More Than Just Symbols: Mental and Neural Representations Related to Symbolic Number Processing in Mathematics

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More than Just Symbols:
Mental and Neural Representations Related to Symbolic Number Processing in Mathematics

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# Table of Contents

**ABSTRACT** ........................................................................................................ III

**CHAPTER 1** ........................................................................................................... 1
- ABSTRACT ........................................................................................................ 2
- A BRIEF HISTORY ........................................................................................... 3
- SYMBOLIC NUMBER PROCESSING .................................................................. 4
- THE PRESENT DISSERTATION ......................................................................... 6
- STUDY 1: SYMBOLIC PROCESSING OF ARITHMETIC FACTS ....................... 8
- STUDY 2: FROM DIGITS TO LITERAL SYMBOLS ........................................... 9
- EDUCATIONAL RESEARCH IMPLICATIONS ................................................... 11
- ACKNOWLEDGMENTS ..................................................................................... 14
- REFERENCES .................................................................................................. 15

**CHAPTER 2** ......................................................................................................... 19
- ABSTRACT ........................................................................................................ 20
- INTRODUCTION ............................................................................................... 22
- METHODS ....................................................................................................... 28
- RESULTS .......................................................................................................... 34
- DISCUSSION .................................................................................................... 37
- LIMITATIONS AND NEXT STEPS .................................................................. 46
- CONCLUSION ................................................................................................... 47
- ACKNOWLEDGMENTS ..................................................................................... 49
- REFERENCES .................................................................................................. 50

**CHAPTER 3** ......................................................................................................... 66
- ABSTRACT ........................................................................................................ 67
- INTRODUCTION ............................................................................................... 68
- THE PRESENT STUDY ..................................................................................... 73
- METHODS ....................................................................................................... 75
- RESULTS .......................................................................................................... 81
- DISCUSSION .................................................................................................... 87
- LIMITATIONS .................................................................................................. 90
- CONCLUSION ................................................................................................... 91
- ACKNOWLEDGMENTS ..................................................................................... 93
- REFERENCES .................................................................................................. 94

**CHAPTER 4** ......................................................................................................... 103
- ABSTRACT ....................................................................................................... 104
- MENTAL & NEURAL REPRESENTATIONS RELATED TO LITERAL SYMBOLS .... 105
- SYMBOLIC NUMBER PROCESSING AND DEVELOPMENTAL DYSCALCULIA .... 110
- HIGHER-ORDER SYMBOLIC REPRESENTATIONS OF NUMBER .................. 112
- CONCLUDING REMARKS ............................................................................... 113
- ACKNOWLEDGMENTS ..................................................................................... 116
- REFERENCES .................................................................................................. 117
Abstract

The ability for students to understand numbers and other mathematical symbols is a crucial part of success in mathematics. Accordingly, it is important for researchers to understand the nature of symbolic number processing – the connections between a symbol or collection of symbols that convey numerical information (e.g., Arabic digits, arithmetic facts, literal symbols) and their related mental and neural representations. Research that joins the mind and brain sciences with education, such as educational neuroscience work, provides a powerful way to examine students’ symbolic number processing. Much of the research in this area has focused on processing of Arabic numerals in adults and children, with relatively less work on symbols common in intermediate and higher-level mathematics. This dissertation contributes two studies that focus on number processing for symbols beyond those used in basic numeracy, arithmetic facts and literal symbols.

The first study uses neuroimaging meta-analysis to examine whether there are brain regions that support both arithmetic and phonological processing. Results suggest that activity in frontal and temporo-occipital brain regions support both types of processing, and that there is recruitment of left temporoparietal areas for each type of processing, but these areas are regionally differentiated. The second study investigates the connection between literal symbols and their mental representations of quantity. Results suggest that there is a cognitive processing cost associated with connecting literal symbols to numerical referents because literal symbols have extant mental referents related to literacy.
Taken together, these studies expand the scope of existing research in educational neuroscience related to mathematics learning, to more fully incorporate notions of symbolic processing in intermediate and higher-level mathematics, and contribute to theory building on the connections between symbols in mathematics and their mental and neural representations. These studies also form the basis of my future work in educational neuroscience related to symbolic number processing, which will build and expand on the studies presented herein. Research on symbolic number processing that spans symbols learned in early numeracy (i.e., Arabic numerals) and in intermediate and higher-level mathematics (e.g., arithmetic facts, literal symbols) can facilitate a more complete picture of student learning, thereby supporting students’ mathematical development from early numeracy through advanced mathematics.
Chapter 1

Introduction
Abstract

The field of Mind, Brain, and Education, and its related discipline, Educational Neuroscience, provide a powerful lens for researching student learning. Educational neuroscience work that concerns mathematics brings together mathematics education research and numerical cognition research. This dissertation contributes two studies to educational neuroscience work that involves mathematics learning that address symbolic number processing, the connection between a symbol or collection of symbols that convey numerical information and the associated mental and neural representations. The first study investigates brain activity in regions that support arithmetic and phonological processing in a developmental sample. The second study examines the connection between literal symbols used in algebra and their associated mental representations. Together these studies expand the scope of existing research in educational neuroscience related to mathematics learning, to more fully incorporate notions of symbolic processing in intermediate and higher-level mathematics. This dissertation contributes to theory building related to symbolic number processing in students, which can in turn impact future research on the role of symbolic number processing in student learning.
Introduction

**A Brief History**

The current movement to integrate knowledge across the fields of neuroscience, psychology, and education began in the late 1990s and early 2000s with the development of the Mind, Brain, and Education (MBE) field (Schwartz, 2015). MBE integrates biological, psychological, and educational research with educational practice and policy to generate research and knowledge on learning (Fischer, 2009; Fischer et al., 2007; Schwartz, 2015). The related but more focal field of Educational Neuroscience utilizes behavioral and neuroimaging techniques from cognitive neuroscience and psychological methods to investigate questions about learning (e.g., De Smedt et al., 2010; Fischer, Goswami, Geake, & the Task Force on the Future of Educational Neuroscience, 2010; Geake, 2011; Patten & Campbell, 2011; Szűcs & Goswami, 2007). Generally, educational neuroscience work spans several domain-general (e.g. attention, working memory) and skill-specific (e.g., reading, mathematics) areas of research related to education. Educational neuroscience work that focuses on mathematics learning brings together research from mathematics education and numerical cognition. Specifically, educational neuroscience work related to mathematics learning is concerned with the mental and neural representations of number, and the mathematical skills built on and with these representations.

Like other work in MBE and Educational Neuroscience, much of the educational neuroscience work related to mathematics learning was theoretical. Scholars argued for the marriage of numerical cognition and mathematics education research and how to conduct research responsibly (e.g., for an extended discussion see De Smedt et al., 2010;
De Smedt & Verschaffel, 2010; Turner, 2011). Over the past several years, empirical educational neuroscience work related to mathematics learning has flourished. This research has provided a more comprehensive look at the cognitive processes involved in doing mathematics, lent additional support to findings from behavioral studies, contributed insights about learning mathematics that are not available using behavioral methods, shed light on the neural underpinnings of atypical development, and examined how instruction affects the brain (Ansari, De Smedt, & Grabner, 2012; De Smedt, 2014; De Smedt, Ansari, et al., 2010; De Smedt & Verschaffel, 2010).

Symbolic Number Processing

A subset of educational neuroscience work related to mathematics learning has focused on symbolic number processing. Symbolic number processing refers to the connection between a symbol or collection of symbols that convey numerical information and their associated mental or neural representations. For example, symbols can be Arabic numerals (e.g., 2), number words (e.g., TWO), higher-order symbolic ‘units’ like arithmetic facts (e.g., 2 x 3), or literal symbols (e.g., x) used in algebra.

Researchers often measure symbolic number processing by how quickly and accurately participants make judgments about individual symbols that represent numbers. Most commonly, symbolic number processing is measured with comparison tasks, such as comparing two Arabic numerals to determine which is larger (e.g., Moyer & Landauer, 1967) or judging if two number symbols such as ‘2’ and ‘ONE’ represent the same value (e.g., Dehaene & Akhavein, 1995). Comparison tasks consistently produce distance effects and ratio effects; judgments take longer and are less accurate on average when the distance between the two numbers is smaller or when the ratio between the two numbers
is higher (Ansari, 2008; De Smedt, Noël, Gilmore, & Ansari, 2013). This is largely attributed to the notion that numbers are represented on a mental number line and that comparison tasks tap that mental number line (for an elaborated discussion, see van Opstal, Gevers, De Moor, & Verguts, 2008).

The ability to make judgments about number symbols, such as by comparing two Arabic numerals, matters for mathematics performance. Indeed, the ability to understand and work with number symbols, particularly Arabic numerals, is related to successful mathematics learning starting in early schooling. In early elementary school, students’ abilities to compare Arabic digits predicts their mathematics performance one year later, including on early arithmetic and word problems (De Smedt, Verschaffel, & Ghesquière, 2009). In fact, there is a consistent, positive relationship between symbolic number processing and mathematics performance on tasks such as arithmetic across development (De Smedt et al., 2013; Schneider et al., 2016). Research on brain activation that is associated with symbolic number processing lends additional evidence to this relationship. For example, Bugden, Price, McLean, and Ansari (2012) found an association between modulation of activity in left intraparietal sulcus (i.e., a brain region that is related to symbolic number processing) during number comparison and 3rd and 4th graders’ arithmetic performance.

As the above literature illustrates, many of the studies on symbolic number processing and mathematics learning have involved Arabic numerals and their relationship with arithmetic for children in early elementary school. There are relatively few studies that examine symbolic number processing for symbols used in intermediate and higher-level mathematics, such as collections of arithmetic facts (e.g., 3 x 4) or literal
symbols (e.g., $x$). The research that exists suggests that fluency with more advanced symbolic number representations is important for more advanced mathematics. For example, activity in brain regions that support arithmetic fact retrieval (i.e., left supramarginal gyrus, bilateral anterior cingulate) predict mathematics (but not reading) PSAT scores (Price, Mazzocco, & Ansari, 2013) and the ability to work with symbolic representations of fraction magnitude positively correlates with algebra readiness (Booth & Newton, 2012).

The relationship between symbolic number processing beyond Arabic digits and students’ mathematics performance suggests the importance of understanding the mechanisms that support symbolic number processing itself, that is, students’ connections between number symbols and their mental and neural representations. There is nascent work that examines mental representations associated with symbolic number processing of fractions (Gabriel, Szücs, & Content, 2013a; Gabriel, Szücs, & Content, 2013b), but this work is scarce, and additional research on the connections between higher-level symbols, such as literal symbols, and their mental representations is lacking (but see Pollack, Leon Guerrero, & Star, 2015 and Rosnick, 1982). As I detail below, researchers have also investigated mental and neural representations associated with arithmetic facts in adults and children (for a review, see Menon, 2015). However, the nature of these representations, particularly how they relate to aspects of language, remains unclear.

**The Present Dissertation: Mental and Neural Representations Related to Symbolic Number Processing in Mathematics**

The two studies in this dissertation directly address the need for additional research on mental and neural representations for symbols in intermediate and higher-
level mathematics. Specifically, this dissertation builds on extant research topics (i.e., arithmetic processing) and examines novel research topics (i.e., literal symbols) in symbolic number processing beyond Arabic numerals. In doing so, this dissertation expands the scope of existing research in educational neuroscience related to mathematics learning, to more fully incorporate notions of symbolic processing in intermediate and higher-level mathematics.

In this chapter, I introduce each study in turn. I first discuss the study in Chapter 2, which examines brain regions that support arithmetic processing in a developmental sample. As I describe in detail below, this study investigates the notion that mental and neural representations related to arithmetic facts rely on verbal processing (e.g., Dehaene, Piazza, Pinel, & Cohen, 2003). I argue that the involvement of verbal processing suggests that there may be brain regions that support both arithmetic and phonological processing. I test this hypothesis using neuroimaging meta-analysis, which quantifies the probability of activation of particular brain regions for both arithmetic and phonological processing tasks. I then discuss the second study, in Chapter 3, which concerns mental representations that relate to literal symbols used in algebra. I argue that students may process literal symbols differently than Arabic numerals because literal symbols have pre-existing mental representations that are related to literacy. Accordingly, this study compares the link between a symbol and its mental representation using symbolic number comparison tasks across three different representations: Arabic numerals, literal symbols, and artificial (i.e., non-alphabetic) symbols. I conclude this introductory chapter by discussing the educational research implications of the dissertation as a whole. I suggest that these studies contribute knowledge of how number is represented in the mind and
brain, and that the basic research in these two studies can facilitate future research on how symbolic number processing contributes to students’ performance in intermediate and higher-level mathematics.

**Study 1: Symbolic Processing of Arithmetic Facts**

When memorizing arithmetic facts, we associate the collection of symbols (e.g., 2 x 3) with a mental representation of the solution (Ansari, 2008; Grabner, Ansari, Koschutnig, Reishofer, & Ebner, 2013). Memorization of arithmetic facts often relies on verbal strategies, such as repeatedly reciting the multiplication tables. Accordingly, mental referents for these arithmetic facts are assumed to be stored as verbal codes (Dehaene et al., 2003). Put another way, the arithmetic fact may function as a symbolic unit whose associated mental referent is a verbal representation of the solution, and these verbal representations are related to cognitive and neural processes that involve language.

Phonological awareness (i.e., awareness of sounds in language) is associated with early arithmetic ability (Simmons, Singleton, & Horne, 2008) and arithmetic fact retrieval in children and adults (De Smedt & Boets, 2010; De Smedt, Taylor, Archibald, & Ansari, 2010). In adults, phonological rehearsal can interfere with arithmetic ability, specifically multiplication (Lee & Kang, 2002). Additionally, left lateralized brain networks associated with phonological processing are engaged during multiplication fact retrieval in children and adults (e.g., Grabner et al., 2009; Prado et al., 2011), suggesting that these networks may support both arithmetic and phonological processing.

A small number of studies have explicitly investigated whether left lateralized brain regions related to phonological processing also support arithmetic, but have not yielded a consistent picture. For example, while one study in adults found a region in left
temporoparietal cortex that supported both types of processing (Simon, Mangin, Cohen, Le Bihan, & Dehaene, 2002), other research suggests regional differentiation. Andin, Fransson, Rönnberg, & Rudner (2015) found that, in adults, multiplication recruited a posterior portion of the angular gyrus (i.e., the PGp) while phonological processing recruited the anterior portion of the angular gyrus (i.e., the PGa). The few studies that have examined the activation of common brain regions for arithmetic and phonological processing have involved adults. However, whether there are brain regions that support both arithmetic and phonological processing in children is also of interest, particularly because of the behavioral relationship between the two processes that exists in children (e.g., De Smedt, Taylor, et al., 2010).

The first study, which I present in Chapter 2, used research at the brain level to shed light on the behavioral relationship between arithmetic and phonological processing. This study also sought to inform the inconsistent findings related to the prior neuroimaging work reviewed above. Using neuroimaging meta-analysis, this study aggregated findings across prior studies in which children completed phonological or arithmetic tasks to examine brain regions with concordant activity across both tasks. Results of this study can shed additional light on the behavioral relationship between symbolic processing related to arithmetic facts and phonological processing that is unavailable using behavioral measures alone. In addition, the findings can contribute to the larger open question in numerical cognition about the relationship between reading and mathematics (Holloway & Ansari, 2015).

**Study 2: From Digits to Literal Symbols**
The second study, which I present in Chapter 3, examined how students form mental representations for literal symbols and artificial digits (i.e., non-alphabetic symbols) compared to Arabic numerals. Important for algebra and higher mathematics, literal symbols and their mental representations pose substantial and prolonged difficulty to students (e.g., Christou & Vosniadou, 2005; Rosnick, 1982). Prior research has asserted a variety of explanations for student difficulty with literal symbols, including inadequate exposure (Nie, Cai, & Moyer, 2009; Philipp, 1999; Rosnick, 1982), the nature of mathematical syntax (e.g., Schoenfeld & Arcavi, 1999), and a shift in notational conventions from arithmetic to algebra (Kieran, 2007).

