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*(Article begins on next page)*

# Safety in Markets: An Impossibility Theorem for Dutch Books\*

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## Abstract

We show that competitive markets protect consumers from many forms of exploitation, even when consumers have non-standard preferences. We analyze a competitive dynamic economy in which consumers have arbitrary time-separable preferences and arbitrary beliefs about their own future behavior. Competition among agents eliminates rents and protects vulnerable consumers, who could have been exploited by a monopolist. In fact, in competitive general equilibrium no consumer participates in a trading sequence that strictly reduces her endowment – there are no Dutch Books. The absence of Dutch Books in and of itself does not distinguish standard and non-standard preferences. However, non-standard preferences do generate qualitatively different equilibrium outcomes than standard preferences. We characterize the testable implications of the standard model with a dynamic generalization of the Strong Axiom of Revealed Preferences.

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# 1 Introduction

## 1.1 OVERVIEW

Economists use Dutch Book — aka money pump — arguments to rule out certain types of tastes and/or beliefs.<sup>1</sup> A typical argument proceeds in three steps.

1. Suppose a consumer had exotic preferences.
2. Then the consumer would participate in a series of self-improverishing trades referred to as a Dutch Book or a money pump.

The third step in the argument comes in two different versions:

- 3a. Self-improverishing trades will bankrupt the consumer, implying that such consumers will not play an important role in the economy.
- 3b. Self-improverishing trades are empirically uncommon, so preferences that imply such trades must also be uncommon.

The current paper focuses on the implicit assumptions that are necessary to support point two in the argument.<sup>2</sup> Although Dutch Book arguments are often made by proponents of the competitive markets model (e.g., Becker (2002)), Dutch Book arguments are rarely embedded in a market context. In practice, step two usually posits the existence of a trader who is able to make offers to an *isolated* consumer. The trader constructing the Dutch Book has a monopoly relationship with the consumer being booked.

This paper shows the limited scope of the Dutch Book argument in *competitive* general equilibrium. Competitive markets protect consumers because suppliers of Dutch Books must compete

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<sup>1</sup>See Becker (2002), Mas-Colell, Whinston, and Green (1995), Yaari (1985), and references therein.

<sup>2</sup>There are also some problems with step 3. Step 3a implies that bankruptcy is not an important topic, but if utility matters instead of wealth, then impoverished consumers — supported by the welfare state or starving for lack of support — should matter for economic analysis. Step 3b implies that swindles are infrequent, but this remains an open question (see Leff 1976).

with one another to attract customers/victims. Though there may exist price vectors that support Dutch Books, none of these price vectors satisfy market-clearing competitive equilibrium conditions. Even if consumers have non-standard preferences (e.g., intransitive, dynamically inconsistent, etc.), they will not get Dutch Booked. To our knowledge, practically all of the preferences that have been described as vulnerable to a Dutch Book are in fact invulnerable once one eliminates the monopoly power of the trader constructing the Dutch Book.

Formally, we consider a dynamic general equilibrium endowment economy with heterogeneous participants characterized by monotonic, concave, and time separable preferences. We allow agents to have arbitrary dynamically inconsistent preferences as well as mistaken beliefs about their own future preferences. For every path of prices, each agent is characterized by a demand profile derived from an intrapersonal game. A general equilibrium is then comprised of a price path and intrapersonal equilibrium demand profiles for all participants, such that all (time-contingent) markets clear. Adapting the methods of Luttmer and Marriotti (2006), we show (in Theorem 1) that a general equilibrium exists.

We define two types of Dutch Books. An equilibrium demonstrates *money making Dutch Books* if there exists a sequence of equilibrium trades that leave at least one agent with a strictly dominating sequence of consumption claims. Analogously, an equilibrium demonstrates *money losing Dutch Books* if there exists a sequence of equilibrium trades that leave at least one agent with a strictly dominated sequence of consumption claims.

As long as preferences are time separable, Walras' law holds, despite the fact that preferences are dynamically inconsistent and beliefs about future behavior may be inaccurate. Walras' law implies that no agent loses wealth, so no agent can gain wealth, eliminating both types of Dutch Books (Theorem 2).

Partial time separability of preferences turns out to be crucial to the argument. We show that Dutch Books *can* exist in competitive equilibrium if preferences are non-separable. Nonetheless, such Dutch Books are fragile in the sense that the Dutch Book can only exist if there are *no* time separable goods. Intuitively, if there is at least one time-separable good, the agent would always

prefer to consume more of it than to dispose of wealth.

Transaction costs provide another conceptual challenge, since Walras' Law does not apply in such settings. The existence of transaction costs will revive Dutch Books in some economies that we study. However, Dutch Books will not arise in economies with transaction costs as long as agents hold accurate beliefs about their future preferences. Such sophisticated agents will anticipate sequences of trade that cause them to be worse off in *every* period and will consequently choose to avoid such sequences in equilibrium.

If markets do not "screen" non-standard preferences and beliefs, it is natural to ask whether the existence of such non-standard behavior will have any effect on competitive market outcomes. If the answer to this question were negative, then the standard time consistent model would serve as a good *as if* model of competitive markets. The last part of the paper is targeted at identifying the testable implications of the general equilibrium model with standard time consistent preferences and accurate beliefs.

As it turns out, given a single set of equilibrium prices and quantities one can always construct a model with dynamically consistent preferences that rationalizes the data. However, such a model would not necessarily correctly predict out of sample behavior. Observing data from numerous economies with identical preferences and different endowments generates testable restrictions for dynamically consistent models.

Intuitively, consider each time period's (single) good as a separate good, so that observing prices and demands over time in a collection of dynamic economies is akin to observing prices and demands for bundles of goods in a collection of static economies. Afriat (1967) provided necessary and sufficient conditions for the existence of a well-behaved utility function that would generate such a data set. These conditions are captured by a generalized version of the axiom of revealed preferences. If economic data satisfy these conditions, then a time-consistent model can be constructed to explain the data. Consequently, the necessary and sufficient conditions for the existence of a time-consistent model that generates a finite data set is a dynamic version of the generalized axiom of revealed preferences. Our results of Section 6 (and Theorem 4 in particular)

illustrate the potential importance of studying dynamically inconsistent preferences and beliefs in economic contexts.

## 1.2 ORGANIZATION OF THE PAPER

The section that follows reviews the related literature on Dutch Books. Section 3 provides an example that motivates our analysis and illustrates our results. Section 4 describes a general endowment economy with complete, competitive futures markets and non-standard agents. It also illustrates the existence of an equilibrium in such environments. Section 5 shows that Dutch Books will not arise in competitive equilibrium, while Section 6 provides a characterization of the restrictions implied by dynamically consistent preferences on observable prices and quantities and demonstrates the observational non-equivalence between dynamically consistent and dynamically inconsistent economies. In Section 7 we discuss the robustness of our results, considering issues like non-separability and transaction costs. Section 8 concludes. Most technical proofs are in the Appendices.

## 2 Dutch Book Literature

Dutch Books were originally applied to probability updating. De Finetti's (1937) treaty on the theory of probability showed that people whose beliefs satisfy the laws of probability are invulnerable to Dutch Books. Ramsey (1931) noted the reverse implication – people whose beliefs are inconsistent with the laws of probability are vulnerable to Dutch Books. The recent literature has identified preferences that yield Dutch Books. Yaari (1985) and Green (1987) study violations of the independence axiom. Mulligan (1996) studies dynamically inconsistent time preferences. These papers assume that a single rational agent can make a sequence of take-it-or-leave-it offers to a second, “irrational” agent.

There have been a few reassessments of the Dutch Book literature. Machina (1989) points out that Dutch Book arguments “snip” the decision tree just before the current choice node and recalculate the optimal continuation. Machina critiques this *consequentialist* assumption and

argues that Dutch Book arguments should only be applied to time separable problems, which is the setting of the current paper.<sup>3</sup>

Border and Segal (1994, 2002) consider an oddsmaking environment, in which odds are chosen strategically against bettors. When either the bookie or the bettors are not standard expected utility maximizers (in the 2002 paper, bettors also do not share a common prior), Border and Segal show that strategic equilibrium behavior by a bookie may lead to betting rates that violate basic laws of probability theory. These papers relate to the recent class of models studying two-sided markets in which firms interact with consumers who may have a variety of psychological biases.<sup>4</sup>

Heifetz, Shannon, and Spiegel (2004) consider evolution as the force restraining preferences. They demonstrate that in almost every two-player continuous-action normal-form game, almost every distortion of a player's perceived payoffs (that shifts the player away from standard payoff-maximization) will not be driven out by any evolutionary process involving payoff-monotonic selection dynamics.

Our impossibility theorem is related to an experimental result in Kluger and Wyatt (2004). They use the Monty Hall problem to induce probability judgement errors in subjects and find that competition among two error-free agents is sufficient to make market prices bias-free.

Kocherlakota (2001) shows that competitive market outcomes do not generally identify dynamically inconsistent time preferences – dynamic arbitrage disables prices of retradable assets from revealing whether underlying preferences are time consistent or not. However, the prices of commitment assets can identify time inconsistent preferences. These results are related to the general identification result we present in Section 6. In addition, we provide some ways of identifying time inconsistency, even when there is no commitment asset.

A similar conceptual question to the one posed in our paper pertains to the plausible beliefs agents can hold in stochastic general equilibrium. Blume and Easley (1992, 2004) and Sandroni (2000) analyze restrictions on agents' behavioral rules as well as market incompleteness that lead participants with inaccurate predictions to be driven out of a competitive market. In a related

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<sup>3</sup>See also Cubitt and Sugden (2001).

<sup>4</sup>For instance, Della Vigna and Malmendier (2006), Gabaix and Laibson (2006), and Spiegel (2006).

spirit, Rubinstein and Spiegel (2007) illustrate the bounds of exploitation of agents who have boundedly rational learning.

Finally, our analysis relies on the general equilibrium existence proof in Luttmer and Marriotti (2003, 2006).

The source of the term Dutch Book remains unclear (Wakker (2001)). The term may have been used to describe 19th century Dutch trading companies that hedged shipping risks so that profits were made whether or not a ship survived the journey. Alternatively, the term may have originated with derisive English expressions adopted during the 17th century rivalry between Holland and England, such as Dutch courage and Dutch treat. Yet another possibility is that the term was coined on the horse track, where it is still commonly used.

