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The 0-brane action in a general D = 4 supergravity background

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Abstract

We begin by presenting the superparticle action in the background of $\mathcal{N}=2$, $D=4$ supergravity coupled to n vector multiplets interacting via an arbitrary special Kähler geometry. Our construction is based on implementing κ-supersymmetry. In particular, our result can be interpreted as the source term for $\mathcal{N}=2$ BPS black holes with a finite horizon area. When the vector multiplets can be associated with the complex structure moduli of a Calabi–Yau manifold, our 0-brane action can then be derived by wrapping 3-branes around 3-cycles of the 3-fold. Our result can be extended to the case of higher supersymmetry; we explicitly construct the κ supersymmetric action for a superparticle moving in an arbitrary $\mathcal{N}=8$ supergravity background with 1/2, 1/4 or 1/8 residual supersymmetry.

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1 Introduction

Recently, a lot of attention has been devoted [1] to the AdS/CFT correspondence between D-dimensional supergravity compactified on manifolds of the form

$$adS_{p+2} \times \mathcal{M}_{D-p-2} \tag{1}$$

and conformal field theories on the $(p+1)$-dimensional boundary $\mathcal{B}_{p+1}$ of anti de Sitter space. In (1), $\mathcal{M}_{D-p-2}$ denotes a compact $(D-p-2)$-dimensional manifold whose geometrical structure largely determines the specific properties of the boundary conformal field theory.

The situation envisaged by this correspondence arises in the context of BPS p-brane backgrounds with a suitable horizon behaviour. These are classical solutions of D-dimensional supergravity that interpolate between two different vacua, namely flat D-dimensional Minkowski space at infinity and the product manifold (1) near the horizon [2]. In addition, they are BPS states in the sense that they preserve some amount of supersymmetry. As a consequence of this there is a saturation of a bound relating the mass density to the charge.

Actually p-branes are solutions of *supergravity plus sources* (see for instance [3]): indeed, one has to supplement the bulk supergravity action with the world-volume action of p-extended objects carrying the charges measured at spatial infinity.

The construction of these world-volume actions for p-branes is mainly based on the principle that they should be κ-supersymmetric, namely that there should be a suitable projection of the target space supersymmetry that is promoted to a local fermionic symmetry on the world-volume. Implementing κ-supersymmetry on the world-volume puts the background supergravity fields on shell and requires that the superspace Bianchi identities be satisfied [4, 5, 6]. Furthermore, by gauge-fixing the κ-symmetry, one halves the target space fermionic coordinates providing the correct number of bosons and fermions for a supersymmetric world-volume theory.

When the κ-supersymmetric action that describes the coupling of the p-brane to generic supergravity backgrounds is known, one can study its properties in any given background solution. Choosing an appropriate κ-gauge one derives a consistent world-volume field theory that inherits as global (super)symmetries the (super)isometry group of the chosen background. For instance, in the background (1) one can derive a *superconformal field theory* on the anti de Sitter boundary $\mathcal{B}_{p+1}$ starting from the p-brane world-volume action [7]. Indeed, in this case the supersymmetric extension of the anti de Sitter isometry group $SO(2, p+1)$ acts as the superconformal group on $\mathcal{B}_{p+1}$. Such a construction has been used recently to investigate the properties of the *singleton conformal field theory* living on the anti de Sitter boundary of M2 [6] and 3-branes [8].

The BPS black-hole solutions of D=4 supergravities [9, 10, 11] fit into the above scheme as instances of 0-branes. However, the κ-supersymmetric action for superparticles has been derived so far only in the case of a pure $\mathcal{N} = 2$ supergravity background [12]. The purpose of this paper is to extend such a construction to more general supergravity backgrounds. Firstly, we present the case of an arbitrary $\mathcal{N} = 2$, $D = 4$ background provided by supergravity coupled to n vector multiplets interacting via a generic *special Kähler geometry*. Secondly, we extend our result to $\mathcal{N}=8$ supergravity.

The main new issue involved in our programme is the coupling of the 0-brane to the scalar fields. As we are going to see this coupling occurs in a very simple and elegant way through a real function

$$|Z(\phi, p^I, q_J)| = \frac{1}{\sqrt{2\nu}} \left(\widehat{Z}_{AB}\,\widehat{\overline{Z}}^{AB}\right)^{1/2} \ , \tag{2}$$

where ν is the number of preserved supercharges, sitting in front of the kinetic term. As will become apparent in the following, the real function (2) is the modulus of the largest skew eigenvalue of the field-dependent central charge tensor Z_{AB}. The hat appearing in (2) denotes a suitable projection operation that extracts the contribution from the largest eigenvalue. The numbers p^I, q_J are the magnetic and electric charges of the black hole and ϕ are the scalar fields. In the $\mathcal{N}=2$ case the scalars belong to the vector multiplets, in the $\mathcal{N}=8$ case they belong to the graviton multiplet, but the form (2) of their coupling to the superparticle action holds in both cases alike. This result could be heuristically justified recalling that for a usual particle the worldline action is multiplied by the particle mass and that in the case of a BPS state the mass should be equal to the central charge. The main difference is that in supergravity the central charge is field dependent.

