Knowing the charge of the parton initiating a light-quark jet could be extremely useful both for testing aspects of the Standard Model and for characterizing potential beyond-the-Standard-Model signals. We show that despite the complications of hadronization and out-of-jet radiation such as pile-up, a weighted sum of the charges of a jet’s constituents can be used at the LHC to distinguish among jets with different charges. Potential applications include measuring electroweak quantum numbers of hadronically decaying resonances or supersymmetric particles, as well as Standard Model tests, such as jet charge in dijet events or in hadronically-decaying $W$ bosons in $t\bar{t}$ events. We develop a systematically improvable method to calculate moments of these charge distributions by combining multi-hadron fragmentation functions with perturbative jet functions and perturbative evolution equations. We show that the dependence on energy and jet size for the average and width of the jet charge can be calculated despite the large experimental uncertainty on fragmentation functions. These calculations can provide a validation tool for data independent of Monte-Carlo fragmentation models.

The Large Hadron Collider (LHC) at CERN provides an opportunity to explore properties of the Standard Model in unprecedented detail and to search for physics beyond the Standard Model in previously unfathomable ways. The exquisite detectors at ATLAS and CMS let us go beyond treating jets simply as 4-momenta to treating them as objects with substructure and quantum numbers. A traditional example is whether a jet was likely to have originated from a $b$-parton. At the LHC, one can additionally explore whether a jet has subjet constituents, as from a boosted heavy object decay [1, 2], or whether it originated from a quark or gluon [3]. See Ref. [4] for a recent review of jet substructure. Here we consider the feasibility of measuring the electric charge of a jet.

The idea of correlating a jet-based observable to the charge of the underlying hard parton has a long history. In an effort to determine the extent to which jets from hadron collisions were similar to jets from leptonic collisions, Field and Feynman [5] argued that aggregate jet properties such as jet charge could be measured and compared. The subsequent measurement at Fermilab [6] and CERN [7] in charged-current deep-inelastic scattering experiments showed clear up- and down-quark jet discrimination, confirming aspects of the parton model. Another important historical application was the light-quark forward-backward asymmetry in $e^+e^-$ collisions, a precision electroweak observable [8]. Despite its historical importance, there seem to have been no attempts yet at measuring the charge of light-quark jets at the LHC.

Most experimental studies of jet charge measured variants of a momentum-weighted jet charge. We define the $p_T$-weighted jet charge for a jet of flavor $i$ as

$$Q^i_\kappa = \frac{1}{(p_T^{jet})^\kappa} \sum_{j \in jet} Q_j (p_T^j)^\kappa$$  \hspace{1cm} (1)

where the sum is over all particles in the jet, $Q_j$ is the integer charge of the color-neutral object observed, $p_T^j$ is the magnitude of its transverse momentum with respect to the beam axis, $p_T^{jet}$ is the total transverse momentum of the jet, and $\kappa$ is a free parameter. A common variant uses energy instead of $p_T$. Values of $\kappa$ between 0.2 and 1 have been used in experimental studies [6, 8].
FIG. 2. Distinguishing $W'$ from $Z'$ with a log-likelihood discriminant, for different values of $\kappa$. Even with only 50 events the samples are extremely well separated.

In hadron-hadron collisions at high energy, such as at the LHC, the particle multiplicities in the final state are significantly larger than at low energy and at $e^+e^-$ or lepton-hadron colliders. Thus, one would expect that measuring the charge of a light-quark jet at the LHC should be difficult, with the primordial quark charge quickly getting washed out. However, this turns out not to be the case. For example, Fig. 1 shows distributions of $Q^\kappa_i$ for $u, \bar{u}, d, \bar{d}$ and $g$ jets for two values of $\kappa$ [9]. One can clearly see that $Q^\kappa_i$ will be useful for identifying the charge of the primordial parton. Moreover, as we will show, the energy and jet-size dependence of moments of jet-charge distributions can be calculated in perturbative QCD.