However, student difficulty with literal symbols may partly be because students have already formed mental referents for literal symbols in another context: literacy. When literal symbols are brought into math, such as when they convey numerical magnitude (e.g., \(x = 1\)), children may confuse the literal symbol with its numerical position in the alphabet (e.g., \(h\) stands for 8, since \(h\) is the 8th letter of the alphabet) (MacGregor & Stacey, 1997). Compared to Arabic numerals, literal symbols that convey numerical magnitude may require the development of more complex mental referents involving both magnitude and language. However, recent research concerning the mechanisms that support the development of mental representations for literal symbols more broadly is lacking (e.g., Philipp, 1992; Rosnick, 1980). A better understanding of mental representations related to literal symbols can further illuminate why students may struggle to work with literal symbols and suggest potential avenues of research for remediation of this difficulty.
To examine the connections between literal symbols and their mental representations, I asked students to perform symbolic number comparison tasks using Arabic numerals, literal symbols, and artificial digits. As I describe in the above section on symbolic number processing, mental representations associated with Arabic numerals are thought to exist on a mental number line, such that reaction time and accuracy for comparing two number symbols vary as a function of the distance or ratio between them (Ansari, 2008; De Smedt et al., 2013). In this study, I tested whether literal symbols that convey numerical magnitude could also access this mental number line, as Arabic numerals do, by looking for the existence of distance effects. Further, to test whether the presence or absence of distance effects may be related to the symbols per se, I also tested for distance effects using artificial digits that convey numerical magnitude. I then correlated the distance effects in the three conditions to investigate whether these effects have similar or different cognitive mechanisms. Results of this study can inform whether there may be cognitive interference associated with processing literal symbols that convey numerical magnitude. These results can also further nuance our understanding of why students continue to struggle to understand literal symbols in mathematics and our understanding of the ways that different symbols in mathematics connect to their mental and neural referents.

**Educational Research Implications**

The focus of this dissertation – mental and neural representations associated with symbols in intermediate and higher-level mathematics – concerns educationally relevant, but basic research in symbolic number processing. As such, it does not address questions about classroom practice or pedagogical approaches related to mathematics learning.
However, this type of basic research provides important opportunities to build theory related to students’ understanding of symbolic representations in mathematics that is an important precursor for, and contributor to, educational advances that promote mathematical development in students.

In particular, the neuroimaging meta-analysis has implications for future research on the relationship between mathematics and reading. Specifically, the results of this study can contribute to future models of the development of arithmetic processing and its relationship to phonological processing. This study can also contribute evidence at the brain level to further elucidate the behavioral relationship between arithmetic and phonological processing (De Smedt & Boets, 2010).

The study on literal symbols provides a novel perspective on mental representations associated with symbolic representations in algebra and beyond and can contribute to a more complete picture in numerical cognition of how we associate quantities with symbols in mathematics, especially symbols that are not unique to numeracy. This study can also contribute to knowledge in mathematics education research related to the source of students’ difficulties in working with literal symbols. Despite the variety of explanations for this difficulty discussed above, research on mental representations associated with literal symbols is largely lacking (e.g., Philipp, 1992; Rosnick, 1980). Further, despite the fact that much of the research on students’ difficulties with literal symbols was conducted in the 1980s, student difficulties in understanding what literal symbols represent persist (McNeil et al., 2010). The present study therefore offers an opportunity to bring new evidence to bear on longstanding questions related to students’ struggles with literal symbols.
Finally, the dissertation as a whole has implications for educational neuroscience work related to mathematics learning. Both studies align with the aims of educational neuroscience work on mathematics learning described above (e.g., De Smedt, 2014; De Smedt, Ansari, et al., 2010). The neuroimaging meta-analysis lends additional support to the behavioral relationship between arithmetic and phonological processing, and provides insights on the neural level that are unattainable through behavioral studies. The study on literal symbols provides a more comprehensive look at the cognitive processes involved in doing mathematics by pushing beyond early numeracy.

Taken together, the studies in this dissertation extend the range of educational neuroscience research related to mathematics learning, to more fully include symbolic processing in intermediate and higher-level mathematics that qualitatively differs from Arabic number processing. Through these and similar studies, we can continue to unpack the step-wise relationship between different levels of symbolic number processing and mathematics performance: between Arabic numerals and arithmetic, arithmetic and high school mathematics, fraction knowledge and algebra, and beyond.
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Chapter 2

Where Arithmetic and Phonology Meet: The Meta-Analytic Convergence of
Arithmetic and Phonological Processing in the Brain
Abstract

Arithmetic processing is complex. Arithmetic operation, strategy use, problem size, and other factors modulate activity in the brain network that supports arithmetic processing. Prior behavioral research has shown that some arithmetic problems, particularly those solved by fact retrieval, are related to phonological processing ability and elicit brain activity in left lateralized brain regions that are known to support phonological processing. However, whether there are common brain regions that support both arithmetic and phonological processing is an open question. In this study, I investigate the functional neural overlap between arithmetic and phonological processing in children using activation likelihood estimation (ALE). I conducted separate meta-analyses for arithmetic and phonological processing, and a conjunction analysis that determines overlapping clusters across the two domains. Results of the individual meta-analyses were in line with prior literature. Arithmetic processing in children produced clusters of concordant activation mainly in left lateralized regions, including inferior and middle frontal gyri, angular gyrus, fusiform gyrus, and in bilateral inferior parietal lobule. Phonological processing in children produced reliable activation in several left lateralized regions including inferior and medial frontal gyri, middle temporal gyrus, and fusiform gyrus. I identified five clusters common to arithmetic and phonological processing in frontal and temporo-occipital regions. Common areas of activation across the two domains appear to support attentional processes, and symbolic processing of arithmetic problems and words. The present meta-analyses contribute novel insights into the relationship of arithmetic and phonological processing in the brain and the brain regions that may support
processing of more complex symbolic representations, such as arithmetic facts and words.
Where Arithmetic and Phonology Meet: The Meta-Analytic Convergence of Arithmetic and Phonological Processing in the Brain

Introduction

Solving an arithmetic problem is a multi-faceted process. Even for single-digit arithmetic problems, fluency involves attending to the problem, recognizing and processing number symbols, understanding arithmetic operations, and either calculating an answer or retrieving an answer from memory. This complexity is also reflected in the neural underpinnings of arithmetic. The triple-code model (Dehaene, Piazza, Pinel, & Cohen, 2003) is a widely cited model of numerical and mathematical processing that describes three neural circuits that underlie number processing, how they support basic arithmetic, and how engagement of these circuits can vary depending on task demands. Any task that requires accessing a mental representation of quantity engages the intraparietal sulcus (IPS), a brain region in the parietal lobe that responds to numerical magnitude broadly. This includes recognizing symbols or dot arrays that convey quantity information, comparing numbers, or mental calculation. Arithmetic problems that are solved using calculation procedures (e.g., multi-digit arithmetic) also recruit the posterior superior parietal lobule, which is involved in visual attention. Finally, the triple-code model posits that overlearned arithmetic problems (e.g., small addition and multiplication problems) are stored in and retrieved from verbal memory, and thus recruit temporoparietal brain regions that are involved in language, such as the angular gyrus (AG), superior temporal gyrus...
Arsalidou and Taylor’s (2011) model of numerical and arithmetical processing, which updates the triple-code model, further supports the notion that multiplication problems recruit left temporoparietal brain regions, specifically the left STG. In addition, Arsalidou and Taylor (2011) include frontal regions that support arithmetic processing across addition, subtraction, and multiplication, such as the left inferior frontal gyrus (IFG). Drawing from these models, the current study focuses on arithmetic processing that is related to verbal processing, specifically phonological processing (e.g., processing the sounds in speech, such as determining whether two words rhyme). In particular, it investigates the overlap between these two symbolic processes using neuroimaging meta-analysis.

Prior behavioral research suggests that arithmetic performance is related to phonological processing. For example, Lee and Kang (2002) found that phonological rehearsal interfered with multiplication (but not subtraction) ability in adults, which may be due to the tendency to rely on verbal strategies to memorize multiplication facts. De Smedt and Boets (2010) found a moderate, positive correlation between healthy adults’ retrieval of multiplication facts from memory and phonological processing, particularly phonological awareness. Research with elementary school children has also shown a relationship between phonological processing and arithmetic performance. For instance, De Smedt, Taylor, Archibald, & Ansari (2010)

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1 Both the parietal and temporal areas named as part of this model support processing beyond numeracy. For instance, these areas constitute the “where” and “what” pathways, respectively, for vision (Goodale & Milner, 1992). However, here I refer specifically to the role of these regions as they support numerical processing.
found that phonological processing predicted accuracy and reaction time for small\(^2\) (versus large) problems across addition, subtraction, multiplication, and division. The authors found a similar relationship for problems that were likely to be solved with retrieval compared to a procedural strategy. Further, adults with severe impairments in phonological processing (i.e., adults with dyslexia) also show difficulty with multiplication arithmetic fact retrieval (De Smedt & Boets, 2010). Taken together, these studies provide evidence that phonological ability is related to performance on arithmetic problems retrieved from memory, such as multiplication problems or small problems.

Brain imaging research in adults has consistently shown that left lateralized brain regions involved in phonological processing are recruited during arithmetic processing. Figure 1 shows a three-dimensional representation of these left lateralized regions. Studies in adults have shown that arithmetic processing recruits the left AG (for a review see Zamarian, Ischebeck, & Delazer, 2009), a temporoparietal area that is also involved in phonological processing and word meaning (Booth et al., 2004; Price, 2000, see Figure 1). The (bilateral) AG is more active for exact addition compared to approximate addition (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999). Furthermore, Grabner, Ansari, Koschutnig, Reishofer, & Ebner (2013) found that the left AG is more active when participants process multiplication problems with incorrect answers that represent the sum of the two operands (e.g., 6 x 5 = 11) compared to incorrect answers not associated with either the sum or product of the

\(^2\) For addition and multiplication, De Smedt et al. (2010) defined small problems as problems in which the product of the operands is less than or equal to 25. Small subtraction and division problems were the inverse of the small addition and multiplication problems.
operands (e.g., 6 x 5 = 32). The left AG is thought to support efficient or fact based retrieval of arithmetic problems (e.g., Grabner et al., 2007; but also see Rosenberg-Lee, Chang, Young, Wu, & Menon, 2011). Indeed, studies with adults that explicitly measure brain activity as a function of strategy use also report greater activity in bilateral AG for problems solved with retrieval-based strategies compared to procedure-based strategies, such as multi-step mental calculation (Delazer et al., 2005; Stanescu-Cosson et al., 2000; Tschentscher & Hauk, 2014). Recruitment of the left AG is also present in training studies with adults, in which participants first overlearn arithmetic problems and subsequently complete arithmetic tasks with both the trained and untrained problems. Results show that there is greater activation in the left AG for trained problems compared to untrained problems (Delazer et al., 2003, 2005; Grabner, Ansari, et al., 2009; Grabner, Ischebeck, et al., 2009; Ischebeck et al., 2006), which further suggests that the left AG supports overlearned arithmetic fact retrieval.

Additional adult research implicates alternative left temporal and frontal brain structures that are involved in verbal processing (see Figure 1). Zhou et al. (2007) found that adult participants had greater activity in the superior temporal gyrus (STG) when solving multiplication problems than when solving addition problems. Prado et al. (2011) found that when adults solved multiplication facts they showed brain activity increases in the left medial temporal gyrus (MTG) and left IFG, two brain areas that participants also recruited when completing phonological tasks. These two studies suggest that left-lateralized brain regions related to phonological processing,
including the AG, STG, MTG, or IFG also support arithmetic processing for small or retrieval-based facts in adults.

Comparatively less work on neural correlates of arithmetic exists for children. Research suggests that over development, left temporoparietal and temporo-occipital regions, including those that relate to language, become increasingly specialized to support arithmetic processing (Ansari, 2008; Zamarian et al., 2009). In a cross sectional study of second to seventh graders, Prado, Mutreja, and Booth (2014) showed that children in higher grades had greater activation in left MTG for single-digit multiplication processing compared to children in lower grades, reflecting greater recruitment of this area to support multiplication processing. Further, they found that the grade-related change in activity was greater for smaller versus larger multiplication problems (e.g., 3 x 4 = 12 versus 6 x 7 = 42). However, Rosenberg-Lee, Barth, and Menon (2011) found that third grade students showed more activity for single-digit addition problems in right, rather than left, temporoparietal and temporo-occipital regions compared to second graders. A meta-analysis of seven fMRI studies with children aged 8-14 involving arithmetic found diffuse bilateral activation related to calculation, but found reliable activation in right, rather than left AG (Kaufmann, Wood, Rubinsten, & Henik, 2011). Across the adult and developmental literature, research suggests that left-lateralized brain areas are recruited for retrieval-based arithmetic processing in adults, and that activation in these areas is driven by increased fluency with arithmetic facts that occurs throughout schooling or with training.
Even if arithmetic and phonological processing both recruit left-lateralized brain areas, the specific areas that support these two processes may be regionally differentiated. Studies that have explored the neural overlap of arithmetic and phonological processing have yielded conflicting results. Simon, Mangin, Cohen, Le Bihan, and Dehaene (2002) investigated overlapping brain regions in adults for calculation and phonological processing tasks and found a region in the left IPS mesial to the AG that was active for both tasks. However, Andin, Fransson, Rönnberg, & Rudner (2015) found neighboring, yet distinct, temporoparietal areas for arithmetic and phonological processing when adults completed multiplication and phonological processing tasks. Both tasks recruited the left AG, however, left AG activity for multiplication was in posterior AG (i.e., PGp) and activity that supported phonological processing was in anterior left AG (i.e., PGA). Together, these studies illustrate the need for more research to inform whether common brain regions support both arithmetic and phonological processing.

Further, the few studies that examine the overlap between arithmetic and phonological processing have been done in adults. Because phonological processing is already related to arithmetic in elementary school students (e.g., De Smedt et al., 2010), investigating potential brain regions that support both processes in children can shed additional light on the behavioral relationship. In addition, examining brain regions that support both arithmetic and phonological processing in children may provide insight into the conflicting findings from adult samples. For example, it may be that both processes rely on common brain regions in children, but recruit more differentiated brain regions in adults as a by-product of learning.
Meta-analysis is a powerful tool that can be used to identify brain regions that support arithmetic and phonological processes by aggregating foci across studies. In the present study, I use activation likelihood estimation (ALE) to investigate the convergence of brain regions that show reliable activity across arithmetic and phonological processing in children. To do so, I conduct three separate meta-analyses: one for arithmetic processing only, one for phonological processing only, and a third that extracts the common results in the first two meta-analyses thereby identifying a set of brain regions serving arithmetic and phonological processing together. The first two meta-analyses identify concordant areas of activation across a set of empirical studies for either arithmetic or phonological processing. The third meta-analysis is a conjunction analysis of the first two and identifies brain regions that are reliably activated across both. I expect that if there are clusters of reliable activation that support both arithmetic and phonological processing, the clusters will be in left temporoparietal areas such as STG, MTG, or AG. Alternatively, in line with Andin et al. (2015), it may be that there are regional similarities in temporoparietal areas for arithmetic and phonological processing, but no shared clusters due to regional differentiation. In addition, I expect that both arithmetic and phonological processing tasks will recruit prefrontal regions, reflecting attentional processes necessary for solving arithmetic problems or phonological tasks.

Methods

Literature Search and Article Selection

To determine areas of overlap between arithmetic and phonological processing across development, I conducted separate literature searches and meta-
analyses for each. The searches yielded a similar number of studies across a range of
tasks targeting arithmetic and phonological processing. I describe the search strategy
according to subject area, starting with arithmetic.

**Arithmetic processing.** For the arithmetic meta-analysis, an initial search of
the PUBMED database using the search terms “fMRI and arithmetic and (child* or
adolescen* or student)” articles published in English resulted in 100 papers. I
screened the set of papers against a set of inclusion criteria and retained studies that
used fMRI, included typically-developing children and adolescents (i.e., participants
under 18), conducted whole brain analyses, had visually-presented stimuli, and
reported within group contrasts between arithmetic processing and baseline
conditions in standard Talairach or Montreal Neurological Institute (MNI) space. I
excluded studies that were reviews, clinical trials, or case studies, studies with non-
symbolic or non-arithmetic experimental tasks (e.g., number matching), studies
whose participants spoke non-alphabetic languages, and other meta-analyses. I
additionally crosschecked references from the other meta-analysis in children
(Kaufmann et al., 2011) in case there were additional papers to include. These
inclusion-exclusion criteria produced a final set of 13 papers, 134 foci, and 388
participants for the developmental meta-analyses for arithmetic.