### 3 Motivating Example

In this section we present a simple example that illustrates and motivates some of our findings. We first consider a case that reproduces classical Dutch Book results and then show why these results vanish in a market context.

#### 3.1 A Dutch Book

Consider an agent who we will refer to as “Naif.” Naif has the following time-inconsistent preferences over time-dated consumption,  $t \in \{1, 2, 3\}$ .<sup>5</sup>

$$\begin{aligned} U_1 &= \ln c_1 + \frac{1}{2}(\ln c_2 + \ln c_3) \\ U_2 &= \ln c_2 + \frac{1}{2} \ln c_3 \\ U_3 &= \ln c_3. \end{aligned}$$

Naif believes (incorrectly) that his preferences in period 2 are dynamically consistent with his preferences in period 1, so Naif believes that  $U_2 = \ln c_2 + \ln c_3$ .

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<sup>5</sup>Strotz (1957), Phelps and Pollak (1968), Akerlof (1991), Laibson (1997), and O’Donoghue and Rabin (1999).



Suppose that Naif has a consumption endowment of

$$(c_1, c_2, c_3) = (3, 2, 1).$$

A second party — call him “Arbitrageur” — offers Naif the opportunity to trade Naif’s original consumption sequence for an alternative consumption sequence:

$$(c_1, c_2, c_3) = \left( 2\sqrt{3/\sqrt{2}}, \sqrt{3/\sqrt{2}}, \sqrt{3/\sqrt{2}} \right).$$

Naif accepts, since the new sequence offers a weakly higher utility than the old bundle:

$$\ln \left( 2\sqrt{3/\sqrt{2}} \right) + \frac{1}{2} \left[ \ln \left( \sqrt{3/\sqrt{2}} \right) + \ln \left( \sqrt{3/\sqrt{2}} \right) \right] = \ln 3 + \frac{1}{2} (\ln 2 + \ln 1).$$

Naif consumes her new claim  $c_1 = 2\sqrt{3/\sqrt{2}}$ , and period 2 begins. Now Arbitrageur returns and offers Naif a new alternative bundle  $(c_2, c_3) = \left( \sqrt[3]{2}\sqrt{3/\sqrt{2}}, \frac{\sqrt[3]{2}}{2}\sqrt{3/\sqrt{2}} \right)$ . Naif accepts. From Naif’s current perspective the new bundle offers a weakly higher utility than the old bundle:

$$\ln \left( \sqrt[3]{2}\sqrt{3/\sqrt{2}} \right) + \frac{1}{2} \ln \left( \frac{\sqrt[3]{2}}{2}\sqrt{3/\sqrt{2}} \right) = \ln \left( \sqrt{3/\sqrt{2}} \right) + \frac{1}{2} \ln \left( \sqrt{3/\sqrt{2}} \right).$$

As a consequence of these two trades, Naif’s final consumption sequence —  $(c_1, c_2, c_3) = \left( 2\sqrt{3/\sqrt{2}}, \sqrt[3]{2}\sqrt{3/\sqrt{2}}, \frac{\sqrt[3]{2}}{2}\sqrt{3/\sqrt{2}} \right)$  — is pointwise strictly dominated by Naif’s initial sequence of claims:  $(3, 2, 1)$ .

This is a classic example of a Dutch Book. A sequence of trades has strictly reduced Naif’s endowment. The numbers in this example may seem unnecessarily complicated, but they are carefully chosen to reflect the unique equilibrium trades that would arise if Arbitrageur was a perfectly rational monopolist. The next subsection provides the relevant formalization.

### 3.2 Game-theoretic formalization

The previous example may leave one wondering about the details of the game being played. To fill in those details, we embed the analysis in a formal game.

We’ve already stated the preferences and endowment of Naif. Assume that Arbitrageur has linear, time-consistent preferences:

$$U^A = c_1^A + c_2^A + c_3^A.$$

Assume also that Arbitrageur has an arbitrary endowment.

At the beginning of every period Arbitrageur is allowed to make a single take-it-or-leave-it offer to Naif. The rules of the game and the preferences of the agents are common knowledge, except for the fact that Naif is mistaken in her forecasts of her own future preferences.<sup>6</sup> One can show that all subgame perfect equilibria of this game are characterized by the sequence of trades that we have described above. These trades are constructed by sequentially offering Naif the least costly bundle – as judged by Arbitrageur – that is at least as good to Naif as the bundle that Naif currently holds.

### 3.3 Markets

Consider now a market economy composed of Naifs with the endowment  $(c_1, c_2, c_3)$  and at least two Arbitrageurs.<sup>7</sup> Assume that instead of take-it-or-leave-it offers, all exchange occurs in spot markets and futures markets that open in every period of the economy. Also, assume that agents may freely dispose of their goods if they prefer not to sell them in the market (this will be defined formally for general dynamic economies in Section 4 that follows).

As usual, an equilibrium is a sequence of prices and actions such that all markets clear and all agents maximize their *perceived* interests, given their beliefs (regarding both prices and individual future choice policies). In this economy it is easy to show that there exists a unique equilibrium consumption sequence for Naifs:

$$\begin{aligned} c_1^* &= \frac{c_1 + c_2 + c_3}{2}, \\ c_2^* &= \frac{c_1 + c_2 + c_3}{3}, \\ c_3^* &= \frac{c_1 + c_2 + c_3}{6}. \end{aligned}$$

It follows that  $\sum c_t^* = \sum c_t$ , so that a Dutch Book does not exist. In other words, agents do *not* engage in trade (or free disposal) that leads to a consumption sequence that is dominated by

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<sup>6</sup>Naif believes that Arbitrageur shares Naif's beliefs about the future.

<sup>7</sup>This is a special case of the general class of markets that we study in this paper.

their endowment. When Arbitrageurs have to compete with one another, they can no longer turn individuals into money pumps.

In this example, the agents in the economy — both Naifs and Arbitrageurs — have non-generic preferences. The crux of the first part of this paper is showing that this example generalizes to a wide class of preferences. Within that class of preferences (namely, time separable and weakly increasing), time inconsistent preferences and associated beliefs do not admit Dutch Books in general equilibrium. However, markets are not panacea. We will also identify preferences for which Dutch Books arise *even in a competitive market equilibrium*. In the second part of the paper we identify the testable implications of time consistent models.

## 4 The Environment

We analyze an exchange economy in which agents trade goods in competitive markets. These agents need not have dynamically consistent preferences or rational expectations. Although all *trade* takes place in a competitive market, we also allow free disposal. The environment is deterministic in the sense that endowments in every period are fixed at the beginning of time.

Most of our assumptions are made without loss of generality. For example, adding risk would not change our results but would significantly complicate our notation. In section 7, we discuss such generalizations.

### 4.1 Goods

We consider an exchange economy with a single consumption good at every date  $t = 1, \dots, T$ . The goods will be represented by the vector  $\{c_1, c_2, \dots, c_T\}$ . There are  $I$  types of consumers, indexed by  $i = 1, \dots, I$ . For every discrete consumer type there is a continuum of individual consumers. Without loss of generality, we assume a unit measure of each type.<sup>8</sup>

At each period  $t$ , the date- $t$  consumer decides how much to consume today and trades future claims to consumption. The action of the date- $t$  consumer is denoted by a vector  $C^t \in \mathbf{C}^t \subseteq \mathbb{R}_+^T$ , where  $C^t$  reflects three types of consumption: past consumption (which is already fixed by date  $t$ ),

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<sup>8</sup>Heterogeneity in the quantity of different types can be captured by allocating some types more than one  $i$  index.

current consumption, and claims to future consumption. More formally, we decompose the action vector  $C^t$  into past consumption,  $\{c_1^t, c_2^t, \dots, c_{t-1}^t\}$ , current consumption,  $c_t^t$ , and claims at date  $t$  to future consumption,  $\{c_{t+1}^t, c_{t+2}^t, \dots, c_T^t\}$ . For notational convenience, we let  $c_t \equiv c_t^t$ . Hence,  $c_t$  refers to the actual consumption that takes place in period  $t$ .

For all  $s \geq t$ , denote by  $p_s^t$  prices of futures contracts traded at time  $t$  for consumption at time  $s$ . To simplify notation let  $p_t \equiv p_t^t$ , where  $p_t^t$  is the spot price for consumption at date  $t$ . Finally, let  $P^t = (p_1, \dots, p_{t-1}, p_t^t, p_{t+1}^t, \dots, p_T^t)$ , which is the vector of past, current and future prices at date  $t$ . All upper-case price vectors have  $T$  cells to facilitate vector operations.

At any period  $t$ , given the price vector  $P^t$ , the agent's action set  $\mathbf{C}^t$  is determined according to her past history of actions, her current budget constraint, and a requirement that she can repay her obligations after current consumption takes place.

$$\mathbf{C}^t = \left\{ C^t \in [-\bar{C}, \infty)^T \mid c_s^t = c_s^s \forall s < t \text{ and } \sum_{s=t}^T p_s^t c_s^t \leq \sum_{s=t}^T p_s^t c_s^{t-1} \text{ and } \sum_{s=t+1}^T p_s^t c_s^t \geq 0 \right\}.$$

Here  $\bar{C}$  is a bound on consumption claims, which is imposed for technical reasons (see our existence proof). Our main results will be applicable to cases in which  $\bar{C}$  is arbitrarily large. The first weak inequality in the expression above is the budget constraint. The second weak inequality is a solvency constraint – agents cannot consume so much at time  $t$  that they leave themselves with negative residual wealth starting at  $t + 1$ .

Note that  $\mathbf{C}^t$  is a compact, convex set, which is a complete separable metric space as long as  $\mathbf{C}^t$  is non-empty. If  $\mathbf{C}^t$  is empty, then the agent declares bankruptcy, exits the economy, and distributes her claims among her creditors on a prorated basis. Given our assumptions, such bankruptcies can only occur if the date-contingent price of a good moves over time. We let  $C^0 = (c_1^0, \dots, c_T^0)$  denote the initial endowment at the outset of the game.

## 4.2 Preferences and Beliefs

A consumer type is defined by an endowment vector  $C_i^0 \in \mathbb{R}_{++}^T$ , a vector of continuous von-Neumann-Morgenstern utility functions  $(U_{i,1}, \dots, U_{i,T})$ , and a belief system.

We make the following assumptions on agents' preferences and beliefs. These assumptions apply to all types  $i$  and dates  $t$ .

