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Is the nature of magnetic order in copper-oxides and in iron-pnictides different?

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Abstract. - We use the results of first-principles electronic structure calculations and a strong coupling perturbation approach, together with general theoretical arguments, to illustrate the differences in super-exchange interactions between the copper-oxides and iron-pnictides. We show that the two magnetic ground states can be understood in a simple manner within the same theoretical foundation. Contrary to the emerging view that magnetic order in the iron-pnictides is of itinerant nature, we argue that the observed magnetic moment is small because of frustration introduced by the electrons of the Fe orbitals as they compete to impose their preferred magnetic ordering.

The copper-oxide layers present in the high-T\textsubscript{c} superconducting families are turned into superconductors by introducing dopants that create electrons or holes in these otherwise antiferromagnetic (AF) insulating layers \cite{1}. The proximity of antiferromagnetism to superconductivity has led to the general view that this form of magnetic order is intimately related to the mechanism of superconductivity in these materials \cite{2}. In the recently discovered iron-pnictide based superconductors \cite{3–8}, which exhibit superconductivity at relatively high-T\textsubscript{c}, the copper-oxide layer is replaced by an iron-pnictide layer. Interest in the new materials, reminiscent of that seen when the cuprates were discovered more than two decades ago, is due to the fact that many unsuccessful attempts were made to replace the copper-oxide layer in high-T\textsubscript{c} materials to facilitate practical applications. The parent compounds of these iron-pnictide materials, like the copper-oxide parent materials, show a spin-density-wave order \cite{9, 10} illustrated in Fig. 1(a). Unlike the copper-oxides, the parent compounds in the iron-pnictides, such as the pure LaOFeAs, are metallic, but are magnetically ordered and non-superconducting and they become superconductors by doping with electrons or holes. The copper-oxide parent compounds are well described as spin-1/2 Heisenberg quantum antiferromagnets \cite{2}. Furthermore, it is widely believed that superconductivity in copper-oxides arises when, by doping the quantum antiferromagnet, the carriers (holes or electrons) form a strongly correlated Anderson-Mott type system with the spin-spin correlations playing a fundamental role in the superconductivity mechanism. After the discovery of the iron-pnictide superconductors, there is an emerging view that the magnetism in these compounds is of itinerant type \cite{11–14} and that these systems are different from the cuprates and in fact weakly correlated. This is an important issue to settle, because any further theoretical analysis of other properties of this new family of materials, including the still unknown superconductivity mechanism, depends on it.

Here we focus on this issue and seek a broader framework to reconcile the different forms of magnetic order and to explain the magnetic properties in both families of materials with the same approach. Our arguments are inspired by the results of Ref. \cite{15}, where first-principles electronic structure calculations based on density functional theory (DFT) were combined with a strong coupling expansion to obtain an effective low-energy Hamiltonian which describes the electrons occupying the five Fe orbitals. In the present work, we address the nature of the magnetic order in the iron-pnictide and the copper-oxide based materials using a more general scheme, which is
correct independently of the conditions of validity and the
details of the calculation presented in Ref. 15. We show
that the origin and nature of magnetism in these two fam-
ilies of materials is the same and both families should be
treated within the same theoretical foundation. Specifi-
cally, if Mott physics is operative in the copper-oxides it
should also be operative in the iron-pnictides, and if the
magnetism in the copper-oxides is of the Anderson-Mott
type it should be of the same-type in these newly discov-
ered materials.

First, we discuss the difference in the structure and mag-
netic order between the iron-pnictide layer and the copper-
oxide layer as shown in Fig. 1. By removing the extra
magnetic ion (i.e., Fe) from the centre of the square unit
 cell of the iron-pnictide layer we obtain not only the same
structure, but identical magnetic order with the copper-
oxide layer. The only important difference, which will be
addressed below, is that the magnitude of the observed
magnetic moment [9] per Fe site is significantly reduced
from its calculated value [11,17–19].

Fig. 2 schematically illustrates the qualitative origin
of the well-known Anderson super-exchange interaction.
This type of processes give rise to a super-exchange con-
tribution to the spin-exchange interaction, \( J_{\nu}(1) \),
between two electrons occupying two nearest neighbour
(NN), or \( J_{\nu}(2) \), between two next nearest neighbour
( NNN) Fe orbitals of a given flavour (where \( \nu \) labels the five Fe or Cu
d-orbitals: \( x^2-y^2, xz, yz \), or \( z^2 \)). The effective spin-spin
interaction Hamiltonian for a particular flavour \( \nu \) is:

\[
\hat{H}_{\nu} = J_{\nu}(1) \sum_{<ij>} \mathbf{S}_i,\nu \cdot \mathbf{S}_j,\nu + J_{\nu}(2) \sum_{<<ij>>} \mathbf{S}_i,\nu \cdot \mathbf{S}_j,\nu, \tag{1}
\]