Our result will be derived requiring κ-supersymmetry: it has a general validity within $\mathcal{N}=2$ or $\mathcal{N}=8$ supergravity. If the $\mathcal{N}=2$ supergravity model can be obtained from the type IIB superstring compactified on a Calabi–Yau (CY) 3-fold a geometric interpretation of our 0-brane action naturally arises: it corresponds to the wrapping of the 3-brane world-volume action on a *supersymmetric cycle* of the Calabi–Yau manifold.

2 $\mathcal{N}=2$ black holes solution

The BPS black hole solutions of $\mathcal{N}=2$ pure supergravity have been introduced in [9]. They have been extended to the coupling with n vector multiplets in [10]. The interplay between the BPS conditions and special Kähler geometry was found by [13]. In particular, in that reference the relation of the black hole entropy with the central charge and the so-called geodesic potential was clarified. These concepts were further discussed in a vast literature; see, for example, [11] for a review. The relevant features of these field configurations are the following ones.

1. The solutions are characterized by the vector of electric (q_J) and magnetic (p^I) charges carried by the black hole and by the values z^i_∞ of the moduli (scalar fields of the vector multiplet) at spatial infinity.

2. The metric has a universal behaviour near the horizon $r=0$, where it approaches the Bertotti–Robinson metric [14]

$$ds^2_{\mathrm{BR}} = -\frac{1}{(G_N m_{\mathrm{BR}})^2} r^2 dt^2 + (G_N m_{\mathrm{BR}})^2 \frac{dr^2}{r^2} + (G_N m_{\mathrm{BR}})^2 \left(\sin^2(\theta)\, d\phi^2 + d\theta^2\right) \ , \tag{3}$$

describing the geometry of $adS_2 \times S^2$. The parameter $m_{\rm BR}$, named the Bertotti–Robinson mass, depends only on p and q.

3. The complex moduli fields z^i, starting at infinity from arbitrary values z^i_∞, flow at the horizon to fixed values $z^i_{\rm fix}$ that depend only on the quantized electric and magnetic charges q and p.

Let us briefly recall the basic ingredients of $\mathcal{N}=2$, $D=4$ supergravity and the way in which its BPS black hole solutions arise. The bosonic part of the supergravity action is (with $\kappa_4^2 = 8\pi G_N = 1$)

$$\begin{aligned}\mathcal{L}_{\rm B} &= \sqrt{-g}\Big[-\tfrac{1}{2}\,R[g] \;-\; g_{i\bar\jmath}(z,\bar z)\,\partial^\mu z^i\,\partial_\mu \bar z^{\bar\jmath} \\ &\quad +\tfrac{1}{4}\,\mathrm{i}\,\Big(\bar{\mathcal{N}}_{IJ}\mathcal{F}^{-I}_{\mu\nu}\mathcal{F}^{-J|\mu\nu} \;-\; \mathcal{N}_{IJ}\mathcal{F}^{+I}_{\mu\nu}\mathcal{F}^{+J|\mu\nu}\Big)\Big]\;. \end{aligned} \tag{4}$$

The action is defined in terms of scalars z^i and vectors A^I_μ whose field strength is the electric 2-form $\mathcal{F}^I$:

$$\mathcal{F}^I = dA^I = \tfrac{1}{2}\mathcal{F}^I_{\mu\nu}dx^\mu\wedge dx^\nu = \tfrac{1}{2}(\partial_\mu A^I_\nu - \partial_\nu A^I_\mu)dx^\mu\wedge dx^\nu\;. \tag{5}$$

Our convention for (anti)selfdual tensors is as follows:

$$\mathcal{F}^{\pm\,I}_{\mu\nu} = \tfrac{1}{2}\left(\mathcal{F}^I_{\mu\nu} \pm \mathrm{i}\tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}\,\mathcal{F}^{I|\rho\sigma}\right)\;. \tag{6}$$

The scalars are in a symplectic section of special geometry (see [15, 16]):

$$V = \begin{pmatrix} X^I \\ F_J \end{pmatrix}\;, \tag{7}$$

where $e^{-\mathcal{K}/2}X^I(z^i)$, $e^{-\mathcal{K}/2}F_J(z^i)$ are holomorphic, and $\mathcal{K}$ is the Kähler potential. The covariant derivative of this section,

$$U_i = \nabla_i V = \left(\partial_i + \tfrac{1}{2}\,\partial_i\mathcal{K}\right)V \equiv \begin{pmatrix} f^I_i \\ h_{J|i}\end{pmatrix}\;, \tag{8}$$

defines f^I_i and $h_{J|i}$, which in turn are sufficient to determine the metric on the scalar manifold $g_{i\bar\jmath}$ and the kinetic matrix $\mathcal{N}$ in (4):

$$\begin{aligned} g_{i\bar\jmath} &= \mathrm{i}\left(f^I_i\bar h_{I|\bar\jmath} - h_{I|i}\bar f^I_{\bar\jmath}\right)\;, \\ \bar F_J &= \bar{\mathcal{N}}_{JI}\,\bar X^I \quad , \quad h_{J|i} = \bar{\mathcal{N}}_{JI}f^I_i\;. \end{aligned} \tag{9}$$