**A1** At all dates  $t$  preferences are time separable:

$$U_{i,t}(c_1, c_2, c_3, \dots, c_T) = \sum_{s=1}^T u_{i,s}^t(c_s),$$

where  $u_{i,s}^t : \mathbb{R}_+ \rightarrow \mathbb{R}$  is continuous, monotonically increasing, and strictly concave for all  $s$ .

**A2** Self  $t$  believes that future selves ( $\tau > t$ ) have time-separable preferences:

$$\widehat{U}_{i,\tau}^t(c_1, c_2, c_3, \dots, c_T) = \sum_{s=1}^T \widehat{u}_{i,s}^t(c_s),$$

where  $\widehat{u}_{i,s}^t : \mathbb{R}_+ \rightarrow \mathbb{R}$  is continuous, monotonically increasing, and strictly concave. Self  $t$  believes that this specification is common knowledge among all future selves.<sup>9</sup>

**A3** Each agent has either *rational price expectations*,  $E_{i,t}p_{s'}^s = p_{s'}^s \quad \forall t \leq s \leq s'$ , or *passive price expectations*,  $E_{i,t}p_{s'}^s = p_{s'}^t \quad \forall t \leq s \leq s'$ . A positive mass of consumers have *rational price expectations*.

The standard general equilibrium framework assumes that agents *all* have rational price expectations. A3 relaxes that standard assumption by allowing (but not requiring) that some agents have passive price expectations. If an agent has passive price expectations, then she assumes that future prices are equal to prices in the *current* futures market – recall that  $p_{s'}^t$  is the price at date  $t$  of consumption at date  $s' \geq t$ . Hence, an agent with passive price expectations uses prices in the futures market to form her beliefs about prices at future dates. Such reliance may arise because of cognitive short-cuts or a recognition that markets aggregate information/insights that the agent may not have on her own. Our analysis admits “mixed” economies in which some types have rational price expectations and other types have passive price expectations.

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<sup>9</sup>All of our results carry through if we allow agents to have a generalized hierarchy of beliefs about future preferences and beliefs about future beliefs about preferences. For example, agent  $t$  could have beliefs about the preferences of agent  $t+2$  that differ from agent  $t$ 's beliefs about agent  $t+1$ 's beliefs about the preferences of agent  $t+2$ . For presentational simplicity, we do not introduce the notation that would be necessary to characterize hierarchies of arbitrary complexity, and instead only admit the “first-order” hierarchies in A2.

Assumptions A1 – A3 will be in force throughout the paper unless otherwise noted. We will occasionally refer to an agent as *non-standard*, if she is characterized by time inconsistent preferences, inaccurate beliefs about her own future behavior, or passive price expectations (or any combination of these properties).

We now discuss two examples of dynamically inconsistent preferences that illustrate our assumptions.

**Example 1** *Non-transitive preferences.* Consider a sequence  $\{U_t\}$  such that:

$$\begin{aligned} U_1(c_1, c_2, \dots, c_T) &= v(c_1, c_2, c_3, c_7, \dots, c_T) + u(c_4) + \frac{1}{2} [u(c_5) + u(c_6)], \\ U_2(c_1, c_2, \dots, c_T) &= v(c_1, c_2, c_3, c_7, \dots, c_T) + u(c_5) + \frac{1}{2} [u(c_4) + u(c_6)], \\ U_3(c_1, c_2, \dots, c_T) &= v(c_1, c_2, c_3, c_7, \dots, c_T) + u(c_6) + \frac{1}{2} [u(c_4) + u(c_5)]. \end{aligned}$$

Define  $\hat{U}_\tau^t(c_1, c_2, c_3, \dots, c_T) = U_\tau(c_1, c_2, c_3, \dots, c_T)$  for all  $\tau > t$ . If  $v$  and  $u$  are monotonic, strictly concave functions, and  $v$  is separable, then the sequences  $\{U_t\}$  and  $\{\hat{U}_\tau^t\}$  satisfy A1 and A2. Let  $\succ^t$  represent the binary preference relation implied by  $U_t$ . The consumer in this example will exhibit the following dynamic non-transitivity.

$$\begin{aligned} \{c_1, c_2, c_3, 1, 0, 0, c_7, \dots, c_T\} &\succ^1 \{c_1, c_2, c_3, 0, 1, 0, c_7, \dots, c_T\} \\ &\succ^2 \{c_1, c_2, c_3, 0, 0, 1, c_7, \dots, c_T\} \\ &\succ^3 \{c_1, c_2, c_3, 1, 0, 0, c_7, \dots, c_T\}. \end{aligned}$$

**Example 2** *Quasi-hyperbolic discounting (Phelps and Pollak (1968), Laibson (1997)).* Consider:

$$U_t(c_1, c_2, \dots, c_T) = u(c_t) + \beta \sum_{s=t+1}^T \delta^{s-t} u(c_s),$$

where  $0 < \delta \leq 1$ ,  $0 < \beta \leq 1$ , and for all  $\tau > t$ ,

$$\hat{U}_\tau^t(c_1, c_2, \dots, c_T) = u(c_\tau) + \hat{\beta} \sum_{s=\tau+1}^T \delta^{s-\tau} u(c_s),$$

where  $\beta \leq \hat{\beta} \leq 1$ . These specifications satisfy A1 and A2. In the standard terminology (see O'Donoghue and Rabin (2000)), when  $\beta = \hat{\beta}$  the agents are sophisticated. When  $\beta < \hat{\beta} < 1$  the agents are partially naive. When  $\beta < \hat{\beta} = 1$ , the agents are completely naive.

### 4.3 Dynamic General Equilibrium

We first describe an *individual* consumer's dynamic decision process assuming that current and future spot prices are known (in a fashion reminiscent of Harris (1985) and Luttmer and Mariotti (2006)). We then embed our consumers in a general equilibrium framework that endogenizes prices. Finally, we present an equilibrium existence theorem that is closely based on the existence theorem of Luttmer and Mariotti. We also characterize prices in equilibrium. We show that futures prices are accurate when a positive measure of agents have rational expectations as prescribed by A3.

#### 4.3.1 The Individual's Maximization Problem: An Intrapersonal Game

In this subsection we present a brief analysis of an individual consumer's equilibrium behavior. For notational simplicity, we suppress the type index throughout.

Denote by  $H_0$  the set of potential initial endowments for our consumer.

The set of actions available to the date- $t$  consumer at each date  $t = 1, \dots, T$  is a set of potential claim vectors, denoted by  $\mathbf{C}^t$ , determined via her budget constraint (see Section 4.2). The set  $\mathbf{C}^t$  is a non-empty complete separable metric space.

A closed subset  $H_t \subset H_0 \times \prod_{s=1}^t \mathbf{C}^s$  encompasses the set of possible histories up to and including date  $t$ . Following any history in  $H_{t-1}$ , the set of actions available to the date- $t$  consumer is given by a correspondence  $A_t : H_{t-1} \rightarrow \mathbf{C}^t$  that is continuous and has non-empty and compact values.

The set of possible histories are defined in a recursive fashion by  $H_t = \text{graph } A_t$  for all  $t = 1, \dots, T$ .

For any history  $h_{t-1} \in H_{t-1}$ , let  $\Gamma_t(h_{t-1})$  be the set of possible continuation histories following history  $h_{t-1}$ . It follows that  $\Gamma_t : H_{t-1} \rightarrow \prod_{s=1}^t \mathbf{C}^s$  is a continuous correspondence with non-empty and compact values.

We will assume that the mixed strategy chosen by the date- $t$  consumer, and not only the outcome of such a mixed strategy, can be observed by her successors. This assumption is not necessary for any of our qualitative results, but makes the exposition easier. This assumption is common in the literature on intrapersonal games (see, e.g., Harris (1985)). In order to consider observable mixed actions, we define extended histories as follows. For any  $t = 1, \dots, T$ , the set of

date- $t$  extended histories  $\tilde{H}_t$  is a closed set given by:

$$\tilde{H}_t = \left\{ (h_t, \sigma_t) \mid h_t \in H_t, \sigma_t \in \prod_{s=1}^t \Delta(A_s(\text{proj}_{H_{s-1}} h_t)) \right\}.$$

That is, a date- $t$  extended history is comprised of a history  $h_t$  as well as a sequence  $\sigma_t$  of sequentially feasible mixed strategies up to and including date  $t$ . For completeness, we set  $\tilde{H}_0 = H_0$  and interpret  $\sigma_0$  as an empty symbol. We can now define strategies.

**Definition 1** *For any  $t = 1, \dots, T$ , a strategy for the date- $t$  consumer is a Borel measurable function  $\gamma_t : \tilde{H}_{t-1} \rightarrow \Delta(\mathbf{C}^t)$  that satisfies*

$$\text{supp} \gamma_t(\cdot \mid \tilde{h}_{t-1}) \subset A_t(\text{proj}_{H_{t-1}} \tilde{h}_{t-1})$$

for all  $\tilde{h}_{t-1} \in \tilde{H}_{t-1}$ .

Equilibrium choices of individuals are time-dated (and possibly mixed) strategies that are optimal at each node of the game.

**Definition 2 (Intrapersonal Equilibrium)** *Given a sequence of price vectors  $\{P^t\}_{t=1}^T$ , an intrapersonal equilibrium is a strategy combination  $\gamma$  such that for all  $t = 1, \dots, T$ , and any history  $\tilde{h}_{t-1} \in \tilde{H}_{t-1}$ , the date- $t$  consumer cannot strictly increase her perceived date- $t$  utility in the subgame  $\tilde{h}_{t-1}$  by using a strategy other than  $\gamma_t$ .*

Intrapersonal equilibria are subgame perfect: strategies are perceived to be optimal at *each* node of the game, including nodes that are not reached with positive probability in equilibrium. In an intrapersonal equilibrium, expectations are based on *perceived* utilities, reflecting possible mistakes in beliefs about future utility functions (cf assumption A2) and mistakes in beliefs about future prices (cf assumption A3).

Harris (1985) ensures the existence of an intrapersonal equilibrium in our setting.

