it yields the property of being Alan, true only of \( a \). Thus, when *that* composes with a simple predicate as in (98), it returns the unique individual identical to the specified object *that*. Simply put, it returns the object *that*.

\[
(98) \quad \begin{align*}
\text{a. } [\text{boy}] &= \{a, b, c\} \\
\text{b. } \text{that} &= a \\
\text{c. } [\text{that boy}] &= \iota y[\text{boy}(y) \land \cup y(a)] \\
&= \iota y[\text{boy}(y) \land \text{IDENT}(y)(a)] \\
&= a
\end{align*}
\]

We now have a generalized semantics for *that* which allows us to specify individuals, and by specifying those individuals to reference the properties that are true of them.

Next, consider what happens with sets of degrees when they serve as the argument of the definite determiner *the*. We take *the* to be a maximality operator, composing with a set and returning its maximal element (Sharvy, 1980; Chierchia, 1998b; Zamparelli, 1998).

Chierchia (1998b, ex. (11a), p.346) defines the iota operator \( \iota \) as in (99).

\[
(99) \quad \iota A = \text{the largest member of } A \text{ if there is one (else, undefined)}
\]

\[8\]Sharvy (1980) observes that the Russellian view of definiteness, whereby its primary function is to signal uniqueness, fails once we expand our coverage beyond singular definite descriptions. He shows that definiteness instead serves to identify maximal elements; uniqueness falls out as a side effect of maximality.
Defining \( \iota \) as a maximality operator allows the to compose with a set of degrees. These degrees are ordered on the basis of a measure, and the returns the largest degree. In (100), we provide a derivation to illustrate this process; \( \text{max} \) stands for the largest possible value in the domain of the measure. The result has the amount of apples denote the maximal apple degree.

\[
\text{(100)} \quad \begin{align*}
\text{a. } \text{[amount of apples]} &= \lambda d. \exists n [d = \bigcap \lambda x. \mu_f(x) = n \land \pi(\text{APPLE})(x)] \\
\text{b. } \text{[the(amount-of-apples)]} &= \iota \lambda d. \exists n [d = \bigcap \lambda x. \mu_f(x) = n \land \pi(\text{APPLE})(x)] \\
&= \bigcap \lambda x. \mu_f(x) = \text{max} \land \pi(\text{APPLE})(x)
\end{align*}
\]

Unmodified, definite amount is often infelicitous. Consider the sentence in (101); its awkwardness arises because definite amount references a maximal degree, which in most circumstances will be impossible to instantiate. In the absence of context, which could establish a salient partition, (101) asserts that John bought some apples that measure the maximal degree, that is, that he bought the totality of apples.

\[
\text{(101)} \quad \text{#John bought the amount of apples.}
\]

To be used felicitously, definite amount must be modified, as in (102). This modification restricts the set of degrees to just those that are relevant. In (102), the degrees are restricted to just those that apply to the apples on the table. Maximality selects the largest such degree.

\[
\text{(102)} \quad \text{John bought the amount of apples on the table.}
\]

Under the existential reading, (102) asserts that John bought some apples equal in amount to the apples on the table. If there are three kilograms of apples on the table, then (102) asserts that John bought three kilograms of apples. But here we must understand how the PP on the table restricts amount of apples to just those apple-degrees that apply to the objects on the table. We start by identifying the ingredients of this modification.

First, we have the NP amount of apples, a set of degrees as in (103-a). To this NP we adjoin the PP on the table, a set of objects as in (103-b). In (103-c), we have the structure that results.
To derive the existential reading of (102), we need the maximal NP in (103-c) to denote a set of apple degrees restricted to just those degrees that apply to objects on the table. As was the case when we had nominalized properties serving as arguments to object-level predicates, here we assume that this restriction involves existential quantification over instances of the de-nominalized properties. This restrictive, existential modification is defined as in (104); the derivation for (102) appears in (105). Note that the effect of composing the modified set of degrees with maximal the is to add the restriction as a presupposition on this degree set.

(104)  **Existential Modification:**
\[ A_{(d,t)} \cap_E P_{(e,t)} = \lambda d. A(d) \land \exists y[P(y) \land \cup d(y)] \]

(105)  John bought the amount of apples on the table.

a.  [the amount of apples on the table]
\[
\begin{align*}
&= \textbf{the} [\lambda d. \exists n[d = \cap \lambda x. \mu_f(x) = n \land \pi(\text{APPLES})(x)]] \cap_E [\lambda x. \text{on-table}(x)] \\
&\text{via Existential Modification} \\
&= \textbf{the} [\lambda d. \exists n[d = \cap \lambda x. \mu_f(x) = n \land \pi(\text{APPLES})(x)] \land \\
&\exists y[\text{on-table}(y) \land \cup d(y)]] \\
&\text{via Maximality} \\
&= \cap \lambda x: \exists y[\mu_f(y) = \text{max} \land \pi(\text{APPLES})(y) \land \text{on-table}(y)]. \mu_f(x) = \text{max} \land \\
&\pi(\text{APPLES})(x)
\end{align*}
\]
b.  [John bought the amount of apples on the table]
\[
= \text{bought}(\text{the-amount-of-apples-on-the-table})(\text{John})
\]
via Generalized DKP
\[
= \exists y [u(\text{the-amount-of-apples-on-the-table})(y) \land \text{bought}(y)(\text{John})]
\]

By making use of existential degree modification, maximality in the semantics of the, and type adjustment via Generalized DKP, the sentences in (105) asserts that John bought some apples equal in amount to the apples that are on the table. These tools, all of them independently justified, thus deliver the existential interpretation for modified amount. Note that by restricting the degrees denoted by amount, its use suddenly describes a much more plausible state of affairs: rather than buying the totality of apples, (105) has John buying merely a small portion of this totality. In the next section, we investigate another way that amount may be modified: relativization.

4.3 Amount relatives

We begin with a note on terminology. The name “amount relative” (sometimes “degree relative”) often indicates a peculiar class of there-existentials that ostensibly flout the Definiteness Restriction (Milsark, 1974; Carlson, 1977a; Heim, 1987; Grosu and Landman, 1998). Examples of these so-called “amount relatives” are provided in (106).

(106)  a.  I bought the books that there were on the table.

b.  I ate the cake that there was in the bakery.

These constructions are analyzed in the following section. For now, our aim is true amount relatives, that is, relative clauses headed overtly by amount. These amount relatives evidence another strategy for modifying degrees (cf. the existential modification of the previous section). The objective here is to show that on the basis of the proposed semantics for degrees and amount, standard takes on relative clauses yield the right interpretations. Consider the sentence in (107).

(107)  John ate the amount of apples that you ate.
We encountered examples like the sentence in (107) at the beginning of this chapter, using them to highlight the existential interpretation of *amount*. Under this reading, (107) asserts that John ate some apples equal in amount to the apples the addressee ate. What follows is an investigation of how this reading results.

Regarding the structure of *amount* relatives, the literature on relative clause syntax is vast and complicated, informed by many nuanced facts from many different languages. Deciding the syntax of relative clauses could fill an entire thesis (in fact it has, many times). But our focus is the semantics of measurement. Therefore, before we make sense of this semantics as it pertains to *amount* relatives, let us make some assumptions about the syntax of relativization.

The literature provides many options for the analysis of relative clause syntax, among them head-external (Montague, 1974; Partee, 1975; Chomsky, 1977), raising (Áfarli, 1994; Kayne, 1994), matching (Sauerland, 1998), and head-raising (Donati and Cecchetto, 2011) analyses. We provide derivations with head-external and raising syntax for explicative purposes; the semantics that derives the existential interpretation for *amount* relatives remains the same: degree abstraction/modification at the CP level, and maximality contributed by definiteness. We begin with head-external syntax.

### 4.3.1 Head-external syntax

Suppose the NP *amount of apples* heads the *amount* relative in (107) and the CP *that you ate* serves as a modifier to this NP; they compose via intersective modification (Scontras and Nicolae, to appear). Composing the resulting NP with *the*, we get the DP in (108). Central to the head-external analysis is the origination of the head NP outside of the relative clause CP. Within the relative clause, a relative operator A’ moves from VP-internal object position to the specifier of CP, binding its trace.
Whether the relative operator $Op$ binds an object-level trace, type $e$, or a degree trace changes
the strategy needed to compose the DP meaning in (108). This choice, however, does not
affect the meaning that results: both types of trace will yield the EXISTENTIAL reading we
are after.

Suppose the relative operator binds an object-level trace. The CP will denote a simple
predicate, the set of objects that the addressee you ate. This CP adjoins to the degree-set
NP $\text{amount of apples}$. In other words, a predicate of degrees composes with a predicate of
individuals. Modification results. Here we need to restrict the degrees in the NP denotation
to just those degrees that apply to things the addressee ate. We encountered this sort of
restrictive, existential degree modification in the previous section for the modification of
$\text{amount}$ by PP predicates. The mechanism is repeated in (109); a set of degrees composes
with a set of objects through a process akin to Generalized DKP. By maximizing these
restricted degrees, the predicate information contributed by the relative clause CP results as
a presupposition on the degree set that we form. To see this process at work, consider the
derivation for the $\text{amount}$ relative in (108), provided in (110). Again, we assume here the
binding of an object-level trace internal to the CP.\(^9\)

\(^9\)We use the shorthand “$\lambda x$” to indicate the result of moving an operator from object position to the
specifier of CP.
Existential Modification:
\[ A_{(d,t)} \cap_E P_{(e,t)} = \lambda d. \, A(d) \land \exists y [P(y) \land \cup d(y)] \]

(109) [the amount of apples that you ate]
\[ = \text{the [amount of apples] [\lambda x. you ate x]} \]
\[ = \text{the [\lambda d. } \exists n [d = \cap \lambda x. \, \mu_f(x) = n \land \pi \text{(APPLES)}(x)] \cap_E [\lambda x. \text{ate}(x)(\text{you})] \]
via Existential Modification
\[ = \text{the } \lambda d. \, \exists n [(d = \cap \lambda x. \, \mu_f(x) = n \land \pi \text{(APPLES)}(x)) \land \exists y [\text{ate}(y)(\text{you}) \land \cup d(y)]] \]
via Maximality
\[ = \cap \lambda x: \exists y [\mu_f(y) = \max \land \pi \text{(APPLES)}(y) \land \text{ate}(y)(\text{you})]. \, \mu_f(x) = \max \land \pi \text{(APPLES)}(x) \]

Through the process of existential modification and the contribution of maximality by definite \textit{the}, the DP \textit{the amount of apples that you ate} references a single degree. Concretely, this referent is an apple-degree presupposed to instantiate as something the addressee ate. Simply put, this degree is the largest amount of apples that the addressee ate. In (111), we complete the derivation for the \textit{amount} relative in (107). The DP denotation in (110) is abbreviated as the degree name \textit{the-amount-of-apples-that-you-ate}.

(111) [John ate the amount of apples that you ate]
\[ = \text{ate(\textit{the-amount-of-apples-that-you-ate})(John)} \]
via Generalized DKP
\[ = \exists y [\uparrow (\textit{the-amount-of-apples-that-you-ate})(y) \land \text{ate}(y)(\text{John})] \]

Generalized DKP delivers the existential interpretation of the \textit{amount} relative in (107): (111) entails that John ate some apples equal in amount to the apples the addressee ate.

Now, suppose that instead of binding an object-level trace, the relative operator \textit{Op} binds a degree trace. At the CP level, instead of a predicate of individuals we would have a predicate of degrees. This degree-denoting CP composes with the NP \textit{amount of apples}, itself a predicate of degrees. Here we implicate run-of-the-mill modification: two elements of the same type compose to yield a new element of the same type (see McNally (to appear) for a discussion of modification). Before seeing how this modification proceeds semantically,
consider the elements involved. We have the denotation for *amount of apples* in (112). In
(113), we derive the set of degrees denoted by the relative CP. Note that Generalized DKP
allows the degree trace to compose with the object-level predicate.

(112)  \[ \text{amount of apples} = \lambda d. \exists n[d = \cap \lambda x. \mu_f(x) = n \land \pi(\text{apples})(x)] \]

(113)  \[ \lambda d. \text{you ate } d \] = \lambda d. \text{ate}(d)(\text{you)}

\text{via Generalized DKP}

\[ = \lambda d. \exists y[\cup d(y) \land \text{ate}(y)(\text{you})] \]

The relative CP in (113) denotes a set of degrees that apply to things that *you* ate. Composing
the two sets of degrees in (112) and (113), we get a new set of degrees: amounts of apples that
the addressee ate. Note that the quantificational force contributed by existential modification
in (110) is now supplied by Generalized DKP. This moves allows us to simply restrict the
set of degrees in (112) by the set in (113). This modification, together with the maximality
contributed by *the*, specifies the maximal amount of apples that the addressee ate. The
derivation of the *amount* relative appears in (114). Again, here we have a RC-internal degree
trace (cf. the object-level trace in (110)).