The field equations of the vector fields involve the magnetic field strengths

$$\mathcal{G}^-_{J|\mu\nu} = \bar{\mathcal{N}}_{JI}\mathcal{F}^{-I}_{\mu\nu}\;;\qquad \mathcal{G}^+_{J|\mu\nu} = \mathcal{N}_{JI}\mathcal{F}^{+I}_{\mu\nu}\;;\qquad \mathcal{G}_{J|\mu\nu} = \mathcal{G}^-_{J|\mu\nu} + \mathcal{G}^+_{J|\mu\nu}\;, \tag{10}$$

defined by the variation of the Lagrangian with respect to the electric field strengths. The electric and magnetic charges of the black hole are defined by the formulae:

$$q_I \equiv \frac{1}{4\pi k}\int_{S^2_\infty}\mathcal{G}_I\;;\qquad p^J \equiv \frac{1}{4\pi k}\int_{S^2_\infty}\mathcal{F}^J\;. \tag{11}$$

Note that p and q are not necessarily integer. Indeed, the Dirac quantization condition gives that for two objects with charges (p,q) and $(\tilde{p},\tilde{q})$

$$8\pi k^2 \left(\tilde{q}_I\, p^I - \tilde{p}^I\, q_I\right) \in \mathbb{Z} \tag{12}$$

To have integer p and q, allowing 1 as the lowest value, we thus have to take $k = 1/\sqrt{8\pi}$. We will, however, leave k arbitrary to facilitate comparison with other papers.

The black hole configuration solves the field equations from the action (4) and also satisfies the BPS conditions, which are just the statement that the supersymmetry transformations of the fermionic fields, the gravitino ψ_μ^A and the gaugino λ^{iA}, are zero:

$$\begin{aligned} 0 &= \mathcal{D}_\mu\, \epsilon_A(x) - \tfrac{\mathrm{i}}{2}\epsilon_{AB}\, T^-_{\mu\nu}\, \gamma^\nu \epsilon^B(x)\ , \\ 0 &= \nabla_\mu\, z^i\, \gamma^\mu \epsilon^A(x) - \tfrac{1}{2} G^{-i}_{\mu\nu}\gamma^{\mu\nu}\epsilon_B(x)\epsilon^{AB}\ , \end{aligned} \tag{13}$$

where $\mathcal{D}_\mu$ contains spin connection and Kähler connection. The graviphoton and matter field strengths, appearing in the above transformations, are

$$\begin{aligned} T^-_{\mu\nu} &= F_J\, \mathcal{F}^{-J}_{\mu\nu} - X^I\, \mathcal{G}^-_{\mu\nu|I}\ , \\ G^{i-}_{\mu\nu} &= -g^{i\bar{\jmath}}\, \bar{f}^J_{\bar{\jmath}}\, (\mathrm{Im}\,\mathcal{N})_{JI} \left(\mathcal{F}^{I-}_{\mu\nu}\right)\ . \end{aligned} \tag{14}$$

BPS black holes are solutions of (13) with a supersymmetry parameter $\epsilon_A(x)$ of the form [17]:

$$\gamma_0 \epsilon_A(x) = \pm \frac{Z}{|Z|}\, \epsilon_{AB}\epsilon^B(x)\ , \tag{15}$$

where

$$Z(z,\bar{z},p,q) = \frac{1}{4\pi k}\int_{S^2} T^- = F_J\, p^J - X^I\, q_I \tag{16}$$

is the central charge. The projection (15) halves the number of components of the spinors, and it is the same projection, as we will see, satisfied by the κ symmetry parameter.

In general the BPS black hole solution depends on radial functions $U(r)$ and $z^i(r)$, to be determined below. The metric is

$$ds^2 = -e^{2U(r)}\, dt^2 + e^{-2U(r)}\, d\vec{x}^2\ , \quad (r^2 = \vec{x}^2) \tag{17}$$

admitting $\mathbb{R} \times SO(3)$ as the isometry group. The spatial parts of the field strengths are

$$\mathcal{F}^I_s = \frac{kp^I}{2r^3}\epsilon_{ijk}x^i dx^j \wedge dx^k\ ; \qquad \mathcal{G}_{s,I} = \frac{kq_I}{2r^3}\epsilon_{ijk}x^i dx^j \wedge dx^k \tag{18}$$

With this parametrization, the explicit form of the first-order BPS conditions (13) is

$$\frac{dz^i}{dr} = \pm k \left(\frac{e^{U(r)}}{r^2}\right) g^{i\bar{\jmath}}\partial_{\bar{\jmath}}|Z(z,\bar{z},p,q)|\ , \tag{19}$$

$$\frac{dU}{dr} = \pm k \left(\frac{e^{U(r)}}{r^2}\right) |F_J p^J - X^I q_I| = \pm k \left(\frac{e^{U(r)}}{r^2}\right) |Z(z,\bar{z},p,q)|\ . \tag{20}$$

Only the upper sign is physical as argued in [18].

