Dynamical spin-electric coupling in a quantum dot

The Harvard community has made this article openly available. Please share how this access benefits you. Your story matters

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Published Version</td>
<td>doi:10.1103/physrevb.67.115324</td>
</tr>
<tr>
<td>Citable link</td>
<td><a href="http://nrs.harvard.edu/urn-3:HUL.InstRepos:25886935">http://nrs.harvard.edu/urn-3:HUL.InstRepos:25886935</a></td>
</tr>
<tr>
<td>Terms of Use</td>
<td>This article was downloaded from Harvard University’s DASH repository, and is made available under the terms and conditions applicable to Other Posted Material, as set forth at <a href="http://nrs.harvard.edu/urn-3:HUL.InstRepos:dash.current.terms-of-use#LAA">http://nrs.harvard.edu/urn-3:HUL.InstRepos:dash.current.terms-of-use#LAA</a></td>
</tr>
</tbody>
</table>
Dynamical spin-electric coupling in a quantum dot

L. S. Levitov and E. I. Rashba

1Department of Physics, Center for Materials Sciences & Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
2Department of Physics, SUNY at Buffalo, Buffalo, New York 14260

(Received 30 September 2002; published 24 March 2003)

Due to the spin-orbital coupling in an anisotropic semiconductor quantum dot, a freely precessing electron spin produces a time-dependent charge density. This creates a sizable electric field outside the dot, leading to promising applications in spintronics. The spin-electric coupling can be employed for noninvasive single-spin detection by electrical methods. We also consider a spin relaxation mechanism due to long-range coupling to electrons in gates and elsewhere in the system, and find a contribution comparable to, and in some cases dominant over, previously discussed mechanisms.

DOI: 10.1103/PhysRevB.67.115324 PACS number(s): 72.25.Dc, 72.80.Rj, 73.21.La

\[ \mathcal{H}_0 = \hbar^2 \mathbf{k}^2 / 2m^* + U(x) + \alpha \sigma_z \mathbf{k}, \]  

with \( U(x) \) the confining potential. For magnetic fields transverse to the y axis, without loss of generality, we write the Zeeman interaction as \( \mathcal{H}_Z = - \mu B \mathbf{r}_3 \). (For the field parallel to the y axis the Zeeman and SO terms commute, and thus the spin-electric coupling is absent.)

We eliminate \( \mathcal{H}_SO \) from \( \mathcal{H}_0 \) by a canonical transformation with a unitary matrix \( S = e^{i \sigma_z \xi / 2} \). Here the length \( \xi = h^2 / m^* \alpha \) coincides with the characteristic size of the Datta-Das device. The transformation shifts \( \mathbf{k} \) by \( \sigma_z / 2 \xi \) and moves the SO coupling to the Zeeman term:

\[ \mathcal{H}_0 = \hbar^2 \mathbf{k}^2 / 2m^* + U(x) - m^* \alpha^2 / \xi^2, \]

\[ \mathcal{H}_Z = - \mu B [ \sigma_z \cos(x / \xi) - \sigma_y \sin(x / \xi) ]. \]  

For a weak magnetic field, the Zeeman term can be treated as a perturbation. In a symmetric potential \( U(x) = U(-x) \), the mean value of the second term in \( \mathcal{H}_Z \) vanishes and the spin Hamiltonian projected onto two Kramers-conjugate states becomes diagonal:

\[ \mathcal{H}_n = \langle n | \mathcal{H}_Z | n \rangle = - \mu B \langle n | \cos(x / \xi) | n \rangle \sigma_3, \]  

where \( n \) labels the orbital wave functions \( \psi_n(x) \). The Zeeman splitting in Eq. (3) depends on the SO coupling \( \alpha \).

For narrow-gap \( A_1B_5 \) quantum wells, the typical values of \( \alpha \) originating from SIA are about \( 10^{-9} \) eV cm. However, larger values of \( \alpha \) up to \( 3 \times 10^{-9} \) eV cm for In0.52Al0.48As/In0.6Ga0.4As heterojunctions and \( 6 \times 10^{-9} \) eV cm for In0.52Al0.48As/In0.5Ga1-xAs structures were reported more recently. Both experiment and theory indicate that the interface asymmetry makes an important, and maybe even dominant, contribution to \( \alpha \), which can be varied by system design. With \( \alpha = 6 \times 10^{-9} \) eV cm and \( m^* / m \approx 0.05 \), we estimate the characteristic length scale as \( \xi \approx 13 \) nm. The dependence of \( \mathcal{H}_n \) on the SO coupling can thus be significant for quantum dots of size comparable or
larger than $\xi$. (In diffusive dots the SO effects are controlled by the ratio of level spacing to the SO scattering spin-flip rate.)