(114)  \[ \text{the amount of apples that you ate} \]

\[ = \text{the } \text{amount of apples } [\lambda d. \text{you ate } d] \]

\[ = \text{the } [\lambda d. \exists n[d = \cap \lambda x. \mu_f(x) = n \land \pi(\text{apples})(x)]] \cap [\lambda d. \exists y[\cup d(y) \land \text{ate}(y)(\text{you})]] \]

\text{via Intersective Modification}

\[ = \text{the } [\lambda d. \exists n[d = \cap \lambda x. \mu_f(x) = n \land \pi(\text{apples})(x)] \land \exists y[\cup d(y) \land \text{ate}(y)(\text{you})]] \]

\text{via Maximality}

\[ = \cap \lambda x: \exists y[\mu_f(y) = \text{max } \pi(\text{apples})(y) \land \text{ate}(y)(\text{you})]. \mu_f(x) = \text{max } \pi(\text{apples})(x) \]

Comparing the derivations in (110) and (114), we see that whether the relative operator
binds an object-level trace or a degree trace, the same denotation results. With object-level
traces, we employ Existential Modification to compose a simple predicate with a predicate
of degrees. With degree traces, RC-internal Generalized DKP contributes existential force,
and simple restrictive modification composes two predicates of degrees. In either case, the maximality contributed by the leaves us with the degree equal to the amount of apples that the addressee ate. Generalized DKP applies at the matrix level, and the sentence in (107) entails that John ate some apples equal in amount to the apples that the addressee ate. We thus observe the success of head-external syntax in the derivation of the existential interpretation for amount relatives.

4.3.2 Raising syntax

Despite the ubiquity of head-external analyses of relative clause syntax, many arguments have been presented in favor of a raising syntax instead (for discussion, see Bhatt, 2002). The latter has the relative clause head originate within the relative CP, then move to a clause-external position. Arguments for this raising approach often involve evidence of semantic reconstruction, where scope-bearing elements interact across a relative clause boundary (e.g. Scontras et al., 2014; Tsai et al., 2014). Here we do not settle the debate between raising and head-external approaches. We merely demonstrate the success of both approaches in the semantics of amount relatives.

Applied to the amount relative in (107), raising syntax yields the structure in (115). Note that the NP amount of apples raises with the relative operator to the specifier of the relative CP, leaving behind a trace. It then raises again, this time to an RC-external position. Reconstructing the moved elements, the DP receives the LF in (116).
(115)  
```
  DP
  /\    
 the NP
 / \    
 NP j CP
|    |   [Op t j] (that)
|    |   TP
|    |
|    | amount of apples
|    |
you ate t i
```

(116)  
```
[the amount of apples that you ate] = the [λd. you ate Op amount of apples]
```

To interpret the reconstructed phrase, we must first adjust its type: amount of apples is a predicate of degrees, but we need an individual to serve as an argument to ate. Note that Generalized DKP will not help us here. Generalized DKP allows nominalized properties (e.g., degrees) to compose with object-level predicates. But amount of apples denotes a set of degrees. Before Generalized DKP can apply, we must convert amount of apples into a single degree. Here we follow Bhatt (2002) in adopting the operation of Trace Conversion from Fox (2002). This process shifts reconstructed predicates (like amount of apples in (116)) into individuals. The operation is described in (117).

(117)  
```
Trace Conversion (Fox, 2002):
```

a. Variable Insertion:
   
(\text{Det}) \text{Pred} \rightarrow (\text{Det}) \text{[Pred } \lambda y. y = x\text{]}

b. Determiner Replacement:
   
(\text{Det}) \text{[Pred } \lambda y(y = x)\text{]} \rightarrow \text{the [Pred } \lambda y. y = x\text{]}

Trace Conversion proceeds in two steps. First, Variable Insertion injects the identity function \( \lambda y. y = x \). The variable \( x \) is free, bound at the CP level as a result of the movement that raises the RC head to a position external to the clause (Heim and Kratzer, 1998). The identity function composes with the reconstructed predicate via restrictive modification. For
this modification to obtain, we must assume that the variable that is inserted is of the same sort as the moved phrase (Chierchia, 1998b); doing so allows the identity function to denote a predicate of degrees, just like *amount of apples*. Next, Determiner Replacement replaces the relative operator *Op* with the, that is, the maximality-seeking *ι* operator. The result of Trace Conversion applied to the (reconstructed) *amount* relative head in (116) appears in the derivation in (118). We abbreviate the predicate of degrees denoted by *amount of apples*, (112), as *amount-of-apples*.

(118) \[
[Op \text{ amount of apples}]
\]
\[
= Op \text{ amount-of-apples}
\]
via Variable Insertion
\[
= Op \text{ amount-of-apples } \lambda d. d = d'
\]
via Intersective Modification
\[
= Op \lambda d. \text{amount-of-apples}(d) \land d = d'
\]
via Determiner Replacement
\[
= \lambda d [\text{amount-of-apples}(d) \land d = d']
\]

After Trace Conversion, the reconstructed RC head denotes the maximal degree identical to the inserted variable, d’. Simply put, the reconstructed head now denotes the degree d’, which is restricted by *amount-of-apples* such that it is a degree of apple-amounts.

With an individual in the object position of the RC predicate, composition may proceed in a familiar fashion. Note that this individual is a degree, so Generalized DKP will apply as *ate* composes with its argument. The derivation for the *amount* relative in (112) appears in (119).

(119) \[
[\text{the amount of apples that you ate}]
\]
\[
= \text{the } [\lambda d'. \text{you ate } \lambda d [\text{amount-of-apples}(d) \land d = d']]
\]
via Generalized DKP
\[
= \text{the } [\lambda d'. \exists y [\lambda d [\text{amount-of-apples}(d) \land d = d']](y) \land \text{ate}(y)(\text{you})]
\]
via Maximality (and unpacking the degree predicate *amount-of-apples*)
\[
= \forall x: \exists y [\mu_f(y) = \text{max } \land \pi(\text{APPLES})(y) \land \text{ate}(y)(\text{you})]. \mu_f(x) = \text{max } \land \pi(\text{APPLES})(x)
\]
The combination of Generalized DKP at the level of the RC predicate with the maximality contributed by the returns the maximal degree that is an amount of apples eaten by you. In other words, the amount relative denotes the amount of apples the addressee ate (i.e., a degree). Generalized DKP at the matrix level delivers the existential interpretation of this amount relative. Once we have a denotation for the amount relative, the process proceeds in (120) exactly as it did in (111) for head-external syntax.

(120) \[ \text{[John ate the amount of apples that you ate]} \]

\[ = \text{ate}(\text{the-amount-of-apples-that-you-ate})(\text{John}) \]

via Generalized DKP

\[ = \exists y[\cup (\text{the-amount-of-apples-that-you-ate})(y) \land \text{ate}(y)(\text{John})] \]

The derivation of the amount relative semantics differs depending on the structure we assume, but the result is the same: (107) entails that John ate some apples equal in amount to the apples the addressee ate.

### 4.3.3 Summary

Relative clauses headed by amount allow for the modification of degrees. We began this section by noting the existential interpretation that arises for amount in (121). The sentence privileges this interpretation, whereby the degree denoted by the amount of apples that you ate is instantiated by different apples, because the alternative, definite interpretation describes a highly unlikely state of affairs: John and the addressee eating the same apples.

(121) John ate the amount of apples that you ate.

\[ \rightarrow \text{John ate some apples equal in amount to the apples that you ate} \]

For the existential interpretation to result, the relative clause in (121) must reference a degree, namely, the amount of apples that the addressee ate. Our focus was therefore the semantic composition of this degree.

We considered two analyses for the syntax of relativization, head-external (e.g., Chomsky, 1977) and raising (e.g., Kayne, 1994). Under the first, head-external approach, the head
amount of apples originates outside of the RC; the relative operator introduces a variable in the RC-internal object position. Determining the semantic type (i.e., sort) of this variable, we faced two options: it either stands for an object (type $e$) or a degree (type $d$). Taking the RC-internal variable to be an object, the RC denotes a simple predicate: the set of things the addressee ate. The process of Existential Modification that we encountered in the previous section (composing, e.g., the amount of apples on the table) allows for the modification by this predicate of the RC head amount of apples. What results is a set of apple-degrees restricted such that they apply to things eaten by the addressee. Maximality and Generalized Derived Kind predication deliver the existential interpretation.

Treating the RC-internal variable as a degree yields identical results, although the mechanism that derives them is slightly different. With a degree variable, we require Generalized DKP at the level of the RC predicate. Abstracting over this degree at the CP level, the result is a predicate of degrees, which may compose with the RC head via simple, intersective modification. Maximality and another instance of Generalized DKP at the level of the matrix predicate deliver the existential interpretation.

Raising syntax necessitates a more complicated derivation for the degree denoted by an amount relative. At issue is the interpretation of the reconstructed head, which originates within the RC and raises out. In its reconstructed position, amount of apples denotes a predicate of degrees. To interpret this predicate as an argument, we apply the operation of Trace Conversion (Fox, 2002). The result is a degree variable bound at the CP level. As was the case when we assumed a degree variable with head-external syntax, this move necessitates RC-internal Generalized DKP and degree abstraction at the CP-level. The remainder of the composition proceeds as above: intersective modification, maximality, and another instance of Generalized DKP deliver the existential interpretation.

While it might seem overly pedantic to go through derivations for each of these possible choices, their success demonstrates the robustness of the proposed program: conceiving of degrees as nominalized properties delivers the existential interpretation that characterizes amount with a minimum of added technology. Regardless of the approach we take in building their structure, the semantic computation of amount relatives involves two basic operations:
Generalized DKP, which contributes existential force; and the maximizing $\epsilon$ operator inherent to the semantics of definiteness. Both moves are independently motivated.

4.4 Degree relatives

This chapter concludes with an investigation of so-called “amount” relatives (Carlson, 1977b). We will see that, despite their suggestive name, these constructions do not involve the noun *amount*. They do, however, involve degrees. For this reason, we follow Grosu and Landman (1998) in adopting the more transparent name “degree” relative. These degree relatives provide another proving ground for our conception of degrees as nominalized properties.

We begin by summarizing the Definiteness Restriction, which precludes individuals from the post-verbal, pivot position of existential sentences (Milsark, 1974; Safir, 1982; Heim, 1987). Understanding the Definiteness Restriction highlights the peculiar behavior of degree relatives, which at least on the surface appear to flout this constraint. We then turn to the account of degree relatives proposed by Grosu and Landman (1998). These authors follow the literature that precedes them in positing degree abstraction in the semantics of degree relatives. Doing so explains the exceptional behavior of degree relatives with respect to the Definiteness Restriction: degrees, unlike individuals, may occur in pivot position. Grosu and Landman show, however, that degrees-as-points are insufficient to account for the behavior of degree relatives. The construction ultimately references individuals, so degrees must contain information about the objects that instantiate them. The authors therefore propose a new, enriched semantics for degrees, similar in spirit to what we have here. Like Grosu and Landman and the work that informs their account, our proposal here will also feature degree abstraction as a means to derive degree relatives. We will see, however, that our conception of degrees as nominalized properties allows for a straightforward account of degree relatives without the ad-hoc machinery of Grosu and Landman (1998).

4.4.1 Existential sentences and the *Definiteness Restriction*

Before we can appreciate the exceptional behavior of degree relatives, we must understand the constraint that they ostensibly violate: the Definiteness Restriction. Milsark (1974) observes
that existential sentences restrict definite and universally quantified NPs from occurring in their postverbal, pivot position. He provides the examples in (122) to illustrate this restriction (Milsark’s examples (64) and (65), p.195). Crucially, the indefinite, existentially quantified NPs in (123) freely serve as pivots to existentials.

(122)  a. *There is 
\[
\begin{array}{l}
\text{the dog} \\
\text{John’s dog} \\
\text{that dog} \\
\text{John} \\
\text{him/he} \\
\end{array}
\]
in the room.

b. *There \[
\begin{array}{l}
\text{are} \\
\text{all dogs} \\
\text{both dogs} \\
\text{every dog} \\
\text{each dog} \\
\end{array}
\]
in the room.

Milsark recasts the Definiteness Restriction as a restriction on quantification. He divides DPs into two classes: those that may express cardinality (headed by, e.g., a, some, few, three), and those that must express quantification (headed by, e.g., each, both, every). By treating definiteness as a special type of universal quantification, to this latter class he adds definite DPs like proper names and those headed by the.

Quantificational DPs are precluded from serving as pivots to existential predicates because of a semantic clash that results from their own quantificational force and the quantificational force contributed by the existential predicate. Treating the semantic contribution of the existential predicate EXIST as simple existential quantification, that is, the operator ∃, Milsark demonstrates that a quantified DP in pivot position yields vacuous binding of the variables introduced. The problem is that two quantifiers, the one internal to the pivot DP and the one in the semantics of EXIST, target the same variable. Consider the LF in (124).
In (124), existential \( \exists \) introduces a variable that either is bound already by the quantificational DP, (124-b), or appears nowhere in the resulting LF, (124-b'). In either case, the contribution of the existential predicate is the vacuous abstraction of a variable. When the pivot is not quantificational, as in (125), abstraction is no longer vacuous and the construction is well-formed.

(125)  
\[ a. \text{There are men in the room.} \]
\[ b. \exists x [\text{men}(x) \land \text{in-room}(x)] \]

Milsark’s Definiteness Restriction thus serves as a constraint on vacuous abstraction, which results when quantificational DPs serve as pivots to existentials.

Heim (1987) also takes up the Definiteness Restriction and shows that it operates not merely at the level of surface representations, giving the contrast between (122) and (123), but at the level of LF. In addition to the patterns originally noted in Milsark (1974), Heim focuses on the following class of examples. In (126), overt bound variable pronouns fail to serve as the pivot to an existential. In (127), indefinite pivots obligatorily receive a narrow scope interpretation. In (128), Heim reproduces judgments from Safir (1982) that demonstrate the inability of definite \( wh \)-phrases to target the pivot position.