The first-order equations (19) have a fixed point, which we put at $r = 0$, by the condition $\nabla_{\bar{\jmath}}\bar{Z}(z,\bar{z},p,q) = 0$. This defines the *fixed* values $z^i_{\rm fix}$, and correspondingly the fixed value of the central charge

$$Z(p,q) = Z(z_{\rm fix}, \bar{z}_{\rm fix}, p, q) \ . \tag{21}$$

The fixed point is the horizon. Indeed, in the vicinity of the fixed point the differential equation for the metric becomes:

$$\frac{dU}{dr} = k\frac{|Z(p,q)|}{r^2}\, e^{U(r)} \ , \tag{22}$$

which has the asymptotic solution:

$$\exp[-U(r)] \stackrel{r\to 0}{\longrightarrow} \text{constant} + k\frac{|Z(p,q)|}{r} \ . \tag{23}$$

Hence, near $r = 0$ the metric (17) becomes of the Bertotti–Robinson type (3), with Bertotti–Robinson mass:

$$m_{\rm BR} = 8\pi k |Z(p,q)| \ . \tag{24}$$

3 The κ supersymmetric action of a 0-brane

In this section we derive a simple formula for the worldline action of a 0-brane moving in the background geometry provided by a generic *on-shell* field configuration of $\mathcal{N} = 2$ supergravity. In particular, our action describes the 0-branes corresponding to the black hole solutions discussed above.

To write the worldline action we use Polyakov first-order formalism. Hence we introduce the worldline *einbein* 1-form

$$e = e_\tau\, d\tau \tag{25}$$

and the auxiliary 0-forms Π^a that, on shell, become the worldline components of the spacetime supervielbein, satisfying the supergravity Bianchi identities [4, 5, 6]:

$$V^a = \Pi^a\, e \ . \tag{26}$$

We write the following ansatz for the worldline action:

$$S_{\rm wl} = 4\pi k \left\{ \int_{M_1} R(z,\bar{z}) \,\left[\left(-\Pi^a\, V^b + \tfrac{1}{2}\,\Pi^a\,\Pi^b\, e\right)\eta_{ab} + \tfrac{1}{2}\, e\right] + \int_{M_2} \left(p^J\, \mathcal{G}_J - q_I\, \mathcal{F}^I\right)\right\} \ , \tag{27}$$

where (p^J, q_I) is the vector of integer magnetic and electric charges (11), while $R(z,\bar{z})$ is a *real* function of the scalar fields that we have to determine in such a way that the action is κ-supersymmetric. Furthermore, in the Wess–Zumino (WZ) term we introduced an integral over a two-dimensional manifold M_2 whose boundary is the worldline $M_1 = \partial\, M_2$. The normalization of the WZ term is fixed by (11) and the vector equation of motion.

The action (27) is to be varied independently with respect to:

1. The auxiliary 0-forms Π^a. Such a variation yields the identification (26) as a field equation.

2. The einbein e. Such an equation yields:

$$\eta_{ab}\,\Pi^a\,\Pi^b = -1 \ , \tag{28}$$

which is the intrinsic way of stating that the worldline metric is the induced one from the target spacetime metric. Indeed equation (28) can be read in the following way:

$$\eta_{ab}\,V^a_\mu\,V^b_\nu\,\frac{dx^\mu}{d\tau}\,\frac{dx^\nu}{d\tau} h^{\tau\tau} = 1 \ , \tag{29}$$

where

$$h^{\tau\tau} = \frac{1}{h_{\tau\tau}} \quad ; \quad h_{\tau\tau} = -e_\tau e_\tau \tag{30}$$

denotes the contravariant worldline metric.

3. The target superspace coordinates x and θ. This yields the second-order field equations.

To discuss κ-supersymmetry we need the ordinary $\mathcal{N}=2$ supersymmetry transformation rules of the bosonic fields [19, 16]:

$$\begin{aligned}
\delta\,V^a_\mu &= \bar{\epsilon}^A\,\gamma^a\,\psi_{A\mu} + \bar{\epsilon}_A\,\gamma^a\,\psi^A_\mu \ , \\
\delta\,A^I_\mu &= 2\bar{X}^I\bar{\psi}_{A\mu}\epsilon_B\epsilon^{AB} + 2X^I\bar{\psi}^A_\mu\epsilon^B\epsilon_{AB} - \left(f^I_i\,\bar{\lambda}^{iA}\gamma_\mu\epsilon^B\,\epsilon_{AB} + \bar{f}^I_{\bar{\imath}}\,\bar{\lambda}^{\bar{\imath}}_A\gamma_\mu\epsilon_B\,\epsilon^{AB}\right) \ , \\
\delta\,z^i &= \bar{\lambda}^{iA}\epsilon_A \ , \\
\delta\,z^{\bar{\imath}} &= \bar{\lambda}^{\bar{\imath}}_A\epsilon^A \ .
\end{aligned} \tag{31}$$