In Table 1, I present an overview of each study in the arithmetic processing
meta-analysis for the developmental sample, including sample size, mean age of
participants, contrasts, and statistical thresholds. In line with prior meta-analyses
(Arsalidou & Taylor, 2011; Kaufmann et al., 2011; Maisog, Einbinder, Flowers,
Turkeltaub, & Eden, 2008; Pollack, Luk, & Christodoulou, 2015; Richlan,
Kronbichler, & Wimmer, 2009, 2011), only one contrast per group per study was included to preserve statistical independence between sets of foci in the analysis. For studies in which there was more than one contrast (see Table 1, studies A7-A10, A13), preference was given to experimental tasks that involved two-operand or single-digit condition, or operations likely to involve fact retrieval (e.g., multiplication rather than subtraction). For one study (Chen et al., 2006), I included two contrasts because each came from a separate participant group who met the inclusion criteria. Accordingly, there were fourteen experiments across the thirteen studies. Eight of the studies (see Table 1, studies A1-A6, A12, A13) employed an experimental task involving single-digit addition. During addition tasks, participants chose between incorrect and correct answers to addition problems, verified addition facts, or added a series of numbers presented sequentially. For controls tasks, participants matched Greek letters or grayscale patterns, or solved simple single-digit addition problems of the form $x + 1 = y$. In three of the studies (see Table 1, studies A9-A11), participants solved addition and subtraction problems (mixed within block) and performed a digit identification control task. Studies A7 and A8 involved verifying multiplication facts and used fixation as the control task. Almost all studies required a button press (see Table 1, studies A1-A7 and A9-A12); in two studies (see Table 1, studies A8 and A13) participants performed mental calculation only.

**Phonological processing.** For the phonological processing meta-analysis, I located articles through PUBMED using the terms “fMRI and reading and (child* or adolescen* or student)” for articles that were published in English and were not reviews or clinical trials. I used to term reading rather than phonological processing
in order to screen a broad set of papers that may have employed phonological processing tasks. Due to the abundance of reading studies, I limited the search to articles that were published within the past ten years (i.e., January 1, 2005 - October 1, 2015). The search yielded 587 studies that I screened against the following inclusion criteria: used fMRI, included typically-developing participants under 18, conducted whole brain analyses, and reported within group contrasts between a phonological processing condition (e.g., word or pseudoword rhyming, decoding) and a baseline condition. I excluded studies with clinical samples or case studies, studies with non-phonological experimental tasks (e.g., semantic judgments), studies with participants who spoke non-alphabetic languages, and other meta-analyses. In addition, I included four studies from our prior meta-analysis on reading in typical and atypical readers (Pollack et al., 2015) that were absent from the current search results, but met the inclusion criteria. This resulted in a final set of 14 studies that contributed 162 foci and 318 participants.

In Table 2, I present an overview of the studies in the phonological processing meta-analysis for the developmental sample including sample demographics, contrasts, and statistical thresholds. Eight of the studies (see Table 2, studies P1-P8) employed a rhyming task such as determining if two visually presented words, pseudowords, or letters rhymed. Seven of the eight studies used a fixation baseline and one used letter matching. Three studies (see Table 2, studies P9-P11) utilized an experimental task involving pseudoword reading; each study had a fixation baseline. In Study P12, participants decoded words and determined if the word referred to an animal. As a baseline, participants identified asterisks embedded in symbol strings.
Study P13 utilized a lexical decision task in which participants first read words or pseudowords, then mentally substituted in a different initial letter, and decided whether the new word was a word or pseudoword. Participants made same-letter substitutions during the control condition. In study P14, participants decided whether a given letter was the same as the previously presented letter. As a baseline, participants completed the same task with a false font. There were fifteen experiments across the fourteen studies. Similar to above, one study (Hoeft et al., 2006) reported contrasts for two control groups that each met the inclusion criteria.

**Data Analysis**

The aim of the present meta-analyses was to quantitatively assess overlapping brain regions that support both arithmetic and phonological processing in a developmental sample. Here I first describe the analytic methods for the two individual meta-analyses. I then describe the conjunction analysis, which quantifies the likelihood of any given brain region recruited for both arithmetic and phonological processing.

**Single-study meta-analyses.** All analyses were conducted using ALE with GingerALE version 2.3.4 (Eickhoff et al., 2009; Eickhoff, Bzdok, Laird, Kurth, & Fox, 2012; Turkeltaub et al., 2012). Prior to the analyses, I transformed all coordinates into a common space; all MNI coordinates were transformed into Talairach space (Talairach & Tournoux, 1988) using the *icbm2tal* transform native to GingerALE (Lancaster et al., 2007).

To conduct the analyses, ALE models the foci from each experiment as a three-dimensional Gaussian probability distribution (Eickhoff et al., 2009). It then
generates three-dimensional activation maps by taking the maximum of each focus’s Gaussian, a non-additive method that limits within-experiment effects (Turkeltaub et al., 2012). ALE then generates a null distribution for the ALE statistic and probabilities associated with the values of the activation maps (Research Imaging Institute UTHSCSA [RII], 2013). The probabilities can then be compared to the null distribution according to a chosen threshold. In this method, GingerALE simulates random data sets for a chosen number of permutations in which each data set retains the same properties as the original data, such as number of foci and subject sample sizes (RII, 2013). The simulated data is first thresholded based on a cluster-forming threshold. Based on the distribution of the cluster sizes, the data are then subject to cluster-level thresholding, which sets a minimum volume cluster size (RII, 2013). In the current analysis, I used 1,000 permutations for the simulated data. Due to the two levels of thresholding, I employed the recommended cluster-forming threshold of uncorrected \( p < .001 \) with a cluster-level threshold of .05 (RII, 2013). GingerALE results were reported in Talairach space, displayed using the anatomical templates native to GingerALE, and were labeled using the Talairach Daemon (talairach.org) and confirmed using the Talairach Daemon in Mango (RII, 2015).

**Conjunction analysis.** I used a conjunction analysis (Eickhoff et al., 2011) to determine areas of overlap between arithmetic and phonological processing. For this analysis, ALE uses results from the individual meta-analyses described above and an additional pooled analysis of all foci. That is, the contrast analysis for the developmental sample involves the meta-analytic results for arithmetic and phonological processing, and a third set of results from the pooled foci from the
arithmetic and phonological processing studies acting as an empirical baseline or “null” distribution. The present analysis meets the criterion for adequate power, which is approximately fifteen studies for each single-study meta-analysis. To determine areas of overlap across the two meta-analyses, GingerALE creates a new ALE map that takes the voxel-wise minimum value from the two original thresholded maps (RII, 2013).

Results

The samples for the arithmetic and phonological processing meta-analyses were comparable. The arithmetic meta-analysis included 388 participants (187 female, 166 male, 34 participants’ gender was not reported) that ranged in age from about 8-17 years. As noted above, the meta-analysis utilized 14 experiments from 13 studies and contributed 134 foci. The phonological processing meta-analysis included 318 participants (179 female, 139 male) that ranged from about 6-14 years old. The meta-analysis used 15 experiments from 14 studies and contributed 162 foci.

Figure 2 and Panel A in Table 3 display the results of the single-study meta-analyses for arithmetic processing. In total, nine clusters show reliable activation when participants engage in arithmetic tasks. The largest clusters are in the right insula (32, 18, 60) and right cingulate gyrus (4, 20, 42). There is also a large cluster in the left insula (-30, 18, 6). There is reliable activation in a set of left-lateralized clusters that include frontal regions (-48, 0, 34), the fusiform gyrus (-46, -56, -14), and angular gyrus (-30, -60, 38). Finally, there are clusters of reliable activation in bilateral inferior parietal lobule (right: 40, -54, 52; left: -42, -46, 42). Note that even if a cluster has a small number of contributing studies (i.e., left inferior parietal lobule,
see Table 3 row 10), it still produces reliable activation across studies. This is because contributing studies have coordinates inside the boundary of the cluster and additional studies may contribute coordinates that lie on or just outside of the cluster boundary (RII, 2013).

To provide additional anatomical specificity, I further examined the clusters using the *whereami* program in AFNI (Cox & Hyde, 1997). This program provides a report on the percent of overlap of each cluster with neighboring anatomical regions from a specified atlas, here the Talairach Daemon. I report on a subset of clusters of interest here. The largest cluster in right insula (Table 3, row 2) showed overlap with right claustrum (7.4%) and right IFG (3.9%), which suggests that this cluster is in the anterior portion of the insula. The cluster in the right inferior parietal lobule (Table 3, row 9) showed overlap with the superior parietal lobule (36.9%) and the cluster in left inferior parietal lobule (Table 3, row 10) showed overlap with the supramarginal gyrus (1.9%).

Figure 3 and Panel B in Table 3 display the results of the phonological processing meta-analysis. Six clusters show reliable activation across studies in which participants complete phonological processing tasks. In line with prior research, most clusters are in left frontal and parietal regions. The largest cluster is in the left middle temporal gyrus (-56, -38, 2) with additional large clusters in the left medial gyrus (-6, 4, 54) and left inferior frontal gyrus (-52, 12, 32). There are also clusters of reliable activation in the right insula (36, 20, 4), the left fusiform gyrus (-44, -52, -12), and a second, smaller cluster in the left inferior frontal gyrus (-42, 24, 14).
Figure 4 and Panel C in Table 3 present the results of the conjunction analysis, which quantitatively assesses clusters of concordant activation for arithmetic and phonological processing tasks. There are five clusters of reliable activation across arithmetic and phonological processing. The largest cluster is in the right insula (36, 20, 4). The remaining four clusters are left-lateralized. Three of the four clusters are in frontal regions, one cluster in the superior frontal gyrus (-4, 6, 52), one in the precentral gyrus (-50, 0, 36), and one in the inferior frontal gyrus (-50, 6, 32). There is also a cluster in the left fusiform gyrus (-46, -54, -14). Similar to above, some clusters show a small number of contributing studies (Table 3, rows 4-6), yet still represent reliable activation across studies, since other studies may contribute foci that sit on or just outside of the cluster boundary (RII, 2013). Note that two clusters common to both arithmetic and phonological processing (i.e., left superior frontal gyrus, left precentral gyrus) are not listed as clusters per se in the phonological processing meta-analysis in Table 3. Rather, these two clusters are submaxima within the large clusters with maxima in left inferior frontal gyrus and left medial frontal gyrus, respectively (see Table 3, rows 13-14). In addition, in contrast to expectations, there was no area of concordant activity in temporoparietal cortex.

Similar to above, I used the whereami program in AFNI (Cox & Hyde, 1997) to provide additional anatomical specificity for a subset of clusters. As above, I used the Talairach Daemon atlas to further specify the large cluster in right insula (Table 3, row 19), which showed overlap with right IFG (15%). This suggests that this cluster is in the anterior portion of the insula. For the cluster in left IFG (Table 3, last row), the Talairach Daemon atlas did not provide additional specificity. However, the
CA_N27_ML atlas showed that the cluster in left IFG overlaps with the pars triangularis (46.2%), precentral gyrus (42.3%), and pars opercularis (11.5%).

Discussion

The present study examined brain functional overlap for arithmetic and phonological processing in a developmental sample. Fourteen experiments involving arithmetic and 15 experiments involving phonological processing were included in separate meta-analyses and a subsequent conjunction analysis. Each meta-analysis produced a set of clusters that are reliably activated across studies, including five clusters common to both arithmetic and phonological processing. I briefly discuss results for each individual meta-analysis and then focus on the clusters common to both arithmetic and phonological processing.

Individual Meta-Analyses

The arithmetic meta-analysis yielded clusters in bilateral frontal, left temporal, and bilateral parietal regions that are in line with prior work on arithmetic processing. Reliable activation in bilateral insula is consistent with prior meta-analyses of arithmetic in both adults (Arsalidou & Taylor, 2011) and children (Kaufmann et al., 2011), though its task-specific role in arithmetic is unclear. As I explain in more detail below, activation in the anterior insula may reflect attentional processes or switching between the executive control and the default mode network (e.g., Craig, 2009; Menon & Uddin, 2010). Reliable activation in bilateral frontal regions is also in line with prior meta-analyses for arithmetic, including clusters in the bilateral cingulate gyri and the left precentral gyrus (Arsalidou & Taylor, 2011), and the middle frontal gyrus (Houdé, Rossi, Lubin, & Joliot, 2010). Recruitment of frontal...
regions during arithmetic may reflect the role of attentional processes (Houdé et al., 2010; Owen, McMillan, Laird, & Bullmore, 2005). Indeed, models of arithmetic processing suggest that children and adolescents engage frontal regions more than adults, and that this activity reflects a developing fluency with arithmetic (Ansari, 2008; Rivera, Reiss, Eckert, & Menon, 2005; Zamarian et al., 2009). The arithmetic meta-analysis also yielded a cluster in right inferior parietal lobe overlapping with right superior parietal lobe, a cluster in left inferior parietal lobe overlapping with the supramarginal gyrus, and a cluster in left angular gyrus, which is consistent with prior literature emphasizing the recruitment of parietal and temporoparietal regions for number processing and arithmetic (Ansari, 2008; Arsalidou & Taylor, 2011; Dehaene et al., 2003; Kaufmann et al., 2011; Zamarian et al., 2009).

The phonological processing meta-analysis produced clusters of reliable activation in left frontal, temporal, and occipital regions, in addition to the right anterior insula. These results are in line with models of reading and phonological processing (e.g., Jobard, Crivello, & Tzourio-Mazoyer, 2003) that outline a left-lateralized fronto-temporo-occipital network. The findings of the phonological processing meta-analysis also largely replicate prior research and meta-analyses that characterize a network of left-lateralized brain regions for typical readers that include the inferior frontal gyrus, middle temporal gyrus, and fusiform gyrus (Jobard et al., 2003; Sebastian et al., 2014; Vigneau et al., 2006). As mentioned above, activation in the insula could be due to attentional processes or shifting between attention and the default mode network (e.g., Craig, 2009; Menon & Uddin, 2010).

Clusters Common to Arithmetic and Phonological Processing
**Right insula.** The largest cluster common to both arithmetic and phonological processing was in the anterior portion of the right insula (see Figure 4, panel d). The presence of a cluster in the right anterior insula is in line with prior meta-analyses of number and arithmetic processing in both adults and children (Arsalidou & Taylor, 2011; Kaufmann et al., 2011). However, a cluster in right insula was not commonly found in prior reading meta-analyses (e.g., Jobard, Crivello, & Tzourio-Mazoyer, 2003; Vigneau et al., 2006); this may be due in part to specific contrast selection criteria (e.g., contrasts that isolate semantic processing). However, reliable activation in the right insula has been found in some meta-analyses associated with atypical reading development. Maisog et al. (2008) found hyperactivity in the anterior insula for atypical readers. They speculated that this activity may be related to atypical readers’ perception of reading-related stimuli as aversive. Barquero, Davis, & Cutting (2014) found that prior to a reading intervention, children and adults showed underactivation in right insula, but after intervention showed consistent activation in this region.

The role of the right anterior insula as it relates to numeracy or literacy, specifically, is currently unclear. It may be that recruitment of the right insula supports arithmetic and phonological processing through domain-general functioning. Models of anterior insula function suggest that it plays an important role in higher level cognitive processing such as task-related attentional capture and control (Craig, 2009; Menon & Uddin, 2010; Nelson et al., 2010), decision-making, or the experience of knowing something before actually recalling it (Craig, 2009). As mentioned above, the insula also is thought to direct cognitive and neural resources to
internally or externally focused attention (Menon & Uddin, 2010). In the present study, recruitment of right anterior insula could represent the direction of externally focused task-specific attention toward arithmetic or phonological tasks. Specifically, recruitment of this region could reflect the experience of knowing the answer to an arithmetic fact or whether two words rhyme.