(126)  
\[ a. \text{*Few people admitted that there had been them at the party.} \]
\[ b. \text{*No perfect relationship is such that there is it.} \]

(127)  
\[ a. \text{Ralph believes that there is someone spying on him.} \]
\[ b. \text{There must be someone in John’s house.} \]

(128)  
\[ a. \text{*Which one of the two men was there drunk?} \]
\[ b. \text{*Which actors were there laughing?} \]
Each type of ruled-out construction has in common a variable standing in pivot position at LF. For example, the impossible wide-scope reading of (127-b) would have the (simplified) LF in (129). The \textit{wh}-movement that yields the ill-formed questions in (128) leaves behind a trace in pivot position, interpreted as a variable as in (130).

\begin{itemize}
  \item (129) \textit{*Someone} x: there must be x in John’s house
  \item (130) \textit{*Which} actors x: there were x laughing
\end{itemize}

Classifying individual variables as the same sort of entity as names, Heim describes the Definiteness Restriction as a prohibition on this sort of entity occurring in pivot position. Her formulation of the Definiteness Restriction appears in (131).

\begin{itemize}
  \item (131) \textit{Definiteness Restriction:} \\
    \begin{itemize}
      \item *There be \textit{x}, when \textit{x} is an individual variable
    \end{itemize}
\end{itemize}

The LFs in (129) and (130) are straightforwardly ruled out by the prohibition in (131): quantifiers and \textit{wh}-phrases leave variables in pivot position, which violate the Definiteness Restriction. Whether we treat proper names as quantifiers like Milsark (1974) or as simple individuals, the prohibition in (131) precludes them from serving as pivots in a similar manner. Once we understand the Definiteness Restriction as a ban on individuals in pivot position, the sentences that avoid this ban become much more interesting.

Heim provides two classes of apparent exceptions to the Definiteness Restriction: certain \textit{wh}-traces in questions, and traces in relative clauses. Consider first the case of questions. We saw in (128) that definite \textit{which} cannot target the pivot of an existential. Heim explains this fact by assuming that \textit{wh}-phrases move and leave behind an individual variable, a configuration ruled out by the constraint in (131). But if all \textit{wh}-phrases leave variables of the same sort, we have no explanation for the contrast between \textit{which} in (128) and \textit{what} or \textit{how many} in (132) (Heim’s examples (14) and (18), p.27).

\begin{itemize}
  \item (132) a. How many soldiers were there in the infirmary?
    \item b. What is there in Austin?
\end{itemize}
To explain the acceptability of the questions in (132), Heim proposes that the traces of these questions are more complex than they at first appear. For *how many* $N$ in (132-a), Heim assumes a trace that takes the form $x$-many $N$. For *what* in (132-b), Heim first notes that the question asks about the kind of things one finds in Austin, and then formulates the trace of *what* to reflect this fact: *something of kind* $x$. Both sorts of trace, Heim argues, avoid the Definiteness Restriction.

Turning to relative clauses, Heim reproduces the following examples from Safir (1982). In each, we have a gap in pivot position due to relativization. Assuming that relativization, like *wh*-movement, leaves an individual trace in gapped pivot position, the relative clauses in (133) pose a puzzle: the Definiteness Restriction should rule them out.

(133)

a. The very few books that there were ___ on his shelves were all mysteries.

b. Every single man that there was ___ in the castle was ready to fight for his life.

c. All of the men that there were ___ in the garrison sallied forth en masse to meet the enemy.

To explain the acceptability of relativization in the existential constructions in (133), Heim draws on the in-depth study of this phenomenon provided in Carlson (1977a).

Carlson (1977a) argues that in addition to restrictive and non-restrictive (i.e., appositive) relative clauses, English contains yet a third class of relative clauses: degree relatives. These relative clauses stand apart from the other two classes on the basis of their limited, peculiar distribution. According to Carlson, only degree relatives may relativize the pivot of an existential as in (133). Additionally, degree relatives may not be introduced by *wh*-form relativizers. To see the interaction of these properties of degree relatives, compare the sentences in (134).

(134)

a. John ate the apples that/Ø there were ___ on the table.

b. *John ate the apples which there were ___ on the table.

In (135-a), where the relative clause is introduced by *that* or Ø, the sentence is acceptable. According to Carlson, (134-a) is acceptable because it features a degree relative. In (135-b), where *which* introduces the relative clause, the sentence is unacceptable. If only degree
relatives can relativize the pivot of an existential, and if degree relatives cannot be introduced by the wh relativizer which, then the relative clause in (134-b) cannot be a degree relative, hence its ungrammaticality.

Carlson (and later Heim) analyzes these exceptions to the Definiteness Restriction as involving a degree variable in the pivot position internal to the relative clause, similar to the x-many N trace Heim posits for how many N in (132-a).\(^{10}\) In other words, the books that there were on the shelves gets treated as the books that there were (d-many books) on the shelves. Crucially, degree variables but not individual variables may serve as pivots to existentials, at least as far as the Definiteness Restriction is concerned.

In essence, Carlson’s analysis of degree relatives has them directly name amounts of stuff. To support this prediction, Heim provides the following example, stressing that it admits a reading where “only identity of amounts, not identity of substances, is required for [its] truth” (Heim, 1987, 38); (135) would be paraphrased as in (136). Heim’s identity-of-amounts reading is our EXISTENTIAL reading.

\[(135)\quad \text{It will take us the rest of our lives to drink the champagne that they spilled that evening.}\]

\[(136)\quad \text{It will take us the rest of our lives to drink the amount of champagne that they spilled that evening.}\]

\[
\leftrightarrow \quad \text{it will take us the rest of our lives to drink an amount of champagne equal to the amount they spilled that evening}\]

It bears noting here that (135) does not involve a there existential. This is no accident. According to Carlson and later accounts his inspires, a degree relative is any relativization structure with a degree variable in the gapped position.

Current theories of degree relatives follow Carlson and Heim in positing degree variables and something like a null many in the gapped position of these relative clauses. In the following subsection, we consider the proposal of Grosu and Landman (1998) in some detail. Understanding their proposal will help to clarify the empirical terrain that ought to be

---

\(^{10}\) Carlson (1977a) uses the term “amount” to describe what we here call degrees, namely, abstract measurements of stuff. Hence his use of the term “amount” relative.
covered, and provide a rich testing ground for degrees-as-kinds.

4.4.2 Grosu and Landman (1998) and enriched degrees

Two factors inform the account of degree relatives in Grosu and Landman (1998): First, as we saw in the previous subsection, degree relatives do not fall under the scope of the Definiteness Restriction; second, the identity-of-amounts reading noted by Heim (1987) for (135) very rarely obtains. To handle the first point, Grosu and Landman follow Carlson (1977a) and subsequent accounts that posit degree variables internal to degree relatives. To handle the second point, they propose a richer notion of degrees. In what follows, we summarize the relevant aspects of their proposal for degree relatives, taking special note of the construction-specific machinery that we will later improve on.

Grosu and Landman start from the observed restriction on degree relative relativizers from Carlson (1977a). Repeated in (137), *that* and the null relativizer Ø may introduce a degree relative, but *wh* relativizers may not.

\[
(137) \quad \begin{align*}
\text{a. } & \text{John ate the apples that/Ø there were } \_ \text{ on the table.} \\
\text{b. } & \text{*John ate the apples which there were } \_ \text{ on the table.}
\end{align*}
\]

Following Heim (1987), Grosu and Landman take this fact to suggest that degree relatives feature a degree variable in the gapped position, a variable which is abstracted over at the CP level. The degree variable avoids the Definiteness Restriction. (137-b) is ruled out because *wh* forms cannot bind a degree variable; the construction in (137-b) features instead an individual variable in gapped position, which violates the Definiteness Restriction.

Now, assuming a degree variable in gapped position, Grosu and Landman propose the LF for degree relatives in (138-b). Note that they also assume a silent *many* to relate the degree variable with the reconstructed head. The resulting denotation for a degree relative appears in (138-c).

\[
(138) \quad \begin{align*}
\text{a. } & \text{(apples) that there were } \_ \text{ on the table} \\
\text{b. } & \text{(apples) that there were (d many apples) on the table} \\
\text{c. } & \{d: \exists x[\text{APPLE}(x) \text{ and } |x| = d \text{ and on-table}(x)]\}
\end{align*}
\]

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In effect, the degree relative *apples that there were on the table* denotes a set of degrees: those degrees that correspond to the cardinality of apples on the table. More accurately, this set will contain every cardinality that applies to the apples and all of their possible subsets. Assuming three apples, those cardinalities will be 3, 2, and 1.

To derive the LF in (138), Grosu and Landman adopt a raising structure for the relative clause. In gapped position, we have the degree phrase *d many apples*. This degree phrase moves to the specifier of CP, and from this position the head *apples* moves to a position external to CP. The structure in (139) results.

(139)

```
(139) DP
    D
    NP
    | the NP
    | apples CP
    | d many apples NP CP
    | (that) C S
    there were [d many apples] on the table
```

Given the structure in (139) and the LF in (138-c), Grosu and Landman propose that the semantic contribution of the head noun is one of a sortal. Here is what sets degree relatives apart from run-of-the-mill relative clauses: the latter compose with their head noun via restrictive modification. But degree relatives cannot compose restrictively, as they denote sets of degrees and the nominal head a set of individuals. The trick, then, is to allow the degree relative head to be interpreted inside the CP, restricting the set of degrees to just
those degrees that apply to objects in the denotation of the head. So far, all of this should sound familiar from our discussion of *amount* relatives in Section 4.3 above.

At this point, Grosu and Landman have simply implemented the proposals from Carlson (1977a) and Heim (1987). Degree variables internal to degree relatives avoid the Definiteness Restriction, and preclude *wh*-form relativizers. A degree relative denotes a set of degrees, restricted by the head noun’s semantics. Here, crucially, we are assuming that degrees are semantic primitives, i.e., basic numbers. This means that a degree relative like *apples that there were on the table* in (138) denotes a set of numbers – nowhere do we have information about apples or the table. Here we confront the problem with the Carlson/Heim analysis: “it just can’t be correct” (Grosu and Landman, 1998, p.132).

Despite Heim’s example of an identity-of-amount reading for the relative clause repeated in (140), Grosu and Landman observe that degree relatives rarely admit such a reading.

(140) It will take us the rest of our lives to drink the champagne that they spilled that evening.

(141) John ate the apples that there were on the table.

In (141), the speaker asserts that John ate the apples on the table, not an amount of apples equal to the amount of apples on the table. Using our terminology, the sentence receives a definite interpretation, not an existential one. But if degree relatives denote just sets of simple degrees, this existential, identity-of-amount interpretation should be only reading available. Something has to give: we need a way to retrieve individuals from degrees.

Grosu and Landman arrive at the same conclusion that we did above: we need richer degrees so that a degree keeps track of what it is a degree of. Doing so will allow the semantics to retrieve from the set of degrees denoted by a degree relative the objects that instantiate those degrees. The authors therefore propose the definition of degrees in (142).

The degree function, \( \text{degree}(x) \), takes a plural individual and maps it to a tuple with three coordinates. The first element is the cardinality of the plural individual, \( |x| \). The second element is the sortal predicate \( P \) to which \( \text{degree}(x) \) is relativized; this sortal predicate constrains the measure domain. The third element is the plural individual itself.
Enriched degrees from Grosu and Landman (1998):

For all plural individuals x: \( \text{DEGREE}_P(x) = \langle |x|, P, x \rangle \)

This enriched notion of degrees includes within degrees themselves information about what the degree is of, just as the authors intended. Note, however, that the notion of degrees in (142) differs fundamentally from our conception of degrees as nominalized quantity-uniform properties. For Grosu and Landman, a degree is a bundle of information that includes a plural individual, its sortal class (a predicate), and its cardinality. An individual is retrieved because it exists internal to the degree itself. For us, a degree is the nominalization of, say, the property of weighing three kilos or measuring ten liters; a degree is the individual correlate of a property that is formed on the basis of a measure. And individual is retrieved via the property it instantiates, which may be reconstructed from the information structure of degrees.

Now, consider how Grosu and Landman handle their sort of degree in the context of degree relatives. In gap position we have \( d \text{ many apples} \), interpreted as in (143). We repeat the LF the authors assume in (144).

\[
(143) \quad \text{[d many apples]} = \lambda x. \text{apples}(x) \land \text{DEGREE}_{\text{apples}}(x) = d
\]

\[
(144) \quad \begin{align*}
\text{a.} & \quad (\text{apples}) \text{ that there were } \underline{-} \text{ on the table} \\
\text{b.} & \quad (\text{apples}) \text{ that there were } (d \text{ many apples}) \text{ on the table} \\
\text{c.} & \quad \{d: \exists x[\text{APPLE}(x) \land \text{DEGREE}_{\text{apples}}(x) = d \land \text{on-table}(x)]\}
\end{align*}
\]

Because degrees are tagged to individuals, \( \text{DEGREE}_{\text{apples}}(x) = \langle |y|, P, y \rangle \) just in case \( x = y \). We therefore unpack the degree relative denotation in (144-c) so that it denotes the set of enriched degrees in (145). The existential quantifier plays no role, so the denotation reduces to (146).

\[
(145) \quad \{\langle |y|, \text{apples}, y \rangle : \exists x[\text{apples}(x) \land x = y \land \text{DEGREE}_{\text{apples}}(x) = \langle |x|, \text{apples}, x \rangle \land \text{on-table}(x)]\}
\]

\[
(146) \quad \{\langle |x|, \text{apples}, x \rangle : \text{apples}(x) \land \text{on-table}(x)\}
\]
To the CP denotation in (146) Grosu and Landman propose that an operation of maximalization applies. Maximalization takes a set of degree triples and selects the singleton set consisting of the unique triple whose coordinates are maximal. The operation is defined as in (147).

\[(147) \quad \text{Maximalization from Grosu and Landman (1998):}\]

\[\text{a. Let CP be a set of degrees of the form } \langle |y|, P, y \rangle,\]
\[\text{max}(CP), \text{the maximal element in CP, is defined by:}\]
\[\text{max}(CP) = \langle \bigcup \{y: \langle |y|, P, y \rangle \in CP\}, P, \bigcup \{y: \langle |y|, P, y \rangle \in CP\} \rangle\]
\[\text{b. MAX}(CP) = \begin{cases} \{\text{max}(CP)\} \text{ if } \text{max}(CP) \in CP \\ \text{undefined otherwise} \end{cases}\]

In (147-a), \(\text{max}(CP)\) builds the maximal degree triple by maximizing its cardinality coordinate, \(|y|\), and its individual correlate, \(y\). Simply put, the operation finds the maximal individual \(y\) and its cardinality, \(|y|\). In (147-b), \(\text{MAX}(CP)\) creates a singleton set containing the maximal degree triple \(\text{max}(CP)\).

