Response thresholds in bacterial chemotaxis

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Stimulation of Escherichia coli by exponential ramps of chemoattractants generates step changes in the concentration of the response regulator, CheY-P. Because flagellar motors are ultrasensitive, this should change the fraction of time that motors spin clockwise, the CWbias. However, early work failed to show changes in CWbias when ramps were shallow. This was explained by a model for motor remodeling that predicted plateaus in plots of CWbias versus [CheY-P]. We looked for these plateaus by examining distributions of CWbias in populations of cells with different mean [CheY-P]. We did not find such plateaus. Hence, we repeated the work on shallow ramps and found that motors did indeed respond. These responses were quantitatively described by combining motor remodeling with ultrasensitivity in a model that exhibited high sensitivities over a wide dynamic range.

INTRODUCTION

Methylation and demethylation of bacterial chemotaxis receptors by CheR and CheB modulate receptor activity and enable precise restoration of response regulator output (1, 2). Because binding of the response regulator, CheY-P, to FliM/FliN motor complexes modulates the probability of clockwise (CW) rotation, this enables perfect adaptation in flagellar motor output (CWbias). Through the action of CheR and CheB, the chemotaxis network effectively computes time derivatives of a logarithm of the external ligand concentration (3–6). Hence, exponential ramps of chemoattractants administered to wild-type cells elicit a step change in [CheY-P] for the duration of the ramp (7). Because the receptor output remains constant during such ramps, they are particularly useful in quantifying the receptor output—motor response characteristics. In such experiments, Block et al. (8) observed thresholds in motor responses when the ramps were small (<±0.01 s⁻¹); changes in CWbias were only seen for ramps >±0.01 s⁻¹. Because no corresponding thresholds to shallow ramps were observed in the levels of CheY-P monitored by a fluorescence resonance energy transfer technique (7), it appeared that flagellar motors filtered small changes in receptor output. Given that the motor is ultrasensitive to changes in [CheY-P] (9), how such filtering could be accomplished remained unclear.

CheR cheB mutants do not adapt, so CheY-P levels are directly set by the receptor activity (10). As a result, step changes in [CheY-P] can be produced in these mutants by applying step changes in attractants (11). Segall et al. subjected such mutants to step changes in a nonmetabolizable attractant α-methyl-DL-aspartate and observed partial adaptation over a time scale of a few minutes. Yuan and co-workers showed that this partial adaptation results from addition to the motor of sub-units of FliM (12). More recent work implicates FliN as well (13, 14), so motor adaptation involves the remodeling of both FliM and FliN. On the basis of these observations, Tu and Berg (15) proposed a model that explained the thresholds in motor responses to shallow ramps. They assumed a nonlinear dependence of the number of FliM molecules, N, on CWbias (Fig. 1A), which resulted in the nonsigmoidal relationship between bias and the concentration of CheY-P (Fig. 1B). Near the region indicated by the dashed lines in Fig. 1B, a small change in [CheY-P] due to slow ramps would result in imperceptible changes in CWbias given the weak dependence on [CheY-P] near the plateau. However, a curve of this kind has not been observed experimentally (9).

Here, we designed a test of motor bias versus [CheY-P] by exploiting the heterogeneity in the distribution of CheY-P levels in cell populations. We did not observe a plateau in the relationship. Next, we subjected wild-type cells to slow exponential ramps (±0.005 s⁻¹), observing partial adaptation in motor response but no thresholds. Finally, we built on a previous model to provide a molecular-level description of how differences in the strengths of binding of FliM/FliN in the motor enable partial adaptation to variations in [CheY-P]. Stochastic simulations of motor responses to step changes in [CheY-P] quantitatively reproduced our experimental results. Our data suggest an important role for remodeling in determining the long-term responses of the motor to ambient conditions.