**Left superior frontal and precentral gyri.** The second and third clusters common to arithmetic and phonological processing were in the left superior frontal gyrus and the left precentral gyrus, respectively. A prior meta-analysis in children showed reliable activation in left superior frontal gyrus for arithmetic, but not reading (Houdé et al., 2010). Both regions were also found in an arithmetic meta-analysis in adults (Arsalidou & Taylor, 2011), however, it appears that reliable activation in these regions may be driven by domain-general task demands. For example, activity in the superior frontal gyrus has been associated with selective attention (e.g., Anderson et al., 2007).

Activity in the precentral gyrus may be due to motor responses associated with scanner tasks. The contrasts for most studies in the phonological processing meta-analysis included fixation (see Table 1), so motor responses were not subtracted out. Contrasts in the arithmetic meta-analysis were more variable (see Table 2). The contrasts for a small number of studies included fixation. Most studies in the arithmetic meta-analysis included contrasts in which both tasks required button presses. However, in these cases, there may have been different levels of interference in generating motor responses for experimental and control tasks. For example, in Kesler, Menon, and Reiss (2006), the experimental task required participants to judge
the correctness of addition and subtraction facts (i.e., a true-false judgment) and the control task involved a button press when a ‘0’ was present (i.e., a go/no-go task).

**Left fusiform gyrus.** There was also a cluster common to arithmetic and phonological processing in the left fusiform gyrus. Recruitment of the left fusiform gyrus during calculation has been found across arithmetic studies and in prior meta-analyses for adults across number processing and arithmetic tasks (Arsalidou & Taylor, 2011; Menon, 2015). The present findings diverge from prior developmental meta-analyses that did not find activation near this region (Houdé et al., 2010) or that found reliable activation in the neighboring inferior temporal gyrus (Kaufmann et al., 2011). Reliable activation in the left fusiform gyrus has also been found in prior meta-analyses related to typically-developing readers (Houdé et al., 2010; Jobard et al., 2003; Pollack et al., 2015).

The left fusiform gyrus may be common to arithmetic and phonological processing because of its functional role in symbol recognition for words and digits. The left fusiform gyrus houses the Visual Word Form Area (VWFA), an area situated near Talairach coordinates -42, -57, -12 that is consistently activated by letters and words (Cohen et al., 2000; Hannagan, Amedi, Cohen, Dehaene-Lambertz, & Dehaene, 2015; McCandliss, Cohen, & Dehaene, 2003). Evidence suggests that an analogue to the VWFA might exist for number recognition. Prior studies suggest there is a number form area (NFA) lateral to the VWFA, in (bilateral) ventral inferior temporal gyrus (Hannagan et al., 2015; Shum et al., 2013). However, some studies do not support the presence of an NFA. Price and Ansari (2011) found activation in left ventral AG for digit recognition compared to letters or scrambled letters during
passive viewing, but found no digit specific activation in temporo-occipital cortex. Another study by Peters, De Smedt, and Op de Beeck (2015) found greater activity in bilateral lateral occipital cortex when comparing digits to number words during a subtraction task. However, this activity was not found in a subsequent experiment that controlled for the number of visual elements presented. This suggests that the greater activation was at best task-specific and at worst due to a confound in the visual stimuli.

One challenge with investigating the proposed NFA is that it is located within the fMRI signal-drop out zone (Shum et al., 2013), which can make activity in this region difficult to detect. A recent study by Grotheer, Herrmann, and Kovács (2016) used novel imaging techniques (i.e., a combination of high spatial resolution and liberal smoothing) to minimize the effect of the signal drop-out zone and show support for the presence of a bilateral NFA. Grotheer et al. (2016) found greater brain activation in inferior temporal gyrus when adult participants completed a 1-back task with numbers compared to when they completed the task with other symbols, such as letters, false numbers or letters, or objects. These findings were consistent across both single-subject and group analyses.

Whether the left VWFA and NFA are separate or merged is unclear (Hannagan et al., 2015; Starrfelt & Behrmann, 2011). Prior research with adults suggests that the VWFA preferentially responds more to words than digits (Polk et al., 2002). Other research suggests that recruitment of this region may depend on task demands associated with symbolic processing. Cohen and Dehaene (1995) reported that lesion patients with pure alexia (i.e., an ability to recognize words) showed
impaired performance when adding numbers or reading them aloud, but not when identifying numbers for magnitude comparison. Further, there appears to be functional specialization of both the VWFA and NFA in adults compared to children (for reviews, see Menon, 2015; Schlaggar & McCandliss, 2007). Indeed, recent research looking across children and adults suggests that the VWFA and NFA may be merged in children and become functionally distinct areas in adulthood (Cantlon, Pinel, Dehaene, & Pelphrey, 2011). Thus, it is plausible that the cluster in left fusiform gyrus found in the present study supports symbolic processing for both number and letter/word identification.

**Left inferior frontal gyrus.** The fifth cluster common to arithmetic and phonological processing was in left inferior frontal gyrus (IFG), overlapping with both pars triangularis and pars opercularis. A cluster in left IFG is in line with prior meta-analyses for arithmetic (Arsalidou & Taylor, 2011; Kaufmann et al., 2011), for studies involving untrained arithmetic problems (Delazer et al., 2003, 2005) and in meta-analyses of reading-related processes (e.g., Houdé et al., 2010; Jobard et al., 2003; Pollack et al., 2015; Vigneau et al., 2006).

Prior studies that have investigated an overlap in arithmetic and phonological processing have shown mixed results related to left IFG activation. Andin et al. (2015) found that both arithmetic and phonological processing recruit left IFG, but recruit different portions. Specifically, they found that multiplication was associated with activity in pars triangularis (BA 45), whereas phonological processing was associated with posterior activity in the pars opercularis (BA 44). Similarly, Fedorenko, Duncan, and Kanwisher (2012) found that different portions of the left
IFG were active during language-specific and domain-general (including arithmetic) tasks. However, they found an area of brain activity on the border of BA 44/45 corresponding to language-specific tasks, with brain activity in both anterior and posterior regions bordering BA 44/45 responding to domain-general tasks, including mental arithmetic. Simon et al. (2002) found a common area of activation in left IFG for arithmetic and phoneme detection tasks, though this area was also common to other tasks (i.e., grasping). Taken together, these studies do not provide evidence of precise overlapping brain regions that support arithmetic and phonological processing in the left IFG. However, the results of the current meta-analysis suggest there may be.

One reason for this discrepancy may be a lack of anatomical specificity across studies, as illustrated above. The discrepancy could also be due to variation in phonological tasks across studies. Tasks that place more demands on working memory may be associated with higher left IFG activity. A third potential reason for this discrepancy may be differences in participants across studies. The above studies all involved adults while the present meta-analysis involved children. There may be differentiation in brain regions that support arithmetic and phonological processing that occurs from learning that is reflected in adult samples that is not yet present in the developmental sample examined here. If this is the case, it may be that the role of the left IFG in supporting both arithmetic and phonological processing in a developmental sample relates to domain-general processes.

**Implications for Future Models**
Counter to expectation, the conjunction analysis did not produce overlapping areas of activation in temporoparietal areas for arithmetic and phonological processing. However, a qualitative comparison of the arithmetic and phonological processing meta-analyses (See Table 3, panels A and B) shows a cluster of concordant activity in left AG for arithmetic only and a cluster of concordant activity in left MTG for phonological processing only. Across studies, the left AG and the left MTG have been implicated in both arithmetic and phonological processing or have been shown to overlap directly (Grabner, Ansari, et al., 2009; Prado et al., 2011, 2014; Simon et al., 2002). However, the results in the present study do not support the notion that the same region in left temporoparietal cortex supports both arithmetic and phonological processing in children.

Most prior research relating arithmetic and phonology in the brain has been conducted with adults; developmental studies are few (e.g., Prado et al., 2014). Therefore, it is possible that with increased arithmetic fluency, temporoparietal areas that support both arithmetic and phonological processing would converge. It may also be that recruitment of left AG in arithmetic only and left MTG in phonological processing only may reflect differences in task difficulty or demands. It may be that the difference in difficulty between the phonological processing and control tasks was smaller than the difficulty between the arithmetic and control tasks.

Looking across the five areas of brain activation concordant to arithmetic and phonological processing may inform how arithmetic and phonological processing are related, cognitively and in the brain. Initial top-down attentional and decision-making resources may be supported by activity in right insula and superior frontal gyrus (e.g.,
Anderson et al., 2007; Menon & Uddin, 2010), and left IFG. Symbol recognition and processing may be supported by left fusiform gyrus activation. For arithmetic, recognition and processing may be at the digit level or may be at the fact level, in which the collection of symbols (e.g., 3 x 4) is treated as a higher-order symbolic representation associated with an answer (e.g., Grabner et al., 2013).

**Limitations and Next Steps**

Two important limitations of the present meta-analyses are the heterogeneity in participant ages and tasks used across studies. While all studies had developmental samples, age ranged from 8-17 years across arithmetic studies and from about 6-14 years across studies involving phonological processing. Research suggests that across development, reading is associated with an increase in brain activity in left lateralized frontal and temporal areas, such as IFG and MTG, and a decrease in activity in right lateralized regions (e.g., Turkeltaub, Gareau, Flowers, Zeffiro, & Eden, 2003), and functional specialization of the VWFA (Schlaggar & McCandliss, 2007). Similarly, brain regions that support arithmetic shift over development, reflecting a decrease in recruitment of frontal regions coupled with an increase in recruitment of temporal and parietal regions as children become more fluent with arithmetic such as multiplication facts (e.g., Prado et al., 2014; Zamarian et al., 2009). As a result, the present results capture what is common across development, but may not capture brain regions that support arithmetic processing during particular points in development. As more developmental arithmetic studies are available, future meta-analyses could consider smaller age ranges or contrast analyses across age groups. Further, since adults recruit temporoparietal regions more than children during arithmetic processing (Zamarian et
al., 2009), it is likely that clusters common to both arithmetic and phonological processing would differ for adults compared to children. For example, it is plausible that adults would show overlapping clusters of reliable activation in left temporoparietal cortex, such as in the AG or MTG. As another example, it may be that adults would not show common activation in left fusiform gyrus across arithmetic and phonological processing if specialization of this area occurs over development (Cantlon et al., 2011). Future meta-analyses could replicate the research in the present study for adults to test these hypotheses.

Heterogeneity across tasks also limits the results of the present study, particularly for the arithmetic meta-analysis, which included tasks that spanned addition, multiplication, and mixed addition and subtraction problems. Due to the substantial number of adult studies of arithmetic, Arsalidou and Taylor (2011) were able to conduct separate meta-analyses for different arithmetic operations; however, not enough developmental studies exist to conduct similar analyses here. Further, whether different arithmetic operations recruit different brain regions is still an open question. Several studies have found differentially active brain regions across arithmetic operations (e.g., Arsalidou & Taylor, 2011; Chochon, Cohen, Moortele, & Dehaene, 1999; Zhou et al., 2007). However, recent research suggests that neural differences attributed to arithmetic operations per se may be due to surface criteria of problems, and that neural differences may instead reflect differences in strategy use (Tschentscher & Hauk, 2014).

Conclusion
In the present study, I investigated whether arithmetic and phonological processing – related but distinct domains – may share overlapping brain regions during development. I first conducted separate meta-analyses for arithmetic and phonological processing using developmental studies, in order to determine concordant areas of activation particular to each meta-analysis. Results largely replicated prior research: arithmetic processing recruited primarily left lateralized frontal and temporal regions, and bilateral parietal regions. Phonological processing primarily recruited left lateralized brain regions in frontal, temporal, and occipitotemporal regions. Second, I conducted a conjunction analysis to determine brain regions that are reliably activated across both arithmetic and phonological processing. The analysis yielded five clusters spanning frontal and temporo-occipital regions. Contrary to expectations, no clusters were found in temporoparietal regions. Results suggest that some common areas of activation correspond to recruitment of attentional or executive functions during task completion, such as right insula, or relate to task demands, such as recruitment of precentral gyrus when tasks require a motor response. Other areas of concordant activation potentially reflect symbolic processing across arithmetic problems and words during development (e.g., left fusiform gyrus). Investigating brain regions that support both arithmetic and phonological processing during development can inform models of how these two processes are related and how the brain may support processing of higher order symbolic representations, such as arithmetic facts or words. Such work can in turn contribute to a better understanding of the neural correlates of learning throughout development.
Acknowledgments

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References


Figure 1. Hypothesized left lateralized brain regions involved in phonological processing and arithmetic. Red indicates the left IFG, light green indicates the left STG, dark green indicates the left MTG, and yellow indicates the inferior parietal lobule, the posterior portion of which is the left AG.
Table 1. Details of the thirteen studies in the arithmetic meta-analysis for the developmental sample, including sample size, mean age, contrast, and statistical threshold.

<table>
<thead>
<tr>
<th>Study</th>
<th>Reference</th>
<th>N</th>
<th>Mean age</th>
<th>Contrast</th>
<th>Statistical threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>Davis et al., 2009a</td>
<td>24</td>
<td>8.2 years</td>
<td>Addition &gt; Greek letter matching</td>
<td>p &lt; .001 uncorrected</td>
</tr>
<tr>
<td>A2</td>
<td>Davis et al., 2009b</td>
<td>19</td>
<td>8.1 years</td>
<td>Addition &gt; Greek letter matching</td>
<td>p = .05 FDR</td>
</tr>
<tr>
<td>A3</td>
<td>Meintjes et al., 2010</td>
<td>16</td>
<td>10.5 years</td>
<td>Addition &gt; Greek letter matching</td>
<td>p &lt; .05 FDR</td>
</tr>
<tr>
<td>A4</td>
<td>Ashkenazi et al., 2012</td>
<td>17</td>
<td>97.41 months</td>
<td>Complex addition &gt; Simple addition</td>
<td>p &lt; .01 FWE</td>
</tr>
<tr>
<td>A5</td>
<td>Metcalfe et al., 2013</td>
<td>74</td>
<td>7.8 years</td>
<td>Complex addition &gt; Simple addition</td>
<td>p &lt; .01 corrected</td>
</tr>
<tr>
<td>A6</td>
<td>Rosenberg-Lee et al., 2014</td>
<td>45 (2nd grade)</td>
<td>7.67 years</td>
<td>Complex addition &gt; Simple addition</td>
<td>p &lt; .01 FWE</td>
</tr>
<tr>
<td></td>
<td></td>
<td>45 (3rd grade)</td>
<td>8.67 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A7</td>
<td>Demir et al., 2014</td>
<td>40</td>
<td>10.9 years</td>
<td>Multiplication &gt; Fixation</td>
<td>p &lt; .05 cluster-wise</td>
</tr>
<tr>
<td>A8</td>
<td>Kawashima et al., 2004</td>
<td>8</td>
<td>11.6 years</td>
<td>Multiplication &gt; Fixation</td>
<td>p &lt; .05 corrected</td>
</tr>
<tr>
<td>A9</td>
<td>Kesler et al., 2006</td>
<td>15</td>
<td>14.6 years</td>
<td>Mixed addition and subtraction &gt; Digit strings</td>
<td>p &lt; .05 corrected</td>
</tr>
<tr>
<td>A10</td>
<td>Rivera et al., 2002</td>
<td>16</td>
<td>16.97 years</td>
<td>Mixed addition and subtraction &gt; Digit strings</td>
<td>p &lt; .01 cluster-wise</td>
</tr>
<tr>
<td>A11</td>
<td>Price et al., 2013</td>
<td>33</td>
<td>17 years 11.5 months</td>
<td>Mixed addition and subtraction &gt; Digit matching</td>
<td>p &lt; .05 FDR</td>
</tr>
<tr>
<td>A12</td>
<td>Kucian et al., 2006</td>
<td>10 (3rd grade)</td>
<td>9.2 years</td>
<td>Addition &gt; Grayscale matching</td>
<td>p &lt; .005 FDR</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10 (6th grade)</td>
<td>12.0 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A13</td>
<td>Chen et al., 2006</td>
<td>8 (abacus experts)</td>
<td>11.75</td>
<td>Serial addition &gt; Viewing numbers</td>
<td>p &lt; .0001 uncorrected</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8 (non-experts)</td>
<td>12.29</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2. Details of the fourteen studies in the phonological processing meta-analysis, including sample size, mean age, contrast, and statistical threshold.

