Essays on Retirement, Savings and Health

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Yizhak Fadlon
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ABSTRACT

As life expectancy rises and the workforce around the developed world ages, questions about retirement, savings, and health are becoming increasingly important. Are households saving enough for retirement? What is the role of employer contributions to savings accounts in determining overall savings? To what extent are households insured against health shocks and are they financially prepared to face them? How should the answers to these questions guide us in designing optimal social insurance policies?

This dissertation addresses these important questions on the economics of aging. In the three chapters of this dissertation, I apply theory in analyzing the behavior of households, firms, and the social planner, and quasi-experimental research designs that use newly available administrative data on labor market behavior and health outcomes.

The first chapter, jointly written with Torben Heien Nielsen, studies how households respond to severe health shocks and the insurance role of spousal labor supply. In the empirical part of the paper, we provide new evidence on individuals' labor supply responses to spousal health and mortality shocks. Analyzing administrative data on over 500,000 Danish households in which a spouse dies, we find that survivors immediately increase their labor supply and that this effect is entirely driven by those who experience significant income losses due to the shock. Notably, widows – who experience large income losses when their husbands die – increase their labor force participation by more than 11%, while widowers – who are significantly more financially stable – decrease their labor supply. In contrast, studying over 70,000 households in which a spouse experiences a severe health shock but survives – for whom income losses are well-insured in our
setting – we find no economically significant spousal labor supply responses, suggesting adequate insurance coverage for morbidity (vs. mortality) shocks. In the theoretical part of the paper, we develop a method for welfare analysis of social insurance using only spousal labor supply responses. In particular, we show that the labor supply responses of spouses fully identify the welfare gains from insuring households against health and mortality shocks. Our findings imply large welfare gains from transfers to survivors and identify efficient ways for targeting government transfers.

The second chapter of this dissertation is jointly written with Jessica Laird and Torben Heien Nielsen. In this chapter we empirically study how firms, which play an increasingly significant role in retirement savings, set their contributions to employees’ savings accounts, and analyze whether employer contributions reflect employees’ savings preferences. Using a reform that decreased the subsidy for contributions to capital retirement savings accounts for Danish workers in the top income tax bracket, we find strong evidence that firms set contributions to employer-provided savings accounts in accordance with their employees’ savings preferences. Specifically, we find that the reform shifted employers’ contributions from capital accounts to the more subsidized annuity accounts. Furthermore, these responses were proportional to the share of employees directly affected by the reform. We also find that employers with more passive savers among the affected workers had stronger reactions, suggesting that firm responses substitute for individual responses.

In the third chapter, jointly written with David Laibson, we theoretically study savings in the presence of non-optimizing agents and the effect of a benevolent planner on overall retirement savings. As equilibrium behavior is jointly determined by the actions of households and social planners, we highlight the distinction between planner optimization and household optimization. We show that planner optimization is a substitute for household optimization and that this is true even when there are information asymmetries, so that households know more about their
preferences than planners. Our analysis illustrates a potential misattribution in economic analysis. Is optimal behavior caused by optimizing households, or is optimal behavior caused by planners who paternalistically manipulate households that would not optimize on their own? We show that widely studied optimality conditions that are implied by household optimization also arise in an economy with a rational planner who uses default savings and Social Security to influence the choices of non-optimizing households. Therefore, many classical optimization conditions do not resolve the question of household optimization. Pseudo-rationality arises when rational planners elicit (seemingly) optimal behavior from non-optimizing households.
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1 HOUSEHOLD RESPONSES TO SEVERE HEALTH SHOCKS AND THE DESIGN OF SOCIAL INSURANCE

1.1 Introduction

Does the labor supply of household members insure against adverse shocks? The answer to this question is important for our understanding of household behavior and is central to the design of social insurance policies.

This paper studies how households respond to severe health shocks and insure against these shocks through spousal labor supply. In the empirical part of the paper, we provide new evidence on how individuals’ labor supply responds to spousal health and mortality shocks. In the theoretical part of the paper, we develop a method for welfare analysis of social insurance that uses only spousal labor supply responses and can be applied to shocks in which the directly affected individual may be at a corner solution. We show that under plausible conditions the labor supply responses of spouses fully identify the welfare gains of insuring households against health and mortality shocks, and map our empirical findings on spousal labor supply responses to the welfare implications of providing more generous social insurance.

For spousal labor supply to provide self-insurance, households must experience sizable income shocks that are otherwise only partially insured. Therefore, our empirical analysis focuses on an extreme shock that leads to significant and permanent income losses – the death of a spouse. To recover the causal effect of this shock we offer a quasi-experimental design that constructs non-parametric counterfactuals to affected households by using households that experience the same shock a few years in the future, and combines event studies for these two experimental groups. The identification strategy we develop relies on the assumption that the exact timing of the shock is as good as random, and is therefore applicable to the analysis of a wide range of other common economic shocks.

Analyzing administrative data on health and labor market outcomes from the years 1980-2011, we study over 500,000 Danish households of married and cohabiting couples in which a spouse

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1This chapter is jointly written with Torben Heien Nielsen.
has died. We find a large increase in the surviving spouses’ labor supply immediately after their spouses die, which amounts to an average increase of 7.6% in labor force participation and 6.8% in annual labor income by the fourth year after the shock. These effects are driven by households that experience significant income shocks due to the loss of a spouse, and therefore have greater need for self-insurance through labor supply. In particular, we show that the average increase in labor supply is entirely attributable to survivors whose deceased spouses had earned a large share of the household’s income, who have less disposable income at the time of the shock, and who are less formally insured by government transfers. We also find that high-earning survivors, who experience smaller relative income losses and face better financial conditions, decrease their labor supply as their high income is no longer necessary to support two people. Notably, widowers – who tend to be financially stable when losing their wives – decrease their labor supply, while widows – who tend to experience considerably larger income losses when losing their husbands – significantly increase their labor supply. By the fourth year after their husbands die, widows increase their participation by 11.3%, which translates into a 10.1% increase in their annual earnings.

In contrast, we show that for shocks that are well-insured in our setting (through social and private insurance) and require no additional informal insurance, there are no economically significant labor supply responses of the unaffected spouse. Studying over 70,000 households in which a spouse experiences a heart attack or a stroke, we find that the earnings of the affected individuals drop by 19% by the third year after the shock, while the household’s post-transfer income declines by only 3.3%. Consistent with this lack of an income drop, there are no significant changes in the unaffected spouses’ participation with an economically small decline in labor earnings (of about 1%). The combination of our quasi-experimental design and rich administrative data allows us to precisely estimate this small response, which has proven difficult in previous studies (e.g., Coile 2004 and Meyer and Mok 2013).

In the theoretical part of the paper, we map these estimates of spousal labor supply responses to predictions about the welfare gains from providing more generous social insurance. Using a collective model of household behavior that assumes decisions are Pareto efficient (Chiappori 1988, 1992), we show that spousal labor supply responses fully identify the benefits of social insurance and develop a new method for welfare analysis that depends only on the spouse’s labor supply
behavior. This result relies on the observation that within each state of nature the spouse’s labor force participation decision reveals the household’s valuation of additional consumption (in the form of labor earnings). Hence, the sensitivity of spousal labor supply to shocks and economic incentives reveals the household’s preference for consumption across different states of nature, which captures the benefits from insurance. We also consider both theoretically and empirically the welfare implications of potential health-state dependence of the unaffected spouse’s willingness to work.\(^2\)

Applying our welfare method to mortality shocks, we find substantial gains from benefit increases for elderly widows. Under our benchmark calibration, an additional dollar to widows over 67 is equivalent to an additional $1.55 to other elderly households, creating a net benefit of $0.55 per $1. However, for younger widows who are more attached to the labor force, we find very small gains from additional benefits through the social insurance system, with a net benefit of only $0.04 per $1.\(^3\) A key implication of our findings, driven by the differential attachment to the labor force over the life-cycle, is that social insurance policies should be age-dependent.

This paper relates to several strands of the literature. First, numerous empirical studies have analyzed spousal labor supply and its responses to shocks in order to uncover the extent to which it is used as insurance. However, while spousal labor supply is commonly modeled as an important self-insurance mechanism against adverse shocks to the household (e.g., Ashenfelter 1980, Heckman and Macurdy 1980, and Lundberg 1985), this prior empirical work has been unable to find evidence of significant increases in spousal labor supply in response to shocks (e.g., Heckman and Macurdy 1980, 1982, Lundberg 1985, Maloney 1987, 1991, Gruber and Cullen 1996, Spletzer 1997, Coile 2004, and Meyer and Mok 2013). The leading explanation for this lack of evidence has been that within the context of temporary unemployment, on which the empirical literature has focused, income losses are small relative to the household’s lifetime income and are already sufficiently insured

\(^2\)As we mentioned above, we find that the increases in surviving spouses’ labor supply are consistently driven by those who experience large income losses. In addition, we find that among survivors who did not work before their spouses died, those who increased their labor force participation were those whose spouses worked before the shock (and not those who consumed more joint leisure with non-working spouses). As we discuss later in the paper, these results strongly suggest that the average increase in labor supply can be attributed to self-insurance and not to a state-contingent preference for social integration.

\(^3\)Nonetheless, we find that younger widows highly value the system in place. The average dollar given to younger widows is equivalent to a $1.54 transfer to other households. See Section 1.6.
through formal social insurance (Heckman and Macurdy 1980; Cullen and Gruber 2000). In order to uncover the self-insurance role of spousal labor supply within unemployment shocks, Cullen and Gruber (2000) study whether it is crowded-out by unemployment insurance benefits and find a large crowd-out effect. We take an alternative empirical approach and directly study the effects of severe health shocks with different degrees of income loss – mortality shocks, which impose large and permanent income losses, and morbidity shocks, which are well-insured.

Second, prior work on estimating welfare gains from insurance has focused on studying the “consumption-smoothing” effects of insurance to identify its welfare benefits.⁴ This consumption-based method has two limitations. First, it is sensitive to the value of risk aversion, for which the literature has a wide range of estimates, as well as to the degree of consumption utility state dependence, which has proven hard to estimate (see Finkelstein, Luttmer, and Notowidigdo 2009 and Chetty and Finkelstein 2013). Second, the choice of the studied consumption measure – most commonly food consumption – is usually driven by data availability rather than theoretical underpinnings. As emphasized by Aguiar and Hurst (2005), focusing on one aspect of expenditure can lead to very misleading conclusions about actual consumption in the presence of home production.⁵

The labor market approach to welfare analysis that we develop addresses these problems by relying solely on directly-observed participation rates and elasticities. Our approach does not involve fragile estimates of preference parameters. In addition, the wide availability of large-scale accurate data from the labor market and the long tradition of studying labor supply decisions render our approach desirable for empirical applications.

Our method relates to and builds on recent work on labor market methods for welfare analysis

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⁵ Even comprehensive and accurate data on overall expenditure across health states, which is rarely available, would have to be accompanied by time-use data (on home production) and would require strong assumptions on its translation into individual consumption. Among other things, this procedure should take into account consumption flows of durable goods as well as economies of scale in the household’s consumption technology. See, e.g., Browning, Chiappori, and Lewbel (2013).
in the context of unemployment.\(^6\) Chetty (2008) recovers gains from social insurance using liquidity and substitution effects in the search effort of the unemployed, and Shimer and Werning (2007) use comparative statics of reservation wages with respect to government benefits.\(^7\) In the shocks we consider, these methods cannot be applied because the directly affected individual may be unresponsive to economic incentives and hence cannot fully reveal the household’s preferences through labor market behavior. Exploiting the household’s collective labor supply decisions, our method uses only the responses of the indirectly affected spouse, and offers a labor market method that is also applicable to any economic shock in which the directly affected individual may be at a corner solution.

The remainder of this paper is organized as follows. Section 1.2 uses a household model of labor force participation to describe the self-insurance role of spousal labor supply and to develop our method for welfare analysis. Section 1.3 describes the institutional environment and the data sources we use to estimate individuals’ labor supply responses to severe spousal health shocks, and section 1.4 specifies our empirical research design. Section 1.5 presents our main estimates for the unaffected spouses’ labor supply responses to shocks and their self-insurance role. In section 1.6 we study the welfare implications of these responses. Section 1.7 concludes.

1.2 A Collective Model of Household Labor Force Participation

1.2.1 Baseline Model

We begin with a baseline static model of extensive labor supply decisions. In Sections 1.2.3 and 1.2.4 we discuss important extensions to the simple framework.\(^8\)

\(^6\) The advantage of the sufficient statistics approach to welfare analysis, to which these methods as well as our own belong, is that it offers results about optimal policy that do not utilize strong assumptions made in structural studies for tractability and identification. The cost is that it can only be used to analyze marginal changes in policy. See Chetty (2009) for a more detailed discussion on this issue.

\(^7\) Following Chetty (2008), who uses variations in severance payments, other recent papers estimate the magnitude of the liquidity effects of social insurance programs — LaLumia (2013) uses variations in the timing of EITC refunds and Landais (forthcoming) uses lags in the schedule of unemployment insurance benefits.

\(^8\) Our model is most closely related to the collective setting analyzed in Blundell, Chiappori, Magnac, and Meghir (2007), in which one spouse is on the participation margin while the other is on the intensive margin, as well as to Immervoll, Kleven, Kreiner, and Verdelin (2011) who study optimal tax-and-transfer programs for couples with extensive-margin labor supply.
Setup. Households consist of two individuals, $w$ and $h$. We consider a world with two states of nature: a “good” state (state $g$) in which $h$ is in good health and works, and a “bad” state (state $b$) in which $h$ experiences a shock and drops out of the labor force. Households spend a share of $\mu^g$ of their adult life in state $g$ and a share of $\mu^b$ in state $b$ ($\mu^g + \mu^b = 1$). In what follows, the subscript $i \in \{w, h\}$ refers to the spouse and the superscript $s \in \{g, b\}$ refers to the state of nature.

Individual preferences. Let $U_i(c_i^s, l_i^s)$ represent $i$’s utility as a function of consumption, $c_i^s$, and labor force participation, $l_i^s$, in state $s$ (such that $l_i^s = 1$ if $i$ works and $l_i^s = 0$ otherwise). We assume that $\frac{\partial}{\partial c_i^s} U_i(c_i^s, l_i^s) = u_i(c_i^s) - v_i \times l_i^s$, where the utility from consumption, $u_i(c_i^s)$, satisfies $u_i'(c_i^s) > 0$ and $u_i''(c_i^s) < 0$, and $v_i$ is $i$’s disutility from labor. The couple’s disutilities from labor $(v_w, v_h)$ are drawn from a continuous distribution defined over $[0, \infty) \times [0, \infty)$. We denote the marginal probability density function of $v_w$ by $f(v_w)$ and its cumulative distribution function by $F(v_w)$.

Household preferences. We follow the collective approach to household behavior (Chiappori 1988, 1992; Apps and Rees 1988) and assume that household decisions are Pareto efficient.\(^9\) Therefore, with equal Pareto weights for both spouses, household decisions can be characterized as solutions to the maximization of $U_w(c_w^s, l_w^s) + U_h(c_h^s, l_h^s)$.\(^10\)

Policy tools. The planner observes the state of nature as well as the employment status of each spouse. Since some spouses work and earn more than others do, the optimal policy is dependent on whether the spouse is employed. We denote the tax on spouse $i$’s labor income in state $g$ by $T_i^g$ and the benefits given to non-working spouses in state $g$ by $b^g$. In state $b$, households in which the unaffected spouse, $w$, works receive transfers of the amount $B^b$ and households in which $w$ does not work receive benefits of the amount $b^b$. This tax-and-benefit structure allows for the analysis of flexible policy designs and mimics features of existing social insurance programs in most developed

\(^9\)We discuss this assumption in Section 1.2.2

\(^10\)More generally, household decisions can be characterized as solutions to the maximization of $\beta_wU_w(c_w^s, l_w^s) + \beta_bU_h(c_h^s, l_h^s)$, where $\beta_w$ and $\beta_b$ are the Pareto weights on $w$ and $h$, respectively. However, setting $\beta_w = \beta_b = 1$ is without loss of generality as long as the spouses’ relative bargaining power is stable across states of nature. Similar to Chiappori (1992), baseline weights do not affect our welfare results.
countries (e.g., income-testing which is common to programs in the US and in Denmark).\footnote{For example, Supplemental Security Income (SSI) within the Old-Age, Survivors and Disability Insurance in the US and the Social Disability Insurance in Denmark.} We denote taxes by $T \equiv (T^g_w, T^h_i)$ and benefits by $B \equiv (b^g, B^h, b^b)$, and let $B(l^s_w)$ represent the actual transfers received by a household as a function of $w$’s participation.\footnote{It is worth mentioning that the exact way in which we model transfers is not necessary for our results, and any system that conditions transfers on the state of nature and employment can be analyzed in our framework.}

**Household’s problem.** The household’s choices reduce to the allocation of consumption to each spouse $i$ in state $s$, $c^s_i$, as well as $w$’s labor force participation in each state, $l^s_w$. Note that there are no savings decisions involved in the baseline static model. We introduce endogenous savings in the dynamic extension to the model in Section 1.2.4. Each choice of $w$’s employment determines the household’s overall income in state $s$, $y^s(l^s_w)$, such that $y^s(l^s_w) = A + \bar{z}^s_h \times \bar{l}^s_h + \bar{z}^s_w \times l^s_w + B(l^s_w)$, where $A$ is the household’s wealth, $z_i$ is $i$’s labor income, and $\bar{z}^s_i = z_i - T^i_w$ is $i$’s labor income net of taxes (with $T^i_w = 0$).\footnote{More generally, the model allows for any type of state-contingent income and assets. These include life insurance and any other source of private insurance, employer-provided insurance, transfers from relatives, social insurance, medical expenses, etc.} At each of $w$’s potential employment statuses, consumption is efficiently allocated across spouses, such that the consumption bundles $c^s_w(l^s_w)$ and $c^s_h(l^s_h)$ are the solutions to

\[
V(y^s(l^s_w)) = \max_{c^s_w, c^s_h} u_w(c^s_w) + u_h(c^s_h) \\
\text{s.t. } c^s_w + c^s_h = y^s(l^s_w). \tag{1.1}
\]

For later reference, we define $y^s_{-w}$ as the household’s resources excluding those directly attributed to $w$’s labor supply decision i.e., $y^s_{-w} \equiv A + \bar{z}^s_h \times \bar{l}^s_h$.

The unaffected spouse, $w$, works in state $s$ if and only if $v_w < \bar{v}^s_w \equiv V(y^s(1)) - V(y^s(0))$.\footnote{The complete formal description of the household’s problem in each state is} That is, the unaffected spouse works if the household’s valuation of the additional consumption of his or her labor income compensates for his or her utility loss from working. Therefore, this simple

\[
\max_{l^s_w \in (0,1), c^s_w(l^s_w), c^s_h(l^s_h)} l^s_w(U_w(c^s_w(1), 1) + U_h(c^s_h(1), l^s_h)) + (1 - l^s_w)(U_w(c^s_w(0), 0) + U_h(c^s_h(0), l^s_h)) \\
\text{s.t. } c^s_w(l^s_w) + c^s_h(l^s_h) = y^s(l^s_w) \\
y^s(l^s_w) \equiv A + \bar{z}^s_h \times \bar{l}^s_h + \bar{z}^s_w \times l^s_w + B(l^s_w).
\]
decision rule reveals the household’s preferences for additional consumption and is the key source for identifying the gains from insurance based on the unaffected spouse’s labor supply (as we show below). We denote w’s participation rate in state s by \( e^s_w = F(\tilde{v}_w^s) \).\(^{15}\)

At this point it is easy to see the self-insurance role of spousal labor supply responses to shocks, which is our main outcome of interest. Denote the income loss from the shock by \( d \equiv y^g_w - y^b_w \).

Then, in each state the participation rate of the unaffected spouses decreases in their unearned income: 
\[
\frac{\partial e^s_w}{\partial y^g_{-w}} = -f(\tilde{v}_w^s)[u'_w(e^s_w(0)) - u'_w(e^s_w(1))] < 0.
\]

This implies that \( e^b_w > e^g_w \) whenever \( d > 0 \) and there is no full insurance — that is, income shocks lead to self-insurance through the unaffected spouse’s labor force participation. Furthermore, the unaffected spouses’ labor supply response to the shock increases in the income loss \( d \) — i.e.,
\[
\frac{\partial(e^b_w/e^g_w)}{\partial d} = \frac{f(\tilde{v}_w^b)}{F(\tilde{v}_w^g)}[u'_w(e^b_w(0)) - u'_w(e^b_w(1))] > 0.
\]

These comparative statics are no more than simple income effects at the household level and are a direct implication of the concavity of \( u_i(e^s_i) \), which translates into the concavity of \( V(y^s(l^s_w)) \).

**Planner’s problem.** Let \( W^s(v_w) \) denote the household’s value function in state \( s \) such that
\[
W^s(v_w) = \begin{cases} 
V(y^s(1)) - v_h \times l^s_h - v_w & \text{if } v_w < \tilde{v}_w^s \\
V(y^s(0)) - v_h \times l^s_h & \text{if } v_w \geq \tilde{v}_w^s.
\end{cases}
\]

Therefore, the household’s expected utility is \( J(B, T) = \mu^g \int_0^\infty W^g(v_w)f(v_w)dv_w + \mu^b \int_0^\infty W^b(v_w)f(v_w)dv_w. \)

The social planner’s objective is to choose the tax-and-benefit system that maximizes the household’s expected utility subject to the requirement that expected benefits paid, \( \mu^g(1 - e^g_w)b^g + \mu^b(e^b_wB^b + (1 - e^b_w)b^b) \), equal expected taxes collected, \( \mu^g(T^g_h + e^g_wT^g_w) \). Hence, the planner chooses the benefit levels \( B \) and taxes \( T \) that solve
\[
\max_{B,T} J(B, T) \quad \text{s.t.} \quad \mu^g(1 - e^g_w)b^g + \mu^b(e^b_wB^b + (1 - e^b_w)b^b) = \mu^g(T^g_h + e^g_wT^g_w). \tag{1.2}
\]

\(^{15}\)There is another natural approach to modeling the household’s decision-making process. One can assert that each individual works if his or her own utility from working is higher than his or her own utility from not working, and then — conditional on the participation decisions — the couple engages in efficient bargaining that allocates resources according to their respective bargaining power (which in our case implies maximizing \( u_w(c^w_w) + u_h(c^w_h) \)). The qualitative theoretical results of our analysis remain unchanged in this alternative model.
1.2.2 Optimal Social Insurance

To solve the planner’s problem we characterize the first-order conditions of (1.2) by perturbing the tax-and-benefit system. For a given level of government revenues, we consider the optimal distribution of benefits to households with non-working spouses across states \(b\) and \(g\). To do so, we consider a small increase in \(b^h\) financed by a corresponding balanced-budget decrease in \(b^g\). In the simple model, this captures the efficient distribution of transfers to low-income households across different health states. Any other perturbation of the system will follow the steps of the analysis conducted below, and the complete optimal system can thus be characterized in the same manner. We focus on this particular aspect of the policy since it captures the essence of insuring households against shocks in a simple and policy-relevant way.

The welfare gain from a $1 (balanced-budget) increase in \(b^h\) is \(\frac{dJ(T,B)}{db^h} = \mu^h \frac{\partial}{\partial b^h} \left( \int_0^\infty W^h(v_w)f(v_w)dv_w \right) + \mu^g \frac{\partial}{\partial b^g} \left( \int_0^\infty W^g(v_w)f(v_w)dv_w \right) \frac{db^g}{db^h}. \) Since this is expressed in utility units with no cardinal interpretation, we follow the recent social insurance literature and normalize it by the welfare gain from a $1 transfer to households with non-working spouses in the good state, scaled by the targeted population. Differentiating the budget constraint to calculate \(\frac{db^g}{db^h}\) and using the household’s choices, which imply that \(\frac{\partial}{\partial b^h} \left( \int_0^\infty W^h(v_w)f(v_w)dv_w \right) = u'_w(c^h_w(0))(1 - e^h_w)\) and \(\frac{\partial}{\partial b^g} \left( \int_0^\infty W^g(v_w)f(v_w)dv_w \right) = u'_w(c^g_w(0))(1 - e^g_w)\), yield the normalized welfare gain

\[
M_W(b^h) = MB(b^h) - MC(b^h),
\]

where the marginal benefit is \(MB(b^h) = \frac{u'_w(c^h_w(0)) - u'_w(c^g_w(0))}{u'_w(c^g_w(0))}\), the marginal cost is \(MC(b^h) = \frac{\varepsilon (1 - e^h_w b^g) - \varepsilon (1 - e^g_w b^g)}{1 + \varepsilon(1 - e^h_w b^g)}\) and \(\varepsilon(1 - e^h_w, b^g) = \frac{\partial(1 - e_w(b))}{\partial b} (1 - e_w(b))\) is the elasticity of the unaffected spouse’s non-participation with respect to government benefits. Note that when the consumption of \(h\) is positive (e.g., when he or she survives the shock), \(MB(b^h)\) is also the gap in his or her marginal utilities due to consumption allocation efficiency in the household, which is determined by the program in (1.1).

Equation (1.3) is a simple variant of Baily’s (1978) and Chetty’s (2006) formula for the optimal

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16 See the recent review by Chetty and Finkelstein (2013).

17 That is, the normalized net gain is \(M_W(b^h) = \frac{dJ(T,B)}{\mu^h(1 - e^h_w)} / \mu^g(1 - e^g_w)\).
level of social insurance. The marginal benefit from a balanced-budget increase in $b^h$ is captured by the insurance value of transferring resources from the good to the bad state, which is measured by the gap in marginal utilities of consumption across the two states. The marginal cost of transferring $\$1$ across states is due to behavioral responses, which capture the fiscal externality that households impose on the government budget when changing their participation decisions. In our case, the government’s revenue could decrease since there are more spouses not working in the bad state due to higher benefits but could increase since there are fewer spouses not working in the good state as they receive fewer transfers.

**Identifying the benefits of social insurance.** While estimating the marginal cost is conceptually straightforward, estimating the marginal benefit is challenging since it requires knowledge of the consumption utility function, particularly of the value of risk aversion, and of each individual’s overall consumption. To circumvent the challenges posed by this consumption-based approach, which we discuss below, we use simple but powerful implications of the household’s labor supply decisions, which allow us to rewrite the marginal benefit solely in terms of the unaffected spouse’s labor supply. The following proposition summarizes this main welfare result and demonstrates the way in which the unaffected spouse’s labor supply behavior fully reveals the gap in the marginal utilities of consumption across states of nature. We provide a simple proof and then discuss the intuition behind the formula; namely, that it identifies the gains of insurance by evaluating changes in the consumption of leisure.

**Proposition 1.** Under a locally linear approximation of $F$, the marginal benefit from raising $b^h$ by $\$1$ is

$$MB(b^h) \equiv L^b + M^b,$$

where $L^b \equiv \frac{e^b_{w^*} - e^{b^h}_{w^*}}{e^{b^h}_{w^*}}$ and $M^b \equiv \left( |\varepsilon(e^h_{w^*,0})|/b^b - 1 \right) \frac{e^b_{w^*}}{e^{b^h}_{w^*}}$, \hspace{1cm} (1.4)

**Proof.** Recall that the unaffected spouse works when the value of additional consumption from his or her labor income, $b^h - V(y^s(1)) - V(y^s(0))$, outweighs his or her disutility from labor, $v_{w^*}$. This decision rule reveals the household’s consumption value of an additional dollar, $V'(y^s(0))$, through the change in the critical labor-disutility threshold below which
the spouse works \((\bar{v}_w^s)\) in response to an increase in benefits, since \(\left| \frac{\partial v^s_w}{\partial b^s} \right| = V'(y^s(0))\). In addition, since (1.1) implies that \(V'(y^s(0)) = u'_w(e^s_w(0))\), we can rewrite the marginal benefit from social insurance using the change in the marginal entrant’s disutility of labor – that is, 
\[ MB(b^s) = \frac{\partial v^s_w}{\partial b^s} \frac{\partial v^s_w}{\partial b^s} \]. The last step to represent \(MB(b^s)\) by using labor supply responses of the unaffected spouse is to map this expression onto directly observable participation rates, 
\[ e^s_w = F(\bar{v}_w^s) \], and their elasticities, \(\varepsilon(e^s_w, b^s)/b^s = \frac{f(\bar{v}_w^s)}{F(\bar{v}_w^s)} \frac{\partial v^s_w}{\partial b^s}\), with simple algebra. Together, the equalities \(MB(b^s) = \left( \left| \frac{\partial v^s_w}{\partial b^s} \right| - \frac{\partial v^s_w}{\partial b^s} \right) / \frac{\partial v^s_w}{\partial b^s} \), \(e^s_w = F(\bar{v}_w^s)\), \(\varepsilon(e^s_w, b^s)/b^s = \frac{f(\bar{v}_w^s)}{F(\bar{v}_w^s)} \frac{\partial v^s_w}{\partial b^s}\) and the approximation in the proposition yield the result.\(^\text{18}\)

This formula shows that the marginal benefit from social insurance can be fully recovered from two moments of the unaffected spouse’s labor supply, which we examine successively. The first term, \(L^b\), is composed of the unaffected spouse’s labor supply response to the shock – or the labor force participation “shock elasticity” – which captures exactly the self-insurance role of the spouse’s labor supply. Recall that the increase in labor force participation across states of nature increases with the income loss due to the shock and therefore reveals the extent to which the household needs to self-insure against this loss.

The second term, \(M^b\), captures the gains from the consumption of leisure by the marginal spouses due to behavioral responses to the policy change. When we increase benefits to non-working spouses in the bad state, \(b^b\), we let more spouses meet their consumption needs if they choose not to work and consume more leisure – which is a welfare gain from the individual’s and hence from the planner’s perspective. The relative share of spouses who are on the labor force participation margin is captured by the semi-elasticity \(\left| \varepsilon(e^b_w, b^b) \right| /b^b\), which quantifies the percent change in labor force participation in state \(b\) when we increase non-participation transfers \(b^b\) by $1. This is illustrated in Figure 1.1: Panel A depicts the pre-perturbation labor force participation in state \(b\), and Panel

\(^{18}\)Specifically, the proposition uses a locally linear approximation of \(F(v_w)\) in the threshold region, \((v_w^a, v_w^b)\). This local first-order expansion of \(F(v_w)\) is supported by the empirical analysis of the spouse’s participation across states of nature, which implies that \(v_w^a\) and \(v_w^b\) are within a small region of the support \([0, \infty)\). If one wishes to avoid this approximation, one can accompany the analysis with assumptions regarding the family of distributions to which \(F\) belongs, and then calibrate its parameters with the participation rates observed in the data. Note that this approximation is isomorphic to a second-order approximation of the search effort function in a search model of participation that we analyze in Appendix A.
FIGURE 1.1
The Unaffected Spouses’ Labor Force Participation Responses to Policy Changes

(a) Spousal Labor Force Participation in the Bad State

\[ f(v_w^b) \]

\[ e_w^b = F(\bar{v}_w^b) \]

(b) The Change in Spousal Labor Force Participation in the Bad State in Response to the Policy Change

\[ \left| \frac{\partial V_w^b}{\partial v_w^b} \right| / b^b = \left( f(\bar{v}_w^b) \times \left| \frac{\partial V_w^b}{\partial b^b} \right| \right) / F(\bar{v}_w^b) \]

Notes: These figures plot a potential probability density function (pdf) for the labor disutility of the unaffected spouse (spouse w) in state b, \( v_w^b \). The x-axis corresponds to \( v_w^b \) and the y-axis corresponds to the pdf, \( f(v_w^b) \). In this figure, \( \bar{v}_w^b \) is the threshold value below which spouse w chooses to work in state b. Therefore, the area between 0 and \( \bar{v}_w^b \) below the pdf is the aggregate labor supply of spouses in state b, \( e_w^b = F(\bar{v}_w^b) \). This is the shaded area in panel A. When government transfers locally change, the threshold changes by \( \frac{\partial e_w^b}{\partial b^b} \) and the approximated change in w’s labor supply is the shaded area in Panel B, \( f(\bar{v}_w^b) \times \left| \frac{\partial V_w^b}{\partial b^b} \right| \). Hence, the relative within-state change in labor force participation can be approximated by \( \left( f(\bar{v}_w^b) \times \left| \frac{\partial V_w^b}{\partial b^b} \right| \right) / F(\bar{v}_w^b) \), which is exactly the semi-elasticity of participation, \( e_w^b \), with respect to benefits, \( b^b \). That is, \( \epsilon(e_w^b, b^b) / b^b = \left( f(\bar{v}_w^b) \times \left| \frac{\partial V_w^b}{\partial b^b} \right| \right) / F(\bar{v}_w^b) \).
B depicts the response of the spouses that are on the participation margin in state $b$. Since we finance the increase in $b^b$ by a decrease in $b^g$, the marginal spouses in state $g$ who now work as a response – and whose relative share is $|\varepsilon(e^g_{b^g}, b^g)|/b^g$ – represent a welfare loss due to their reduced consumption of leisure. Therefore, the net gain through the change in the consumption of leisure due to the policy change is captured by $\frac{|\varepsilon(e^g_{b^g}, b^g)|}{\varepsilon(e^g_{b^g}, b^g)} - 1$. To scale these within-state elasticities into cross-state terms (which are relevant for our cross-state perturbation), we multiply this gain by the relative labor supply across states, $\frac{e^b_g}{e^g_b}$. This results in the second term of the formula: $M^b \equiv \left( \frac{|\varepsilon(e^g_{b^g}, b^g)|}{\varepsilon(e^g_{b^g}, b^g)} - 1 \right) \frac{e^b_g}{e^g_b}$. Note that whenever we transfer resources from the good to the bad state, the formula adjusts through the semi-elasticity ratio that enters this term; it is always the ratio of the responses to the specific policy tools that we consider changing.

**Discussion.** The alternative method for recovering welfare gains from social insurance is consumption-based and aims at directly identifying the gap in marginal utilities of consumption across states of nature. The reduced-form literature uses the approach developed by Baily (1978) and Chetty (2006) and was first implemented by Gruber (1997) in the context of unemployment insurance. This approach is based on analyzing consumption fluctuations across states, which are transformed to utility losses with estimates for the curvature of the utility function. The structural literature follows a similar approach but with the additional complexity of estimating the full set of the economic model’s primitives. Our approach maps the identification problem from the consumption domain to the labor supply domain. By doing so, it does not rely on assumptions regarding the appropriate value of risk aversion about which there is tremendous uncertainty in the literature and to which the consumption-based calculations of gains from insurance are highly sensitive (Chetty and Finkelstein 2013). Additionally, it requires only data from the labor market.

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19 Note that within a state, marginal spouses are indifferent between working and not working. In the absence of full insurance, this is not the case across states, which is the relevant comparison for our policy change and is represented by the semi-elasticity ratio.

20 Recall that we study the welfare implications of transferring resources from state $g$ to state $b$. This transfer induces behavioral responses within each state of nature, which are expressed here in terms of semi-elasticities, since it changes the economic incentives within each state. However, to evaluate the implications of these elasticities in “cross-state” rather than “within-state” terms, we need to scale the elasticity ratio by the relative labor supply flow across states, $\frac{e^b_g}{e^g_b}$. Note that the first term of the welfare formula is already in cross-state terms so that no scaling is required.

21 See examples for these papers in Footnote 4.
which is typically more precise and widely available than is consumption data. While consumption measures are usually partial (and cover only a sub-set of goods, such as expenditure on food), and strong assumptions are needed to translate overall expenditure into individuals’ consumption bundles, labor market data exactly matches the theoretical behaviors of interest, namely, participation and earned income.

Two other labor-market methods have been developed in the context of unemployment in the modern literature on social insurance. These are based on the labor supply responses of the directly affected individual. Chetty (2008) recovers gains from social insurance using liquidity and substitution effects in the search effort of the unemployed, while Shimer and Werning (2007) rely on comparative statics of reservation wages with respect to government benefits. However, these methods are not applicable to the case of a severe health shock in which the sick individual’s labor supply can no longer identify preferences. This is because a non-negligible share of those experiencing severe health shocks (and whose ability to work is directly affected) may be forced out of the labor market and become unresponsive to economic incentives. Their implied small behavioral responses to changes in policy tools and other economic incentives may wrongly imply a low value of additional insurance, while they are actually driven by significant shocks to their ability to work. Our approach falls within this group of labor market approaches but extends the scope of identifying welfare gains from social insurance using labor supply responses. In particular, it can be applied to important cases in which the directly affected individual may be at a corner solution, such as a severe health shock or the extreme case of death.\(^{22}\)

The analysis above has also shown that, in contrast to conventional wisdom, the level of optimal benefits does not necessarily decrease in the crowd-out of self-insurance by social insurance. It is indeed the case that increased benefits to non-working spouses in the bad state impose a fiscal externality on the government’s budget through an increase in this group’s non-participation rate, which is captured by the non-participation elasticity \(\varepsilon(1 - e^b_w, b^p)\) in \(MC(b^p)\). However, at the same time, the decreased participation entails a gain from consumption of additional leisure, which is captured by the participation elasticity \(\varepsilon(e^b_w, b^p)\) in \(MB(b^p)\). Therefore, our analysis formalizes

\(^{22}\)We discuss additional examples of such shocks in the Conclusion.
Gruber’s (1996) argument that in any assessment of net welfare gains from social insurance both effects have to be taken into account and weighted appropriately.

**Identifying assumption: efficiency.** Before we proceed with extensions to the basic model, it is worth emphasizing the source of identification of the household’s preferences by using the unaffected spouse’s labor supply responses. The key assumption underlying our analysis is that household decisions are Pareto efficient. This implies that on the margin, all members of the household exhibit the same returns to additional resources; hence any member not at a corner solution can reveal the preferences of each member of the household.

This approach relies on the premise that when spouses have symmetric information about each other’s preferences and consumption (because they interact on a regular basis) we would expect them to find ways to exploit any possibilities of Pareto improvements. Importantly, as emphasized by Browning, Chiappori, and Weiss (2014), this does not preclude the possibility of power issues such that the allocation of resources within the household can depend on its members’ respective Pareto weights. The approach simply assumes that no resources are left on the table. An additional advantage of the collective model is that it does not require specifying the mechanism that households use, e.g., the bargaining process, but only assumes such a mechanism exists. Note that the unitary model is a special case of our collective framework, and therefore our results readily apply to the unitary assumption that is widely used in models of the household.23

### 1.2.3 State-Dependent Preferences

There are several important ways in which preferences can be directly affected by the shocks that we analyze. In this section, we consider different potential types of state dependence in the household’s preferences and illustrate how they affect the analysis. Since our welfare method identifies gains from the labor supply behavior of the unaffected spouse, the sort of state dependence that affects the theoretical analysis is confined to potential changes in the unaffected spouse’s labor disutility as we show below. We assess its empirical implications in Section 1.5.2.

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23There are some cases in which the efficiency assumption fails (see discussion in Browning, Chiappori, and Weiss 2014). To model these cases, one would need to specify the underlying model of household decision making and make additional assumptions in order to identify one spouse’s preferences from the other spouse’s behavior.
Let $U^s_i(c^s_i, l^s_i)$ represent $i$’s utility in state $s$ as a function of consumption, $c^s_i$, and labor force participation, $l^s_i$, in state $s$ and assume that $U^s_i(c^s_i, l^s_i) = u^s_i(c^s_i, l^s_i) - v^s_i \times l^s_i$. This formulation generalizes preferences in the following important ways. First, it allows for a completely flexible dependence of consumption utility on the state of nature. Note in particular that this allows us to study the death of $h$ within our framework since it corresponds to setting $u^h_i(c^h_i, l^h_i) = 0$. Second, it allows for flexible consumption-leisure complementarities by allowing the consumption utility to depend freely on participation. These two extensions to the baseline model have no effect on the welfare formulas since we mapped the welfare evaluation problem from the consumption domain completely onto the labor force participation domain. This simplifies the analysis tremendously since the estimation of consumption utility state dependence has proven very challenging (Finkelstein, Luttmer, and Notowidigdo 2009) and also allows us to avoid the common practice of assuming consumption-leisure independence.\footnote{For later reference in Section 1.5.2, we denote the household’s “consumption value function” for this extension by $V^s(y^s(w^s)) \equiv \max u^s_w(c^s_w) + u^s_i(c^s_i) \text{ s.t. } c^s_w + c^s_i = y^s(w^s).$}

Furthermore, we allow labor disutility, $v^b_i$, to change across states of nature. Since we identify welfare gains from the behavior of the unaffected spouse, allowing the labor disutility of the affected spouse to change completely across states of nature does not affect the analysis. It is indeed the underlying motive for studying the unaffected spouse’s behavior in the first place since the affected spouse’s preferences can change in many unidentifiable ways as a result of the shock.