RESULTS

Test of CWbias versus [CheY-P] relationships

A heterogeneous distribution of CheY-P levels in a population of cells is expected to result in a range of CWbias in the corresponding population of motors (9, 16). Such heterogeneity in CheY-P levels arises due to stochastic noise associated with the output of the chemotaxis network (17, 18). On the other hand, the adapted or steady-state CWbias versus CheY-P relationship is an intrinsic property of the motor, and is independent of the chemotaxis network. Hence, the distribution of population CWbias at a particular mean CheY-P level (µCheY-P) contains information regarding the spread of the CheY-P distribution (σ) and the CWbias versus CheY-P relationship. For example, if the plateau shown in Fig. 1B indeed exists, then for a narrow distribution of CheY-P around the mean indicated by the dashed line, a significant fraction of the population of motors will have a bias similar to the one predicted by the plateau (CWbias ~0.5). On the other hand, for a wide distribution of CheY-P, a significant fraction of the motors will have a CWbias = 0 (or 1), with a smaller fraction exhibiting CWbias ~0.5.

Measurements of CWbias distributions

To test the nature of CWbias versus [CheY-P], we measured CWbias in populations of tethered cheR cheB cells. CheY was expressed from the genome under control of its native promoter. We set the mean level of CheY-P in each population via treatment with the nonmetabolizable

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attractant α-methyl-dl-aspartate (MeAsp), as outlined in our Materials and Methods section. Cell rotation was recorded via phase-contrast microscopy, and CWbias was calculated over a minimum of 2 min. As seen in Fig. 2A, when treated with 0.1 mM MeAsp, a large fraction of the motors had a CWbias ~1. When treated with 0.25 mM MeAsp (Fig. 2B), a sizeable fraction of the motors switched (0 < CWbias < 1). Upon treatment with 0.35 mM MeAsp (Fig. 2C), most of the motors had a CWbias ~0. Note that no peaks were discernible between 0 < CWbias < 1 in any of the experimental distributions. As we discuss below, such distributions are consistent with a model that excludes any plateaus in the relationship between CWbias and [CheY-P]. A surprising observation was that in each experiment, CW-only and counterclockwise (CCW)–only motors were both seen, suggesting that the spread in [CheY-P] levels is considerable in cheR cheB strains.

**Measurements of FliM numbers versus CWbias**

Because FliM/FliN remodeling undoubtedly plays a role in determining the CWbias versus [CheY-P] relationship, we directly measured the FliM content in motors in individual tethered cells using methods described previously (19, 20). In brief, single tethered cells of strains carrying fliM-eyfp on the genome were imaged via TIRF (total internal reflection fluorescence) microscopy and phase-contrast microscopy. CheY-P levels were controlled via an inducible plasmid (cheY-pBAD34). FliM numbers (NfliM, where the subscript S indicates steady state) and CWbias for each motor were calculated. These data (n = 45 motors) were combined with data sets previously collected (n = 36, Fig. 1C in Lele et al. (19)) to increase the sample size and enable more accurate estimations. The values for CWbias were binned (bin size = 0.15), and the corresponding values of NS were used to determine the average NS for each bin. These averages were then divided by NfliM (the number of FliM molecules measured in CW-only motors) and plotted as a function of CWbias in Fig. 2D. The data show a consistent decrease in these adapted NS values with CWbias.

**Interpretation of CWbias distributions**

Early measurements at the level of single cells demonstrated that the relationship between steady-state motor bias and [CheY-P] could be very well fitted with a Hill curve (Hill coefficient ~10), indicating a steep, sigmoidal relationship (9). To distinguish between the sigmoidal and nonsigmoidal relationships for motor bias (Fig. 1), we predicted the distributions of CWbias expected from either model in a population of cells containing a heterogeneous distribution of [CheY-P]. The distributions were calculated as a function of μCheY-P and σ in the manner described in Appendix A. Qualitatively, the predictions from the sigmoidal model were consistent with experimental data over a range of values of μCheY-P and σ, given the absence of peaks in distributions between 0 < CWbias < 1. To quantitatively describe the data, we estimated the mean CheY-P levels and spread of CheY-P levels by fitting predictions to experimental density functions (Fig. 3A). The symbols are Gaussian kernel density estimates from the experimental data shown in Fig. 2 (A to C), and the shaded regions are probability densities for the predicted μCheY-P. We find that an SD for CheY-P of ~40 to 50% of the mean CheY-P level enabled the best fits to data as determined by χ² minimization. This suggests that the spread in CheY-P levels in cheR cheB strains ranges from ~1 to 2.5 μM.