<table>
<thead>
<tr>
<th>Study</th>
<th>Reference</th>
<th>N</th>
<th>Mean age</th>
<th>Contrast</th>
<th>Statistical threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>Bitan et al., 2007</td>
<td>36</td>
<td>11.7 years</td>
<td>Rhyme words &gt; Fixation</td>
<td>$p &lt; .0001$ uncorrected</td>
</tr>
<tr>
<td>P2</td>
<td>Cao et al., 2008</td>
<td>12</td>
<td>12.3 years</td>
<td>Rhyme words &gt; Fixation</td>
<td>$p &lt; .001$ uncorrected</td>
</tr>
<tr>
<td>P3</td>
<td>Hoeft et al., 2007</td>
<td>64</td>
<td>10 years</td>
<td>Rhyme words &gt; Fixation</td>
<td>$p &lt; .01$ FDR</td>
</tr>
<tr>
<td>P4</td>
<td>Hoeft et al., 2006</td>
<td>10 (5th grade)</td>
<td>10.95 years</td>
<td>Rhyme words &gt; Fixation</td>
<td>$p &lt; .001$ uncorrected</td>
</tr>
<tr>
<td></td>
<td>10 (3rd grade)</td>
<td>8.75 years</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P5</td>
<td>Cao et al., 2006</td>
<td>14</td>
<td>11.5 years</td>
<td>Rhyme words &gt; Fixation</td>
<td>$p &lt; .001$ uncorrected</td>
</tr>
<tr>
<td>P6</td>
<td>McNorgan et al., 2011</td>
<td>14 (young group)</td>
<td>9.3 years</td>
<td>Rhyme words &gt; Fixation</td>
<td>$p &lt; .05$ FDR</td>
</tr>
<tr>
<td></td>
<td>12 (older group)</td>
<td>13.5 years</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P7</td>
<td>Temple et al., 2001</td>
<td>15</td>
<td>10.5 years</td>
<td>Rhyme letter &gt; Match letter</td>
<td>$p &lt; .025$</td>
</tr>
<tr>
<td>P8</td>
<td>Backes et al., 2002</td>
<td>8</td>
<td>11.6 years</td>
<td>Pseudoword rhyming &gt; Fixation</td>
<td>$p &lt; .05$</td>
</tr>
<tr>
<td>P9</td>
<td>Noble et al., 2006</td>
<td>38</td>
<td>7 years 11 months</td>
<td>Pseudoword reading &gt; Fixation</td>
<td>$p &lt; .0001$ uncorrected</td>
</tr>
<tr>
<td>P10</td>
<td>van der Mark et al., 2009</td>
<td>24</td>
<td>11.3 years</td>
<td>Pseudoword reading &gt; Fixation</td>
<td>$p &lt; .05$ FDR</td>
</tr>
<tr>
<td>P11</td>
<td>Georgiewa et al., 1999</td>
<td>17</td>
<td>14.4 years</td>
<td>Pseudoword reading &gt; Font strings</td>
<td>$p &lt; .05$</td>
</tr>
<tr>
<td>P12</td>
<td>Bach et al., 2013</td>
<td>19</td>
<td>6.4 years</td>
<td>Word decoding &gt; Symbol</td>
<td>$p &lt; .005$ cluster extent threshold</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>identification</td>
<td></td>
</tr>
<tr>
<td>P13</td>
<td>Bach et al., 2010</td>
<td>18</td>
<td>8.3 years</td>
<td>Different letter substitution</td>
<td>$p &lt; .005$ cluster extent threshold</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Same letter substitution</td>
<td></td>
</tr>
<tr>
<td>P14</td>
<td>Yamada et al., 2011</td>
<td>7</td>
<td>5.7 years</td>
<td>Letter 1-back &gt; False fonts</td>
<td>$p &lt; .05$ uncorrected</td>
</tr>
</tbody>
</table>
Table 3. Activation likelihood estimation results for arithmetic and phonological processing, and the contrast analysis in the developmental sample, including cluster, Talairach coordinate, ALE value, volume, and contributing studies.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Talairach coordinates</th>
<th>ALE value</th>
<th>Volume (mm$^3$)</th>
<th>Contributing studies</th>
</tr>
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<tbody>
<tr>
<td>(A) Arithmetic processing</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Right Insula (BA 13)</td>
<td>32 18 6</td>
<td>0.034677282</td>
<td>2096</td>
<td>A1, A2, A3, A5, A6, A11, A12</td>
</tr>
<tr>
<td>Right Cingulate Gyrus (BA 32)</td>
<td>4 20 42</td>
<td>0.019645344</td>
<td>2024</td>
<td>A1, A2, A3, A4, A6, A10, A11, A13</td>
</tr>
<tr>
<td>Left Insula (BA 13)</td>
<td>-30 18 6</td>
<td>0.030592252</td>
<td>1440</td>
<td>A4, A5, A6, A11, A12</td>
</tr>
<tr>
<td>Left Precentral Gyrus (BA 6)</td>
<td>-48 0 34</td>
<td>0.016053366</td>
<td>696</td>
<td>A7, A8, A11, A12</td>
</tr>
<tr>
<td>Left Angular Gyrus (BA 39)</td>
<td>-30 -60 38</td>
<td>0.01573684</td>
<td>528</td>
<td>A3, A11, A12</td>
</tr>
<tr>
<td>Left Middle Frontal Gyrus (BA 6)</td>
<td>-26 -4 58</td>
<td>0.015526843</td>
<td>496</td>
<td>A4, A11, A12</td>
</tr>
<tr>
<td>Left Fusiform Gyrus (BA 37)</td>
<td>-46 -56 -14</td>
<td>0.016054135</td>
<td>424</td>
<td>A9, A10, A12</td>
</tr>
<tr>
<td>Right Inferior Parietal Lobule (BA 40)</td>
<td>40 -54 52</td>
<td>0.01471246</td>
<td>344</td>
<td>A4, A9, A12</td>
</tr>
<tr>
<td>Left Inferior Parietal Lobule (BA 40)</td>
<td>-42 -46 42</td>
<td>0.014944481</td>
<td>320</td>
<td>A5, A10</td>
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<tr>
<td>(B) Phonological processing</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Left Middle Temporal Gyrus (BA 22)</td>
<td>-56 -38 2</td>
<td>0.022555953</td>
<td>1152</td>
<td>P1, P2, P6, P8, P12, P14</td>
</tr>
<tr>
<td>Left Inferior Frontal Gyrus (BA 9)</td>
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<td>0.014821018</td>
<td>1072</td>
<td>P3, P5, P6, P9</td>
</tr>
<tr>
<td>Left Medial Frontal Gyrus (BA 6)</td>
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<td>0.017439213</td>
<td>1056</td>
<td>P1, P3, P4, P5, P10</td>
</tr>
<tr>
<td>Right Insula (BA 13)</td>
<td>36 20 4</td>
<td>0.017645713</td>
<td>944</td>
<td>P1, P4, P6, P14</td>
</tr>
<tr>
<td>Left Fusiform Gyrus (BA 37)</td>
<td>-44 -52 -12</td>
<td>0.020551741</td>
<td>744</td>
<td>P4, P6, P9</td>
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<tr>
<td>Left Inferior Frontal Gyrus (BA 45)</td>
<td>-42 24 14</td>
<td>0.017423127</td>
<td>528</td>
<td>P1, P2, P5</td>
</tr>
<tr>
<td>Region</td>
<td>X</td>
<td>Y</td>
<td>Z</td>
<td>p-Value</td>
</tr>
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<td>---------------------------------------------</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>-------------</td>
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<tr>
<td>Right Insula (BA 13)</td>
<td>36</td>
<td>20</td>
<td>4</td>
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<td>Left Superior Frontal Gyrus (BA 6)</td>
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<td>Left Precentral Gyrus (BA 6)</td>
<td>-50</td>
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<td>36</td>
<td>0.012467423</td>
</tr>
<tr>
<td>Left Fusiform Gyrus (BA 37)</td>
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<td>-54</td>
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<tr>
<td>Left Inferior Frontal Gyrus (BA 9)</td>
<td>-50</td>
<td>6</td>
<td>32</td>
<td>0.010398336</td>
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</table>
Figure 2. Clusters of concordant activation across arithmetic tasks: (a) right cingulate gyrus, left insula, left inferior frontal gyrus, left superior frontal gyrus, left inferior parietal lobule and left angular gyrus (adjacent clusters), (b), left fusiform gyrus (lower left), and (c) right inferior parietal lobule, right insula, and right cingulate gyrus.
Figure 3. Clusters of concordant activity across phonological processing tasks: (a) left medial frontal gyrus, left inferior frontal gyrus (BA 46), left inferior frontal gyrus (BA 46), left middle temporal gyrus, (b) fusiform gyrus (lower left), and (c) right insula, left medial frontal gyrus, and left inferior frontal gyrus (BA 46).
Figure 4. Five clusters of reliable brain activation across studies of arithmetic and phonological processing: (a) superior frontal gyrus, inferior frontal gyrus and precentral gyrus (adjacent clusters); (b) right insula and left fusiform gyrus (lower left), and (c) right insula and left superior frontal gyrus.
Chapter 3

Mental Representations of Numerical Magnitude Across Symbolic Formats
Abstract

The ability to understand numerical symbols is important for success in mathematics. In particular, literal symbols (e.g., $x$) in higher-level mathematics such as algebra are often used to represent numerical magnitude. However, compared to Arabic numerals, literal symbols may require more complex mental representations because these symbols have strong pre-existing associations in literacy that may interfere with numerical referents.

The present study tested this notion using the same-different distance paradigm that typically produces longer reaction times and higher error rates for different magnitudes that are closer together compared to farther apart (i.e., a same-different distance effect). Twenty-four high school-aged adolescents completed three same-different tasks using Arabic numerals, literal symbols, and artificial symbols. All three symbolic formats produced a same-difference distance effect, suggesting that both literal and artificial symbols can access mental representations of magnitude. However, there was longer reaction time to process literal symbols compared to artificial symbols, which suggests a processing cost associated with literal symbols that may reflect interference with extant mental representations related to literacy. Further, while there was a moderate positive correlation for reaction time between Arabic numeral and artificial symbol distance effects, reaction times in the literal symbols distance effect did not correlate with reaction times in the other representations. Taken together, the results of the present study suggest that different mechanisms may support literal symbol processing in a mathematics context.
Mental Representations of Numerical Magnitude Across Symbolic Formats

**Introduction**

The ability to understand numerical symbols and their relation to one another is important for success in mathematics. Indeed, prior research has shown a link between the ability to efficiently compare two Arabic numerals and mathematics achievement in children and adults (e.g., Castronovo & Göbel, 2012; De Smedt, Noël, Gilmore, & Ansari, 2013; Holloway & Ansari, 2009; Schneider et al., 2016). However, the ability to be successful in higher-level mathematics relies on other symbolic representations of numerical magnitude besides Arabic numerals. In particular, fluency with literal symbols (e.g., “x”) is necessary for learning algebra and subsequent mathematics such as calculus (Rosnick, 1980, 1982; Schoenfeld & Arcavi, 1999). The present study concerns differences in mental representations associated with these symbols compared to Arabic numerals.

Students experience substantial and persistent difficulty working with literal symbols in math, from first exposure through college algebra (Akgün & Özdemir, 2006; Booth, 1999; Christou & Vosniadou, 2005; MacGregor & Stacey, 1997; McNeil et al., 2010; Philipp, 1992; Rosnick, 1982; Trigueros & Ursini, 2003; Ursini & Trigueros, 2004). Prior research points to myriad causes for this difficulty. For example, students’ difficulty in working with literal symbols may be due in part to mathematical syntax (Schoenfeld & Arcavi, 1999), novel algebraic notational conventions (Kieran, 2007), or vague or absent explanations in mathematics curricula (Rosnick, 1982).

Research also suggests that difficulty in working with literal symbols may be due to the mental representations associated with these symbols (MacGregor & Stacey, 1997;
Philipp, 1992; Rosnick, 1982, 1999; Stacey & MacGregor, 1999). Literal symbols in mathematics require more complex mental representations that may compete with existing mental referents related to language or objects. For instance, young students may confuse the magnitude of a literal symbol with its numerical position in the alphabet (MacGregor & Stacey, 1997). Students may also incorrectly associate letters with objects (e.g., \( a = \text{apples} \)) rather than the related magnitude (e.g., \( a = \text{the number of apples} \)) or confuse the two associations (McNeil et al., 2010; Rosnick, 1982). Associating letters with objects can interfere with the idea that literal symbols can have a numerical referent (Booth, 1999) and can lead to difficulties with word problems that persist into college (McNeil et al., 2010; Philipp, 1992; Rosnick, 1982).

In addition, forming mental representations for literal symbols is difficult because of properties inherent to how literal symbols function in math. Students may see that Arabic numerals and literal symbols co-occur in algebra problems (e.g., \( 3x = 12 \)), but the two symbolic representations are qualitatively different in three main ways. First, literal symbols are not specific to numeracy, as Arabic numerals are. Children first form mental representations related to literal symbols in the context of literacy several years before learning about them in math. When children see literal symbols in math, they already have strong associations for what those symbols represent for reading and writing. Second, literal symbols do not have a consistent magnitude across mathematical contexts as numbers do (e.g., \( x \) can be 3, -4, or \( \frac{1}{2} \) across mathematics problems). Third, literal symbols do not have a single magnitude, as numbers do (e.g., ‘3’ never stands for both 3 and 4 objects; \( x \) can stand for two numbers, all real numbers, or no number at all [e.g., Usiskin, 1999]).
As symbols that represent numerical magnitude, literal symbols signify a substantial departure from the quantity-symbol mapping that students have learned previously with Arabic numerals or number words. It is possible that even for the most restricted use of literal symbols - when they convey one particular magnitude (e.g., \( Q = 1 \)) as Arabic numerals do - pre-existing mental representations of literal symbols may interfere with the numerical referent. Yet, little is known about how mental representations for literal symbols compare to analogous representations for Arabic numerals, even when they convey a single magnitude. Additional research in this area can shed light on fundamental cognitive mechanisms underlying student difficulties with literal symbols.

**Measuring Mental Representations of Numerical Magnitude**

Number comparison tasks are a standard way to investigate mental representations associated with numerical magnitude (e.g., Dehaene, Dehaene-Lambertz, & Cohen, 1998; Schneider et al., 2016). In these tasks, participants may view two arrays of dots or two Arabic numerals and judge which one is larger in number. Error rates and reaction times decrease as the distance between the two numbers increase, a behavioral signature known as the Comparison Distance Effect (CDE) (Moyer & Landauer, 1967). The CDE may arise from overlapping mental representations of numerical magnitude that are arranged on a mental number line (e.g., Restle, 1970). Numbers that are closer together are harder to compare since their mental representations overlap more than numbers that are farther apart. However, several studies have challenged the notion that the CDE is caused by an overlap in mental representations of numerical magnitude *per se*, arguing that it may instead be due to decision-making processes associated with
comparison itself (Cohen Kadosh, Brodsky, Levin, & Henik, 2008; van Opstal, Gevers, De Moor, & Verguts, 2008; Verguts, Fias, & Stevens, 2005). Researchers have used alternative tasks to elicit distance effects that they believe reflect mental representations of numerical magnitude rather than decision-making processes, such as a priming task (e.g., Reynvoet, de Smedt, & Van den Bussche, 2009) or a same-different task (e.g., van Opstal & Verguts, 2011).

In a priming paradigm, participants view two numbers sequentially and compare the second number in the pair (i.e., the target) to a fixed standard (e.g., 5). Reaction times and error rates are smaller when the distance between the prime and target is closer together. This is known as the priming distance effect (PDE), which occurs because the first number acts as a prime for the second (e.g., Defever, Sasanguie, Gebuis, & Reynvoet, 2011; Van Opstal et al., 2008). For example, if a participant sees 3 and then 4, she will be faster to compare 4 to 5 than if she saw 1 and then 4, since thinking about 3 also partially activates her mental representation for 4.