An extension that affects the welfare analysis and that we consider here is the potential state dependence of the unaffected spouse’s labor disutility. For example, when the bad state is $h$’s sickness, $v^b_w$ might be greater than the baseline labor disutility $v^b_w$ if $w$ places greater value on time spent at home — e.g., to take care of his or her sick spouse. When the bad state is $h$’s death, working may become less desirable if the surviving spouse experiences depression and has difficulties working, or conversely, working may become more desirable if the surviving spouse wishes to seek social integration. For simplicity, we model this type of state dependence as $v^b_w = v_w$ and $v^b_w = \theta^b \times v^b_w$, such that $\theta^b$ captures the mean percent change in the utility cost of labor compared to the baseline state $g$.\footnote{In Appendix A we show that this is a simplification and that it is not necessary to define such a global parameter. We}

The adjustment of the welfare formula to this extension is presented in
the following proposition.

**Proposition 2.** Under a locally linear approximation of $F$, the marginal benefit from raising $b^b$ by $\theta$ is

$$MB(b^b) \equiv L^b + M^b + S^b,$$

where $L^b \equiv \frac{e^b_w - e^g_w}{e^w_w}$, $M^b \equiv \left( \frac{\varepsilon(e^b_w, b^b)}{\varepsilon(e^b_w, b^b)} / |\theta| - 1 \right) \frac{e^b_w}{e^w_w}$, and $S^b \equiv (\theta^b - 1) \left(1 + L^b + M^b\right)$.

**Proof.** With these preferences, it is straightforward to show that $MB(b^b) = \left(\theta^b \left| \frac{\partial v_w^b}{\partial b^b} \right| - \left| \frac{\partial \theta^b}{\partial b^b} \right| \right)$. Combining this equality with $e^s_w = F(\bar{v}_w^s)$, $\varepsilon(e^s_w, b^s) / b^s = f(\bar{v}_w^s) \frac{\partial \bar{v}_w^s}{\partial b^s}$, and the approximation in the proposition yields the result.\(^{26}\)

The additional component, $(\theta^b - 1) \left(1 + L^b + M^b\right)$, essentially “prices” in utility terms the cost of the first two labor supply “quantity” expressions, $L^b$ and $M^b$. The unaffected spouse’s labor supply is more costly by $\theta^b - 1$ percent. This additional cost needs to be applied to the overall relative labor supply response across health states, i.e., the sum of the baseline participation rate (normalized to 1) and the two quantity components: $1 + L^b + M^b$. Since our welfare method identifies the gains from insurance by evaluating the change in the consumption of leisure, higher valuation of leisure, that is, a higher $\theta^b$, renders leisure more valuable in state $b$, which makes the transfer of resources from state $g$ to state $b$ more socially desirable. We offer a way to assess $\theta^b$ in our empirical analysis below (see Section 1.5.2).

### 1.2.4 Additional Generalizations and Extensions

**Dynamic life-cycle model.** In the context of social insurance over the life-cycle, it is important to consider households’ self-insurance through ex-ante mechanisms such as precautionary savings. In Appendix A, we analyze life-cycle participation decisions using a dynamic search model that illustrate how it can be locally and non-parametrically defined in the more general dynamic search model. In addition, in Appendix C we offer an example for allowing heterogeneity in $\theta^b$.

\(^{26}\) Specifically, the proposition uses a locally linear approximation of $F(v_w)$ in the threshold region, $(\bar{v}_w^g, \bar{v}_w^b)$.
allows for endogenous savings. The general result of this analysis is that our formulas extend to the dynamic case with the adjustment that post-shock responses in the static case are replaced by mean responses at the onset of a shock.\textsuperscript{27} This is exactly what we recover in our empirical analysis. Hence, our results as well as the welfare analysis we conduct readily apply to the dynamic case. The intuition behind this theoretical result is that responses of forward-looking households to shocks internalize the full expected path of future consumption and leisure. Therefore, responses in periods right after a shock occurs reveal the household’s life-time welfare implications of additional transfers.\textsuperscript{28}

The dynamics of the life-cycle analysis likewise enter the marginal costs of social insurance. A household in state $g$ not only decreases its labor supply due to higher taxes in the present, but also in response to increased benefits in the hitherto unencountered state $b$. The prospect of higher benefits in the case that the household experiences a shock lowers its need to save for that scenario, which translates into a decrease in labor supply in state $g$.

**Intensive-margin model of labor supply.** One can construct similar formulas for the case in which the household’s intensive labor supply decisions are considered. Since there are no individuals on the participation margin, the formulas consist only of the labor supply changes across states of nature and the potential change in the utility cost of labor. See Appendix B for an analysis of this model. Note that the choice of the appropriate model for welfare analysis should depend on the data. For example, studying a sub-population with full employment before a shock occurs calls for the intensive-margin model because in such a case work intensity is the operative margin.\textsuperscript{29}

\textsuperscript{27} The robustness of our approach to the inclusion of additional margins of response is a general feature of the sufficient statistic approach to welfare analysis (see Chetty 2006).

\textsuperscript{28} The setting we analyze in the appendix also extends the model by allowing for multiple and sequential shocks. In particular, we analyze a model in which $h$ can experience a health shock and may die as a consequence. This illustrates our analysis in a more complex and realistic setting that can be applied to different types of sequential shocks.

\textsuperscript{29} Studying the discrete participation decision rather than the intensive-margin decision has several important advantages. First, it allows for flexible consumption-leisure complementarities. Second, it captures additional moral hazard responses that the social insurance literature discusses. By modeling means-tested transfers that can condition on household-level income we can study the welfare effects of the potential crowd-out of spousal labor force participation. Third, labor market frictions (such as hour requirements set by employers) can limit employees’ ability to optimize; hence participation decisions may reveal preferences more accurately (since the potential costs of non-optimization are higher).
1.3 Data and Institutional Background

To study labor supply responses to severe spousal health shocks we turn to the Danish institutional setting and its rich administrative data on health and labor market outcomes. In this section, we describe the Danish insurance environment as it relates to sick individuals and surviving spouses as well as our data sources. It is useful to distinguish between two types of insurance: health insurance (coverage of medical care) and income insurance (insurance against income losses in different health states). Health insurance in Denmark is a universal scheme in which all costs are fully covered by the government.\textsuperscript{30} Therefore, the Danish setting allows us to concentrate on (social and private) income insurance for losses that go beyond immediate medical expenses, as we describe below.\textsuperscript{31}

\textbf{Institutional background.} In Denmark, income insurance against severe health shocks and the death of a spouse consists of four main components that are typical of systems in developed countries: temporary sick-pay benefits, permanent Social Disability Insurance, privately purchased insurance policies, and other indirect social insurance programs.

During the first four weeks after a health shock occurs, workplaces are obliged to provide the sick employee with sick-pay benefits, which fully replace wages as long as the employee is ill within this period. Some common agreements and work contracts insure wage earnings against sicknesses of longer duration. For example, some blue-collar common agreements in the private sector provide wages during periods of sickness for up to one year. If the sick worker’s contract does not provide such a scheme, then the local government must provide flat-rate sick-pay benefits from the fifth up to the fifty-second week after the worker has stopped working.\textsuperscript{32}

If the worker remains sick and is unable to work, he or she can apply at the municipality level for Social Disability Insurance (Social DI) benefits that will provide income permanently.

\textsuperscript{30}There are a few exceptions such as dental care, chiropractic treatments and prescription drugs which entail out-of-pocket expenses.

\textsuperscript{31}Note, however, that the theory allows for medical expenses (and any other state-contingent expenses) and that our method is robust to any degree of medical coverage.

\textsuperscript{32}During this period the sick worker receives a fixed daily rate that in 2000 added up to DKK 11,400 ($1,425) per month (exactly the same as the unemployment benefit rate).
For example, in 2000, subject to income-testing against overall household income, a successful application amounted to DKK 110,400 ($13,800) per year for married or cohabiting individuals and DKK 144,500 ($18,000) for single individuals.

The Danish Social DI program has a broad social insurance scope since it can be awarded for “social reasons”. In 1984 the notion of “social reasons” came to replace a complex mix of programs, such as survivors benefits for women and special old-age pensions for single women. The motive behind this rule change was that the pre-1984 rules discriminated between genders, which did not comply with EU legislation. Social DI is therefore the relevant insurance mechanism for surviving spouses who are unable to maintain their standard of living after losing their partners. Indeed, we find sharp increases in the Social DI benefits received by survivors immediately after their spouses die.

While Social DI is a state-wide program, it is locally administered. Regional councils (in a total of 15 regions) decide whether to approve or reject an individual’s application, and municipal caseworkers (in a total of 270 municipalities) administer the application and handle all aspects of each case – including any contact with the applicant, preparation of the application, collection of physician records, communication with previous employers, etc. The local administration of the program has led to differential application behavior across municipalities, which has resulted in substantial variation in rejection rates across municipalities – ranging from 7% to 30% – and thus in the mean receipts of Social DI benefits across the different municipalities (Bengtsson 2002). We exploit this cross-municipality variation in DI awards over time later in the paper.

An additional source of income to a household that experiences health shocks or in which a member dies is payments from an employer-based insurance policy, an element that is standard in labor-market pension plans. Since 1993, most sectors covered by common agreements (75% of the labor force) have mandatory pension savings, part of which consists of life insurance and insurance against specific health shocks. These pay out a lump-sum to the sick worker, as long as he or she is making contributions to the pension plan, or to the surviving spouse in case the plan member dies. The rates of these payouts are set by the individual pension funds. In addition, individuals can purchase private insurance policies of a similar structure.

Lastly, there are social insurance programs that can indirectly protect survivors or households
that experience other shocks. When crossing into their 60s and until they reach their old-age pension retirement age, individuals who have (voluntarily) been members of an unemployment fund for a sufficiently long period (10 years before 1992 and gradually increasing to 20 years thereafter) are eligible for the Voluntary Early Retirement Pension (VERP). Approximately 80% of the population is eligible for VERP, which provides a flat-rate annual income of roughly DKK 130,000 ($16,250). At age 67 (or 65 for those born after July 1, 1939) all residents become eligible for the Old-Age Pension (OAP), which provides income-tested annuities of up to DKK 99,000 ($12,375) per year for singles and DKK 75,000 ($9,375) for coupled individuals (at 2000 rates). The VERP and OAP pension schemes indirectly serve as social insurance against shocks for those eligible, who can decide to take them up at different ages according to their financial needs. Note that the dependence of OAP and Social DI benefits on the structure of the household (with higher benefits for singles as compared to married or cohabiting individuals) further insures the standard of living of surviving spouses.

Data sources. We have merged data from several administrative registers to obtain annual information on Danish households of married and cohabiting couples from 1980 to 2011. We use the following registers: (1) the national patient register, which covers all hospitalization records (from both private and public hospitals), and from which we extract information on all the individuals that experienced a heart attack or a stroke; (2) the cause of death register, from which we identify death dates; (3) income registers, which include all sources of household income – e.g., labor income, capital income, annuity payouts, and government benefits from any program – as well as annual measures of gross wealth and liabilities; and (4) the Integrated Database for Labor Market Research, which includes measures from which we construct full-time and part-time labor supply variables and extract demographic variables.33 All nominal values are deflated based on the consumer price index and are reported in 2000 prices. In that year the exchange rate was approximately DKK 8 per US $1.34

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33 We postpone describing the summary statistics of the analysis sample to the next section since they directly relate to the discussion on the advantages of our research design.

34 In our sample, the net assets of the median household amount to only DKK 13,236 ($1,655) while the median annual household-level income is DKK 229,922 ($29,900). Therefore, our analysis of labor supply responses focuses on income losses, and we use the wealth data for robustness checks.
1.4 Research Design

In this section we describe our empirical strategy for identifying the causal effect of spousal health and mortality shocks on individuals’ labor supply, $e_{w}/e_{w}^{*}$. Our method relies on the simple intuition that within a short period of time the exact timing of a severe health shock or death is as good as random. In particular, we construct non-parametric counterfactuals to affected households using households that experience the same shock a few years in the future, and recover the treatment effect by performing event studies for these two experimental groups. Before formally describing our research design, we illustrate its basic intuition with a concrete example.

Illustration. Let us focus on a treatment group of individuals born between 1930 and 1950 who experienced a severe health shock, in particular, a heart attack or a stroke, in 1995. Consider studying the effect of the shock on some economic outcome of these individuals, e.g., their labor force participation. Panel B of Figure 1.2 plots the outcome for these households as well as for households that experienced the same shock in 2010 (15 years later), in 2005 (10 years later), in 2000 (5 years later) and in 1996 (1 year later). Studying the behavior of households that experienced the shock in different years reveals increasingly comparable patterns to those of the treatment group’s behavior – in trends before 1995 – the closer the year in which the individual experienced the shock was to 1995. These patterns confirm our intuition and suggest using households that experienced a shock in $1995+\Delta$ as a control group for households that experienced a shock in 1995. Our method, which we describe formally below, generalizes this example by aggregating different calendar years.

The trade-off in the choice of $\Delta$ can be immediately seen in Panel C of Figure 1.2. On the one hand, we would want to choose a smaller $\Delta$ such that the control group is more closely comparable to the treatment group, e.g., year 1996 which corresponds to $\Delta = 1$. On the other hand, we would want to choose a larger $\Delta$ in order to be able to identify longer-run effects of the shock, up to period $\Delta – 1$. For example, using those who experienced a shock in 2005 ($\Delta = 10$) will allow us to estimate the effect of the shock for up to 9 years. However, this entails a potentially larger bias since the trend in the behavior of this group is not as tightly parallel to that of the treatment group. Our choice of $\Delta$ is five years, such that we can identify effects up to four years after the shock. Perturbations to this choice are inconsequential to our results.
FIGURE 1.2
Illustration of the Empirical Research Design

(a) Health Shocks in Year 1995 vs. No Shock
(b) Health Shocks in Different Years and No Shock
(c) Health Shocks in Years 1995, 1996 and 2005

Notes: These figures compare the labor force participation of a treatment group of individuals who were born between 1930 and 1950 and experienced a heart attack or a stroke in 1995 to that of potential control groups. Panel A compares the treatment group to those who belong to the same cohorts but did not experience a shock in our data window, years between 1985 and 2011, and shows that the pre-1995 patterns of these groups are far from parallel. Panel B adds the behavior of households that experienced the same shock but in different years, and shows that the groups are becoming increasingly comparable to the treatment group—in terms of parallel trends before 1995—the closer the year in which the individual experienced the shock was to the year the treatment group experienced the shock (1995). The figures suggest using households that experienced a shock in year 1995+$\Delta$ as a control group for households that experienced a shock in 1995. The trade-off in the choice of $\Delta$ is presented in Panel C. On the one hand, we would want to choose a smaller $\Delta$ such that the control group would be more closely comparable to the treatment group, e.g., year 1996 which corresponds to $\Delta=1$. On the other hand, we would want to choose a larger $\Delta$ in order to be able to identify longer-run effects of the shock, up to period $\Delta-1$. Using those that experienced a shock in 2005, which corresponds to $\Delta=10$, will allow us to estimate up to the 9-year effect of the shock. However, this entails a potentially greater bias since the trend in the behavior of this group is not as tightly parallel to that of the treatment group.
Formal description. Fix a group of cohorts, denoted by $\Omega$, and consider estimating the treatment effect of a shock experienced at some point in the time interval $[\tau_1, \tau_2]$ by individuals who belong to group $\Omega$. We refer to these households as the treatment group and divide them into sub-groups indexed by the year in which they experienced the shock, $\tau \in [\tau_1, \tau_2]$. We normalize the timing of observation such that the time period, $t$, is measured with respect to the year of the shock — that is, $t = \text{year} - \tau$ (where year is the calendar year of the observation). As a control group, we match to each treated group $\tau$ the households among cohorts $\Omega$ that experienced the same shock at $\tau + \Delta$ for a given choice of $\Delta$. For these households we assign a “placebo” shock at $t = 0$ by normalizing timing in the same way as we do for the treatment group ($t = \text{year} - \tau$).\textsuperscript{35} Denote the mean outcome of the treatment group at time $t$ by $y_{T}^{T}$ and the mean outcome of the control group at time $t$ by $y_{C}^{T}$ and choose a baseline period (or periods) prior to the shock (e.g., period $t = -2$), which we denote by $p$ (for “prior”). For any $n > 0$, the treatment effect can be simply recovered by the differences-in-differences estimator $\beta_{n} = (y_{n}^{T} - y_{n}^{C}) - (y_{p}^{T} - y_{p}^{C})$. The treatment effect in period $n$ is measured by the difference in outcomes between the treatment group and control group at time $n$, purged of the difference in their outcomes at the baseline period, $p$. Note that the choice of $\Delta$ puts an upper bound on $n$ such that $n < \Delta$.

Simply put, our design conducts event studies for two experimental groups: a treatment group composed of households that experience a shock in year $\tau$, and a matched control group composed of households from the same cohorts that experience the same shock in year $\tau + \Delta$.

Identifying assumption. The identifying assumption is that, absent the shock, the outcomes of the treatment and control groups would run parallel. In particular, in accordance with the differences-in-differences research design, there is no requirement regarding the levels of outcomes. The plausibility of this assumption relies on the intuition that within the short window of time of length $\Delta$ the exact time at which the shock occurs is as good as random. To test the validity of our assumption, we accompany our empirical analysis with the treatment and control groups’ behavior in the five years prior to the shock year $0$ in order to assess their co-movement in the pre-shock period.

\textsuperscript{35}By construction, their actual shock occurs at $t = \Delta$. 

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Other papers that use similar identifying assumptions include earlier studies in the context of the long-run effects of job displacement (Ruhm 1991) and the effect of arrests on employment and earnings (Grogger 1995), as well as more recent studies such as that by Hilger (2014), who exploits variation in the timing of fathers’ layoffs in order to study the effect of parental income on college outcomes. More generally, our quasi-experimental design can be applied to any shock of which the exact timing is random, which can be easily validated in any particular setting by studying the pre-trends of the experimental groups.

Comparison to pure event studies. Pure event studies, which analyze the evolution of outcomes of a treated group around the time of a shock, suffer from three main shortcomings in our application. First, they identify short-run responses by relying on immediate and sharp responses at the onset of a shock. However, we are interested in identifying longer-run effects because of potential delays in adjustment due to, e.g., labor market frictions. A pure event study would misleadingly attribute gradual responses and delays in adjustment to the outcome’s trend and would overlook the treatment effect. Second, potentially complex life-cycle trends in, e.g., spouses’ labor force participation as depicted in Figure 1.3, may lead to biased extrapolations of the counterfactual behavior of an outcome in the absence of a shock if based on pre-shock behavior. Third, potential time trends in outcomes are a common confounding factor and a concern to any event study design. Our research design addresses these concerns by constructing a control group that recovers non-parametrically the treatment group’s counterfactual behavior.

Summary statistics. Table 1.1 displays key summary statistics for the analysis sample. Our main analysis sample of households in which one spouse died between ages 45 and 80 is comprised of 310,720 households in the treatment group and 409,190 households in the control group. The table reveals the advantage of our research design – the comparability of the year of observation and the age of unaffected spouses across experimental groups. The average survivor in the treatment group loses his or her spouse in 1993 at age 62.86 and the average unaffected spouse in the control group experiences the placebo shock in year 1993 at age 62.27, with even closer similarities in the sub-sample of survivors under age 60.36 The sample for our secondary analysis of severe health

36By construction, the research design nets out calendar year effects non-parametrically. However, due to the randomness of the exact timing of the shock, it also nets out life-cycle effects by comparing groups of very similar ages so that we effectively
FIGURE 1.3

Life-Cycle Labor Force Participation of the Unaffected Spouses in the Death Event Sample

Notes: This figure displays the life-cycle labor force participation of the unaffected spouses that are included in the death event sample (i.e., individuals whose spouses died between ages 45 and 80 from 1985 to 2011). The observations include the pre-shock periods (specifically, periods -5 to -2). The sharp drop at age 60 corresponds to eligibility for the Voluntary Early Retirement Pension (VERP). The figure shows the complex life-cycle trends in labor supply and illustrates why an extrapolation based on behavior in previous years is a poor predictor of future behavior.
# TABLE 1.1
Summary Statistics of Analysis Sample

<table>
<thead>
<tr>
<th></th>
<th><strong>Death Event Sample</strong></th>
<th></th>
<th></th>
<th><strong>Health Shock Sample</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>All Ages</strong></td>
<td><strong>Under 60</strong></td>
<td></td>
<td><strong>Under 60</strong></td>
</tr>
<tr>
<td></td>
<td>Treatment</td>
<td>Control</td>
<td>Treatment</td>
<td>Control</td>
</tr>
<tr>
<td><strong>Characteristics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>62.86</td>
<td>62.27</td>
<td>47.60</td>
<td>47.48</td>
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<tr>
<td>Education (months)</td>
<td>118.66</td>
<td>119.94</td>
<td>129.19</td>
<td>129.38</td>
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<tr>
<td>Percent female</td>
<td>0.6937</td>
<td>0.6632</td>
<td>0.7485</td>
<td>0.7485</td>
</tr>
<tr>
<td><strong>Affected Spouse</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>64.84</td>
<td>64.01</td>
<td>52.51</td>
<td>52.14</td>
</tr>
<tr>
<td>Education (months)</td>
<td>123.57</td>
<td>124.05</td>
<td>131.80</td>
<td>132.22</td>
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<td><strong>Outcomes</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Participation</td>
<td>0.3474</td>
<td>0.3719</td>
<td>0.7389</td>
<td>0.7445</td>
</tr>
<tr>
<td>Earnings (DKK)</td>
<td>62,455</td>
<td>67,452</td>
<td>160,799</td>
<td>162,094</td>
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<tr>
<td><strong>Affected Spouse</strong></td>
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<td></td>
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<td></td>
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<tr>
<td>Participation</td>
<td>0.2723</td>
<td>0.3211</td>
<td>0.6033</td>
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<td>Earnings (DKK)</td>
<td>51,579</td>
<td>61,791</td>
<td>143,118</td>
<td>158,447</td>
</tr>
<tr>
<td><strong>Number of Households</strong></td>
<td>310,720</td>
<td>409,190</td>
<td>55,103</td>
<td>80,578</td>
</tr>
</tbody>
</table>

Notes: This table presents means of key variables in our analysis sample. All monetary values are reported in nominal Danish Kroner (DKK) deflated to 2000 prices using the consumer price index. In this year the exchange rate was approximately DKK 8 per US $. For each event, the treatment group comprises households that experienced a shock in different years, to which we match households that experienced the same shock five years later as a control group (Δ=5). Columns 1 and 2 report statistics for the death event sample of households in which a spouse died of any cause between ages 45 and 80 from 1985 to 2011. Column 1 reports statistics for the entire sample, and Column 2 reports statistics for the sub-sample of surviving spouses under age 60. Column 3 reports statistics for the health event sample. It includes households in which one spouse experienced a heart attack or a stroke between 1985 and 2011 and survived for at least three years, and in which both spouses were under age 60. The values reported in the table are based on data from two periods before the shock occurred (period t = -2).
shocks includes households in which one spouse experienced a heart attack or a stroke (for the first time) and survived for at least three years. We focus on households with both spouses under 60 to ensure that the results we document are driven only by the health shock and not by eligibility for retirement benefits.\textsuperscript{37} The sample consists of 37,432 households in the treatment group and 54,926 households in the control group. The unaffected spouse is on average 45.7 years old in the treatment group at the time of the shock and 45.3 years old in the control group, where the mean calendar year of the shock is around 1992 for both groups.\textsuperscript{38}

1.5 Spousal Labor Supply Responses

1.5.1 Labor Supply Responses to the Death of a Spouse

In this section, we present our main empirical analysis and study the survivors’ labor supply responses to the death of their spouses. We begin by estimating the average labor supply responses. Then, we analyze the heterogeneity of these responses by the degree of income loss imposed by the loss of a spouse in order to study the self-insurance role of spousal labor supply.

\textit{Mean responses.} Figure 1.4 plots the average labor supply response of individuals whose spouse died between ages 45 and 80.\textsuperscript{39} Panel A reveals an immediate increase in labor force participation following the death of a spouse. By the fourth year after the shock, the surviving spouses’ participation increases by 7.6% – an increase of 1.6 percentage points (pp) on a base of 20.6 pp. Panel B of Figure 1.4 shows that this response translates into a 6.8% increase in annual earnings, which represents an annual increase of DKK 2,572 ($322) from a low base of DKK 37,952 ($4,744).\textsuperscript{40}

\begin{itemize}
  \item \textsuperscript{37}We constrain the sample for this shock (and not for the death events) because the average age of the unaffected spouses at the time of the shock in the unconstrained sample is very close to sixty (60.67), and because there are large life-cycle responses in labor force participation exactly at this age (when the majority of individuals become eligible for early retirement benefits as displayed in Figure 1.3).
  \item \textsuperscript{38}We also report the means of main labor supply outcomes in Table 1.1 for completeness. Note that participation and earnings are slightly higher for the control group, which poses no threat to the validity of the design since comparability requires similar trends and not similar levels.
  \item \textsuperscript{39}We define participation as having any positive level of labor income during the calendar year.
  \item \textsuperscript{40}These means include zeros for those who do not work.
\end{itemize}
FIGURE 1.4
Survivors’ Labor Supply Responses to the Death of Their Spouse

(a) Labor Force Participation

(b) Annual Earnings

Notes: These figures plot the labor supply responses of survivors to the death of their spouse. The sample includes individuals whose spouses died between ages 45 and 80 from 1985 to 2011. Panel A depicts the behavior of labor force participation, and Panel B depicts the behavior of annual earnings. The x-axis denotes time with respect to the shock, normalized to period 0. For the treatment group, period 0 is when the actual shock occurs; for the control group period 0 is when a “placebo shock” occurs (while their actual shock occurs in period 5). The dashed gray line plots the behavior of the control group. To ease the comparison of trends, we normalize the level of the control group’s outcome to the pre-shock level of the treatment group’s outcome. This normalized counterfactual is displayed by the blue line and squares. The red line and circles plot the behavior of the treatment group.
Since men and women may face substantially different financial distress when they lose their spouse we analyze widowers and widows separately. Panel A of Figure 1.5 reveals the stark differences in responses. While on average widowers do not change their labor force participation when their wives die, widows immediately and significantly increase their labor force participation when they lose their husbands. Four years after the shock, widows’ labor force participation increases by 2.2 pp from a baseline participation rate of 19.5 pp, which amounts to a large increase of 11.3% in their labor force participation.

This differential response reveals that female survivors have greater need to self-insure through labor supply and suggests that they experience greater income losses when they lose their spouses as compared to their male counterparts. To see this, we plot the evolution of overall household income (from any source) around the death of a spouse, including earnings, capital income, annuity payouts and benefits from social programs. We begin by plotting the household’s income in the absence of behavioral responses from the unaffected spouse in order to capture the income loss directly attributable to the loss of an earning spouse. In Panel A of Figure 1.6, we plot the household’s overall income, holding the unaffected spouse’s earnings and social benefits at their pre-shock level.\textsuperscript{41} The graph shows that widowers who lose their wives experience a 32% loss in household income, while widows who lose their husbands experience a significantly larger loss of 40%. Panel B of Figure 1.6 studies the actual change in household income, taking into account the surviving spouses’ labor supply responses and any change in the benefits they may receive from social or private insurance. The figure shows that widowers experience an actual loss of 31% and that widows manage to decrease their potential loss to incur an actual lower loss of 35%.

Note that surviving spouses do not fully compensate for a loss in household income since as singles they do not need the full pre-shock level of income. However, potential economies of scale in the household’s consumption technology may make half of the pre-shock level of household income insufficient for maintaining the pre-shock level of utility (see, e.g., Nelson 1988 and Browning, Chiappori and Lewbel 2013). The share of household income that keeps consumption utility at its pre-shock level is usually assumed to lie between 0.5 and 1 and is commonly referred to as the adult

\textsuperscript{41} Specifically, we fix the surviving spouse’s labor income, Social Disability and Social Security benefits as well as sick-pay benefits at their level in $t = -1$. 

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FIGURE 1.5

Survivors’ Labor Supply Responses to the Death of Their Spouse by Gender

(a) Labor Force Participation

Widowers (wife dies)                                           Widows (husband dies)

0.356 0.2324 0.2317 0.307 0.2176 0.1954

Notes: These figures plot the labor supply responses of survivors to the death of their spouse by the gender of the surviving spouse. The sample includes individuals whose spouses died between ages 45 and 80 from 1985 to 2011. Panel A depicts the behavior of labor force participation, and Panel B depicts the behavior of annual earnings. The x-axis denotes time with respect to the shock, normalized to period 0. For the treatment group period 0 is when the actual shock occurs; for the control group period 0 is when a “placebo shock” occurs (while their actual shock occurs in period 5). The dashed gray line plots the behavior of the control group. To ease the comparison of trends, we normalize the level of the control group’s outcome to the pre-shock level of the treatment group’s outcome. This normalized counterfactual is displayed by the blue line and squares. The red line and circles plot the behavior of the treatment group.

(b) Annual Earnings

Widowers (wife dies)                                           Widows (husband dies)

78,876 49,816 48,776 48,855 49,816 36,881 33,500

Notes: These figures plot the labor supply responses of survivors to the death of their spouse by the gender of the surviving spouse. The sample includes individuals whose spouses died between ages 45 and 80 from 1985 to 2011. Panel A depicts the behavior of labor force participation, and Panel B depicts the behavior of annual earnings. The x-axis denotes time with respect to the shock, normalized to period 0. For the treatment group period 0 is when the actual shock occurs; for the control group period 0 is when a “placebo shock” occurs (while their actual shock occurs in period 5). The dashed gray line plots the behavior of the control group. To ease the comparison of trends, we normalize the level of the control group’s outcome to the pre-shock level of the treatment group’s outcome. This normalized counterfactual is displayed by the blue line and squares. The red line and circles plot the behavior of the treatment group.
FIGURE 1.6
Household Income in the Death Event Study

(a) Potential Household Income

(b) Actual Household Income

Notes: These figures plot different measures of household-level income for households in the death event study by the gender of the surviving spouse. The sample includes individuals whose spouses died between ages 45 and 80 from 1985 to 2011. Panel A plots an adjusted measure of household income. Specifically, we fix the surviving spouse’s labor income, Social Disability and Social Security benefits as well as sick-pay benefits at their pre-shock levels (in period -1). Hence, this measure captures the income loss that is directly attributed to the loss of a spouse. Panel B plots the actual household income that is observed in the data, which takes into account the surviving spouse’s behavioral responses. The x-axis denotes time with respect to the shock, normalized to period 0. For the treatment group period 0 is when the actual shock occurs; for the control group period 0 is when a “placebo shock” occurs (while their actual shock occurs in period 5). The dashed gray line plots the behavior of the control group. To ease the comparison of trends, we normalize the level of the control group’s outcome to the pre-shock level of the treatment group’s outcome. This normalized counterfactual is displayed by the blue line and squares. The red line and circles plot the behavior of the treatment group.
“equivalence scale”. We return to this issue in Section 1.5.2 below.

We continue with further investigation of the heterogeneity in the survivors’ labor supply responses across different subgroups and show that the responses are proportional to the survivors’ degree of financial stability and to the income loss they experience. First, we focus on the subsample of surviving spouses under 60, who have a stronger attachment to the labor force and are therefore more financially resilient after the loss of an earning spouse.42 The overall mean response for this group is plotted in Panel A of Figure 1.7. Consistent with the view that their higher participation rates and annual earnings effectively insulate them against losing an earning spouse, survivors under 60 exhibit a smaller relative increase in labor force participation compared to the universe of survivors – only 2.1% (1.4 pp on a base of 67.2). Similar to the overall treatment effect, this increase is entirely driven by women. As seen in Panel B of Figure 1.7, widows increase their labor force participation by 3.3%, while widowers – who experience the shock at a significantly higher participation rate (0.78) as compared to widows (0.715) – decrease their participation by 1.1%. Panel A of Figure 1.8 shows that these responses translate to a 3.2% increase in annual earnings for the lower-earning widows and a decrease of 4.1% in annual earnings for the higher-earning widowers, who as singles do not need their entire higher pre-shock levels of income. As before – and as displayed in Panel B of Figure 1.8 – these differential responses reveal the differential financial shock that they experience, with men experiencing a decline of 31% in household income and women experiencing a striking loss of 44%.

We report estimates for the regression counterparts of these figures in Table 1.2, which replicates our results. As we alluded to in Section 1.4, the treatment effect can be recovered by a simple differences-in-differences regression of the form:

\[ l_{w,i,t} = \beta_0 + \beta_1 treat_i + \beta_2 post_{i,t} + \beta_3 treat_i \times post_{i,t} + \beta_4 X_{i,t} + \alpha_i + \epsilon_{i,t}. \]  

(1.6)

In this regression \( l_{w,i,t} \) denotes an indicator for the labor force participation or annual earnings of the unaffected spouse \( w \) in household \( i \) at time \( t \); \( treat_i \) denotes an indicator for whether a

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42 Recall that at 60 there is a sharp drop in participation when most of the labor force becomes eligible for early retirement benefits.
FIGURE 1.7

Labor Force Participation Responses of Survivors under Age 60 to the Death of Their Spouse

(a) Both Genders

(b) By Gender

Notes: These figures plot the labor force participation responses of survivors under age 60 to the death of their spouse by the gender of the surviving spouse. The sample includes individuals under 60 whose spouses died between ages 45 and 80 from 1985 to 2011. Panel A depicts the behavior of the overall sample; Panel B divides the sample by the gender of the surviving spouse. The x-axis denotes time with respect to the shock, normalized to period 0. For the treatment group period 0 is when the actual shock occurs; for the control group period 0 is when a “placebo shock” occurs (while their actual shock occurs in period 5). The dashed gray line plots the behavior of the control group. To ease the comparison of trends, we normalize the level of the control group’s outcome to the pre-shock level of the treatment group’s outcome. This normalized counterfactual is displayed by the blue line and squares. The red line and circles plot the behavior of the treatment group.
FIGURE 1.8
Annual Earnings and Potential Household Income of Survivors under Age 60 by Gender

(a) Annual Earnings

(b) Potential Household Income

Notes: These figures plot different outcomes for survivors under age 60 around the death of their spouse by the gender of the surviving spouse. The sample includes individuals under 60 whose spouses died between ages 45 and 80 from 1985 to 2011. Panel A plots annual earnings. Panel B plots an adjusted measure of household income. Specifically, we fix the surviving spouse's labor income, Social Disability and Social Security benefits as well as sick-pay benefits at their pre-shock levels (in period -1). Hence, this measure captures the income loss that is directly attributed to the loss of a spouse. The x-axis denotes time with respect to the shock, normalized to period 0. For the treatment group period 0 is when the actual shock occurs; for the control group period 0 is when a “placebo shock” occurs (while their actual shock occurs in period 5). The dashed gray line plots the behavior of the control group. To ease the comparison of trends, we normalize the level of the control group’s outcome to the pre-shock level of the treatment group’s outcome. This normalized counterfactual is displayed by the blue line and squares. The red line and circles plot the behavior of the treatment group.


TABLE 1.2
Survivors’ Labor Supply Responses to the Death of Their Spouse

| A. Surviving Spouses of All Ages | | |
|------------------------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| | Widowers | | Widows | | | | | |
| Dependent variable: | Participation | Participation | Earnings | Earnings | Participation | Participation | Earnings | Earnings |
| Treat × Post | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Treat × Post | -.0016 | -.0017 | -.939* | -.906** | .0188*** | .0164*** | 2,957*** | 2,707*** |
| Household FE | X | X | X | X | X | X | X | X |
| Year and Age FE | X | X | X | X | X | X | X | X |
| Number of Obs. | 1,397,030 | 1,397,030 | 1,397,030 | 1,397,030 | 2,919,946 | 2,919,946 | 2,919,946 | 2,919,946 |
| Number of Households | 232,973 | 232,973 | 232,973 | 232,973 | 486,890 | 486,890 | 486,890 | 486,890 |

| B. Surviving Spouses under 60 | | |
|--------------------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| | Widowers | | Widows | | | | | |
| Dependent variable: | Participation | Participation | Earnings | Earnings | Participation | Participation | Earnings | Earnings |
| Treat × Post | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Treat × Post | -.0075** | -.0071** | -7.902*** | -7.730*** | .0207*** | .0219*** | 4.093*** | 4.423*** |
| Household FE | X | X | X | X | X | X | X | X |
| Year and Age FE | X | X | X | X | X | X | X | X |
| Number of Obs. | 203,569 | 203,569 | 204,438 | 204,438 | 607,437 | 607,437 | 608,742 | 608,742 |
| Number of Households | 34,104 | 34,104 | 34,118 | 34,118 | 101,529 | 101,529 | 101,562 | 101,562 |

Notes: This table reports the differences-in-differences estimates of the surviving spouses’ labor supply responses (equation (6)). The sample includes individuals whose spouses died between ages 45 and 80 from 1985 to 2011. The treatment group comprises households that experienced the shock in different years, to which we match households that experienced the same shock five years later as a control group (Δ=5). Panel A reports the responses of all survivors by gender, where widowers are those who lost their wives and widows are those who lost their husbands. Panel B reports the responses of survivors under 60 by gender. The pre-shock periods include periods -5 to -3. The post-shock periods include periods 2 to 4. Robust standard errors clustered at the household level are reported in parentheses.

*** p<0.01, ** p<0.05, * p<0.1.
household belongs to the treatment group; \( post_{i,t} \) denotes an indicator for whether the observation belongs to post-shock periods; \( X_{i,t} \) denotes a vector of controls, and \( \alpha_i \) is a household fixed effect. The parameter \( \beta_3 \) represents the causal effect of the death of a spouse on the labor supply of the unaffected spouse.

As we show in Figure 1.9, in periods 0 and 1 there are temporary transitions to part-time work, consistent with spending time with the dying spouse and mourning his or her loss. These transitions stabilize thereafter such that the active decision margin becomes full-time work vs. non-participation.\(^{43}\) Throughout the analysis, \( post_{i,t} \) therefore assumes the value 1 for periods 2 to 4.

**Within-gender regression analysis.** Next, we study the effect of the death of a spouse on labor force participation by the degree of income loss for each gender separately. To this end, for each household we calculate the potential income loss due to the shock in the following way.

First, similarly to Panel A of Figure 1.6 and Panel B of Figure 1.8, we calculate for each household the overall income holding the unaffected spouse’s earnings and social benefits at their pre-shock level. Second, we calculate the ratio of this “potential income” measure in \( t = 1 \) to the household’s income in \( t = -1 \). Third, we normalize this ratio for the treated households by the mean ratio of the control households in order to purge life-cycle and time effects. This leaves us with an ex-ante measure of the potential income replacement rate for each treated household, which we denote by \( rr_i \), that captures the change in household income directly attributed to (and only to) the loss of a spouse.

