Chiral Prethermalization in Supersonically Split Condensates

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We study the dynamics of phase relaxation between a pair of one-dimensional condensates created by a supersonic unzipping of a single condensate. We use the Lorentz invariance of the low energy sector of such systems to show that dephasing results in an unusual prethermal state, in which right- and left-moving excitations have different, Doppler-shifted temperatures. The chirality of these modes can be probed experimentally by measuring the interference fringe contrasts with the release point of the split condensates moving at another supersonic velocity. Further, an accelerated motion of the release point can be used to observe a spacelike analog of the Unruh effect. A concrete experimental realization of the quantum zipper for a BEC of trapped atoms on an atom chip is outlined.

Introduction.—Trapped gases of ultracold atoms today provide the most remarkable examples of nearly isolated quantum systems. While traditional condensed matter systems are typically difficult to isolate from external noise, these artificial systems have been successfully engineered to be sufficiently decoupled [1,2] from the environment, so that one can assume a unitary evolution of the system over long time scales. These developments, have, in turn, reinvigorated theoretical interest in the study of out-of-equilibrium dynamics of isolated quantum systems.

Amidst such interest, one-dimensional (1D) systems have garnered particular attention because the nonequilibrium behavior in these systems is enhanced [3] due to limited phase space available for scattering and equilibration. Further, in many cases, these systems exhibit an emergent Lorentz invariance—for example, while the constituent atoms of 1D gases obey only Galilean invariance, the collective modes, described by the Luttinger liquid (LL) theory, exhibit a richer Lorentz invariance. These symmetries make calculations feasible, and lead to many universal features in nonequilibrium dynamics, such as scaling laws for the growth of domains [4,5], light-cone spreading of correlations [6,7], and the relaxation of observables [8]. Such features have also been seen in experiments [6,7,9–12].

One of the basic tools for studying nonequilibrium dynamics is the “quantum quench”—a system is prepared in the ground state of a Hamiltonian \( H_0 \), and subsequently evolved with another Hamiltonian \( H \), for which the initial state is not an eigenstate. Typical scenarios for quenches include a sudden change in the confining potential of a 1D gas [13–15], the interaction between the constituent particles [16], or, in a more abstract case, the excitation gap in the system [17,18]. Importantly, most previous studies have focused on sudden (and spatially homogeneous) quenches, that is, the transition from \( H_0 \) to \( H \) happens over a time scale shorter than any other time scale in the problem. The study of such quenches, however, does not fully utilize the rich symmetry of the low energy physics of these systems.

The aim of this Letter is to describe how a new class of quantum quenches, wherein the Hamiltonian is continually perturbed or “quenched” along a relativistic, supersonic trajectory, can be analyzed by using this Lorentz symmetry to full effect. In particular, due to the supersonic character of the quench, and the Lorentz invariance of ground state correlations, we show that such quenches can be mapped to the usual sudden quench case via a Lorentz boost. To motivate the analysis of such a quench, we describe our theoretical problem in the context of recent experiments [19] studying the relaxation of the phase difference between the halves of a coherently split quasi-1D condensate. After the splitting, the phase difference evolves freely and can be described by a Luttinger Hamiltonian [20] (which has a linear dispersion, and an associated speed of sound \( c \)), with an initial state that appropriately represents the squeezing of the relative phase to zero (bar quantum fluctuations) at the time of the quench. We propose a modification of this sudden quench protocol to one where the splitting happens along a “knife edge” that travels through the condensate at a supersonic velocity \( v_s \) (see Fig. 1). Such a protocol may be realized by using an inhomogeneous set of rf fields that create a propagating splitting potential (see Supplemental Material [21]).

An interesting prediction of previous studies [20,22] describing the sudden quench was that the system eventually evolves to a state with thermal-looking correlations at a temperature \( T_0 = \mu/2 \), where \( \mu \) is the chemical potential of each half-condensate. Such predictions were confirmed by experiments [19]. In our case, we find that the system prethermalizes in an asymmetric fashion—left- and right-moving excitations prethermalize at different effective temperatures \( T_0/\eta \), and \( \eta T_0 \), respectively, where \( \eta \) is a relativistic Doppler shift; however, equal-time correlations...
are described by an average of these two temperatures, \( T = \gamma_s T_0 \). To bring out the chirality, we propose probing unequal-time correlations \([23]\) of the phase. In particular, releasing the condensate at a supersonic velocity \( v_s \) (see Fig. 1) gives us access to correlations of the phase field at space-time points obeying the relation \( t = x/v_r \). We find that such correlations (characterized by different \( \nu_r \)) exhibit the entire range of effective temperatures \( T_0/\eta_r < T < \eta_r T_0 \). We note that many of these interesting relativistic effects arise due to the fact that our supersonic splitting can be mapped to a sudden quench in a moving frame—thus, our protocol effectively creates a moving relativistic body with thermal-looking correlations (at a temperature \( T_0 \)) in its rest frame and can be used to probe various aspects of relativistic thermodynamics \([24–30]\) in a controlled experimental way. Finally, we discuss how a spatial analog of the Unruh effect \([31]\) can be realized in these experiments by releasing the split condensates in an accelerated fashion.