Now we consider the time-dependent electric charge density arising due to electron spin precession. It is given by the off-diagonal component in the spin element of the density matrix $N_{i1}(x,t) = e^{-i\omega_{z,n}t}N_{i1}^{(n)}(x)$ with the $n$th orbital state Zeeman frequency $\omega_{z,n}$ defined by Eq. (3). Here

$$N_{i1}^{(n)}(x) = \langle \Psi_{n1}(x) | \Psi_{n1}(x) \rangle_{\text{spin}},$$  

(4)

with $\Psi_{n\sigma}(x)$ the exact Zeeman-split Kramers-doublet spinor wave functions. The partial trace in Eq. (4) is taken over spin. To the first order in $\omega_z$ one obtains

$$N_{i1}(x) = 2\mu B \sum_{n' \neq n} \frac{\psi_{n}(x) \psi_{n}(x)}{E_n - E_{n'}} \langle n' | \sin(x'/\xi) | n \rangle,$$  

(5)

and the corresponding dipole moment equals

$$P_n = -2e\mu B \sum_{n' \neq n} \frac{\langle n | x' \rangle \langle n' | x'' \rangle \langle n' | \sin(x'/\xi) | n \rangle}{E_n - E_{n'}}$$  

(6)

(here $x'$ and $x''$ are integration variables). To complete this general discussion, let us consider free spin precession in a quantum dot holding an odd number of electrons, one electron with unpaired spin at the $n$th level, with all lower states $E_{n'} < E_n$ fully filled. Taking an arbitrary superposition state for the spin configuration of the electron on the $n$th level, the time-dependent dipole is

$$P(t) = \sin \theta P_n \cos(\omega_{z,n}t + \varphi) \hat{x}$$

where $\theta$ is the angle between the magnetic field and the spin precessing around it. The quantities $\omega_{z,n}$ and $P_n$ are given by Eq. (3) and Eq. (6).

Now we focus on two practically interesting confinement models: a square well with hard walls, describing a quantum wire segment, and a parabolic quantum dot. For a wire segment of length $L$, using sinusoidal standing wave states, the Zeeman frequency (3) is

$$\hbar \omega_{z,n}(\gamma) = 2\mu B \frac{\pi^2 n^2 \sin \gamma}{(\pi^2 n^2 - \gamma^2)}, \quad \gamma = L/2\xi,$$  

(7)

with $n \geq 1$. (Zeros in the denominator do not cause divergence because of $\sin \gamma$.) The electric dipole (6) is

$$P_n(\gamma) = e L (\mu B/\Delta_n) \sum_{n' \neq n, n' \equiv \text{odd}} \frac{\langle n | x' \rangle \langle n' | x'' \rangle \langle n' | \sin(x'/\xi) | n \rangle}{(n^2 - n'^2)^3 \{[n^2 + n'^2 - (2\gamma/\pi)^2]^2 - 4(nn')^2\}}.$$  

(8)

with $\Delta_n = E_{n+1} - E_{n} = (2n+1)\pi^2 \hbar^2 / 2m^* L^2$ the separation between the energy levels $E_{n+1}$ and $E_n$. The sum, evaluated exactly for $n = 1$ and $\gamma \ll 1$, gives

$$P_1 = \frac{15 - \pi^2}{8 \pi^2} \frac{\mu B L}{\Delta_1} e L.$$  

(9)

This result is similar to the matrix element of electric-dipole transitions in 3D donor centers. The two factors multiplying the geometric dipole $eL$ in Eq. (9) have the following meaning. The factor $\mu B/\Delta_1 \ll 1$ reflects that the matrix element of $x$ between two Kramers-conjugate states vanishes at $B = 0$ due to the time-reversal symmetry. The factor $L/\xi \ll 1$ makes $P_1$ vanish at zero SO coupling. Despite the small factors, the electric dipole can still be much larger than the Bohr’s magneton $\mu = 0.5 e\alpha C$, where $\alpha C$ is the electron Compton length.

For a parabolic dot with a confining potential $U(x) = m\omega^2 x^2 / 2$, the Zeeman frequency (3) is

$$\hbar \omega_{z,n}(\gamma_2) = 2\mu B e^{-\gamma_2^2} L_n(\gamma_2), \quad \gamma_2 = \frac{1}{2}(x_0/\xi)^2,$$  

(10)

where $n \geq 0$, $x_0 = (\hbar/m\omega)^{1/2}$, and $L_n$ are Laguerre polynomials. Similar to Eq. (7), the frequency (10) is a sign-changing function of the SO coupling $\alpha$, vanishing at $\alpha \to \infty$. The summation in Eq. (6), performed exactly using harmonic oscillator selection rules, gives

$$P_n(\gamma_2) = e x_0^2 (\mu B/\hbar \omega) (2 \gamma_2)^{1/2} e^{-\gamma_2^2} L_n(\gamma_2).$$  

(11)

For the ground state, the dipole moment (11) is similar to $P_1$ of Eq. (9) with $x_0$ replacing $L$. 