### 4.3.2 Competitive Equilibrium

We use the following terminology:



**Definition 3** For all  $(x_1, x_2, \dots, x_t), (y_1, y_2, \dots, y_t) \in R^t$ , we say  $(x_1, x_2, \dots, x_t)$  weakly dominates  $(y_1, y_2, \dots, y_t)$  if  $x_i \geq y_i$  for all  $i$ . When a vector  $x$  weakly dominates a vector  $y$  we write  $x \geq y$ . We say  $(x_1, x_2, \dots, x_t)$  strongly dominates  $(y_1, y_2, \dots, y_t)$  if  $x_i \geq y_i$  for all  $i$ , with at least one strict inequality. When a vector  $x$  strongly dominates a vector  $y$  we write  $x > y$ .

Our exchange economy is said to be in equilibrium if all consumers are following equilibrium strategies, and all markets clear. Formally,

**Definition 4 (Dynamic General Equilibrium)** A dynamic general equilibrium of the economy is a set of sequences of claim vectors  $\left\{ \{C^t(i)\}_{t=1}^T \right\}_{i \in I}$  and a sequence of price vectors  $\{P^t\}_{t=1}^T$ , such that

(i) All agents choose (intrapersonal) equilibrium strategies at all dates.

(ii) Markets clear at all dates: for all  $0 < t \leq T$

$$\int_i C^t(i) \leq \int_i C^{t-1}(i).$$

Our first theorem implies that our environment possesses a dynamic general equilibrium.

**Theorem 1** There exists a competitive equilibrium in which consumers of the same type follow the same strategies.<sup>10</sup>

The proof follows the lines of the existence proof presented by Luttmer and Mariotti (2006). Our setting differs because we allow for free disposal, passive price expectations, and perceived preferences of future selves that do not match actual preferences of future selves. Since free disposal does not alter the topological attributes of the action sets, this weakening in itself does not complicate the argument. Passive price expectations and inaccurate perceived future preferences enter the calculations needed to compute the backward induction process consumers use to choose their actions at each stage. Our assumption A2 assures that demands remain well-behaved.

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<sup>10</sup>This theorem relies on the assumption that agents observe their own past mixed actions. Dropping this observability assumption would not sabotage existence, but would potentially eliminate symmetric equilibria.

For the sake of completeness, we include a detailed description of the appropriately modified proof in Appendix A. We show that the intrapersonal equilibrium paths of this game are well-behaved. Specifically, these paths take non-empty convex and compact values and have closed and bounded graphs (upper hemicontinuous). These attributes carry, in turn, to the excess demand function, which is the crucial object for the analysis of equilibrium existence here. In fact, Debreu (1982) drew a connection between existence and the limit values well-behaved excess demand functions take at the boundary of the potential price set. Namely, one needs to show that excess demand explodes as one or more prices reach the boundary. Our assumptions A1 and A2 ensure this is indeed the case.

#### 4.4 Equilibrium Dutch Booking

The presence of a Dutch Book is checked by comparing successive profiles of consumption claims. In rough terms, the economy admits *money losing Dutch Books* if there exists a type  $i$  that executes a sequence of equilibrium trades that leaves her with a *strictly dominated* profile of claims. Our assumptions on the agent's utility functions imply such domination will indeed leave the agent weakly worse off at any date, and strictly worse off in at least one period (compared to the autarkic consumption path).

**Definition 5** *An economy exhibits money losing Dutch Books if there exists an agent type  $i$  and a sequence of equilibrium vectors  $\{C^t(i)\}_{t=0}^T$  such that*

$$C^t(i) > C^{t+s}(i),$$

for some  $s > 0$ .

Analogously, the economy admits *money making Dutch Books* if there exists a type  $i$  that executes a sequence of equilibrium trades that leaves her with a *strictly dominant* profile of claims.

**Definition 6** *An economy exhibits money making Dutch Books if there exists an agent type  $i$  and a sequence of equilibrium vectors  $\{C^t(i)\}_{t=0}^T$  such that*

$$C^t(i) < C^{t+s}(i),$$

for some  $s > 0$ .

We will show that neither type of Dutch Book is possible in our environment.

Another natural definition for Dutch Books would be based on agents' wealth. Namely, a money losing Dutch Book could be defined as a situation where a sequence of trades leaves the agent with less consumed and residual wealth than the amount of wealth with which she started. A money making Dutch Book would be defined analogously. Our definition is technically more restrictive. We choose Definitions 5 and 6 for two reasons. First, given our assumptions on preferences, lower wealth may not strictly reduce equilibrium welfare (see examples below). In contrast, a dominated sequence of claims always leaves the player strictly worse off relative to autarky. Second, choosing a wealth-based definition does not alter the impossibility results as will be seen in Section 5.

## 5 Impossibility of Dutch Books

Our impossibility theorem, Theorem 2, shows that as long as there is a positive measure of agents with rational price expectations, neither money losing Dutch Books nor money making Dutch Books can be features of the dynamic general equilibrium.

Our proof follows two simple Lemmas (whose full proofs appear in Appendix B). We start by showing that equilibrium futures prices must be positive and equal to their associated future spot prices. The proof is based on a standard no-arbitrage argument. We then show that Walras' law holds for all agents at each point in time. The combination of these claims leads to our impossibility theorem.

Indeed, Assumption A2 guarantees that any sufficiently large increase in future wealth will increase perceived current utility. Thus, if prices are not rational these agents can arbitrage the market, predicting that upon a sufficient increase of future wealth, future behavior will assure an increase in all future consumption levels. Formally,

**Lemma 1** *For sufficiently large  $\bar{C}$ , all equilibrium futures prices satisfy rational expectations and are strictly positive.*

Our second Lemma essentially illustrates the fact that Walras' law holds in the economy. That is, agents do not freely dispose of wealth in the economy. As we show in the robustness section below, the crucial assumption for this result is the separability of preferences. All agents would strictly prefer to consume more in the present – increasing current instantaneous utility and not affecting future utility – rather than throwing away wealth. Specifically, we have:

**Lemma 2 (Walras' Law)** *For sufficiently large  $\bar{C}$ , equilibrium prices and allocations satisfy  $P^T \cdot C^s = P^T \cdot C^t$  for all agents and all  $s, t = 1, \dots, T$ .*

Our main result follows. The existence of any Dutch Book implies that wealth is gained or lost at some point in the sequence of trades. But, this violates Walras' Law. Hence, as long as the cap on individual sales  $\bar{C}$  is large, the exchange economy is free of Dutch Books.

**Theorem 2 (Impossibility of Dutch Books)** *For sufficiently large  $\bar{C} > 0$ , there is no money losing or money making Dutch Book.*

**Proof :** Assume that a money losing Dutch Book did exist. Then, for some agent (suppressing her index),  $C^s > C^t$ , where  $s < t$ . Consider any  $T$ -length vector,  $x \gg 0$ . It follows that  $x \cdot C^s > x \cdot C^t$ . However, for sufficiently large  $\bar{C}$ , Lemma 1 implies that  $P^T \gg 0$  while Lemma 2 implies that  $P^T \cdot C^s = P^T \cdot C^t$ . Hence, we have a contradiction.

Similarly, assume that a money making Dutch Book existed. Then, for some agent (suppressing her index),  $C^s < C^t$ , where  $s < t$ . Consider any  $T$ -length vector,  $x \gg 0$ . It follows that  $x \cdot C^s < x \cdot C^t$ . As before, for sufficiently large  $\bar{C}$ , the Lemmas above imply that  $P^T \cdot C^s = P^T \cdot C^t$  while  $P^T \gg 0$ , which leads to a contradiction. ■

The theorem implies that in general equilibrium, non-standard agents do not engender Dutch Books.

We next pursue two separate directions. In Section 6 we ask whether market mechanisms restrain prices and demands in ways that would enable an outside observer to *detect* agents with time inconsistent preferences or inaccurate beliefs about future behavior. In Section 7 we provide robustness analysis for our results.

## 6 Identification of Non-standard Preferences

Our impossibility theorem establishes that consumers in competitive markets will not be Dutch Booked if they have preferences/beliefs that satisfy assumptions A1–A3. These assumptions admit dynamically inconsistent preferences and inaccurate beliefs about future preferences. We now ask whether the existence of such consumers would affect empirically measurable market transactions. Specifically, we identify price and quantity profiles that are consistent with the standard model — i.e., dynamically consistent preferences and rational expectations — and those that are not.

We start by showing that essentially any quantities and (strictly positive) prices can be explained with a general equilibrium model of a heterogeneous population comprised of dynamically consistent agents with preferences satisfying assumption A1 and rational expectations.

**Theorem 3** *For any  $P \gg 0$ , and demand correspondences  $\{c_t(i)\}_{i=1}^I$ , there are time-consistent utilities  $\{U_{it}\}_{i=1}^I$  satisfying A1 such that there exists a general equilibrium of the economy, comprised of agents with utilities  $\{U_{it}\}_{i=1}^I$  and rational price expectations, generating the observed prices and demands.*

**Proof:** Let  $P \gg 0$  and assume  $\{c_t(i)\}_{i=1}^I$  is the consumption profile observed in the economy. For each  $i = 1, \dots, I$ , consider the following utility indices:

$$u_{i,t}(c) = -\frac{p_t}{2}(c - c_t(i))^2, \quad t = 1, \dots, T.$$

Then  $U_{i,t}(c_1, \dots, c_T) = \sum_{s=1}^T u_{i,s}(c_s)$ , for all  $t$ , satisfies assumption A1 (with strict concavity of  $u_{i,t}(c)$  for all  $i, t$ ). Furthermore, a deviation constituting  $\Delta > 0$  less consumption units at time  $s$  for the benefit of an additional  $\frac{p_s}{p_t} \Delta$  units at time  $t$  leads to a loss of  $p_s \Delta$  at time  $s$  and a gain of  $p_t \frac{p_s}{p_t} \Delta = p_s \Delta$  at time  $t$ . In particular,  $(P, \{c_t(i)\}_{i=1}^I)$  would indeed constitute a dynamic general equilibrium of the economy with time consistent consumers identified by preferences  $\{U_{i,t}(c_1, \dots, c_T)\}_{i,t}$  and rational price expectations.<sup>11</sup> ■

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<sup>11</sup>In fact, we could take  $u_{i,t}(c) = p_t c$  for all  $i$  and  $t$  and still arrive at the result, but  $u_{i,t}$  would be only weakly concave.