To study the heterogeneity in labor supply responses by the income replacement rate we estimate the following augmented differences-in-differences model

\[
l_{w,i,t} = \beta_0 + \beta_1 treat_i + \beta_2 post_{i,t} + \beta_3 i \times treat_i \times post_{i,t} + \beta_4 X_{i,t} + \alpha_i + \varepsilon_{i,t}, \tag{1.7}
\]

where

\[
\beta_{3i} = \beta_{30} + \beta_{31} rr_i + \beta_{32} Z_{i,t}.
\]

\(^{43}\) Indeed, this is one of our reasons for focusing the theoretical analysis on the participation margin rather than work intensity.
FIGURE 1.9
Labor Supply Responses of Survivors under Age 60 to the Death of Their Spouse

Notes: These figures plot labor supply responses of survivors under age 60 to the death of their spouse. The sample includes individuals under 60 whose spouses died between ages 45 and 80 from 1985 to 2011. Panel A depicts labor force participation; Panels B and C depict the fraction of surviving spouses who are employed full time and part time, respectively. The pictures are constructed from ATP data available for workers under 60. Full-time employment is defined as working at least 30 hours per week all 12 months of the calendar year (“full-time full-year”); part-time employment is defined as working at some point during the year, but either fewer than 30 hours per week or fewer than 12 months within the calendar year. The x-axis denotes time with respect to the shock, normalized to period 0. For the treatment group, period 0 is when the actual shock occurs; for the control group, period 0 is when a “placebo shock” occurs (while their actual shock occurs in period 5). The dashed gray line plots the behavior of the control group. To ease the comparison of trends, we normalize the level of the control group’s outcome to the pre-shock level of the treatment group’s outcome. This normalized counterfactual is displayed by the blue line and squares. The red line and circles plot the behavior of the treatment group.
In this regression $l_{w,i,t}$ denotes an indicator for the labor force participation of the unaffected spouse $w$ in household $i$ at time $t$. We augment the basic differences-in-differences design by allowing the treatment effect, $\beta_{3i}$, to vary across households and model it as a function of the household’s potential replacement rate $rr_i$. Our parameter of interest is $\beta_{31}$, which captures the extent to which the surviving spouse’s labor supply response correlates with the income loss he or she experiences. Since $\beta_{31}$ can capture other dimensions of heterogeneity beyond the income replacement rate, we let the treatment effect vary with additional household-level characteristics, $Z_{i,t}$, such that $\beta_{31}$ further isolates the treatment effect’s partial correlation with the loss of household income.\footnote{The variables we include in $Z_{i,t}$ are age dummies for the surviving spouse, dummies for the age of the deceased at the year of death, year dummies, indicators for the number of children in the household as well as the surviving spouse’s months of education (and its square). The results are also robust to the inclusion of a quadratic in the household’s net wealth. Note that $X_{i,t}$ always includes the variables in $Z_{i,t}$ as well as their interaction with $treat_i$ and $post_{i,t}$.}

Table 1.3 reports the results of estimating (1.7) separately for each gender, with and without $Z_{i,t}$, for the entire sample of surviving spouses and only the sub-sample of survivors under age 60. The results consistently show throughout the specifications the strong correlation between labor supply responses and income losses; survivors in households with lower potential income replacement rates (lower $rr_i$) who experience larger income losses are much more likely to increase their labor force participation in response to the shock. Since controlling for the additional interactions with $Z_{i,t}$ does not change the results much, the evidence suggests that the heterogeneous responses are indeed driven by differential income replacement rates. In addition, the estimation results reveal quite similar sensitivity to income losses across genders and verify that gender differences in preferences do not drive the average labor supply responses.

\textbf{Responses by own earnings.} The heterogeneity in responses due to the household’s degree of income insurance that we have analyzed so far has focused on income losses relative to pre-shock income flows. An additional strategy for studying this sort of heterogeneity focuses on the levels of the surviving spouses’ disposable income available at the time of the shock. To do this, we turn to analyze how labor supply responses of surviving spouses may vary with their own level of earnings when their spouses die, since higher-earning survivors have more disposable income and are therefore better insured.

We constrain the sample in the following way. First, we exclude surviving spouses whose average
### TABLE 1.3
Survivors’ Labor Force Participation Responses to the Death of Their Spouse by the Degree of Income Loss

**A. Surviving Spouses of All Ages**

<table>
<thead>
<tr>
<th></th>
<th>Both Genders</th>
<th>Widowers</th>
<th>Widows</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td><strong>Baseline Regression</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treat × Post</td>
<td>0.1265***</td>
<td>0.1220***</td>
<td>0.1170***</td>
</tr>
<tr>
<td></td>
<td>(0.0023)</td>
<td>(0.0042)</td>
<td>(0.0027)</td>
</tr>
<tr>
<td>Treat × Post ×</td>
<td>-0.1889***</td>
<td>-0.1894***</td>
<td>-0.1744***</td>
</tr>
<tr>
<td>Replacement Rate</td>
<td>(0.0035)</td>
<td>(0.0061)</td>
<td>(0.0044)</td>
</tr>
<tr>
<td>Number of Obs.</td>
<td>4,288,621</td>
<td>1,387,615</td>
<td>2,901,006</td>
</tr>
<tr>
<td>Number of Households</td>
<td>714,892</td>
<td>231,318</td>
<td>483,574</td>
</tr>
</tbody>
</table>

**Regression with Interactions**

<table>
<thead>
<tr>
<th></th>
<th>Both Genders</th>
<th>Widowers</th>
<th>Widows</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Treat × Post ×</td>
<td>-0.1989***</td>
<td>-0.2021***</td>
<td>-0.1927***</td>
</tr>
<tr>
<td>Replacement Rate</td>
<td>(-.0045)</td>
<td>(.0081)</td>
<td>(.0056)</td>
</tr>
<tr>
<td>Number of Obs.</td>
<td>2,741,690</td>
<td>821,742</td>
<td>1,919,948</td>
</tr>
<tr>
<td>Number of Households</td>
<td>459,622</td>
<td>137,724</td>
<td>321,898</td>
</tr>
</tbody>
</table>

**B. Surviving Spouses under 60**

<table>
<thead>
<tr>
<th></th>
<th>Both Genders</th>
<th>Widowers</th>
<th>Widows</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td><strong>Baseline Regression</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treat × Post</td>
<td>0.0883***</td>
<td>0.0652***</td>
<td>0.0954***</td>
</tr>
<tr>
<td></td>
<td>(0.0054)</td>
<td>(0.0125)</td>
<td>(0.0063)</td>
</tr>
<tr>
<td>Treat × Post ×</td>
<td>-0.1270***</td>
<td>-0.1081***</td>
<td>-0.1338***</td>
</tr>
<tr>
<td>Replacement Rate</td>
<td>(0.0083)</td>
<td>(0.0168)</td>
<td>(0.0101)</td>
</tr>
<tr>
<td>Number of Obs.</td>
<td>803,158</td>
<td>201,487</td>
<td>601,671</td>
</tr>
<tr>
<td>Number of Households</td>
<td>134,199</td>
<td>33,720</td>
<td>100,479</td>
</tr>
</tbody>
</table>

**Regression with Interactions**

<table>
<thead>
<tr>
<th></th>
<th>Both Genders</th>
<th>Widowers</th>
<th>Widows</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Treat × Post ×</td>
<td>-0.1481***</td>
<td>-0.1375***</td>
<td>-0.1490***</td>
</tr>
<tr>
<td>Replacement Rate</td>
<td>(0.0091)</td>
<td>(0.0186)</td>
<td>(.0110)</td>
</tr>
<tr>
<td>Number of Obs.</td>
<td>704,370</td>
<td>173,620</td>
<td>530,750</td>
</tr>
<tr>
<td>Number of Households</td>
<td>118,812</td>
<td>29,288</td>
<td>89,524</td>
</tr>
</tbody>
</table>

Notes: This table reports the interaction of the treatment effect of the death of a spouse with the household’s post-shock income replacement rate (equation (7)). The sample includes individuals whose spouses died between ages 45 and 80 from 1985 to 2011. The treatment group comprises households that experienced the shock in different years, to which we match households that experienced the same shock five years later as a control group (Δ=5). Panel A reports estimates for the sample of all survivors by gender; Panel B reports estimates for the sample of survivors under age 60 by gender. In each panel, we report estimates of two specifications. The upper half of each panel estimates a baseline differences-in-differences specification which interacts the treatment effect with the replacement rate variable. This replacement rate is calculated as follows. First, we fix the surviving spouse’s labor income, Social Disability and Social Security benefits as well as sick-pay benefits at their pre-shock levels (in period -1). Then, we calculate the ratio of this adjusted household income in period 1 (post-shock) to that in period -1 (pre-shock), and normalize it by the average ratio for the control group in order to account for calendar year trends as well as for life-cycle effects. The lower half of each panel extends this specification to include interactions of the treatment effect with additional household characteristics: age dummies for the surviving spouse, dummies for the age of the deceased at the year of death, year dummies, indicators for the number of children in the household as well as the surviving spouse’s months of education (and its square). The results are also robust to the inclusion of a quadratic in the household’s net wealth. All the variables that are interacted with “Treat × Post” are interacted with “Treat” and “Post” and enter the regressions separately as well. All specifications include year, age and household fixed effects. The pre-shock periods include periods -5 to -3. The post-shock periods include periods 2 to 4. Robust standard errors clustered at the household level are reported in parentheses.

*** p<0.01, ** p<0.05, * p<0.1.
labor income before the shock (in periods -5 to -2) was lower than that of their experimental-group-specific 20th percentile. Then, for each household we calculate the pre-shock labor income share of the deceased spouse out of the household’s overall labor income and include only households in which both spouses were sufficiently attached to the labor force. Specifically, we keep households for whom the average share was between 0.20 and 0.80. These restrictions allow us to focus on households in which there has been some loss of income due to the death of a spouse and in which the surviving spouse has earned non-negligible labor income both in levels and as a share within the household.45

We divide the remaining sample into five equal-sized groups according to their pre-shock level of earnings and plot in Panel A of Figure 1.10 the average labor income response (as well as its 95-percent confidence interval46) against the pre-shock mean earnings for each group. The figure reveals a strong gradient of labor supply responses with respect to the survivors’ own level of earnings when the shock occurs. Survivors at the bottom of the income distribution increase their earnings by 7.8% in order to meet their consumption needs, while those at the top decrease their earnings by 2.93% as their high income is no longer necessary to support two people.

Since the household’s pre-shock labor income is composed of two earners, we need to account for the pre-shock earnings of the dying spouse. Hence, we divide the sample into two groups – households in which the dying spouse’s pre-shock labor income fell within the bottom three quintiles of its group-specific distribution, to which we refer as “low-earners”, and households in which the dying spouse’s pre-shock labor income fell within the top two quintiles, to which we refer as “high-earners”. Panels B and C of Figure 1.10 reveal that the gradient prevails in both sub-samples, such that surviving spouses with lower earnings are much more likely to increase their labor supply when their spouse dies, regardless of whether their spouse was a high- or low-earner. Panel A of Table 1.4 shows that the relationship is robust to the inclusion of controls (dummy variables for age and year) by separately estimating the corresponding differences-in-differences equation for each surviving

45 Furthermore, to guarantee that our results are not driven by outliers, we exclude households with dying spouses whose mean pre-shock earnings did not fall within their group-specific 5th and 95th percentiles or households with unaffected spouses whose mean pre-shock earnings were higher than that of their group-specific 95th percentile.

46 Standard error are calculated using the Delta method.
FIGURE 1.10

Survivors’ Annual Earnings Responses to the Death of Their Spouse by the Level of their Own Pre-Shock Earnings

(a) All Households

Notes: These figures include individuals whose spouses died between ages 45 and 80 from 1985 to 2011, where we constrain the sample in the following way. First, we exclude surviving spouses whose average labor income before the shock (in periods -5 to -2) was lower than their experimental-group-specific 20th percentile. Then, we calculate for each household the pre-shock labor income share of the deceased spouse out of the household’s overall labor income and include only households in which both spouses were sufficiently attached to the labor force; specifically, we keep households for whom the average share was between 0.20 and 0.80. These restrictions allow us to focus on households for which there has been some loss of income due to the death of a spouse and in which the surviving spouse has earned non-negligible labor income both in levels and as a share within the household. In addition, to guarantee that our results are not driven by outliers, we exclude households with dying spouses whose mean pre-shock earnings did not fall within their group-specific 5th and 95th percentiles as well as households with unaffected spouses whose mean pre-shock earnings were higher than those of their group-specific 95th percentile. We divide the remaining sample into five equal-sized groups by their pre-shock level of earnings and plot the average labor income response as well as its 95-percent confidence interval (in which standard error are calculated using the Delta method) against the pre-shock mean earnings for each group. Panel A includes all households; Panel B includes households in which the dying spouses’ pre-shock labor income fell within the bottom three quintiles of its group-specific distribution, to which we refer as “low earners”; Panel C includes households in which the dying spouses’ pre-shock labor income fell within the top two quintiles, to which we refer as “high earners”. The pre-shock periods include periods -5 to -3. The post-shock periods include periods 2 to 4.
TABLE 1.4
Survivors’ Annual Earnings Responses to the Death of Their Spouse

<table>
<thead>
<tr>
<th>A. Mean Responses by Quintiles of Own Pre-Shock Earnings</th>
<th>All Survivors</th>
<th>Low-Earning Deceased</th>
<th>High-Earning Deceased</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quintile 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treat × Post</td>
<td>6,062***</td>
<td>8,847***</td>
<td>5,105***</td>
</tr>
<tr>
<td>Mean Earnings</td>
<td>(1,211)</td>
<td>(978)</td>
<td>(1,481)</td>
</tr>
<tr>
<td>Percent Change</td>
<td>8.07%</td>
<td>11.78%</td>
<td>6.06%</td>
</tr>
<tr>
<td>Quintile 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treat × Post</td>
<td>5,946***</td>
<td>7,283***</td>
<td>4,919***</td>
</tr>
<tr>
<td>Mean Earnings</td>
<td>(1,348)</td>
<td>(1,070)</td>
<td>(1,641)</td>
</tr>
<tr>
<td>Percent Change</td>
<td>5.13%</td>
<td>6.26%</td>
<td>5.54%</td>
</tr>
<tr>
<td>Quintile 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treat × Post</td>
<td>1,154</td>
<td>3,744***</td>
<td>1,370</td>
</tr>
<tr>
<td>Mean Earnings</td>
<td>(1,369)</td>
<td>(1,049)</td>
<td>(1,674)</td>
</tr>
<tr>
<td>Percent Change</td>
<td>0.78%</td>
<td>2.52%</td>
<td>0.88%</td>
</tr>
<tr>
<td>Quintile 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treat × Post</td>
<td>-2,203</td>
<td>-934</td>
<td>-2,644</td>
</tr>
<tr>
<td>Mean Earnings</td>
<td>(1,495)</td>
<td>(1,157)</td>
<td>(1,818)</td>
</tr>
<tr>
<td>Percent Change</td>
<td>-1.19%</td>
<td>-0.50%</td>
<td>-1.37%</td>
</tr>
<tr>
<td>Quintile 5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treat × Post</td>
<td>-7,494***</td>
<td>-5,846***</td>
<td>-8,877***</td>
</tr>
<tr>
<td>Mean Earnings</td>
<td>(1,765)</td>
<td>(1,399)</td>
<td>(2,170)</td>
</tr>
<tr>
<td>Percent Change</td>
<td>-3.12%</td>
<td>-2.45%</td>
<td>-3.60%</td>
</tr>
<tr>
<td>Household FE</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Age and Year FE</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Mean Responses by Gender</th>
<th>Both Genders</th>
<th>Widowers</th>
<th>Widows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treat × Post</td>
<td>585</td>
<td>-6,623***</td>
<td>3,403***</td>
</tr>
<tr>
<td>(667)</td>
<td>(1,342)</td>
<td>(729)</td>
<td></td>
</tr>
<tr>
<td>Counterfactual Earnings</td>
<td>150,994</td>
<td>163,010</td>
<td>145,969</td>
</tr>
<tr>
<td>Household FE</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>No. of obs.</td>
<td>686,521</td>
<td>220,125</td>
<td>466,392</td>
</tr>
<tr>
<td>No. of Households</td>
<td>114,462</td>
<td>36,705</td>
<td>77,756</td>
</tr>
</tbody>
</table>

Notes: This table reports the differences-in-differences estimates of the surviving spouses’ annual earnings by the level of their own earnings when their spouses died. The sample includes individuals whose spouses died between ages 45 and 80 from 1985 to 2011, where we constrain the sample in the following way. First, we exclude surviving spouses whose average labor income before the shock (in periods -5 to -2) was lower than their experimental-group-specific 20th percentile. Then, we calculate for each household the pre-shock labor income share of the deceased spouse out of the household’s overall labor income and include only households in which both spouses were sufficiently attached to the labor force; specifically, we keep households for whom the average share was between 0.20 and 0.80. These restrictions allow us to focus on households for which there has been some loss of income due to the death of a spouse and in which the surviving spouse has earned non-negligible labor income both in levels and as a share within the household. In addition, to guarantee that our results are not driven by outliers, we exclude households with dying spouses whose mean pre-shock earnings did not fall within their group-specific 5th and 95th percentiles or households with unaffected spouses whose mean pre-shock earnings were higher than their group-specific 95th percentile. We divide the remaining sample into five equal-sized groups by their pre-shock level of earnings. Panel A separately estimates a differences-in-differences specification for each surviving spouses’ quintile. Column 1 includes all surviving spouses; Column 2 includes households in which the dying spouses’ pre-shock labor income fell within the bottom three quintiles of its group-specific distribution, to which we refer as “low earners”; Column 3 includes households in which the dying spouses’ pre-shock labor income fell within the top two quintiles, to which we refer as “high earners”. The gradient is also robust to the inclusion of a quadratic in the household’s net wealth. Panel B reports the average treatment effect for this sample. The second row reports the counterfactual outcome based on the differences-in-differences estimation. The pre-shock periods include periods -5 to -3. The post-shock periods include periods 2 to 4. Robust standard errors clustered at the household level are reported in parentheses.

*** p<0.01, ** p<0.05, * p<0.1.
spouses’ quintile.\textsuperscript{47} Note that merely analyzing the average earnings response in this sample would have masked the substantial heterogeneity we documented. Panel B of Table 1.4 shows that the average labor income increase for this sample is DKK 585 (0.39\%) and is not statistically different from zero.

In summary, the results reveal a clear pattern: there are significant increases in labor supply in response to losing a spouse, which are entirely driven by households that experience large income losses. The results provide clear evidence of the self-insurance role of spousal labor supply in the extreme case of the death of a spouse, which translates into large and permanent income losses for most households.

\subsection*{1.5.2 Labor Disutility State Dependence}

In Section 1.2.3 we discussed the theoretical implications of state dependence of the unaffected spouse’s labor supply. If labor supply becomes more costly due to the shock (that is, $\theta^b > 1$), then for any given increase in labor supply it is more socially desirable to transfer resources to spouses in state $b$ to avoid their loss of (more valued) leisure. On the other hand, lower cost of labor supply ($\theta^b < 1$) can lead to an increase in labor force participation even if households are well insured. The welfare implications of labor supply responses in this case are different since they are driven by preferences and not by under-insurance. One empirical motivation to account for this sort of state dependence is the striking change in the surviving spouse’s health-care utilization following the loss of a spouse. Figure 1.11 shows that the overall expenditure on primary medical care (Panel A) as well as the prescription rate for antidepressants (Panel B) exhibit sharp increases in the year of bereavement. While part of these phenomena may be purely driven by changes in take-up of medical care and supply-side responses rather than in actual changes in health, it calls for an empirical investigation of labor disutility state dependence.

In this section, we provide a formal method to assess the extent to which survivors’ labor disutility changes in response to the death of their spouse and then apply it to our setting. Our key result, which we derive in detail below, is that individuals do not fully adjust their post-shock

\footnote{The gradient is also robust to the inclusion of a quadratic in the household's net wealth.}
Survivors’ Health-Care Utilization around the Death of Their Spouse

(a) Health-Care Costs

(b) Prescriptions for Antidepressants

Notes: These figures plot measures of survivors’ health-care use around the death of their spouse. The sample includes individuals born between 1930 and 1950 (for whom we have data on drug prescriptions) whose spouses died between ages 45 and 80 from 1985 to 2011. Panel A depicts overall expenditure on primary medical care, and Panel B depicts the prescription rate for antidepressants (Psycholeptics and Psychoanaleptics). The x-axis denotes time with respect to the shock, normalized to period 0. For the treatment group, period 0 is when the actual shock occurs; for the control group, period 0 is when a “placebo shock” occurs (while their actual shock occurs in period 5). The dashed gray line plots the behavior of the control group. To ease the comparison of trends, we normalize the level of the control group’s outcome to the pre-shock level of the treatment group’s outcome. This normalized counterfactual is displayed by the blue line and squares. The red line and circles plot the behavior of the treatment group.
consumption to their pre-shock level of consumption utility, implying that their labor disutility increases in response to the shock.

**Calibrating $\theta^b$: method.** Consider the following thought experiment. First, assume that we could mimic a full income insurance environment. Normally, this would imply fully insuring the household’s pre-shock level of income. However, in our case, as the composition of the household changes, we need to ask how much income does a surviving spouse need as a single to achieve the same level of consumption utility that he or she enjoyed before the shock? The classic answer to this question is the adult “equivalence scale”, which is commonly assumed to lie within the interval $(0.5,1)$. It is less than 1 since the household becomes a one-person household and is more than 0.5 due to economies of scale in consumption within a two-person household. We denote the equivalence scale by $r^0$. A direct implication of its definition is that in the absence of labor disutility state dependence, when $\theta^b = 1$, the labor force participation of the surviving spouses would not change across states of nature if they receive $r^0$ of their pre-shock household income.

Second, assume we observe the replacement rate – denoted by $r^{eq}$ – that surviving spouses are implicitly willing to accept in equilibria when their labor supply remains unchanged after they experience the shock. In that case, the comparison of the two replacement rates, $r^0$ and $r^{eq}$, can reveal the degree of state dependence. Intuitively, if $r^{eq} < r^0$, survivors are willing to accept less than “full insurance” to avoid self-insuring through labor supply, which implies an increase in its utility cost, $\theta^b > 1$. That is, incomplete adjustment of post-shock consumption (captured by $r^{eq}$) to the consumption level which achieves the pre-shock level of utility (captured by $r^0$) implies that labor disutility increased. If $r^{eq} \cong r^0$, then state dependence on average is likely to be negligible.

To formalize this procedure we begin by stating the following lemma:

---


49 We focus on the adult equivalence scale since we study older households. The median age of the youngest child of our treated individuals born after 1930 (for whom we have data on children) is 30, with only 10% having a youngest child under 18.
Lemma. Let $V^s(y^s(1))$ denote the household’s consumption value function when $w$ works in state $s$,\footnote{See Footnote 24 for its formal definition.} $\theta^u \equiv V^{b^u}(y^b(1))/V^{g^u}(y^g(1))$ denote the change in the marginal value of household income, and $\gamma \equiv -[V^{g^u}(y^g(1))/V^{g^u}(y^g(1))] \times y^g(1)$ denote the household-level pre-shock relative risk aversion. Then, in equilibria in which $w$’s labor supply is the same in state $g$ and state $b$ the following holds

$$\theta^u(1 + \gamma(1 - r^{eq})) \approx \theta^b,$$

(1.8)

where $r^{eq} \equiv y^b(1)/y^g(1)$ is the steady state replacement rate that satisfies this relationship.

Proof. The proof relies on the necessary relationship between household income streams across states of nature in equilibria where labor supply remains unchanged such that $\bar{v}^b_w = \bar{v}^b_w$, where $\bar{v}^b_w \equiv \frac{1}{\theta^b}[V^s(y^s(1)) - V^s(y^s(0))]$. See Appendix D for details.

The relationship in (1.8) has a simple intuition: if labor supply is unchanged when the shock occurs, then the change in the cost of labor must equal the change in the marginal utility from income. The right-hand side of the equation captures the change in the marginal entrant’s labor disutility by the definition of $\theta^b$. The left-hand side evaluates the marginal utility from income in the new state. It is the baseline pre-shock marginal utility from income (normalized to one), augmented by the change in the marginal utility due to income changes, $1 - r^{eq}$, and the curvature of the consumption value function $V^s(y^s(1))$, $\gamma$. Then, we multiply the resulting expression by the change in the marginal value of household income across states, $\theta^u$.

When $r^{eq}$ is directly observed (i.e., revealed by individuals’ choices) – as in the case of widowers who do not change their mean participation rate when their wives die – we can recover $\theta^b$ with two simple steps, which correspond to the two steps of the intuitive explanation above. First, since (1.8) is satisfied when $\theta^b = 1$ and $r^{eq} = r^0$, we can recover $\theta^u$ by $\theta^u = 1/(1 + \gamma(1 - r^0))$, if we borrow estimates for $r^0$ from the literature as we discuss below. Second, we can use $\theta^u$ and the observed $r^{eq}$ to recover $\theta^b$ using (1.8), such that $\theta^b \approx \frac{1 + \gamma(1 - r^{eq})}{1 + \gamma(1 - r^0)} = 1 + \frac{\gamma(r^0 - r^{eq})}{1 + \gamma(1 - r^0)}$. This formalizes our intuition: whenever $r^{eq} < r^0$ – that is, whenever survivors are willing to accept less than what they need – it
follows that self-insurance became more costly when the shock occurred, $\theta^b > 1$.

When $r^{eq}$ is not directly observed by choices – e.g., when participation increases in response to a shock – we can use the equilibrium responses to construct a bound on $\theta^b$ with the additional identifying assumption of monotonicity as defined below.

Assumption (monotonicity). Define the potential outcome $Y_i(0)$ to be i’s participation decision that would be realized were he or she not to experience a shock and $Y_i(1)$ to be i’s participation decision that would be realized if he or she were to experience a shock. If $Y_i(1) \geq Y_i(0)$ for every $i$, we say that monotonicity is satisfied.

Under monotonicity, the mean increase in the participation of spouses is driven by individuals who switch from working to not working (“compliers”), while the remaining spouses either keep working (“always-takers”) or stay out of the labor force (“never-takers”). Given this response, we observe an aggregate income replacement rate in the data, denoted by $r'$, which is composed of the rate among compliers, denoted by $r'_c$, and a replacement rate for the rest of the sample. Now, assume that we change the environment only by offering the compliers a higher income if they do not work. Since working is costly, there must exist $r''_c < r'_c$ such that compliers prefer receiving $r''_c$ without working to receiving $r'_c$ and working. Therefore, under monotonicity, in an equilibrium in which the mean participation rate does not change when the shock occurs, $r^{eq}$ (which involves $r''_c$) must be smaller than the $r'$ that we actually observe (which involves $r'_c$). This imposes a lower bound on $\theta^b$ such that $\theta^b \leq 1 + \frac{\gamma(r^0 - r^{eq})}{1 + \gamma(1 - r^0)} \geq 1 + \frac{\gamma(r^0 - r')}{1 + \gamma(1 - r^0)}$.

Calibrating $\theta^b$: results. We begin by studying the implications of a commonly used equivalence scale – the modified OECD equivalence scale which implies $r^0 = 0.67$. Other widely used adult equivalence scales deliver similar approximations.\textsuperscript{51} Combining widows and widowers in Panel B of Figure 1.6 yields an average post-shock replacement rate of $r' = 0.665$. Given the increase in mean labor force participation and using the bound we derived above, these estimates imply that $\theta^b \geq 1$ and suggest that state dependence is negligible.

Next, we consider model-based estimates for adult equivalence scales. In particular, we use

\textsuperscript{51} For example, the square-root scale which implies $r^0 = 0.71$ (see, e.g., Cutler and Katz 1992 and OECD 2011). Note that the implicit equivalence scale in the Danish Social DI is approximately 0.65 and is 0.66 in the Old-Age Pension. See Section 1.3 for institutional details.
recent estimates from Browning, Chiappori, and Lewbel (2013), which offer separate estimates for “indifference scales” for men and women.\footnote{Their notion of “indifference scales” is an individual-based version of equivalence scales, which aims at identifying the fraction of the household’s income a member would need in order to buy a bundle of privately consumed goods at market prices that put him or her on the same indifference curve over goods that he or she attained as a member of the household. Their method relies on recovering the consumption demand functions of individuals within a household based on a collective household model, which they estimate by using the Canadian Survey of Family Expenditures.} Since widowers do not change their mean participation rate when their wives die, we can directly observe their \( r^{eq} \). Recall from Panel B of Figure 1.6 that widowers experience an actual loss of 31\% in household income and hence for them \( r^{eq} = 1 - 0.31 = 0.69 \). This implies that they are willing to accept 69\% of their pre-shock level of household income to avoid increasing their labor supply. Browning, Chiappori, and Lewbel (2013) find that in households with equal sharing of income among the two spouses, the indifference scale for males is about 0.80. This suggests that for widowers \( r^0 = 0.80 > r^{eq} = 0.69 \) and thus on average their labor disutility increases when they lose their wives – that is, \( \theta^{b} \approx 1 + \frac{0.112}{1 + 0.69} > 1 \). For widows, who increase their labor force participation, we can recover a bound for state dependence using Browning, Chiappori, and Lewbel’s (2013) indifference ratio of 0.72. Recall from Panel B of Figure 1.6 that for widows \( r' = 1 - 0.35 = 0.65 \). This implies a lower bound of \( \theta^{b} \geq 1 + \frac{0.072}{1 + 0.72} > 1 \), which suggests that on average labor disutility likewise increases for widows when they lose their husbands.

While these calibrations are suggestive, they provide further evidence that our results for the surviving spouses’ labor force participation are driven by self-insurance and large income losses.\footnote{In section 1.6, we also show that the increase in widows’ participation due to the shock declines in the formal insurance they receive from the government, which further strengthens the self-insurance hypothesis.} The data is inconsistent with the conjecture that the increases in labor supply are driven by lower cost of labor (e.g., due to the desirability of social integration) since in that case we would expect to observe noticeably larger actual replacement rates than the “needed” ones suggested by equivalence scales.\footnote{In addition, we test a potential implication of the specific hypothesis that seeking social integration after losing a partner with whom the surviving spouse has spent his or her leisure time may drive his or her labor supply response. Consider surviving spouses who did not work before the shock in a model where time is divided between labor and leisure. A spouse in a household in which the deceased spouse did not work before his or her death consumed more joint leisure and may have a higher chance of experiencing loneliness. On the other hand, a spouse in a household in which the deceased spouse worked before his or her death consumed less joint leisure and experience larger income losses. The social integration hypothesis is consistent with the former household increasing its labor supply more than the latter does, while the self-insurance hypothesis is consistent with the opposite pattern. The data reveals that labor supply increases are driven by the latter group, which provides additional evidence for the social integration hypothesis.}
1.5.3 Labor Supply Responses to Spousal Health Shocks

In this section we briefly study individuals’ labor supply responses to severe health shocks to their spouses. The purpose of studying this additional shock is to provide further evidence for the self-insurance hypothesis of spousal labor supply. Recall that our analysis sample for this shock consists of households in which a spouse experienced a heart attack or a stroke (for the first time) and survived for at least four years (until \( t = 3 \)), and in which both spouses were under age 60.

Panel A.1 of Figure 1.12 shows that within three years of the shock, the affected spouse’s participation sharply falls, which translates into a large loss of annual earnings as shown in Panel A.2. Table 1.5 quantifies these effects by estimating a differences-in-differences regression, in which we allow for differential treatment effects in the “short run” (periods 1 and 2) and the “medium run” (period 3), to account for the gradual responses documented in Panel A of Figure 1.12.\(^{55}\) Columns 2 and 4 of Table 1.5 reveal that by the third year after the shock the labor force participation of the sick spouse drops by 12 pp – about 17% – and that annual earnings drop by DKK 36,015 ($4,500) – a significant drop of 19%.

However, while there is a significant drop in the sick spouse’s earnings, Columns 5 and 6 of Table 1.5 show that the actual loss of income that the household experiences is much smaller and amounts to only 3.3% of overall household income. That is, taking into account the entire household income, including any transfers from social or private sources, reveals that these shocks are very well-insured in our Danish setting. Therefore, as shown in Panel B of Figure 1.12 and Columns 7 to 10 of Table 1.5, there are no economically significant labor supply responses among unaffected spouses as there is no significant need to self-insure.

Note that the rich data-set and our research design allow for a precise estimation of these eco-

---

\(^{55}\) We estimate the following specification

\[
y_{i,t} = \beta_0 + \beta_1 \text{treat}_i + \beta_{2a} \text{post}^{a}_{i,t} + \beta_{3a} \text{treat}_i \times \text{post}^{a}_{i,t} + \beta_{2b} \text{post}^{b}_{i,t} + \beta_{3b} \text{treat}_i \times \text{post}^{b}_{i,t} + \alpha_i + \epsilon_{i,t}. \tag{1.9}
\]

where \( y_{i,t} \) denotes an outcome of household \( i \) at time \( t \), \( \text{post}^{a}_{i,t} = 1 \) in periods 1 and 2 and zero otherwise, and \( \text{post}^{b}_{i,t} = 1 \) in period 3 and zero otherwise. Therefore, \( \beta_{3a} \) captures the “short-run” effect, and \( \beta_{3b} \) captures the “medium-run” effect.
FIGURE 1.12
Household Labor Supply Responses to Severe Health Shocks in which the Affected Spouse Survived

(a) Affected Spouse

(1) Labor Force Participation

(2) Annual Earnings

(b) Unaffected Spouse

(1) Labor Force Participation

(2) Annual Earnings

Notes: These figures plot the labor supply responses of households in which an individual experienced a heart attack or a stroke between 1985 and 2011 and survived for at least three years. The sample includes households in which both spouses were under age 60. Panels A.1 and A.2 depict the labor force participation and annual earnings of the individual that experienced the shock, respectively. Panels B.1 and B.2 depict the labor force participation and annual earnings of the unaffected spouse, respectively. The x-axis denotes time with respect to the shock, normalized to period 0. For the treatment group, period 0 is when the actual shock occurs; for the control group, period 0 is when a “placebo shock” occurs (while their actual shock occurs in period 5). The dashed gray line plots the behavior of the control group. To ease the comparison of trends, we normalize the level of the control group’s outcome to the pre-shock level of the treatment group’s outcome. This normalized counterfactual is displayed by the blue line and squares. The red line and circles plot the behavior of the treatment group.
### TABLE 1.5
Household Responses to Severe Health Shocks in which the Affected Spouse Survived

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Affected Spouse</th>
<th>Household Income</th>
<th>Unaffected Spouse</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Participation</td>
<td>Earnings</td>
<td>Participation</td>
</tr>
<tr>
<td></td>
<td>Short Run</td>
<td>Medium Run</td>
<td>Short Run</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Treat × Post</td>
<td>-0.0861***</td>
<td>-0.1212***</td>
<td>-29.012***</td>
</tr>
<tr>
<td></td>
<td>(.0023)</td>
<td>(.0027)</td>
<td>(741)</td>
</tr>
<tr>
<td>Household FE</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Counterfactual Post-Shock Mean of Dependent Var.</td>
<td>.7328</td>
<td>.7147</td>
<td>195,433</td>
</tr>
<tr>
<td>Percent Change</td>
<td>-12%</td>
<td>-17%</td>
<td>-15%</td>
</tr>
<tr>
<td>Percent Change Excluding the Unaffected Spouse’s Responses</td>
<td>-2.1%</td>
<td>-3.3%</td>
<td></td>
</tr>
<tr>
<td>Number of Observations</td>
<td>644,699</td>
<td>646,272</td>
<td>645,817</td>
</tr>
<tr>
<td>Number of Households</td>
<td>92,349</td>
<td>92,358</td>
<td>92,356</td>
</tr>
</tbody>
</table>

Notes: This table reports the differences-in-differences estimates of household labor supply responses to severe health shocks in which the affected spouse survived and the effect of these shocks on overall household income (equation (9) in footnote 54). The sample includes households in which one spouse experienced a heart attack or a stroke and survived for at least three years, and in which both spouses were under age 60. The treatment group comprises households that experienced the shock in different years, to which we match households that experienced the same shock five years later as a control group (Δ=5). We allow for differential treatment effects for the “short run” – periods 1 and 2 – and the “medium run” – period 3, to account for the gradual responses documented in Panel A of Figure 11. The pre-shock periods include periods -5 to -2. Household income (Columns 5 and 6) includes income from any source – including earnings, capital income, annuity payouts, and benefits from any social program. The third row reports the counterfactual outcome based on the differences-in-differences estimation. Robust standard errors clustered at the household level are reported in parentheses.

*** p<0.01, ** p<0.05, * p<0.1.
nomically insignificant spousal responses to shocks. In particular, our results imply a small but positive degree of complementarity in spouses’ labor supply in response to health shocks, with an estimate of 0.065 for the unaffected spouse’s earnings elasticity with respect to the affected spouse’s earnings. Since the household’s income is not perfectly insured, this response implies – in the context of our theoretical framework – health-state dependence of the household’s utility. Intuitively, the fact that given a small loss in income the unaffected spouse’s decrease in labor supply involves an additional (very small) loss is consistent with two main state dependence channels. First, it is consistent with households in the bad state valuing income less than do households in the good state – i.e., a consumption utility state dependence. Second, it is consistent with an increase in the unaffected spouse’s utility loss from time spent away from home either because he or she would like to take care of his or her sick spouse or due to his or her preference for joint leisure – i.e., a labor disutility state dependence. With no additional assumptions, we can only reach conclusions about the ratio of these two types of potential state dependence. See Appendix E for a formal analysis.

1.6 Welfare Implications

We now turn to illustrate our method for welfare analysis and to study the welfare implications of the surviving spouses’ labor supply responses. In accordance with the theoretical analysis in Section 1.2.2, consider the following policy question: how should we divide a given budget between households of widows and non-widows? This is essentially a comparison of the social “returns” of two “investment” vehicles – a $1 transfer to non-working spouses in state $b$ that yields a return of $u'_w(c^b_w(0))$ vs. a $1 transfer to non-working spouses in state $g$ that yields a return of $u'_w(c^g_w(0))$. We therefore focus on the marginal benefit from increasing $b^b$ by lowering $b^g$, and abstract from the associated costs on which the literature has focused. We employ the formula of Proposition 1 (equation (1.4)) and abstract from labor disutility state dependence by assuming $\theta^b = 1$. Since the data is consistent with $\theta^b \geq 1$, this approach delivers a lower bound on the welfare gains from this policy change (as implied by Proposition 2).

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56 This may explain the survey-based noisy estimates of Cole (2004) and Meyer and Mock (2013), who study responses to health shocks in the US. Note that Meyer and Mock (2013) similarly find that the typical disabled individual in the US loses about 21% in earnings but only 6.75% in post-transfer household income by the fourth year after the shock.
In order to assess the marginal benefit from this policy perturbation, we need to calibrate the ratio \( \frac{\varepsilon(b^h, b^w)}{\varepsilon(b^w, b^w)} \). Here we make the simplifying assumption of equal elasticities and use the approximation that this ratio is locally constant, which allows us to illustrate our method in the simplest possible way.\(^57\) Assuming that \( \frac{\varepsilon(b^h, b^w)}{\varepsilon(b^w, b^w)} = 1 \), the formula for the welfare benefits is reduced to

\[
MB(b^b) \cong \frac{b^g}{y^b} \times \frac{e^w}{e^w} - 1.
\]

To study existing social programs in Denmark, we divide the analysis into two sub-populations. First, we consider widows over age 67, who are eligible for the Danish Old-Age Pension (the equivalent of Social Security in the US) and analyze the perturbation within this program. Second, we consider widows younger than 67 who are more attached to the labor force and analyze changes to Social DI benefits for which they can apply.