where the \( \mathbf{S}_i \) is a three component vector of a spin-1/2
quantum spin operator and \( <ij> \) and \( <<ij>> \) stand
for NN and NNN pairs of magnetic ion sites. Here we have
assumed that the particular Fe d-orbital is singly occupied.
The case in which an orbital is occupied by two electrons is
discussed separately. Other models in the literature, as in
Refs. [16–18], are similar to ours with the very important
difference that our Hamiltonian, Eq.1 treats differently
the various Fe d-orbitals labelled by \( \nu \). The condition
for quantum- Néel (checkerboard) ordering (shown in Fig.
5(b)) is

\[ J_{\nu}^{(1)} > 2 J_{\nu}^{(2)}; \tag{2} \]

if the coupling between spins of electrons occupying the
orbital \( \nu \) do not satisfy the above condition, the SDW
order of Fig.1(a) is energetically favourable against the
checkerboard order (Fig. 5(b)).

By fitting the first-principles electronic structure results
to a tight-binding model, we have established that, with
the exception of the \( d_x^2-y^2 \) orbital, for any given Fe d-
orbital, there is only one orbital of the intervening As (of
s or p mixed character) with which the hybridization ampli-
itude is most significant. Therefore, the most significant

Fig. 1: (a) The spin-density-wave order (columnar antiferro-
magnetic) as observed by neutron diffraction. The Fe magnetic
moments along the (1,1) direction are aligned, while two near-
est neighbouring such chains are antiferromagnetically aligned.
(b) The familiar antiferromagnetic ordering of the cuprous ox-
ides. The shaded square denotes the unit cell. With the map-
ing \( Fe \rightarrow Cu, As \rightarrow O \), the \( CuO_2 \) unit cell differs from the
\( Fe_2As_2 \) one by a magnetic atom missing from the centre.

Fig. 2: Super-exchange between two nearest neighbour (NN)
or next nearest neighbour (NNN) Fe atoms through electron-
hopping (hybridization) process between any Fe d-orbital and
any p or s orbital of an intervening As atom. One of the As
spin-up electrons (shown as red) hops to a singly occupied Fe
d-orbital (process 2), then the spin-down electron (shown as
blue) from the same As orbital hops to a NN or NNN Fe d-
orbital (process 3). With two successive hops (processes 4 and
5) the other electrons that were initially occupying the same
two Fe d states, hop to the same As site, restoring its doubly
occupied status. The initial configuration 1 differs from the
final configuration 5 by the spin orientation on the two Fe or-
bital. Such processes lead to AF spin-spin interaction
between electron spins occupying any two d orbitals belonging to NN
or NNN Fe atoms.
super-exchange processes involve the same intervening As orbital for both of the exchanged electrons. The super-exchange process illustrated in Fig. 2 requires four steps because two electrons, each occupying one of the two NN or NNN magnetic (Fe) ions, need to move through hopping to the intervening non-magnetic ion (As) and, thus, each electron has to make two steps. To obtain the total spin-spin interaction we need to consider also the direct hopping of electrons between these two neighbouring Fe or Cu d-orbitals. The results of Ref. [15] show that the direct hopping matrix element between two NN Fe d-orbitals, indicated as $t$ in Fig. 3, is smaller but not negligible. In the case of copper-oxides this direct process is non-applicable because two NN Cu atoms correspond to NNN Fe atoms in the Fe-pnictides.

Fig. 3 illustrates paths that give rise to the most significant contribution to $J^{(1)}_p$ and $J^{(2)}_p$, and compares the super-exchange interactions between the iron-pnictide layer and the copper-oxide layer through the intervening non-magnetic As or O atom respectively. For example, for iron-pnictides, $J^{(1)}_{xz}$ has two contributions: (i) one from the direct hopping $t_{xz}$ between two NN Fe $d_{xz}$ orbitals, (shown as the red path in Fig. 3), and (ii) a super-exchange contribution due to processes such as the one illustrated by the combined green-blue path in Fig. 3. The hybridization $V_{xz}^{\nu}$ of the $d_{xz}$ orbital of the Fe atom M1 and the $p_x$ orbital of the As atom N1 is very large ($\sim 1.4$ eV), while its hybridization $V_{yz}^{\nu}$ with the $p_y$ orbital of the same atom N1 is negligible ($\sim 0.1$ eV). The reverse is true for the hybridization of the same Fe d-orbital with the $p_x$ and $p_y$ orbitals of atom in position N2 because of the 90° relative orientation. The contribution to $J^{(1)}_{xz}$ due to the green-red process illustrated in Fig. 3 is proportional to

$$J^{(1)}_{xz} \sim (V_{xz}^{\nu} V_{xz}^{\nu})^2.$$  (3)

On the other hand, the contribution to $J^{(2)}_{xz}$ due to the green-yellow process illustrated in Fig. 3 is proportional to

$$J^{(2)}_{xz} \sim (V_{xz}^{\nu} V_{xz}^{\nu})^4.$$  (4)

Therefore, $J^{(2)}_{xz}/J^{(1)}_{xz} \sim (V_{xz}^{\nu} V_{xz}^{\nu})^2 \sim 0.01$ and the super-exchange contribution to $J^{(1)}_{xz}$ is negligible as compared to that of $J^{(2)}_{xz}$ between NNN Fe atoms. Hence, we can conclude that in the subspace formed by $d_{xz}$ and $d_{yz}$, $J^{(1)}_{xz}$ has contributions mainly through direct hopping while $J^{(2)}_p$ has significant super-exchange contributions similar to the copper-oxide case, as shown in Fig. 3b.