Using a similar spread of CheY-P level (0.45 μCheY-P), we calculated the distributions from the nonsigmoidal model shown in Fig. 1B. These distributions are plotted in Fig. 3B. The Tu-Berg model assumed that the plateau is observed near a CWbias ~0.5, because the original reports of thresholds in motor responses were observed near a prestimulus CWbias ~0.5. As anticipated, the distributions predicted from the nonsigmoidal relationship indicate a peak around the plateau, CWbias ~0.5. The absence of corresponding peaks in the experimental data suggests that the explanation for the observed thresholds is unlikely to be due to the model suggested by Tu and Berg (15). We note here that in the CWbias versus [CheY-P] curves measured by Cluzel et al. (9), a small number of data points appeared to form a plateau near CWbias ~0.15. Our analysis for motor bias distributions (Appendix A) predicts a sharp peak for such a plateau near CWbias ~0.15, not seen in the data of Fig. 3. In any event, such a plateau, assuming it is not an artifact, is not relevant to the Tu-Berg model, because it cannot explain thresholds originally observed at CWbias ~0.5.
Cell stimulation with exponential ramps

To determine conditions under which thresholds to exponential ramps of attractants might arise, we repeated the experiments of Block et al. (8) by subjecting wild-type cells to ramps of 0.005 s⁻¹, where thresholds were observed earlier for both up- and down-ramp. For these measurements, we used the protocols of Shimizu et al. (7), applied at room temperature. Tethered cells were subjected to repeated cycles of ramps: they were exposed to fixed concentrations of attractant for ~400 s, exposed to an exponential ramp in concentration either up or down for ~350 s, and then rapidly returned to the prestimulus level of attractant for another 400 s, before beginning the next ramp. The response of a motor to a single up-ramp is shown in Fig. 4A (top panel). As is evident, the CWbias decreased immediately after the start of the ramp and recovered partially to a new CWbias for the duration of the ramp. Because of the nonlinear relationship between CWbias and [CheY-P], any two cells undergoing an identical change in CheY-P level during a ramp will exhibit different changes in motor CWbias depending on the prestimulus level of CheY-P. Hence, we selected motors that had similar prestimulus biases (~0.5) before averaging. The average of four such motors (n = 4, two replicates) is shown in Fig. 4A (middle panel).

The response of a motor to a single down-ramp is shown in Fig. 4B (top panel). As is evident, the CWbias increased immediately after the start of the ramp and recovered partially to a new CWbias for the duration of the ramp. The average of five motors (n = 5, each motor subjected to a single ramp cycle) with similar prestimulus CWbias is shown in Fig. 4B (middle panel), with the corresponding attractant profile indicated in the bottom panel. As is evident, thresholds were not observed for either up- or down-ramp, in contrast to the previous report (8).

Fig. 4. Adaptation in wild-type cells subjected to exponential ramps of attractant. (A) Top panel: Response of a single motor to an up-ramp (0.005 s⁻¹). Middle panel: Average response of four motors each subjected to two ramp cycles. Bottom panel: Ramp profile. (B) Top panel: Response of a single motor to a down-ramp (~0.005 s⁻¹). Middle panel: Average response of five motors each subjected to a single ramp cycle. Bottom panel: Ramp profile. Dashed lines indicate prestimulus bias. Arrows indicate beginning of ramps. Faint gray lines indicate SD.
MODEL FOR RAMP RESPONSES

FliM/FliN remodeling as a function of bias

To predict the dependence of $N_S$ on CW$_{bias}$ shown in Fig. 2D, we modeled FliM/FliN dynamics based on a previous approach (19). We assumed that the total number of binding sites for FliM/FliN protomers (1 protomer = 1 FliM/4 FliN subunits) within the motor (M) remains the same irrespective of rotor conformation (CW or CCW). These sites are of two kinds: strong and weak. FliM/FliN protomers bound to weak sites continuously exchange with a pool of subunits in the cell even at steady state, consistent with experiments (19, 21, 22). Protomers bound to the strong sites are irreversibly attached [dissociation constant $\sim 0$ (19)], and these sites remain fully occupied with little or no exchange of FliM/FliN.