In the above formulae $\bar{\lambda}^{iA}$ and $\bar{\lambda}^{\bar{\imath}}_A$ denote the two chiral projections of the (conjugate) gaugino field. In addition to A^I_μ we need to introduce a dual magnetic potential $B_{J|\mu}$ whose field strength is $\mathcal{G}_J$:

$$\mathcal{G}_J = dB_J = \tfrac{1}{2}\mathcal{G}_{J|\mu\nu}dx^\mu\wedge dx^\nu = \tfrac{1}{2}(\partial_\mu B_{J|\nu} - \partial_\nu B_{J|\mu})dx^\mu\wedge dx^\nu \ . \tag{32}$$

By consistency, the supersymmetry transformation rule of the dual magnetic potential B_I is defined as

$$\delta\,B_{I|\mu} \;=\; 2\bar{F}_I\bar{\psi}_{A\mu}\epsilon_B\epsilon^{AB} + 2F_I\bar{\psi}^A_\mu\epsilon^B\epsilon_{AB} - \left(h_{I|i}\,\bar{\lambda}^{iA}\gamma_\mu\epsilon^B\,\epsilon_{AB} + \bar{h}_{I|\bar{\imath}}\,\bar{\lambda}^{\bar{\imath}}_A\gamma_\mu\epsilon_B\,\epsilon^{AB}\right) . \tag{33}$$

The κ transformation is simply a supersymmetry transformation where the supersymmetry parameter $\epsilon_A = \kappa_A$ is projected on the 0-brane through the following equations:

$$\begin{aligned}
\kappa_A \,+\, \epsilon_{AB}\Pi^a\,\gamma_a\,\kappa^B e^{i\varphi} &= 0 \ , \\
\kappa^A \,+\, \epsilon^{AB}\Pi^a\,\gamma_a\,\kappa_B e^{-i\varphi} &= 0 \ ,
\end{aligned} \tag{34}$$

where φ is an appropriate phase that we identify below. Note that the above 0-brane projection on the κ-supersymmetry parameter is identical in form to the condition (15) imposed on the parameter of the supersymmetry preserved by the BPS black holes. This can be easily checked by going to a static gauge where the worldline time τ is identified with the coordinate time t.

Now we prove the following statement: *the action (27) is κ-supersymmetric if the real function is chosen as follows:*

$$R(z,\bar{z}) = -2|Z(z,\bar{z})| \ . \tag{35}$$

We obtain the proof by direct verification using the '1.5-order formalism'. This means that we vary the action (27) only in the superspace coordinates x and θ and after variation we implement the field equations of the auxiliary fields Π^a and e.

With this proviso there are only three relevant variations, namely that of the vielbein V^a, that of the gauge fields A^I, B_J and that of the scalars z^i, $\bar{z}^{\bar{\imath}}$. In this way we produce two kinds of terms, those containing the gravitino ψ_A, ψ^A and those containing the gaugino $\bar{\lambda}^{iA}, \bar{\lambda}^{\bar{\imath}}_A$. Such terms have to cancel separately. The variation of the gauge fields contains both type of terms, while the variation of the vielbein contains only the gravitino and the variation of the scalars contains only the gauginos (see (31) and (33)). Let us concentrate first on the gravitino terms. We obtain:

$$\begin{aligned}\frac{1}{4\pi k}\delta_\kappa\, S_{\mathrm{wl}} \;=\;& \int_{M_1} R(z,\bar{z})\,\left[\bar{\psi}_A\,\gamma_a\kappa^A + \bar{\psi}^A\,\gamma_a\kappa_A\right]\Pi^a \\ & -2q_I\int_{M_1}\left(\bar{X}^I\,\bar{\psi}_A\,\kappa_B\,\epsilon^{AB} + X^I\,\bar{\psi}^A\,\kappa^B\,\epsilon_{AB}\right) \\ & +2p^J\int_{M_1}\left(\bar{F}_J\,\bar{\psi}_A\,\kappa_B\,\epsilon^{AB} + F_J\,\bar{\psi}^A\,\kappa^B\,\epsilon_{AB}\right) \ .\end{aligned} \tag{36}$$