**FIG. 1.** Electric dipole of a Zeeman-split state vs SO coupling. The state $|n\rangle$ with $n = 9$ is used. The dipole (8) for a square well is scaled by $\frac{1}{\xi} e L (\mu B/\Delta_1)$; the dipole (11) for a parabolic well is scaled by $e x_0^2 (\mu B/\hbar \omega)$. 

115324-2
The dipole moments (8) and (11) for the two models (Fig. 1) behave as follows. For a square well \( P_n(\gamma) \) has a peak at \( \gamma = \pi n \) of width \( \delta \gamma = 2\pi \) independent of \( n \). The peak is positive or negative depending on the parity of \( n \), with weak oscillations on both sides. For a parabolic well the oscillations of \( P_n(\xi) \) increase in amplitude as \( \xi^{-1} \) up to \( \gamma_z = 4n \) and then abruptly disappear as \( P_n(\gamma_z) \) drops.

The dependence of the dipole on the level number is quite interesting (Fig. 2). For the square well, since the dipole (8) peaks near \( \gamma = \pi n \), \( P_n \) is large only for specific levels, while for the harmonic potential it is a smooth oscillatory function of level number. This dramatic difference is explained as follows. In a square well each wave function is characterized by a specific wavelength, equal to \( 2L/n \) for the \( n \)th level, while in a harmonic potential the wavelength is position dependent. The SO effect will be strong when the wavelength matches the spatial period \( 4\pi \xi \) of the matrix \( e^{i\sigma_2/2} \) used to gauge the SO interaction. Thus one expects the dipole in a square well to be large for states with \( 2L/n = 4\pi \xi \), which is exactly the above condition \( \gamma = \pi n \). In a harmonic potential, on the other hand, there should be no specific levels with enhanced dipole.

A much stronger spin-electric coupling arises for non-Kramers states brought to degeneracy at the Zeeman energy \( \mu B \) matching the level separation \( E_n - E_{n'} \). The spin-electric coupling for SO-split avoided crossings of levels with different orbital wave functions and opposite spin is different from that for a Zeeman split Kramers doublet. The matrix element of electric dipole in this case does not contain the factor \( \mu B/\Delta \) that appeared in Eq. (10) due to time-reversal symmetry at \( B = 0 \). The SO-split level crossings in small elongated dots were reported by Rokhinson et al.\(^{17}\) and in 2D dots by Folk et al.\(^{18}\).

For such a pair of states \(|\psi_n,\rangle, |\psi_{n'},\rangle\) the energy separation in the absence of the SO coupling is \( \Delta = E_n - E_{n'} - 2\mu B \). The avoided crossing of levels split by the SO matrix element \( V \) is described by \( \delta E = (\Delta^2 + 4V^2)^{1/2} \). The off-diagonal charge density matrix element is

\[
N_{n'\rightarrow n}^\alpha(x) = u_+ u_- \psi_n^*(x) + v_+ v_- \psi_{n'}^2(x),
\]

where

\[
u_\pm = [(\delta E \pm \Delta)/2\delta E]^{1/2}
\]

and \( v_\pm = \pm u_- \) are the components of the two states participating in the avoided crossing. (The quantities \( \psi_n, \psi_{n'}, \) and \( V \) are real due to the absence of magnetic orbital coupling in \( D = 1 \).) Evaluating \( u_+ u_- = -v_+ v_- = V/(\Delta^2 + 4V^2)^{1/2} \) we obtain

\[
N_{n'\rightarrow n}^\alpha(x) = \frac{V}{(\Delta^2 + 4V^2)^{1/2}} \left[ \psi_n^2(x) - \psi_{n'}^2(x) \right].
\]

This charge density oscillates with frequency \( \delta E/\hbar \).

We note that the distribution (14) possesses a dipole moment only for an asymmetric confining potential \( U(x) \neq U(-x) \), while the quadrupole moment \( Q_{nn'} = e \int x^2 N_{n'\rightarrow n}^\alpha(x) dx \) exists even for symmetric dots:

\[
Q_{nn'} = e x_0^2(n-n') V/(\Delta^2 + 4V^2)^{1/2}.
\]

The factor \( e x_0^2 \) has the scale of the quadrupole moment of the \( n \)th quantum state. The enhancement of the electrical signal from a spin precessing between two non-Kramers levels resembles a similar effect for the electric-dipole spin resonance at acceptor centers.\(^{16}\)

**Electrical detection of a single-electron spin precession** is attractive because of high sensitivity of electrical measurements.\(^{19}\) Moreover, electrical detection can be performed locally, e.g., by a single-electron transistor\(^{20}\) (SET) or just by measuring the time-dependent potential induced on the gates around the dot holding spin.