Pollack, Leon Guerrero, & Star (2015) used a priming paradigm in an initial attempt to investigate the differences in mental representations between literal symbols and Arabic numerals in adults. To do so, they tested whether a PDE was present for literal symbols when the symbols were assigned a particular numerical magnitude (e.g., $Y = 9$) to use during comparison. Literal symbols could either be the prime (i.e., the first number in the pair) or the number that is compared to the fixed standard (i.e., the second number in the pair). In line with prior literature (e.g., Defever et al., 2011), Pollack et al. (2015) found a PDE when Arabic number pairs were used. However, they did not find a PDE when literal symbols were used, either as a prime or when compared to the fixed
standard, even though participants learned the numerical magnitudes with a high level of accuracy. The authors suggested that literal symbols may have fundamentally different mental representations, even in this limited context. However, it was also possible that a PDE was absent because the priming paradigm did not force participants to process numerical magnitude. Participants compared each number or literal symbol to ‘5’ by pressing a left keyboard button for ‘less than 5’ and a right keyboard button for ‘more than 5.’ It is possible that instead of making the numerical magnitude judgment, participants merely associated particular numbers or literal symbols (e.g., 4, x) with ‘left’ (i.e., less than 5) or ‘right’ (i.e., more than 5) button presses.

To address this limitation, the present study employs a same-different task rather than a priming task. In same-different tasks, participants simultaneously view two number symbols and decide whether the number symbols represent the same or different numerical magnitudes. These tasks produce a Same-Difference Distance Effect (SDDE) in which reaction times and error rates decrease as numerical distance increases. A same-different task is well suited to investigate differences in mental representations between Arabic numerals and literal symbols. As noted above, researchers believe that the same-different task taps mental representations of numerical magnitude rather than decision-related processes (e.g., van Opstal & Verguts, 2011). Prior studies have employed these tasks with two dot arrays, two number words, two Arabic digits, or a cross-format combination, for example comparing ‘TWO’ to ‘7’ (Defever, Sasanguie, Vandewaetere, & Reynvoet, 2012; Dehaene & Akhavein, 1995; Duncan & McFarland, 1980; van Opstal & Verguts, 2011).
The present study uses cross-format comparisons, since they require participants to process numerical magnitude and consistently elicit an SDDE. For instance, Defever et al. (2012) found an SDDE in children who compared dot arrays and Arabic numerals. Van Opstal and Verguts (2011) found an SDDE for adults during cross-format symbolic comparisons (e.g., ONE - 2) that were grouped into near trials (distance of 1) and far trials (distances of 5 to 7). Importantly, van Opstal & Verguts (2011) used the same-different task to look for an SDDE with both numerical stimuli and alphabetic, non-numerical stimuli (e.g., g - T). They found an SDDE for the numerical stimuli only, which suggests that letters devoid of numerical magnitude do not produce an SDDE based on their position in the alphabet. Taken together, these findings suggest that a cross-format same-different task can inform the nature of mental representations related to literal symbols when they correspond to particular numerical magnitudes.

The Present Study

Building on Pollack et al. (2015), the present study sought to further investigate whether literal symbols that have been given one numerical magnitude can access the same mental representations as Arabic numerals or whether extant mental representations for literal symbols may interfere with numerical magnitude processing. In this study, I contrasted high school-aged adolescents’ performance on a series of cross-format symbolic same-different judgments. Participants first completed a same-different task with Arabic numerals and number words (number condition). In line with prior literature (e.g., van Opstal & Verguts, 2011), I expected to find an SDDE on reaction times and error rates in the number condition. Participants then memorized assigned numerical magnitudes for literal symbols and performed another same-different task with Arabic
numerals and literal symbols (literal symbols condition). If literal symbols can access the same mental representations as Arabic numerals, I expected an SDDE for reaction times in the literal symbols condition. However, if literal symbols do not access the same mental representations as Arabic numerals, I expected no SDDE to be present.

In order to test whether there may be competing mental representations for literal symbols, participants also completed a same-different task with Arabic numerals and artificial symbols that have no prior meaning in mathematics (artificial symbols condition). If existing mental representations (e.g., linguistic associations) interfere with numerical magnitude processing of literal symbols, then I expected differences in the SDDE between the literal symbols and artificial symbols conditions. For instance, it may be that literal symbols do not access mental representations of magnitude, but artificial symbols do, in which case an SDDE would be present for artificial symbols only. Alternatively, it may be that both literal and artificial symbols access mental representations of magnitude, but that competing mental representations for literal symbols manifest as increased error rates or reaction times in the literal symbols condition compared to the artificial symbols condition, since the latter symbols do not have existing mental representations.

Across all conditions, it is possible that participants’ performance would vary based on working memory capacity. For instance, it may be that participants with higher working memory capacity may respond more accurately and quickly than participants with low working memory capacity. Recent research suggests that taxing working memory may affect performance on some number comparison tasks (e.g., van Dijck & Fias, 2011; van Dijck, Gevers, & Fias, 2009). In addition, because participants learned
novel associations between literal and artificial symbols and then performed same-different judgments with those symbols, working memory may play a role. Therefore, participants completed a measure of working memory, described below, that was considered in the analysis for each of the three conditions.

Finally, if there were an SDDE in each of the three conditions, different mechanisms may underlie each. Specifically, it may be that Arabic numerals and artificial symbols map to numerical referents in a similar way, since both are symbols that are only associated with magnitude. In contrast, literal symbols may associate with their numerical referents via a different mechanism, since these symbols have extant non-numeric mental representations. To test this, I examined the correlations among the SDDEs across the three conditions for both error rate and reaction time.

**Methods**

**Participants**

Twenty-four typically developing students aged 14-18 ($M = 16.63$, $SD = 1.28$, 67% female, 87% right-handed) from the Boston area participated. Participants were recruited via flyers, online postings, and through schools. All participants were native English speakers, since the language in which numbers are learned (e.g., Chinese versus English) can contribute to differences in mathematics performance (Dehaene, 1997). To ensure that participants had adequate introductory exposure to literal symbols, all participants had previously passed Algebra I and either had taken, or were concurrently enrolled in, a subsequent mathematics class, such as Geometry, Algebra II, or Calculus. However, participants’ highest level of mathematics was not recorded. All participants who were 18 years old provided consent and those under 18 years old provided assent.
along with parental permission. The study was approved by the Committee on the Use of Human Subjects at Harvard University and participants received small monetary compensation for participating.

**Stimuli and Procedures**

The study took place over one one-hour testing session in which each participant completed six tasks: three same-different numerical magnitude comparison tasks, two training tasks, and a working memory task. The five comparison and training tasks were designed using OpenSesame 2.8.3 (Mathôt, Schreij, & Theeuwes, 2012) and were presented on a Google Nexus 7 tablet using the OpenSesame Experiment Runtime Application. To assess working memory, participants completed the backward digit span task (Mueller, 2011), which was administered using The Psychology Experiment Building Language (Mueller & Piper, 2014) on a Macbook Pro running the OSX operating system.

Participants completed the tasks while sitting in a quiet space either at their school, in a lab at a university campus, or at a public library. For the tablet tasks, students held the tablet in landscape orientation with both hands and touched each side of the screen using their thumbs. For the backward digit span task, participants typed digits using the laptop keyboard. All participants began with a number comparison task using Arabic numerals and number words. In-between, students completed two sets of tasks that involved learning pairs of symbols and numbers and performing subsequent symbolic comparison. One set involved a learning task in which participants learned numerical equivalents for literal symbols (e.g., $H = 8$) and then compared the literal symbols to Arabic numerals. The second set involved a learning task with artificial
symbols (e.g., \( \mathcal{H} = 1 \)) and then a comparison of the artificial symbols to Arabic numerals. The artificial symbols were Gibson figures from prior artificial symbol learning paradigms (e.g., Cohen Kadosh, Soskic, Iuculano, Kanai, & Walsh, 2010; Tzelgov, Yehene, Kotler, & Alon, 2000). Table 1 provides the complete set of literal and artificial symbols, and their numerical equivalents. The literal symbols and artificial symbols conditions were counterbalanced across participants; half were randomly assigned to complete the literal symbols set first and the other half completed the artificial symbols set first. All participants ended the experiment with the backward digit span task.

[Insert Table 1]

**Same-Different Judgment with Arabic Numerals**

In the first task, participants saw cross-notation pairs of Arabic numerals and number words for the numbers 1, 2, 7, and 8, and the number words “ONE,” “TWO,” “SEVEN,” and “EIGHT.” Both symbols were Arial font size 72 and number words were uppercase. Cross-notation pairs were used to eliminate a visual matching strategy that can interfere with semantic processing (Defever et al., 2012; van Opstal & Verguts, 2011) and for consistency with the other same-different tasks. Participants judged whether the pairs represented the same or different magnitude. “Same” pairs (e.g., ONE - 1) had a distance of zero and different pairs were either Near, with a distance of one (e.g., TWO - 1), or Far, with a distance of five, six, or seven (e.g., 8 - ONE). There were 8 trials of Same pairs, each shown four times, 8 trials of Near pairs, each shown twice, and 16 trials of Far pairs, each shown twice, for a total of 80 trials per block. As in van Opstal & Verguts (2011, see Experiment 2), participants saw Near and Far pairs equally often;
Same pairs were shown twice as often to provide approximately the same number of left and right responses.

Figure 1 illustrates a sample trial. Each trial began with a fixation dot displayed for 500 ms, followed by the number pair, which remained on the screen until response. Participants touched the left side of the screen if the number pair represented the same magnitude and touched the right side of the screen if the number pair symbolized a different magnitude. The trial ended with an intertrial interval of 500 ms.

[Insert Figure 1]

**Literal symbols**

In the literal symbols condition, participants first learned four symbol-magnitude associations, similar to a procedure used in Pollack et al. (2015; see Table 1). The goal was for participants to equate the literal symbols with numerical magnitude and to use the symbols in a subsequent same-different task. Initially, participants took as much time as needed (minimum 20 seconds) to learn the four associations (e.g., $H = 8$). To test the associations, participants then saw a symbol and were asked to recall the associated number, both aloud and by selecting the number from a choice screen containing the four possible numbers. The positions of the numbers on the choice screen changed for each trial. The task began with 8 practice trials, two of each association. Trials continued until the participant reached an accuracy threshold of 95% with at least 21 additional trials, indicating they had sufficiently learned the associations. On average, participants completed 29 trials ($SD = 23$, Range: 21-109). This process took approximately five minutes in total.
A trial began when a literal symbol was displayed for 750 ms. The participant recalled the corresponding magnitude aloud and selected it from a choice screen that displayed 1, 2, 7, and 8, each in one quadrant of the screen, until response. For each trial, the participant received feedback of “Good job!” or “Oops!,” displayed for 750 ms, The correct symbol-magnitude association was reinforced (e.g., $H = 8$) for 750 ms and the trial ended with an intertrial interval of 500 ms. Trials were presented in a pseudorandom order that was the same for all participants to test all associations approximately equally.

Once the participants had learned the associations, they completed a same-different task, which used the same number pairs as the numerical same-different task above. However, in the different number pairs, the number words “one,” “two,” “seven,” and “eight” were instead the literal symbols Q, G, R, and H (letters are adopted from Van Opstal & Verguts, 2011). Half of the same pairs had the literal symbol on the left (e.g., 1-Q) and the other half had the literal symbol on the right (e.g., Q-1). All procedures were the same as the numerical same-different task.

**Artificial Digits**

As in the literal symbols condition, participants completed a learning task and performed a subsequent same-different task, using the same pairs as above. In the learning task, each participant successfully completed the task with 8 practice and 21 additional trials. In the same-different task, different number pairs consisted of the numbers 1, 2, 7, and 8, and four artificial digits, which were the equivalent Gibson figures adopted from Tzelgov et al. (2000; see Table 1). As in the literal symbols condition, half of the same pairs had the artificial digit on the left and the other half had
the artificial digit on the right. All other procedures were the same in the literal symbols condition.

**Backward Digit Span**

Finally, each participant completed a backward digit span task (Mueller & Piper, 2014) as a control for working memory capacity. In this task, each participant saw a sequence of numbers 3 to 10 digits in length. After the sequence ended, the participant typed the sequence in reverse order. If the participant gave the correct response, one number was added to the next sequence. If incorrect, the participant got another sequence of the same length. Length and number of correct items were recorded; participants’ backward digit span was measured as the previous string length after two unsuccessful recalls at the same string length.

**Data Analysis**

Following Sasanguie, Defever, Van den Bussche, & Reynvoet (2011), same pairs were excluded from further analysis since the SDDE only manifests between same and different pairs. In order to test the SDDE, mean error rate was calculated for each participant, separately for near and far pairs for each of the three conditions (i.e., numerical, literal symbols, artificial symbols). Median reaction time was also calculated for each participant for correct responses, separately for the same and different trial types in each of the three conditions. Error rate and reaction time were modeled as separate outcomes for the number condition and for the non-numeric symbol conditions using random-effects multi-level modeling. I describe the model for each analysis in the results section below. All statistical tests were conducted at the \( \alpha = .05 \) level.
In order to investigate whether there were correlations among the SDDE for numbers, literal symbols, and artificial symbols, individual distance effects were first calculated for each participant for both error rate and reaction time for each condition. Then, both Pearson and Spearman correlations were calculated. In line with prior research (Sasanguie et al., 2011), for reaction time distance effects, median reaction time for near trials was subtracted from median reaction time in far trials and then divided by the median reaction time for near trials. For error rate distance effects, the rate for near trials was subtracted from the error rate for far trials.

**Results**

This study sought to use the SDDE to test whether non-numeric symbols could access the same mental representations as Arabic numerals. To do so, the first aim was to replicate the SDDE using Arabic numerals and number words. The second aim was to test for the presence of an SDDE using four literal symbols and four artificial symbols that had been assigned arbitrary magnitudes. The final aim was to explore whether similar mechanisms may underpin the SDDEs across the three conditions. I discuss each analysis in turn.

**A Same-Different Distance Effect with Arabic Numerals**

**Model.** The relationship between distance and the two outcomes was estimated using a multi-level modeling approach. This approach is well suited to account for repeated-measures in which distance (i.e., near, far) is nested within participants, while also allowing for the inclusion of covariates. Equation (1) describes the random-effects two-level multi-level model:

\[ Y_{ij} = \alpha_0 + \alpha_1 \text{NEAR}_{ij} + \alpha_2 X_j + (e_i + u_j) \]  

(1)
in which distance is nested within participant. In Equation 1, \( Y_{ij} \) represents each outcome (i.e., error rate, reaction time) for each distance \( i \) and each participant \( j \). \( \text{NEAR}_{ij} \) is a dichotomous predictor representing near trials (with far trials as the reference category), and \( X_j \) represents a vector of participant level covariates. Effects of backward digit span, age, and gender were tested to ensure that any performance differences were not related to these demographic characteristics. The parameter of interest from Equation 1 is \( \alpha_1 \), which represents the difference between near trials and far trials.

**Findings.** Across all participants, mean error rate for the number condition was 3.56 (\( SD = 3.29 \)) and average median reaction time was 842.85 ms (\( SD = 173.81 \) ms). Participants had a mean backward digit span of 6.71 (\( SD = 1.55 \), range: 4 – 10). The left two columns of Table 2 display fitted models for error rate and reaction time in the number condition, including parameter estimates, standard errors, and goodness-of-fit statistics.

[Insert Table 2]

The first column of Table 2 shows the estimated effect of distance on error rate. Shapiro-Wilk tests for residual normality showed a violation of level 1 (\( z = 3.91, p < .0001 \)) and level 2 (\( z = 3.42, p < .001 \)) residual normality, so the model was fit with bootstrapped standard errors (200 replications). There was a statistically significant effect of distance on error rate, such that the error rate for near trials was 1.78 percentage points higher than for far trials, on average (\( z = 3.65, p < .0001 \)). A second model was fit including backward digit span, gender, and age, and none of the covariates individually had a statistically significant relationship with error rate (\( ps \) ranged from .07 to .93). A General Linear Hypothesis (GLH) test was conducted to test the null hypothesis that
these three covariates jointly did not predict error rates. By testing the effect of these three predictors as a set, the GLH test conserves Type-I error. The null hypothesis could not be rejected ($\chi^2(df=3) = 3.42, p = 0.33$) and so the covariates were not retained in the analysis. Panel A in Figure 2 illustrates the fitted relationship between distance and error rate in the number condition.