Suppose there were three apples on the table: \(a, b, c\). The derivation in (148) yields the denotation for the degree relative \textit{apples that there were on the table}.

\[(148) \quad [(\text{apples}) \text{ that there were (d many apples) on the table}]\]
\[= \text{MAX}\{\langle |x|, \text{apples, } x \rangle: \text{apples}(x) \land \text{on-table}(x)\}\]
\[= \text{MAX}\{\langle 1, \text{apples, } a \rangle, \langle 1, \text{apples, } b \rangle, \langle 1, \text{apples, } c \rangle, \]
\[\quad \langle 2, \text{apples, } a+b \rangle, \langle 2, \text{apples, } a+c \rangle, \langle 2, \text{apples, } b+c \rangle, \]
\[\quad \langle 3, \text{apples, } a+b+c \rangle\}\]
\[= \{\langle 3, \text{apples, } a+b+c \rangle\}\]

After maximalization, a degree relative denotes a maximal degree triple. In (148), this is the triple consisting of the maximal apple individual on the table, \(a+b+c\), the \text{apples} predicate to which this individual belongs, and the cardinality of this individual, 3. Now, what do we do with this triple?
As mentioned above, Carlson/Heim approaches to degree relatives fail by supposing that these constructions denote degrees. Crucially, *the apples that there were on the table* refers to the apples that were on the table, not to their cardinality. At this point, Grosu and Landman face a similar problem: their degree relatives also denote degrees, even if these degrees keep track of what they are degrees of. To retrieve the relevant individual from the degree denoted, Grosu and Landman therefore propose the SUBSTANCE operator, which turns a degree relative from a (singleton) set of degrees into a set of individuals. The operation is defined in (149).

(149)  

Shift the set of degrees to a set of individuals:

\[
\text{SUBSTANCE}(\text{CP}) = \{x: (|x|, P, x) \in \text{CP}\}
\]

According to Grosu and Landman, SUBSTANCE applies in the unmarked case. This move means that degree relatives behave as they ought to: after SUBSTANCE transforms a degree into an individual, *the apples there were on the table* references apples, not a degree that applies to them. The full derivation appears in (150); again, suppose three apples were on the table.

(150)  

[the (apples) that there were (d many apples) on the table]

= the(\{(|x|, apples, x): apples(x) \land on-table(x)\})

via obligatory MAX

= the(\{(3, apples, a+b+c)\})

via obligatory SUBSTANCE

= the(\{a+b+c\})

= a+b+c

Recall the steps leading to the derivation in (150): degree abstraction in the relative CP avoids the Definiteness Restriction, degrees-as-triples allows degrees to keep track of the objects they are degrees of, MAX ensures we retrieve the maximal degree from the relative CP, and SUBSTANCE takes this maximal degree triple and returns its individual coordinate. The result has *the apples that there were on the table* refer to the apples that were on the
table, albeit in a circuitous manner that requires constructing and then decomposing degree triples.

The question now is whether we can make do without the added machinery of MAX(CP) and SUBSTANCE(CP) to derive degree relatives. In the following subsection, we see that the answer to this question is yes. Conceiving of degrees as nominalized quantity-uniform properties, degree relatives fall out in a relatively straightforward manner without the construction-specific machinery needed by Grosu and Landman.

4.4.3 A novel account: property-denoting degrees

Recall the facts: Relative clauses introduced by that or the null relativizer Ø may participate in existential constructions, ostensibly flouting the Definiteness Restriction. Relative clauses introduced by the wh-form relativizers cannot participate in existential constructions. We thus get the contrast in (151).

(151) a. John ate the apples that/Ø there were __ on the table.
   b. *John ate the apples which there were __ on the table.

Heim (1987) expands on the Definiteness Restriction from Milsark (1974) and conceives of it as a ban on the sorts of entities that may occur in the post-verbal pivot position of an existential. Individuals (in our terms, objects) are not allowed in this position. (151-b) is ruled out because an individual variable sits in gap position, a configuration rule out by the Definiteness Restriction.

The work lies in explaining the success of (151-a). Following Carlson (1977b) and Heim (1987) (and later Grosu and Landman (1998)), we should take the relativizer fact as informative: wh-forms necessitate individual abstraction in the degree relative, hence the violation of the Definiteness Restriction. However, that and Ø are more permissive in the abstraction they sanction, admitting more than just individual abstraction. Following the Carlson/Heim approach to degree relatives, these constructions succeed because they feature variables of a different sort in pivot position: they feature degree variables.11 Now, let us see whether

11 For this reason alone, we should settle on a degrees-as-kinds approach as a means to capture the parallels in behavior between degrees and kinds in pivot position.
conceiving of degrees as nominalized properties sheds any light on the analysis of degree relatives.

First, to see that *that* but not *which* may bind degree variables, compare (152-a) with (153-a). Furthermore, note that kind variables pattern with degree variables: with *that* in (152-b) kind abstraction succeeds, but with *which* in (153-b) kind abstraction fails.

(152)  
\[
\begin{align*}
\text{a. } & \text{I ate the amount of apples that you ate.} \\
\text{b. } & \text{I ate the kind of apple that you ate.}
\end{align*}
\]

(153)  
\[
\begin{align*}
\text{a. } & \text{#I ate the amount of apples which you ate.} \\
\text{b. } & \text{#I ate the kind of apple which you ate.}
\end{align*}
\]

The facts in (152) and (153) confirm that degree abstraction lies at the heart of the relativizer restriction on degree relatives and, moreover, align degrees yet again with kinds. Both are entities of the same sort: nominalized properties.

We therefore posit a degree variable in pivot position. Unlike Grosu and Landman (1998) and their predecessors, here we need not assume additional material in this position (cf. the silent *many* that allows the degree to compose). Our degrees contain information about the measure that determines them, so there is no need for a silent *many* or something similar that would deliver this measure information. As nominalized properties – individuals – our degrees may also sit in argument position, composing with an object-level predicate via Generalized DKP. All we need, then, is a degree in pivot position and degree abstraction at the CP level, as in (154).

(154)  
\[
\text{the apples } \lambda d. \text{ (that) there were } d \text{ on the table}
\]

To hold on to the assumption that something moved leaves behind a trace of the same sort, here we adopt a head-external syntax for degree relatives.\textsuperscript{12} The structure for (154) appears in (155); note that the structure is largely the same as what Grosu and Landman propose (cf. (139)).

\textsuperscript{12}Raising syntax that moves the NP head from RC-internal position and leaves behind a degree trace would work equally well.
Before we can make sense of the semantics of the degree relative, we ought to understand better the structure of predication internal to the existential CP. As we did in the case of settling on a syntax for relative clauses in the previous section, here we make assumptions about structure in service of making explicit the semantics at play. These assumptions should by no means be taken as the final word on the complex and well-studied topic of existentials.

Suppose the existential predicate takes a small clause (SC) argument (Stowell, 1981). This small clause contains the degree variable and its PP modifier. Suppose further a symmetric structure internal to the small clause: the degree is a sister of its PP modifier (Moro, 2000; Citko, 2011b; see Citko, 2011a, for discussion).
Internal to the small clause we have a degree composing with an object-level predicate. In Section 4.2, we developed the operation of Existential Modification to compose a set of degrees with an object-level predicate, allowing for the modification of degrees as in *the amount of apples on the table*. Here, however, we must modify a single degree, the degree variable, with an object-level predicate, the modifying PP. To do so, we denominalize the degree, turning it into a property, and intersect this property with the property denoted by the degree’s sister.

The process of Degree Modification is defined as in (157). Note that we are dealing with a single degree, so the value of its measure \( \mu \) is fixed to some value \( n_i \).

\[
(157) \quad \text{Degree Modification:}
\]

\[
d \cap P_{(e,t)} = \cap (\cup d \cap P) = \cap \lambda x. \mu_f(x) = n_i \land P(x)
\]
The denominalized degree, $\cup d$, intersects with the predicate $P$. Re-nominalizing the product of this intersection, $\cap (\cup d \cap P)$, the result is a complex degree that contains the information contributed by $P$. In (158), we have the derivation for the complex degree that results as the denotation of the small clause in (156).

(158) \[
[d \text{ on the table}]
= d \cap \lambda x. \text{on-table}(x)
\] via Degree Modification

= $\cap (\cup d \cap \lambda x. \text{on-table}(x))$

= $\cap \lambda x. \mu_f(x) = n_i \land \text{on-table}(x)$

In the small clause we now have the complex degree which applies to quantities of things on the table. The existential predicate takes this complex degree as an argument; supposing that the existential predicate applies at the level of objects, Generalized DKP mediates the composition of this predicate with its degree argument. Its effect has the degree relative assert the existence of an instantiation of the complex degree, as in (159).\footnote{Chierchia (1998b, p.378) provides a similar treatment of kinds in existential constructions.}

(159) \[
[\text{be d on the table}]
= \text{exist}(\cap \lambda x. \mu_f(x) = n_i \land \text{on-table}(x))
\] via Generalized DKP

= $\exists y[\mu_f(y) = n_i \land \text{on-table}(y)]$

Now we have asserted that some quantity of stuff exists on the table. But recall the relativizer facts, which suggest degree abstraction within the relative clause. This abstraction should target the complex degree that results when the material within the small clause composes, as schematized in (160).

(160) \[
\lambda d. \exists y[\cup d \text{on-table}(y)]
\]

As in Grosu and Landman (1998), after degree abstraction our degree relative denotes a set of degrees. These degrees are complex, incorporating the predicate information within
the small clause. Therefore, these degrees will apply only to things for which this predicate information is true – in other words, to objects on the table. Unlike Grosu and Landman, we have not yet interpreted the head of the degree relative, which sits external to the clause. Because Grosu and Landman interpret the head within the degree relative, they must go through the trouble of extracting from the set of degrees that results a set of individuals via the SUBSTANCE operator. We can do without SUBSTANCE.

First recall the motivation behind SUBSTANCE. Grosu and Landman take the name “degree relative” to heart, such that degree relatives denote degrees. They want degree relatives to denote degrees in order to capture the identity-of-amount interpretation noticed by Heim (1987) for sentences like (161).

(161) It will take us the rest of our lives to drink the champagne that they spilled that evening.

\[ \leftrightarrow \text{it will take us the rest of our lives to drink an amount of champagne equal to the amount they spilled that evening} \]

But, as Grosu and Landman themselves observe, this reading is not available to degree relatives. Uttering (162), the speaker does not convey that John ate some apples equal in amount to the apples on the table. Instead, he conveys that John ate the apples on the table – those objects that are indicated. Whence comes SUBSTANCE, which applies in the general case: to the complex degree Grosu and Landman construct in the denotation of the degree relative, SUBSTANCE applies and extracts the relevant individual.

(162) John ate the apples that there were on the table.

Grosu and Landman (1998) observe that the identity-of-amount reading is a marginal phenomenon requiring a special interpretation strategy. To the example in (161) they add the following sentences.

(163) We will never be able to recruit the soldiers that the Chinese paraded last May Day.

(164) At passover I drink the four glasses of wine that everybody drinks.
The authors speculate that the presence of a modal, generic, or habitual plays a non-trivial role in all of these examples. For our purposes, note simply that none of these examples are clear degree relatives. That is, none features an existential construction.

It would appear, then, that Heim’s champagne is a red herring. So why go to so much trouble to allow an identity-of-amount reading when it does not arise for degree relatives? In other words, why posit the construction-specific SUBSTANCE operator, rather than derive the fact that a degree relative names objects, not degrees?

We are in a position to avoid the unnecessary stipulation of SUBSTANCE or something similar. Our degree relative does denote a set of degrees, but the head (e.g., *apples*) is a simple predicate, denoting a set of objects. To compose this predicate with a set of degrees, we need only appeal to the operation of Existential Modification. In this case, it is a predicate of degrees that restrictively modifies an object-level predicate. The result of this modification is itself an object-level predicate. This process of Existential Modification is defined as in (165). Note that Existential Modification is head-driven, so that when the head is a predicate of degrees we create a predicate of degrees, (165-a), and when the head is an object-level predicate we create an object-level predicate, (165-b). It is this latter situation that we face in the case of degree relatives.\(^\text{14}\)

\begin{align*}
(165) & \quad \text{Existential Modification:} \\
& \quad \text{a. } A_{(d,t)} \cap E P_{(e,t)} = \lambda d. A(d) \land \exists y [P(y) \land \cup d(y)] \\
& \quad \text{b. } P_{(e,t)} \cap E A_{(d,t)} = \lambda x. P(x) \land \exists d [A(d) \land \cup d(x)]
\end{align*}

Consider how Existential Modification as in (165-b) applies in the case of degree relatives. The head noun gets modified by the relative CP, itself a set of complex degrees.