The fraction of strong sites depends on motor conformation ($M_{NE}^{CW}/M$ $\sim 0.2$ and $M_{NE}^{CCW}/M$ $\sim 0.6$), where the subscript $NE$ refers to nonexchanging and the superscript refers to the motor conformation. When a motor switches from CW to CCW, the fraction of strongly bound FliM/FliN protomers changes from $\sim 0.2$ to $\sim 0.6$. This variation drives stochastic fluctuations in the total number of FliM/FliN protomers, $N$, in a motor rotating with a constant CW$_{bias}$ as explained below.

Consider the case of a predominantly CW rotating motor with a steady-state number of FliM/FliN protomers ($N_S$). At $t = 0$, the motor switches to a CW-only direction of rotation, resulting in an immediate increase in the number of strongly bound FliM. At this instant, among the previously attached $N_S$ subunits, $N_{SCW}^{CCW}$ are now strongly bound causing a perturbation in the number of weakly bound protomers (see Appendix B). This perturbation forces the system to a new steady state: the process of binding and unbinding to the available weak sites continues, but the rate of binding is greater than the rate of unbinding (because there are very few weakly bound protomers after the switch). The imbalance in the rates of unbinding/binding drives an increase in $N$, with the increment limited by the amount of time the motor spends in the CCW conformation. If the motor rotates CCW indefinitely after the switch, a new steady-state value, $N_{SCW}^{CCW}$, will be reached. The time scales over which this remodeling occurs are insensitive to CW$_{bias}$ and agree very well with the observed time scales of remodeling ($1/K_{off} \sim 50$ s) (12). The process is reversible. The entropic forces that drive the increase in FliM/FliN content when the motor switches from CW to CCW are also responsible for the decrease in FliM/FliN content when the motor switches from CCW to CW. The variation in $N_S$ as a function of the CW$_{bias}$ can be derived as (Appendix C)

$$N_S/N_{SCW}^{CW} = CW_{bias} + (1 - CW_{bias}) N_{SCW}^{CCW}/N_{SCW}^{CW} \quad (1)$$

The predictions from Eq. 1 are shown in Fig. 2D (black curve), consistent with the observed trend in experimental data.

Model for ultrasensitivity

CheY-P is predicted to bind with a higher affinity to the FliM/FliN complex in the CW conformation than in the CCW conformation, characterized by two dissociation constants $K_{CW}$ and $K_{CCW}$, with $K_{CCW} \sim 4 K_{CW}$ (23, 24). The number of CheY-P bound to the motor at any instant is governed by the ligand concentration and the number of FliM/FliN protomers within the motor. We calculated the number of CheY-P bound to the motor ($Y_B$) using simple first-order binding relations

$$Y_B = (1 - CW_{bias}) \frac{c}{c + K_{CCW}^{CW}} N_{SCW}^{CCW} + CW_{bias} \frac{c}{c + K_{CCW}^{CW}} N_{SCW}^{CW} \quad (2)$$

where $c$ is the CheY-P concentration, and $N_{SCW}^{CCW}$ and $N_{SCW}^{CW}$ refer to the number of FliM/FliN protomers in the two states. The first term in Eq. 2 accounts for binding to the motor when it rotates CCW, and the second accounts for binding to the motor when it rotates CW.

On the basis of a previous model (25), we assume that the number of CheY-P bound to the motor determines the offset to the free-energy difference in the CW and CCW states of the FliG ring. We assume that the FliG ring contains a fixed number of subunits ($N_{FliG}$), independent of the direction of motor rotation (26, 27). When a very low number of CheY-P is bound, the CCW state is energetically more favorable and the motors have a lower probability of rotating CW. Each CheY-P molecule bound reduces the free-energy difference by a constant energy offset, $\Psi$, and the probability of CW rotation is calculated using a two-state model