We simplify the terms in (36) that contain the γ_a matrix by use of the projection property (34):

$$\begin{aligned}\bar{\psi}_A\,\gamma_a\kappa^A\,\Pi^a &= -e^{-\mathrm{i}\,\varphi}\,\epsilon^{AB}\bar{\psi}_A\,\gamma_a\,\gamma_b\kappa_B\,\Pi^a\,\Pi^b = -e^{-\mathrm{i}\,\varphi}\,\epsilon^{AB}\bar{\psi}_A\,\kappa_B\,\Pi^a\,\Pi^b\,\eta_{ab} \\ &= e^{-\mathrm{i}\,\varphi}\,\epsilon^{AB}\bar{\psi}_A\,\kappa_B \\ \bar{\psi}^A\,\gamma_a\kappa_A\,\Pi^a &= -e^{\mathrm{i}\,\varphi}\,\epsilon_{AB}\bar{\psi}^A\,\gamma_a\,\gamma_b\kappa^B\,\Pi^a\,\Pi^b = -e^{\mathrm{i}\,\varphi}\,\epsilon_{AB}\bar{\psi}^A\,\kappa^B\,\Pi^a\,\Pi^b\,\eta_{ab} \\ &= e^{\mathrm{i}\,\varphi}\,\epsilon_{AB}\bar{\psi}^A\,\kappa^B \ ,\end{aligned} \tag{37}$$

where we have also used the first-order equation (28). Inserting (37) into (36) we see that all the $\bar{\psi}_A, \bar{\psi}^A$ terms cancel if

$$\begin{aligned}\bar{Z} \equiv \left(p^J\,\bar{F}_J - q_I\,\bar{X}^I\right) &= -\tfrac{1}{2}R(z,\bar{z})\,e^{-\mathrm{i}\,\varphi} \ , \\ Z \equiv \left(p^J\,F_J - q_I\,X^I\right) &= -\tfrac{1}{2}R(z,\bar{z})\,e^{\mathrm{i}\,\varphi} \ .\end{aligned} \tag{38}$$

Hence we conclude that

$$R(z,\bar{z}) = -2\,|\,Z\,| \quad ; \quad \varphi(z,\bar{z}) = \tfrac{1}{2}\mathrm{i}\,\log\frac{\bar{Z}}{Z} \ . \tag{39}$$

At this point the action is completely fixed and no further parameters can be adjusted. The very non-trivial check of κ-supersymmetry is that the same choices (39) needed to cancel the gravitino terms guarantee the cancellation of gaugino terms as well. To verify this, we need to consider the variation of the real function $R(z,\bar{z})$. We obtain:

$$\begin{aligned}
\delta_\kappa R(z,\bar{z}) &= -\frac{1}{|Z|}\left[\bar{Z}\,\nabla_i Z\,\delta z^i + Z\,\nabla_{\bar{\imath}}\bar{Z}\,\delta\bar{z}^{\bar{\imath}}\right] \\
&= -e^{-\mathrm{i}\varphi}\left(p^I\,h_{I|i} - q_J\,f_i^J\right)\bar{\lambda}^{iA}\,\kappa_A - e^{\mathrm{i}\varphi}\left(p^I\,\bar{h}_{I|\bar{\imath}} - q_J\,\bar{f}_{\bar{\imath}}^J\right)\bar{\lambda}^{\bar{\imath}}_A\,\kappa^A\ . \qquad (40)
\end{aligned}$$

Note that in deriving (40) we have used the property that the central charge is covariantly holomorphic $\nabla_{\bar{\imath}}Z = 0$. Using (40) we conclude that at the level of the gaugino terms the κ-supersymmetry variation of the worldline action is as follows:

$$\begin{aligned}
\frac{1}{4\pi k}\delta_\kappa\,S_{\rm wl} &= \int_{M_1}\left[\left(p^I\,h_{I|i} - q_J\,f_i^J\right)\bar{\lambda}^{iA}\,\kappa_A\,e^{-\mathrm{i}\varphi} + \left(p^I\,\bar{h}_{I|\bar{\imath}} - q_J\,\bar{f}_{\bar{\imath}}^J\right)\bar{\lambda}^{\bar{\imath}}_A\,\kappa^A\,e^{\mathrm{i}\varphi}\right]\Pi^a\,V^b\,\eta_{ab} \\
&- \left[\left(p^I\,h_{I|i} - q_J\,f_i^J\right)\bar{\lambda}^{iA}\,\gamma_a\kappa^B\,\epsilon_{AB} + \left(p^I\,\bar{h}_{I|\bar{\imath}} - q_J\,\bar{f}_{\bar{\imath}}^J\right)\bar{\lambda}^{\bar{\imath}}_A\,\gamma_a\kappa_B\,\epsilon^{AB}\right]V^a\ . \qquad (41)
\end{aligned}$$