\(^{14}\)Note that if the first option,(i-a), were used instead, such that the result was a set of degrees, we would derive the identity-of-amount reading for Heim-style sentences: the degree relative would denote a degree, DKP would allow this degree to compose with the rest of the sentence, and the resulting assertion would be an existential one.
This modification restricts the set of apples to just those apples for which there is an on-table degree that faithfully applies. In other words, the modification restricts the set to just those apples that are on the table. Definiteness, conceived of as a maximality operator, selects from this set the maximal apple individual. Concretely, maximality selects the apples that there were on table. Using a perhaps more familiar semantics for degrees that captures the striking parallels in behavior between degrees and kinds, we have successfully derived the object-level interpretation for degree relatives. This interpretation results without stipulating SUBSTANCE.

What about MAX? Recall the claim from Grosu and Landman that MAX applies at the CP-level in degree relatives to return the maximal degree. This move allows the authors to account for one final peculiarity of behavior that degree relatives exhibit, namely the restrictions they impose on the determiners that may compose with them. Picking up on an observation from Carlson (1977a), Grosu and Landman point out that only universal and definite determiners are felicitous in degree relatives. Moreover, the authors remark on the cross-linguistic stability of these determiner restrictions. Compare the sentences in (167).

\[(167) \quad \text{a. I took with me } \begin{cases} \text{every book} \\ \text{any books} \\ \text{the books} \\ \text{the three books} \\ \text{three of the books} \end{cases} \text{ that there was/were } \underline{\text{—}} \text{ on the table.}\]
b. #I took with me \{ three books \\
  few books \\
  many books \\
  some books \\
  most books \\
  no books \} that there were ___ on the table.

(Grosu and Landman, 1998, ex.(22), p.136)

By requiring MAX to apply at the CP level, degree relatives will always denote a singleton (i.e., the maximal degree). In other words, MAX ensures that uniqueness is built into the analysis of degree relatives. It is this property of uniqueness that Grosu and Landman use to derive Carlson’s determiner restrictions. However, uniqueness alone will not limit the set of possible determiners to just definites or universals. Two additional constraints are proposed.

The first constraint prohibits existentials from applying to singleton sets, like the Definiteness Restriction, presumably because the result would be definite anyway. This move rules out most of the determiners in (167). To handle most, the authors need a more verbose story. Simply put, they claim that the effect of quantification in the specific case of degree relatives must be cardinality-preserving. The details appear in (168).

(168)  Grosu and Landman (1998, p.146) on determiner restrictions:

a.  Definition: Given a quantificational DP D(NP) based on a degree relative NP, \( \text{max} \) is preserved into the quantification iff for every predicate P: in normal contexts for D(NP, P), \( |\text{MAX}_A| = \text{max} \).\(^{15}\)

b.  Constraint: An NP based on a degree relative can only be combined with determiners that preserve max into the quantification.

c.  Consequence: The only determiners that preserve max into the quantification are the universals like EVERY and definites like the. Hence, these are the only determiners that can head a DP with a degree relative.

\(^{15}\)MAX\(_A\) is the object of quantification. In the books that there were on the table, MAX\(_A\) would be the set of books that were on the table.
Suppose Grosu and Landman are on the right track with their strategy to force a singleton denotation for degree relatives in order to preclude indefinite (i.e., existential) determiners. We may do the same. What we need is the means to intelligently link the apples on the table; the partitioning function internal to the semantics of a degree stands to deliver this result.

Our goal is to restrict the set of determiners that may apply to degree relatives to just universals or definites. Following Grosu and Landman, the strategy is to ensure that the degree relative denotes a singleton set. Ideally, this restriction falls out from the machinery we already have; that is, we make do without MAX. Recall that under our proposal, *apples that there were on the table* will denote the set of apples that were on the table, as in (169) (the derivation appears in (166) above).

\[(169) \quad [\text{apples that there were on the table}] = \lambda x. \text{apples}(x) \land \exists! d'((\lambda d. \exists n \exists k[d = \land \lambda y. \mu_{\text{card}}(y) = n \land \pi(\text{Apple})(y) \land \text{on-table}(y)])(d') \land \cup d'(x)]\]

Now, consider what the maximizing partition function internal to the semantics of the degree variable contributes to the denotation of the entire phrase. We motivated the function by the requirement that degrees receive suitable objects to measure. The partition, \(\pi\), applies to a kind and returns maximal instances of the kind supported by context.

Imagine a context where three apples and four bananas sit on a table; the table is otherwise empty. In this context, without any additional structure to further divide the fruit, there are two amounts of stuff on the table, corresponding to the two kinds of entities that are there: the amount instantiated by apples (with cardinality 3) and the amount instantiated by bananas (with cardinality 4). In other words, at the CP internal to the degree relative denotes the set in (170).

\[(170) \quad \lambda d. \text{there were } d \text{ on the table} = \{\land \lambda x. \mu_{\text{card}}(x) = 3 \land \pi(\text{Apple})(x) \land \text{on-table}(x),
\land \lambda x. \mu_{\text{card}}(x) = 4 \land \pi(\text{Banana})(x) \land \text{on-table}(x)\}\]

\(^{16}\)If we ultimately do require an operator like MAX in the semantics of degree relatives, we will have at least made do without SUBSTANCE. More importantly, we will have derived using standard machinery the existential reading of degrees.
Now, suppose the degree relative head *apples* denotes the +-closed predicate in (171), where the apples a, b, and c are on the table together.

\[(171) \quad \text{[apples]} = \{a, b, c, a+b, a+c, b+c, a+b+c\}\]

The degree relative head and the degree relative CP compose via Existential Modification; the result restricts the head *apples* to just those apple individuals to which a degree denoted by the CP applies. There is only one such individual, namely the totality of apples on the table: a+b+c. Thus, the degree relative denotes the singleton set consisting of the apples on the table.

\[(172) \quad \text{[apples that there were on the table]} = \lambda x. \text{apples}(x) \wedge \exists d'[(\lambda d. \exists n \exists k[d = \cap \lambda y. \mu f(y) = n \wedge \pi(k)(y) \wedge \text{on-table}(y))](d') \wedge \cup d'(x)] = \{a+b+c\}\]

Using only the maximizing partition function internal to the semantics of degrees, we have derived the fact that a degree relative denotes a singleton. In other words, we have achieved the goal of Grosu and Landman (1998) without the construction-specific MAX operator. Assuming that only a limited set of determiners may apply to a singleton set, we have additionally derived the determiner restrictions that characterize degree relatives. Now, spelling out how these restrictions fall out will likely require many additional assumptions (as in (168)), but for our purposes it suffices to show that our semantics for degrees gets us at least as much coverage for degree relatives as the degree-triple approach from Grosu and Landman without the need of SUBSTANCE or MAX. Moreover, our degrees-as-kinds approach gives us a straightforward account of the existential interpretation, something that eludes the degree-triple approach.

### 4.5 Discussion

This chapter provides a case study of the quantizing noun *amount*. We began with the observation that *amount* stands apart with other degree nouns in its ability to deliver what
we termed an existencial interpretation, as in (173).

(173) I ate that amount of apples every day for a year.

→ every day for a year I ate some apples that measured the relevant amount

The existencial interpretation of amount derives from the direct interpretation, under which amount names an abstract measurement (i.e., an amount). We identified these entities amount names as degrees. Given the behavior of the existencial interpretation, we must have enough information in the semantics of a degree to determine the objects that instantiate them.

Here we found inspiration for the semantics of amount from one of the few nouns that also admits an existencial reading: kind, as in (174).

(174) I ate that kind of apple every day for a year.

→ every day for a year I ate some apples of the relevant kind

The nouns kind and amount behave similarly because they reference the same sort of thing: degrees, like kinds, are nominalized properties. Degrees stand apart because the properties from which they are built are quantity-uniform, formed on the basis of a measure. We thus arrive at the definition for degrees in (175) and the semantics for amount in (176), which names a set of degrees.

(175) \text{DEGREE} := \bigcap \lambda x. \exists k[\mu_f(x) = n \land \pi(k)(x)]

where \(\mu_f\) is a contextually-specified measure,

\(n\) is some number in the range of the measure \(\mu_f\),

and \(\pi\) is a contextually-supplied partition

(176) \text{[amount]} = \lambda k \lambda d. \exists n[d = \bigcap \lambda x. \mu_f(x) = n \land \pi(k)(x)]

For our degrees to compose with the structures that embed them, we adopted the following technology. Given the conception of degrees as kinds, the tools we develop here apply generally in the realm of kind semantics. First, we take the \(\iota\) operator to be a maximality-seeking function, selecting from a set its maximal member; uniqueness results as a side effect of max-
imality (Sharvy, 1980). Next, we proposed the following semantics for demonstratives; this semantics identifies nominalized properties through the salient objects that instantiate them. In *that amount of apples* or *that kind of dog*, we establish a pointer to a real-world object and access the kind it instantiates.

\[(\text{that}) = \lambda A. \ i\ y[\lambda (y) \land \cup (\text{that})]\]

where \(A\) is a set of individuals, either nominalized properties or objects, and \(\text{that}\) is the salient object indicated in the use of the demonstrative.

We deliver the existential interpretation for degrees the same as for kinds by generalizing the operation of Derived Kind Predication, which quantifies over instantiations of nominalized properties to allow these entities to serve as arguments to object-level predicates (Chierchia, 1998b).

\[(\text{Generalized DKP}):\]

If \(P\) apples to objects and \(y\) denotes a nominalized property, then

\[P(y) = \exists x[\cup (x) \land P(x)]\]

To see these tools at work, consider the derivation in (179). Suppose there are three salient apples: a, b, c. Suppose also that the kilogram measure, \(\mu_{kg}\), is relevant, and that \(\mu_{kg}(a+b+c) = n_{a+b+c}\).

(179) John bought that amount of apples.

a. \[\text{[that]}([\text{[amount of apples]}])\]

\[\text{[that]}(\lambda d. \exists n[d = \cap \lambda x. \mu_{kg}(x) = n \land \pi(\text{APPLE}(x))])\]

\[\iota y[\lambda d. \exists n[d = \cap \lambda x. \mu_{kg}(x) = n \land \pi(\text{APPLE}(x))(y) \land \cup y(\text{that})]]\]

\[\iota y[\lambda d. \exists n[d = \cap \lambda x. \mu_{kg}(x) = n \land \pi(\text{APPLE}(x))(y) \land \cup y(a+b+c)]]\]

*via Maximaly*

\[\cap \lambda x. \mu_{kg}(x) = n_{a+b+c} \land \pi(\text{APPLE})(x)\]
b. [John bought that amount of apples]  
\[ \text{bought}(\bigcap \lambda x. \mu_{kg}(x) = n_{a+b+c} \land \pi(\text{APPLE})(x))(\text{John}) \]

via Generalized DKP  
\[ \exists y[\bigcup(\bigcap \lambda x. \mu_{kg}(x) = n_{a+b+c} \land \pi(\text{APPLE})(x))(y) \land \text{bought}(y)(\text{John})] \]

We have derived the existential interpretation for amount. Through Generalized DKP, (179) asserts that John bought some apples equal in weight to the salient apple individual a+b+c. In other words, John bought apples equal in amount to the salient apples indicated.

We next considered how degrees are modified, for example by prepositional phrases as in (180) or by relative clauses as in (182). For a predicate of degrees – amount of apples – to get modified by an object-level predicate – on the table – we apply Generalized DKP in a point-wise manner as the two predicates intersect. This operation of Existential Modification is head-driven, defined as in (181); if a predicate of individuals is modified by a predicate of degrees, restrict the first predicate to just those instances for which a degree applies.

(180) John ate the amount of apples on the table.

(181) Existential Modification:

a. \[ A_{(d,t)} \cap_E P_{(e,t)} = \lambda d. A(d) \land \exists y[P(y) \land \bigcup d(y)] \]

b. \[ P_{(e,t)} \cap_E A_{(d,t)} = \lambda x. P(x) \land \exists d[A(d) \land \bigcup d(x)] \]

With relative clauses, we can make do with simple, intersective modification once we assume degree abstraction at the level of the relative CP. This degree abstraction is schematized in (182-b).

(182) a. John ate the amount of apples that you ate.

b. John ate the [amount of apples] [\lambda d. you ate d]

Finally, we saw how degree abstraction also applies in degree relatives, constructions where a degree variable sits in the pivot position of an existential predicate, as in (183). By conceiving of degrees as nominalized properties and applying the operation of Existential Modification in (181-b) to compose its head with the degree relative, we derive the desired result that the apples that there were on the table refers to the apples that there were on the table (and not
to a degree; cf. Carlson, 1977a; Heim, 1987; Grosu and Landman, 1998).

(183) John ate the [apples] $\lambda d. \text{that there were } d \text{ on the table}$

Because degrees apply to contextually-supported maximal instances of stuff on the basis of a partition function $\pi$, we derive the fact that degree relatives denote a singleton set and therefore compose only with a limited set of determiners.