$$CW_{bias} = \frac{1}{1 + \exp(N_{FliG}E - \Psi Y_B)} \quad (3)$$

Here, $E$ refers to the free-energy difference between the CCW and CW states of the FliG ring (Fig. 5A) in the absence of [CheY-P]. Fitting Eq. 3 using an iterative $\chi^2$ minimization (Levenberg-Marquardt algorithm) to previous data for steady-state motor bias versus [CheY-P] (9) yielded three parameters: $K_{CW} = 2.61 \pm 1.02$, $E = 0.22 \pm 0.02$ $k_BT$, and $\Psi = 0.35 \pm 0.01$ $k_BT$. Residuals calculated using Eq. 3 were smaller (sum of residuals = 0.28) than calculated using the Hill equation (sum of residuals = 0.32). Thus, our model fits previous data better than the Hill function, which is expected considering that we use three fitting parameters compared to two in the Hill function ($K_D$ and $h$). The measurements by Cluzel et al. (9) represent adapted motor responses after remodeling (12, 28), and hence, these fitted values reflect the steady-state motor behavior. Here, we make a very reasonable assumption that the values of $\Psi$, $E$, and $K_{CCW}$ remain constant irrespective of the variations in [CheY-P] or $N$, which enable us to use these values to predict transient motor responses observed during ramps.

Remodeling occurs on a slower time scale compared to the time scales over which motors respond to changes in [CheY-P]. Therefore, the instantaneous motor sensitivity is in fact determined by the number of FliM/FliN subunits already present in the motor at that instant. Remodeling of FliM/FliN shifts the instantaneous motor response versus [CheY-P] curve, similar to the methylation/demethylation-mediated shifts in kinase activity versus ligand curves. The true sensitivity of the motor, which determines its short-term response to fluctuations in [CheY-P], has been recently measured for fixed values of FliM/FliN (28). For the values we obtained by fitting Eq. 3, we predict responses for fixed values of FliM/FliN (that is, transient motor responses) comparable with those reported by Yuan and Berg (28) (see fig. S1). Finally, because Eq. 2 calculates the increase or decrease in the number of CheY-P bound to the motor as a result of remodeling, it connects remodeling with the ultrasensitivity of the switch.

Stochastic simulations of motor responses to exponential ramps

To test if our model could quantitatively explain the role of FliM/FliN remodeling in motor responses in wild-type cells subjected to exponential ramps, we carried out stochastic simulations of motor dynamics, as discussed here. The model equations and algorithms are detailed in Appendix B and in the Supplementary Materials. Briefly, the simulations are initialized at $t = -80$ s, with a motor at an initial CW$_{bias}$
and containing $N_S$ protomers (as determined by Eq. 1). Motor switching traces (Fig. S2) were then generated for the initial $CW_{bias}$ by sampling wait times for CCW ($\tau_{CCW}$) and CW rotations ($\tau_{CW}$) from exponential distributions of respective intervals. The mean values of these distributions have been experimentally determined to vary with $CW_{bias}$ (24), as shown in Fig. S3. Each time a switch occurs, the number of strongly bound FliM/FliN changes to one of the only two possible values ($N_{NE}$). This perturbs the number of weakly bound protomers, leading to a fluctuation in the number of protomers even under steady-state conditions (no attractants added). These stochastic fluctuations in each interval (CCW or CW) were modeled via a simple Monte-Carlo scheme (29) using the equations in Appendix B and the parameters in Table 1. For very short intervals, often no fluctuations occurred because FliM/FliN exchange occurs on relatively slower time scales. For longer intervals, subunits attached more often (compared to detachment) when the motor rotated CCW, and detached more often (compared to attachment) when the motor rotated CW. We assumed that $Y_B$ varied instantaneously with each FliM/FliN addition or removal, according to Eq. 2 (see the Supplementary Materials). The probability of CW rotation was updated via Eq. 3 each time a fluctuation in $Y_B$ occurred, which resulted in a change in the mean interval times for both CW and CCW rotation, according to the experimental measurements shown in Fig. S3. The simulations were then repeated by sampling from exponential wait time distributions with the updated means. The time-varying $CW_{bias}$ was calculated from such motor switching traces with a 20-s moving-average window (12).