Thus, generically, non-standard agents cannot be identified by observing only one arbitrary sequence of quantities and strictly positive prices. We note that observations of the entire dynamics of the game, i.e., the timed profiles of claims  $\{C^t(i)\}_{i=1}^I$  rather than the stream of “spot” consumption, does not qualitatively change the result, as long as a dynamic Walras law holds. That is, when observed prices are positive, and claims satisfy

$$\sum_{s=t}^T p_s C_s^{t-1}(i) = \sum_{s=t}^T p_s C_s^t(t)$$

for all  $t$  and  $i$ , the results of Theorem 3 carry through.

While the identification analysis thus far makes the point that any data set can be generated with a model using time consistent preferences, it has little to say about the consistency of these induced models across data sets. For example, a model that matches the data for the observed behavior in the economy in a setting in which agents have one set of endowments may not provide a good fit for the data generated in the same economy endowed with a different profile of endowments. Put another way, restricting the set of acceptable models to ones that explain behavior under one set of fundamentals (i.e., endowments) provides too many degrees of freedom and would generate a class of models, some of which will have very little predictive power in environments with different fundamentals.

In the remainder of the section, we put more restrictions on observed behavior. We now look *across*  $n$  different environments that are characterized by different endowments. As in static demand theory, a crucial consistency requirement is that if one profile of consumption  $C_1$  is chosen in one environment when another profile  $C_2$  is affordable (i.e., it is revealed preferred), then in any environment in which  $C_2$  is chosen, it should be the case that  $C_1$  is not affordable. This is the Weak Axiom of Revealed Preferences. In static, multiple good environments, Afriat (1967) proved that a stronger condition is necessary and sufficient to explain a finite data set with a locally non-satiated concave utility model.<sup>12</sup> Viewing each timed good in our setup as a different good, we can apply Afriat’s Strong Axiom of Revealed Preference as follows. Assume that an economist has data from  $n$  different economies, including both price and consumption sequences:  $\{P_j, \{C_j(i)\}_{i=1}^I\}_{j=1}^n$ .

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<sup>12</sup>For two new proofs of Afriat’s Theorem see Fostel, Scarf, and Todd (2004).

**Definition 7 (Dynamic Axiom of Revealed Preferences)** *The data set  $\{P_j, \{C_j(i)\}_{i=1}^I\}_{j=1}^n$  satisfies the Dynamic Axiom of Revealed Preferences (DARP) if for all  $i$ , for any  $\{j_1, \dots, j_k\} \subseteq \{1, \dots, n\}$  such that*

$$P_{j_l} \cdot C_{j_{l+1}}(i) \leq P_{j_l} \cdot C_{j_l}(i), \quad l = 1, \dots, k - 1,$$

*then  $P_{j_k} \cdot C_{j_1}(i) \geq P_{j_k} \cdot C_{j_k}(i)$ .*

The Dynamic Axiom of Revealed Preferences, henceforth DARP, is certainly a necessary condition for the existence of a time-consistent model that generates the observable data.

Thinking about consumption in each period as a different good, the existence of a well-behaved utility function of the form  $U(c_1, \dots, c_T)$  that generates the observed data guarantees a time consistent well-behaved model that generates the data. Indeed, define the time  $t$  utility by  $U_t(c_1, \dots, c_T) = U(c_1, \dots, c_T)$  for all  $t$ . We can thus use Afriat's theorem to provide necessary and sufficient conditions for the existence of a time consistent, piecewise linear, strictly monotonic, and concave model that generates the data.

**Theorem 4 (Afriat's Theorem)** *The observed data set  $\{P_j, \{C_j(i)\}_{i=1}^I\}_{j=1}^n$  satisfies DARP if and only if there exist a collection of  $I$  time consistent, piecewise linear, strictly monotonic, and concave utilities that generate the observed data.*

Note that while Theorem 4 provides necessary and sufficient conditions for a time consistent model, it does not provide conditions for the time separability assumed in A1 and A2.<sup>13</sup>

## STATIONARY EXPONENTIAL PREFERENCES

The first part of this section analyzed preferences that are time-separable, monotonic, strictly concave, and dynamically consistent. We now consider a much more restrictive class of dynamic preferences.

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<sup>13</sup>When observing only aggregate consumption data, but all individual endowments, Kubler (2003) illustrated that a slight relaxation of time separability yields practically no restrictions on observables.

**Definition 8 (Stationary Exponential Preferences)** *A stationary, time-separable exponential consumer is a time consistent consumer with preferences of the form:*

$$U_t(c_1, c_2, c_3, \dots, c_T) = \sum_{s=1}^T \delta^s u(c_s),$$

for all  $t$ , where  $u : \mathbb{R}_+ \rightarrow \mathbb{R}$  is continuous, monotonically increasing, and concave. Furthermore,  $0 < \delta \leq 1$ .

Stationary exponential preferences have a time-invariant felicity function and discount felicities exponentially. As a consequence of these assumptions, stationary exponential preferences are dynamically consistent. Stationary exponential preferences imply that demand tracks prices in the following sense.

**Definition 9 (Discounted Monotonic Demand)** *For any price vector  $P$ , we say the profile  $\{c_t(i)\}_{i=1}^I$  exhibits discounted monotonic demand if for all  $i$ , there exists  $\delta_i \in (0, 1]$ , such that for all  $s, t = 1, \dots, T$ ,  $\frac{p_t}{p_s} \delta_i^{t-s} \geq 1 \Leftrightarrow c_s(i) \geq c_t(i)$ .*

Clearly, a dynamic equilibrium of an economy comprised of consumers with stationary exponential preferences with rational price expectations will satisfy discounted monotonic demand. Moreover, if an economy satisfies discounted monotonic demand, then there exist stationary exponential preferences that would generate those demand profiles:

**Theorem 5** *Demand profile  $\{c_t(i)\}_{i=1}^I$  exhibits discounted monotonic demand if and only if there exists a set of consumers with stationary exponential preferences that would reproduce the economy's equilibrium.*

The sufficiency part of the theorem's proof is constructive, and appears in Appendix B. We use parameters  $\{\delta_i\}$  that correspond to the discounted monotonic demand condition and construct a piecewise linear utility function that assures that forgoing an amount  $\Delta > 0$  in any period  $s$  generates a loss in utility terms that is identical to the potential utility gain of shifting  $\Delta$  to any other period's consumption. Namely, forgoing  $\Delta > 0$  in any period  $s$  translates into a loss of



$p_s \Delta$  in utility terms.  $\Delta$  can then be transformed into  $\frac{p_s}{p_t} \Delta$  wealth units in any other period  $t$ , which translates into  $p_t \frac{p_s}{p_t} \Delta = p_s \Delta$  in utility terms. The observed demand profile is then part of an equilibrium.

## 7 Robustness

We have made several assumptions that limit the scope of our analysis. In the current section we discuss those assumptions and explain which of them can be relaxed without affecting our results. Our discussion covers separability of preferences, transaction costs, and risk.

### 7.1 Separability of preferences

Our analysis relies on the assumption that preferences (actual and perceived) are separable over time, as assured by *A1* and *A2*. In this subsection, we illustrate why our impossibility result cannot be extended generically to economies populated by agents with non-separable preferences.

Consider an economy comprised of agents with the following dynamically inconsistent preferences<sup>14</sup>

$$\begin{aligned} U_1 &= c_3 \\ U_2 &= \ln(c_1 + \alpha c_2 + c_3) + c_2 \\ U_3 &= c_3 \end{aligned}$$

where  $\alpha \in (0, 1)$ . As we will show below, self 1 and self 2 generally disagree about how much of the endowment to spend on  $c_2$ . But this disagreement vanishes when self 2 has sufficiently *little* wealth. At low levels of wealth, self 2 wants to spend all wealth on  $c_3$ . When wealth rises above a key threshold,  $c_3$  becomes an inferior good for self 2. Consequently, there is an incentive for self 1 to *partially impoverish* self 2, thereby leading self 2 to cut consumption of  $c_2$  and implicitly raise consumption of  $c_3$ .

At stage 0, each agent in the economy is endowed with a vector of claims  $(c_1^0, c_2^0, c_3^0)$ .

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<sup>14</sup>These analytically tractable preferences do not satisfy our regularity assumptions — e.g., strict concavity — but the example would go through if we perturbed the preferences so that they did satisfy all of our regularity conditions.

We now show that there exists an equilibrium that exhibits money losing Dutch Books, and which has  $p_s^t = 1$  for all  $s, t$ . Assume that all spot and future prices are equal to 1 and denote each agent's wealth endowment at date zero by  $W$ :

$$W \equiv \sum_{t=1}^3 c_t^0.$$

To solve for the interpersonal equilibrium choices, consider the decision of self 2. With wealth  $W'$ , self 2 will choose

$$c_2 = \begin{cases} 0 & \text{if } 0 \leq W' < 1 - \alpha - c_1 \\ \frac{1}{1-\alpha} (W' - 1 + \alpha + c_1) & \text{if } 1 - \alpha - c_1 \leq W' < \frac{1}{\alpha} (1 - \alpha - c_1) \\ W' & \text{if } \frac{1}{\alpha} (1 - \alpha - c_1) \leq W' \end{cases} .$$

Using the policy function  $c_2(W')$ , we can solve for  $c_3(W') = W' - c_2(W')$ .

$$c_3 = \begin{cases} W' & \text{if } 0 \leq W' < 1 - \alpha - c_1 \\ \frac{1}{1-\alpha} (-\alpha W' + 1 - \alpha - c_1) & \text{if } 1 - \alpha - c_1 \leq W' < \frac{1}{\alpha} (1 - \alpha - c_1) \\ 0 & \text{if } \frac{1}{\alpha} (1 - \alpha - c_1) \leq W' \end{cases} .$$

Note that  $c_3$  is falling in  $W'$  and falling in  $c_1$ . From the perspective of self 1, the optimal policy would be to set  $c_1 = 0$  and  $W' = 1 - \alpha$ . So if  $W > 1 - \alpha$ , the equilibrium path of the game will be

$$\begin{aligned} c_1 &= 0 \\ c_2 &= 0 \\ c_3 &= 1 - \alpha. \end{aligned}$$

Note that the market clearing condition holds as long as  $W \geq 1 - \alpha$ , so that the above demand profile is indeed part of an equilibrium for all  $W \geq 1 - \alpha$ . Furthermore, if  $W > 1 - \alpha$  self 1 of each agent engages in free disposal. Self 1 would rather freely dispose of her wealth, than pass it along to self 2 or spend it on  $c_1$ . In this sense, self 1 can be Dutch Booked. She would rather give wealth away, than sell it (regardless of how high the price).