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Fig. 4: The Hilbert space of the five Fe d-orbitals is divided into three subspaces ($S_1$ spanned by $d_{x^2-y^2}$, $S_2$ spanned by $d_{x^2}$, $d_{yz}$, and $S_3$ spanned by $d_{xy}$, $d_{z^2}$ ). These subspaces are coupled through the Hund’s rule coupling $J^H$ which is indicated by the blue arrows.

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On the other hand, the NN spin-spin interaction $J^{(1)}_p$, involving the other three Fe d-orbitals ($yz, xy, z^2$), has contributions from super-exchange processes in which the intervening As orbital is the $sp_z$ (a linear combination of the As 4s and 4p$_z$). In these processes the magnitude of the hybridization of the $yz, xy, z^2$ Fe d-orbitals with the $sp_z$-As orbital does not change with a 90° rotation which necessarily occurs in the red-green path. Therefore, for these three orbitals, $J^{(1)}_p$ is significantly larger than $J^{(2)}_p$, which implies that the condition (2) for checkerboard order is fulfilled.
the subspace $S$ (which prefers the checkerboard AF order) only type quantum-Néel order shown in Fig. 5(b). These two while the other two subspaces prefer the checkerboard- the case of the undoped material where there are six elec- the energy in one subspace and at the same time frustrate the global choice within the same atom will minimize the en- any such atoms, forces the electrons on these different subspaces stronger that the AF coupling between NN and NNN composite. The Hund’s rule coupling, which is significantly the undoped iron-pnictides. This state is strongly favoured E. Manousakis et al. the familiar checkerboard Néel ordering which is preferre d by the Fe atoms along the green lines are frustrated because the Fe atom: 

$$H = \sum_{\nu=1}^{5} \nu - J^{H} \sum_{i,\nu \neq \nu'} S_{i,\nu} \cdot S_{i,\nu'}.$$ (5)

The NN AF couplings $J^{(1)}_{\nu}$ in the subspaces $S_1$ and $S_3$ are greater than the NNN AF couplings $J^{(2)}_{\nu}$; however, in the subspace $S_2$ the NNN AF coupling $J^{(2)}_{xx}$ (and $J^{(2)}_{xy}$) is large, that is, the condition (2) is not satisfied for the arguments presented above. As a result the subspace $S_2$ alone prefers the observed SDW state illustrated in Fig. 4(a), while the other two subspaces prefer the checkerboard-type quantum-Néel order shown in Fig. 4(b). These two competing orders create a magnetic frustration illustrated in Fig. 5. Namely, in the two ordered states illustrated in Fig. 4, the magnetic moments of the electrons on the iron atoms along the alternating green lines in Fig. 5 are opposite. The Hund’s rule coupling, which is significantly stronger that the AF coupling between NN and NNN atoms, forces the electrons on these different subspaces to choose a common spin orientation. However, any such global choice within the same atom will minimize the energy in one subspace and at the same time frustrate the other subspaces as discussed in the caption of Fig. 5. In the case of the undoped material where there are six electrons occupying the five Fe d-orbitals, the $S_1$ subspace has zero spin; hence, the $S_2$ subspace competes with the $S_3$ subspace (which prefers the checkerboard AF order) only and because $J^{(2)}_{zz}$ is larger than the AF couplings in $S_3$, it imposes the observed SDW state. The local magnetic moment is expected to be small due to this frustration. If we neglect the influence of the other subspaces, and we restrict ourselves to $S_2$, the maximum expected order should be less than $2\mu_B$ because of the reduction of the magnitude of the order parameter from its classical value due to zero-point spin fluctuations. If we turn on the interaction with the other subspaces, this has a frustrating effect of the magnetic moment. Therefore, the present analysis gives a natural explanation for the observed small magnitude of the magnetic moment [9], while the calculations based on the itinerant picture [11,12,17-19] produce values for the Fe magnetic moment greater than $2\mu_B$.

From this analysis, we have shown that the different ordering in the iron-pnictides and copper-oxides can be understood within the same theoretical foundation. The observed small Fe magnetic moment arises from the fact that only the electrons occupying the subspace spanned by $d_{xz}$, $d_{yz}$ prefer the observed SDW order (Fig. 4(a)), while the electrons occupying the other three Fe orbitals prefer a checkerboard ordering (Fig. 4(b)), which create large zero-point spin fluctuations. Including these frustrating effects, we expect a significantly reduced local moment in agreement with neutron scattering experiments.

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