We focused on the up-ramp experiments, shown in Fig. 4A. At time $t = 35$ s, the CheY-P level was changed to a lower value and held constant, representing a wild-type cell subjected to exponential ramps in attractant level. The change in CheY-P level generated a drop in the probability of CW rotation, calculated from Eqs. 2 and 3. This change in CheY-P level ($\Delta$CheY-P) was used as a free parameter to ensure that the drop in the predicted CW$_{bias}$ matched the dip in the experimental curve. The motor now predominantly rotated CCW. As discussed previously, the numbers of FliM/FliN protomers in the motor grow because the motor spends more time in the CCW conformation. Upon each addition of a subunit, the probabilities of CW rotation were recalculated and the means of the sampling distributions were updated. The process was run in a loop (~250 iterations). The average of simulated traces of $N$ versus time (Fig. 5B) and the corresponding time variations in $CW_{bias}$ (Fig. 5C) are shown by the solid black curve ($n = 11$ motors). Next, for the same value of $\Delta$CheY-P, we repeated the simulations for a case in which the FliM/FliN complex does not remodel (gray curve in Fig. 5C). The open symbols indicate experimental data from Fig. 4A (middle panel). The simulated results (with FliM/FliN remodeling) are in close agreement with experimental observations, whereas predictions without FliM/FliN remodeling fail to capture observed trends. Similarly, simulation results for the down-ramp are also in agreement with the experiments (see the Supplementary Materials).

**DISCUSSION**

Our measurements of single-motor responses to exponential ramps in attractants failed to show the thresholds previously observed by Block et al. (8). Block et al. subjected the same cells to repeated cycles of up- and down-ramp over periods of an hour or more. The thresholds

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**Table 1. Parameters used in the study.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
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<tbody>
<tr>
<td>$k_{off} = k_{on}$ (for CW and CCW)</td>
<td>$\sim 0.02 \text{ s}^{-1}$ [Lele et al. (19)]</td>
</tr>
<tr>
<td>$M$</td>
<td>56 [Lele et al. (19)]</td>
</tr>
<tr>
<td>$N^C_{CW}$</td>
<td>34 [Thomas et al., 2006]</td>
</tr>
<tr>
<td>$N^C_{NE}$, $N^C_{CCW}$</td>
<td>12, 34 [Lele et al. (19)]</td>
</tr>
<tr>
<td>$E$</td>
<td>0.22 $k_B T$ (this work)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.35 $k_B T$ (this work)</td>
</tr>
<tr>
<td>$K_{CW}$</td>
<td>2.61 (this work)</td>
</tr>
</tbody>
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that appeared in those experiments at the ramp rates used here (0.005 s−1) might have been due to desensitization of receptor adaptation owing to exhaustion of intermediates not replenished by growth, a problem that might be revealed by measurements of [CheY-P]. Our analysis, on the other hand, was conducted on data sets no more than 25 min long.

The addition/removal of FlIM/FlIN subunits and the consequent shifts in motor response curves appear similar in principle to the effect of methylation/demethylation on kinase activity. This raised the possibility that motor remodeling plays an analogous role by enabling precise adaptation at the motor level, an ability that would be manifested in the form of a plateau in CWbias versus [CheY-P] relationship. However, our measurements of motor dynamics as well as adapted motor response curves rule out such a possibility. Instead, remodeling enhances motor sensitivity. For a fixed number of FlIM/FlIN protomers, the CWbias versus [CheY-P] relationship is relatively steep [Hill coefficient ~20; (28)]. This sensitivity controls short-term responses of the motor to even weak signals: a small drop in [CheY-P] results in a tran-
the motor consists of a steady number of protomers \( (N_S) \), and \( M, B, C \), and \( N_{NE} \) are related as

\[
M = N_{NE} + C_S + B_S \quad (B1)
\]

The total protomers in the motor are

\[
N = N_{NE} + C_S \quad (B2)
\]