Our semantics for amount yields a new semantics for degrees, which aligns degrees with kinds on the basis of the sort of entity they reference: a nominalized property. As we have seen, this new semantics delivers otherwise elusive interpretations (e.g., the existential reading), interacts with standard theories of syntax (e.g., relativization structures), and provides a straightforward account of problematic constructions (e.g., degree relatives).
Chapter 5

General Discussion

In our investigation of measurement in the nominal domain, we have focused on three themes: 1) number marking as a morphological reflex of measurement, 2) the semantics of nouns that perform or facilitate measurement, and 3) the linguistic representation of measurement itself (i.e., degrees). The account of the linguistic phenomena considered relies primarily measure functions in the compositional semantics. These measures are introduced by various items in the lexicon, and serve to map individuals to numbers. In other words, measures relate individuals to points on a scale.

For number marking, we saw how nominal semantics interacts with morphology via the one-ness presupposition attributed to the singular form of nouns. Flexibility in the selection of the measure $\mu$ determining this presupposition allows for a unified system of number marking cross-linguistically. Languages vary in what they attend to as they check for singularity. For example, English firsts evaluates the structure of a nominal predicate, checking to see whether there are measures that uniformly apply to the members of the predicate; if every member evaluates to 1 by these measures, singular morphology surfaces. Turkish, on the other hand, checks always for relative atomicity via the measure in P-atoms.

Measures also feature prominently in the semantics of nouns. In English, measure terms like $kilo$ directly name a measure and, through the named measure, delimit sets of individuals. Other quantizing nouns facilitate measurement by specifying discrete quantities. For example, container nouns like $glass$ package our surroundings on the basis of the quantities that they contain. Similarly, atomizers like $grain$ partition substances into stable minimal units. Once
packaged or partitioned, these elements may be counted by the measure in cardinality.

In addition to performing measurement, language allows us to reference the outcome of this process: degree nouns like *amount* name the abstract representations of measurement. These representations, degrees, contain four pieces of information. A degree contains information about the measure \( \mu \) that determines it, the value \( n \) to which this measure evaluates, the kind of thing \( k \) which gets measured, and the means by which the kind gets instantiated \( \pi \) for the purpose of being measured. A degree is thus a collection of coordinates, the 4-tuple \( \langle \mu, n, k, \pi \rangle \); the degree named by *three kilos of apples* may be represented as \( d_{\langle \mu_{kg}, 4, \text{APPLE}, \pi_c \rangle} \). By conceiving of degrees as nominalized properties, we may manipulate them as we do kinds and access their instantiations.

What results is a program for representing and making claims about the world that is centered around measurement. Measurement specifies individuals, builds properties and sorts, and determines the form of words as we speak them. In the semantics, we partition the world into discrete chunks that serve as arguments to measure functions, which translate these chunks onto a specified scale. Scalar representations of real-world objects allow for a richer understanding not only of the objects themselves, but of their relationships to other objects in the world. Although we have considered mostly nouns in English to shape the proposal, the architecture of and machinery within this system of measurement should be viewed as constituting a theory of language broadly speaking. Its proving ground will be the application of this theory to other languages and domains of linguistic phenomena. What follows is a discussion of three such applications.

### 5.1 Extending the system

Given our new conception of degrees, we should check to see that this system is compatible with the existing accounts of degree constructions. We start there. Then we turn to mass nouns, which have received relatively little attention so far in this thesis. By explicating the process that determines number marking in the absence of numerals, we will see that our system of number marking stands to derive the lack of plural morphology on mass nouns. Finally, we turn to the similarities and differences between classifiers and classifier languages.
on the one hand, and quantizing nouns and number marking languages on the other.

5.1.1 Degree semantics

Under the current proposal, degrees are nominalized quantity-uniform properties. Internal to these nominalized properties is the measure $\mu_f$, which maps individuals to numbers; degrees are ordered on the basis of this measure. The template for a degree appears in (1).

(1) $\text{DEGREE} := \bigcap \lambda x. \exists k[\mu_f(x) = n \land f_p(k)(x)]$

where $\mu_f$ is a contextually-specified measure,

$n$ is some number in the range of the measure $\mu_f$,

and $f_p$ is a contextually-supplied partition.

This notion of degree stands as a perhaps drastic departure from standard theories of degrees. For clarity’s sake, we distinguish the two approaches as ‘degrees-as-kinds’ vs. ‘degrees-as-points’. In what follows, we review standard theories of degrees-as-points and their applications, then consider how the degrees-as-kinds approach fares. We will see that our notion of degrees-as-kinds merely enriches traditional conceptions. Nothing is lost by this move, and we spent the previous chapter spelling out what is gained.

Degrees feature prominently in much of the work on the formal semantics of gradability and comparison. Degrees enter into the ontology as abstract entities; they are points (or intervals) ordered along some dimension. In other words, degrees are numbers tagged with information about the dimension to which they pertain (e.g., height, width, cost, beauty, etc.). Along a given dimension, the set of ordered degrees constitutes a scale. Scales provide the structure for comparison: By establishing a correspondence between individuals and degrees, we map individuals onto scales; the relative position of these individuals on the scale determines comparison.

According to degree approaches to gradability, lexical predicates establish the correspondence between individuals and degrees (Kennedy, 1999; see also Seuren, 1973; Cresswell, 1976; von Stechow, 1984; Heim, 1985). Concretely, gradable predicates denote relations between individuals and degrees. For example, the predicate tall expresses the relation between individuals and degrees of height, as in (3-a).
(2) \([\text{six feet}] = 6 \text{ft}\)

(3) 
   a. \([\text{tall}] = \lambda d \lambda x: d \text{ is suitable for height. } \text{tall}(d)(x)\)
   b. \([\text{tall}] = \lambda d \lambda x: d \text{ is suitable for height. } \mu_{\text{tall}}(x) \geq d\)

(4) \([\text{six feet tall}] = \lambda x. \mu_{\text{tall}}(x) \geq 6 \text{ft}\)

For many degrees-as-points approaches, the \text{tall} relation decomposes as in (3-b) into a measure function, \(\mu_{\text{tall}}\), which maps individuals onto the height scale. Composing with the degree of height \text{six feet}, \text{tall} in (4) returns the set of individuals that are six feet tall.

Consider how degrees-as-kinds interacts with the semantics of gradability assumed in (3-a) for \text{tall}. First, let us continue to suppose that gradable predicates take a degree argument. These degrees are no longer simple points on a scale; \text{six feet} would denote the degree in (5), which is the nominalization of the property of measuring six feet (in height).\(^1\)

(5) \([\text{six feet}] = \cap \lambda x. \exists k[\mu_{\text{ft}}(x) = 6 \land \pi(k)(x)]\)

Now, for \text{tall} to relate individuals with a degree-kind as in (5), it no longer needs to perform the height measurement itself: degrees contain measure functions. Therefore, \text{tall} may compose individuals and degrees directly, presupposing that the degrees are degrees of height. The modified semantics for \text{tall} appears in (6).

(6) \([\text{tall}] = \lambda d \lambda x: d \text{ is suitable for height. } ^{\cup}d(x)\)

(7) \([\text{six feet tall}] = \lambda x. \exists k[\mu_{\text{ft}}(x) = 6 \land f_{p}(k)(x)]\)

Just as before with degrees-as-points, once \text{tall} in (7) composes with the degree of height \text{six feet}, it returns the set of individuals that are six feet tall. However, there is an important difference between the sets denoted by the predicates in (4) and (7): In the former, where degrees are construed as points, we return the set of individuals that are \textbf{at least} six feet tall; in the latter, where degrees are construed as kinds, we return the set of individuals that are \textbf{exactly} six feet tall. An ‘exactly’ semantics for degree predicates has been supplanted by the ‘at least’ semantics in order to capture facts concerning modified numerals and scope-bearing

\(^1\)The measure internal to this degree is, more precisely, the height measure in feet.
To derive an ‘at least’ semantics for our degrees-as-kinds, we could take one of two approaches. The first approach builds \( \leq \) ‘at least’ into the basic semantics for degrees, as in (8).

\[
(8) \quad \text{DEGREE} := \cap x. \exists k [\mu_f(x) \leq n \land f_p(k)(x)]
\]

Where exactness is required of degree constructions, some other mechanism (like the maximality-seeking \( \iota \) operator) would have to deliver it. Rather than construct sub-maximal degrees as the default, we could instead follow the degree literature and derive sub-maximal degrees from properties of lexical predicates.

The other approach to an ‘at least’ semantics for degrees-as-kinds reconsiders the relationship between individuals and degrees that gets supplied by gradable predicates. First, let us define an interval of ‘at least’ degrees as in (9).

\[
(9) \quad D_n^{\geq} = \bigcup_{i=n} d_i
\]

The degree interval \( D_n^{\geq} \) contains every degree at least as great as \( d_n \). For example, suppose \( d_n \) is the degree of height \textit{six feet}. \( D_n^{\geq} \) will be the set of degrees of height at or above six feet. A degree interval holds of an individual just in case it contains a degree true of that individual.

\[
(10) \quad D_n^{\geq}(x) = 1 \text{ iff } \exists d \in D_n^{\geq}[\cup d(x)]
\]

Now, suppose gradable predicates relate individuals with these degree intervals. Composition would proceed as in (11). Note that the basic semantics for degrees-as-kinds has not changed: \textit{six feet} continues to denote the degree in (5).

\[
(11) \quad \begin{align*}
\text{a. } [\text{tall}] &= \lambda d_n \lambda x. \text{ } d_n \text{ is suitable for height. } D_n^{\geq}(x) \\
\text{b. } [\text{six feet tall}] &= \lambda x. D_{\text{6ft}}^{\geq}(x) \\
&= \lambda x. \exists d \in D_{\text{6ft}}^{\geq}[\cup d(x)]
\end{align*}
\]

By construing gradable predicates as relations between individuals and degree intervals, the semantics in (11-b) for \textit{six feet tall} matches that found in the degrees-as-points approach:
the denotation identifies the set of individuals whose height measure at least six feet. Here we have the ‘at least’ semantics for gradable predicates.

We have captured gradability in degree constructions using degrees-as-kinds. Now let us consider comparison. Here we assume a standard A-NOT-A analysis of the semantics of comparatives (see Schwarzschild, 2008, for a primer on comparative semantics). A comparative sentence like in (12-a) receives the paraphrase in (12-b), with the logical translation in (12-c).

(12) a. John is taller than Bill is.
   b. There is some degree of height true of John that is not true of Bill.
   c. $\exists d [\text{tall}(d)(j) \land \neg \text{tall}(d)(b)]$

With degrees-as-points, (12) amounts to the assertion that John attains a degree of height that Bill does not. This semantics translates straightforwardly into our degrees-as-kinds framework. Recall that we have defined the relation specified by gradable predicates as one that exists between individuals and degree intervals. As a result, comparatives quantify not over single degrees (points or kinds), but over degree intervals as in (13).

(13) a. $\exists D_n^\geq [\text{tall}(D_n^\geq)(j) \land \neg \text{tall}(D_n^\geq)(b)]$
   b. $\exists D_n^\geq [(\exists d \in D_n^\geq [\cup d(j)]) \land \neg (\exists d \in D_n^\geq [\cup d(b)])]$

Despite looking a great deal more complex than (12-c), (13-b) delivers the same truth conditions: John is taller than Bill just in case there is some degree interval that contains John’s height and does not contain Bill’s. Because the height degree interval is lower bounded, (13-b) amounts to the assertion that John’s height is greater than Bill’s.

A more straightforward way to capture the same behavior would have the gradable adjective quantify over degrees directly, as in (14). Six feet tall would be a predicate true of individuals whose height is at least six feet.

(14) $[\text{tall}] = \lambda d \lambda x: d$ is suitable for height. $\exists d' [d' \geq d \land \cup d'(x)]$

Whether we adopt the more complex notion of degree intervals, or we suppose that gradable adjectives create these intervals, we arrive at the same result: an ‘at least’ semantics for gradability.
In summary, we have seen how degree-as-kinds are compatible with standard approaches to gradability and comparison. Because they contain more information than simple degrees-as-points, no ground is lost by switching to degrees-as-kinds. A host of additional issues arise within the domain of degree semantics, but for now it suffices to show how degrees-as-kinds behave within this framework. Crucially, in addition to losing nothing with respect to standard theories of gradability, degrees-as-kinds capture the parallels in behavior between degrees and kinds and deliver the existential interpretation.

5.1.2 Mass nouns

Mass nouns like water and rice feature prominently in the typology of quantizing nouns. Without stable minimal parts or well-defined notions of what counts as a whole entity, they must be parcelled out to be referenced. Put differently, mass nouns stand apart because they lack stable atoms, which is why mass nouns so often provide the substance to be quantized. Quantizing nouns package the substance denoted by mass nouns into stable wholes so that they may serve as arguments to object-level predicates and, more importantly, numeral quantifiers. In Chapters 3 and 4, we considered in detail the way that mass noun interact with the semantics of quantizing nouns; here we review the properties of mass nouns and consider them in light of the proposed system for number marking developed in Chapter 2.