To get an impression of how much data is needed for $Q^\kappa_i$ to be useful, we consider measurements designed to distinguish charged from neutral vector resonances. To be concrete, we consider scaled-up $W$ and $Z$ bosons at a mass of 1 TeV decaying into light quark jets. Simply cutting on the sum of the $Q^\kappa_i$ of the hardest two jets in each event we can distinguish the two samples (assuming no background) with 95% confidence using around 30 events. We find that the best discriminating power is achieved for $\kappa \sim 0.3$. A more sophisticated log-likelihood discriminant based on the two-dimensional jet charge distribution is shown in Fig. 2 where $\sim 4\sigma$ separation of the two samples is achievable with 50 events.

For another phenomenologically relevant application of jet charge consider a simplified supersymmetric model with squarks pair produced through $t$-channel gluino exchange and decaying as $\tilde{q} \rightarrow q + \chi^0_1$. At $m_{\tilde{q}} = m_{\tilde{g}} = 1.5$ TeV such a model is still allowed [10], although it will come under scrutiny with the next round of 8 TeV data. Due to the high concentration of up-type valence quarks at large $x$, the di-squark production process yields many events with two hard up-type jets and missing energy, in contrast to the background (dominated by $V +$jets) where the two hardest jets are rarely both ups. Adopting a set of cuts similar to those of Ref. [10], we estimate if an excess is seen in 2 jets and missing energy channel, the increased concentration of up quarks could be measured above the $2\sigma$ level with 25 fb$^{-1}$ of 8 TeV data, providing unique insights into the flavor structure of the new physics.

To trust a measurement of jet charge, it is important to test it on samples of known composition. While proton collisions do not generally provide clean samples of pure up- or down-quark jets, there are still ways to validate the method on data. For example, dijet production has an enormous cross section at the LHC and the fraction of jets originating from different partons is directly determined by the parton distribution functions (PDFs). At larger energies the valence quark PDFs dominate over gluon or sea quark PDFs, producing more charged final states, as can be seen in see Fig. 3. The mean total jet charge in dijet events is also shown for various values of $\kappa$.

FIG. 3. Top: final state composition in dijet production. Bottom: Sum of the two jet charges in dijet events, for various $\kappa$. The growth with dijet invariant mass reflects the larger fraction of valence quark PDFs at large $x$ and corresponding decrease in $gg$ final states.
k. Verifying the trend in this plot on LHC data would help validate jet charge.

Another sample of interest for validating jet charge is hadronically decaying W bosons coming from top decays. In a semi-leptonic t\bar{t} sample, the leptonically decaying W can be used to determine the two charges of the jets from the hadronically decaying W. The distributions of these charges can then be compared to expectations, an example comparison is shown in Fig. 4. Validating this simulation on data would establish weighted jet charge as a trustworthy tool, which could then be used for new physics applications. Perhaps it could even be employed within the context of W decays to help with top-tagging or W polarization measurements.

Next, we consider the effects of pile-up and contamination on jet charge. One might worry that at high luminosity jet charge would be diluted by pile-up events, as up to O(100) proton-proton collisions can take place in the same bunch crossing. However, the products of these interactions tend to be soft, and are thus assigned little weight as long as \( \kappa \) is not too small. Further, charged particles can be traced to their collision vertex allowing weighted jet charge calculations. While Monte-Carlo programs like PYTHIA often provide an excellent approximation to full quantum chromodynamics, they are only valid to leading-order in perturbation theory including the resummation of leading Sudakov double logarithms [12].

A precise calculation of jet charge is challenging because it is not an infrared-safe quantity. Jet charge is sensitive to hadronization and cannot be calculated without knowledge of the fragmentation functions \( D^{h}_{\nu}(x, \mu) \). These functions give the average probability that a hadron \( h \) will be produced by a parton \( j \) with the hadron having a fraction \( z \) of the parton’s energy. Fragmentation functions, like parton distribution functions, are non-perturbative objects with perturbative evolution equations which simplify in moment space. The Mellin moments are defined by

\[
\tilde{D}^{h}_{\nu}(\nu, \mu) = \int_{0}^{1} dx \, x^{\nu} D^{h}_{\nu}(x, \mu),
\]

which evolve through local renormalization group equations, just like the moments of parton distribution functions.