This counterexample is very fragile. We now break the example by introducing a second good —  $\hat{c}_1$  — in period 1. Preferences are now

$$\begin{aligned} U_1 &= \hat{c}_1 + c_3 \\ U_2 &= \ln(c_1 + \alpha c_2 + c_3) + c_2 \\ U_3 &= c_3. \end{aligned}$$

In this case, the Dutch Book vanishes, the period 1 incarnation of each agent in the economy will take whatever wealth she would have previously discarded, and instead use that wealth to consume  $\widehat{c}_1$ .

These two examples demonstrate that our impossibility result does *not* depend on complete time separability of preferences. Rather our impossibility result is implied by the existence of at least *one* good that is time separable.

## 7.2 Transaction costs

Our analysis assumes that exchange takes place without any transaction costs. If we introduce such transaction costs our results will change, but *only* if agents do not have accurate beliefs about their own future preferences.

For example, consider an economy comprised of agents of two types. Each type constitutes half of the population. Type  $i = 1$  agents share the following preferences:<sup>15</sup>

$$U_{1,1} = c_3 + \ln c_2$$

$$U_{1,2} = \ln c_3$$

$$U_{1,3} = \ln c_3$$

Type 1 agents are endowed with consumption claims  $(c_1, c_2, c_3) = (0, 0, 1)$ . In period  $t = 1$ , agents of type 1 want to shift at least part of their consumption from period  $t = 3$  to period  $t = 2$  (since  $\lim_{c \rightarrow 0} \frac{d \ln c}{dc} = \infty$ ). But this preference is only transitory (vanishing in period  $t = 2$ ).

Type  $i = 2$  agents share the following preferences:

$$U_{2,1} = c_2 + \ln c_3$$

$$U_{2,2} = \ln c_2$$

$$U_{2,3} = \ln c_2$$

Type 2 agents are endowed with consumption claims  $(c_1, c_2, c_3) = (0, 1, 0)$ . In period  $t = 1$ , agents

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<sup>15</sup>Again, the analytically tractable utility specification could be perturbed to satisfy our regularity assumptions.

of type 2 want to shift at least part of their consumption from period  $t = 2$  to period  $t = 3$ . But this preference is also only transitory (vanishing in period  $t = 2$ ).

Assume all agents are naive, so they do not anticipate their dynamically inconsistent preferences. Hence,  $\widehat{U}_{i,\tau}^t = U_{i,t}$  for all  $i, t, \tau$ .<sup>16</sup>

Assume that agents face proportional transaction costs. At each period, any claims of market value  $w$  can be traded for claims of value  $(1 - \beta)w$ , where  $\beta \in (0, 1)$ . For the purposes of this example, we assume that this transaction cost is real: a portion  $\beta$  of the goods and claims being traded is destroyed during the transfer.

It is straightforward to see that all equilibria imply that  $p_2^t = p_3^t$  for all  $t$  ( $p_1^t$  is arbitrary since agents never value  $c_1$ ). Furthermore, there exists an equilibrium in which agents of type 1 end up with a claim vector of  $(0, 0, (1 - \beta)^2) < (0, 0, 1)$  in periods 2 and 3, while agents of type 2 end up with a claim vector of  $(0, (1 - \beta)^2, 0) < (0, 1, 0)$ . In particular, there is a money losing Dutch Book in equilibrium.<sup>17</sup>

At the other extreme, if we assume all agents are perfectly forward looking, so that  $\widehat{U}_{i,\tau}^t = U_{i,\tau}$  for all  $i, t, \tau$ , there exists a unique equilibrium that involves no trade and no Dutch Book.

This example illustrates two points. First, if consumers are naive and transaction costs exist, then agents may get Dutch Booked — a “money losing Dutch Book.” Second, if consumers anticipate their future preferences accurately, no Dutch Book occurs. The following Proposition, proven in Appendix B, establishes the second point generally.

**Proposition (Impossibility - Transaction Costs)** *Consider an economy in which transaction costs are proportional at each period  $t$  and are defined by the fraction  $\beta_t \in (0, 1)$ ,  $t = 1, \dots, T$ . If all agents hold accurate beliefs about their own future preferences, so that  $\widehat{U}_{i,\tau}^t = U_{i,\tau}$  for all  $i, t, \tau$ , then there are no Dutch Books in equilibrium.*

In the absence of transaction costs, agents of any type were not losing wealth through equilibrium trades. It is important to note that in the presence of transaction costs, wealth may in fact be lost

<sup>16</sup>We maintain A3, so a positive mass of agents have rational *price* expectations for the economy.

<sup>17</sup>There are many equilibria in this economy. Only one has no Dutch Books. In fact, in this equilibrium no trade occurs at all.

even by agents with accurate beliefs about future behavior. Indeed, such consumers may engage in equilibrium trades leading them to lower overall wealth. However, markets put restrictions on the patterns by which wealth can be lost. In particular, wealth cannot be lost in a way that makes all incarnations of an agent worse off, thereby ruling out money losing Dutch Books.

### 7.3 Risk

Throughout the analysis we have only considered environments that are riskless. This assumption is made for convenience. Analogous arguments rule out Dutch Books in risky economies. In a risky economy a money making Dutch Book is a series of trades that improve one's claims in every state of nature.<sup>18</sup> Likewise, a money losing Dutch Book is a series of trades that reduce one's claims in every state of nature.

The only problem that arises in such a risky framework is the challenge of restricting the contract space so that agents are able to repay their equilibrium obligations.<sup>19</sup> To resolve this problem, we adopt the assumption that contracts can only be written if parties to the contract can repay in all states of nature. With this assumption, an equilibrium will exist and the impossibility theorems from the deterministic economy of the current paper extend one-for-one to the risky environment.

## 8 Conclusion

Competitive pressures protect non-standard agents – economic actors with dynamically inconsistent preferences, or inaccurate beliefs about their own future behavior – from being exploited by a Dutch Book. More generally, in equilibrium agents with non-standard preferences and beliefs will not give up wealth without engaging in consumption of equal market value. Non-standard consumers may still make bad choices (depending on the welfare function that is applied). They may consume their wealth too early or too late, or consume the wrong bundle of static goods. Nevertheless, whatever their equilibrium choices, they will not get Dutch Booked or tricked into losing wealth.

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<sup>18</sup>The improvement needs to be strict in one state of nature.

<sup>19</sup>For example, if an agent is certain that it will, say, rain tomorrow, she will try to bet more than all of her wealth on that state of nature. But, if she loses the bet, she cannot repay.

Many of the preferences that were considered to be vulnerable to Dutch Books are not in fact vulnerable once agents with those preferences are embedded in a competitive economic market.

However, the existence of non-standard consumers may nevertheless have identifiable effects on market outcomes. If the Dynamic Axiom of Revealed Preferences does not hold, then a time consistent model will not explain observed data.

## 9 Appendix A – Existence of Equilibria

For any initial  $h_0 \in H_0$ , a strategy profile  $\gamma = (\gamma_1, \dots, \gamma_T)$  generates a probability distribution over the set of potential histories in the subgame  $h_0$ . We term this probability distribution as the path induced by  $\gamma$  in the subgame  $h_0$ .

Recall that  $\Gamma_{t+1}(h_t)$  denotes the set of all possible continuation histories following history  $h_t$ . Assume now that  $\Theta_{t+1} : H_t \rightarrow \Delta(\Gamma_{t+1}(\cdot))$  is an upper hemicontinuous correspondence with non-empty convex and compact values that describes intrapersonal equilibrium continuation paths following any history in  $H_t$ .

We work backward in order to obtain continuation paths for histories in  $H_{t-1}$ . We first need to mesh continuation paths in  $\Theta_{t+1}(H_t)$  with mixed actions of the date- $t$  consumer. We start by specifying a sure superset of continuation paths following a history  $h_{t-1} \in H_{t-1}$ :

$$\mathcal{P}\Theta_{t+1}(h_{t-1}) = \{\mu : \mu = \nu \otimes \lambda, \nu \in \Delta(A_t(h_{t-1})), \lambda(\cdot | C^t) \in \Theta_{t+1}(h_{t-1}, C^t)\}.$$

This is the set of all probability distributions over  $\Gamma_t(h_{t-1})$  that are consistent with marginal distributions in  $\Delta(A_t(h_{t-1}))$  and conditionals that map the  $t$ -dated claims profile  $C^t$  into  $\Theta_{t+1}(h_{t-1}, C^t)$ .

As in Luttmer and Mariotti (2006), we have the following:

**Lemma 3**  $\mathcal{P}\Theta_{t+1} : H_{t-1} \rightarrow \Delta(\Gamma_t(\cdot))$  is an upper hemicontinuous correspondence with non-empty values that are convex and compact.

For any  $h_t \in H_t$ , the utility of the date- $t$  consumer from the worst possible continuation path in  $\Theta_{t+1}(h_t)$  is:

$$W_t(h_t) = \min \left\{ \int U_t(c_1, \dots, c_t, z) d\mu(z) \mid \mu \in \Theta_{t+1}(h_t) \right\}.$$

denote by  $\mathcal{A}W_t(h_t)$  the set of probabilities for which the minimum is attained. For any history  $h_{t-1} \in H_{t-1}$ , the date- $t$  consumer can guarantee an approximate utility level of:

$$V_t(h_{t-1}) = \sup \{W_t(h_{t-1}, C^t) \mid C^t \in A_t(h_{t-1})\}.$$

We can now define

$$\mathcal{G}_{\Theta_{t+1}}(h_{t-1}) = \left\{ \mu \in \mathcal{P}_{\Theta_{t+1}}(h_{t-1}) \mid \int U_t(c_1, \dots, c_{t-1}, z) d\mu(z) \geq V_t(h_{t-1}) \right\}.$$

Roughly speaking,  $\mathcal{G}_{\Theta_{t+1}}(h_{t-1})$  are all of the conceivable continuation paths that are individually rational, in the sense of achieving a utility level of at least  $V_t(h_{t-1})$ . The convexity of  $\mathcal{P}_{\Theta_{t+1}}(h_{t-1})$  implies that  $\mathcal{G}_{\Theta_{t+1}}(h_{t-1})$  is convex as well.