Whenever a switch in conformation occurs (say at \( t = 0 \) s), \( N_{NE} \) assumes a new value, \( N_{NE} (j \neq i) \), thereby perturbing \( C_S \). This perturbation is expressed as

\[
C(0) = N(0) - N_{NE} \quad (B3)
\]

where \( N(0) \) refers to the total FliM/FliN protomers existing in the motor at the instant of the switch in motor conformation. The system evolves to a new steady state after the perturbation. This evolution is governed by the dynamics of the weakly bound protomers and is limited by the amount of time that the motor spends in the new conformation. The kinetics driving the relaxation are

\[
\frac{dC}{dt} = k_{on}B - k_{off}C \quad (B4)
\]

and at steady state

\[
B_S = \frac{k_{off}}{k_{on}}C_S \quad (B5)
\]

Here, \( k_{on} \) is the pseudo-first-order on-rate and \( k_{off} \) is the off-rate for FliM subunits attached to weak sites.

**APPENDIX C**

**Deriving \( N_S \) versus CWbias**

Consider a motor that switches between the two directions of rotation with a constant CWbias. Let the mean wait times for the CW and CCW events be represented by \( \lambda_{CCW}, \lambda_{CW} \), \( \lambda_{CW} < 1/\lambda_{off} \). The number of FliM/FliN protomers will fluctuate around a time-averaged value \( N_0 \) due to the switches between CW and CCW. Starting with \( N_0 = N_0 \) at the instant when the motor switches to the CCW conformation \( (t = t_0) \) for \( t \) seconds, the positive fluctuation or the number of FliM/FliN protomers added over that interval can be calculated from Eqs. B1 to B5 in Appendix B. Solving the equations yields the relation for the added subunits

\[
\Delta N_{CCW} = (N_{CCW}^* - N_0)(1 - e^{-(k_{off} + k_{on})\tau}) \quad (C1)
\]

First, consider only the positive fluctuations (each time a motor switches to CCW). The probability density function (pdf) for \( \Delta N_{CCW} \) can be calculated from the pdf for \( \tau \), which is exponentially distributed with a mean wait time of \( \lambda_{CCW} \). The expression for the pdf of \( \Delta N_{CCW} \) is

\[
f(\Delta N_{CCW}) = \frac{\lambda_{CCW}}{(k_{off} + k_{on})(N_{CCW}^* - N_0)} \left( \frac{\Delta N_{CCW}}{N_0 - N_{CCW}^*} + 1 \right)^{\frac{\Delta N_{CCW}}{k_{off} + k_{on}} - 1} \quad (C2)
\]

Similarly, the pdf for negative fluctuations (each time the motor switches to CW) is

\[
f(\Delta N_{CW}) = \frac{\lambda_{CW}}{(k_{off} + k_{on})(N_{CW}^* - N_0)} \left( \frac{\Delta N_{CW}}{N_0 - N_{CW}^*} + 1 \right)^{\frac{\Delta N_{CW}}{k_{off} + k_{on}} - 1} \quad (C3)
\]

The positive and negative fluctuations cancel each other over time, such that on an average \( N_S = N_0 \). That is, the means of the two fluctuations are equal and opposite in sign, which enables determination of the relationship between \( N_S \) and CWbias

\[
\Delta N_{CCW} + \Delta N_{CW} = 0
\]

\[
\int_0^{\infty} f(\Delta N_{CCW}) d\Delta N_{CCW} - \int_0^{\infty} f(\Delta N_{CW}) d\Delta N_{CW} = 0 \quad (C4)
\]

Integrating the above equation and algebraically manipulating the resulting expression yield Eq. 1 shown in the main text.

**SUPPLEMENTARY MATERIALS**

Supplementary material for this article is available at http://advances.sciencemag.org/cgi/content/full/1/9/e1500199/DC1

Fig. S1. CWbias versus CheY-P for fixed values of \( N \) (gray dotted and solid curves).

Fig. S2. Motor switching trace simulated by alternately sampling exponential distributions of CW and CCW wait-time intervals.

Fig. S3. Variations in mean intervals with CWbias.

Fig. S4. Comparisons between experimental and simulation results for a down-ramp (\( -0.005 \text{ s}^{-1} \)).

**REFERENCES AND NOTES**


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