We first consider the average value of the jet charge

\[
\langle Q_{h}^{i} \rangle = \frac{1}{\sigma_{\text{jet}}} \int d\sigma \, Q_{h}^{i} = \int dz \, z^{\kappa} \sum_{h} Q_{h}^{i} \frac{1}{\sigma_{\text{jet}}} \frac{d\sigma_{h \in \text{jet}}}{dz},
\]

where \( z = E_{h}/E_{\text{jet}} \) is the fraction of the jet’s energy the hadron carries. For narrow jets \( z \sim p_{T}^{h}/p_{T}^{\text{jet}} \).

To connect to the fragmentation functions, we first observe that for \( \kappa > 0 \) the charge is dominated by collinear and not soft radiation. Thus the contributions of the hard and soft sectors of phase space, while contributing to the formation of the jet, should have a suppressed effect on \( Q_{h}^{i} \). We can therefore use the fragmenting jet functions introduced in Refs. [13, 14] to write

\[
\frac{1}{\sigma_{\text{jet}}} \frac{d\sigma_{h \in \text{jet}}}{dz} = \frac{1}{16\pi^{3}} \sum_{j} \int_{z}^{1} \frac{dJ_{ij}(E, R, x, \mu)}{J_{ij}(E, R, \mu)} D^{h}_{j}(x, \mu).
\]

Here \( J_{ij}(E, R, \mu) \) is a jet function and \( J_{ij}(E, R, x, \mu) \) a set of calculable coefficients which depend on the jet definition and flavor \( i \) of the hard parton originating the jet. The hard and soft contributions conveniently canceled in
\[ \langle Q^j_\kappa \rangle = \frac{1}{16\pi^3} \frac{\bar{J}_{ij}(E, R, \kappa, \mu)}{J_q(E, R, \mu)} \sum_h Q_h \bar{D}^h_j(\kappa, \mu), \quad (5) \]

with $\bar{J}_{ij}$ related to $J_{ij}$ by a Mellin-transform as in Eq. (2). By charge conjugation $\sum_h Q_h \bar{D}^h_j(\kappa, \mu) = 0$, so in particular $\langle Q^j_\kappa \rangle = 0$. We have checked that the $\mu$-dependence of $\bar{J}_{ij}/J_i$ exactly compensates for the $\mu$-dependence of the fragmentation functions at order $\alpha_s$. We have written both $J_i(E, R, \mu)$ and $J_{ij}(E, R, x, \mu)$ as if they depend on the energy $E$ and size $R$ of the jet, however, these functions only give a valid description as if they depend on the energy $E$ and size $R$ of the jet. Here we use the $e^+e^-$ version of anti-$k_T$ jets of size $R$, for which the natural scale is $\mu_1 = 2E \tan(R/2)$. We can therefore calculate the average jet charge by evaluating the Mellin-moments of fragmentation functions at the scale $\mu_j$ and multiplying by the jet functions.

Since only one linear combination of fragmentation functions appears in Eq. (3), the theoretical prediction is not significantly limited by the large uncertainty on $D^j_q(\kappa, \mu)$. One can simply measure $D^h_j(\kappa, \mu)$ by observing the average jet charge for each flavor at one value for $\mu$ and then using the theoretical calculation to predict it at other values. In the absence of data, we simulate such a comparison using PYTHIA. The result is shown in Figure 6 for various values of $\kappa$ and $R$, and normalized at a reference point. Already we can see a clear agreement between the theory and PYTHIA.

To calculate other properties of the jet charge distribution requires correlations among hadrons. For example, we can consider the width of the jet charge, $(\Gamma_\kappa^i)^2 = \langle Q^i_\kappa \rangle^2 - \langle (Q^i_\kappa)^2 \rangle$. This depends on the moment
\[ \langle (Q^i_\kappa)^2 \rangle = \sum_n \sum_{h_1, \ldots, h_n} \int dz_1 \cdots dz_n (Q_1 z_1^\kappa + \cdots + Q_n z_n^\kappa)^2 \times \frac{1}{\sigma_{\text{jet}}} d^3\sigma_{h_1 \cdots h_n \in \text{jet}}, \quad (6) \]

where the sum runs over all hadronic final states. After integrating over most of the $z_i$ and including a factor of $\frac{1}{2}$ for identical hadrons, this simplifies to
\[ \langle (Q^i_\kappa)^2 \rangle = \int dz z^{2\kappa} \sum_h Q_h^2 \frac{1}{\sigma_{\text{jet}}} \frac{d\sigma_{h \in \text{jet}}}{dz} \quad (7) \]