[Insert Figure 2]

The second column of Table 2 shows the estimated relationship between distance and reaction time. Because Shapiro-Wilk tests for residual normality showed a violation of level 1 ($z = 2.50, p = .006$) and level 2 ($z = 1.67, p = .047$) residual normality, bootstrapped standard errors were used (200 replications). There was a statistically significant relationship between distance and reaction time, such that participants took an average of about 38 ms longer to respond to near trials compared to far trials ($z = 3.58, p < .0001$). As above, a model including backward digit span, gender, and age showed that none has a statistically significant relationship with reaction time (all $ps > .33$). A GLH test showed that their joint effect on reaction time was statistically indistinguishable from zero ($\chi^2(df=3) = 1.37, p = 0.71$), and so the covariates were not included further. Panel B of Figure 2 illustrates the relationship between distance and reaction time in the number condition. Together, the error rate and reaction time results contribute additional support for an SDDE in adolescents using Arabic numerals and number words.

**Same-Different Distance Effects Across Literal and Artificial Symbols**

**Model.** To investigate whether there is an SDDE for non-numeric symbols, a two-level random-effects multi-level model was used. Equation (2) describes the model:

$$Y_{ij} = \beta_0 + \beta_1 \text{NEAR}_{ij} + \beta_2 \text{LS}_{ij} + \beta_3 X_j + (e_{ij} + u_j)$$  (2)
in which distance is nested within participant. In this model, $Y_{ij}$ represents each outcome (i.e., error rate, reaction time) for each distance $i$ and each participant $j$. $NEAR_{ij}$ is a dichotomous predictor that represents distance, $LS_{ij}$ is a dichotomous predictor that indicates whether the distances are for the literal symbols condition ($LS_{ij} = 1$) or artificial symbols condition, and $X_j$ represents the participant level covariates, which include backward digit span score, age, gender, and whether the participant completed the artificial symbols or literal symbols condition first. The latter was included to test for potential order effects beyond counterbalancing the non-numeric same-different tasks across participants. There are two parameters of interest; $\beta_1$, which captures the effect of distance on error rate and reaction time, respectively, and $\beta_2$, which represents the average difference in error rate or reaction time between the literal and artificial symbol conditions.

Findings. Due to a software malfunction, one participant’s data for the literal symbols same-different condition was not recorded. For the literal symbols condition, the overall sample mean error rate was 4.91 ($SD = 5.44$) and sample average median reaction time was 1073.38 ms ($SD = 331.41$ ms). For the artificial symbols condition, the overall sample mean error rate was 4.58 ($SD = 4.05$) and the overall sample average median reaction time was 948.16 ms ($SD = 197.65$ ms), both lower in comparison to performance in the literal symbols condition. The right two columns in Table 2 display the fitted models for error rate and reaction time, and include parameter estimates, standard errors, and goodness-of-fit statistics.

For error rate, Shapiro-Wilk tests showed violations of level 1 ($z = 4.23, p < .0001$) and level 2 ($z = 2.70, p = .003$) residual normality, so bootstrapped standard errors
were used (200 replications). The third column of Table 2 shows the estimated relationship between error rate and distance and condition (i.e., literal or artificial symbols). There was a statistically significant relationship between distance and error rate, such that error rate for near trials is 2.68 percentage points higher than far trials, on average, controlling for condition \(z = 5.80, p < .0001\). There was not a statistically significant main effect of condition \(z = -0.74, p = 0.46\). A subsequent model showed that there were no statistically significant relationships between backward digit span, age, gender, or testing order and error rate (all \(p_s \geq .2\)). A GLH test showed there was no joint effect of these covariates on error rate \(\chi^2(df = 4) = 1.91, p = .75\), so they were excluded from the model. The fitted relationship between distance and error rate for literal and artificial symbols is illustrated in Figure 3.

[Insert Figure 3]

The final column of Table 2 shows the estimated relationship of condition and distance with reaction time. Because Shapiro-Wilk tests showed a violation of level 1 \(z = 4.58, p < .0001\) and level 2 \(z = 3.50, p = .001\) normality, models were fit using bootstrapped standard errors (200 replications). There was a statistically significant relationship between distance and reaction time, such that it took participants 23 ms longer to respond to near trials than to far trials, on average, controlling for condition \(z = 3.05, p = .002\). The main effect of condition had a statistically significant relationship with reaction time, such that reaction time to literal symbols trials was 79 ms longer compared to artificial symbols trials \(z = 2.75, p = .006\), on average, controlling for distance. Similar to error rate, a subsequent model showed no statistically significant relationships between backward digit span, testing order, gender, or age and reaction
time, on average (all \( ps > .10 \)). A GLH test showed that the joint effect of these covariates was statistically indistinguishable from zero (\( \chi^2(df = 4) = 3.59, p = .46 \)) and so they were not included in the model. The fitted relationship between distance and reaction time for literal and artificial symbols is illustrated in Figure 4. Taken together, these results show that both literal and artificial symbols produced a SDDE for error rate and reaction time, but that there is a processing cost to comparing with literal symbols, which manifests as a longer reaction time, on average, regardless of distance.

[Insert Figure 4]

**Different Mechanisms for Same-Different Distance Effects**

To explore the underlying mechanisms across the SDDEs in the numerical, literal symbols, and artificial symbols conditions, both parametric (Pearson) and non-parametric (Spearman) correlation coefficients were calculated across the three conditions using participant level distance effects. For error rate, there were no statistically significant correlations between the numerical and literal symbol SDDEs (\( r = -.29, p = .18; \rho = -.32, p = .14 \)), numerical and artificial symbol SDDEs (\( r = .26, p = .22; \rho = .25, p = .51 \)), or literal and artificial symbol SDDEs (\( r = -.18, p = .42; \rho = -.14, p = .14 \)), which may be due to low error rates for both near and far trials across conditions. For reaction time, there was not a statistically significant correlation between the numerical and literal symbol SDDEs (\( r = -.07, p = .76; \rho = -.05, p = .81 \)) or between the literal and artificial symbol SDDEs (\( r = -.30, p = .16; \rho = -.17, p = .45 \)). The correlations between the numerical and artificial symbol SDDEs differed for Pearson and Spearman correlations. There was a moderate, positive, statistically significant Pearson correlation between the numerical and artificial symbol SDDEs (\( r = .44, p = .03 \)) but the associated Spearman
correlation was not statistically significant ($\rho = .28, p = .20$). The reaction time correlations provide limited evidence that similar mechanisms, such as the unique mapping between symbol and magnitude in both conditions, may support the numerical and artificial symbol SDDEs. However, the error rate and reaction time correlations do not support the notion that similar mechanisms underlie the numerical and literal symbol SDDEs.

**Discussion**

The present study investigated the nature of mental representations of magnitude as they relate to numeric and non-numeric symbols. Specifically, this study sought to replicate the SDDE with Arabic numerals and explore whether an SDDE would arise with literal and artificial symbols. Further, this study examined differences in literal and artificial symbol SDDEs as a way to investigate the notion that competing mental representations for literal symbols may interfere with their connection to numerical referents. Each aim is discussed in turn.

**Same-Different Distance Effects with Arabic Numerals**

Participants compared cross-notation pairs of number words and Arabic digits that either represented the same magnitude or differed in magnitude by a small or large amount. An SDDE was present for both error rate and reaction time. Error rate was lower and reaction times were shorter on average for number pairs with greater distance. These results lend further support to the presence of a symbolic SDDE in adolescents, which is in line with prior research on the symbolic SDDE in children (e.g., Defever et al., 2012) and adults (e.g., Dehaene & Akhavein, 1995; van Opstal & Verguts, 2011), and with
research on the non-symbolic SDDE in adults (e.g., Sasanguie et al., 2011; Smets, Gebuis, & Reynvoet, 2013).

**Same-Different Distance Effects with Non-Numeric Symbols**

Participants learned novel associations between Arabic numerals and two sets of non-numeric symbols: literal and artificial symbols. Subsequently, participants completed same-different tasks with each. There were SDDEs for error rate and reaction time in both the literal symbol and artificial symbol conditions. Since the presence of a symbolic SDDE suggests access to mental representations of magnitude (Dehaene & Akhavein, 1995; van Opstal & Verguts, 2011), these results suggest that both non-numeric symbolic representations were able to access mental representations of magnitude.

The presence of an SDDE for non-numeric symbols is in line with related research that replicates similar behavioral signatures of numerical processing with novel symbols. Prior studies have shown congruity effects for numerical Stroop tasks after participants learned ordinal relationships between nine artificial symbols (Cohen Kadosh et al., 2010; Tzelgov et al., 2000). In addition, Lyons and Ansari (2009) found distance effects when participants associated novel symbols with non-symbolic quantities and subsequently completed a number comparison task. The present results add to this research and suggest that non-numeric symbols can access mental representations of magnitude when the symbols are associated with symbolic, cardinal representations of number.

**Connecting Literal Symbols to Numerical Magnitudes**

The results of the present study shed new light on whether literal symbols can access mental representations related to numerical magnitude. In particular, the presence
of an SDDE in the literal symbols condition stands in contrast to prior related work with literal symbols. As discussed above, Pollack et al. (2015) used a priming paradigm to elicit a PDE with Arabic numerals, but did not elicit a PDE with literal symbols. This may be due to a methodological issue whereby participants did not process numerical magnitude. Alternatively, this may be because literal symbols might not access mental representations of numerical magnitude as Arabic numerals do or may do so in a qualitatively different way. The fact that the present study elicited an SDDE with literal symbols suggests that these symbols can indeed access mental representations of numerical magnitude.

As discussed above, one reason for the discrepancy in findings could be that participants did not process numerical magnitude during the PDE task (Pollack et al., 2015) but did during the same-different task. A second reason may be that priming and same-different comparison tasks require a different level or strength of association between symbols and their numerical referents. Arabic numerals have strong associations with numerical magnitudes due to overtraining through formal schooling, whereas the associations between literal symbols and numerical magnitudes are temporary. It may be that these temporary associations are not encoded strongly enough to elicit a PDE as with Arabic numerals, but are strong enough to elicit an SDDE. Third, and related, it may be that literal symbols can elicit an SDDE, but not a PDE because the two effects arise from different mechanisms. Indeed, a recent study found that the SDDE and PDE were not correlated within individuals (Sasanguie et al., 2011).

Even though an SDDE was present for literal symbols for both error rate and reaction time, results of the present study suggest that literal symbols may access mental
representations of numerical magnitude in a qualitatively different way than other symbolic representations. Participants took 79 ms longer, on average, to compare literal symbols than to compare artificial symbols, controlling for distance. The sole difference between the literal and artificial symbols conditions was the symbolic representation, which suggests the reaction time difference between the literal symbol and artificial symbol conditions is due to mapping literal symbols to numerical magnitudes per se.

There are several potential explanations for the difference in reaction time between the two conditions. As one possibility, a longer reaction time for literal symbols may reflect interference that is associated with competing linguistic and numerical representations for literal symbols. Another possibility is that there are different mechanisms for processing literal symbols that have numerical referents. This interpretation is supported by the lack of correlations in the present study between the literal symbol and number conditions. Further research is needed to explore the role of interference or separate mechanisms for linking literal symbols to numerical magnitude.

**Limitations**

There are several limitations to the present study. First, it may be that participants had heterogeneous levels of literal symbol knowledge. All participants had baseline exposure to literal symbols in Algebra I and a subsequent math class. Participants’ instruction regarding literal symbols likely also varied. Participants came from different schools and may have had different (or no) formal instruction related to the role of literal symbols in mathematics (e.g., see Nie, Cai, & Moyer, 2009). A second limitation is the relatively large age range, which implies that participants spanned a large grade range as well, from beginning high school to beginning college. Age was not a statistically
significant predictor of error rate or reaction time, which is in line with prior research on the numerical SDDE (Defever et al., 2012; Duncan & McFarland, 1980). However, there may be differences across age, grade, or level or type of exposure to literal symbols. Future studies might address this limitation with a cross-sectional study that tests groups of participants from different grades in the same school, as one example. A third limitation pertains to the stimuli in the number condition. As noted in prior studies on symbolic number processing (Peters, De Smedt, & Op de Beeck, 2015), numerosity is confounded with the length of number words and this may affect decision making in the numerical condition of the same-different task. For example, ‘1’ is a small numerosity and ‘ONE’ has relatively few letters, whereas ‘8’ is a larger numerosity and ‘EIGHT’ has relatively many letters.

**Conclusion**

This study examined the relationship between numeric and non-numeric number symbols and their mental referents. Specifically, it investigated potential differences between how literal symbols and artificial symbols access mental representations of magnitude using the same-different distance effect. Participants completed three same-different tasks in which they compared Arabic numerals to number words, literal symbols, or artificial symbols. To do so, they learned associations between the non-numeric symbols and arbitrary magnitudes to a high degree of accuracy. All three conditions elicited an SDDE, suggesting that the non-numeric symbols accessed mental representations related to magnitude. Interestingly, participants took longer, on average, to complete the literal symbol trials compared to the artificial symbol trials. Additionally, while the SDDE for artificial symbols was correlated with the SDDE for Arabic
numerals, the literal symbol SDDE was not. Taken together, these findings suggest that there is a processing cost associated with mapping literal symbols, which have pre-existing linguistic mental representations, to numerical magnitude, and that there may be different mechanisms supporting the connection between literal symbols and their numerical referents.

In order to succeed in learning algebra, students must develop fluency and flexibility with literal symbols, including when they represent magnitude. Yet, students persistently show difficulty with literal symbols. The present study suggests this difficulty may be due in part to different cognitive mechanisms that support literal symbol processing in mathematics contexts. Additional research is needed to examine the nature of such mechanisms and how they may impact students’ understanding of literal symbols and their use in higher-level mathematics such as algebra.
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References


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Table 1
Arabic numerals used in all three conditions with the associated literal and artificial symbols.

<table>
<thead>
<tr>
<th>Arabic numeral</th>
<th>Literal symbol</th>
<th>Artificial symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Q</td>
<td>△</td>
</tr>
<tr>
<td>2</td>
<td>G</td>
<td>§</td>
</tr>
<tr>
<td>7</td>
<td>R</td>
<td>¥</td>
</tr>
<tr>
<td>8</td>
<td>H</td>
<td>Ｃ</td>
</tr>
</tbody>
</table>

Figure 1. Illustration of a trial in the number condition of the same-different task.
Table 2
Taxonomy of fitted models for error rate and reaction time in the number condition (left two columns) and in the literal and artificial symbols conditions (right two columns), including parameter estimates, standard errors (in parentheses), and goodness of fit statistics ($n = 24$).