**Old-Age Pension.** In Panel A of Figure 1.13 we plot the responses of widows over 67. Panel A.1 reveals that even the elderly need to self-insure and increase their participation by 1.08 pp on a very low base of 1.19 pp. This implies that the participation rate of widows over 67 almost doubles when their husbands die with \( \frac{e^h}{e^w} = \frac{0.0227}{0.0119} = 1.91 \). As the Old-Age Pension includes adjustments to the household’s composition (as explained in Section 1.3 and seen in practice in Panel A.2 of Figure 1.13), widows during our sample period received on average DKK 87,454 ($10,932) and their non-widow counterparts received DKK 70,684 ($8,836) such that for this population \( \frac{b^h}{b^w} = \frac{70,684}{87,454} = 0.81 \). Together, these imply that

\[
MB(b^b) \cong \frac{b^g}{y^b} \times \frac{e^w}{e^w} - 1 = 0.81 \times 1.91 - 1 = 0.55.
\]

That is, an additional $1 transferred to widows through the Old-Age Pension creates a net benefit equivalent to 55 cents as compared to transferring $1 to non-widows. This large marginal benefit from an additional dollar to elderly widows is driven by their significant relative increase in participation, which reveals their high valuation of additional insurance. This suggests that in-

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\(^57\) In Appendix F we provide suggestive estimates – that require some additional structure – for these elasticities in the case of surviving spouses under 60 that imply a ratio of 1.375.
FIGURE 1.13
Widows’ Labor Force Participation and Government Transfers around the Death of Their Spouse by Age Group

(a) Widows over Age 67

(1) Labor Force Participation

(2) Old-Age Pension Benefits

(b) Widows under Age 67

(1) Labor Force Participation

(2) Take-Up of Social Disability Insurance

Notes: These figures plot outcomes for survivors around the death of their spouse by age group. Panel A plots outcomes for widows over age 67 whose husbands died between 1985 and 2011. Panel A.1 plots their labor force participation, and Panel A.2 plots the benefits they received from the Old-Age Pension program. Panel B plots outcomes for widows under age 67 (the age at which the Social Disability Insurance transitions into the Old-Age Pension) in years prior to 1994 (when there is a data break in the reporting method of benefits received through Social Disability Insurance) whose husbands died between 1985 and 2011. Panel B.1 plots their labor force participation, and Panel B.2 plots their take-up of the Social Disability Insurance program. The x-axis denotes time with respect to the shock, normalized to period 0. For the treatment group, period 0 is when the actual shock occurs; for the control group, period 0 is when a “placebo shock” occurs (while their actual shock occurs in period 5). The dashed gray line plots the behavior of the control group. To ease the comparison of trends, we normalize the level of the control group’s outcome to the pre-shock level of the treatment group’s outcome. This normalized counterfactual is displayed by the blue line and squares. The red line and circles plot the behavior of the treatment group.

55
creasing the relative compensation to older widows within the Old-Age Pension beyond the current household-composition adjustment entails significant welfare improvement.

**Social Disability Insurance.** To focus on the value of Social DI, we constrain the sample to widows under 67 (the age at which the program transitions into the Old-Age Pension). In addition, we constrain the sample to the period prior to 1994 due to a data break in the reporting method of benefits received through Social DI. Panel B.1 of Figure 1.13 plots the labor force participation behavior of this sample and shows that 
\[
\frac{e_{bw}}{e_{bw}} = \frac{0.4718}{0.4537} = 1.04,
\]
which is smaller than the effect among the elderly as well as among the overall sample of widows as shown in Section 1.5.1. Panel B.2 of Figure 1.13 clearly displays the insurance role of Social DI for widows, whose take-up of the program increases by more than 50% in the year that their husbands die. For this time period, the mean benefits received from Social DI by those on the program are the same for widows as for non-widows and, therefore, \( \frac{\beta}{\theta} \simeq 1 \). Combining these estimates, it follows that

\[
MB(b^h) = \frac{\beta^g}{\beta^b} \times \frac{e_{bw}}{e_{bw}} - 1 = 1 \times 1.04 - 1 = 0.04.
\]

That is, an additional $1 transfer to widows through Social DI is worth 4 cents more to each household than is transferring this additional $1 to non-widows. These small (but positive) welfare gains are a direct result of the relative increase in labor force participation among widows that are eligible for this program (under 67), which is smaller than the effect among the universe of widows.

Therefore, a key implication of our findings, driven by the differential attachment of individuals to the labor force over the life-cycle, is that the social insurance policy should be age-dependent.

An additional valuable welfare exercise allows us to use our method to assess how far the benefits are from their optimal levels by evaluating the local rate at which marginal benefits change, \( MB'(b^h) \). To estimate this derivative we take advantage of spatial variation in the administration of Social DI. Recall that while Social DI is a state-wide program, it is locally administered so that regional councils decide whether to approve or reject an application and municipal caseworkers (in a total of 270 municipalities) administer the application and handle all aspects of each case. Since this structure has led to substantial variation in rejection rates across municipalities, it has created

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58 The exact figures are \( b^h = 8,115 \) for widows and \( b^g = 8,016 \) for non-widows (in 2000 dollars).
significant variation in the mean receipts of Social DI benefits across the different municipalities over time (Bengtsson 2002). We use these year by municipality average receipts as an instrument for actual receipts. In particular, we calculate for each municipality, the average benefits received by non-working surviving spouses through Social DI in each year. Then, we assign to each widow of household \(i\) in the treatment group the respective mean in municipality \(m\) at time \(t\) excluding her own benefits (the “leave-one-out” mean), denoted by \(\overline{\text{DI}}_{-i,t,m}\). We estimate the following augmented differences-in-differences regression

\[
l_{w,i,t} = \beta_0 + \beta_1 \text{treat}_i + \beta_2 \text{post}_i,t + \beta_3 \text{treat}_i \times \text{post}_i,t + \beta_4 X_{i,t} + \varepsilon_{i,t},
\]

where

\[
\beta_{3i} = \beta_{30} + \beta_{31} \text{DI}_{i,t}.
\]

In this regression, \(l_{w,i,t}\) denotes the participation of individual \(w\) of household \(i\) at time \(t\), and \(X_{i,t}\) includes municipality \(m\)’s unemployment rate and average earnings, as well as age, year and municipality fixed effects. \(\text{DI}_{i,t}\) are actual Social DI receipts for which we instrument using \(\overline{\text{DI}}_{-i,t,m}\). The identifying assumption is that, given our set of controls, the average of Social DI benefits transferred to widows in a municipality in a given year affects a widow’s participation only through its influence on her own DI receipts. Note that the source of variation we use is within municipalities over time since we include municipality and calendar year fixed effects as controls. The two-stage least squares results are presented in Table 1.6. The estimate for our parameter of interest, \(\beta_{31} = \frac{\partial(e_{t}^b - e_{w}^b)}{\partial b^b}\), is -.0057. \(^{60}\)

Using this estimate, Figure 1.14 plots the behavior of \(MB(b^b)\) around the sample mean of DKK 65,000 ($8,115). The figure shows that additional DKK 1,500 ($188) in annual benefits decrease the excess benefit to zero. Converting these monetary values into net replacement rates out of

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\(^{59}\)The F-statistic on the excluded instrument in the first stage is 24.3.

\(^{60}\)The benefits \((b^b)\) are measured in annual DKK 1,000 ($125) terms. With an average of DKK 23,362 ($2,908) in actual DI receipts by widows in the analysis sample (including zeros for those not on the program) and a participation rate of 0.5054, this estimate implies a participation elasticity of \(e(e_{w}^b, b^b) = -0.26\) for widows under 67.
**TABLE 1.6**
Widows’ Labor Force Participation Responses to the Death of Their Spouse by Social Disability Benefits

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>Widows’ Participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treat × Post × DI</td>
<td>-.0057***</td>
</tr>
<tr>
<td>Average Treatment Effect</td>
<td>(0.0020)</td>
</tr>
<tr>
<td>Counterfactual Participation</td>
<td>48.7 pp</td>
</tr>
<tr>
<td>No. of obs.</td>
<td>364,100</td>
</tr>
<tr>
<td>No. of clusters</td>
<td>268</td>
</tr>
</tbody>
</table>

Notes: This table reports the interaction of the treatment effect of the death of a spouse with the actual Social Disability Insurance (Social DI) benefits widows received (equation (10)). The regression is estimated by two-stage least squares, where the instrument for actual benefits is constructed as follows. In each year we calculate for each municipality the average benefits received by non-working surviving spouses through Social DI. Then, we assign to each widow in the treatment group her respective municipality-year leave-one-out mean. The sample includes widows under age 67 (the age at which the program transitions into the Old-Age Pension) in years prior to 1994 (when there is a data break in the reporting method of benefits received through Social DI). The controls included in the estimation are municipality unemployment rate and average earnings as well as age, year and municipality fixed effects. The identifying assumption is that, given our set of controls, the average Social DI benefits transferred to widows in a municipality in a given year affects a widow’s participation only through its influence on her own DI receipts. Note that the source of variation we use is within municipalities over time since we include municipality and calendar year fixed effects as controls. The pre-shock periods include periods -5 to -3. The post-shock periods include periods 2 to 4. Robust standard errors clustered at the municipality level are reported in parentheses.

*** p<0.01, ** p<0.05, * p<0.1.
FIGURE 1.14

Welfare Gains from Survivors Benefits within the Social Disability Insurance Program

Notes: This figure plots the marginal benefit from transfers to widows within the Social Disability Insurance program. The x-axis denotes the benefit level, \( b^b \), measured in Danish Kroner (DKK), and the y-axis denotes the marginal benefit, \( MB(b^b) \). The vertical dashed line at DKK 65,000 ($8,115) denotes the mean benefits transferred to widows who are on the program. It represents a net replacement rate (denoted by “net rr” in the figure) of 0.648 relative to the mean pre-shock annual earnings of deceased spouses who worked before they died. To convert the monetary values into net replacement rates out of the deceased spouse’s pre-shock earnings, we calculate the average earnings in \( t = -1 \) for affected spouses who had positive earnings in the year before they died. The average is DKK 170,000 ($21,250), which implies net wage earnings of DKK 100,300 ($12,538) using an average labor income tax rate of 41% (OECD estimates). The vertical dashed line at DKK 66,500 ($8,300) denotes the benefit level that sets the marginal benefit to zero. It represents a net replacement rate (denoted by “net rr” in the figure) of 0.663. This suggests that for widows under 67 the current levels are near optimal.
the deceased spouse’s pre-shock earnings,\textsuperscript{61} the current system stands at 0.648 and the optimal allocation of benefits across states stands at 0.663, suggesting that for younger widows the current levels are near optimal. To evaluate the overall value of the program, we can approximate the integral \( \int_0^{65,000} MB(b)b^b \) by using our estimates. This integral answers the question: within the Social DI system in Denmark, what is the welfare gain from the benefits given to widows relative to non-widow beneficiaries of the program? The estimate amounts to DKK 99,942 (\$12,500) annually, which means that transfers to widows relative to non-widows create a benefit of (\$12,500/\$8,115-1=) 54\%. That is, on average, each dollar given to younger widows through Social DI generates a net benefit equivalent to 54 cents relative to a dollar given to non-widow recipients, which reveals the large social value of survivors benefits.

1.7 Conclusion

This paper provides clear evidence of household self-insurance through labor supply in response to large and persistent income losses and develops a new labor market method for welfare analysis of social insurance. Studying the critical event of the death of a spouse, we find large increases in the surviving spouses’ labor force participation rate driven by households for whom this event imposes significant income losses. We show that the unaffected spouse’s self-insurance response fully reveals the household’s marginal utility from consumption. As the gap in marginal utilities across states of nature captures the value of insurance, we offer a way to recover the gains from social insurance based solely on spousal labor supply responses. Applying this method to spousal mortality shocks, we show that allocation of additional resources to elderly widows has significant welfare gains and that survivors benefits should be age-dependent.

We additionally exploit the Danish setting to analyze households in which an individual has experienced a severe health shock but survived, for which income losses are well-insured. Together, the results point to a potential explanation for the elusiveness of the insurance role of spousal labor

\textsuperscript{61} We calculate the average earnings in \( t = -1 \) for affected spouses who had positive earnings the year before they passed away. The average is DKK 170,000 (\$21,250), which implies net wage earnings of DKK 100,300 (\$12,538) using an average labor income tax rate of 41\% (OECD estimates).
supply in previous literature. In support of the hypotheses raised by Heckman and MaCurdy (1980) and Cullen and Gruber (2000), we find that spousal labor supply plays a significant self-insurance role when the income loss incurred by the shock is large relative to the household’s lifetime income – as in the death of a spouse – and is irrelevant when the loss is sufficiently insured through formal social insurance – as in spousal health shocks.

Our findings have further implications for potentially improving efficiency in the distribution of government benefits. The significant heterogeneity in responses we find across different pre-shock dimensions of household characteristics suggests that enriching the policy tools to condition transfers on these observable characteristics may be welfare improving. For example, since increases in the surviving spouse’s labor supply are strongly correlated with the income shock that he or she experiences after losing an earning spouse, it may be welfare improving to let survivors benefits increase in the deceased spouse’s pre-shock share of annual household earnings.62

More broadly, our quasi-experimental design for identifying the effect of shocks as well as our method for welfare analysis can be applied to other important economic questions. Our research design, which relies on comparing households that are affected only a few years apart, can be applied to estimating the effect of a shock in any setting in which its exact timing is likely to be random. Our welfare analysis method, which relies on spousal labor supply, can be applied to evaluating the welfare gains from social insurance in any setting in which the directly affected individual may be at a corner solution. For example, relevant to the debate on the privatization of Social Security, the value of protecting against pension-wealth losses in the 401(k) account of a working individual can be recovered by the labor supply response of his or her spouse. Spousal labor supply can also be used to evaluate the welfare losses caused by the discontinuation of an employee’s compensation, such as health insurance, as well as the value of unemployment insurance for the long-term unemployed (whose long durations of unemployment significantly harm their employment prospects).63

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62 A similar feature is implicit in the US system, where survivors are eligible for their deceased spouses’ Social Security benefits, which are a function of the deceased’s work history.

63 See Kroft, Lange, and Notowidigdo (2013) on the adverse effect of longer unemployment spells.
2 DO EMPLOYER PENSION CONTRIBUTIONS REFLECT EMPLOYEE PREFERENCES? EVIDENCE FROM A RETIREMENT SAVINGS REFORM IN DENMARK64

2.1 Introduction

With the decrease in defined-benefit pension plans, individual savings are becoming an increasingly important income source for post-retirement consumption. Recent research shows that employer contributions and default contribution rates to workers’ retirement savings accounts have large effects on individuals’ overall saving since most workers do not actively deviate from the default (Madrian and Shea 2001, Choi et al 2004, Beshears et al. 2009, Gelber 2011, and Chetty et al. 2014). This research has led policymakers to consider introducing policies that encourage employer contributions to pension accounts in order to increase individuals’ retirement savings. However, it is theoretically unclear how profit-maximizing employers should manage their employees’ pension contributions and portfolios, and it has not been empirically studied how employers set contribution rates in practice. The increased reliance on employer-based savings accounts necessitates our understanding of how they are designed and whether important parameters of these accounts – e.g., the contribution rate and the plans’ portfolio – are set to reflect individuals’ preferences.65

This paper takes a first step at addressing these important issues by testing whether employer savings contributions reflect employees’ saving incentives. To do so, we exploit a reform to the Danish retirement savings system. This reform differentially affected employees according to their location on the labor-income tax schedule and differentially changed tax deductions for contributions to capital savings accounts, which are paid out in full at retirement, and annuity savings accounts, which are paid out as an annuity. Specifically, in 1999, the Danish government decreased the subsidy for capital pension contributions for workers in the top income tax bracket, while the capital pension

64 This chapter is jointly written with Jessica Laird and Torben Heien Nielsen.

65 It is not theoretically obvious ex-ante that employers have the proper incentives to do so, given that most employees do not actively save in their individual accounts and may have only imperfect knowledge about their employer pension plans (Mitchell 1988, Gustman et al. 2009). Thus, employees may not even value their employers’ actively setting pensions contribution.
tax incentives for workers in lower tax brackets remained unchanged. There was also no change in the subsidies for annuity pension contributions.

This quasi-experimental setting provides us with three empirical tests. First, if employers set contribution rates to cater to their employees' savings incentives, then employer-based contributions to capital accounts should decrease in response to the reform. Second, employers' responses should be in proportion to the fraction of their workers who are directly affected by the reform. Third, since workers care about their overall level of savings, contributions by optimizing firms should depend on their workers' individual pension savings behavior. Therefore, firms with a larger share of workers that were affected by the reform and were passive savers – that is, did not actively save through private accounts – would be predicted to have a larger response. Furthermore, firms with a larger share of workers who are affected by the reform and are passive savers – that is, they do not actively save through private accounts – should have a larger response.

We find strong evidence that employers set their contribution rates in accordance with their workers' savings incentives. On average, employers decreased capital contributions by 25% in response to the reform – a decrease of 0.7 percentage points (pp) from a baseline contribution rate of 2.9 pp. Importantly, we find that firms' responses to the policy change were continuously increasing in the share of affected workers, with no responses in workplaces where all the employees had earnings below the top income tax bracket and a 1.8 pp decrease in employer capital contributions in workplaces where all the employees had earnings above the top bracket. Furthermore, we find that the elasticity of the firm's response to the policy change with respect to the share of affected employees who actively saved in private (non-employer-based) accounts was -0.7, suggesting that firms substitute for workers' passive saving behavior.

In addition to the empirical literature on employer-based retirement savings, this paper relates to theoretical work on optimal savings policies and defaults.\(^{66}\) Choi et al. (2003) analyze the optimal

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\(^{66}\) Note that while our analysis focuses on employer contributions, which are different from defaults in that there are more restrictions on the ability of the individual to change the level of contributions, their difference from savings default rates is insubstantial in the Danish context. Since individual savings accounts have the same tax benefits as employer-based savings accounts, individuals can achieve their optimal savings level and distribution between accounts by increasing or decreasing their individual savings contributions. Since individual and employer-based savings accounts are perfect substitutes in our setting, our results can be directly mapped into implications regarding default savings rates. The only case in which individuals cannot fully offset employer contributions is when they are at a corner solution of zero private contributions and they want to decrease their private savings in response to an increase in employer savings. However, this scenario is not relevant in our analysis since
default level in an environment of workers with heterogeneous savings preferences who are susceptible to procrastination and have the option to opt-out of defaults. Carroll et al. (2009) consider a similar environment, but derive conditions under which the optimal enrollment regime is default enrollment, default non-enrollment, or compulsory choice. Other papers analyze optimal pension schemes (other than defaults) when agents are heterogeneous in their rationality and sometimes productivity (Cremer, De Donder, Maldonado, and Pestieau, 2008; Cremer and Pestieau 2011; Roeder 2014). Additionally, Goda and Manchester (2013) consider a firm that is choosing a default of the type of pension plan for employees - defined benefit or defined contribution - and find that there considerable welfare gains if defaults vary by observable characteristics (in their case age). This relates to our theoretical results which show that if firm’s maximize their workers welfare, there should be heterogeneity in default rates based on the fraction of workers who are above the top tax threshold.

While testing for optimality is beyond the scope of this paper, our results provide clear evidence for conditions that are necessary for the existence of such optimal decision-making process by the firm – namely, the responsiveness of employer contributions to their workers’ savings incentives. This is a necessary condition for any model which assumes that employer contributions are maximized with respect to workers’ welfare — whether directly if employers act as benevolent planners or indirectly through explicit bargaining or implicit contracts between the worker and the firm induced by competition between firms.

The paper proceeds as follows. In Section 2.2 we set the conceptual framework for the firm’s problem. We study a simple stylized model in which firms choose an optimal contribution rate for heterogeneous workers and analyze its predictions in the context of the 1999 reform. In Section 2.3, we discuss the institutional setting of the policy change and the data that we use. In Section 2.4, we test the theoretical predictions of the model and present our main empirical results on firms’ responses to the policy change and their heterogeneity with respect to workplace composition. Section 2.5 concludes.

Employers generally decrease their level of capital contributions in response to the reform, which individuals can completely offset by increasing their private capital contributions.
2.2 Conceptual Framework

To model how optimizing employers choose contribution rates when workers have heterogeneous preferences, we begin with a generalized model that could be applied to other common settings. We then review the particular institutional framework of our context and adjust the model to the specific quasi-experimental design we analyze. We solve the stylized model and derive predictions that we test in the empirical section.

2.2.1 A Generalized Model

Setup. Consider workers in firm $f$ at time $t$ and denote the total pension savings contribution rate of individual $i$ in firm $f$ by $s_{it}$. $s_{it}$ is a sum of individual $i$’s own contribution rate, $s^{I}_{it}$, and her employer’s contribution rate, $s^{E}_{it}$, such that $s_{it} = s^{I}_{it} + s^{E}_{it}$. Since we are analyzing a particular firm’s problem we drop the subscript $f$ for simplicity.

There are different types of workers, each with a different optimal pension savings rate at time $t$. We denote this savings rate by $s^{m}_{it}$, where $M$ is the set of “savings types” and $m \in M$. Therefore, the savings rate at time $t$ that maximizes individual $i$’s utility, $s^{*}_{it}$, satisfies $s^{*}_{it} = s^{m(i)}_{it}$, where $m(i) \in M$ is individual $i$’s savings type. We denote the fraction of individuals of type $m \in M$ by $\alpha_{m}$. Each period, individuals experience a loss from deviating from their optimal savings rate. For tractability, we assume a quadratic loss function such that the per-period loss is equal to the square of the difference between individuals’ optimal savings rate in that period, $s^{*}_{it}$, and their actual savings rate, $s_{it}$, such that $L(s^{*}_{it}, s_{it}) = (s^{*}_{it} - s_{it})^2$.

Individuals face a cost of adjusting their pension savings contributions. These can be actual monetary costs, such as fees to the pension fund account manager; opportunity costs, such as the loss of time spent re-optimizing; as well as cognitive costs, such as the effort of acquiring the necessary information for “solving” for the optimal savings rate. In addition, these adjustment costs can capture psychological biases which render active decision making costly. For example, present-biased individuals will tend to outweigh the cost of making savings choices relative to the benefits in any given period of time. They would, however, value reaching their optimal savings levels if
the cost of doing so were lower, which motivates the firm to substitute for their employees’ passive behavior.\textsuperscript{67}

To model these adjustment costs and their potential heterogeneity in the simplest way, we assume that within each savings type, \( m \), a fraction \( \lambda_m \) has infinite costs of adjustment, while a fraction \( 1 - \lambda_m \) has zero costs of adjustment.

\textit{Individual’s Behavior.} Since individuals determine their contribution to private savings accounts, those with infinite costs of adjustment – to whom we refer as “passive savers” – have zero private saving contributions upon joining the firm and in each period thereafter. Assuming there are no restrictions on the level of individual savings contributions, \( s^I_{it} \), individuals with zero adjustment costs will choose their individual savings such that their total level of savings is equal to their optimal level. That is, for them, \( s_{it} = s^*_{it} \) in each period. We refer to these individuals as “active savers.”

\textit{Employer’s Problem.} Since employer contributions do not affect the welfare of active savers in equilibrium (as they completely offset the firm’s contribution rate), the firm’s objective reduces to choosing a contribution rate, \( s^E_t \), that minimizes the weighted average of only its \textit{patent} workers’ loss from having a sub-optimal savings rate. If all workers are weighted equally by the firm, the firm chooses the contribution rate \( s^E_t \) that solves:

\[
\min_{s^E_t} \sum_{m=1}^{M} \alpha_m \lambda_m (s^E_t - s^m_t)^2.
\]

2.2.2 Adapting the Model to the Empirical Setting

Our empirical analysis studies the Danish population from 1996 to 2001, surrounding a 1999 reform in retirement savings. As a part of the reform, the tax deduction of contributions to capital pension accounts was reduced from 59 cents per Danish Kroner (DKr) to 45 cents per DKr for individuals in the top income tax bracket, while the deduction remained unchanged for workers in lower tax brackets. The tax on capital pension payouts remained unchanged at 40\% throughout the

\textsuperscript{67}This is the case when present-biased agents are sophisticated to some degree such that they are aware of their bias and value active firms that “correct” for their behavior.
observation window. To see the effect of the reform on the capital subsidy visually, Figure 2.1 plots
the tax deduction value of capital pension contributions net of the payout tax by year and whether
an individual is in the middle income tax bracket or in the top income tax bracket. Before the
reform in 1999, the subsidy from contributing to capital pensions was approximately three times
higher for workers above the top income tax threshold. In 1999, there was a large decrease for
the workers above the threshold, such that the subsidy became equal to that of the workers in the
middle tax bracket, and the subsidy for individuals in the middle tax bracket stayed constant.

To adapt our model to this empirical setting, we consider only two different savings types, \( h \)
(high) and \( l \) (low), each with a different level of optimal savings, such that for the high type
\( s^h_t = h_t \)
and for the low type \( s^l_t = l_t \). A firm has a fraction \( \alpha \) of the high type workers and a fraction \( 1 - \alpha \)
of low type workers. There are two periods, 0 and 1, which respectively capture the periods before
and after the reform. The optimal level of savings for the low type is the same in both periods,
such that \( l_0 = l_1 = l \), while the optimal level of savings for the high type decreases from period 0 to
period 1, such that \( h_0 > h_1 \). Hence, low-type savers correspond to the workers below the top tax
threshold, while high-type savers correspond to the workers above the threshold. Before the reform
(period 0), there were different levels of subsidies on capital accounts for the two groups, which
resulted in different levels of optimal savings. After the reform (period 1), the subsidy on savings in
capital accounts decreased for the workers above the threshold, which decreased their optimal level
of savings, but the subsidy remained the same for the workers below.\(^{68}\)

In this adapted model, firms set their savings contributions in the first period, \( s^E_0 \), to minimize
\( \alpha \lambda_h (s^E_0 - h_0)^2 + (1 - \alpha) \lambda_l (s^E_0 - l)^2 \), which results in the first-order condition:
\( \alpha \lambda_h (s^E_0 - h_0) + (1 - \alpha) \lambda_l (s^E_0 - l) = 0 \). This implies that:

\[
s^E_0^* = \frac{\alpha \lambda_h}{\alpha \lambda_h + (1 - \alpha) \lambda_l} h_0 + \frac{(1 - \alpha) \lambda_l}{\alpha \lambda_h + (1 - \alpha) \lambda_l} l.
\]

The optimal level of employer contributions is the weighted average of the optimal level of savings

\(^{68}\)While the subsidy for employees in the top income tax bracket decreased to the level of that for the employees below the
top tax threshold, we do not assume that this caused the optimal savings rate to be equal in the second period for the two
groups (\( l = h_1 \)). Workers above the top tax threshold may have different optimal levels of saving for other reasons.
FIGURE 2.1
Marginal Tax Gain to Contributing to Capital Pension Accounts by Year and Whether a Worker is below or above the Top Tax Threshold

Notes: This figure plots the marginal tax gain to contributing to capital pension accounts by year, separately for workers above the top tax threshold (the blue curve) and workers below the threshold (the red curve) from years 1996 to 2001. The marginal tax gains reflects the government subsidy for capital pension contributions for the individual, i.e., the tax deduction value of capital pension contributions in municipality and state taxes on personal income net of the tax to be paid on the pay-out of capital pensions (40%).
for type $h$ and type $l$, where the weight on each type is its relative share among the firm’s passive workers whose total level of savings are affected by the firm’s choices.

Given the decrease in $h_t$ in period 1, the firm chooses savings contributions in period 1, $s_1^E$, to minimize $\alpha \lambda_h (s_1^E - h_t)^2 + (1 - \alpha) \lambda_l (s_1^E - l_t)^2$ such that:

$$s_1^{E*} = \frac{\alpha \lambda_h}{\alpha \lambda_h + (1 - \alpha) \lambda_l} h_t + \frac{(1 - \alpha) \lambda_l}{\alpha \lambda_h + (1 - \alpha) \lambda_l} l_t.$$

Defining the change from period 0 to period 1 for a given variable by $\Delta x = x_1 - x_0$, it follows that

$$\Delta s_1^{E*} = \frac{\alpha \lambda_h}{\alpha \lambda_h + (1 - \alpha) \lambda_l} \Delta h + \frac{(1 - \alpha) \lambda_l}{\alpha \lambda_h + (1 - \alpha) \lambda_l} \Delta l = \frac{\alpha \lambda_h}{\alpha \lambda_h + (1 - \alpha) \lambda_l} \Delta h.$$

That is, the change in the employer’s capital contribution due to the reform is a function of the change in the optimal savings rate for the high types, scaled by the share of them who are passive savers. Three theoretical predictions immediately follow from the solution to $\Delta s_1^{E*}$ as we state below.

**Prediction 1.** When $h_t$ decreases from period 0 to period 1, such that $\Delta h < 0$, the change in the employer’s contribution rate is negative. That is, $\Delta s_1^{E*} < 0$ whenever $\Delta h < 0$.

**Prediction 2.** The firm’s response is increasing in the share of workers above the top-income threshold, $\alpha$. When $\Delta h < 0$, it implies that the decrease in the firm’s contribution increases in magnitude with $\alpha$ according to

$$\frac{\partial \Delta s_1^{E*}}{\partial \alpha} = \frac{\lambda_h \lambda_l}{(\lambda_h \alpha + \lambda_l (1 - \alpha))^2} \Delta h < 0.$$

**Prediction 3.** The change in employer contributions is proportional to the fraction of passive workers who are above the threshold, $\lambda_h$. Thus, firms with a larger fraction of passive workers above the threshold have a larger decrease in employer contributions for any given decrease in the optimal savings for type $h$, such that

$$\frac{\partial \Delta s_1^{E*}}{\partial \lambda_h} = \frac{\alpha (1 - \alpha) \lambda_l}{(\lambda_h \alpha + \lambda_l (1 - \alpha))^2} \Delta h < 0.$$
These theoretical predictions suggest that if firms are responsive to workers’ incentives, and there is a decrease in the optimal capital contribution rate for workers above the threshold, then empirically, we should expect a decrease in employer capital contributions after the reform. We should also expect larger reductions for firms with a larger fraction of workers above the threshold. Additionally, if firms respond to workers’ individual pension savings behavior (that is, whether they are active or passive savers) as well as to their pension incentives, then firms with more passive workers above the threshold should have a larger decrease in employer capital contributions. We test these predictions in the empirical analysis section below.

2.3 Data and Institutional Background

2.3.1 Institutional Background

This section provides background on Danish retirement institutions important for our empirical analysis. The Danish pension system is based on three central pillars, similar to retirement savings systems in many developed countries: a state-provided defined benefit (DB) plan, employer-provided defined contribution (DC) accounts, and individual retirement (DC) accounts.

Our analysis focuses mainly on employer-provided DC accounts, though we do some additional analysis of individual DC accounts. Since individual DC accounts have equivalent tax properties to employer accounts but are completely independent, employer-based and individual DC accounts perfect substitutes. Within both the employer and individual DC pension plans, there are two types of accounts: capital pension accounts and annuity pension accounts.\(^{69}\) Capital pension accounts are paid out as a lump sum and taxed at 40\% on payout, while annuity pension accounts are paid out over several years and are taxed as labor income.\(^{70}\) Balances in capital pension accounts can be converted to annuity pensions, but the reverse is not allowed. Contributions to both types of accounts are tax deductible at the time of contribution, and capital gains are taxed at 15\%, compared to approximately 29\% for assets in taxable accounts.

Our empirical research design exploits a 1999 tax reform, which aimed at reducing the generosity

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\(^{69}\)Note that there are also two different types of annuity pensions: “rate” annuity pensions that pay a fixed amount for the 10 years following retirement, and “life” annuity pensions that pay out in annuity each year after retirement. For now, we refer to both accounts as annuity pensions. We return to discuss their differences later in Section.

\(^{70}\)Note that the lowest marginal labor income tax rate at the bottom bracket during this time period was 44\%.
of capital accounts and incentivizing a shift to annuity accounts. To do so, the reform reduced the deduction for contributions to capital pensions from 59 cents per DKr to 45 cents per DKr for individuals in the top income tax bracket. The deduction for those in the lower tax bracket remained the same at 45 cents per DKr. We study how firms respond to this reform as a function of the fraction of their workers who were directly affected by the reform. We focus mainly on the firm’s response through changing employer capital contributions, since the reform directly affected the incentives for this type of accounts.

Most jobs in Denmark (roughly 80%) are covered by collective bargaining agreements between worker unions and employer associations. Collective bargaining agreements that set wage rates often include a pension plan in which a fixed proportion of an individual’s earnings is paid into a retirement account that is managed by an independent occupation-specific pension fund (of which there are approximately 30). We focus our analysis on the 20% of jobs that are not covered by collective bargaining for two reasons. First, since our data does not allow us to match workers to unions and firms to employer associations, we are unable to explicitly analyze how employer contributions are set in the collective bargaining process. Second, by focusing on jobs that are not covered by collective bargaining, we concentrate our analysis on firms that are more similar to firms in other countries such as the US, which are predominantly private and not covered by collective bargaining.

For the 20% of jobs that are outside the common agreements, employers set contribution rates to capital and/or annuity accounts for their workers. While individuals cannot change the total contribution rate, they can choose a different allocation across capital and annuity accounts, but only if their pension fund allows both types of accounts. Since pension funds are occupation specific, firms often set contribution rates separately by occupation, and usually under the advice of the occupation-specific pension funds that invest the savings for them. To isolate the 20% of jobs that have employer-based contributions set by the employer, we exclude public sector and blue collar workers, since they are more likely to be covered by collective bargaining. Note, however, that 50% of private white collar jobs are still covered by collective bargaining. Therefore, in the empirical section, we additionally assess the potential bias that inadvertently including workers covered by collective bargaining may create and show that it likely attenuates our estimates.
Since firms generally set contributions separately by occupation, we run our analysis at the occupation-firm level, and differentiate occupations at the 2-digit occupation code level.\footnote{Due to measurement error in many-digit occupation codes, our choice for the analysis is the 2-digit code level; however, actual employer contributions may be set at a higher- (or lower-) digit occupation code level. This causes some measurement error in our identified decision unit. Our results stay the same if we aggregate at the 1-digit occupation code level, or even at the firm level.}

2.3.2 Data Sources, Sample Selection, and Variable Definitions

We merge data from several administrative registers of the Danish population – the income tax register, the population register, and the Danish Integrated Database for Labor Market Research (IDA) – to obtain annual information on Danish employees and firms from 1996 to 2001.\footnote{During this time period, DKK 6.5≈USD 1.} Starting from the population dataset, we impose three restrictions to obtain our primary analysis sample. First, we exclude observations in which individuals are below the age of 20 or over 60, at which point the majority of the Danish workforce is eligible for early retirement benefits and retirement savings are eligible for withdrawal. Second, we exclude observations of workers with self-employment income because their employer contributions are not set by the firm. Third, we exclude occupation-firm cells with fewer than 5 individuals in order to decrease measurement error since such small cells are unlikely to be treated as an independent unit by employers. As mentioned previously, for most of our analysis, we exclude public and blue collar workers since their employer contributions are less likely to be determined at the occupation-firm level.

We measure the employer contribution rate as the employer contribution we observe for a worker divided by her taxable labor income. Since our taxable income measure is not the exact income measure that firms use to calculate contributions, and since individuals may have some degree of discretion in choosing a different distribution of contributions between capital and annuity accounts than the default that the employer set, employer contribution rates may be measured with error. Therefore, to reduce error in identifying the default contribution rate, we use the median percent of contributions within an occupation-firm cell as our measure of the default employer contribution
rather than the mean.\textsuperscript{73}

We consider a worker to be “active” if the worker has positive individual contributions to private, non-employer-provided capital accounts in the previous year.\textsuperscript{74} Note that all income and savings variables used in the analysis are based on third-party reports. Earnings and pension contributions are reported directly by employers and pension funds to the tax authority.

\subsection*{2.3.3 Summary Statistics}

Table 2.1 presents summary statistics for the sample of private white collar wage earners between ages 20-60 who are in occupation-firm cells with at least five workers.\textsuperscript{75} Our restricted sample leads to analysis of 8,945,365 worker-year observations from 1996-2001.

Mean individual labor income in the full sample is DKr 296,585 (equivalent to US $45,600), and the share of individuals above the top tax threshold is .49. The standard deviation of the fraction of workers above the threshold across occupation-firm cells is .36. Given the restriction to at least five workers per occupation-firm cell, the average cell size is 21. Note that there is large variance in the size of the occupation-firm cells, which is due to a large upper tail – the median cell size is 8.

The fraction of active workers above the top tax threshold (i.e., those with positive capital savings) is .27.

To give an idea of what pensions looked like before the reform, we present summary statistics for employer and individual capital and annuity contributions calculated from 1996-1998. Before 1999, employer capital contributions were on average 3.2\% of income and 61\% of workers had positive employer capital contributions. Average employer annuity contributions were slightly higher

\textsuperscript{73}Chetty et al. (2014) find that only 15\% of Danish workers are active savers within their own individual accounts. Even if all 15\% were also active within their employer accounts (which is unlikely), it would not affect the median contribution within the firm-occupation cell.

\textsuperscript{74}While the results presented here use this measure of activeness, our findings are robust to defining activeness as workers with any positive individual pension contribution, or the fraction of years the worker has changed contributions to her individual pension accounts.

\textsuperscript{75}Note that during our sample period, 57\% of the total sample of wage earners are in the private sector, and 70\% are in white collar occupations.
TABLE 2.1
Summary Statistics of Analysis Sample

<table>
<thead>
<tr>
<th></th>
<th>Mean (1)</th>
<th>SD (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income in DKr (DKr 6.5 ≈ USD 1)</td>
<td>296,585</td>
<td>175,674</td>
</tr>
<tr>
<td>Fraction Above Top Tax Threshold</td>
<td>0.49</td>
<td>0.36</td>
</tr>
<tr>
<td># in Occupation/Firm Cell</td>
<td>21</td>
<td>97</td>
</tr>
<tr>
<td>Fraction With Pos Cap. Below Threshold</td>
<td>0.17</td>
<td>0.25</td>
</tr>
<tr>
<td>Fraction With Pos Cap. Above Threshold</td>
<td>0.27</td>
<td>0.30</td>
</tr>
<tr>
<td><strong>Employer (Before 1999)</strong></td>
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<td></td>
</tr>
<tr>
<td>Capital Contributions</td>
<td>9,851</td>
<td>11,923</td>
</tr>
<tr>
<td>% Capital Contribution</td>
<td>0.032</td>
<td>0.038</td>
</tr>
<tr>
<td>% With Positive Capital Contr.</td>
<td>0.61</td>
<td>0.49</td>
</tr>
<tr>
<td>Annuity Contributions</td>
<td>13314</td>
<td>42936</td>
</tr>
<tr>
<td>% Annuity Contribution</td>
<td>0.036</td>
<td>0.046</td>
</tr>
<tr>
<td>% With Annuity Capital Contr.</td>
<td>0.72</td>
<td>0.45</td>
</tr>
<tr>
<td><strong>Individual (Before 1999)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital Contributions</td>
<td>3,225</td>
<td>7,715</td>
</tr>
<tr>
<td>% Capital Contribution</td>
<td>0.012</td>
<td>0.028</td>
</tr>
<tr>
<td>% With Positive Capital Contr.</td>
<td>0.29</td>
<td>0.45</td>
</tr>
<tr>
<td>Annuity Contributions</td>
<td>1780</td>
<td>10993</td>
</tr>
<tr>
<td>% Annuity Contribution</td>
<td>0.005</td>
<td>0.020</td>
</tr>
<tr>
<td>% With Annuity Capital Contr.</td>
<td>0.15</td>
<td>0.36</td>
</tr>
<tr>
<td># Worker-Year Obs</td>
<td>8,945,365</td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table presents means and standard deviations of key variables from 1996-2001 for private white collar workers, which compose the main sample of workers we use in our analysis. White collar workers are those that have first digit occupation codes of 1, 2, 3, 4 or 5. For the individual and employer contributions, we report means and standard deviations from the years 1996-1998, which include the periods before the reform. All monetary values are reported in nominal Danish Kroner; the exchange rate was approximately 6.5 DKr per US $1 during the period we study. Income is total pre-tax wage earnings plus employer pension contributions. We top code all pension contribution levels at the maximum allowable level for that year.
at 3.6% of income, and 72% had positive employer annuity contributions. Individual capital contribu-
tions were 1.2% of income and approximately 29% of workers had positive individual capital contributions. Individual annuity contributions were fairly low at .5% of income and only 15% of individuals had positive annuity contributions.

2.4 Empirical Evidence

In this section we test the three predictions from the theoretical section: Prediction 1 – the decrease in subsidy for workers above the threshold causes a decrease in the employer capital contribution rate; Prediction 2 – the magnitude of the firm’s response is increasing in the share of workers above the top-income tax threshold, such that there is a larger decrease in employer capital contribution rates for firms with more workers above the threshold; and Prediction 3 – firms with a larger fraction of passive workers above the threshold have a larger decrease in the employer capital contribution rate. If we find evidence consistent with these predictions, it suggests that firms set employer contributions with respect to their workers’ pension contribution incentives.