**Model.**—To describe the splitting process, we consider the condensate as a system of two intercoupled 1D LLs, whose mutual coupling is destroyed in the process of splitting. The Hamiltonian that describes the dynamics of the phase difference field \( \phi \) is

\[
H(t) = \int dx \left[ \frac{\rho}{4m} \left( \partial_x \phi \right)^2 + gn^2 + J \Theta(x - v_r t) \phi^2 \right].
\]

Here, \( n \) is the number fluctuations conjugate to the phase \( \phi \), \( 2\rho \) is the density of the unsplit condensate, \( g \) characterizes the strength of contact interactions, and \( v_r > c \) is the supersonic velocity at which the splitting perturbation travels. We choose \( J = gn^2 \) so that the initial correlations [in the ground state of \( H(-\infty) \)] match the correlations one expects at the time of splitting [Gaussian, with moment \( \langle n(x) n(x') \rangle = \rho/2\delta(x - x') \); see Bistritzer et al. \([22]\)]. Since \( v_s \) is supersonic, light-cone physics guarantees that correlations of \( H(-\infty) \) persist at every point until the time of splitting. Thus, \( H(t) \) captures the two essential features of our problem—(i) correlations at the time of splitting and (ii) LL dynamics after the splitting.

**Dynamics.**—To simplify calculations, we analyze the problem in a frame that is Lorentz-boosted from the lab frame by velocity \( u_s = (c/v_r)c < c \). Strictly speaking, for finite systems (such as in experiments) there is no Lorentz symmetry. However, the notion of a light-cone effect allows us to think of our finite system as formally infinite for the time over which the portion of the condensate we are interested in is unaffected by perturbations traveling from the edge (see Supplemental Material \([21]\)). Then, the Lorentz transformation \( L: \{x_i, t_i\} \rightarrow \{x'_i, t'_i\} \) with \( x' = \gamma_s(x - u, t), \ t' = \gamma_s(t - u x/c^2), \) and \( \gamma_s = \frac{1}{\sqrt{1 - (u/c)^2}} = \frac{1}{\sqrt{1 - (c/v_r)^2}} \), which achieves this boost, results in a convenient form of the splitting perturbation \( \Theta_s(x - v_r t) \rightarrow \Theta(x - v_r t) \), which appears to be spatially uniform and sudden in this frame \([32]\). The dynamics of the system are then governed by the following well-studied quench \([17]\):

\[
H'(t') = \int dx \left[ \frac{\rho}{4m} \phi'^2 + gn^2 + J \Theta(-t') \phi'^2 \right].
\]

where the field \( \phi' \) satisfies the relation \( \phi'(x'_i, t'_i) = \phi(x_i, t_i) \), and \( n' \) is conjugate to \( \phi' \).

In general, such a coordinate transformation may simplify the dynamics, but complicates the form of initial correlations. In our case, prior to any splitting, the system resides in the ground state of a Lorentz invariant system described by the Hamiltonian \( H(t < -\infty) \). Now, all vacuum correlations of a scalar field governed by a Lorentz invariant action are invariant under Lorentz transformations \([33]\). Equivalently, \( \langle \phi(x_1, t) \phi(x_2, t) \rangle = \langle \phi(L(x_1, t)) \phi(L(x_2, t)) \rangle = \langle \phi'(x_1, t) \phi'(x_2, t) \rangle \). This implies that our system is also in the ground state of \( H'(t' < 0) \) for \( t' < 0 \). At precisely \( t' = 0 \), which is the trajectory of the splitting perturbation in the boosted frame, the system is perturbed and subsequently evolves under \( H'(t' > 0) \). Formally, the problem in this boosted frame is identical to that of the sudden quench case considered previously.