<table>
<thead>
<tr>
<th>Condition</th>
<th>Error Rate (%)</th>
<th>Reaction Time (ms)</th>
<th>Error Rate (%)</th>
<th>Reaction Time (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number condition</td>
<td></td>
<td></td>
<td>Literal and artificial symbols conditions</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>1.17***</td>
<td>770.42***</td>
<td>1.91***</td>
<td>847.31***</td>
</tr>
<tr>
<td></td>
<td>(0.31)</td>
<td>(26.97)</td>
<td>(0.50)</td>
<td>(33.08)</td>
</tr>
<tr>
<td>Near</td>
<td>1.78***</td>
<td>38.08***</td>
<td>2.68***</td>
<td>23.40**</td>
</tr>
<tr>
<td></td>
<td>(0.49)</td>
<td>(10.63)</td>
<td>(0.46)</td>
<td>(7.67)</td>
</tr>
<tr>
<td>Literal symbols</td>
<td></td>
<td></td>
<td>-0.43</td>
<td>79.04**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.58)</td>
<td>(28.77)</td>
</tr>
<tr>
<td>$\hat{\sigma}_u^2$</td>
<td>4.79</td>
<td>20120.29</td>
<td>6.30</td>
<td>30677.04</td>
</tr>
<tr>
<td>$\hat{\sigma}_e^2$</td>
<td>3.09</td>
<td>1258.33</td>
<td>6.54</td>
<td>8747.46</td>
</tr>
<tr>
<td>$\hat{\rho}$</td>
<td>0.61</td>
<td>0.94</td>
<td>0.49</td>
<td>0.78</td>
</tr>
<tr>
<td>$\text{Pseudo} - R_u^2$</td>
<td>-0.18</td>
<td>-0.02</td>
<td>-0.09</td>
<td>-0.03</td>
</tr>
<tr>
<td>$\text{Pseudo} - R_e^2$</td>
<td>0.32</td>
<td>0.35</td>
<td>0.26</td>
<td>0.18</td>
</tr>
</tbody>
</table>

$^p = .05, \ * p < .05, \ ** p < .01, \ *** p < .001$

Note: For goodness-of-fit statistics, $e$ subscripts refer to level 1 (distance) and $u$ subscripts refer to level 2 (participant).
Figure 2. Fitted relationship between distance and the two outcome measures in the number condition. Panel (a) displays the relationship between distance and error rate and Panel (b) displays the relationship between distance and reaction time.
Figure 3. Fitted relationship between distance and error rate for the literal symbol condition (solid line) and artificial condition (dashed line). Note that the difference in error rate between the literal symbols and artificial symbols conditions is not statistically significant.
Figure 4. Fitted relationship between distance and reaction time (ms) for the literal symbol condition (solid line) and artificial condition (dotted line).
Chapter 4

The Path Forward
Abstract

This dissertation contributed two studies related to symbolic number processing in intermediate and higher-level mathematics. This discussion chapter focuses on three areas for future research concerning symbolic number processing that builds on, and moves beyond, the dissertation studies. First, future studies on literal symbols can harness methodologies related to brain and behavior to illuminate the cognitive processing costs associated with literal symbols compared to artificial symbols. Studies using eye tracking and event-related potentials can shed additional light on whether and when cognitive processing differences occur, respectively. Second, future work on the etiology of Developmental Dyscalculia offers an opportunity to research symbolic number processing as it relates to foundational mathematical competency and the importance of processing number symbols compared to the development of their underlying neural representations. Third, research on higher-order symbolic processing, such as with arithmetic facts, offers the opportunity to further investigate common areas of brain activation for arithmetic and phonological processing. Through these studies, I seek to conduct research that spans brain and behavior to further elucidate symbolic number processing and its role in mathematics learning. Such research provides an important source of knowledge on how to facilitate students’ learning in mathematics from basic numeracy through arithmetic and into higher-level mathematics such as algebra.
The Path Forward

This dissertation contributed two studies related to symbolic number processing in intermediate and higher-level mathematics. This discussion chapter focuses on future research concerning symbolic number processing that builds on, and moves beyond, the dissertation studies. I use this chapter to frame a research agenda with three main components that all address the connections between symbols that represent number and their mental and neural representations. First, I focus on literal symbols and their mental and neural representations. I offer several avenues for future research that span behavioral studies and studies that integrate brain and behavior. I envision that research on mental and neural representations associated with literal symbols will play a major role in my future work, and accordingly I allot more space to potential future work in this area. Second, I introduce and outline research on symbolic number processing that is a departure from the two dissertation studies. As an aspiring researcher who studies symbolic number processing, I seek to expand my work into the role that number symbols play in atypical development, which is an open and big question in educational neuroscience work related to mathematics learning. Specifically, I discuss next steps related to the role of symbolic number processing in the etiology of Developmental Dyscalculia (DD). Third, I briefly discuss future work on arithmetic facts as higher-order symbolic representations related to language. I conclude by discussing the implications of symbolic number processing research broadly for research related to mathematics learning, both within and outside of educational neuroscience.

Mental & Neural Representations Related to Literal Symbols
The beginning research on symbolic number processing related to literal symbols has yielded conflicting findings. The results of Pollack, Leon Guerrero, and Star (2015) showed that literal symbols given arbitrary numerical magnitudes did not produce priming distance effects. This may have been due to response-related processes of the paradigm that did not require participants to process numerical magnitude, because literal symbols did not access the same mental representations as Arabic numerals, or because literal symbols did not access the mental representations in the same way. The study in Chapter 3 suggests that when literal symbols are given arbitrary magnitudes, they can indeed access the mental number line and produce same-different distance effects as Arabic numerals and number words, despite having extant mental representations related to reading. However, as discussed at length in Chapter 3, the results of this study also suggest a processing cost associated with literal symbols because of their extant mental representations related to reading. These findings support the notion that literal symbols access mental representations of magnitude differently from Arabic numerals. These results, along with the conflicting findings and methodological differences across the above two studies, provide several opportunities for future research to further elucidate the nature of mental and neural representations related to literal symbols.

**Behavioral Studies**

Additional behavioral studies are needed to further understand how literal symbols connect to mental representations of magnitude. The study in Chapter 3 involved participants across a relatively large developmental range, from early high school to early college. Though same-different distance effects (SDDEs) are thought to be stable across age (Defever, Sasanguie, Vandewaetere, & Reynvoet, 2012; Duncan & McFarland,
1980), and I found no effects of age in the study in Chapter 3, it is possible that the size of SDDEs for literal symbols compared to the SDDE for artificial symbols may differ depending on students’ level of literal symbol and/or literacy knowledge. Accordingly, a future study could directly compare participants’ efficiency with same-different comparisons across different age groups, similar to research on other numerical cognition paradigms such as the comparison distance effect (e.g., Duncan & McFarland, 1980) and the priming distance effect (Defever, Sasanguie, Gebuis, & Reynvoet, 2011).

Related, the notion that literal symbols have existing mental representations related to reading that may interfere with their numerical referent raises intriguing questions about how these mental representations may form in atypical populations. For instance, might students with dyslexia have increased difficulty forming numerical referents for literal symbols? Or in contrast, might there be less interference if students’ mental representations for literal symbols related to reading are not as strong? Whether students with dyscalculia may have increased difficulty forming mental or neural representations for literal symbols is similarly unclear. The etiology of dyscalculia is an open question (more on this below). However, I speculate that students with dyscalculia would have increased difficulty mapping mental representations of magnitude to literal symbols regardless of whether the etiology relates to an impaired approximate number system or an impaired ability to link symbols and their associated mental and neural representations.

Future behavioral studies could also contrast performance on priming and comparison paradigms to test the level of encoding of literal symbols compared to Arabic numerals. Arabic numerals have stable mappings to their associated mental
representations while literal symbols do not. It may be that literal symbols can map onto the same mental representations as Arabic numerals, but only through a shallower level of encoding. If this were the case, it may be that the same group of participants could show an SDDE with literal symbols, but not a priming distance effect with literal symbols, for example. In order to test this type of hypothesis, I could use the same-different task from Chapter 3 and a priming paradigm that forces numerical magnitude processing. One possibility is a paradigm that combines matching with priming, such as in Notebaert, Pesenti, and Reynvoet (2010). In brief, in this study, participants saw sequentially presented pairs of cross-notation number symbols (i.e., a prime digit and then a target number word or vice versa) and responded with a button press if the target matched a predefined numerosity. However, Notebaert et al., (2010) did not find evidence of behavioral priming effects, potentially due to a small number of trials. Whether this paradigm could be modified to be suitable for a study with literal symbols would require further investigation.

**Beyond Behavior**

The results of Chapter 3 also open the door to investigate mental and neural representations related to literal symbols using measures beyond behavior. The longer reaction time for making same-different judgments with literal symbols compared to artificial symbols and the lack of correlation between the literal symbol SDDE and the SDDEs in the other two conditions provide initial evidence that different cognitive mechanisms may be at work. A follow up study could employ eye-tracking using the paradigm from Chapter 3 to further investigate this notion. Specifically, I could compare fixation duration, fixation count, and number of saccades (i.e., small eye movements)
when participants make same-different judgments with literal symbols and artificial symbols. As one example, I can look for an effect of distance within each condition, such as a greater number of saccades for near versus far trials, and compare these effects across conditions. I can also correlate behavioral distance effects with distance effects obtained through eye measurements. Prior research suggests that behavioral and eye measurement distance effects with Arabic numerals may measure different levels of symbolic number processing (Merkley & Ansari, 2010). This could also be true for literal symbols.

A second way to examine the average reaction time difference between the literal symbol and artificial symbol conditions in Chapter 3 is using event-related potentials (ERP). ERP uses electrodes on the scalp to measure electrical activity in the brain. This method is well suited to investigate when cognitive processes take place. Generally, differences in cognitive processing across conditions are associated with differences in latencies or peak amplitudes of waveforms that relate to different cognitive processes. A future study could compare ERPs across the literal symbol and artificial symbol conditions in order to elucidate at what stage there is a difference in cognitive processing across the two conditions. Additional ERP studies involving literal symbols could look to replicate the ERP distance effects with literal symbols and artificial symbols that have been found in number comparison tasks involving Arabic numerals (Libertus, Woldorff, & Brannon, 2007; Turconi, Jemel, Rossion, & Seron, 2004).

**Expanding the Notion of Literal Symbols**

Lastly, there are several other ways that literal symbols are used in mathematics besides representing a single magnitude. For instance, literal symbols can also be used to
generalize relationships (e.g., $a + b = b + a$) or even be symbols to just manipulate, such as trigonometric identities (Usiskin, 1999). To provide a more complete picture of students’ understanding of literal symbols, future work should incorporate other uses of literal symbols beyond representing a single magnitude. For example, future studies could involve developing novel paradigms to examine mental representations associated with literal symbols that have a range of values (e.g., $0 < x < 4$) or literal symbols that are “semantically laden letters,” (Rosnick, 1982) with both a quantitative and qualitative meaning (e.g., $b = \text{an unknown number of books}$).

I speculate that mental and neural representations associated with literal symbols will differ depending on the particular mathematical context in which literal symbols are used. A better understanding of the cognitive and neural mechanisms that support literal symbols across these contexts can provide insight into how to support students’ understanding of the ways that literal symbols are used in higher-level mathematics.

**Symbolic Number Processing and Developmental Dyscalculia**

While research into symbolic number processing associated with literal symbols is novel, I also plan to focus on more foundational and well-established lines of research on symbolic number processing related to mathematics learning in atypical populations. Specifically, the second part of my research on symbolic number processing will examine the role that connecting a symbol to its neural representation of magnitude plays in developmental dyscalculia (DD), also known as mathematics disability. There are currently two competing hypotheses for the etiology of DD, one of which focuses on the connection between a number symbol and its neural representation. The first, the defective number module (DNM) hypothesis, focuses on the quality of neural
representations of magnitude, separate from the mapping of these representations to number symbols. In brief, imprecise or noisy neural representations of numerical magnitude lead to impaired performance on basic numerical processing tasks - both non-symbolic and symbolic tasks, such as comparing arrays of dots or Arabic digits, respectively (Butterworth, 2005; Wilson & Dehaene, 2007). The second hypothesis, the access-deficit (AD) hypothesis, specifically involves number symbols. According to the AD hypothesis, DD stems from an impaired connection between intact neural representations of magnitude and number symbols (i.e., Arabic digits).

Directly comparing these two hypotheses will provide an opportunity to investigate the role of symbolic number processing in mathematics performance. Currently, only a handful of behavioral studies have directly contrasted these two hypotheses, investigating group differences in performance on symbolic and non-symbolic number comparison tasks. Some results have supported the DNM hypothesis, showing that children with DD perform poorly on non-symbolic tasks compared to their typically developing peers (Landerl, Fussenegger, Moll, & Willburger, 2009; Mussolin et al., 2010). Others, however, have favored the AD hypothesis, showing that children with DD do not show impaired performance on non-symbolic tasks, but do show impairment on tasks that require extracting meaning from number symbols, such as comparing Arabic digits (Castro Cañizares, Reigosa Crespo, & González Alemañy, 2012; De Smedt & Gilmore, 2011; Iuculano, Tang, Hall, & Butterworth, 2008; Landerl & Kölle, 2009; Rousselle & Noël, 2007). Accordingly, there is no consensus on whether the DNM or AD hypothesis best explains the cause of DD.
One way to investigate the merit of the AD hypothesis in comparison to the DNM hypothesis is using fMRI. This methodology may be particularly helpful here, since studies on DD that have used both behavioral and neuroimaging methods have revealed differences in brain activation between children with and without DD, even when their behavioral performance is equivalent (Mussolin et al., 2010; Price, Holloway, Räsänen, Vesterinen, & Ansari, 2007). Additionally, to date, no studies exist that directly contrast the DNM and AD hypotheses at the neural level. This research could therefore make a novel contribution to begin to adjudicate between these two competing hypotheses, which will further illuminate the role of symbolic number processing in mathematics learning.

**Higher-Order Symbolic Representations of Number**

Both of the above areas of research concern individual symbols (e.g., x, 2) that convey magnitude and their associated mental and neural representations. Research in symbolic number processing also encompasses higher-order symbols that convey numerical information, such as arithmetic facts. The term higher-order refers to the fact that arithmetic facts may function as a symbolic unit in which the arithmetic fact as a whole (e.g., 3 x 4) is associated with a neural representation of the answer (Ansari, 2008; Grabner, Ansari, Koschutnig, Reishofer, & Ebner, 2013) and with verbally stored representations rather than magnitudes (Dehaene, Piazza, Pinel, & Cohen, 2003).

Research on higher-order symbolic representations such as arithmetic facts provides an opportunity to examine the precise overlap between arithmetic and phonological processing, which is an open area of research in the field (Holloway & Ansari, 2015). I began to investigate this overlap in a developmental sample in the study in Chapter 2, which found areas of concordant brain activity for arithmetic and
phonological processing related to attentional processes and symbolic processing of letters, words, and numbers. This study also showed that both arithmetic and phonological processing engage left-lateralized brain areas related to verbal representations. However, there was regional differentiation in the specific areas associated with arithmetic and phonological processing, respectively, meaning there were not common brain regions associated with verbal representations across arithmetic and phonological processing.

As mentioned in Chapter 2, one next step in this research would be to conduct a similar meta-analysis to examine brain regions that support arithmetic and phonological processing across studies done with adults. Then, I could qualitatively compare whether brain regions that support arithmetic and phonological processing differ across adult and developmental samples. Because patterns of brain activation associated with arithmetic shift from frontal regions to left-lateralized language areas (i.e., parietal and temporoparietal regions) across development (e.g., Zamarian, Ischebeck, & Delazer, 2009) it may be that there are common regions of activation in left-lateralized brain regions in adults, but not in children. This and related work can further illuminate the behavioral relationship between phonological processing and children’s ability with arithmetic facts (e.g., De Smedt, Taylor, Archibald, & Ansari, 2010) and speak to a potential language-calculation network in the brain that may have regional differentiation (e.g., Andin, Fransson, Rönnberg, & Rudner, 2015).

**Concluding Remarks**

Over the course of this dissertation, I have argued that educational neuroscience research related to mathematics learning should attend more to symbolic number
processing associated with intermediate and higher-level mathematics. However, I am not arguing that this work should be done at the expense of additional research on symbolic processing of Arabic numerals, number words, and other foundational representations of number. Though we have learned much about how number symbols are processed in the mind and brain, many open questions remain related to early numeracy, such as what changes in the brains of young children facilitate the ability for number symbols to initially acquire their meaning (Holloway & Ansari, 2015).

Both of the studies in this dissertation align with open questions related to symbolic number processing that can inform mathematics learning. One open question relates to the connection between number symbols and their mental and neural representations when a symbol’s referent is more abstract (Holloway & Ansari, 2015). The current work on literal symbols is one way to speak to this question. As mentioned above, Holloway & Ansari (2015) speak to the need for additional research on the connection between symbolic number processing and cognitive processing that supports reading ability, the topic of Chapter 2 of this dissertation.

The studies in this dissertation and the next steps I have outlined in this chapter seek to bring together research that spans brain and behavior to further elucidate symbolic number processing and its role in mathematics learning. By providing a more complete picture of the cognitive processes involved in mathematics and providing additional research evidence unavailable through behavioral methods alone, the above research agenda speaks to the aims of educational neuroscience work focused on mathematics learning (De Smedt, Ansari, et al., 2010). Such research provides an important source of knowledge on how to facilitate students’ learning in mathematics.
from basic numeracy, through arithmetic, and into higher-level mathematics such as algebra.
Acknowledgments

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