2.4.1 Average Response of Firms to the Retirement Savings Reform

To test Prediction 1, whether the decrease in the capital subsidy for workers above the threshold causes a decrease in the employer capital contribution rate, Figure 2.2a plots the mean employer capital contribution as a fraction of workers’ income by year for private white-collar occupation-firm cells. In 1999, when the capital subsidy decreased for workers above the threshold, mean employer capital contributions decreased by 0.7 percentage points (pp) on a base of 2.9 pp. That is, in response to the decrease in incentives for capital contributions for a share of their employees, employers decreased their capital contributions by 25%. Throughout the rest of the paper, we discuss the heterogeneity of this response across different characteristics of the occupation-firm cells in our analysis sample.
Notes: Panel (A) plots the average employer capital contribution rate (defined as a fraction out of labor earnings) for private white collar occupation-firm cells for the years 1996 to 2001. The employer capital contribution rate is estimated as the median employer capital contribution rate within an occupation-firm cell. Panel (B) plots the average estimated employer capital contribution rate for private white collar occupation-firm cells for the years 1996 to 2001, separately for firm-occupation groups with less or more than 50% of workers above the top tax threshold in 1998.
2.4.2 Heterogeneity by the Fraction of Workers above the Top Tax Threshold

Next, we test Prediction 2, whether the magnitude of a firm’s response to the reform is increasing in the share of workers above the top income threshold.

**Graphical Analysis.** We begin with a simple graphical analysis to see how the change in employer capital contributions after the reform differs between occupation-firm cells in which the majority of workers is above the threshold and those in which the majority of workers is below the top tax threshold. If employers are responsive to changes in their workers’ incentives, we would expect to see a larger decrease in capital contributions in 1999 for firms with more workers above the threshold.

Figure 2.2b plots the average employer capital contribution rate over time for occupation-firm cells with more than 50% of workers above the threshold (the solid line), and for cells with less than 50% of workers above the threshold (the dashed line). Note that the groups are defined by the fraction of workers above the threshold in 1998. Therefore, this is essentially a simple differences-in-differences design where the experimental groups are defined by treatment intensity. As we can see from Figure 2.2b, before the reform, the contribution rates in the two groups move in parallel. After the reform, occupation-firm cells with more than 50% of workers above the threshold have a sharp 1.4 pp *decrease* in capital contributions from 1998 to 1999, while there is a .1 pp *increase* for occupation-firm cells that have less than 50% of workers above the threshold. With the differences-in-differences estimate for the decrease in employer capital contributions being 1.5 pp, this indicates that the size of the employers’ response to the reform is based on whether the median worker was affected by the reform.

To explore the full relationship between a firm’s response to the reform and the fraction of workers above the threshold, in Figure 2.3a, we divide individuals into deciles by the fraction of employees above the threshold within occupation-firm cell. We then plot the mean employer capital contribution rates in each bin for the years 1996-2001. The lines in red shades plot employer contributions for the years before the reform, and the lines in blue shades plot employer capital contributions for the years after the reform. Note that before the reform, the relationship between employer capital contributions and the fraction of workers above the threshold is increasing, while the relationship is decreasing in the years after the reform. To understand the effect of the
FIGURE 2.3
Employer Capital Contributions by the Fraction of Workers above the Top Tax Threshold

Notes: Panel (A) plots the employer capital contribution rate versus the fraction of employees above the threshold within an occupation-firm cell for the years 1996-2001. Specifically, we bin the sample into 10 equal-sized groups by the fraction of employees above the threshold in each year, and for each bin we plot the mean of the employer capital contribution rate (on the y-axis) against the mean of the fraction of workers above the top tax threshold (on the x-axis). The circles and lines in red shades plot the years before the reform, and the circles and lines in blue shades plot the years after the reform. Panel (B) is also a binned scatter plot, but where the outcome on the y-axis is the change in employer capital contribution rate from the previous year calculated at the occupation-firm level.
reform explicitly, Figure 2.3b displays a binned scatter-plot of the difference in employer capital contributions from the previous year within an occupation-firm cell versus the fraction of employees above the threshold by year. In years 1996-1998, there is no noticeable relationship between the change in employer capital contributions and the fraction of employees above the threshold in that year. However, in 1999, there is a large decreasing (and approximately linear) relationship between the change in employer capital contributions and the fraction of employees above the threshold. Occupation-firm cells with 0% of workers above the threshold had almost no change in employer capital contributions, while occupation-firm cells with 100% of workers had more than a 1.8 pp decrease in employer capital contributions from the previous year. For the years 2000 and 2001, there is a slightly decreasing relationship between the difference in employer capital contributions and the fraction of workers above the threshold, but the slope is much smaller than that of 1999. This is consistent with a delayed or gradual response to the reform due to adjustment costs in updating the employees’ savings portfolio.

Overall, the graphical evidence clearly indicates that employers with more workers above the threshold had a larger response to the reform, suggesting that employers are indeed responsive to changes in their employees’ saving incentives in proportion to the fraction of their workers whose incentives changed.

**Regression Analysis.** To formally estimate the relationship between the change in employer capital contributions and the fraction of workers above the threshold, we turn to regression analysis and estimate the following equation:

\[
\Delta \left( \frac{Capital}{Income} \right)_{ft} = \beta_0 + \beta_1 AboveThreshold_{ft} + \sum_{s=1996, s \neq 1998}^{2001} \left[ \beta_s (I_{t=s} * AboveThreshold_{ft}) + \mu_s \right] + X_{ft} + \epsilon_{ft}.
\]

\(^{76}\) Note that if we had just looked at the cross-sectional difference in employer capital contributions by the fraction of workers above in the pre-period, we would have underestimated the relationship between employer capital contributions and the fraction of workers above the threshold that is due to the different capital subsidy for workers below and above the top tax threshold. This is likely because, aside from the differences in the capital subsidy before the reform, workers above and below the top tax threshold have different preferences for capital contributions.
In our main specification, the outcome variable is the difference in median employer capital contribution rate from the previous year grouped at the occupation-firm level, $f$, in a particular year, $t$. The right-hand side variables include the fraction of employees above the threshold in an occupation-firm-year cell ($\text{AboveThreshold}_{ft}$), year fixed effects ($\mu_s$), and year dummies interacted with the fraction of employees above the threshold ($I_{t=s} \ast \text{AboveThreshold}_{ft}$). In all the specifications, we omit 1998 so that all the coefficients, $\beta_s$, are calculated in reference to 1998. Hence, $\beta_1$ estimates the linear relationship between the change in employer capital contributions and the fraction above the threshold in 1998. We choose a specification linear in the fraction of employees above the top threshold, since Figure 2.2b exhibits an approximately linear relationship between the change in employer capital contributions and the fraction above the threshold. The main coefficient of interest is $\beta_{1999}$, the coefficient on the fraction of workers above the threshold interacted with the year 1999. This estimates the difference in the relationship between the change in employer capital contributions and the fraction of workers above the threshold in 1999 compared to their relationship in 1998, thus estimating the effect of the reform on this relationship. This regression framework also allows us to include various sets of controls, $X_{ft}$, to verify the robustness of the estimated relationship.

Table 2.2 presents the coefficients on the fraction of employees above the threshold, to which we refer in the remainder of the paper as “the fraction above,” and the interaction of the fraction above with the indicators for years 1996 through 2001 (omitting 1998) in regressions that include various sets of controls. In all columns, we include year fixed effects, cluster standard errors at the occupation-firm level, and multiply the coefficients by 100 to convert them into percentage-point units.

The coefficient on fraction above in Column (1) is -.05 and not statistically significant at the 5% level. The interaction between the fraction above and year 1996 and 1997 are also precise zeros. This suggests that before the reform there was no relationship between the annual change in employer capital contributions and the fraction of workers above the threshold. On the other hand, the coefficient on the fraction above interacted with 1999 is -2.1 pp and is statistically significant at any conventional significance level. This implies that in 1999, an occupation-firm cell with 100% of employees above the top-income tax threshold on average decreased its employer capital contribution
### TABLE 2.2
Regression Analysis of the Heterogeneity in the Response to the Reform by the Fraction of Workers above the Top Tax Threshold

<table>
<thead>
<tr>
<th>Dep. Variable:</th>
<th>Δ Employer Capital Contributions</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>Fraction Above</td>
<td>-0.05</td>
<td>-0.11</td>
<td>0.13</td>
<td>0.18</td>
<td>-0.02</td>
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<tr>
<td></td>
<td>(0.03)</td>
<td>(0.07)</td>
<td>(0.14)</td>
<td>(0.14)</td>
<td>(0.17)</td>
</tr>
<tr>
<td>Above Interacted With:</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1996</td>
<td>0.04</td>
<td>0.10</td>
<td>0.20</td>
<td>0.24</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.10)</td>
<td>(0.14)</td>
<td>(0.16)</td>
<td>(0.27)</td>
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<tr>
<td>1997</td>
<td>0.06</td>
<td>0.07</td>
<td>0.07</td>
<td>0.10</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.10)</td>
<td>(0.13)</td>
<td>(0.14)</td>
<td>(0.24)</td>
</tr>
<tr>
<td>1999</td>
<td>-2.13 ***</td>
<td>-2.18 ***</td>
<td>-2.19 ***</td>
<td>-2.36 ***</td>
<td>-1.88 ***</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.13)</td>
<td>(0.18)</td>
<td>(0.20)</td>
<td>(0.31)</td>
</tr>
<tr>
<td>2000</td>
<td>-0.61 ***</td>
<td>-0.57 ***</td>
<td>-0.57 ***</td>
<td>-0.64 ***</td>
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</tr>
<tr>
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<td>(0.05)</td>
<td>(0.10)</td>
<td>(0.15)</td>
<td>(0.16)</td>
<td>(0.28)</td>
</tr>
<tr>
<td>2001</td>
<td>-0.56 ***</td>
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<td>-0.60 ***</td>
<td>-0.63 ***</td>
<td>-0.85 ***</td>
</tr>
<tr>
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<td>(0.05)</td>
<td>(0.10)</td>
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<td>(0.16)</td>
<td>(0.26)</td>
</tr>
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<td>Year FE</td>
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<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Income Controls &amp; Cell Size</td>
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<td></td>
<td></td>
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<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>2-Digit Occupation/Firm FE</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-Digit Occupation/Year FE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-Digit Occupation/Year FE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm/Year FE</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Obs.</td>
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<td>60,643</td>
<td>60,643</td>
<td>60,643</td>
<td>60,643</td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.08</td>
<td>0.07</td>
<td>0.39</td>
<td>0.39</td>
<td>0.70</td>
</tr>
</tbody>
</table>

*** p<0.01, ** p<0.05, * p<0.1

Notes: In column (1) we regress the change in employer capital contributions on the fraction of workers above the top tax threshold, year fixed effects, and the fraction above interacted with year fixed effects. Year 1998 is omitted as the base year. Column (2) also includes a 5-order polynomial of the average income above the threshold and below the threshold interacted with year fixed effects, and includes the cell size interacted with year fixed effects. Column (3) additionally includes 2-digit occupation-firm fixed effects. Column (4) adds 1-digit occupation-year fixed effects. Column (5) includes year fixed effects, income controls, and 2-digit occupation-year fixed effects as well as firm-year fixed effects. Each regression is run at the 2-digit occupation-firm-year level. The size of occupation-firm units is restricted to be more than or equal to 5. All regressions are restricted to firms in the private sector, and only white collar occupations are included. Changes in employer capital contributions are calculated as the change in the median worker's employer contribution rate from the previous year. A specific worker's employer contribution rate is calculated as a fraction of his or her labor income. All standard errors are clustered at the 2-digit occupation-firm level. Coefficients are multiplied by 100 so that they are in percentage point units.
rate by 2.1 pp more than an occupation-firm cell with 0% of employees above the threshold. Given that the average fraction of workers above in 1998 was 50% and that the average decrease in employer capital contributions in 1999 was 0.7 pp, the implied elasticity is 1.5 – indicating that a 1% increase in the fraction above leads to a 1.5% larger decrease in employer capital contributions. For years 2000 and 2001, both coefficients on fraction above are approximately -0.6 pp. While these are approximately a third as large as the 1999 interaction coefficient, they are still statistically significant at the 1% level. As in Figure 2.3b, these patterns are consistent with delayed or gradual responses to the reform among some firms.77

While these results are consistent with the model’s predictions, given that the fraction of employees above the threshold may be correlated with other characteristics of the firm that affect the change in contribution rates, we proceed to verify that our results are not driven by an omitted variable bias. To do so, we add a variety of controls and see if our estimated coefficient is stable across specifications. We might be especially concerned that the results are driven by other moments of the income distribution. For example, we might worry that “good” firms, those with higher than average wages, are more likely to respond to the reform and are also more likely to have a higher fraction of workers above the top tax threshold.

Since our identification relies on the discontinuity of crossing the top tax threshold and since any other effect of income should be continuous across the top tax threshold, we can account for these potential confounding effects by flexibly controlling for the average income of workers in occupation-firm cells. In particular, in columns (2)-(5), of Table 2.2 we include high order polynomials of the average income of workers below and above the threshold, and also include interactions of the polynomial with each year fixed effect.78 Note that this limits our sample to occupation-firm cells that have at least one person above and below the threshold. Column (2) presents the results for a specification that controls for income and a linear term of the size of the

77 While we did not model it here, this could be explained by firms having time varying costs of adjusting employer capital contributions. Empirically, we find that for firms with more than 50% of workers above, approximately one half of the decrease in employer capital contribution after 1999 is “delayed” and due to firms who did not respond in 1999, and the other half is “gradual” and due to additional responses by firms who responded in 1999.

78 The reported estimates are for polynomials of degree five, but the results are robust to higher- and lower-degree polynomials.
occupation-firm cell interacted with year dummies. In column (3), we also include fixed effects for each 2-digit occupation-firm cell. In column (4), we add 1-digit occupation-year fixed effects. In column (5), instead of the 2-digit occupation-firm fixed effects and the 1-digit occupation-year fixed effects, we include firm-year fixed effects and 2-digit occupation-year fixed effects. Across all the specifications, the coefficients on the fraction above and fraction above interacted with year 1996 and 1997 remain small and insignificant. Importantly, the coefficient on year 1999 is also very stable across the different specifications, and only decreases slightly when firm-year fixed effects and 2-digit occupation-year fixed effects are included in column (5). One reason the coefficient may decrease slightly when firm-year fixed effects are included is that some firms may not set contributions separately by occupation but instead set them at the firm level. When we control for firm-year fixed effects, we exclude this variation which results in a slightly smaller coefficient on the fraction above.

The stability of the estimated effect across the different specifications suggests that the estimated relationship is not driven by omitted variables and gives additional evidence that firms react to the 1999 reform in proportion to the number of their workers who are directly affected by the reform. Throughout the rest of this paper, we explore this further by conducting additional robustness checks, by looking at other responses of firms to the reform, and finally by looking at whether this effect is heterogeneous across different sets of firms. Our preferred specification is the one from column (4), since it has a flexible set of controls and since we are interested in both the firm level responses and the occupation-firm responses (as opposed to specification (5)). We will thus focus on variations of the specification in column (4).

2.4.3 Robustness Checks

**Income vs. Capital Responses.** Since we are analyzing employer capital contributions as a fraction of income, the heterogeneous response of firms by the fraction of workers above the threshold in 1999 could be due to differential changes in the numerator (employer capital contributions) or the denominator (taxable labor income). Figure 2.4a breaks down the response by plotting the natural log of taxable income (in blue) and employer capital contributions (in red) for occupation-
FIGURE 2.4
Firm Responses by the Fraction of Workers above the Top Tax Threshold

Notes: Panel (A) plots the average log of labor earnings (in blue), and the average log of employer capital contributions (in red) by year, separately for occupation-firm cells with more or less than 50% of workers above the top tax threshold in 1998. To include occupation-firm cells that have 0 employer capital contributions we calculate the log as: ln(x+1).
Panel (B) plots the difference in the average employer capital contribution rate (in red) and the difference in the average employer annuity contribution rate (in blue) from the previous year, separately for firm-occupation cells with more or less than 50% of workers above the top tax threshold in 1998.
firms cells with more than 50% of workers above the threshold (solid line) and for those with less than 50% of workers above the threshold (dashed line). There is no noticeable differential change in income for the two different groups. On the other hand, there is a large decrease in employer capital contributions for occupation-firm cells with more than 50% of workers above the threshold, and a slight increase in employer capital contributions in occupation-firm cells with less than 50% of workers above the threshold. This provides evidence that the changes in the employer capital contribution rate are driven by changes in employer capital contributions and not changes in income.

To formalize these results, column (1) of Table 2.3 replicates column (4) of Table 2.2 but has the log change in taxable labor income as the dependent variable. Given that the dependent variable is income, this regression, and all others in this table, do not include controls for income. The coefficient on the fraction above interacted with year 1999 is essentially zero and is not statistically significant. This suggests that in the year after the reform, income did not change differently for occupation-firm cells with more workers above the threshold. On the other hand, if we look at column (2) of Table 2.3, which has the change in log employer capital contributions as the dependent variable (not as a fraction of income), the coefficient on fraction above interacted with year 1999 is -1.04 and is statistically significant at the 1% level. Thus, occupation-firm cells with a higher fraction of employees above the threshold had a much larger decrease in employer capital contributions. This formalizes the results from Figure 2.4a, showing that the estimated effect of the fraction above in 1999 on the change in the rate of employer capital contributions is due to the changes in capital contributions rather than changes in income.

**Assessing Bias from Collective Bargaining.** So far our analysis has restricted the sample to white collar workers in the private sector, since their employer contributions are more likely to be determined by the firm and less likely to be determined by collective bargaining (as compared to public or blue collar workers). However, despite this restriction, we still do not entirely isolate occupation-firm cells that have employer contributions determined solely by firms. This could bias our results in two different ways. First, given that there are union representatives within the collective bargaining process, employer contributions for these workers may be more closely related to workers’ preferences than employer contributions that are set exclusively by firms, and thus employer capital contributions may respond more to the reform and to the fraction of workers above
TABLE 2.3
Labor Income vs. Employer Capital Contribution Responses

| Dep. Variable: | Δ Log |       |       |
|               |       | Labor Income | Capital |
|               |       | (1)          | (2)     |
| Fraction Above| 0.39  | ***          | 0.46    | ***     |
|               | (0.01)|              | (0.14)  |
| Above Interacted With: |     |       |       |
| 1996          | 0.02  |       | -0.29  | **      |
|               | (0.02)|       | (0.13) |
| 1997          | 0.01  |       | -0.40  | ***     |
|               | (0.01)|       | (0.12) |
| 1999          | 0.00  |       | -1.04  | ***     |
|               | (0.01)|       | (0.13) |
| 2000          | -0.01 |       | -0.17  |         |
|               | (0.01)|       | (0.13) |
| 2001          | -0.05 | ***    | -0.30  | **      |
|               | (0.01)|       | (0.12) |
| Number of Obs.| 84,809|       | 84,809 |
| R-Squared     | 0.45  |       | 0.387  |

*** p<0.01, ** p<0.05, * p<0.1

Notes: The dependent variable in column (1) is the log difference in average labor income from the previous year calculated at the 2-digit occupation-firm level, whereas the dependent variable in column (2) is the log difference in average capital contributions calculated at the 2-digit occupation-firm level. To include occupation-firm cells that have zero employer capital contributions we take the natural log of the average capital contribution plus one. The results are robust to using just the log as well. All columns include year fixed effects, 2-digit occupation-firm fixed effects, 1-digit occupation-year fixed effects, and the size of the occupation-firm cell interacted with year fixed effects. Note that unlike other tables, income controls are not included because income is an outcome. All standard errors are clustered at the 2-digit occupation-firm level.
the threshold. Therefore, inadvertently including occupation-firm cells that are covered by collective bargaining could increase the magnitude of the coefficients. On the other hand, in these occupation-firm cells, employer contributions are set at the union level, and thus at the pure occupation level, rather than at the occupation-firm level. Since we use the occupation-firm cell as the decision cell for our analysis, there is measurement error in the decision cell for workers covered by collective bargaining. Therefore, inadvertently including these workers could lead to attenuation of the coefficients.

In order to empirically quantify how unintentionally including cells covered by collective bargaining biases our estimates, we analyze the relationship between employer capital contributions and the fraction of workers above for groups that are either more or less likely to be covered by collective bargaining as compared to private white collar workers. Specifically, we first restrict the analysis to blue collar and public workers who are more likely to be covered by collective bargaining, and then restrict the analysis to highly educated private white collar workers who are even less likely to be covered by collective bargaining than our main sample of private white collar workers.

Column (2) of Table 2.4 replicates the main specification of column (1) but restricts the regression to public or blue collar workers. The coefficient on the interaction of the fraction above with 1999, -0.38, is an order of magnitude smaller than the coefficient of the main specification (-2.4) and their difference is statistically significant at the 1% level. In column (3) of Table 2.4, we restrict the analysis to private white collar occupation-firm cells where more than 20% of workers have at least 17 years of education. The point estimate on the fraction above interacted with year 1999 increases in magnitude to -3.77. Computing the coefficients using a fully interacted regression, we find that the difference in the coefficient on fraction above is significantly larger in magnitude, at the 5% level, for the highly educated sample than the full sample of private white collar workers.

These results suggest that inadvertently including some workers who are covered by collective bargaining in our main specification may attenuate our results, which reassures us that our estimates do not overestimate the relationship between the change in employer capital contributions and the fraction of workers above the threshold.

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70 That is, for workers covered by collective bargaining, employer contributions should be set based on the fraction above within the occupation (i.e. union), and not the fraction above within the particular occupation-firm cell.
### TABLE 2.4
Heterogeneity in Responses to the Reform by Worker Characteristics

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<thead>
<tr>
<th>Dep. Variable:</th>
<th>( \Delta ) Employer Capital Contributions</th>
</tr>
</thead>
<tbody>
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<td>(1)</td>
</tr>
<tr>
<td>Fraction Above</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
</tr>
<tr>
<td>Above Interacted With:</td>
<td></td>
</tr>
<tr>
<td>1996</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
</tr>
<tr>
<td>1997</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
</tr>
<tr>
<td>1999</td>
<td>-2.36</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
</tr>
<tr>
<td>2000</td>
<td>-0.64</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
</tr>
<tr>
<td>2001</td>
<td>-0.63</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
</tr>
<tr>
<td>Public</td>
<td>x</td>
</tr>
<tr>
<td>Blue Collar</td>
<td>x</td>
</tr>
<tr>
<td>Highly Educated</td>
<td></td>
</tr>
<tr>
<td>Number of Obs.</td>
<td>60,643</td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.39</td>
</tr>
</tbody>
</table>

*** p<0.01, ** p<0.05, * p<0.1

Notes: Column (1) replicates column (4) from Table 2.2. Column (2) replicates column (1) except that the sample is restricted to public or blue collar workers instead of private white collar workers. Column (3) replicates column (1) except that the sample is restricted to 2-digit occupation-firm cells where more than 20% of workers have 17 or more years of education. All columns include year fixed effects, 2-digit occupation-firm fixed effects, 1-digit occupation year fixed effects, extensive income controls interacted with year fixed effects, and the size of the occupation-firm cell interacted with year fixed effects. All standard errors are clustered at the 2-digit occupation-firm level. Coefficients are multiplied by 100 so that they are in percentage point units.
2.4.4 Additional Outcomes

The empirical analysis up to this point has focused on one aspect of the effect of the reform – namely, the effect on the median employer capital contribution within an occupation-firm cell. To understand the broader effects of the reform, we look at additional outcomes that are relevant for employees’ pension savings.

Portfolio Response. In the previous section, we provided evidence that employer capital contributions decreased in response to the reform in proportion to the fraction of workers who were above the threshold. However, there are annuity accounts that employers can contribute to, and hence we next analyze how the portfolios of employer pension contributions adjust to the reform.

The 1999 reform made contributions to savings accounts more “costly” by decreasing the subsidy for contributions to capital accounts and caused an income effect that lowered the optimal level of total pension savings. At the same time, there is also a substitution effect due to the decrease in the relative price of contributing to annuity accounts. Therefore, as long as the substitution effect is not dominated by the income effect, we would expect employer contributions to annuity accounts to increase in response to the reform and in proportion to the share of employees directly affected by the reform.

To test this hypothesis, in Figure 2.4b we plot the change in employer contributions as a fraction of income separately by capital (in red) and annuity contributions (in blue) for occupation-firm cells with more than 50% of workers above the threshold (solid lines) and less than 50% of workers above the threshold (in dashed line). The lines for employer annuity contributions are essentially mirror images of those for capital contributions. In 1999, when there is a large decrease in capital contributions, annuity contributions increase sharply for occupation-firm cells with more than 50% of workers above the threshold while they remain the same for those with less than 50% of workers above the threshold. This reveals that employers compensated for the decrease in capital contributions with an increase in annuity contributions. Column (1) of Table 2.5 replicates the response in capital contributions from column (4) of Table 2.2, and column (2) of Table 2.5 formalizes the graphical result from Figure 2.4b by replicating column (1), but instead using the change in employer annuity contributions as the dependent variable. For annuity contributions, the coefficient on the
# TABLE 2.5
Portfolio Analysis

<table>
<thead>
<tr>
<th>Dep. Variable:</th>
<th>Δ Employer Contributions</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Capital</td>
<td>Δ</td>
<td>Annuity</td>
<td>Rate</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>All</td>
<td>(2)</td>
<td>Life</td>
</tr>
<tr>
<td>Fraction Above</td>
<td>0.18</td>
<td>-0.14</td>
<td>-0.45 ***</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.14)</td>
<td>(0.11)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>Above Interacted With:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1996</td>
<td>0.24</td>
<td>0.02</td>
<td>0.14</td>
<td>-0.09</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.13)</td>
<td>(0.11)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>1997</td>
<td>0.10</td>
<td>-0.11</td>
<td>0.04</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.12)</td>
<td>(0.11)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>1999</td>
<td>-2.36 ***</td>
<td>1.89 ***</td>
<td>0.13</td>
<td>1.34 ***</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
<td>(0.17)</td>
<td>(0.12)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>2000</td>
<td>-0.64 ***</td>
<td>1.05 ***</td>
<td>0.35 ***</td>
<td>0.72 ***</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.16)</td>
<td>(0.13)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>2001</td>
<td>-0.63 ***</td>
<td>0.66 ***</td>
<td>0.10</td>
<td>0.60 ***</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.17)</td>
<td>(0.14)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>Number of Obs.</td>
<td>60,643</td>
<td>60,643</td>
<td>60,643</td>
<td>60,643</td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.39</td>
<td>0.39</td>
<td>0.35</td>
<td>0.46</td>
</tr>
</tbody>
</table>

*** p<0.01, ** p<0.05, * p<0.1

Notes: Column (1) replicates column (4) from Table 2.2. The dependent variable in column (2) is the difference from the previous year in the median employer annuity contributions (as a fraction of income) calculated at the 2-digit occupation-firm level. The dependent variable in column (3) is the difference in the employer life-annuity contribution rate as a fraction of income, and the dependent variable in column (4) is the difference in the employer rate-annuity contribution rate as a fraction of income. The different types of annuities are described in the text in Section 2.4.4. All columns include year fixed effects, 2-digit occupation-firm fixed effects, 1-digit occupation-year fixed effects, extensive income controls interacted with year fixed effects, and the size of the occupation-firm cell interacted with year fixed effects. All standard errors are clustered at the 2-digit occupation-firm level. Coefficients are multiplied by 100 so that they are in percentage point units.
year 1999 interacted with the fraction above is 1.89 and is significant at the 1% level. This suggests that firms with a larger fraction above in 1999 compensate almost fully for the bigger decrease in capital contributions with a larger increase in annuity contributions. In fact, we cannot reject the hypothesis that the sum of the coefficients for capital and annuity contributions on fraction above interacted with 1999 is significantly different from zero at the 5% significance level, consistent with a full substitution of contributions across the different types of accounts.

Annuity contributions can be further decomposed into “life” and “rate” annuities. Life annuities pay out as a true annuity with payments for the rest of a person’s life after retirement; while rate annuities pay a fixed amount for the 10 years after retirement, making rate annuities a closer substitute for capital accounts than life annuities. Prior to the reform, 58% of individuals had employer-based life annuity contributions and 15% had employer-based rate annuity contributions. To see whether the increase in annuity contributions is attributable to an increase in life or rate annuities, columns (3) and (4) of Table 2.5 show the results for when employer life annuities and rate annuities are respectively the dependent variable. In column (3), the coefficient on fraction above interacted with year 1999 is .13, which is small in magnitude and not significant. This suggests that employers are not substituting to life annuities when they decrease capital contributions in proportion to the fraction of workers above. On the other hand, in column (4), the coefficient on fraction above is 1.34 and is statistically significant at the 1% level. This suggests that employers respond to the reform by increasing rate annuities for occupation-firm cells that have more workers above the threshold. Given that rate annuities have a shorter payout period, employers are shifting contributions to accounts that are closer substitutes to capital accounts. These results provide additional evidence that firms take into account their workers’ preferences when they set employer capital contributions, in particular, by shifting contributions to the account that has a payout schedule their workers (who previously had employer capital contributions) are more likely to prefer.

**Externalities from Employers Setting Contributions at Aggregate Levels.** Recall that our analysis focuses on the response of employer contributions for the median worker. In order to

---

80Note that while annuity contributions are equal to the sum of life and rate annuities, the coefficients from columns (3) and (4) of Table 2.5 do not necessarily need to sum to the coefficients in column (2). This is because we consider the median employer pension contribution rather than the mean.
understand whether there is a heterogeneous response *within* the occupation-firm cell, in this section we calculate the median level of employer capital contributions *separately* for workers below and above the threshold. This will allow us to see how each separate group is affected by the fraction of workers above, while controlling for their own individual status.\footnote{If we identified the decision cells perfectly, and individuals are unable to make any adjustments, then the median contribution rate should be the contribution rate for everyone within the decision cell (or close given some measurement error). However, as mentioned above, our measure of the decision cell is imperfect, and it is possible, for example, that employers set contributions separately for workers above and below the threshold. In addition, workers whose employer pension fund allows both capital and annuity contributions can shift their employer contributions between capital and annuity accounts.} The effect of the fraction of workers above in this setting explicitly estimates the externality from the firm setting contributions en masse, rather than separately for workers above and below the threshold.

To this end, we calculate the change in capital contributions separately for workers above and below the threshold, and in Table 2.6, we regress these outcomes on the fraction above, an indicator for whether the individual is above the threshold, an interaction of this indicator with the fraction above (all interacted with year fixed effects), and our usual set of controls. To see if the fraction above affects the change in employer contributions for workers below the threshold, we can look at the coefficient on the fraction above interacted with 1999, which is -0.72. This is substantially smaller in magnitude compared to the original estimate of -2.4, but is statistically significant at the 1% level. The estimated effect of the fraction above on the change in employer contributions in 1999 for workers above the threshold is -1.17 (=-.72-.45), where its difference from the effect for workers below the threshold is statistically significant.

This evidence indicates that an individual’s employer contribution is affected by the saving incentives of other workers within their occupation-firm group, which is due to externalities from the firm setting employer contributions as a whole for occupation-firm groups. Since employers do not set the contributions individually, employer contributions are not perfectly reflective of each individual’s incentive. Rather, their behavior is consistent with maximizing some aggregate function of their employees’ individual preferences using a limited set of tools.

### 2.4.5 Heterogeneity by the Activeness of Workers

Lastly, we test our third prediction: whether firms respond more to the reform if the workers
## TABLE 2.6
Separate Analysis for Workers Above and Below the Threshold

<table>
<thead>
<tr>
<th></th>
<th>Dep. Variable: Δ Employer Capital Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td><strong>Fraction Above Interacted with:</strong></td>
<td></td>
</tr>
<tr>
<td>1997</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
</tr>
<tr>
<td>1999</td>
<td>-0.72</td>
</tr>
<tr>
<td></td>
<td>***</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
</tr>
<tr>
<td>2000</td>
<td>-0.09</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
</tr>
<tr>
<td><strong>Fraction Above Interacted with Above and:</strong></td>
<td></td>
</tr>
<tr>
<td>1997</td>
<td>-0.07</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
</tr>
<tr>
<td>1999</td>
<td>-0.45</td>
</tr>
<tr>
<td></td>
<td>***</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
</tr>
<tr>
<td>2000</td>
<td>-0.09</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
</tr>
</tbody>
</table>

| Number of Obs.       | 136,118                                       |
| R-Squared            | 0.22                                          |

*** p<0.01, ** p<0.05, * p<0.1

Notes: In Column (1) the dependent variable is the change in the median employer capital contributions, where the median is calculated separately in each 2-digit occupation-firm cell for the workers above and the workers below the threshold. Along with the normal sets of controls, which are described in the notes for Table 2.4, we include a dummy variable that indicates whether workers are located above or below the threshold interacted with year fixed effects, as well as the interaction between this dummy variable, the fraction above, and year fixed effects. All standard errors are clustered at the 2-digit occupation-firm level. Coefficients are multiplied by 100 so that they are in percentage point units.
above the threshold are passive savers and have smaller individual pension contributions. The intuition behind this prediction is that if the directly affected workers are active within their individual accounts (which are perfect substitutes for employer accounts), then they will set their individual contributions optimally, and the firm would not need to respond to the reform. Note that we remain agnostic about the reason workers are passive in their capital accounts. It can, for example, be driven by non-optimizing workers’ inattention or by the behavior of optimizing workers who expect in equilibrium the firm to react to the change in incentives.

To test this prediction, we compare the change in employer capital contributions in occupation-firm cells that have different fractions of active workers above the threshold. We identify active workers above the threshold as those who had positive contributions in privately-held capital accounts in the previous year. Note that while we use this measure of activeness, our findings are robust to defining activeness in different ways, such as defining it as the fraction of previous years individuals changed their pension contribution levels, which is the activeness measure used by Chetty et al. (2014). We also restrict the sample to occupation-firm cells with more than 50% of workers above the threshold to see if the response in the cells with the largest reaction to the reform is moderated by the number of active workers above the threshold.

Figure 2.5 plots the employer capital contribution rate by year, splitting the sample into two groups based on whether the occupation-firm cell has less (in solid lines) or more (in dashed lines) than the median percent of active workers among those in the top income tax bracket in 1995. Note that the patterns in the pre-period are consistent with employer and individual capital contributions being substitutes: occupation-firm cells with fewer workers above the threshold with positive individual capital contributions have employer capital contributions that are approximately 2.5 pp higher than occupation-firm cells with more workers above the threshold with positive capital contributions. This also leads to a disparity in their response to the reform: occupation-firm cells with fewer active workers above the threshold – the solid line – have an approximately 2.1 pp decrease in employer capital contributions, while occupation-firm cells with more active workers above the threshold – the dashed line – have a mere .7 pp decrease. This suggests that firms are responsive to their workers’ individual contributions behavior.

Given that the share of active workers above the threshold may be correlated with the fraction
FIGURE 2.5
Change in Employer Capital Contributions by the Percent of Workers above the Threshold with Positive Individual Capital Contributions

Notes: This figure plots the average employer capital contribution rate for private white collar workers in firm-occupation cells with more than 50% of workers above the threshold by year, separately by the activeness of workers within the occupation-firm cell. We measure activeness by whether workers in 1995 (the earliest year in our sample period) have positive individual capital contributions. The solid line plots the average employer capital contribution rate for occupation-firm cells with fewer active workers (that is, fewer workers with capital contributions in 1995). The dashed line plots employer capital contributions for occupation-firm cells with more active workers above the threshold (that is, more workers above the threshold that have capital contributions in 1995). The two groups are split by the median fraction of workers above the threshold who had capital contributions in 1995.
of workers above, we augment this analysis with regressions that explicitly control for the fraction of workers above the threshold. Column (1) of Table 2.7 shows the results for when we restrict the sample to occupation-firm cells with more than 50% of workers above the threshold, and include the lagged fraction of workers above and below the threshold with positive individual capital contributions interacted with year dummies in our standard regression from column (4) of Table 2.2. In column (1) of Table 2.7, the coefficient on the fraction of workers above the threshold with positive individual capital contributions interacted with the year 1999 is 3.2 and is statistically significant at the 1% level. This results in an elasticity of 1.4 so that a 1% increase in the fraction of passive workers above the threshold leads to a 1.4% increase in the firm’s response to the 1999 reform.\textsuperscript{82} This result implies that the firm substantially substitutes for the individuals’ passive behavior.

This heterogeneous response across firms with different fractions of active workers is largely due to different levels of contributions in the pre-period as seen in Figure 2.5. Given that we are looking at an equilibrium result, this substitution pattern could be due to employer responses to individual contributions or vice versa. To try and disentangle these two explanations, we instrument for whether a worker has positive capital contributions with the fraction of years they made positive capital contributions at their previous firm. If we still find a significant effect when using this instrument, it would suggest that firms are indeed choosing contributions based on their workers’ individual contributions.\textsuperscript{83}

Column (2) of Table 2.7 reports the reduced-form effect, while column (3) reports the estimates from two-stage least squares. When we run two-stage least squares, the coefficient on the fraction of workers above with positive contributions interacted with 1999 decreases in magnitude from column (1) to 2.2, though this is not statistically different from the estimate in column (1) at the 5% level, and it is statistically significant at the 1% level. Therefore, the results from the IV provide evidence

\textsuperscript{82}This is calculated given that the average fraction of passive workers above the threshold was .66 and the average decrease in employer capital contributions in 1999 for occupation-firm cells with more than 50% of workers above the threshold was 1.5 percentage points.

\textsuperscript{83}This instrumental variable strategy requires the assumption that workers’ pension contribution preferences do not change discretely when they switch firms, which is similar to the assumption Chetty et al. (2014) make in their analysis of firm switchers.
**TABLE 2.7**  
Heterogeneity in Responses to the Reform by Activeness of Workers

<table>
<thead>
<tr>
<th>Dep. Variable:</th>
<th>Δ Employer Capital Contributions</th>
<th>Reduced Form</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Fraction Above:</td>
<td></td>
<td>0.35</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.37)</td>
<td>(0.41)</td>
</tr>
<tr>
<td>Above Interacted With:</td>
<td></td>
<td>1997</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.42)</td>
<td>(0.46)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1999</td>
<td>-2.84***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.59)</td>
<td>(0.67)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2000</td>
<td>-0.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.49)</td>
<td>(0.53)</td>
</tr>
<tr>
<td>Fraction with Positive Individual Capital Contributions Above the Threshold</td>
<td>-0.29</td>
<td>0.04</td>
<td>3.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.20)</td>
<td>(0.26)</td>
</tr>
<tr>
<td>Fraction with Positive Individual Capital Contributions Above the Threshold Interacted With:</td>
<td>1997</td>
<td>0.19</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.20)</td>
<td>(0.24)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1999</td>
<td>3.16***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.27)</td>
<td>(0.34)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2000</td>
<td>1.08***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.24)</td>
<td>(0.29)</td>
</tr>
<tr>
<td>Number of Obs.</td>
<td></td>
<td>23,526</td>
<td>21,725</td>
</tr>
<tr>
<td>R-Squared</td>
<td></td>
<td>0.45</td>
<td>0.45</td>
</tr>
</tbody>
</table>

*** p<0.01, ** p<0.05, * p<0.1

Notes: This table displays regressions of the change in employer capital contributions for 2-digit occupation-firm cells with more than 50% of workers above the threshold. Along with the usual set of controls (described in the notes of Table 2.4) and the fraction above, column (1) includes the fraction of workers with positive individual capital contributions in the previous year above and below the threshold interacted with year fixed effects. Only the coefficients on the fraction of workers with positive individual capital contributions above the threshold are reported in the table. In column (2), this measure is replaced by the fraction of years workers had positive capital contributions at their previous firm, which is then averaged among all workers who are above (or below) the threshold. In column (3), the percent with positive individual capital contributions (the measure from column (1)) is instrumented with the measure from column (2). All standard errors are clustered at the 2-digit occupation-firm level. Coefficients are multiplied by 100 so that they are in percentage point units.
that firms set employer contributions taking into account individual contributions.\footnote{This evidence is consistent with Chetty et al. (2014), which showed that most workers do not adjust their individual pension contributions when they move to a firm with a different level of employer contributions.}

Altogether, our results suggest that firms set pensions contributions not only considering changes in their workers’ saving incentives, but also considering workers’ behavior with respect to their individual pension contributions, which affects their preferred level of employer contributions. That is, employers not only respond to changes in their employees’ savings incentives (their preferred choices), but also to the degree to which the employees actively respond on their own (their actual decisions).