**Distribution functions.**—To characterize the dynamics, we calculate the distribution function (DF) \( P(\alpha, t) \) of the spatially integrated phase contrast \( \alpha = \int_{-l/2}^{l/2} dx |\phi(x, t)|^2 \). Since we are also interested in unequal-time correlations, the phase \( \phi(x, t) \) in \( \alpha \) will generally correspond to times \( t \) related to the position \( x \) as \( t = t_0 + x/v_r \), with some \( |v_r| > c \). The DFs for equal-time correlations were measured in experiments by repeatedly recording the value of the integrated phase contrast over many experimental runs \([7,19,34,35]\).

Theoretically, the distributions characterizing the correlations of the field \( \phi(x', t'_i) \) at equal times \( t'_i \) were evaluated in \([20]\) in their analysis of the sudden splitting case. This approach can be directly generalized to evaluate unequal-time correlations as long as the points \( \{x'_i, t'_j\} \) (or equivalently \( \{x_i, t_j\} \)) are...
spacelike separated. Mathematically, this ensures that field operators at various points commute making calculations feasible. Physically, this is necessary, because experiments cannot record correlations of operators at spacelike separated points, without altering the system during the measurement process. The condition $|v_r| > c$ precisely ensures that $\{(x_i, t_i)\}$ are spacelike separated. Once correlations are calculated for the field $\phi'$, we simply reexpress the results in the relevant lab frame coordinates to arrive at the DFs $P(\alpha, t, t_0, v_r, v_s)$. Following the procedure of [20], we find (see Supplemental Material [21])

$$P = \prod_k \int \frac{d\theta_k}{2\pi} \frac{dr_k^2}{2} e^{-r_k^2/2} \left( \Delta - \int_{-\Delta/2}^{\Delta/2} dx \, e^{i\theta_k} \right)^2,$$

$\chi_d = \sum_{c=\pm 1} \int \frac{dk}{\sqrt{2\pi}} \sqrt{\frac{g}{c|k|}} A(k, e) r_k \sin [\eta_r k(x - ct) + \theta_k], \quad (3)$

where $\eta_r(v_s) = (1 + u_s/c)/(1 - u_s/c) = \sqrt{(1 + c/v_s)/(1 - c/v_s)} > 1$ is a relativistic Doppler shift, associated with $u_s$, and $A(k, e) = \text{sgn}(k)/2(\sqrt{2g\rho/c|k|} + \text{sgn}(k)e\sqrt{c/k}/|k|/2g)$. This result has a clear physical interpretation—the splitting process generates waves of momentum $k$ with a magnitude proportional to $r_k$ and a random phase $\theta_k$. Each configuration $\{r_k, \theta_k\}$ serves as an initial condition for which we can predict the exact evolution of the phase field $\phi(x, t)$, which shows up as the real function $\chi_d(x, t)$. The probability that the integrated phase contrast takes a given value $\alpha$ is found by performing an integral over the space of all initial $\{r_k, \theta_k\}$ with appropriate statistical weights. The factors of $\eta_r$ accompanying right-moving waves $\propto (x - ct)$ and $1/\eta_r$ accompanying left-moving waves $\propto (x + ct)$ simply indicate the relativistic blue and red shifting of the corresponding sets of waves. However, it is important to note that when we measure unequal-time correlations (characterized by finite $v_r$), we will set $t(t_0, x) = t_0 + x/v_r$, which modifies these dilation factors to $a_R = \eta_r(1 - c/v_r)$ and $a_L = (1 + c/v_r)/\eta_r$. These factors will figure prominently when we see how these DFs behave in the long time limit $t_0 \to \infty$.

**Prethermalization.**—The tendency of an integrable system to flow into a state with correlations describable by a generalized Gibbs ensemble is called “prethermalization.” To explore the process of prethermalization in our problem, we compare the DFs obtained for our dynamical system to that obtained for two independent, thermal condensates. The DF for two thermal condensates at a temperature $T$ can be obtained using a similar approach to the one used above [36]. The thermal result is simply a modification of results of Eq. (3) with $\chi_d$ being replaced by $\chi_T$—

$$\chi_T(x, r_k, \theta_k) = \int \frac{dk}{\sqrt{2\pi}} r_k \sqrt{f_T(k)} \sin (kx + \theta_k), \quad (4)$$

where

$$f_T(k) = \frac{g}{c|k|} \coth \left( \frac{c|k|}{2k_BT} \right) \approx \frac{g}{c} \frac{2k_BT}{c|k|}. \quad (5)$$

Here, we note that in the classical limit of $T$ being large, the amplitude of each of the waves, $f_T(k) \sim T/k^2$, in accordance with the equipartition theorem, which guarantees that each mode carries an energy $k_BT$ distributed equally in phase and density fluctuations. Notably, the $1/k^2$ scaling of amplitudes implies that DFs are determined primarily by low momentum fluctuations.