To estimate the magnitude of the effect, we consider an electron in a square well defined in a quantum wire. The potential at a distance \( r \) from the dot has the order of magnitude \( \varphi(r) = \beta r^2 \) with \( \beta \) given by Eq. (9). For an estimate, taking \( L = 20 \text{ nm}, \quad r = 50 \text{ nm}, \quad e = 13, \) and \( L/\xi \approx 1 \), we obtain

\[
\varphi(r) = (\mu B/\Delta_0) \times 0.1 \text{ mV}.
\]

The detection of a signal with the amplitude (17) oscillating at the Zeeman frequency is certainly feasible.\(^{19}\)

We expect a stronger effect in the two other situations considered above. In a parabolic dot the dipole is typically larger than in a square well, mainly because of its smooth dependence on level number (Fig. 2). In the case of non-Kramers level crossing, the effect is enhanced due to the absence of the small factor \( \mu B/\Delta_n \). The leading effect is dipolar for asymmetric and quadrupolar for symmetric dots. Although exact estimates are problematic because of a large number of independent parameters, we expect the effect in
the surface can be related with that in the bulk: \( \kappa_{12} = K_{12} + K_{12} \), where \( 2' \) is a mirror image of the point 2. We then rewrite Eq. (18) as \( W = (e^2/\hbar) \sum_k \sigma(\mathbf{k})^2 \Im \kappa(\omega_z, \mathbf{k}) \). In a clean metal, using \( \Im \kappa(\omega, \mathbf{k}) = (\pi/2)\nu \omega|\mathbf{k}|v_F \), with \( \nu \) the density of states,

\[
W = \frac{e^2\nu}{4\hbar} \omega_z \left( \frac{2r_z^2}{1 + k_z^2 r_z^2} \right)^2 \frac{dk}{v_F |k|} = (e^2/\hbar v_F) \ln(r/r_0) (Pr/re^2)^2 \omega_z.
\]

(19)

The logarithm arises due to particles incident at small angles that interact more strongly with the screened dipole field.

For a diffusive metal, with \( K(\omega, \mathbf{k}) = -\nu DK^2/(Dk^2 - i\omega) \), integrating over the normal component of \( \mathbf{k} \), we obtain

\[
W = \frac{e^2\nu r_s^4}{2\pi^2 \hbar D} \int \frac{d^2 \mathbf{q}}{(q^2 - i\omega_z D)^{1/2}} \left( Pr/r^2 \right)^2 \left( \omega_z D/\hbar \right) \approx \left( Pr/r^2 \right)^2 \omega_z \approx \omega_z \operatorname{d}l/r^2.
\]

(21)

(\( \mathbf{q} \) is the parallel-to-surface component of \( \mathbf{k} \). We find that diffusion enhances \( W \) by the number \( N \) of returns to the surface during the coherence time, estimated as \( N \approx \min[(\omega_z \tau)^{-1/2}, r/l] \), with \( \tau \) and \( l \) the elastic mean free time and path. These bounds restrict the diffusion time to \( \omega_z^{-1} \) and the displacement of diffusing quasiparticles to the distance \( r \) from the dot, respectively.

Estimating the relaxation rate (19) with the above parameter values, one has \( W \approx (\mu B/\Delta)^2 \times 10^{-5} \omega_z \). Recently, electron spin relaxation due to coupling to fluctuating magnetic fields and to nuclear spins was considered. Although a direct numerical comparison is difficult due to a wide spread of parameter values, Eq. (21) of Ref. 4 gives a number similar to ours obtained for \( \mu B/\Delta \approx 10^{-5} \). In the case of non-Kramers level crossing, due to the absence of the factor \( \mu B/\Delta \), the electric mechanism can dominate. Generally, the electrical and magnetic spin relaxation rates depend on different combinations of parameters and, therefore, should be considered as complementary mechanisms. We note also that the estimated spin relaxation time is long compared to \( \omega_z^{-1} \), allowing for many cycles of free spin precession.

In summary, the SO-induced electric field around a freely precessing spin can be employed for single-spin detection. It may also significantly contribute to spin relaxation. The reverse spin-electric effect is also of interest in view of spintronics applications. It can serve as a mechanism for independent spin monitoring and control in different dots by local electric field sources.

This work is supported by the MRSEC Program of the National Science Foundation under Grant No. DMR 98-08941 (L.L.) and by DARPA/SPINS, Office of Naval Research Grant No. 000140010819 (E.R.).
DYNAMICAL SPIN-ELECTRIC COUPLING IN A QUANTUM DOT


115324-5