Using the projection property (34) of the κ-supersymmetry parameter we can reduce the terms containing the γ_a matrix in complete analogy to (37):

$$\begin{aligned}
\bar{\lambda}^{iA}\,\gamma_a\,\kappa^B\,\epsilon_{AB}\,V^a &= -e^{-\mathrm{i}\varphi}\,\bar{\lambda}^{iA}\,\gamma_a\,\epsilon^{BC}\,\Pi^b\,\gamma_b\,\kappa_C\,\epsilon_{AB}\,V^a = e^{-\mathrm{i}\varphi}\bar{\lambda}^{iA}\kappa_A\,V^a\Pi^b\,\eta_{ab}\ , \\
\bar{\lambda}^{\bar{\imath}}_A\,\gamma_a\,\kappa_B\,\epsilon^{AB}\,V^a &= -e^{\mathrm{i}\varphi}\,\bar{\lambda}^{\bar{\imath}}_A\,\gamma_a\,\epsilon_{BC}\,\Pi^b\,\gamma_b\,\kappa^C\,\epsilon_{AB}\,V^a = e^{\mathrm{i}\varphi}\bar{\lambda}^{\bar{\imath}}_A\kappa^A\,V^a\Pi^b\,\eta_{ab}\ . \qquad (42)
\end{aligned}$$

Inserting the identity (42) into (41) we verify that all the gaugino terms cancel identically. This concludes the proof of our statement.

Now that we have shown that the action (27) is supersymmetric with the choice (39), we can use the first-order field equations (28) and (26) to recast the action in second-order form. To do this explicitly at the level of both the fermionic and bosonic coordinates we would need an explicit parametrization of the spacetime supervierbein V^a in terms of both x and θ. For the bosonic part we obtain

$$S = 4\pi k\left[-2\int_{M_1}|Z|\sqrt{-h_{\tau\tau}}d\tau + \int_{M_2}\left(p^J\,\mathcal{G}_J - q_I\,\mathcal{F}^I\right)\right]\ , \qquad (43)$$

where the induced metric $h_{\tau\tau}$, defined by (30), is given in (29). Note that the mass we find here as a source term for gravity agrees with (24).

4 Derivation from 3-brane wrapping

The 0-brane action constructed in the previous section has a fully general validity. Indeed, as we emphasized above, its κ supersymmetry relies only on the general identities of special Kähler geometry, irrespective of whether the vector multiplet complex scalars $z^i, \bar{z}^{\bar{\imath}}$ can be interpreted as *moduli* of a Calabi–Yau manifold or not.

In the case where vector multiplets are associated with *complex structure moduli* of a Calabi–Yau 3-fold $\mathcal{M}_{\rm CY}$, giving the compactification of type IIB string theory from 10 to

4 dimensions, then our 0-brane action admits a geometrical interpretation as the result of wrapping 3-branes along suitable 3-cycles. For simplicity, we restrict our attention to the bosonic action (43). The full identification is guaranteed by κ-supersymmetry that has already been proven.

We start from the bosonic world-volume action of a 3-brane in type IIB string theory; for our purpose we can limit ourselves to its 'Nambu–Goto' kinetic term and its coupling to the Ramond–Ramond 4-form, namely

$$S_3 = -\frac{T_3}{\kappa_{10}} \int_{M_4} d^4\xi \sqrt{-\det h_{mn}} + \mu_3 \int_{M_5} F_5^+ \ . \tag{44}$$

Here $h_{mn}(\xi)$ is the induced metric

$$h_{mn} = \frac{\partial X^M}{\partial \xi^m} G_{MN} \frac{\partial X^N}{\partial \xi^n} \ , \tag{45}$$

where G_{MN} is the 10-dimensional metric, and the fields $X^M(\xi)$ describe the embedding of the brane in the 10-dimensional space. The tension of the 3-brane is $T_3 = \sqrt{\pi}$ and it is related to its RR charge by the BPS condition $\mu_3 = \sqrt{2}T_3$. Further, κ_{10} is the 10-dimensional gravitational coupling constant which appears in the Einstein–Hilbert part of the 'bulk' type II supergravity action as

$$S_{\rm IIB} = -\frac{1}{2\kappa_{10}^2} \int d^{10}X \sqrt{G}\, R(X) + \dots \ . \tag{46}$$

We wrote the Wess–Zumino term as an action on a five-dimensional manifold M_5, having the 3-brane world-volume M_4 as its boundary.

We compactify the theory to 4 dimensions on a Calabi–Yau space; accordingly, we decompose the 10-dimensional coordinates as $X^M = (x^\mu, y^\alpha, \bar{y}^{\bar{\alpha}})$. The metric has a direct product form: the only non-zero entries are $G_{\mu\nu}$, the 4 dimensional metric, and $G_{\alpha\bar{\beta}}$, the metric on the CY space. Notice that upon compactification, the integral over the Calabi–Yau space contributes in (46) with a factor

$$V_{\rm CY} = \int_{\mathcal{M}_{\rm CY}} {\rm i}\, d^3y\, d^3\bar{y}\, \left|\det G_{\alpha\bar{\beta}}\right| \ , \tag{47}$$

the volume of the CY manifold. Therefore, the effective gravitational constant in 4 dimensions, that, as usual, we have conventionally taken to be 1, is

$$\kappa_4 = \frac{\kappa_{10}}{V_{\rm CY}^{1/2}} = 1 \ . \tag{48}$$

This will be important below.