Now we show that DF resulting from $\chi_d$ for our dynamical problem reaches a steady state (at long times $t \sim 1/c$, $l$ being the integration length) resembling the thermal DF realized from $\chi_T$. First note that evaluating different moments $\langle a^m \rangle$ from $P(\alpha)$ corresponds to evaluating correlators of the form $\langle \prod_i \chi_d(x_i, t_i) \rangle$ and integrating over $x_i$'s. The averages here are taken over the measure $\Pi_k \int dr_k e^{-r_k^2/2} \int d\theta_k/2\pi$. Thus, to compare the DF of our dynamical problem to a thermal DF, we compare these correlators of $\chi_d$ to those of $\chi_T$. We find (see Supplemental Material [21]) that in the long-time limit, these correlators factorize pairwise into correlators that can be characterized by an amplitude $f_d(k)$ in analogy to the amplitude $f_T(k)$ of Eq. (5) that arises in the thermal case with

$$f_d(k) = \frac{g}{c|k|} \frac{g\rho}{c|k|}(a_R + a_L) \left[ 1 + \gamma_T^2 \frac{\zeta k^2}{2\pi} \right]^2. \quad (6)$$

Here, $a_R = \eta_r(1 - c/v_r)$ and $a_L = (1 + c/v_r)/\eta_r$ are effective dilation factors of right and left moving waves as discussed above, $\gamma_T^2 = 1/a_R a_L$, and $\zeta$ is the healing length. We see that the amplitudes $f_d(k)$ and $f_T(k)$ [in Eqs. (5) and (6)] are of the same form, but for an extra $k$-independent factor. This factor gives a UV-dependent contribution to correlations, which is small since typically $k\zeta \ll 1$ for low $k$ modes that come with the largest amplitudes (differences in the “thermal” and “steady” state distributions in Fig. 2 can be traced to this extra term; see Supplemental Material [21]). Ignoring this extra term, and comparing the results for the dynamical and thermal amplitudes, we arrive at the most significant result of this Letter—the temperature $T$ characterizing the prethermalization of equal- and unequal-time correlations is

$$T(v_s, v_r) = \gamma_s \left( 1 - \frac{c^2}{v_s v_r} \right) T_0; \quad T_0 = \frac{g\rho}{2}. \quad (7)$$

where $\gamma_s = 1/\sqrt{1 - u_s^2/c^2} = 1/\sqrt{1 - c^2/v_s^2}$ is the Lorentz factor corresponding to boost velocity $u_s$. We first examine some general features of this result. In the limit $v_s \to \infty$, $v_r \to \infty$, that is, measurement of equal-time correlations when the condensate is split suddenly, we reproduce the sudden quench result $T = T_0 = g\rho/2$. In the
Interestingly, for the case factor, in agreement with the conclusions of Ott \cite{26}. implies that the moving body seems hotter, by a Lorentz diverges, and Eq. (7) is not expected to hold. To estimate $h T \leq e$ of temperatures collapses when $v_r > c$. In the general case of finite $v_r > c$ and $|v_r| > c$, we find that the temperature $T$ satisfies the relation $T_0 / \eta s \leq T \leq \eta u T_0$. The limits $T = \eta u T_0$ and $T = T_0 / \eta s$ are reached for the cases $v_r = -c$ and $v_r = c$, respectively—in such cases, we effectively measure correlations of only right- or left-moving waves, that correspond to the temperatures $\eta u T_0$ and $T_0 / \eta s$, respectively. We also note that this range of temperatures collapses when $\eta s = 1$. Thus, the precise notion of a temperature is only meaningful for a body at rest \cite{28}.

Finally, we note that these results are valid when the LL description holds. As $v_r \to c$, the effective temperature diverges, and Eq. (7) is not expected to hold. To estimate where our approach breaks down, we discuss the effect of the cutoff dependent contributions on two-point correlations $\langle e^{i\phi(x,t)-\phi(x',t')} \rangle$. First, it is important to note that the cutoff itself is not Lorentz invariant—if modes up to $k_c = 2 \pi / \xi s$ are occupied in the Lorentz boosted frame, then some modes are Doppler shifted beyond the cutoff in the lab frame. Thus, cutoff dependent contributions cannot be precisely computed. Nevertheless, we provide an estimate by assuming that all modes up to $k_c$ are occupied in the boosted frame. For two-point correlations, the result is a constant multiplicative factor of $e^{-1/2} a^2 (a^{3/2} - 1) / [16 K] \geq e^{-r^2 k^2 / 4 K}$. Thus, we expect that for $r^2 k^2 \leq 2 \sqrt{K}$, the effect of UV physics is negligible, and our results valid. For experimentally relevant $K \sim 50$, this yields a large range of splitting velocities and effective temperatures.