\subsection{Conclusion}

This paper provides clear evidence that employers set savings contributions in accordance with their employees’ saving incentives. In particular, we find that the change in employer capital contributions in response to the 1999 savings reform is strongly related to the fraction of workers who are above the threshold and are directly affected by the change in pension subsidies. We find that occupation-firm cells with a 1% larger fraction of affected workers have on average 1.5% larger decrease in employer capital contributions. This fact remains robust to a large set of controls and various specifications. We additionally find that firms in which there are more passive workers among those affected by the reform respond more strongly to the reform. This pattern suggests that employer and employee responses to savings incentives are substitutes. For example, if workers’ passive saving behavior is due to behavioral biases, our evidence suggests that firms are more active in response to changes in incentives for these workers to correct for their bias.

More generally, our results suggest that firms are responsive to workers’ preferences in setting contributions to employer-based savings accounts within the private sector. Even though (and possibly because) a significant share of workers is passive in their individual savings decisions, the evidence is consistent with an equilibrium in which employees value firms setting savings contributions in accordance with their preferences.

Since employer contributions and defaults are extremely effective at increasing individuals’ total
level of savings, some governments are considering implementing policies that incentivize employer-based savings accounts and default contribution rates. Given the increasing reliance of individual retirement savings on employers' contributions, our findings are promising and encouraging preliminary evidence that they are set carefully and in accordance with workers’ savings preferences. Whether firms’ behavior is due to employers acting as benevolent planners or indirectly through bargaining and competition has important implications for the optimal design of retirement savings policies and is a fruitful direction for future research.
3 ENDOGENOUS PATERNALISM AND PSEUDO-RATIONALITY

3.1 Introduction

Equilibrium behavior is jointly determined by many actors, including households, firms, and government institutions. In this paper we study the interaction between households and a well-intentioned government: “the social planner.” We ask what would happen if (i) the social planner were rational, (ii) the households were heterogeneous in their degree of rationality, and (iii) the social planner had some scope to intervene (e.g., Social Security). We show that in equilibrium, planner optimization is a substitute for household optimization. This is true even when there are information asymmetries, so that households know more about their preferences than planners.

Our analysis illustrates a potential misattribution in economic analysis. Is optimal behavior caused by optimizing households, or is optimal behavior caused by planners who paternalistically manipulate households? We find that some widely studied classical tests for household optimization, which rely on the Euler Equation and other optimality conditions, do not resolve this question. Equilibrium properties that are implied by household optimization are also implied by planner optimization (even when households are not optimizers). The actions of rational planners may result in optimal equilibrium allocations, e.g., by causing the Euler Equation to hold on average in the population.

To demonstrate these ideas, we study a classic life-cycle model in which agents work until reaching retirement age and can save a fraction of their labor income for consumption after retirement. The planner has two policy levers: forced retirement savings accounts, similar to Social Security, and defaults within private voluntary savings accounts. We consider an economy with three types of households: optimizing households, who behave optimally throughout their life-cycle, myopic households, who opt out of all defaults and consume their entire disposable income in each period, and passive households, who accept the planner’s defaults and consume their residual income. We include myopes – an extreme type – to emphasize that our results hold even when the deck is stacked against social efficiency. Additionally, we allow for agents to have privately observed pref-

85 This chapter is jointly written with David Laibson.
erence parameters of arbitrary structure. Our planner designs a Social Security system and chooses default levels of savings and payouts within the voluntary savings accounts in order to maximize average social welfare, taking into account the optimal behavior of optimizing households and the suboptimal behavior of myopic and passive households.

We show that in equilibrium average marginal utility before retirement is equal to average marginal utility after retirement for any distribution of optimizing, myopic, and passive households. Such marginal utility smoothing applies for all concave utility functions with a general class of taste shocks. Our main argument is that Euler Equations, which characterize an economy with optimizing households, also arise in an economy with optimally-designed institutions. As such, these equilibrium properties do not differentiate between household optimization and planner optimization.

Marginal utility smoothing is closely related to consumption smoothing, which is among the most common tests for household optimization. The precise form of consumption smoothing depends on the curvature of the utility function and the structure of taste shocks. We illustrate that when average consumption smoothing arises in a world with optimizing households, it also arises in our economy with an optimizing planner (for any distribution of optimizing, myopic, and passive households). Moreover, in some leading cases consumption smoothing does not arise with optimizing households, but does arise with an optimizing planner and non-optimizing households. These results imply that consumption smoothing is a more robust property of the model without optimizing households, in contrast to the common view.

Although we show that the average Euler Equation and average consumption smoothing are not generally diagnostic of household optimization, we demonstrate that other moments of economic behavior – both in the cross section and in response to policy changes – identify the extent of household optimization. For example, bunching (i.e., mass points) at the default savings or payout levels in the cross sectional distribution identify passive households. In addition, household-level elasticities in response to changes in policy tools reveal household-level optimization. For instance, less-than-full crowd out in economic aggregates in response to changes in default or forced savings levels entails information on the mass of passive and myopic households, respectively. Importantly, these tests are not confounded by the equilibrium link between household and planner optimization. As such, they provide ways to overcome the misattribution problem that we highlight.
Our choice of the particular framework for illustrating our arguments – namely, savings over the life-cycle – is motivated by the seemingly contradictory findings of the recent research on savings for retirement. Analyzing equilibrium allocations, some papers find evidence that is consistent with optimal savings by households and is perceived as support for household optimization. For example, using a structural life-cycle analysis and accounting for government transfers and Social Security benefits, Scholtz et al. (2006) estimate that less than twenty percent of households in the Health and Retirement Study under-save. Moreover, Scholtz et al. (2006) find that the wealth deficits, when they exist at all, are generally a small fraction of lifetime wealth. Aguiar and Hurst (2005) report that the apparent drop in food expenditure at retirement is illusory, in the sense that caloric consumption and other measures of meal quality do not change when households enter retirement. This evidence suggests that food expenditure and leisure are substitutes, pointing to a home production channel for meal preparation and shopping. Studying overall nondurable expenditure, Aguila, Attanasio, and Meghir (2011) find no evidence of a significant drop in consumption at retirement.

However, a growing body of research on retirement savings finds evidence for economic behavior that is not consistent with household optimization. In particular, recent research has shown that automatic employer contributions and default contribution rates to workers’ retirement savings accounts have large effects on individuals’ savings. The vast majority of households do not actively deviate from the default contribution rates, as well as from the default fund, and are unresponsive to subsidies for contributions to retirement savings accounts (Madrian and Shea 2001, Choi et al 2004, Beshears et al. 2009, and Chetty et al. 2014). Our analysis reconciles the different findings by showing that household-level sub-optimization, which is identified by natural experiments like the change in a firm-level default, will be masked in the total population (due to paternalistic, national policies).

The remainder of the paper is organized as follows. In section 3.2, we study a baseline case of multiplicative taste shifters and general concave utility functions, which yields the result summarized in the introduction. Section 3.2 also analyzes several special cases – i.e., some commonly used parametric utility functions. In section 3.3, we discuss the issue of identification – how can the distribution of optimizing, passive, and myopic households be identified? In section 3.4, we generalize our results by studying a larger class of (non-parametric) utility functions that allow for
very general forms of taste shocks. Section 3.5 concludes.

3.2 Baseline Model

Setup. We consider a continuous-time life-cycle model that focuses on the decision of saving for retirement. Agents work from date 0 to date $T$ and live forever. We assume that the instantaneous interest rate, $r$, is fixed. The life-time budget constraint household $i$ faces is

$$
\int_{t=0}^{\infty} e^{-rt} c_i(t) dt \leq \int_{t=0}^{T} ye^{-rt} dt = \frac{y}{r} (1 - e^{-rT}),
$$

(3.1)

where $c_i(t)$ is consumption in period $t$, $y$ is income during working life, and all variables are in real terms.

Preferences. Agents have instantaneous utility function $u(c_i(t))$ when they are working ($0 \leq t < T$) and instantaneous utility function $\theta_i u(c_i(t))$ when they are retired ($T \leq t$). The discount rate is denoted by $\rho$ and we assume that $r = \rho$. Total lifetime utility is given by

$$
U_i = \int_{t=0}^{T} e^{-\rho t} u(c_i(t)) dt + \int_{t=T}^{\infty} e^{-\rho t} \theta_i u(c_i(t)) dt.
$$

The taste parameter $\theta_i \in \mathbb{R}$ varies across agents. This parameter can be thought of as a taste shifter for retirement consumption. Across households, $\theta_i$ is independently drawn from the same cumulative distribution function $F(\theta_i)$ over $\Theta$ with a probability density function $f(\theta_i)$, and is assumed to be known to the households at time 0.

Institutions. There are two kinds of institutions: a voluntary savings account and a forced retirement savings account. We assume that the planner sets a default level of savings in the voluntary savings account, $s_D$, as well as a default annuity payout from the voluntary savings account after retirement, $a_D$. The Households are able to opt-out of either (or both) of these defaults at zero cost. In addition, the planner sets forced retirement savings (from labor income) of the amount $s_F$, which is deposited into the forced savings account during working life.\(^{86}\) Upon retirement, the balance of this account remains illiquid except for a fixed annuity payout, $a_F$, that

---

\(^{86}\)This is essentially what has been adopted by Australia, Israel, and Singapore, and has similarities to Social Security in the US.
is set by the planner. We denote the planner’s policy tools by \( \tau \equiv (s_D, a_D; s_F, a_F) \).

The savings and payout rates introduced in the previous paragraph \(- s_D, a_D, s_F, a_F \) are constants (as opposed to being age-dependent) because (i) labor income is constant during working life and (ii) \( r = \rho \). If either of these assumptions does not hold, then the planner would choose to make the vector \( \tau \) age-dependent and all of our key results would continue to hold. We omit this generalization to keep the analysis as simple as possible. In this sense, the assumptions of constant labor income and \( r = \rho \) are without loss of generality.

**Household types.** There are three types of households (on a continuum from 0 to 1): *Optimizers*, *Passives*, and *Myopes*. We explain each of these in turn.

*Optimizers* (notated \( O \)) choose the optimal level of consumption in all time periods, taking into account their private information about \( \theta_i \) and the institutional constraints that they face. Optimizing households may not be able to achieve their first best allocation if their optimal level of savings is lower than the forced level of savings. In other words, optimizing households choose the life-time consumption path \( \{c_i(t)\}_{t=0}^{\infty} \) that solves

\[
\max_{\{c_i(t)\}_{t=0}^{\infty}} U_i \text{ subject to (3.1) and}
\]

\[
c_i(t) \leq y - s_F \text{ if } x_i(t) = 0 \text{ and } t < T,
\]  

\[
c_i(t) \leq a_F \quad \text{if } x_i(t) = 0 \text{ and } t \geq T,
\]

where \( x_i(t) \) denotes the accrued balances in the voluntary savings account at time \( t \).

To characterize the equilibrium behavior of optimizers, first note that since \( r = \rho \) optimizing households with any value of \( \theta_i \) choose to annuitize their overall savings at period \( T \) so that their consumption flow at retirement is constant. Next, we consider the unconstrained problem – that it, we focus on optimizing households that are unconstrained during their working life (for whom \( x_i(t) \neq 0 \) for all \( t < T \)). We denote the solutions to this problem by \( UO \) (for *Unconstrained Optimizers*). In this case, \( r = \rho \) implies that households’ consumption when working is constant at \( c_{w1}^{UO} \leq y - s_F \). Additionally, optimization requires that \( u'(c_{w1}^{UO}) = \theta_i u'(c_{r1}^{UO}) \), where \( c_{r1}^{UO} \) is the constant consumption flow in retirement (periods \( t \geq T \)). Households of type \( \theta_i \) for whom \( y - s_F - c_{w1}^{UO}(\theta_i) \geq 0 \) can achieve the unconstrained optimum. This allows us to divide optimizers into
two distinct groups in any equilibrium—constrained and unconstrained households—as summarized in the following lemma.

**Lemma 1.** For given institutions $\tau$, there are exactly two cases for optimizing households during working life ($t < T$): (1) Constrained Optimizers, for whom $c_i(t) = y - s_F$, and (2) Unconstrained Optimizers, for whom $c_i(t) < y - s_F$ and is constant during working life. Households for whom $\theta_i < \theta^*(\tau)$ are constrained and households for whom $\theta_i \geq \theta^*(\tau)$ are unconstrained, where $\theta^*(\tau)$ is the value of $\theta_i$ that solves $y - s_F - c_{ui}^{UO}(\theta_i) = 0$.

**Proof.** First, from $u'(c_{ui}^{UO}) = \theta_i u'(c_{ri}^{UO})$ it follows that $c_{ui}^{UO}$ declines in $\theta_i$. Therefore, households with $\theta_i \geq \theta^*(\tau)$ are unconstrained as they want to defer consumption to the future. Second, for households with $\theta_i < \theta^*(\tau)$ it is never optimal to save. Transferring $\$1$ from period $t_0 < T$ for consumption at period $t_1 > T$ after retirement would amount to a benefit of at most $e^{(r - \rho)(t_1 - t_0)}\theta_i u'(c_{ri}^{UO}(\theta^*)) < e^{(r - \rho)(t_1 - t_0)}\theta^* u'(c_{ri}^{UO}(\theta^*))$ at a cost of at least $u'(c_{ui}^{UO}(\theta^*))$, which is undesirable since $u'(c_{ui}^{UO}(\theta^*)) = \theta^* u'(c_{ri}^{UO}(\theta^*))$. In addition, intertemporal substitution of resources before retirement is never necessary for reaching the optimal path. This is because with no voluntary savings for retirement, with constant income flows before retirement, and with $r = \rho$, it must be optimal for households with $\theta_i < \theta^*(\tau)$ to consume their entire disposable income in each period before $T$. That is, for them $c_i(t) = y - s_F$ for any $t < T$ and their consumption while working is constrained by their resources that are left after the government-mandated savings. $\blacksquare$

We denote the consumption of Constrained Optimizers by the superscript $CO$, with $c_{ui}^{CO} = y - s_F$ in each period before retirement ($t < T$) and $c_{ri}^{CO} = a_F$ in each period after they retire ($t \geq T$).

*Passives* (notated $P$) accept all defaults and consume the residual income flows. That is, for them $c_{ui}^{P} = y - s_F - s_D$ in periods $t < T$ and $c_{ri}^{P}(t) = a_F + a_D$ in periods $t \geq T$. *Myopes* (notated $M$) opt out of all defaults and consume as much as possible in every period. Hence, myopes are constrained only by the forced savings, so that they consume $c_{ui}^{M} = y - s_F$ in periods $t < T$ and $c_{ri}^{M}(t) = a_F$ in periods $t \geq T$, similarly to the constrained optimizers.

*The rational planner’s problem.* Define $V^O(\tau; \theta_i)$, $V^P(\tau; \theta_i)$, and $V^M(\tau; \theta_i)$ to be the value
functions of optimizers, passives, and myopes, respectively, such that

\[
V^O(\tau; \theta_i) = \begin{cases} 
V^{CO}(\tau; \theta_i) & \text{if } \theta_i < \theta^*(\tau) \\
V^{UO}(\tau; \theta_i) & \text{if } \theta_i \geq \theta^*(\tau)
\end{cases},
\]

\[
V^{UO}(\tau; \theta_i) \equiv \int_{t=0}^{\tau} e^{-\rho t} u(c_{\tau i}(\theta_i)) dt + \int_{t=\tau}^{\infty} e^{-\rho t} \theta_i u(c_{\tau i}(\theta_i)) dt,
\]

\[
V^{CO}(\tau; \theta_i) = V^M(\tau; \theta_i),
\]

\[
V^P(\tau; \theta_i) \equiv \int_{t=0}^{\tau} e^{-\rho t} u(y - s_F - s_D) dt + \int_{t=\tau}^{\infty} e^{-\rho t} \theta_i u(a_F + a_D) dt,
\]

\[
V^M(\tau; \theta_i) \equiv \int_{t=0}^{\tau} e^{-\rho t} u(y - s_F) dt + \int_{t=\tau}^{\infty} e^{-\rho t} \theta_i u(a_F) dt.
\]

The shares of optimizing, passive, and myopic households are \( \mu_O, \mu_P \), and \( \mu_M \), respectively, where \( 0 \leq \mu_O, \mu_P, \mu_M \leq 1 \) and \( \mu_O + \mu_P + \mu_M = 1 \). We denote the distribution of these "decision" types by \( \mu \equiv (\mu_O, \mu_P, \mu_M) \). The utilitarian social planner’s objective is to choose the policy tools \( \tau \) that maximize the households’ average utility,

\[
W(\tau) \equiv \mu_O \int V^O(\tau; \theta_i) dF(\theta_i) + \mu_P \int V^P(\tau; \theta_i) dF(\theta_i) + \mu_M \int V^M(\tau; \theta_i) dF(\theta_i),
\]

taking into account the behavior and resource constraint of each household. Hence, the planner solves

\[
\max_{\tau} W(\tau; \theta_i).
\]

### 3.2.1 Equilibrium with Optimal Institutions

We now turn to characterize the equilibrium allocation in our economy with a rational social planner. We prove that some basic equilibrium features that are commonly attributable to household optimization also appear in our highly irrational economy as a result of the planner’s intervention.

To present these results we make repeated use of the expectation operator, \( E[\cdot] \), which is the expectation taken over the entire population. Specifically, for any random variable \( x \), this expectation operator integrates jointly over decision types (optimizers, passives, and myopes) and
over taste shocks \((\theta_i)\):

\[
E [x] \equiv \mu_O \int_{\Theta} x_i^O dF(\theta_i) + \mu_P \int_{\Theta} x_i^P dF(\theta_i) + \mu_M \int_{\Theta} x_i^M dF(\theta_i).
\]

**Marginal-Utility Smoothing.** We begin by analyzing the basic Euler Equation for consumption allocation before and after retirement. In an economy where *all* agents are optimizers, \(u'(c_{wi}) = \theta_i u'(c_{ri})\) for each household. In principle, if an econometrician knew each household’s value of \(\theta_i\), then it would be possible to test this equation at the household level. However, \(\theta_i\) (which is a taste shock) is not observable and all variables are measured with noise (which prevents equations from holding exactly). Accordingly, empirical analysis tends to focus on whether the Euler Equation holds on average, that is, whether:

\[
E \left[ u'(c_{wi}) \right] = E \left[ \theta_i u'(c_{ri}) \right].
\]

Our first theorem proves that this last equation *also* holds in our economy, in which optimizing households can represent any fraction of the economy (i.e., \(\mu_O \in [0, 1]\)).

**Theorem 1.** Assume a rational planner. Then for any distribution of optimizing, passive, and myopic households, a classical Euler Equation will hold on average in the population:

\[
E \left[ u'(c_{wi}) \right] = E \left[ \theta_i u'(c_{ri}) \right].
\]

**Proof.** Since labor income is constant and \(r = \rho\), the planner will choose \(a_F\) and \(a_D\) that are not age-dependent. The budget constraint implies that \(a_F = s_F(e^{\rho T} - 1)\) and \(a_F + a_D = (s_F + s_D)(e^{\rho T} - 1)\). Next, we turn to the necessary conditions for the optimal levels of \(s_D\) and \(s_F\). At the optimum, \(\frac{dW(\tau)}{ds_D} = 0\). As passives are the only group that is affected by \(s_D\) in equilibrium, we have that

\[
\frac{dW(\tau)}{ds_D} = \mu_P \int_{\Theta} \frac{dV^P(\tau; \theta_i)}{ds_D} dF(\theta_i) = 0. \tag{3.4}
\]

\(^{87}\)These annuities are calculated as \(s_F \int_{t=0}^{T} e^{-\tau t} dt = a_F \int_{t=0}^{\infty} e^{-\tau t} dt\) and \((s_F + s_D) \int_{t=0}^{T} e^{-\tau t} dt = (a_F + a_D) \int_{t=0}^{\infty} e^{-\tau t} dt\).
Since \( \frac{dV^P(\tau; \theta_i)}{ds_D} = \frac{(1-e^{-\rho T})}{\rho} (-u'(c^P_{w1}) + \theta_i u'(c^P_{r1})) \), it follows that

\[
\int_{\Theta} (-u'(c^P_{w1}) + \theta_i u'(c^P_{r1})) dF(\theta_i) = 0. \tag{3.5}
\]

Next, consider the necessary condition for the forced savings,

\[
\frac{dW(\tau)}{ds_F} = \mu_O \left[ \int_{\theta_i < \theta^*(\tau) \atop \theta_i < \theta^*(\tau)} \frac{dV^{CO}(\tau; \theta_i)}{ds_F} f(\theta_i) d\theta_i \right. \\
+ \mu_O \left. \left[ \int_{\theta_i \geq \theta^*(\tau)} \frac{dV^{UO}(\tau; \theta_i)}{ds_F} f(\theta_i) d\theta_i - V^{UO}(\tau; \theta^*(\tau)) f(\theta^*(\tau)) \right] \right] \\
+ \mu_P \left[ \int_{\Theta} \frac{dV^P(\tau; \theta_i)}{ds_F} f(\theta_i) d\theta_i \right] + \mu_M \left[ \int_{\Theta} \frac{dV^M(\tau; \theta_i)}{ds_F} f(\theta_i) d\theta_i \right] = 0. \tag{3.6}
\]

To simplify this formula, first note that \( V^{CO}(\tau; \theta^*(\tau)) = V^{UO}(\tau; \theta^*(\tau)) \). Second, since passives are affected symmetrically by forced and default savings, we get that \( \frac{dV^P(\tau; \theta_i)}{ds_F} = \frac{dV^P(\tau; \theta_i)}{ds_D} \) and, hence,

\[
\mu_P \int_{\Theta} \frac{dV^P(\tau; \theta_i)}{ds_F} dF(\theta_i) = \mu_P \int_{\Theta} \frac{dV^P(\tau; \theta_i)}{ds_D} dF(\theta_i) = 0. \quad \text{Third, recall that unconstrained optimizers can completely offset the effect of forced savings and are, therefore, unaffected by the government’s tools in equilibrium — that is, } \frac{dV^{UO}(\tau; \theta_i)}{ds_F} = 0. \quad \text{Put together, equation (3.6) implies that}
\]

\[
\frac{dW(\tau)}{ds_F} = \mu_O \left[ \int_{\theta_i < \theta^*(\tau)} \frac{dV^{CO}(\tau; \theta_i)}{ds_F} f(\theta_i) d\theta_i \right. \\
+ \mu_O \left. \left[ \int_{\theta_i \geq \theta^*(\tau)} \frac{dV^{UO}(\tau; \theta_i)}{ds_F} f(\theta_i) d\theta_i - V^{UO}(\tau; \theta^*(\tau)) f(\theta^*(\tau)) \right] \right] \\
+ \mu_P \left[ \int_{\Theta} \frac{dV^P(\tau; \theta_i)}{ds_F} f(\theta_i) d\theta_i \right] + \mu_M \left[ \int_{\Theta} \frac{dV^M(\tau; \theta_i)}{ds_F} f(\theta_i) d\theta_i \right] = 0, \tag{3.7}
\]

which combined with \( \frac{dV^{CO}(\tau; \theta_i)}{ds_F} = \frac{dV^M(\tau; \theta_i)}{ds_F} = \frac{(1-e^{-\rho T})}{\rho} (-u'(y - s_F) + \theta_i u'(s_F(e^{\rho T} - 1))) \) implies that

\[
\mu_O \int_{\theta_i < \theta^*(\tau)} (-u'(c^{CO}_{w1}) + \theta_i u'(c^{CO}_{r1})) dF(\theta_i) + \mu_M \int_{\Theta} (-u'(c^{M}_{w1}) + \theta_i u'(c^{M}_{r1})) dF(\theta_i) = 0. \quad \text{\tag{3.8}}
\]
Lastly, recall that for the unconstrained optimizers $u'(c_{wi}^{UO}) = \theta_i u'(c_{ri}^{UO})$ and, therefore

$$
\mu_O \int_{\theta_i \geq \theta^*(\tau)} \left( -u'(c_{wi}^{UO}) + \theta_i u'(c_{ri}^{UO}) \right) dF(\theta_i) = 0. \quad (3.9)
$$

Equations (3.5), (3.8), and (3.9) yield the result. ■

Theorem 1 implies that

$$
E \left[ u'(c_{wi}) \right] = E \left[ \theta_i u'(c_{ri}) \right],
$$

whether or not all households are optimizers. Theorem 1 is true for any mass vector $\mu$ that characterizes the fraction of optimizing, passive, and myopic households. All of the theorems that follow have a shared property: a classical optimality condition holds (on average in the population) whether or not all of the households are optimizers.

A different representation of our result in the baseline model is also useful. In particular, for a given prior for (or estimate of) $E[\theta_i]$ one may wish to explore the mean of the ratio of marginal utilities before and after retirement, $E \left[ \frac{u'(c_{wi})}{u'(c_{ri})} \right]$. In a fully rational world, the equality $E \left[ \frac{u'(c_{wi})}{u'(c_{ri})} \right] = E[\theta_i]$ holds since the average ratio of marginal utilities must reflect the average taste shock for consumption at retirement. This representation emphasizes the case in which researchers assume away individual-level or mean consumption-leisure complementarities, which implies that $E[\theta_i] = 1$, and has been assumed in most papers. In this case, marginal utility is smoothed such that the mean ratio of marginal utilities equals one, i.e., $E \left[ \frac{u'(c_{wi})}{u'(c_{ri})} \right] = 1$. In the next theorem we prove that this exactly holds in our economy as well.

**Theorem 2.** Assume a rational planner. Then for any distribution of optimizing, passive, and myopic households, a classical Euler Equation Ratio will hold on average in the population:

$$
E \left[ \frac{u'(c_{wi})}{u'(c_{ri})} \right] = E[\theta_i].
$$

**Proof.** Equation (3.5) implies that $\int_{\Theta} \left( -\frac{u'(c_{wi}^{CO})}{u'(c_{ri}^{CO})} + \theta_i \right) dF(\theta_i) = 0$, equation (3.8) and $c_{ri}^{CO} = c_{ri}^{M} = \theta_i$ imply that $\mu_O \int_{\theta_i \geq \theta^*(\tau)} \left( -\frac{u'(c_{wi}^{CO})}{u'(c_{ri}^{CO})} + \theta_i \right) dF(\theta_i) + \mu_M \int_{\Theta} \left( -\frac{u'(c_{wi}^{M})}{u'(c_{ri}^{M})} + \theta_i \right) dF(\theta_i) = 0$, and $u'(c_{wi}^{UO}) = u'(c_{wi}^{CO})$.
Lemma 2 [Special Cases]. Assume any distribution of optimizing, passive, and myopic house-
holds. The results are summarized in the following lemma.

\[
E[\theta_i] = 1.
\]

Given any distribution of optimizing, passive, and myopic households, we analyze consumption moments under two assumptions: first, when \( \theta_i = 1 \) for every \( i \) as implicitly assumed in most papers, and second, when there are no taste shocks on average at retirement, i.e., \( b > 0 \).

We now introduce our notation. We study the cases of: (i) quadratic utility, i.e., \( u(c) = c - \frac{1}{2} rc^2 \), and (ii) constant relative risk aversion, i.e., \( u(c) = \frac{1}{1-\gamma} c^{1-\gamma} \) where \( \gamma > 1 \). For each case, we analyze consumption moments under two assumptions: first, when \( \theta_i = 1 \) for every \( i \) as implicitly assumed in most papers, and second, when there are no taste shocks on average at retirement, i.e., \( b > 0 \).

For the case of quadratic utility, we get analogous results studying the growth rate of consumption at retirement (and using a first-order approximation). Together, these results imply that consumption smoothing is a more robust property of the model without optimizing households.

For the case of constant relative risk aversion, we get analogous results studying the growth rate of consumption at retirement (and using a first-order approximation). Together, these results imply that consumption smoothing is a more robust property of the model without optimizing households.

For the other case, when \( \theta_i = 1 \) for every \( i \) as implicitly assumed in most papers, and second, when there are no taste shocks on average at retirement, we get analogous results studying the growth rate of consumption at retirement (and using a first-order approximation). Together, these results imply that consumption smoothing is a more robust property of the model without optimizing households.

For the case of quadratic utility, we get analogous results studying the growth rate of consumption at retirement (and using a first-order approximation). Together, these results imply that consumption smoothing is a more robust property of the model without optimizing households.

For the case of constant relative risk aversion, we get analogous results studying the growth rate of consumption at retirement (and using a first-order approximation). Together, these results imply that consumption smoothing is a more robust property of the model without optimizing households.

\[
\begin{align*}
E[\theta_i] \neq 0. &
\end{align*}
\]

\( E[\theta_i] = 1 \) implies that \( \frac{\partial u(c)}{\partial \theta_i} = 0 \) and \( \frac{\partial u(c)}{\partial \theta_i} = 0 \) only if there are no optimizing households. Therefore, the presence of optimizing households causes average consumption to fall at retirement. For both \( \theta_i = 1 \) and \( \theta_i = 0 \), we show that \( \theta_i = 0 \).

\[
\theta_i = 0.
\]

\( \theta_i = 0 \) implies that \( \frac{\partial u(c)}{\partial \theta_i} = 0 \) and \( \frac{\partial u(c)}{\partial \theta_i} = 0 \) only if there are no optimizing households. Therefore, the presence of optimizing households causes average consumption to fall at retirement. For both \( \theta_i = 1 \) and \( \theta_i = 0 \), we show that \( \theta_i = 0 \).

\[
\theta_i = 0.
\]

\( \theta_i = 0 \) implies that \( \frac{\partial u(c)}{\partial \theta_i} = 0 \) and \( \frac{\partial u(c)}{\partial \theta_i} = 0 \) only if there are no optimizing households. Therefore, the presence of optimizing households causes average consumption to fall at retirement. For both \( \theta_i = 1 \) and \( \theta_i = 0 \), we show that \( \theta_i = 0 \).

\[
\theta_i = 0.
\]
holds.

(A) Quadratic utility implies:

1. With no taste shocks at retirement \((\theta_i = 1\) for every \(i\)),

\[
E[cr_i - c_{wi}] = 0;
\]

2. With no taste shocks on average at retirement \((E[\theta_i] = 1)\),

\[
E[cr_i - c_{wi}] = -\text{cov}(\theta_i, cr_i) \leq 0.
\]

(B) Constant relative risk aversion utility implies (to a first-order approximation):

1. With no taste shocks at retirement \((\theta_i = 1\) for every \(i\)),

\[
E\left[ \frac{cr_i - c_{wi}}{c_{wi}} \right] \approx 0;
\]

2. With no taste shocks on average at retirement \((E[\theta_i] = 1)\),

\[
E\left[ \frac{cr_i - c_{wi}}{c_{wi}} \right] \approx -\text{cov}\left( \frac{\theta_i}{c_{wi}}, \frac{cr_i}{c_{wi}} \right) \leq 0.
\]

**Proof.** (A) Plugging \(u(c) = c - \frac{\gamma}{2} c^2\) into Theorem 1 implies that \(E[c_{wi} - \theta_i c_{ri}] = E[1 - \theta_i]/b\). Assuming that \(\theta_i = 1\) for every \(i\) yields \(E[c_{ri} - c_{wi}] = 0\), and assuming that \(E[\theta_i] = 1\) yields \(E[c_{ri} - c_{wi}] = -\text{cov}(\theta_i, c_{ri})\).

(B) Similar to the proof of Theorem 2, one can show that \(E\left[ \frac{\theta_i u'(c_{wi})}{u'(c_{wi})} \right] = 1\). This is because equation (3.5) implies that \(\int_{\Theta} \left(-1 + \frac{\theta_i u'(c'_{wi})}{u'(c_{wi})}\right) dF(\theta_i) = 0\), equation (3.8) and \(c_{wi}^{CO} = c_{wi}^M = y - s_F\) imply that \(\mu_O \int_{\Theta} \left(-1 + \frac{\theta_i u'(c'_{wi})}{u'(c_{wi})}\right) dF(\theta_i) + \mu_M \int_{\Theta} \left(-1 + \frac{\theta_i u'(c^M_{wi})}{u'(c_{wi})}\right) dF(\theta_i) = 0\), \(u'(c_{wi}) = \theta_i u'(c_{ri}')\) implies that \(\frac{\theta_i u'(c_{wi}')}{u'(c_{wi})} = 1\) and \(\mu_O \int_{\Theta} \left(-1 + \frac{\theta_i u'(c^{CO}_{wi})}{u'(c_{wi})}\right) dF(\theta_i) = 0\), which together yield \(E\left[ \frac{\theta_i u'(c_{wi})}{u'(c_{wi})} \right] = 1\).

Linearization of \(u'(c_{ri})\) around \(u'(c_{wi})\) in the latter equality results in \(E\left[ -\theta_i \gamma \frac{c_{ri} - c_{wi}}{c_{wi}} \right] \approx 1 - E[\theta_i]\).\(^{88}\)

Assuming that \(\theta_i = 1\) for every \(i\) yields \(E\left[ \frac{c_{ri} - c_{wi}}{c_{wi}} \right] \approx 0\), and assuming that \(E[\theta_i] = 1\) yields

\(^{88}\)The coefficient of relative risk aversion is \(\gamma \equiv \frac{u''(c)}{u(c)} c\).
Lemma 2 demonstrates that when there are no taste shocks, our economy is characterized by mean consumption smoothing for any distribution of optimizing, myopic, and passive households. Therefore, in this commonly studied case, consumption smoothing is not a diagnostic test for household optimization. In addition, with no average taste shocks, the covariance expressions in Lemma 2 are weakly positive and proportional to the mass of optimizing households. This introduces some counter-intuitive properties. First, the average change in consumption (entering retirement) will be exactly zero only when the fraction of optimizing households is zero (since \( \text{cov}(\theta_i, c_{ri}) = \text{cov}(\theta_i, \hat{c}_{ri}) = 0 \) if and only if \( \mu_O = 0 \)). Second, with a positive share of optimizing households, the consumption drop at retirement increases (since \( \mu_O > 0 \) implies that \( \text{cov}(\theta_i, c_{ri}) > 0 \) and \( \text{cov}(\theta_i, \hat{c}_{ri}) > 0 \)). That is, not only are consumption smoothing tests not conclusive for analyzing household optimization, in the important case of \( E[\theta_i] = 1 \) average consumption smoothing actually rejects the null hypothesis of household optimization.

### 3.3 Identification

While average Euler Equation and average consumption smoothing cannot generally reveal the distribution of household types, tests that rely on other moments of households’ economic behavior can be indicative of household optimization.

Consider the cross-sectional distribution of a key economic behavior, e.g., savings in our analysis. Figure 3.1 characterizes the distribution of overall savings when the distribution of taste shocks, \( F(\theta_i) \), is continuous, \( 0 < \mu_O, \mu_P, \mu_M < 1 \), and \( \mu_O + \mu_P + \mu_M = 1 \).

The first type of test for household optimization relies on the cross section. One example for such a test is that bunching (excess mass) around the combined levels of forced and default savings, \( s_F + s_D \), reveals the presence of passive households. Therefore, evidence of a discrete jump in the savings’ cumulative distribution function at this point would reject the null hypothesis of a fully rational economy.

Another type of test employs behavioral responses to policy variations, which can be measured by changes to the cross-sectional distribution or by estimating household-level elasticities. First, if a change in the default level of savings from \( s_D^0 \) to \( s_D^1 \) creates bunching around \( s_F + s_D^1 \), it is
FIGURE 3.1
The Cumulative Distribution Function of Household Savings

Notes: This figure plots the cumulative distribution function of household savings in our economy with a continuous distribution of taste shocks, $F(\theta_i)$, and with positive shares of optimizing, passive, and myopic households ($0 < \mu_O, \mu_P, \mu_M < 1$). The x-axis denotes overall household savings, which are defined as the difference between each period’s income ($y_i$) and consumption ($c_{wi}$) before retirement. The y-axis denotes the share of households with savings levels that are equal to or lower than any given level on the x-axis.
indicative of passive households even if before the policy change bunching at \( s_F + s_D^0 \) was induced by potential discontinuities in \( F(\theta_i) \). A different measure for testing the same hypothesis is the change in average (or overall) savings. If all households are optimizers, then a change in defaults will keep the distribution of savings, and hence average savings, unchanged. Any change in average savings would reject the hypothesis of full rationality, and in our world, would be indicative of the share of passive households.

Second, changes to the level of forced savings can shed light on the presence and fraction of myopic households. To see this, consider a change in the level of forced savings and evaluate the change in the mass of households that locate at the mandatory savings level, \( s_F \):

\[
\frac{d\Pr(s_i = s_F)}{ds_F} = \mu_O \times f(\theta^*(\tau)).
\]

This change in the mass of households at the forced savings level is positive if and only if there are optimizing households.

A related global test can be developed by studying average savings. With a decrease in the forced savings level, close to one-for-one decline in average savings would imply that households are not fully rational (and are either myopic or passive), whereas less-than-full adjustment would imply a positive share of optimizing households.

While these tests rely on the positive model that we assume, the general point is that a thoughtful look at the distribution of important economic outcomes can provide tests for household optimization that are not confounded by the presence of a rational planner.

### 3.4 Generalization

In this section we prove that our main result applies to a general class of preferences. In particular, we relax the multiplicativity assumption on taste shifters and show that marginal utility smoothing is achieved for all concave utility functions with a general class of taste shocks.

We assume that agents have instantaneous utility function \( u_w(c_i(t); \phi_i) \) when they are working \((0 \leq t < T)\) and instantaneous utility function \( u_r(c_i(t); \phi_i) \) when they are retired \((T \leq t)\). The \( k+1 \)-dimensional taste vector \( \phi_i \in \Phi \), which varies across households, can arbitrarily and differentially affect consumption utility of working and retired individuals. Note that indexing consumption
utility by work status allows for flexible consumption-leisure dependencies.

Across households, \( \phi_i \) is independently drawn from the same joint cumulative distribution function \( Q(\phi_i) \) over \( \Phi \) with a joint probability density function \( q(\phi_i) \), and is assumed to be known to the households at time 0. We partition the vector \( \phi_i \) into a scalar parameter \( \theta_i \in \Theta \subseteq \mathbb{R} \) and a \( k \)-dimensional vector \( \beta_i \in \mathbb{B} \subseteq \mathbb{R}^k \). We denote the marginal distribution of \( \beta_i \) by \( G(\beta_i) \) and its joint probability density function by \( g(\beta_i) \). We allow the conditional distribution of \( \theta_i \) to depend on \( \beta_i \) and denote the probability density function of this conditional distribution by \( f_{\beta_i}(\theta_i) \). Throughout this section, we make the following assumption:

**Assumption 1.** There exists at least one entry in \( \phi_i \), denoted by \( \theta_i \in \Theta \subseteq \mathbb{R} \), such that

\[
\frac{\partial^2 u_w(\phi_i)}{\partial \theta_i \partial \theta_i} \leq 0 \quad \text{and} \quad \frac{\partial^2 u_w(\phi_i)}{\partial \beta_i \partial \theta_i} \geq 0 \quad \text{with at least one strict inequality for any } \beta_i \in \mathbb{B} \subseteq \mathbb{R}^k.
\]

Intuitively, Assumption 1 means that there is some dimension that indexes an individual’s relative preference for additional consumption at retirement, which is a generalization of the particular form of consumption-leisure dependence we assumed in Section 3.2. This assumption allows us to repeat the one-dimensional analysis we conducted so far for any given \( \beta_i \), and then achieve the same equilibrium result by integrating over all possible values of this vector. To see this, we begin with the multi-dimensional version of Lemma 1. As before, we denote the solution to the unconstrained problem by \( UO \) (for Unconstrained Optimizers).