The first term on the right hand side can be expressed in terms of products of fragmentation functions and jet functions as for $\langle Q^j_\kappa \rangle$. The second term can be expressed in terms of something we call a dihadron fragmenting jet function, $\mathcal{G}^h_{ij,h_2}$. Its matching onto (dihadron) fragmentation functions is given by
\[ \mathcal{G}^h_{ij,h_2}(E, R, z_1, z_2, \mu) \]
\[ = \sum_j \int \frac{du}{u^2} J_{ijk}(E, R, u, \mu) D_j^{h_1,h_2} \left( \frac{z_1}{u}, \frac{z_2}{u}, \mu \right) \]
\[ + \sum_{j,k} \int \frac{du}{u} \frac{dv}{v} J_{ijk}(E, R, u, v, \mu) D_j^{h_1}(\frac{z_1}{u}, \mu) D_k^{h_2}(\frac{z_2}{v}, \mu), \]

The second term is due to a perturbative parton splitting before hadronization and only starts at 1-loop order,
\[ J^{(1)}_{ijk}(E, R, u, v, \mu) = J^{(1)}_{ij}(E, R, u, \mu) \delta(1-u-v) \delta_{k,a(ij)}, \quad (9) \]

where $\delta_{k,a(ij)}$ indicates that the flavor $k$ is completely fixed by $ij$. E.g. $a(qq) = g$, $a(gq) = q$. We then find
\[ \langle (Q^i_\kappa)^2 \rangle = \frac{1}{16\pi^3} \sum_j \frac{\bar{J}_{ij}(E, R, 2\kappa, \mu)}{J_q(E, R, \mu)} \left[ \sum_h Q_h^2 \bar{D}^h_j(2\kappa, \mu) \right] \]
\[ + \sum_{h_1, h_2} Q_{h_1} Q_{h_2} \bar{D}^{h_1,h_2}_j(\kappa, \kappa, \mu), \quad (10) \]

(For a gluon jet, which we do not consider here, there is a contribution from the last line of Eq. (5) corresponding to
a perturbative $g \to q\bar{q}$ splitting.) We have checked that this equation is renormalization-group invariant at order $\alpha_s$.

Unfortunately, the dihadron fragmentation functions are even more poorly known than the regular fragmentation functions. However, we can use the same trick as for the average jet charge to calculate the $E$ and $R$ dependence of the width, given measurements at some reference scale. As with the average jet charge, we can now calculate the width by fitting one parameter for each flavor, corresponding to the term in brackets in Eq. (10), and predicting the $E$ and $R$ dependence. Results compared to PYTHIA for the width are shown in Fig. 6 and show good agreement. The gluon mixing contribution is not included in these figures since it requires additional matching; a discussion of the effect of gluon mixing can be found in Ref. [16].

To go beyond the average and the width, for example to the 3rd or higher moments, multi-hadron fragmentation functions would be needed. From a practical point of view, such functions are nearly impossible to measure with any precision. However, we have found that the discriminating power of jet charge is nearly as strong using Gaussians based on the average and width as it is with the full differential jet charge distribution. It follows that accurate calculations of the phenomenologically relevant part of jet charge distributions are achievable with the formalism we have introduced in this paper. The full fragmenting jet functions, both for the single hadron and dihadron case, and the evolution kernels, are now known at 1-loop order. To see whether higher precision is required, and to explore the importance of power corrections, requires some LHC data to compare with. The calculations and issues discussed here are expanded on in Ref. [10].

As we have shown, the weighted jet charge, and its moments, are measurable and testable already at the LHC. With potential to uniquely determine quantum number of certain new physics particles, should they show up, it is important to verify jet charge on standard model processes. Thus jet charge holds promise as a measurable, calculable and useful observable.

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