Chierchia (2010) identifies three properties of mass nouns that are (‘tendentiously’) constant across languages. The first property Chierchia calls the ‘signature property’ of mass nouns, namely their inability to compose directly with numerals. Compare the phrases in (15); unlike count nouns, mass nouns resist direct counting.

\[(15) \quad \text{a. three apples/hamburgers/grapes.} \]
\[(15) \quad \text{b. *three waters/silvers/oils} \]

To count instances of the substance denoted by mass nouns, we make use of a quantizing noun as in (16). Note that even when a quantizing noun mediates counting, mass nouns never appear as morphologically plural.
(16) a. three cups of water  
b. three pounds of silver  
c. three quantities of oil

Next, mass nouns track the language-independent substance/object contrast. When two objects, say two books, meet, the objects retain their identities; we are left with two books. When two quantities of some substance, say water, meet, a single quantity of that substance results. Moreover, substances are continuous: a portion of some quantity of water is still water, whereas a portion of a book is not necessarily a book. Infants are attuned to these properties of their environment (Spelke, 1991; Carey, 1992), and languages are too: substances are named by mass nouns.²

Lastly, the mass/count distinction is flexible. A canonical mass noun may admit count uses, as in (17) and (18).³ However, this flexibility is firmly (and illuminatingly) constrained. The interpretations of count uses of mass nouns fall into two categories: either some contextually-supplied partition (i.e., a silent atomizer) quantizes mass nouns into discrete and stable portions for counting, (17), or counting proceeds over subkinds of the substance named, (18).⁴

(17) John ordered three waters.
    ↦ John ordered three glasses/bottles of water

(18) a. The hospital has three bloods on hand.
    ↦ the hospital has three kinds of blood on hand

    b. You will find ten beers on tap at the bar.
    ↦ you will find ten kinds of beer on tap at the bar

In sum, mass nouns name substances which cannot be directly counted. They also preclude plural morphology. However, mass nouns may be coerced into count uses; when they are,

²Chierchia (2010) is careful to delimit the scope of this claim: whereas substances are coded as mass, objects are not always coded as count (cf. fake mass nouns like furniture).

³Conversely, count nouns admit mass uses, as in John ate a salad with apple in it or there is table all over the floor. We ignore count→mass shifts for present purposes, but see Pelletier (1975) for discussion of the ‘Universal Grinder’ that handles these shifts.

⁴Note that this second shift, into subkinds, also applies to count nouns. For example, three dogs could be interpreted as referencing three kinds of dogs.
they deliver either quantized (i.e., ATOMIZING) or subkind interpretations and can appear morphologically plural. Now, how do these properties conspire to inform the semantics of number marking on mass nouns?

We want to exclude mass nouns from composing with the PL #-head. One move would be to say that DPs containing mass nouns lack #P altogether. In other words, mass nouns lack a number feature. However, this move would have undesirable consequences for the number agreement these DPs effect in the sentences that contain them (e.g., the water is/*are boiling). Moreover, conjunctions of mass nouns display regular agreement (e.g., the water and rice *is/are in the pot). It would seem, then, that we cannot simply omit number features from these nominals.

Recall that our system of number marking decides number morphology via competition between SG and PL: when SG, which has a stronger meaning, can be used, it is; otherwise PL must be used. In other words, to preclude the appearance of plural morphology, mass nouns must always satisfy the one-ness presupposition of SG. The semantics of the #-heads is repeated in (19). Recall that English number morphology is sensitive to measures that determine quantity-uniform predicates, defined in (20).

(19) English #-heads:
   a. \( [\text{SG}] = \lambda P : \forall \mu \forall x \in P[ \text{QU}_\mu(P) \rightarrow \mu(x) = 1 ] \). P
   b. \( [\text{PL}] = \lambda P. P \)

(20) Quantity-uniform:
\[
\text{QU}_\mu(P) = 1 \text{ iff } \forall x\forall y[ P(x) \land P(y) \rightarrow \mu(x) = \mu(y) ]
\]

For mass nouns to always appear singular, their denotation must be quantity-uniform with respect to some measure, and by that measure every element in the denotation must evaluate to 1. Here mass nouns place into focus a more general question concerning the semantic account of number marking, namely how the one-ness presupposition of SG gets checked in the absence of a numeral, that is, in the absence of a M(easure) head.

Before tackling mass nouns, let us take a step back and consider number marking on count nouns proceeds in the absence of M. Count nouns denote simple predicates that come
as either semantically singular, (21-a), or semantically plural, that is, closed under sum-formation, (21-b).

(21)  

\textit{Semantic number (assuming three books):}

a. \([\text{book}] = \{a, b, c\}\)

b. \([\ast \text{book}] = \{a, b, c, a+b, a+c, b+c, a+b+c\}\)

Semantically singular \textit{book} straightforwardly satisfies the one-ness presupposition of \textit{sg}: the predicate denotation in (21-b) is quantity-uniform with respect to the measure in relative atoms, \(\mu_{\text{P-atom}}\), and every member of this denotation measures 1 P-atom. However, semantically plural \(*\text{book}\) also satisfies the one-ness presupposition of \textit{sg}: the predicate is not quantity-uniform, so there is no measure against which the one-ness presupposition must be checked. In other words, a semantically plural predicate vacuously satisfies the one-ness presupposition by virtue of there being no measure which this presupposition checks. But if this process of vacuously satisfying the one-ness presupposition actually transpired, then singular \textit{the book} should be able to refer to a plurality of books; it cannot.

It would appear that we need to say something additional about the semantics of \textit{sg} to rule out singular morphology on semantically plural predicates (in the absence of numerals), namely that a predicate must be quantity-uniform before it may satisfy the one-ness presupposition of \textit{sg}. Put differently, the one-ness presupposition should require that there exist at least one measure by which a predicate counts as quantity-uniform, and that every member of the predicate evaluates to 1 with respect to this measure. The revised semantics for the English \#-heads appears in (22).

(22)  

\textit{English \#-heads (revised):}

a. \([\text{sg}] = \lambda P: \exists \mu[QU_\mu(P)] \& \forall \mu \forall x \in P[ QU_\mu(P) \rightarrow \mu(x) = 1 ]\]. P

b. \([\text{pl}] = \lambda P: P\)

Applied to a numeral-less semantically singular predicate like \textit{book} in (21-a), the revised one-ness presupposition of \textit{sg} is satisfied: the predicate is quantity-uniform with respect to the P-atom measure, and every element of the predicate denotation evaluates to 1 with
respect to this measure. Thus, we correctly predict singular *book to reference only individual
books.

Applied to a numeral-less semantically plural predicate like *book in (21-b), the one-
ess presupposition of SG is not satisfied: the predicate is not quantity-uniform. Without a
measure to check its presupposition against, SG cannot appear. The elsewhere condition, PL,
therefore applies and we correctly predict plural books to include sums in its reference.

Now, let us return to mass nouns. Without shifting to a count interpretation, mass nouns
never appear plural.\footnote{The lack of plural marking on mass nouns is a relatively stable phenomenon cross-linguistically. However, a handful of languages do pluralize their mass nouns. Tsoulas (2006) observes that Modern Greek allows the pluralization of mass nouns, and Gillon (2010) observes the same for Innu-aimun. Crucially, mass noun pluralization in these languages retains the mass character of the interpretation; in other words, the meaning that results is not a packaged, count interpretation. However, pluralization of mass nouns does delimit a narrow range of possible meanings, for example signaling that a striking amount of the relevant substance gets referenced.} To derive this fact, our system of number marking must allow SG to compose with mass nouns; because SG may compose, it must. For SG to compose, every
member of a mass noun’s denotation must measure 1 P-atom.

When referring to the corresponding kind, mass nouns trivially satisfy the one-ness pre-
supposition of SG. Ontologically, a kind is an individual, and there is only one such individual
in the denotation of a mass noun. The semantics in (23) illustrates this fact for the mass
noun water.

\begin{equation}
\text{[water]} = \text{WATER (i.e., the water kind)}
\end{equation}

The problem with number marking on mass nouns centers around uses where concrete in-
stances of the kind are referenced, as in (24).

\begin{enumerate}
\item John drank the water that you poured for him.
\item The oil in Sue’s car needs changing.
\end{enumerate}

Now, consider the denotation of a mass noun. We could align such denotations with count
nouns. Just like book denotes a set of minimal book individuals (i.e., a set of books), water
denotes a set of minimal water quantities; these quantities would be unstable across worlds
(Chierchia, 2010). In other words, the size of these minimal quantities would vary across
situations. However, given that they are minimal, the elements in the denotation of a mass
noun would each constitute a single P-atom: no element has a proper part that is also an element (if it did, the element would not be minimal. While this approach would deliver singular morphology on mass nouns, it would also shrink the conceptual and formal distance between mass nouns and count nouns. For example, with minimal instances in its denotation, we might expect to be able to count mass nouns directly with numerals; like count nouns, mass nouns would have stable (relative) atoms suitable for counting.

Another option, attributed by Chierchia (2013) to G. Magri, treats mass nouns as naming singleton properties. The result has mass nouns name the contextually relevant totality of some substance. Using the language of mereotopology from Chapter 3, a mass noun would contain in its denotation only the maximally self-connected instance of the corresponding kind. Under this singleton property approach to mass noun semantics, water would receive the denotation in (25).

(25)  \[ \text{water} = \lambda x. x = \bigcup \text{water} \]

The mass noun water names the property of being the supremum of the water property, which is always a singleton. In this way, the semantics in (25) will satisfy the numerical presupposition of sg: the predicate is quantity-uniform (there is some measure by which every member evaluates to the same value, namely the measure in P-atoms) and every member of the predicate measures 1 with respect to this quantity-uniform measure (as a singleton, its one member is necessarily a minimal element). To summarize: In order to ensure singular morphology on mass nouns in the general case, we may take advantage of the cumulative nature of the substances that they so often name and mandate that their denotation is always a singleton containing the (relevant) totality of substance. These current musings are not intended to decide the issue of mass noun semantics, but rather to demonstrate possible directions in which to pursue an account.

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6Zamparelli (2008) suggests a similar move.

7Note that by necessarily denoting a singleton, we would expect the same determiner restrictions that were observed for degree relatives in the previous chapter.
5.1.3 Classifier languages

In our discussion of quantizing nouns in Chapter 3, we saw that classifiers are taken to be an epiphenomenon of classifier languages. In these languages, counting proceeds on the basis of a closed, contrasting set of morphemes that designate countable units (Greenberg, 1972; Allan, 1977a,b; Denny, 1976, 1979; Adams and Conklin, 1973). These morphemes, classifiers, mediate the relationship between numerals and nouns. Consider the obligatory status of the classifier *ge* in (26).

(26) san *(ge) ren
    three  CL people
    ‘three people’  (Chinese Mandarin)

Measure heads, both CARD and measure terms like kilo, mirror true classifiers: they are a set of morphemes that mediate the relation between numerals and nouns. But classifiers also resemble atomizers, partitioning instances of kinds into discrete units for the purpose of counting. In fact, we will see that classifiers subsume most of what we have considered quantizing nouns. Furthermore, any instance of counting (or measuring more broadly) appeals to one of these elements, both in classifier and number marking languages.

Viewed through the lens of the system described in Chapter 2, classifier languages stand apart because they lack CARD: what number marking languages can do covertly with CARD (i.e., compose numerals with nouns for the purpose of counting), classifier languages must do overtly with a classifier. But there is more to the dissimilarity between classifier and number marking languages, as evidenced by the following cross-linguistic generalizations: First, if a language has obligatory classifiers, then it freely allows bare arguments (Chierchia, 1998b); and second, if a language has obligatory classifiers, then it lacks obligatory number marking (Greenberg, 1972). Both of these generalizations receive an account once we augment our semantics of number marking with the assumption that nouns in classifier languages refer to kinds, whereas nouns in number marking languages may denote predicates. The semantic import of number marking always yields redundant information in a classifier language. We therefore settle on the claim that obligatory number marking is only allowed if it delivers
otherwise unrecoverable information.\textsuperscript{8} To see how this claim falls out within the current system, we first consider in more detail the relevant cross-linguistic generalizations concerning classifiers and number marking. We then see how classifiers conform with the syntax and semantics of nominals proposed in the previous section.

Cross-linguistic generalizations

The first generalization concerns the lack of number marking in classifier languages. Greenberg (1972) reproduces the following claim, attributed to an unpublished manuscript by Slobin, later appearing in Sanches and Slobin (1973).

(27) \textit{Slobin–Greenberg–Sanches Generalization:}

“If a language includes in its basic mode of forming quantitative expressions numeral classifiers, then […] it will not have obligatory marking of the plural on nouns.”

(Greenberg, 1972, 286)

In other words, classifier languages do not have obligatory systems of number marking: if a language requires classifiers in the presence of numerals, morphological number will not be (necessarily) expressed.\textsuperscript{9} Conversely, if a language has obligatory number marking, then it will not have a generalized system of classifiers. Specifying obligatory number marking is crucial: the Slobin–Greenberg–Sanches generalization does not rule out number marking altogether in classifier languages, allowing for optional number marking in a classifier language, as with Chinese \textit{men} or Japanese \textit{tati} (e.g., Li, 1999; Kurafuji, 2004).

Knowing what we do about the semantic import of number marking – namely, that it indexes the one-ness of nominal predicates – our task is to understand the connection between classifiers and number marking such that the two are incompatible. Before exploring this connection, however, we consider another property of classifier languages that will quickly become relevant to the task at hand.

\textsuperscript{8}This section expands on a proposal put forth in Scontras (2013a).