**Lemma 3.** For given institutions \( \tau \), there are exactly two cases for optimizing agents during working life \( (t < T) \): (1) Constrained Optimizers, for whom \( c_i(t) = y - s_F \), and (2) Unconstrained Optimizers, for whom \( c_i(t) < y - s_F \) and is constant during working life. Households of type \( \beta_i \) for whom \( \theta_i < \theta^*(\tau; \beta_i) \) are constrained and households for whom \( \theta_i \geq \theta^*(\tau; \beta_i) \) are unconstrained, where \( \theta^*(\tau; \beta_i) \) is the value of \( \theta_i \) that solves \( y - s_F - c_{UO_{wi}}(\beta_i; \theta_i) = 0 \) for a given value of \( \beta_i \).

**Proof.** In the unconstrained problem, optimality requires \( \frac{\partial u_w(c_{UO_{wi}}; \phi_i)}{\partial c} = \frac{\partial u_w(c_{UO_{wi}}; \phi_i)}{\partial \phi_i} \). Assumption 1 implies that \( c_{UO_{wi}} \) declines in \( \theta_i \) for any vector \( \beta_i \) and, therefore, that households with \( \theta_i \geq \theta^*(\tau; \beta_i) \) are unconstrained as they want to defer consumption to the future. Second, note that for households with \( \theta_i < \theta^*(\tau; \beta_i) \) it is never optimal to save. Transferring \$1 from period \( t_0 < T \) for consumption at
period $t_1 > T$ after retirement would amount to a benefit of at most $e^{(r-\rho)(t_1-t_0)} \frac{\partial u_c(v^{(O)}(\theta^*); \beta_1, \theta_1)}{\partial c} \leq e^{(r-\rho)(t_1-t_0)} \frac{\partial u_c(v^{(O)}(\theta^*); \beta_1, \theta_1)}{\partial c}$ at a cost of at least $\frac{\partial u_c(v^{(O)}(\theta^*); \beta_1, \theta_1)}{\partial c} \geq \frac{\partial u_c(v^{(O)}(\theta^*); \beta_1, \theta_1)}{\partial c}$ with at least one strict inequality, which is undesirable since $\frac{\partial u_c(v^{(O)}(\theta^*); \beta_1, \theta_1)}{\partial c} \neq \frac{\partial u_c(v^{(O)}(\theta^*); \beta_1, \theta_1)}{\partial c}$. In addition, intertemporal substitution of resources before retirement is never necessary for reaching the optimal path. This is because with no voluntary savings for retirement, with constant income flows before retirement, and with $r = \rho$, it must be optimal for households with $\theta_i < \theta^*(\tau; \beta_i)$ to consume their entire disposable income in each period before $T$. That is, for them $c_i(t) = y - s_F$ for any $t < T$ and their consumption while working is constrained by their resources that are left after the government-mandated savings. ■

We denote the consumption of constrained optimizers by the superscript $CO$, with $c^{CO}_{ui} = y - s_F$ in each period before retirement ($t < T$) and $c^{CO}_{ri} = a_F$ in each period after they retire ($t \geq T$). As in Section 3.2, Passives accept all defaults and consume the residual income flows such that for them $c^{P}_{ui} = y - s_F - s_D$ in periods $t < T$ and $c^{P}_{ri}(t) = a_F + a_D$ in periods $t \geq T$. Myopic households opt out of all defaults and consume as much as possible in every period and, therefore, consume $c^{M}_{ui} = y - s_F$ in periods $t < T$ and $c^{M}_{ri}(t) = a_F$ in periods $t \geq T$, similarly to the constrained optimizers. We denote the value functions of optimizers, passives, and myopes by $V^O(\tau; \phi_i), V^P(\tau; \phi_i)$, and $V^M(\tau; \phi_i)$, respectively.

The utilitarian social planner’s objective is, once again, to choose the policy tools $\tau$ that maximize the households’ average utility,$^89$

$$W(\tau) = \mu_O \int V^O(\tau; \phi_i) dQ(\phi_i) + \mu_P \int V^P(\tau; \phi_i) dQ(\phi_i) + \mu_M \int V^M(\tau; \phi_i) dQ(\phi_i)$$

$$= \mu_O \int \left[ \int_{\theta_i < \theta^*(\tau; \beta_i)} V^{CO}(\tau; \beta_i, \theta_i) f_{\beta_i}(\theta_i) d\theta_i + \int_{\theta_i \geq \theta^*(\tau; \beta_i)} V^{UO}(\tau; \beta_i, \theta_i) f_{\beta_i}(\theta_i) d\theta_i \right] dG(\beta_i)$$

$$+ \mu_P \int V^P(\tau; \phi_i) dQ(\phi_i) + \mu_M \int V^M(\tau; \phi_i) dQ(\phi_i).$$

$^89$To simplify notation, we define the multi-dimensional expectation of a function of a vector $x_i = (x_{i1}, ..., x_{ik})$, $\Gamma(x_i)$, with a cumulative distribution function $M(x_i)$ over a region $A$ and a probability density function $m(x_i)$ by:

$$\int_{\Gamma(x_i)} m(x_{i1}, ..., x_{ik}) dx_{i1} ... dx_{ik} \equiv \int_A \Gamma(x_i) dM(x_i).$$

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In this generalized case the expectation operator, \( E[\cdot] \), for any random variable \( x \) integrates jointly over decision types (optimizers, passives, and myopes) and over the taste shocks vector \( \phi_i = (\beta_i, \theta_i) \):

\[
E[x] = \mu_O \int_{\Phi} x^O_i dQ(\theta_i) + \mu_P \int_{\Phi} x^P_i dQ(\theta_i) + \mu_M \int_{\Phi} x^M_i dQ(\theta_i).
\]

The next theorem proves that Theorem 1 holds for this generalization.

**Theorem 3.** Assume a rational planner. Then for any distribution of optimizing, passive, and myopic households, a classical Euler Equation will hold on average in the population:

\[
E \left[ \frac{\partial u_w(c_{w}; \phi_i)}{\partial c} \right] = E \left[ \frac{\partial u_r(c_r ; \phi_i)}{\partial c} \right].
\]

**Proof.** The steps of this proof follow those of the proof of Theorem 1. Since labor income is constant and \( r = \rho \), the planner will choose \( a_F \) and \( a_D \) that are not age-dependent. The budget constraint implies that \( a_F = s_F (e^{rT} - 1) \) and \( a_F + a_D = (s_F + s_D)(e^{rT} - 1) \). At the optimum \( \frac{dW(r)}{ds_D} = 0 \), and since passives are the only group that is affected by \( s_D \) in equilibrium, we have that

\[
\frac{dW(r)}{ds_D} = \mu_P \int_{\Phi} \frac{dV^F(r; \phi_i)}{ds_D} dQ(\phi_i) = 0.
\]

From

\[
\frac{dV^F(r; \phi_i)}{ds_D} = \frac{1-e^{-\rho T}}{\rho} \left( -\frac{\partial u_w(c_{w}; \phi_i)}{\partial c} + \frac{\partial u_r(c_r ; \phi_i)}{\partial c} \right)
\]

it follows that

\[
\int_{\Phi} \left( -\frac{\partial u_w(c_{w}; \phi_i)}{\partial c} + \frac{\partial u_r(c_r ; \phi_i)}{\partial c} \right) dQ(\phi_i) = 0.
\]

The necessary condition for forced savings implies that

\[
\frac{dW(r)}{ds_F} = \mu_O \int_{B} \left[ \frac{d}{ds_F} \right]_{\theta_i < \theta^*(r; \beta_i)} V^{CO}(r; \beta_i, \theta_i) f_{\beta_i}(\theta_i) d\theta_i
\]

\[
+ \frac{d}{ds_F} \int_{\theta_i \geq \theta^*(r; \beta_i)} V^{UO}(r; \beta_i, \theta_i) f_{\beta_i}(\theta_i) d\theta_i
\]

\[
+ \mu_P \int_{\Phi} \frac{dV^P(r; \phi_i)}{ds_F} dQ(\phi_i) + \mu_M \int_{\Phi} \frac{dV^M(r; \phi_i)}{ds_F} dQ(\phi_i).
\]
Since $V^{CO}(\tau; \beta_t, \theta^*(\tau; \beta_t)) = V^{UO}(\tau; \beta_t, \theta^*(\tau; \beta_t))$ we have that for each value of $\beta_t$:  

$$
\frac{d}{dt} \int_{\theta_t < \theta^*(\tau; \beta_t)} V^{CO}(\tau; \beta_t, \theta_t) f_{\beta_t}(\theta_t) d\theta_t + \frac{d}{dt} \int_{\theta_t \geq \theta^*(\tau; \beta_t)} V^{UO}(\tau; \beta_t, \theta_t) f_{\beta_t}(\theta_t) d\theta_t
$$

In addition, since passives are affected symmetrically by forced and default savings, we have that  

$$
\frac{dV^F(\tau; \phi)}{ds_F} = \frac{dV^{P}(\tau; \phi)}{ds_D} 
$$

and, hence,  

$$
\mu_P \int_{\Phi} \frac{dV^P(\tau; \phi)}{ds_F} dQ(\phi) = \mu_P \int_{\Phi} \frac{dV^P(\tau; \phi)}{ds_D} dQ(\phi) = 0. 
$$

Unconstrained optimizers can completely offset the effect of forced savings and are, therefore, unaffected by the government’s tools, so that  

$$
\frac{dV^{UO}(\tau; \phi)}{ds_F} = 0. 
$$

Put together, we get  

$$
\frac{dW(\tau)}{ds_F} = \mu_O \int_{B} \left[ \int_{\theta_t < \theta^*(\tau; \beta_t)} \frac{dV^{CO}(\tau; \beta_t, \theta_t)}{ds_F} f_{\beta_t}(\theta_t) d\theta_t \right] dG(\beta_t) + \mu_M \int_{\Phi} \frac{dV^M(\tau; \phi)}{ds_F} dQ(\phi) = 0, 
$$

which combined with  

$$
\frac{dV^{CO}(\tau; \beta_t, \theta)}{ds_F} = \frac{dV^M(\tau; \phi)}{ds_F} = \left( 1 - \frac{e^{-\rho T}}{\rho} \right) \left( - \frac{\partial u_w(y - s_F; \beta_t, \theta)}{\partial c} + \frac{\partial u_r(c^{M}; \beta_t, \theta)}{\partial c} \right) 
$$

implies that  

$$
\mu_O \int_{B} \left[ \int_{\theta_t < \theta^*(\tau; \beta_t)} \left( - \frac{\partial u_w(c^{CO}; \beta_t, \theta_t)}{\partial c} + \frac{\partial u_r(c^{CO}; \beta_t, \theta_t)}{\partial c} \right) f_{\beta_t}(\theta_t) d\theta_t \right] dG(\beta_t) \quad (3.11) 
$$

$$
+ \mu_M \int_{\Phi} \left( - \frac{\partial u_w(c^{M}; \beta_t, \theta_t)}{\partial c} + \frac{\partial u_r(c^{M}; \beta_t, \theta_t)}{\partial c} \right) dQ(\phi) = 0. \quad (3.12) 
$$

Lastly, recall that for the unconstrained optimizers  

$$
\frac{\partial u_w(c^{UO}; \phi)}{\partial c} = \frac{\partial u_r(c^{UO}; \phi)}{\partial c}. 
$$

The latter equality and equations (3.10) and (3.11) yield the result. ■

3.5 Conclusion  

We study a simple setting that illustrates the interactions between optimizing social planners and heterogeneous households, some of whom are optimizing, some of whom are passive, and some of whom are myopic. In this setting, planner optimization is a substitute for household optimization. This substitution arises because the social planner has the ability to design institutions – e.g., Social Security and savings default – that influence the consumption profiles of households. In
equilibrium, classical Euler Equations hold on average in the cross-section of households (but not for each household). These Euler Equation properties arise generally, whether or not households are optimizers.

These results imply that Euler Equation tests and related consumption-smoothing tests – e.g., the lack of an average drop in consumption at retirement – do not differentiate between an optimizing social planner and optimizing households. However, even in the economy that we have studied, planner optimization is distinguishable, in principle, from household optimization. Under the maintained assumption of household optimization, the Euler Equation will hold for each household, but, with planner optimization, it will only hold on average in the cross-section. Although such household-by-household tests are theoretically determinative, such fine-grained analysis is practically problematic if data is measured with noise or if some variables are unobservable (e.g., household-level taste shocks). However, other tests do distinguish between planner optimization and household optimization. Exogenous changes in policy (e.g., a default change at the level of a firm, or a natural experiment in the Social Security system), reveal more about household rationality than averages of observational data in the cross-section.

Our conclusions are limited by our modeling assumption that the government is public-spirited. We assumed that the government is a fully rational, utilitarian social planner. It is likely that flesh-and-blood governments fall short of this optimistic benchmark, despite (or because of) the pressures that they face to get re-elected.
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4 Appendix to Chapter 1

Appendix A: A Dynamic Collective Model of Household Labor Force Participation

The model we analyze in this appendix generalizes our baseline model in two ways. Most importantly, it analyzes life-cycle participation decisions using a dynamic search model, which allows for endogenous savings. Second, we extend the one-shock model we analyzed in the text to include different and potentially sequential shocks. In addition, we use the generalized preference structure we analyzed in Section 1.2.3 and allow for additional extensions to it as we describe below.

Setup. We consider a discrete-time setting in which households live for $T$ periods $\{0, 1, ..., T - 1\}$ (where $T$ is allowed to go to infinity) and set both the interest rate and the agents’ time discount rate to zero for simplicity. Households consist of two individuals, $w$ and $h$. We assume that at time 0 households are in the “good health” state (state $g$) in which $h$ is in good health and works. In each period, the household transitions with probability $\rho_t$ to the “bad health” state (state $b$) in which $h$ experiences a health shock and drops out of the labor force. Conditional on being sick, $h$ may die in period $t$ with probability $\lambda_t$ in which case the household transitions to the state where $w$ is a widow – state $d$. In what follows, the subscript $i \in \{w, h\}$ refers to the spouse and the superscript $s \in \{g, p, d\}$ refers to the state of nature.

At the beginning of the planning period, $w$ does not work and searches for a job. When $w$ enters period $t$ in state $s$ without a job she chooses search intensity, $e_{wt}^s$, which we normalize to equal the probability of finding a job in the same period. If $w$ finds a job, the job begins at time $t$ and is assumed to last until the end of the planning period once found.90

Individual preferences. Let $u_t^i(c_{it}^s, l_{it}^s, l_{jt}^s)$ represent $i$’s flow consumption utility at time $t$ in state $s$ as a function of consumption, $c_{it}^s$, labor force participation, $l_{it}^s$, and the other spouse’s labor force participation, $l_{jt}^s$, where $\frac{\partial u_t^i}{\partial c_{it}^s} > 0$ and $\frac{\partial^2 u_t^i}{\partial(c_{it}^s)^2} < 0$. We denote $w$’s cost of search effort at time $t$ in state $s$ by $\kappa^s(e_{wt}^s)$, which we assume to be strictly increasing and convex. The relative cost of time invested in search effort across states is captured by $\theta^s(e_{wt}^s) \equiv \kappa^s(e_{wt}^s)/\kappa^s(e_{wt}^d)$, where $\sigma \in \{b, d\}$.

Household preferences. We assume that the household’s per-period utility weights individual

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90 This simplifies the algebra of the analysis. We later allow for job separations such that employment is absorbing within a health state but not across health states.
utilities according to their respective Pareto weights $\beta_w$ and $\beta_h$, such that the household’s flow utility at time $t$ in state $s$ is $\beta_w u^h_{wt} + \beta_h u^h_{ht}$ augmented by $w$’s weighted search cost $\beta_w \kappa^s(e^s_{wt})$ when she is unemployed. We assume equal Pareto weights and normalize $u^d_{ht} = 0$. In the following analysis we suppress the dependence of the consumption utility on participation for ease of notation only.

**Policy tools.** The planner observes the state of nature as well as the employment status of each spouse. Since some spouses work and earn more than others do, the optimal policy is dependent on whether the spouse is employed. We denote the tax on spouse $i$’s labor income in state $g$ by $T_i^g$ and the benefits given to non-working spouses in state $g$ by $b^g$. In state $\sigma \in \{b, d\}$, households in which the unaffected spouse, $w$, works receive transfers of the amount $B^\sigma$ and households in which $w$ does not work receive benefits of the amount $b^\sigma$. We denote taxes by $T \equiv (T_w^g, T_h^g)$ and benefits by $B \equiv (b^g, B^b, B^d, b^d)$, and let $B(t^s_{wt})$ represent the actual transfers received by a household as a function of $w$’s participation.

**Household’s problem.** The household’s choices include the allocation of consumption to each spouse, $c^s_{it}$, as well as $w$’s search effort if she is unemployed, $e^s_{wt}$. In each period, $w$’s employment status, $l^s_{wt}$, determines the household’s income flow, $y^s_{it}(l^s_{wt})$, such that $y^s_{it}(l^s_{wt}) = z^s_{ht} \times l^s_{ht} + z^s_{wt} \times l^s_{wt} + B(l^s_{wt})$, where $z_{it}$ is $i$’s labor income and $z^s_{it} = z_{it} - T^s_i$ is $i$’s labor income net of taxes in state $s$ (with $T^s_i = 0$). This implies that each period’s consumption as well as the next period’s wealth – where we denote assets in period $t$ by $A_t$ – are functions of $w$’s participation, which we denote by $c^s_{it}(l^s_{wt})$ and $A_{t+1}(l^s_{wt})$, respectively. Therefore, the value function for households in state $s$ who enter period $t$ when $w$ is without a job and with household assets $A_t$ is

$$V_t^{s, 0}(B, T, A_t) \equiv \max_{c^s_{wt}} \left( u^s_w(c^s_{ht}(1)) + u^s_w(c^s_{wt}(1)) + W^{s, 1}_{t+1}(B, T, A_{t+1}(1)) \right) \nonumber$$

$$+ (1 - e^s_{wt}) \left( u^s_h(c^s_{ht}(0)) + u^s_w(c^s_{wt}(0)) + W^{s, 0}_{t+1}(B, T, A_{t+1}(0)) \right) - \kappa^s_w(e^s_{wt}),$$

where the budget constraints satisfy

$$c^s_{ht}(l^s_{wt}) + c^s_{wt}(l^s_{wt}) + A_{t+1}(l^s_{wt}) = A_t + y^s_{it}(l^s_{wt}),$$

and $W^{s, 0}_{t+1}(B, T, A_{t+1})$ are the continuation value functions which depend on whether the job search
was successful or not in time $t$. The continuation functions are defined by

$$W^{g,t}_{t+1} (B,T,A_{t+1}) \equiv (1 - \rho t) V^{g,t}_{t+1} (B,T,A_{t+1}) + \rho t V^{g,t}_{t+1} (B,T,A_{t+1}),$$

$$W^{b,t}_{t+1} (B,T,A_{t+1}) \equiv (1 - \lambda t) V^{b,t}_{t+1} (B,T,A_{t+1}) + \lambda t V^{b,t}_{t+1} (B,T,A_{t+1}),$$

$$W^{d,t}_{t+1} (B,T,A_{t+1}) \equiv V^{d,t}_{t+1} (B,T,A_{t+1}),$$

where $V^{s,1}_t (B,T,A_t)$ is the value of entering period $t$ when $w$ is employed in state $s$ which is defined by

$$V^{s,1}_t (B,T,A_t) \equiv \max \left\{ u^s_w (c^s_{ht} (1)) + u^s_w (c^s_{wt} (1)) + W^{s,1}_{t+1} (B,T,A_{t+1}(1)) \right\}.$$

The optimal search effort is chosen according to the first-order condition

$$\left( u^s_w (c^s_{ht} (1)) + u^s_w (c^s_{wt} (1)) + W^{s,1}_{t+1} (B,T,A_{t+1}(1)) \right) - \left( u^s_w (c^s_{ht} (0)) + u^s_w (c^s_{wt} (0)) + W^{s,0}_{t+1} (B,T,A_{t+1}(0)) \right) = \kappa^s_w (e^s_{wt}),$$

(4.1)

where the effect of a $1$ increase in the benefit level $b^s$ on search intensity in state $s$ is

$$\frac{\partial e^s_{wt}}{\partial b^s} = - \frac{1}{\kappa^s_w (e^s_{wt})} \left( u^s_w (c^s_{wt} (0)) + \frac{\partial W^{s,0}_{t+1}}{\partial b^s} \right),$$

(4.2)

**Planner’s problem.** We define the household’s expected utility at the beginning of the planning period by $J_0 (B,T) \equiv (1 - \rho_0) V^{b,0}_0 (B,T,A_0) + \rho_0 V^{b,0}_0 (B,T,A_0)$. The social planner’s objective is to choose the tax-and-benefit system that maximizes the household’s expected utility subject to a balanced-budget constraint. For simplicity, we assume there is some expected revenue collected from each household and study the optimal redistribution of this revenue. We abstract from the specific way in which revenue is collected (or, similarly, assume a lump-sum tax that is determined outside of our problem) since our focus is on the benefits from social insurance and not its fiscal-externality costs. The perturbations we study involve increasing $b^\sigma$, $\sigma \in \{b,d\}$, by lowering $b^\theta$. Therefore, to
further simplify the analysis we assume that $B^b = B^d = 0$, as well as that $b^d = 0$ when we perturb $b^b$ and that $b^b = 0$ when we perturb $b^d$.

Let $D^s$ denote the expected share of the household’s life-time in state $s$ and let $\hat{e}_w^s$ denote the conditional probability of $w$ being employed if she is observed in state $s$. To construct the budget constraint, consider randomly choosing a household at a random point in its life-cycle. The probability of choosing a household in state $s$ is $D^s$ and, hence, the probability of choosing a household in state $s$ in which $w$ is unemployed is $D^s \times (1 - \hat{e}_w^s)$. If the government collects revenues of the amount $r$ per household, a balanced budget requires that the expected transfer to a random household is equal to this amount. That is, $D^g (1 - \hat{e}_w^g) b^g + D^b (1 - \hat{e}_w^b) b^b + D^d (1 - \hat{e}_w^d) b^d = r$. Hence, the planner chooses the benefit levels $B$ that solve

$$\max_B J_0(B, T) \text{ s.t. } D^g (1 - \hat{e}_w^g) b^g + D^b (1 - \hat{e}_w^b) b^b + D^d (1 - \hat{e}_w^d) b^d = r. \quad (4.3)$$

**Optimal Social Insurance**

We consider the optimal distribution of benefits to households with non-working spouses across health states $\sigma \in \{b, d\}$ and $g$. First, consider a $1$ increase in $b^b$ financed by lowering $b^g$. The net welfare gain from this perturbation is

$$\frac{dJ_0(T, B)}{db^b} = Q_1^b + Q_2^b \frac{db^g}{db^b}, \quad (4.4)$$

where $Q_1^b = \left( \rho_0 \frac{\partial V_0^{g,b}}{\partial b^b} + (1 - \rho_0) \frac{\partial V_0^{g,d}}{\partial b^b} \right)$ and $Q_2^b = \left( \rho_0 \frac{\partial V_0^{g,b}}{\partial b^b} + (1 - \rho_0) \frac{V_0^{g,d}}{\partial b^b} \right)$. The following proposition provides an approximated formula for the normalized version of this gain.

**Proposition A1.**

Under a locally quadratic approximation of the effort function $\kappa_w^g(e_{w,0}^g)$ around $e_{w,0}^g$ and assuming that the ratio $\theta(b, e_{w,0}^b)$ is locally constant at $e_{w,0}^b$, the marginal benefit from raising $b^b$ by $\$1$ is

$$M_w(b^b) \approx MB(b^b) - MC(b^b),$$

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with

1. \( MB(b^t) \equiv L^b + M^b + S^b \), where \( L^b \equiv \frac{e^b_w - e^b_{w_0}}{e^b_{w_0}} \), \( M^b \equiv \left( \frac{\rho^b(e^b_w - b^t)}{\rho^b(e^b_w - b^t)} \right) e^b_{w_0} \), \( S^b \equiv (\theta^b - 1)(1 + L^b + M^b) \), \( \varepsilon(x, y) \equiv \frac{\partial x y}{\partial y} \), \( \theta^b \equiv \theta^b(e^b_{w_0}) \), \( e^b_{w_0} \) is \( w^t \)'s participation rate at the beginning of the planning period, and \( e^b_{w_0} \) is \( w^t \)'s mean participation rate in households that transition to state \( b \).

2. \( MC(b^t) \equiv \beta^0 + \beta^b_1(1 - e^b_w, b^t) + \beta^b_2(1 - e^b_w, b^t) \), where the coefficients \( \beta^0 \), \( \beta^b_1 \), and \( \beta^b_2 \) are functions of the transition probabilities, average participation rates, and benefits and \( \varepsilon(x, y) \equiv \frac{\partial x y}{\partial y} \).

Proof.

The general logic of the proof is to characterize the derivatives of the value functions in their sequential problem representation – that is, as a sum of derivatives over time and over different states of nature. To do so, we work backwards from period \( T - 1 \) to period 0. Taylor approximations then lead to our results.

We begin by providing expressions for \( \frac{\partial V^{b,0}}{\partial b} \) and \( \frac{\partial V^{g,0}}{\partial b} \) in order to characterize \( Q^b_1 \). First, we have that \( \frac{\partial V^{b,0}}{\partial b} = (1 - e^b_w) \left( u_w^b(c_w^b(0)) + \sum_{t=1}^{T-1} \left( \prod_{j=t+1}^{T} (1 - b_{w_{j+1}}(1 - \lambda_j)) \right) \left( u_w^b(c_w^b(0)) \right) \right) \).

Next, since \( \frac{\partial W^{g,1}}{\partial b} = 0 \) we obtain \( \frac{\partial V^{g,0}}{\partial b} = (1 - e^g_w) \frac{\partial W^{g,0}_{t+1}}{\partial b} \), where \( \frac{\partial W^{g,0}_{t+1}}{\partial b} = (1 - \rho_{t+1}) \frac{\partial W^{g,0}_{t+1}}{\partial b} + \rho_{t+1} \frac{\partial W^{g,0}_{t+1}}{\partial b} \). Therefore, we get that \( \frac{\partial V^{g,0}}{\partial b} = (1 - e^g_w)(1 - \rho_{t+1}) \frac{\partial W^{g,0}_{t+1}}{\partial b} + (1 - e^g_w) \rho_{t+1} \frac{\partial W^{g,0}_{t+1}}{\partial b} \), which implies by working backwards from period \( T - 1 \) to period 0 that \( \frac{\partial V^{b,0}}{\partial b} = (1 - e^b_w) \sum_{i=1}^{T-1} \left( \prod_{j=t+1}^{T} (1 - e^y_{w_{j+1}})(1 - \lambda_j) \right) \rho_{t+1} \frac{\partial V^{b,0}}{\partial b} \).

Specifically, \( \beta^0 \equiv \frac{\sigma^b_{1\rho}(1 - e^b_w) - \rho^b(1 - e^b_w)}{\rho^b(1 - e^b_w)} \), \( \beta^1 \equiv \rho^b + \rho^b - 1 \), \( \beta^b_1 \equiv \sigma^b_{1\rho}(1 - e^b_w) \), and \( \beta^b_2 \equiv \sigma^b_{1\rho}(1 - e^b_w) \), where \( \sigma^b \equiv (1 - \rho_0)(1 - e^b_w) \rho_{t+1} \) and \( \rho \equiv \sum_{i=0}^{T-1} \left( \prod_{j=t+1}^{T} (1 - \rho_{t+1}) \right) \). Note that the elasticities in \( MC(b^t) \) consist of the total effect of increasing \( b^t \), which takes into account the effect of lowering the level of the financing tool, \( b^t \). Also note that with forward-looking households, transfers in states not yet encountered can have effects through ex-ante responses. For example, individuals in state \( g \) can lower labor supply and savings today in response to larger benefits in state \( b \).
Putting the terms together, it follows that

$$Q^b_t = \left( \rho_0 \frac{\partial V_{t0}^{b,0}}{\partial b} + (1 - \rho_0) \frac{\partial V_{t0}^{g,0}}{\partial b} \right) = \sum_{i=0}^{T-1} \prod_{j=0}^{i-1} (1 - e_{wj}^g)(1 - \rho_j) \rho_i \left[ \frac{\partial V_{t0}^{b,0}}{\partial b} \right]. \quad (4.5)$$

Using equation (4.2) and \( \frac{\partial V_{t0}^{h,0,b}}{\partial b_w} = (1 - e_{wt}^b) \left( u_w \left( e_{wt}(0) \right) + \frac{\partial W_{t0}^{b,0}}{\partial b_w} \right) \), we get that \( \frac{\partial V_{t0}^{h,0,b}}{\partial b} = -\epsilon_w \frac{\partial^2 \overline{\epsilon}_{wt}}{\partial b_w} \). Plugging this expression into (4.5) yields the following result

$$Q^b_t = -\sum_{i=0}^{T-1} \prod_{j=0}^{i-1} (1 - e_{wj}^g)(1 - \rho_j) \rho_i \left[ (1 - e_{wt}) \kappa_w \frac{\partial^2 \overline{\epsilon}_{wt}}{\partial b_w} \right]. \quad (4.6)$$

To understand the meaning of this formula let us break it down into its components. First, note that it is a weighted sum of a function of the change in effort (or participation rate), \( \frac{\partial b_w}{\partial b} \). The weight, the term in brackets, is the probability of reaching period \( i \) with \( w \) unemployed and transitioning to state \( b \) exactly in that period. For households that transition to state \( b \) in period \( i \) when \( w \) is employed, the change in effort and participation rates is zero (because they stay employed and do not engage in search effort). Therefore, dividing the probability weights by the chance of transitioning to state \( b \) at some point throughout the planning horizon, \( \rho \equiv \sum_{i=0}^{T-1} \left( \prod_{j=0}^{i-1} (1 - \rho_j) \rho_i \right) \), and rewriting (4.6) in terms of elasticities (with \( \epsilon(x, y) \equiv \frac{\partial e}{\partial x} \)) yield \( Q_t^b = \rho E_b \left\{ (1 - e_{wt}) \kappa_w \frac{\partial^2 \overline{\epsilon}_{wt}}{\partial b_w} \right\} \equiv \rho E_b(g(\widehat{e}_{w0}^b)) \), where \( \widehat{e}_{w0}^b \) denotes participation in the period the household transitions to state \( b \) and \( E_b \) is the expectation operator conditional on being in state \( b \). By expanding \( g(e) \) around \( w \)'s average participation in households in which \( h \) becomes sick – which we denote by \( \widehat{e}_{w0}^b \) – such that \( g(e) \equiv g(\widehat{e}_{w0}^b) + g'(\widehat{e}_{w0}^b)(e - \widehat{e}_{w0}^b) \), we approximate \( E_b(g(\widehat{e}_{w0}^b)) \equiv E_b(g(\widehat{e}_{w0}^b)) = g(\widehat{e}_{w0}^b) \) and obtain the approximation

$$Q_t^b \approx \rho (1 - \widehat{e}_{w0}^b) \kappa_w \frac{\partial^2 \overline{\epsilon}_{wt}}{\partial b_w} \left| \frac{e_{w0}^b}{b} \right| \quad (4.7)$$

We now turn to provide expressions for \( \frac{\partial V_{t0}^{h,0}}{\partial b_w} \) and \( \frac{V_{t0}^{g,0}}{\partial b_w} \) in order to characterize \( Q^b_t \). Since households that transitioned to state \( b \) either stay in state \( b \) or transition to state \( d \), we have that \( \frac{\partial V_{t0}^{h,0,b}}{\partial b_w} = 0 \). In addition, \( \frac{V_{t0}^{g,0}}{\partial b_w} = (1 - e_{wt}) \left( u_w \left( e_{wt}(0) \right) + \frac{\partial W_{t0}^{b,0}}{\partial b_w} \right) \), which combined with equation (4.2)
yields \( \frac{V^{g,0}}{\partial \theta} = -(1 - e^{g}_{w0}) \left( \kappa^{g}_{w}''(e^{g}_{w0}) \frac{\partial e^{g}_{w}}{\partial \theta} \right) \). Put together, we get that

\[
Q_{2}^{b} = (1 - \rho_{0}) (1 - e^{g}_{w0}) \kappa^{g}_{w}''(e^{g}_{w0}) |\varepsilon(e^{g}_{w0}, b^{g})| \frac{e^{g}_{w0}}{b^{g}}.
\] (4.8)

To complete the proof we need to calculate \( \frac{db^{g}}{db^{b}} \). Total differentiation of the simplified budget constraint \( D^{g} (1 - e^{g}_{w}) b^{g} + D^{b} (1 - e^{b}_{w}) b^{b} = r \) with respect to \( b^{b} \) gives us

\[
\frac{db^{g}}{db^{b}} = \frac{D^{g} (1 - e^{b}_{w}) \varepsilon(1 - e^{g}_{w}, b^{b}) - D^{b} (1 - e^{b}_{w}) \varepsilon(1 - e^{g}_{w}, b^{b})}{D^{g} (1 - e^{g}_{w}) - D^{b} (1 - e^{b}_{w})},
\] (4.9)

where \( \varepsilon(x, y) \equiv \frac{dy}{dx} \). Plugging (4.7), (4.8) and (4.9) into (4.4), using a quadratic approximation of the effort function \( \kappa^{g}(e^{g}_{w0}) \) around \( e^{g}_{w0} \) and assuming that the ratio \( \theta^{b}(e^{b}_{w0}) \) is locally constant at \( e^{b}_{w0} \), we obtain the approximated formula for the normalized welfare gain \( M_{w}(b^{g}) \equiv \frac{dJ_{0}(T, B)}{db^{g} \partial b^{g} / (1 - \rho_{0})(1 - e^{g}_{w0})} \) that is stated in the proposition, which completes the proof. ■

Next, consider a $1$ increase in \( b^{d} \) financed by lowering \( b^{g} \). We analyze this perturbation separately from the former since the sequential nature of the model requires a more careful investigation of transfers to different “bad” states (as shown in the following proof), although the approximated formulas turn out to be conceptually similar. The net welfare gain from this perturbation is

\[
\frac{dJ_{0}(T, B)}{db^{d} \partial b^{d}} = Q_{1}^{d} + Q_{2}^{d} \frac{db^{g}}{db^{d}},
\] (4.10)

where \( Q_{1}^{d} = \left( \rho_{0} \frac{\partial y^{b,0}}{\partial b^{g}} + (1 - \rho_{0}) \frac{\partial y^{d,0}}{\partial b^{d}} \right) \) and \( Q_{2}^{d} = \left( \rho_{0} \frac{\partial y^{b,0}}{\partial b^{g}} + (1 - \rho_{0}) \frac{V^{g,0}}{\partial \theta} \right) \). We present the approximated formula in the following proposition.

**Proposition A2.**

Under a locally quadratic approximation of the effort function \( \kappa^{g}_{w}(e^{g}_{w0}) \) around \( e^{g}_{w0} \) and assuming that the ratio \( \theta^{d}(e^{d}_{w0}) \) is locally constant at \( e^{d}_{w0} \), the marginal benefit from raising \( b^{d} \) by $1$ is

\[
M_{w}(b^{d}) \equiv MB(b^{d}) - MC(b^{d}),
\]

with
1. \( MB(b^d) \equiv L^d + M^d + S^d \), where \( L^d \equiv \frac{\varepsilon^d - c^d_{w0}}{e_{w0}} \), \( M^d \equiv \left( \frac{e(e_{w0}, b^d)}{e(b^d, 0)} - 1 \right) \frac{\varepsilon^d}{e_{w0}} \), \( S^d \equiv (\theta^d - 1)(1 + L^d + M^d) \), \( \varepsilon(x, y) \equiv \frac{\partial u_y}{\partial y} \), \( \theta^d \equiv \theta^d(e_{w0}, \lambda) \), \( e_{w0} \) is w’s participation rate at the beginning of the planning period, and \( \varepsilon^d_{w0} \) is w’s mean participation rate in households that transition to state d.

2. \( MC(b^d) \equiv \beta^d + \beta_1^d(e(1 - \varepsilon^d_w, b^d) + \beta_2^d(e(1 - \varepsilon^d_w, b^d), where the coefficients \( \beta^d_0 \), \( \beta^d_1 \), and \( \beta^d_2 \) are functions of the transition probabilities, average participation rates, and benefits and \( \varepsilon(x, y) \equiv \frac{dx}{dy}^2 \).

Proof.

We first find expressions for \( \frac{\partial V_i^{b, 0}}{\partial \theta} \) and \( \frac{\partial V_i^{b, 0}}{\partial \theta} \) in order to characterize \( Q^d_i \). With \( \frac{\partial V_i^{b, 0}}{\partial \theta} = (1 - e_{w1}^b) \left( \frac{\partial V_i^{b, 0}}{\partial \theta^b} \right) + (1 - \lambda_{t+1})(\frac{\partial V_i^{b, 0}}{\partial \theta^b} + \lambda_{t+1})(\frac{\partial V_i^{b, 0}}{\partial \theta^b}) \) we have that \( \frac{\partial V_i^{b, 0}}{\partial \theta} = (1 - e_{w1}^b) \left( \frac{\partial V_i^{b, 0}}{\partial \theta^b} \right) + (1 - \lambda_{t+1})(\frac{\partial V_i^{b, 0}}{\partial \theta^b} + \lambda_{t+1})(\frac{\partial V_i^{b, 0}}{\partial \theta^b}) \).

Working backwards from period \( T - 1 \) to period 0 one can show that \( \frac{\partial V_i^{b, 0}}{\partial \theta} = \sum_{t=1}^{T-1} \Pi_{t+1}^{T-1}(1 - e_{w1}^b) \left( \frac{\partial V_i^{b, 0}}{\partial \theta^b} \right) \), which imply that \( \frac{\partial V_i^{b, 0}}{\partial \theta} = (1 - e_{w1}^b) \left( \frac{\partial V_i^{b, 0}}{\partial \theta^b} \right) + (1 - \lambda_{t+1})(\frac{\partial V_i^{b, 0}}{\partial \theta^b}) \). Define the probability of transitioning to state d exactly at time i while w is unemployed by \( \mu_i^{d, 0} \) (which takes into account all the possible transition paths). Then, combining the results so far one can show by working backwards that \( Q^d_i = \left( \rho_0 \left( \frac{\partial V_i^{b, 0}}{\partial \theta} \right) + (1 - \rho_0) \frac{\partial V_i^{b, 0}}{\partial \theta} \right) = \sum_{t=1}^{T-1} \mu_i^{d, 0} E_{\mu_i^{d, 0}} \left( \frac{\partial V_i^{b, 0}}{\partial \theta^b} \right) \), where \( E_{\mu_i^{d, 0}} \) is the expectation operator conditional on arriving at period i with w unemployed and transitioning to state d then (taken over all possible paths).