**Lemma 4**  *$W_t : H_t \rightarrow \mathbb{R}$  and  $V_t : H_{t-1} \rightarrow \mathbb{R}$  are lower semicontinuous. Moreover,  $\mathcal{G}_{\Theta_{t+1}} : H_{t-1} \rightarrow \Delta(\Gamma_t(\cdot))$  is an upper hemicontinuous correspondence with non-empty values that are convex and compact.*

The proof is standard and makes use of Lemma 3.

For any correspondence  $F$  defined on  $H_t$ , an *extended Borel measurable selection* from  $F$  is a Borel measurable function  $f$  defined on  $\tilde{H}_t$  is for which  $f(\tilde{h}_t) \in F(\text{proj}_{H_t} \tilde{h}_t)$  for all  $\tilde{h}_t \in \tilde{H}_t$ . Extended measurable selections exist if measurable selections exist.

**Lemma 5** *Suppose  $\theta_t$  is an extended Borel measurable selection from  $\mathcal{G}_{\Theta_{t+1}}$ . Then there exists a Borel measurable function  $\gamma_t : \tilde{H}_{t-1} \rightarrow \Delta(\mathbf{C}^t)$  and an extended Borel measurable selection  $\theta_{t+1}$  from  $\Theta_{t+1}$  such that:*

1.  $\gamma_t$  is optimal for the date- $t$  consumer given the continuation  $\theta_{t+1}$ ;
2.  $\theta_t(\cdot \mid \tilde{h}_{t-1}) = \int \theta_{t+1}(\cdot \mid \tilde{h}_{t-1}, C^t, \gamma_t(\cdot \mid \tilde{h}_{t-1})) d\gamma_t(C^t \mid \tilde{h}_{t-1})$ .

The proof follows that of Lemma 3 in Luttmer and Mariotti (2006) and is thereby omitted. We do note, however, that the proof uses the observability of past mixed actions in the construction of an equilibrium path. Indeed, the agent is not indifferent between actions in the support of her strategy at each date, since she knows choosing one of them may lead to a punishment at a later date if a mix was prescribed. As mentioned in the body of the paper, the observability assumption is, in fact, not necessary for existence, but makes the proofs far simpler, and allows us to concentrate on symmetric equilibria.



We now use backward induction to deduce the consumer's potential actions at each stage. At date  $T$ , for any  $h_{T-1}$  in  $H_{T-1}$ , define the maximizing actions by:

$$M_T(h_{T-1}) = \arg \max \left\{ \int U_T(c_1, \dots, c_{T-1}, c) d\mu(c) \mid \mu \in \Delta(A_T(h_{T-1})) \right\}.$$

Using the Maximum Theorem,  $M_T(h_{T-1})$  is an upper hemicontinuous correspondence with non-empty, convex, and compact values.

Similarly, for all  $t = 1, \dots, T - 1$ ,  $\hat{M}_T^t$  can be defined to be the maximizing actions at time  $T$  from the perspective of the date- $t$  consumer. Formally,

$$\hat{M}_T^t(h_{T-1}) = \arg \max \left\{ \int \hat{U}_T^t(c_1, \dots, c_{T-1}, c) d\mu(c) \mid \mu \in \Delta(A_T(h_{T-1})) \right\}.$$

$\hat{M}_T^t$  is characterized by the same topological attributes of  $M_T$ .

For all  $t$ , we can define recursively  $\hat{M}_s^t = \mathcal{G}\hat{M}_{s+1}^t$  for all  $s = t, \dots, T - 1$ .

Define  $M_1 = \mathcal{G}\hat{M}_2^1$ .

**Lemma 6** *The correspondence  $M_1 : H_0 \rightarrow \Delta(\Gamma_1(\cdot))$  is upper hemicontinuous with non-empty values that are convex and compact. For every  $h_0 \in H_0$ ,  $M_1(h_0)$  is the set of intrapersonal equilibrium paths given initial condition  $h_0$ .*

The Lemma is essentially a replication of Proposition 1 in Luttmer and Mariotti (2006) – the proof is thus omitted.

Under the above assumptions, there exists an intrapersonal equilibrium in the intrapersonal game corresponding to each type of consumer (see Harris (1985)). We denote by  $M_{i,1}(P) = M_1(P, C^0)$  the set of intrapersonal equilibrium paths for a consumer of type  $i$  in the economy with (rational) prices  $P$  and initial endowment of  $C^0$ . From Lemma 6,  $M_{i,1}$  is upper hemicontinuous with non-empty convex and compact values.

Aiming at illustrating the existence of a symmetric equilibrium, assume that consumers of type  $i$  all follow strategies that implement the same path  $\mu_i(\cdot \mid P) \in M_{i,1}(P)$ . The aggregate consumption claim vector of type  $i$  is then  $\int C d\mu_i(C \mid P)$ . Define  $\Psi_i(P) : M_{i,1}(P) \rightarrow \mathbb{R}^T$  by

$$\Psi_i(P)(\mu_i) = \int C d\mu_i(C \mid P) - C_i^0.$$

$\Psi_i(P)$  is a continuous mapping.

We define the *excess demand correspondence* as:

$$\zeta_i(P) = \text{Im } \Psi_i(P).$$

Upper hemicontinuity of  $M_{i,1}$  implies the upper hemicontinuity of  $\zeta_i$ .

The implied *aggregate excess demand correspondence* is:

$$\zeta(P) = \left\{ \sum_{i=1}^I z_i \mid z_i \in \zeta_i(P) \text{ for all } i \right\}.$$

Note that for each  $i$ ,  $\zeta_i(P)$  is convex-valued since  $M_{i,1}$  is convex-valued. In addition, it is bounded below by  $-C_i^0$ .

The strict monotonicity and separability of preferences combined with price rationality imply that  $P \cdot C = P \cdot C_i^0$  for all  $C$  in the support of some path  $\mu_i(\cdot \mid P) \in M_{i,1}(P)$  (the proof follows the lines of that of Lemma 2). Thus, for all  $z \in \zeta(P)$ ,

$$P \cdot Z = \sum_{i=1}^I \int (C - C_i^0) d\mu_i(C \mid P) = 0.$$

In particular, the aggregate excess demand  $\zeta$  satisfies Walras' law.

The proof of existence of a competitive equilibrium follows the lines of the proof in Luttmer in Mariotti (2006), which is in itself based on a theorem in Debreu (1982). In essence, we need to check that  $\zeta$  satisfies the following boundary condition: if a sequence  $\{P^n\}$  in  $\Delta_T$  converges to a price in  $\partial\Delta_T$ , then  $\inf_{e \in \zeta_i(P)} \|e\|$  goes to  $+\infty$ , where  $\|e\| = \sum_{t=1}^T e_t$ .

We start by showing that the expected utility of the date-1 consumer goes to  $+\infty$  uniformly across intrapersonal equilibria as  $\{P^n\}$  converges to  $P \in \partial\Delta_T$ .

**Lemma 7** *For any  $(U_{i,1}, \dots, U_{i,T})$  satisfying A1 and A2, if a sequence  $\{P^n\}$  in  $\Delta_T$  converges to  $P \in \partial\Delta_T$ , then the expected utility of the date-1 consumer goes to  $+\infty$  uniformly across intrapersonal equilibria as  $\{P^n\}$  converges to  $P \in \partial\Delta_T$ .*

**Proof :** We use induction on the length  $T$ . For  $T = 2$ , the date-1 consumer faces a standard decision problem with strictly increasing utility indices and the expected utility of the date-1 consumer indeed goes to  $+\infty$  as  $\{P^n\}$  approaches  $P \in \partial\Delta_T$ .

Suppose that the result holds for any game of length  $T$ . We now confirm the claim for games of length  $T + 1$ . Suppose first that  $\{P_1^n\}$  approaches zero. Then the date-1 consumer can guarantee a utility approaching  $+\infty$  in the limit by spending her entire wealth on current consumption, regardless of the behavior of her successors. This implies that the expected utility of the date-1 consumer must go to  $+\infty$  uniformly across equilibria, as desired. Formally, for all sequences  $\{\mu_i^n\}$  that satisfy  $\mu_i^n \in M_{i,1}(P^n)$ :

$$\lim_{n \rightarrow \infty} \int U_{i,1}(C) d\mu_i^n(C) = +\infty.$$

Suppose alternatively that  $\{P_1^n\}$  is bounded away from zero and some other price goes to zero. Assume, toward a contradiction, that there is a sequence of intrapersonal equilibria along which expected date-1 utility remains bounded. Note that the induction hypothesis implies that along this same sequence of intrapersonal equilibria the amount of nominal wealth left by date-1 consumer to the date-2 consumer must go to zero. Indeed, if the date-1 consumer were to leave a positive nominal wealth  $\varepsilon > 0$  to the date-2 consumer, then the induction hypothesis would imply that the utility of the date-2 consumer would go to  $+\infty$  uniformly across intrapersonal equilibria along the considered price sequence. More specifically,

$$\lim_{n \rightarrow \infty} \max_{t \in \{2, \dots, T+1\}} \int u_{i,t}^2(c_t) d\mu_{i,\varepsilon}^n(C) = +\infty$$

uniformly across equilibrium continuation paths  $\mu_{i,\varepsilon}^n \in \Delta(\Gamma_2(P^n, C_i^0, \frac{P^n C_i^0 - \varepsilon}{P_1^n}))$  that follow the date-1 consuming  $\frac{P^n C_i^0 - \varepsilon}{P_1^n}$  and leaving nominal wealth  $\varepsilon > 0$  to the date-2 consumer. But this implies that the date-1 consumer could guarantee for herself infinite utility in the limit, in contradiction. ■

Let  $\{P^n\}$  in  $\Delta_T$  converge to a price in  $\partial\Delta_T$ . Lemma 7 asserts that for all sequences  $\{\mu_i^n\}$  that satisfy  $\mu_i^n \in M_{i,1}(P^n)$ :

$$\lim_{n \rightarrow \infty} \int U_{i,1}(C) d\mu_i^n(C) = +\infty.$$

The concavity of  $u_{i,t}^1$  together with Jensen's inequality imply:

$$\int U_{i,1}(C) d\mu_i^n(C) = \int \left( \sum_{t=1}^{T+1} D_{i,t}^1 u_{i,t}^1(c_t) \right) d\mu_i^n(C) \leq \sum_{t=1}^{T+1} D_{i,t}^1 u_{i,t}^1 \left( \int c_t d\mu_i^n(C) \right),$$

which leads to:

$$\lim_{n \rightarrow \infty} \sum_{t=1}^{T+1} \int c_t d\mu_i^n(C) = +\infty.$$

In particular,  $\inf_{e \in \zeta_i(P)} \|e\|$  goes to  $+\infty$ .