**Accelerating probes.**—A simple extension of the measurement protocol is to release the condensates in an accelerated fashion, instead of a constant velocity $v_r$. One can then ask if additional conformal symmetries of the LL allow us to interpret such accelerated unequal-time correlations as thermal correlations at some effective temperature. An answer to such a question was first provided by Unruh \cite{37}, who showed that a relativistically accelerating detector traveling through vacuum [on a trajectory $x^2 = (c t)^2 + c^4 / a^2$, with proper acceleration $a$] finds a thermal flux of particles at temperature $T_U = a/(2 \pi c)$. Unfortunately, the Unruh trajectory is timelike, and since only correlations of spacelike separated points are maintained in the interference process, such an effect cannot be directly measured. To circumvent this difficulty, we suggest a simple modification of the Unruh trajectory to the following spacelike trajectory: $x^2 = (c t)^2 - c^4 / a^2$. To see how the Unruh effect shows up here, we calculate the correlations of a zero temperature LL, in a set of conformal coordinates $\{\xi, \eta\}$ that are “natural” to the observer moving along such a trajectory. These coordinates are given by the transformation rule $x = (1/a) e^\eta \sinh \xi, t = (1/a) e^\eta \cosh \xi$ and are natural in the sense that the proper interval $d s = \sqrt{d^2 \xi^2 - d^2 t}$ for an observer traveling along such a trajectory ($\eta = 0$) is equal to the length $d \xi$. It is now easy to see that power law correlations of a $T = 0$ LL in coordinates $\{x, t\}$, decay exponentially in coordinates $\{\xi, \eta\}$, since $\langle \phi(\xi, \eta = 0) \phi(\xi', \eta = 0) \rangle \propto a/(c^2 \eta - \xi^2), \xi$, with a temperature $T = T_U = a/(2 \pi c)$, (see Supplemental Material [21]) which is the Unruh result. Thus, we have here a spacelike analog of the Unruh effect.

The experimental protocol to measure this effect is to (i) split the condensate slowly, preparing the phase difference field in a very low temperature \cite{38} state with $T \ll 2 \pi c / L$ and $L$ is the length of the condensate and (ii) measure the correlations along the specified trajectory, expressing the result in the conformal coordinate $\xi$.

**Summary.**—In this Letter we described a new class of quantum quenches wherein the quench procedure occurs along supersonic trajectories. We showed that such quenches can be analyzed using the emergent conformal symmetries present in the low-energy theory of many 1D systems, and that they give rise to a number of interesting relativistic and conformal effects. To ground our theoretical work in the context of experiments, we proposed how such effects can be seen in experiments involving the coherent splitting of 1D quasicondensates [7,19]. Specifically, we showed that when the quasicondensate is split at a supersonic velocity, we realize a moving (at a related subsonic velocity) relativistic system that is prethermalized at a set temperature in its rest frame. In the laboratory frame, this system appears to have excitations that are prethermalized at different Doppler-shifted temperatures, according to their chirality. This chirality can be probed by

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**FIG. 2** (color online). Comparison of contrast distributions in the steady or long-time limit with thermal distributions at temperatures predicted by Eq. (7) for (left) equal-time correlations with different $v_r$, and (right) unequal-time correlations with fixed $v_r/c = 4$. For all plots, system size $L = 4000 \xi s$, integration length $l = 200 \xi s$, Luttinger parameter $K = 10$. Dynamical distributions were evaluated at $t_0 = 2 / c$, which is long enough for prethermalization to have occurred.
interfering the condensates by releasing them at another supersonic velocity. To probe the (additional) conformal symmetries of our 1D system, we proposed a spatial analog of the Unruh effect, and then discussed how it can also be measured in the same set of experiments. Since our theoretical considerations are general to many 1D systems, we expect a wide array of experimental setups (such as 1D Josephson Junction arrays, etc. [31]) to be able to probe the physics we discuss in this work. Also, while our discussion was geared towards analyzing the problem of a quench that occurs at a fixed supersonic velocity (for the purposes of clarity), our methods are general, and can be used analyze quenches along arbitrary spacelike trajectories.

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