Since \( \frac{\partial V_i^{d, 1}}{\partial \theta^d} = 0 \) we have that \( \frac{\partial V_i^{d, 0}}{\partial \theta} = (1 - e_{w1}^d) \left( \frac{\partial V_i^{d, 0}}{\partial \theta^d} \right) + \frac{\partial V_i^{d, 0}}{\partial \theta} \right) \). Combined with (4.2) it can be expressed as \( \frac{\partial V_i^{d, 0}}{\partial \theta} = -(1 - e_{w1}^d) \frac{\partial V_i^{d, 0}}{\partial \theta} + \frac{\partial V_i^{d, 0}}{\partial \theta} \). Putting the terms together we obtain

\[
Q^d_i = \sum_{t=1}^{T-1} \mu_i^{d, 0} E_{\mu_i^{d, 0}} \left( (1 - e_{w1}^d) \kappa_{w}^{d, 0}(e_{w1}^d) \frac{\partial V_i^{d, 0}}{\partial \theta} \right). \tag{4.11}
\]

Specifically, \( \beta_{i+1}^d \equiv \frac{\sigma^d D^d(1 - e_{w1}^d) - \sigma^d(1 - e_{w1}^d)}{D^d(1 - e_{w1}^d)} \), \( \beta_1^d \equiv \frac{\sigma^d D^d(1 - e_{w1}^d)}{D^d(1 - e_{w1}^d)} \), and \( \beta_2^d \equiv \frac{\sigma^d D^d(1 - e_{w1}^d)}{D^d(1 - e_{w1}^d)} \) where \( \sigma^d \equiv (1 - \rho_0)(1 - e_{w0})/\lambda(1 - \varepsilon^d_{w0}) \), \( \lambda \equiv \sum_{i=0}^{T-1} \mu_i^{d, 0} \), and \( \mu_i^{d, 0} \) is the probability of transitioning to state d in period i.
Define the probability of transitioning to state $d$ in period $i$ by $\mu_i^d$ and note that for those households who arrive at this period with $w$ employed the change in participation is zero. Dividing the probabilities in (4.11) by the chance of transitioning to state $d$ at some point, $\lambda \equiv \sum_{i=0}^{T-1} \mu_i^d$, we can rewrite $Q_i^d$ as $Q_i^d = \lambda E_\lambda \left\{ (1 - \tilde{e}_{w0}^d) \kappa_w^d \nu(\tilde{d}_{w0}^d) \left| \varepsilon(\tilde{e}_{w0}^d, b^d) \right| \frac{\partial \varepsilon_{w0}^d}{\partial b^d} \right\} = \lambda E_\lambda (g(\tilde{e}_{w0}^d))$, where $\tilde{e}_{w0}^d$ denotes participation in the period the household transitions to state $d$ and $E_\lambda$ is the expectation operator conditional on being in state $d$. Expanding $g(e)$ around $w$’s average participation upon the transition to state $d$ – which we denote by $\tilde{e}_{w0}^d$ – we can approximate $Q_i^d$ by

$$Q_i^d \approx \lambda (1 - \tilde{e}_{w0}^d) \kappa_w^d \nu(\tilde{d}_{w0}^d) \left| \varepsilon(\tilde{e}_{w0}^d, b^d) \right| \frac{\varepsilon_{w0}^d}{b^d}. \tag{4.12}$$

In addition, as in the proof of Proposition A1

$$Q_i^d = \left( \rho_0 \frac{\partial V_{0,b,0}^{g,0}}{\partial b^d} + (1 - \rho_0) \frac{V_{0,b,0}^{g,0}}{\partial b^d} \right) = (1 - \rho_0) \left( 1 - e_{w0}^g \right) \kappa_w^g \nu(\tilde{d}_{w0}^g) \left| \varepsilon(e_{w0}^g, b^d) \right| \frac{e_{w0}^g}{b^d}. \tag{4.13}$$

To complete the proof we differentiate the budget constraint with respect to $b^d$ which yields

$$\frac{db^g}{db^d} = -\frac{b^g}{b^d} (1 - \tilde{e}_{w0}^g, b^d) - \frac{D^d (1 - \tilde{e}_{w0}^d)}{D^g (1 - \tilde{e}_{w0}^g)} \varepsilon(1 - \tilde{e}_{w0}^d, b^d) - \frac{D^d (1 - \tilde{e}_{w0}^d)}{D^g (1 - \tilde{e}_{w0}^g)} \frac{D^d (1 - \tilde{e}_{w0}^d)}{D^g (1 - \tilde{e}_{w0}^g)}. \tag{4.14}$$

Plugging (4.12), (4.13) and (4.14) into (4.10), using a quadratic approximation of the effort function $\kappa^g(e_{wt})$ around $e_{wt}^g$ and assuming that the ratio $\theta^d(e_{wt}^d)$ is locally constant at $\tilde{e}_{w0}^d$, we obtain the approximated formula for the normalized welfare gain $M_w(b^d) \equiv \frac{dJ_T(T,B) / \partial b^d}{\rho_0(1 - \rho_0)(1 - \rho_0)}$ that is stated in the proposition, which completes the proof. □

**Extension: Exogenous Separations**

One natural extension of our search model is to allow for $w$’s employment status to change at state transitions. For example, a working $w$ is state $g$ may want to decrease her labor supply in state $b$ to take care of the ill $h$ and may decide to quit her job and start searching for a job again in a year or two after the shock occurs. We can extend the model such that employment is only absorbing within each health state, but can be exogenously terminated at rate $\delta_t$ at heath-state transitions. To demonstrate how to include this sort of separation, let us reconsider the value of entering period $t$ in state $g$ when $w$ is unemployed. In this case, the household’s value function is
\[ V^{g,0}_t(B, T, A_t) \equiv \max e^g_{wt} \left( u^g_{h_t}(c^g_{ht}(1)) + u^g_w(c^g_{wt}(1)) + W^{g,1}_{t+1}(B, T, A_{t+1}(1)) \right) \\
+ (1 - e^g_{wt}) \left( u^g_{h_t}(c^g_{ht}(0)) + u^g_w(c^g_{wt}(0)) + W^{g,0}_{t+1}(B, T, A_{t+1}(0)) \right) - \kappa^g_w(c^g_{wt}), \]

where as before

\[ W^{g,0}_{t+1}(B, T, A_{t+1}) \equiv (1 - \rho_{t+1})V^{g,0}_{t+1}(B, T, A_{t+1}) + \rho_{t+1}V^{b,0}_{t+1}(B, T, A_{t+1}), \]

but with the adjustment that now

\[ W^{g,1}_{t+1}(B, T, A_{t+1}) \equiv \rho_{t+1} \left( \left(1 - \delta_{t+1} \right)V^{b,1}_{t+1}(B, T, A_{t+1}) + \delta_{t+1}V^{b,0}_{t+1}(B, T, A_{t+1}) \right) + \left(1 - \rho_{t+1} \right)V^{g,1}_{t+1}(B, T, A_{t+1}). \]

That is, if \( h \) becomes sick when \( w \) works, there is a probability of \( \delta_{t+1} \) that \( w \) stops working and then renews her search effort. In this case, it is no longer true that \( \frac{\partial W^{g,1}_{t+1}}{\partial \delta^t_{t+1}} = 0 \), but rather \( \frac{\partial W^{g,1}_{t+1}}{\partial \delta^t_{t+1}} = \rho_{t+1} \delta_{t+1} \frac{\partial V^{b,0}_{t+1}}{\partial \delta^t_{t+1}} \). In turn, this implies that in equation (4.5) one needs to take into account additional paths to reach period \( i \) with \( w \) unemployed and transition to state \( b \) exactly in that period. It is no longer merely the probability of becoming exactly sick in period \( i \) and staying unemployed until that period. Rather, it is also the probability of being employed before period \( i \) and then transitioning into state \( b \) and becoming unemployed in that period (with probability \( \delta_i \)). However, recall that our final formulas include expected values and averages. Before, those who were employed contributed a value of zero to the integrals. But, now, with a positive probability they contribute a non-zero value (because a fraction \( \delta_i \) responds on the effort margin as for them employment is not absorbing). Therefore, our formulas remain the same under this extension such that welfare is still identified by the means stated in our formulas. The change is that conceptually these means include additional individuals that respond. The sample moments that one needs to calculate to recover welfare remain the same.

**Appendix B: A Collective Intensive-Margin Model of Household Labor Supply**

In this appendix we present a baseline static model that is the intensive-margin counterpart
to the participation model in the text. The analysis of the dynamic version of the model follows the logic of the analysis in Appendix A and is available from the authors on request. The general conclusion of the dynamic model in the intensive-margin case is similar to that in the extensive-margin case – the labor supply responses that identify the marginal benefits from social insurance are replaced by average labor supply responses. For completeness, we describe the full setup of the model although it has close similarities to the model of Section 1.2.1.

Setup. Households consist of two individuals, $w$ and $h$. We consider a world with two states of nature: a “good state” (state $g$) in which $h$ is in good health, and a “bad state” (state $b$) in which $h$ experiences a shock. Households spend a share of $\mu_g$ of their adult life in state $g$ and a share of $\mu_b$ in state $b$ ($\mu_g + \mu_b = 1$). In what follows, the subscript $i \in \{w, h\}$ refers to the spouse and the superscript $s \in \{g, b\}$ refers to the state of nature.

Individual preferences. Let $U_i(c^s_i, l^s_i)$ represent $i$’s utility as a function of consumption, $c^s_i$, and labor supply, $l^s_i$, in state $s$. We assume that $\frac{\partial U_i}{\partial c^s_i} > 0$, $\frac{\partial^2 U_i}{\partial (c^s_i)^2} < 0$, $\frac{\partial U_i}{\partial l^s_i} < 0$ and $\frac{\partial^2 U_i}{\partial (l^s_i)^2} < 0$.

Household preferences. We follow the collective approach to household behavior and assume that household decisions are Pareto efficient and can be characterized as solutions to the maximization of $\beta_w U_w(c^s_w, l^s_w) + \beta_h U_h(c^s_h, l^s_h)$, where $\beta_w$ and $\beta_h$ are the Pareto weights on $w$ and $h$, respectively. For simplicity, we assume equal Pareto weights ($\beta_w = \beta_h = 1$), which is without loss of generality as long as the spouses’ relative bargaining power is stable across states of nature.

Policy tools. Households in state $b$ receive transfers of the amount $B$, which are financed by a linear tax rate $\tau^s_i$ on $i$’s labor income in state $s$. We denote taxes by $T \equiv (\tau^g_w, \tau^g_h, \tau^b_w, \tau^b_h)$ and actual transfers by $B^s$ such that $B^g = 0$ and $B^b = B$.

Household’s problem. In each state $s$ the household solves the following problem

$$V^s(B, T, A) \equiv \max_{c^s_i, l^s_i} U_h(c^s_h, l^s_h) + U_w(c^s_w, l^s_w)$$

s.t.: $c^s_h + c^s_w = A + w^s_h (1 - \tau^s_h) l^s_h + w^s_w (1 - \tau^s_w) l^s_w + B^s,$

where $A$ is the household’s wealth, $w^s_h$ is $h$’s wage rate in state $s$ and $w^s_w$ is $w$’s wage rate. The household’s first-order conditions imply that $\frac{\partial U_h}{\partial c^s_h} = \frac{\partial U_w}{\partial c^s_w} = -\frac{\partial U_w}{\partial l^s_w} \frac{1}{w^s_w(1-\tau^s_w)}$. Importantly, note that we allow $h$ to be at a corner solution in state $b$ – that is, $l^b_h = 0$ – and use only $w$’s labor supply
first-order conditions.

**Planner’s problem.** The social planner’s objective is to choose the tax-and-benefit system that maximizes the household’s expected utility, \( J(B, T) \equiv \mu^gV^g(B, T, A) + \mu^bV^b(B, T, A) \), subject to the requirement that expected benefits paid, \( \mu^bB \), equal expected taxes collected, \( \mu^g(\tau^g_{w_h}w^g_{w_h} + \tau^g_w w^g_w) + \mu^b(\tau^b_{W_h}w^b_{W_h} + \tau^b_w w^b_w) \). Hence, the planner chooses the benefit level \( B \) and taxes \( T \) that solve

\[
\max_{B, T} J(B, T) \quad \text{s.t.} \quad \mu^bB = \mu^g(\tau^g_{w_h}w^g_{w_h} + \tau^g_w w^g_w) + \mu^b(\tau^b_{W_h}w^b_{W_h} + \tau^b_w w^b_w). \tag{4.15}
\]

**Optimal Social Insurance**

Consider a $1 increase in \( B \) financed by an appropriate increase in taxes, e.g., through \( \tau^g_h \). To simplify notation we assume that \( \tau^g_w = \tau^b = 0 \), which allows us to obtain concise welfare formulas.\(^{93}\) The welfare gain from this perturbation is \( \frac{\partial J(B, T)}{\partial B} = \mu^b \frac{\partial \tau^b}{\partial B} + \mu^g \frac{\partial \tau^g_{w_h}}{\partial B} \frac{\partial \tau^g_w}{\partial B} \), which we normalize by the welfare gain from raising \( h \)’s net-of-tax labor income in state \( g \) by $1 (scaled by the targeted population) to gain a cardinal interpretation.\(^{94}\) Exploiting the Envelope theorem (in the differentiation of the household’s value functions) and using the household’s first-order conditions, we obtain \( \frac{\partial V^g}{\partial \tau^g_h} = -w^g_{W_h} \frac{\partial \tau^g_w}{\partial w^g_w} \) and \( \frac{\partial V^b}{\partial B} = \frac{\partial \tau^b}{\partial B} \). Differentiating the budget constraint with respect to \( B \) we get \( \frac{\partial \tau^g_w}{\partial B} = \frac{\mu^b}{1 - \tau^g_h} \left( 1 + \frac{\varepsilon (z^g_{h,1 - \tau^g_h}) \tau^g_w}{1 - \varepsilon (z^g_{h,1 - \tau^g_h}) \tau^g_w} \right) \), where \( z^g_{h,i} \equiv w^g_{W_h,h} i \) is \( h \)’s taxable income and \( \varepsilon (z^g_{h,1 - \tau^g_h}) \equiv \frac{\partial z^g_{h,1 - \tau^g_h}}{\partial (1 - \tau^g_h)} \frac{1 - \tau^g_h}{\tau^g_h} \) is the commonly estimated net-of-tax taxable income elasticity.\(^{95}\) Put together, it follows that the normalized welfare gain from a marginal increase in \( B \) is \( MW(B) = MB(B) - MC(B) \), where \( MB(B) \equiv \frac{\partial \tau^g_w}{\partial \tau^g_h} \frac{\partial \tau^g_w}{\partial w^g_w} \) and \( MC(B) \equiv \frac{\varepsilon (z^g_{h,1 - \tau^g_h}) \tau^g_w}{1 - \varepsilon (z^g_{h,1 - \tau^g_h}) \tau^g_w} \).

**Identifying the benefits from social insurance.** The identification of the gap in marginal utilities of consumption using the unaffected spouse’s labor supply responses in the intensive margin model is summarized in the following proposition.

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\(^{93}\)Relaxing this assumption results in additional elasticities in \( MC(B) \) which is defined below. See Footnote 95.

\(^{94}\)The formula for the normalized gain is \( MW(B) \equiv \frac{\partial J(B, T) / \partial B}{\partial \tau^g_h \partial w^g_w / \partial w^g_w} \), where \( z^g_{h,i} \equiv w^g_{W_h,h} i \).

\(^{95}\)Note that when calculating the change in government revenues, we need to take into account any possible margin that can respond to the change and is being taxed. For example, if we added taxes on \( w \), we would need to include her labor supply responses to changes in \( h \)’s tax rate.
Proposition B1. Assuming consumption-leisure separability, the marginal benefit from raising $B$ in §1 is approximately

$$MB(B) \equiv L^b + M^b,$$

where $L^b \equiv \frac{L^b - L^h}{L^b}$, $M^b \equiv (\varphi - 1) \frac{L^b - L^h}{L^b}$, and $\varphi \equiv \frac{\partial^2 U_w / \partial (l^b_w)^2}{\partial U / \partial l^b_w}$. This allows us to map i's marginal utility from consumption to the unaffected spouse's marginal disutility from labor, such that $MB(B) = \left| \frac{\partial U_w / \partial c^b_w}{\partial c^b_w} \right| - \left| \frac{\partial U_w / \partial l^b_w}{\partial l^b_w} \right|$. Following Gruber's (1997) analysis for estimating the consumption representation of the welfare formulas (see also Chetty and Finkelstein 2013), we take a second-order approximation of $w$'s labor disutility function around $l^b_w$. The consumption-leisure separability assumption yields the result. 

Identification of $\varphi$

In this section we derive a relationship between $\varphi$ and observable labor supply elasticities. The analysis uses a similar strategy as that introduced by Chetty (2006b) to recover risk aversion – i.e., we recover the curvature of the labor disutility function in the same way that Chetty (2006b) recovers the curvature of the consumption utility function. The intuition for the method is that the extent to which an individual responds to changes in economic incentives (wages and income) is directly linked to the rate at which preferences change (over consumption or labor). To conduct the analysis at the individual level, we use the “sharing-rule” interpretation of the collective model as defined by Chiappori (1992). That is, we assume that non-labor income in state s, denoted by $y^s$, is shared between the members such that $y^s_w \equiv \pi^s_w(w_w, w^s_h, A)$ is the amount received by $w$ and $y^s_h \equiv y^s - \pi^s_w(w_w, w^s_h, A)$ is the amount received by $h$. With these definitions, one can write $w$'s program in state $g$ as

$$\max_{c^g_w, l^g_w} U_w(c^g_w, l^g_w)$$

96 Recent research finds supportive evidence for consumption-leisure separability – e.g., Agência, Attanasio, and Meghir (2011) who find no change in consumption (defined as non-durable expenditure) around retirement. However, complementarities between consumption and leisure can be handled by estimating the cross-partial using the technique in Chetty (2006b).
s.t.: $\ell^{g}_{w} = y^{g}_{w} + w_{l}p^{g}_{l}$.

Since we are focusing on the analysis of spouse $w$ in state $g$, we drop spouse subscripts and state superscripts for convenience.

The first-order conditions of this program imply that $wU_{c}(y + w l, l) = -U_{l}(y + w l, l)$, where $U_{x}$ denotes the partial derivative of $U$ with respect to $x$. Partially differentiating the latter equation with respect to $y$ and $w$ yields $\frac{\partial l}{\partial y} = -\frac{wU_{c} + wU_{l}}{w^{2}U_{c} + wU_{l} + 2wU_{cl}}$ and $\frac{\partial l}{\partial w} = -\frac{U_{c} + wU_{c} + wU_{l}}{w^{2}U_{c} + wU_{l} + 2wU_{cl}}$. It follows that $\varphi \equiv \frac{U_{l}}{U_{l}^{2}} = \frac{1 + \varepsilon(l, y) w}{\varepsilon(l, w)} + \varepsilon(U_{c}, l)$, where $\varepsilon(l, y) \equiv \frac{\partial y}{\partial y} l$, $\varepsilon(l, w) \equiv \frac{\partial l}{\partial w}$, $\varepsilon(U_{c}, l) \equiv \frac{U_{c}}{U_{c}} l$ and $\varepsilon(c, l, w) \equiv \varepsilon_{l,w} - \varepsilon_{l,g} \frac{w_{l}}{y}$. With consumption-leisure separability the formula reduces to $\varphi \equiv \frac{1 + \varepsilon(l, y) w}{\varepsilon_{l,w} - \varepsilon_{l,g} \frac{w_{l}}{y}}$.

Appendix C: Heterogeneity in $\theta^{b}$

In this section we return to our participation model of Section 1.2.3 and provide an approximated formula for the case in which the labor disutility state dependence is heterogeneous. Denote the joint distribution of the vector of $w$’s labor disutility and labor disutility state dependence, $(v_{w}, \theta^{b})$, by $\Gamma(v_{w}, \theta^{b})$, the marginal distribution of $\theta^{b}$ by $K(\theta^{b})$ and the marginal distribution of $v_{w}$ by $F(v_{w})$ as before. In addition, denote the distribution of $v_{w}$ conditional on $\theta^{b}$ by $F_{\theta^{b}}(v_{w})$ and the corresponding probability density function by $f_{\theta^{b}}(v_{w})$. Define $y^{s} \equiv \theta^{s}v_{w}$ (where $\theta^{s} = 1$ by normalization) and denote its distribution by $G^{s}(y^{s})$ for $s \in \{g, b\}$ with a probability density function $g^{s}(y^{s})$. Using this notation, $w$ works in state $s$ whenever $y^{s} < \bar{y}^{s}$ where

$$\bar{y}^{s} \equiv \left[u_{h}^{s}(c_{h}^{s}(1)) + u_{w}^{s}(c_{w}^{s}(1))\right] - \left[u_{h}^{s}(c_{h}^{s}(0)) + u_{w}^{s}(c_{w}^{s}(0))\right].$$

It follows that we can rewrite the marginal benefit in labor disutility terms as $MB(\theta^{b}) = \left|\frac{\partial \varphi^{b}}{\partial \theta^{b}}\right|^{\theta^{b}}$. Define participation by $e^{s}_{w} \equiv G^{s}(\bar{y}^{s})$ and note that $\frac{\partial \varphi}{\partial \theta} = g^{b}(\bar{y}^{b})\frac{\partial \varphi}{\partial \theta^{b}}$. To continue, we would want to express $g^{b}(\bar{y}^{b})$ in terms of the marginal distribution of $v_{w}$. Since $G^{b}(y^{b}) = \int_{0}^{\infty} k(\theta^{b}) \left[f_{\theta^{b}}(v_{w})dv_{w}\right] d\theta^{b}$ we have that $g^{b}(y^{b}) = \int_{0}^{\infty} \left[f_{\theta^{b}}(\bar{y}^{b})\frac{\partial \theta^{b}}{\partial \theta^{b}}\right] k(\theta^{b})d\theta^{b} = E_{\theta^{b}}\left[f_{\theta^{b}}(\bar{y}^{b})\frac{1}{\theta^{b}}\right].$ Next, consider approximating $g^{b}(\bar{y}^{b})$. Define $\mu(\theta^{b}) \equiv f_{\theta^{b}}(\bar{y}^{b})\frac{1}{\theta^{b}}$ and take a first-order Taylor expansion around $E\theta^{b}$ to get

\textit{Note the subtlety that we focus on partial derivatives of the unaffected spouse’s behavior with respect to $y$ and $w$. In particular, $y$ is held fixed when we change $w$.}
\( \mu(\theta^b) \cong \mu(E\theta^b) + \mu'(E\theta^b)(\theta^b - E\theta^b) \). Hence, to a first approximation \( g^b(\bar{y}^b) = E_{\theta^b} [\mu(\theta^b)] \cong \mu(E\theta^b) = \int g^b(\bar{y}^b) \frac{1}{E\theta^b} \). Define \( \bar{v}^b \) to be \( v_w \) which satisfies \( v_w E\theta^b = \bar{y}^b \). This implies that \( g^b(\bar{y}^b) \cong \frac{1}{E\theta^b} \int g^b(\bar{y}^b) \frac{\partial \bar{y}^b}{\partial \theta^b} \) and hence that \( \frac{\partial \bar{y}^b}{\partial \theta^b} = g^b(\bar{y}^b) \frac{\partial \bar{v}^b}{\partial \theta^b} \). If, for example, \( v_w \) is distributed independently of \( \theta^b \), such that \( f_{\theta^b}(\bar{v}^b) = f(\bar{v}^b) \), a first-order approximation of \( F \) in the threshold region \( (\bar{v}^b_0, \bar{v}^b) \) will yield the same approximated formula for \( MB(b) \) as in Proposition 2 where \( \theta^b \) is replaced by its mean value, \( E\theta^b \).

Appendix D: Calibration of \( \theta^b \)

In this section we provide a proof for the Lemma in the text (Section 1.5.2). We begin with the baseline model and then provide a proof for the dynamic model. Similar analysis can be conducted for the intensive-margin case and is available from the authors on request.

**Static Extensive Margin Model**

Recall that \( V^s(y^s(l^s_w)) \equiv \max u^s_w(c^s_w) + u^s_h(c^s_h) \) s.t. \( c^s_w + c^s_h = y^s(l^s_w) \), where \( y^s(l^s_w) \equiv A + \bar{z}_h^s \times \bar{z}_w^s + \bar{z}_w^s \times \bar{l}_w^s + B(l^s_w) \). Since we are interested in analyzing steady-state equivalence scales we account for transitory labor income shocks and later employ conditions under which the scales we study are not sensitive to these shocks. We decompose \( w \)'s net labor income, \( \bar{z}_w \), into its permanent component, \( z_w \), and its transitory component, \( \bar{z}_w \), such that \( \bar{z}_w = z_w + \bar{z}_w \) and \( y^s(l^s_w) = A + \bar{z}_h^s \times \bar{z}_w^s + \bar{z}_w^s \times \bar{l}_w^s + B(l^s_w) \).

Next, recall that \( w \) works when \( v^s_w < \bar{v}^s_w \equiv V^s(y^s(1)) - V^s(y^s(0)) \), where \( v^s_w = v_w \) and \( v^b_w = \theta^b \times v^s_w \). In equilibria in which \( w \)'s participation rate in state \( g \) and in state \( b \) are the same it must be that \( \bar{v}^g_w \equiv \bar{v}^b_w / \theta^b \), or: \( V^g(y^g(1)) - V^g(y^g(0)) = \frac{1}{\theta^b} [V^b(y^b(1)) - V^b(y^b(0))] \). This implies a necessary condition that the household income flows - \( y^g(0), y^g(1), y^b(0), \) and \( y^b(1) \) - must satisfy when labor supply is unchanged across states of nature. In a steady state, this equality is insensitive to local income shocks. By equating the derivative of both sides with respect to the transitory income shock, \( \bar{z}_w \), we get the relationship

\[
V^g'(y^g(1)) = \frac{1}{\theta^b} \left\{ V^b'(y^b(1)) \right\}.
\] (4.17)
Let \( \theta^u \equiv V^b(y^b(1))/V^g(y^g(1)) \) denote the change in the marginal value of household income, and let \( \gamma \equiv -[V^g(y^g(1))/V^g(y^g(1))] \times y^g(1) \) denote the household-level pre-shock relative risk aversion. A second-order expansion of the value function \( V^b \) on the right-hand side of (4.17) around \( y^g(1) \) yields the result in the Lemma

\[
\theta^u(1 + \gamma(1 - r^{eq})) \cong \theta^b,
\]

(4.18)

where \( r^{eq} \equiv y^b(1)/y^g(1) \) is the steady state replacement rate that satisfies this relationship.

**Dynamic Search Model**

The notation and definitions we use here are described in Appendix A. To simplify the analysis we assume two states of nature as in the baseline model, \( s \in \{g, b\} \). Recall from Appendix A that

\[
c^s_{ht}(l^s_{wt}) + c^s_{wt}(l^s_{wt}) + A_{t+1}(l^s_{wt}) = A^s_t + y^s_t(l^s_{wt}) \quad \text{and} \quad y^s_t(l^s_{wt}) \equiv \bar{z}^s_{ht} \times l^s_{wt} + \bar{z}_{wt} \times l^s_{wt} + B(l^s_{wt}).
\]

As in the baseline case, we decompose \( w' \)'s net labor income, \( \bar{z}_{wt} \), into its permanent component, \( \bar{z}_{wt} \), and its transitory component, \( \bar{z}_{wt} \), such that \( \bar{z}_{wt} = \bar{z}_{wt} + \bar{z}_{wt} \) and \( y^s_t(l^s_{wt}) = \bar{z}^s_{ht} \times l^s_{wt} + (\bar{z}_{wt} + \bar{z}_{wt}) \times l^s_{wt} + B(l^s_{wt}) \). For each period in which \( w' \) is not working define the flow consumption utility at the optimal choices as a function of the period’s wealth and income by

\[
U^s(A_t, y^s_t(l^s_{wt})) \equiv u^s_h(c^s_{wt}(l^s_{wt})) + u^s_w(c^s_{wt}(l^s_{wt})),
\]

where

\[
(c^s_{wt}(l^s_{wt}), A^s_t) \equiv \arg \max_{c^s_{wt}(l^s_{wt})} \left\{ c^s_{wt}\left( u^s_h(c^s_{wt}(l^s_{wt})) + u^s_w(c^s_{wt}(l^s_{wt}) + W^s_{t+1}(B, T, A^s_{t+1}(1)) + \left(1 - c^s_{wt}\right) \left( u^s_h(c^s_{wt}(0)) + u^s_w(c^s_{wt}(0)) + W^s_{t+1}(B, T, A^s_{t+1}(0)) - c^s_{wt}\right) \right) \right\}
\]

We can, therefore, rewrite the first-order condition for \( w' \)'s effort as

\[
\left( U^s(A_t, y^s_t(1)) + W^s_{t+1}(B, T, A^s_t(1)) \right) - \left( U^s(A_t, y^s_t(0)) + W^s_{t+1}(B, T, A^s_t(0)) \right) = \kappa^s_t(c^s_{wt}(l^s_{wt})).
\]

(4.19)

In equilibrium in which \( w' \)'s participation rate in state \( g \) and state \( b \) are the same it must be that \( e^g_{wt} = e^b_{wt} \). For a given period, which we normalize to 0, define \( \theta^b \equiv \kappa^g(c^b_{wt(0)})/\kappa^g(c^b_{wt(0)}) \), which
implies that
\[
\left(U^g(A_0, y^g_0(1)) + W^{g,1}_1(B, T, A^*_1(1)) \right) - \left(U^g(A_0, y^g_0(0)) + W^{g,0}_1(B, T, A^*_1(0)) \right) = \frac{1}{\theta^b} \left\{ \left(U^b(A_0, y^b_0(1)) + W^{b,1}_1(B, T, A^*_1(1)) \right) - \left(U^b(A_0, y^b_0(0)) + W^{b,0}_1(B, T, A^*_1(0)) \right) \right\}. 
\]
(4.20)

Differentiating both sides with respect to the transitory shock \(\varsigma_{w0}\) yields
\[
U^g_y(A_0, y^g_0(1)) = \frac{1}{\theta^b} U^b_y(A_0, y^b_0(1)),
\]
where \(U^g_x\) is the partial derivative of \(U^g\) with respect to \(x\). Let \(\theta^u \equiv \frac{U^b_y(A_0, y^b_0(1))}{U^g_y(A_0, y^g_0(1))}\) denote the change in the marginal value of household income, and let \(\gamma \equiv \frac{-U^g_{y y}(A_0, y^g_0(1))}{U^g_y(A_0, y^g_0(1))} \times y^g_0(1)\) denote the household-level pre-shock relative risk aversion. A second-order expansion of the consumption flow “value function” \(U^b\) around \(y^g_0(1)\) yields the result in the Lemma

\[
\theta^u (1 + \gamma (1 - r^{eq})) \equiv \theta^b, \quad (4.21)
\]
where \(r^{eq} \equiv y^g_0(1)/y^g_0(1)\) is the steady state replacement rate that satisfies this relationship.

Appendix E: Implications for Health-State Dependence of the Household’s Preferences

In this section we formalize the discussion in Section 1.5.3 on health-state dependence. Since we found the unaffected spouse’s labor supply response to spousal health shocks to be on the intensive margin, we refer to the intensive-margin model of the household behavior developed in Appendix B. We generalize preferences such that each spouse’s preferences in state \(s\) can be represented by the utility function \(U^h_i(c^h_i, l^h_i)\), where \(c^h_i\) and \(l^h_i\) are spouse \(i\)’s consumption and labor supply in state \(s\), respectively. Efficiency requires the marginal utility of \(h\)’s consumption, \(\frac{\partial U^h_i}{\partial c^h_i}\), to equal \(w\)’s marginal disutility of labor, \(-\frac{\partial U^h_i}{\partial l^h_i}\). This is the basic logic behind the welfare result for the intensive margin case, which implies that \(\frac{\partial U^b}{\partial c^h} - \frac{\partial U^g}{\partial c^h}\). Define \(\theta^u \equiv \frac{\partial U^b}{\partial c^h} / \frac{\partial U^g}{\partial c^h}\) at \(c^h\) and \(\theta^b \equiv \frac{\partial U^b}{\partial l^w} / \frac{\partial U^g}{\partial l^w}\) at \(l^w\) to be the local consumption utility and labor disutility state dependence parameters, respectively. With consumption-leisure separability it follows that \(\theta^u \gamma \frac{\Delta w}{c^h} + \theta^b \varphi \frac{\Delta l^w}{l^w} \equiv \theta^b - \theta^u\), where \(\gamma \equiv -\frac{\partial U^g}{\partial c^h} / \frac{\partial U^g}{\partial c^h}\) is \(h\)’s risk aversion parameter,
\[ \varphi \equiv \frac{\partial^2 U_g}{\partial (c_h)^2} l_w^g \] is the curvature of \( w \)'s disutility from labor, and \( \frac{\Delta x}{x} \equiv \frac{x^g - x^b}{x^g} \). Since we find that \( \frac{\Delta l_w}{l_w} > 0 \), since \( \theta^u, \theta^b, \gamma, \varphi > 0 \), and if \( \frac{\Delta c_h}{c_h} > 0 \) due to the small income loss the household experiences, it must be that \( 0 < \theta^u \gamma \frac{\Delta c_h}{c_h} + \theta^b \varphi \frac{\Delta l_w}{l_w} \equiv \theta^b - \theta^u \). This implies that \( \frac{\theta^b}{\theta^u} > 1 \), which includes the extreme cases of \( \theta^u = 1 \) with \( \theta^b > 1 \) and \( \theta^b = 1 \) with \( \theta^u < 1 \). More generally, our results imply that labor disutility state dependence is greater than the potential state dependence in the sick spouse's consumption utility.

\[ ^{98} \text{This is achieved by taking a Taylor expansion of } \theta^u \frac{\partial U_g}{\partial c_h} \text{ around } c_h^g \text{ and of } \theta^b \frac{\partial U_g}{\partial l_w} \text{ around } l_w^g. \]
Appendix F: An Empirical Model of Labor Force Participation

In this section we estimate an empirical counterpart to the theoretical model of household labor force participation in order to provide suggestive estimates for $\varepsilon(e_{w,i}^{h}, b^{h})$ and $\varepsilon(e_{w}^{h}, b^{h})$. We model $w$’s participation such that in the years before the event her decision is conditional on $h$’s behavior. Specifically, the income $h$ contributes to the household – whether through transfers or through labor income – is perceived as non-labor income in $w$’s decision making. We constrain the sample to individuals who are younger than 60 to avoid retirement transitions that are due to eligibility for early retirement benefits and Social Security.

Labor force participation. We let $w$’s labor supply depend on her potential wage if she decides to work, on the potential transfers she would receive if she decides not to work, as well as on her unearned income. Denote the participation decision and the latent index of spouse $w$ in household $i$ at time $t$ in state $s$ by $l_{w,i,t}^s$ and $I_{w,i,t}^s$, respectively. Then, $l_{w,i,t}^s = 1$ if $I_{w,i,t}^s > 0$ and $l_{w,i,t}^s = 0$ otherwise. We assume the following linear form for the participation latent index

$$I_{w,i,t}^s = \delta_0 + \delta_1 z_{w,i,t}^s + \delta_2 b_{w,i,t}^s + \delta_3 y_{w,i,t}^s + \delta_4 wealth_{i,t} + controls + \varepsilon_{i,t}^s,$$

where

$$\delta_0 = \delta_{00} + \delta_{01} treat_i + \delta_{02} post_{i,t} + \delta_{03} treat_i \times post_{i,t},$$

$$\delta_k = \delta_{k0} + \delta_{k1} treat_i \times post_{i,t} \ for \ k = 1, ..., 4.$$  

In this specification $z_{w,i,t}^s$ denotes $w$’s potential labor income in state $s$, $b_{w,i,t}^s$ denotes her potential government transfers if she decides not to work in state $s$, $y_{w,i,t}^s$ denotes $w$’s unearned income as well as any income (earned or unearned) that is attributed to $h$ before his death, and $wealth_{i,t}$ denotes the household’s net wealth. The coefficients are allowed to freely change across states of nature, since $treat_i \times post_{i,t}$ is the differences-in-differences interaction variable. The controls include dummies for $w$’s age, calendar year, and municipality of residence before the shock occurs.

Wage equations. Following Blundell, Chiappori, Magnac, and Meghir (2007), we take the stan-
ard human capital approach to wages and additionally allow for the relative prices of education to change over time. In particular, we assume\textsuperscript{100}

\[ z_{w, i, t} = \pi_0 + \pi_{1t} educ_i + \pi_{2t} educ_i^2 + \pi_{3t} gender_i + \pi_{4t} age_{i,t} \]
\[ + \pi_{5} local labor market_{i,t} + \pi_{6} health_{i,t} + \pi_{7} X_{i,t} + \kappa_{i,t}. \]

This assumes that wage offers are a function of calendar year, education (and its square), gender, age indicators, local labor market conditions (which include municipality fixed-effects and municipality-level unemployment rate and average labor income), health (current and lagged hospitalization), and additional characteristics \( X_{i,t} \) in which we include a dummy variable for whether the person is a native or an immigrant and indicators for the number of children (of any age). The coefficients on education are allowed to vary over time. To account for selection into the labor force in the imputation of wage offers, we employ the (two-stage) Heckman correction (1979). The analysis is repeated separately for each combination of timing (before/after the shock) and experimental group (treatment/control).

**Potential transfers.** In the same manner we need to impute the expected potential government transfers in the case an individual chooses not to work. The labor-supply-dependent transfers are Social Disability Insurance (Social DI) benefits, which are awarded in Denmark for medical reasons as well as for social reasons. Recall that Social DI is a state-wide means-tested program that is locally administered (at the municipality level). Hence, we model expected benefits as a function of calendar year dummies (which capture overall national trends in benefits), municipality fixed effects, and interactions of municipality dummies with year dummies. The source of variation we use to identify the effect of potential transfers on participation is within municipalities over time since we include municipality and calendar year fixed effects as controls in the participation equation (4.22). We also include deciles of gross wealth, liabilities, and home value since some portion of DI is asset-tested, as well as age dummies, gender, and health indicators (hospitalization and lagged

\textsuperscript{100}For expositional reasons we use the notation that whenever the variable is multidimensional (e.g., \( age_{i,t} \), which denotes a complete set of age dummies), the corresponding coefficient is a vector of the same dimension (e.g., \( \pi_4 \) has as many entries as the number of unique ages observed in our sample).
hospitalization). We use the following specification

\[ b_{u,i,t}^s = \sigma_0 + \sigma_1 \text{municipality}_i + \sigma_2 \text{municipality}_i \times \text{year}_{i,t} + \sigma_3 \text{age}_{i,t} + \sigma_4 \text{gender}_i + \sigma_5 \text{health}_{i,t} + \sigma_6 \text{gross wealth}_{i,t} + \sigma_7 \text{liabilities}_{i,t} + \sigma_8 \text{home value}_{i,t} + \omega_{i,t}^s. \]

We estimate this equation using the sample of individuals that do not participate in the labor force, separately for different combinations of timing (before/after the shock) and experimental groups (treatment/control). In this way we construct the transfers an agent who decides not to work expects to receive at time \( t \) in state \( s \).

**Non-labor income and net-wealth.** We want a measure for non-labor income that is exogenous to other decisions such as take-up of social benefits (beyond direct government transfers that are captured by \( b_{u,i,t}^s \)), withdrawals from savings accounts, claims from private insurance policies, etc. Therefore, we treat \( w \)’s component of unearned income \( y_{t\text{,}i}^u \) as endogenous (following Blundell, Chiappori, Magnac and Meghir 2007), and use predictions based on reduced-form projections, which we run for each combination of timing and experimental group for the effective unearned income on a series of pre-shock household economic variables and characteristics.\(^{101}\) We then construct non-labor income \( y_{t\text{,}i}^u \) as the sum of \( h \)’s income and \( w \)’s predicted non-labor income. To account for potential endogeneity in household-level net wealth (excluding home value), we use pre-shock wealth levels as the right-hand side variable for wealth.

**Stochastic specification and estimation.** We estimate the model as a probit and hence assume that the error in the latent index, \( \varepsilon_{t\text{,}i}^u \), is normally distributed with unit variance. The participation equation is estimated using the imputed wages, the expected government benefits, the household-level non-labor income, pre-shock net wealth, and the additional controls (age, year, and municipality dummies).

\(^{101}\)To improve the fit of this reduced-form we included a rich set of predictors. These include age and year dummies as well as their interaction, deciles of pre-shock wealth, liabilities, and home value, pre-shock income flows from different private and social sources available in the register-based data, occupation, employment and earnings history, health indicators, education, cohort dummies, as well as gender and municipality fixed effects.
Elasticity Estimates

The estimation of the model above provides us with the following elasticities, evaluated at sample means: $\varepsilon(e^b_w, b) = -0.1937$ with a confidence interval of $[-0.2031, -0.1842]$ and $\varepsilon(e^g_w, b^g) = -0.1409$ with a confidence interval of $[-0.1468, -0.1350]$. The estimate for their ratio is $\varepsilon(e^b_w, b^b)/\varepsilon(e^g_w, b^g) = 1.375$ with a confidence interval of $[1.292, 1.457]$. 
References