We can now apply Debreu (1982, Theorem 8) to obtain our desired existence result, Theorem 1.

## 10 Appendix B – Proofs

**Proof of Lemma 1:** We start by showing that if in equilibrium for some  $1 \leq s \leq T$ ,  $p_t^s = p_t^{s'}$  for all  $t \geq s' \geq s$ , then  $p_t^{s'} > 0$  for all  $t \geq s' \geq s$  in that equilibrium. We use induction on the length of the game starting at period  $s$ ,  $T - s + 1$ . Consider  $T = s$ . Since utility is increasing, prices have to be strictly positive for A1 to hold. Suppose the claim holds for  $T - s + 1 = \tau - 1$  and consider a situation in which  $T - s + 1 = \tau$ . From the induction step,  $P^{s+1}, P^{s+2}, \dots, P^T \gg 0$ . From the rationality assumption, we only need to show that  $p_s^s > 0$ . The strict monotonicity and separability entailed by A1 as well as the perceived separability captured by A2, assure that this is indeed the case.

Now, suppose prices are not rational in some equilibrium. Let  $s$ ,  $1 \leq s \leq T$ , be the last period for which there exists  $t > s$  such that  $p_t^s \neq p_t^{s+1}$ . In particular, in the subgame starting at period  $s + 1$ , prices are rational and hence, using the above, are all strictly positive. Assume first that  $p_t^s < p_t^{s+1}$ . Any agent can engage in riskless arbitrage by purchasing  $c_t^s \leq \bar{C}$  units of the  $t$ -timed good in period  $s$  and selling them in period  $s + 1$ , thereby increasing time  $s + 1$  wealth by  $\Delta(c_t^s) \equiv (p_t^{s+1} - p_t^s)c_t^s$ . Denote by  $(\hat{c}_{s+k}^s(w_k), \dots, \hat{c}_T^s(w_k))$ ,  $k > 0$ , the expected consumption stream at time  $s$ , from time  $s + k$  and on, when the wealth at the beginning of period  $s + 1$  is  $w_k$ . We now show that under A2, for any  $w_{k,1} > w_{k,2} \geq 0$ ,  $(\hat{c}_{s+k}^s(w_{k,1}), \dots, \hat{c}_T^s(w_{k,1})) > (\hat{c}_{s+k}^s(w_{k,2}), \dots, \hat{c}_T^s(w_{k,2}))$ .

We use induction on the length of the remainder of the game,  $T - s$ .

For  $T - s = 1$ , the claim follows directly since utilities are strictly monotonic.

Assume the claim holds for  $T - s = \tau - 1$  and consider the case of  $T - s = \tau$ . From the induction hypothesis, for any  $w'_{2,1} > w'_{2,2} \geq 0$ ,

$$(\hat{c}_{s+2}^s(w'_{2,1}), \dots, \hat{c}_T^s(w'_{2,1})) > (\hat{c}_{s+2}^s(w'_{2,2}), \dots, \hat{c}_T^s(w'_{2,2})).$$

Let  $w_{1,1} > w_{1,2}$ . The optimality of  $(\hat{c}_{s+1}^s(w_{1,2}), \dots, \hat{c}_T^s(w_{1,2}))$  implies that for all  $x \in [-\hat{c}_{s+1}^s(w_{1,2})p_{s+1}, w_{1,2} - \hat{c}_{s+1}^s(w_{1,2})p_{s+1}]$ ,

$$\hat{U}_{s+1}^s(\hat{c}_{s+1}^s(w_{1,2}) + \frac{x}{p_{s+1}}, \hat{c}_{s+2}^s(w_{1,2} - x), \dots, \hat{c}_T^s(w_{1,2} - x)) \leq \hat{U}_{s+1}^s(\hat{c}_{s+1}^s(w_{1,2}), \dots, \hat{c}_T^s(w_{1,2})).$$

From the strict concavity implied by A2, it follows that  $\widehat{c}_{s+1}^s(w_{1,1}) < \widehat{c}_{s+1}^s(w_{1,2}) + \frac{w_{1,1}-w_{1,2}}{p_{s+1}}$  so that  $(\widehat{c}_{s+2}^s(w_{1,1}), \dots, \widehat{c}_T^s(w_{1,1})) > (\widehat{c}_{s+2}^s(w_{1,2}), \dots, \widehat{c}_T^s(w_{1,2}))$ . Similarly, strict concavity together with the induction hypothesis imply that  $\widehat{c}_{s+1}^s(w_{1,1}) < \widehat{c}_{s+1}^s(w_{1,2})$ .

From strict monotonicity, agents will therefore choose  $c_t^s = \bar{C}$ . In particular, for sufficiently large  $\bar{C}$ , markets will not clear.

A parallel argument holds for  $p_t^s > p_t^{s+1}$ .

Note that for any given endowment, for sufficiently large  $\bar{C}$ , *any* form of irrational expectations (corresponding to any two periods) will lead to a violation of market clearing. ■

**Proof of Lemma 2:** Using Lemma 1 it suffices to show that for all  $t$ ,  $P^t \cdot C^t = P^t \cdot C^{t-1}$ . Indeed, if for some  $t, t_1 \leq t \leq t_2$ ,  $P^t \cdot C^t < P^t \cdot C^{t-1}$  then the agent can deviate by purchasing  $\frac{P^t \cdot C^{t-1} - P^t \cdot C^t}{P_t^t}$  units of the good to be consumed at time  $t$ , thereby strictly increasing her time  $t$  utility and not affecting her future allocations, in contradiction to condition (i) for a general equilibrium allocation. ■

**Proof of Theorem 5:** Assume that  $\{c_t(i)\}_{i=1}^I$  exhibits discounted monotonic demand with parameters  $\{\delta_i\}$ . For each  $i = 1, \dots, I$ , let  $\pi_i : \{1, \dots, T\} \rightarrow \{1, \dots, T\}$  be a permutation such that  $c_{\pi(1)}(i) \leq c_{\pi(2)}(i) \leq \dots \leq c_{\pi(T)}(i)$ . Define:

$$w_i(c) = \begin{cases} \frac{p_{\pi_i(1)}}{\delta_i^{\pi_i(1)}} c & c \in [0, \frac{c_{\pi_i(1)}(i) + c_{\pi_i(2)}(i)}{2}] \\ \frac{p_{\pi_i(1)}}{\delta_i^{\pi_i(1)}} c + \sum_{s=1}^{t-1} \left( \frac{p_s}{\delta_i^s} - \frac{p_{s+1}}{\delta_i^{s+1}} \right) \frac{c_{\pi_i(s-1)}(i) + c_{\pi_i(s)}(i)}{2} & c \in \left[ \frac{c_{\pi_i(t-1)}(i) + c_{\pi_i(t)}(i)}{2}, \frac{c_{\pi_i(t)}(i) + c_{\pi_i(t+1)}(i)}{2} \right] \\ & \text{and } 2 \leq t \leq T \\ \frac{p_{\pi_i(T)}}{\delta_i^{\pi_i(T)}} c + \sum_{s=1}^{T-1} \left( \frac{p_s}{\delta_i^s} - \frac{p_{s+1}}{\delta_i^{s+1}} \right) \frac{c_{\pi_i(s-1)}(i) + c_{\pi_i(s)}(i)}{2} & c \in \left[ \frac{c_{\pi_i(t)}(i) + c_{\pi_i(t+1)}(i)}{2}, \infty \right) \end{cases}$$

The monotonicity of demand assures that  $U_{i,t}(c_1, \dots, c_T) = \sum_{s=1}^T \delta_i^s u_i(c_s)$  satisfies assumption A1. Furthermore, forgoing an amount  $\Delta > 0$  in period  $s$ , translating into  $p_s \Delta$  in utility terms, could be transformed into  $\frac{p_s}{p_t} \Delta$  wealth units in any other period  $t$ , which translates into  $p_t \frac{p_s}{p_t} \Delta = p_s \Delta$  in utils. In particular, agents  $i$  are using equilibrium strategies when consuming profiles  $\{c_t(i)\}_{i=1}^I$ .

The converse direction follows definitionally.<sup>20</sup> ■

<sup>20</sup>Note that an analogous construction could be used had we considered the case  $\delta_i > 1$ .

**Proof of Proposition (Impossibility - Transaction Costs):** Similar arguments to those used in Lemmas 1 and 2 assure that in any equilibrium, all prices are strictly positive. Assume that a money losing Dutch Book did exist. Then, in some equilibrium, for some agent (suppressing her index),  $C^s > C^t$ , where  $s < t$ . Given the equilibrium prices, if the agent were to start in period  $s + 1$  with a claim vector  $C^t$ , there would be an interpersonal equilibrium of the truncated game starting at period  $s + 1$  specifying no trade between periods  $s$  and  $t$ . Indeed, otherwise there would necessarily be a profitable deviation in the original game, in contradiction. Consider now the following deviation at period  $s$  (in the original economy): consume  $c_s + \frac{(1-\beta_s)P^s \cdot (C^s - C^t)}{p_s}$  in period  $s$  and assure the  $s + 1$ 'th incarnation receives current and future claims coinciding with  $C^t$ . That is, consider the deviation from  $C^s$  to  $\hat{C}^s$ , where  $\hat{C}^s$  is given by:

$$\hat{C}^s = (c_1^s, \dots, c_{s-1}^s, c_s + \frac{(1-\beta_s)P^s \cdot (C^s - C^t)}{p_s}, c_{s+1}^t, \dots, c_{t-1}^t, c_t, c_{t+1}^t, \dots, c_T^t).$$

$\hat{C}^s$  clearly provides self  $s$  with higher utility than does  $C^s$  and does not affect future consumption, contradicting the optimality of  $C^s$ .

Regarding money making Dutch Books, note that similar arguments to those used in Lemmas 1 and 2 would imply that for sufficiently large  $\bar{C}$ , for any agent (suppressing their index) and for all  $s < t$ ,  $P^T \cdot C^s \geq P^T \cdot C^t$  while  $P^T \gg 0$ . Money making Dutch Books would suggest that for some agent (suppressing her index),  $C^s < C^t$ , where  $s < t$ , which as before would lead to a contradiction. ■

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