\textsuperscript{9}See Doetjes (2012) for a fuller discussion of this generalization and potential counterexamples to it.
In addition to necessitating classifiers for counting, classifier languages freely allow bare nominal arguments: nouns appear bare (i.e., determiner-less) in argument position (cf. dogs in the sentence dogs are widespread). Chierchia (1998b; see also Chierchia 1998a, 2010) provides an account of bare arguments in classifier languages via his Nominal Mapping Parameter, whereby nouns in classifier languages are born argumental, referring at the kind level. Contrasting with classifier languages, English and other number marking languages map their nouns to predicates (of type \( \langle e, t \rangle \)), while functional structure (e.g., determiners or other methods of type-shifting) transforms nouns into arguments.¹⁰

Because nouns are born as kinds in classifier languages, classifiers are required to access the members of a kind for the purpose of counting. A classifier transforms a kind – a name for the maximal plural individual (i.e., the supremum) instantiating that kind – into the set of individuals belonging to the kind. We consider the semantics of classifiers in more detail presently; for now it suffices to adopt the view under which nouns in classifier languages are kind-denoting unless they appear with a classifier, which shifts kind-denoting nouns into predicates. In number marking languages, nouns are born as predicates and shift to kinds as needed. Thus, bare arguments are restricted in number marking languages: a predicate-denoting noun must shift to an argumental type. In classifier languages, bare arguments are freely allowed: a kind-denoting noun is born argumental. To see how this nominal mapping interacts with the Slobin–Greenberg–Sanches generalization, we turn now to the semantics of classifiers.

Classifier semantics

The structure attributed to classifiers should look familiar from our discussion of Measure Phrases in Chapters 2 and 3: classifiers compose first with a nominal and then with a numeral, projecting a classifier phrase (e.g., Li, 2011; Jiang, 2012; Li and Rothstein, 2012).

¹⁰This description of the Nominal Mapping Parameter is a simplification for the sake of perspicuity; the reader is referred to Chierchia (1998b) for the details.
(28) Cl(assifier)P(hrase)

\[
\begin{array}{c}
\text{Num} \\
\text{san} \ 'three' \quad \text{Cl} \\
\text{NP} \\
\text{ge} \quad \text{ren} \ 'people'
\end{array}
\]

Note the similarity between the classifier phrase in (28) and MP: in both cases the classifier/M\(^0\) heads the structure, intervening between a nominal and a numeral. Semantically, the type we attribute to measure heads also applies to classifiers (Li, 2011; Krifka, 1995). In light of our discussion of nominal mapping, the only difference between the M\(^0\)-head \text{CARD} and classifier semantics is that the former composes first with predicates (type \(\langle e, t \rangle\)), whereas the latter composes with kinds (type \(k\)).\(^{11}\) A candidate classifier semantics appears in (29-a); the semantics for \text{CARD} is repeated in (29-b).

(29) a. \[ [\text{CL}] = \lambda k \lambda n \lambda x. \lor k(x) \land \mu_{\text{CARD}}(x) = n \]
b. \[ [\text{CARD}] = \lambda P \lambda n \lambda x. P(x) \land \mu_{\text{CARD}}(x) = n \]

Given the parallels in both structure and semantics between classifiers and measure heads, it seems no great stretch to align the two: classifiers are yet another instantiation of M\(^0\). However, classifiers also serve the role of atomizers; they deliver maximal instances of the kind supported by context for the purpose of counting. Thus, rather than merely instantiating a kind via the \(\lor\)-operator in (29-a), the template for classifier semantics should include a partitioning function \(\pi\), as in (30). The partitioning function will necessarily vary across classifiers, which delivers their broad range of meaning.

(30) \[ [\text{CL}] = \lambda k \lambda n \lambda x. \pi(k)(x) \land \mu_{\text{CARD}}(x) = n \]

With a semantics for classifiers and an understanding of the nominal system in languages that use them, we stand to account for the lack of number marking in classifier languages.

\(^{11}\)Recall that assigning to kinds the type \(k\) is merely a shorthand; kinds are individuals (or individual concepts) just like the \text{president} or \text{John}.
Having aligned classifier and number marking languages such that both appeal to MP in the formation of a numeral-modified nominal, we turn now to the difference between the two types of languages that precludes obligatory number marking in classifier languages. We begin by recapping the discussion thus far.

Assume a version of the Nominal Mapping Parameter whereby languages either map their nouns to kinds (type $k$) or to predicates (type $\langle e, t \rangle$) (Chierchia, 1998b). In an $N_k$ language, nouns are born argumental, so we predict nouns to freely appear bare, without determiners, in argument positions. However, assuming that counting proceeds over members of a set and that kinds are individuals, classifiers are required in the presence of a numeral to shift the kind-denoting noun to a predicate, that is, a set of individuals (the maximal instances of the kind supported by context and meeting the restrictions of the classifier).

Next, consider the role of number marking: number morphology is realized on nouns and gives information about the quality of the nominal denotation. Only when every member of a noun’s denotation evaluates to 1 by the relevant measure does singular morphology surface. In an $N_{\langle e, t \rangle}$ language where nouns denote predicates, number marking is (at least sometimes) informative. For example, in the case of the boy ate the cake, we know on the basis of the singular morphology expressed on boy that the intended referent is a singular individual, not a plurality; only singular individuals may be included in the denotation of the singular-marked boy because of the one-ness presupposition of $sg$. In this way, number marking in an $N_{\langle e, t \rangle}$ language provides information about the denotation of nominals that would otherwise be unrecoverable from the larger linguistic structure; only the number marking clues us in to the number of boys referenced.

In a classifier language, the singular/plural distinction is uninformative: nouns in these languages denote kinds, and (intensionalized) individuals are not something that can be closed under sum-formation. Moreover, kinds are concepts that require more than one instantiation, so it should never be the case that the kind’s instantiation has cardinality 1 (Chierchia,
Evaluating the one-ness of a kind therefore necessarily fails, owing both to the type-mismatch between predicate-selecting #-heads and to the conceptual difficulty associated with evaluating the one-ness of a kind. Thus, indexing kind-denoting nouns with number morphology is nonsensical. In the general case, introducing a system of number marking into a $N_k$ language makes no semantic contribution: Nouns denote kinds and so the one-ness presupposition always fails (assuming it could apply at all), necessitating plural morphology in all cases.

Note, however, that nominal predicate semantics may be derived in a classifier language via a numeral-classifier construction. The role of a classifier is to mediate between a noun’s kind referent and a numeral, forming a predicate of individuals (type $(e, t)$). However, in any such construction, the resulting denotation will be quantity-uniform, determined by the numeral present. For example, the predicate $\text{one-CL-person/people}$ will denote the quantity-uniform set of people pluralities, each with cardinality 1; the semantics for this construction and its parts appears in (31).

\[(31)\] $\text{yi ge ren ‘one person’}$

\begin{enumerate}
\item $[yi] = 1$
\item $[ge] = \lambda k \lambda n \lambda x. \pi(k)(x) \land \mu_{\text{CARD}}(x) = n$
\item $[ren] = \text{PERSON (i.e., the people kind)}$
\item $[yi ge ren] = \lambda x. \pi(\text{PERSON})(x) \land \mu_{\text{CARD}}(x) = 1$
\end{enumerate}

The semantic contribution of number morphology on a derived predicate like in (31) is redundant: the numeral delivers the information that the resulting denotation is both quantity-uniform and has members all with cardinality 1. Number morphology on such a derived predicate would therefore be uninformative – the information it could convey is already present in the numeral ‘one’ (and similarly with all other numerals; in $\text{three-CL-person}$ the numeral ‘three’ clues us in to the fact that more than one person is referenced, namely three). We see that in a $N_k$, that is, a classifier language, number morphology fails to contribute meaningful information both in the general case of kind-denoting bare nouns and in the case of derived nominal predicates.
These facts lead to the following constraint, meant to explain the lack of obligatory number marking in classifier languages: **Only allow a system of number marking in a language if there are instances where the system delivers otherwise unrecoverable information** (about nominal denotations). We have seen that in \( N_{(e,t)} \) languages there are cases, namely non-quantified nominals, where number morphology is informative. We therefore correctly predict the presence of number marking in such a language. In \( N_k \) languages, either the noun refers directly to a kind and is not eligible to be checked by the one-ness presupposition of singular morphology, or a numeral-classifier construction derives a nominal predicate and the numeral itself provides the information about one-ness that number morphology would have delivered. Therefore, given the constraint just stated, in \( N_k \) languages we predict the absence of obligatory systems of number marking. Fig. 5.1 diagrams the implicational connections that lead to this conclusion.

First, the Nominal Mapping Parameter determines whether a language maps its nouns to kinds or to predicates. If the former holds, classifiers are required for the purpose of counting with numerals. However, once there is a generalized classifier system, number marking loses its informativity and so obligatory number marking is ruled out. If the Nominal Mapping Parameter has a language map its nouns to predicates, number marking stands to provide information about the one-ness of these nominal predicates and so number marking is allowed.

Note that we do not necessitate number marking in \( N_{(e,t)} \) languages, which map their nouns to predicates; we merely rule out obligatory number marking in \( N_k \) languages. Our typology therefore predicts languages that we have heretofore not considered: ones in which nouns map to predicates, type \( (e,t) \), thus precluding classifiers, but in which number marking is also absent. In other words, we predict languages that lack both classifiers and obligatory number marking. Fortunately, such languages are attested (e.g., Dène Suliné, Wilhelm, 2008;
One last aspect of the implications diagramed in Fig. 5.1 warrants further scrutiny: We have said that classifiers are required in $N_k$ languages to retrieve the members of kind-denoting nominals for the purpose of counting. However, we have aligned true classifiers with all measure heads, including CARD. Functionally, classifiers and CARD serve a similar purpose: to mediate the relation between numerals and nouns. Classifiers perform the added step of accessing, or partitioning the members of a kind. Now we return to the point that began this section: what number marking languages may do covertly with CARD, classifier languages must do overtly with classifiers. So what prohibits a null measure head like CARD from entering into the functional lexicon of classifier languages? While the answer to this question requires future study, consider the following observation, which will likely constrain the set of possible explanations.

Classifiers and CARD differ in two ways. First, classifiers are overt while CARD is silent. Second, classifiers take a kind-denoting argument, whereas CARD selects for predicates. Could these differences be related? Consider the possibility that only covert measure heads like CARD compose with predicates, whereas overt measure heads and other quantizing nouns necessarily compose with kinds. The quantizing nouns we have considered—crucially, measure terms and atomizers—support this link between phonologically realized measure heads and kind-selection. Measure terms and atomizers thus align with classifiers to the exclusion of CARD: the former take kind-denoting arguments, while CARD composes with a predicate.

It would appear, then, that only covert CARD selects for predicates; overt classifiers and quantizing nouns compose with kinds. While the reason why overt measure heads should select for kinds and covert ones for predicates remains an open question, this tendency stands to clarify the implication between nominal mapping and the presence/absence of classifiers in Fig. 5.1. $N_k$ languages lack covert measure heads like CARD because their nouns, the arguments of measure heads, denote kinds, not predicates, and overt measure heads are required in the presence of kind-denoting nouns. Put differently, kind-selection in nominal semantics requires an overt element that determines how the kind will instantiate. In an $N_{(c,t)}$ language, nouns denote predicates at base and so for counting to proceed over members of
their denotation we make do with a covert, predicate-selecting measure head like CARD.

5.2 Looking forward

In addition to the extensions the current program suggests in the three domains outlined in the previous section, this thesis has advanced a semantics of measurement that stands to inform future work on nominal semantics and on natural language more generally. The most interesting – and most elusive – answers such work can uncover concern the nature of the linguistic (i.e., mental) representations we create for the world that surrounds us. Let us highlight two points from this thesis that make predictions about these representations.

First, we have seen that speakers employ diverging criteria to evaluate whether or not something counts as singular (i.e., as one thing), at least grammatically so. In languages like Turkish, which require singular morphology with a quantized predicate, singular-marked nominals enjoy plural reference. Taking singular morphology as an unambiguous cue for one-ness cross-linguistically, we saw that speakers of these languages evaluate one-ness in a relative manner, on a predicate by predicate basis. If a predicate (e.g., iki çocuk ‘two boy’) contains in its denotation only minimal elements, that is, relative atoms, then that predicate will spell out as singular. However, in languages like English, one-ness is evaluated with respect to the measures that are relevant, for example basic cardinality or the measure named by the linguistic expression used. We might therefore expect to find a behavioral reflex of these grammatical strategies, such that one-ness judgments beyond the domain of number marking show an influence of these strategies.

Second, on the basis of the existential interpretation observed for degree nouns, we have seen that abstract representations of measurement are richer than mere points on a scale. The result of this observation is in fact a simplification of the ontology: rather than positing degree primitives, we make do with independently-motivated semantic objects, namely nominalized predicates, or kinds. Here the claims are even more far reaching than in the case of grammatical number. Whenever language encodes reference to these abstract representations of measurement, the representations themselves must contain information about the objects that instantiate them.
This new semantics for degrees highlights the four basic elements of the semantics of measurement that are hypothesized to be stable across languages. First, and perhaps most obviously, we have measure functions in our semantics. These measure functions translate objects onto a scale, allowing for the encoding of gradability. Scales are composed of the second element in our measurement semantics: numbers. Numbers, specifically non-negative real numbers, are taken as semantic primitives. The third semantic element, kinds, often provides the objects of measurement. Kinds are abstract, intensional entities, the nominalizations of properties, so the fourth element in our measurement semantics, partitions, delivers maximal instances of the kind (i.e., real-world objects) for the purpose of measurement. With measures, numbers, kinds, and partitions, we now have a semantics of measurement.
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