We consider the case in which the 3-brane is wrapped on a non-trivial 3-cycle of the internal CY manifold. Then the embedding of the brane is as follows:

$$x^\mu(\xi^0) \ ; \qquad y^\alpha(\xi^i) \ ; \qquad \bar{y}^{\bar{\alpha}}(\xi^i) \ , \tag{49}$$

having split the coordinates on the world-volume as $\xi^m = (\xi^0, \xi^i)$, with $i = 1, 2, 3$.

We now consider separately the kinetic terms and WZ-terms.

Wrapping the kinetic term. Although it is very simple, we do not write the explicit form of the first-order 3-brane action since we can reach our conclusion by working directly in the second-order formalism. The second-order bosonic action of the 3-brane (44) is given in terms of the induced metric (45). Due to the direct product structure of the metric G, the metric h_{mn} is block diagonal, and $\det(-h_{mn})$ is a product of $-h_{00} = -\dot{x}^\mu G_{\mu\nu}\dot{x}^\nu$ times the determinant of the 3×3 matrix

$$h_{ij} = 2\frac{\partial y^\alpha}{\partial \xi^{(i}}\frac{\partial y^{\bar\beta}}{\partial \xi^{i)}} G_{\alpha\bar\beta} \ , \tag{50}$$

that is the pullback of the CY Kähler metric $G_{\alpha\bar\beta}$ on the 3-cycle.

As a next step we show that the integral on a 3-cycle $\mathcal{C}^3$ of the 'spatial' part of the kinetic action (44) is proportional (through a constant) to the modulus of the central charge

$$\int_{\mathcal{C}^3} d^3\xi \sqrt{\det h_{ij}} = 8\sqrt{\pi} k\, V_{\rm CY}^{1/2}\, |Z| \ , \tag{51}$$

if $\mathcal{C}^3$ is a *supersymmetric* cycle [20]. A supersymmetric 3-cycle of a CY is described by an embedding $f:\ \mathcal{C}^3 \longrightarrow \mathcal{M}_{\rm CY}$ satisfying two conditions.

1. The cycle is a Lagrangian submanifold, namely the pull-back of the Kähler 2-form J vanishes:
$$0 = f^*J = \frac{\mathrm{i}}{2\pi} G_{\alpha\bar\beta}\frac{\partial y^\alpha}{\partial \xi^i}\frac{\partial y^{\bar\beta}}{\partial \xi^j} d\xi^i \wedge d\xi^j \ . \tag{52}$$

2. Introduce the function ϕ as follows:
$$f^*\Omega = \Omega_{\alpha\beta\gamma}\frac{\partial y^\alpha}{\partial \xi^i}\frac{\partial y^\beta}{\partial \xi^j}\frac{\partial y^\gamma}{\partial \xi^k}\, d\xi^i \wedge d\xi^j \wedge d\xi^k \equiv 6\phi\, d^3\xi \ , \tag{53}$$
where Ω is the unique holomorphic 3-form on the CY space, and in our conventions $d\xi^i \wedge d\xi^j \wedge d\xi^k = \epsilon^{ijk} d^3\xi$. Then *the phase of ϕ has to be constant.* As will become evident, this is a minimal volume condition.

It is not guaranteed *a priori* that supersymmetric representatives exist in any cohomology class; their existence may thus restrict the allowed values of electric and magnetic charges. The latter specify indeed, as we shall see below, the cohomology class of the 3-cycle.

In order to compute $\det h_{ij}$ we need to consider the induced metric on the world-volume

$$\det h_{ij} = 8\,|\det(\partial_i y^\gamma)|^2 \det G_{\alpha\bar\beta} \ , \tag{54}$$

using the definition (50), combined with the first of the two conditions defining a supersymmetric cycle, equation (52). Since Ω is covariantly constant on the CY 3-fold, its norm

$$\|\Omega\|^2 \equiv \tfrac{1}{6} G^{\alpha\bar\alpha'} G^{\beta\bar\beta'} G^{\gamma\bar\gamma'} \Omega_{\alpha\beta\gamma}\bar\Omega_{\bar\alpha'\bar\beta'\bar\gamma'} = (\det G_{\alpha\bar\beta})^{-1} |\Omega_{123}|^2 \tag{55}$$

is a constant. From the above relation we can obtain the expression of the determinant of the Calabi–Yau Kähler metric and insert it into (54). We find

$$\det h_{ij} = \frac{8}{\|\Omega\|^2} |\det(\partial_i y^\alpha)\Omega_{123}|^2 = 8\frac{|\phi|^2}{\|\Omega\|^2} \ , \tag{56}$$

where in the last step we have used $\phi = \Omega_{123}\det(\partial_i y^\alpha)$, as follows from (53).

Next we recall that for the special Kähler geometry the Kähler potential $\mathcal{K}$ is defined as

$$e^{-\mathcal{K}} = \int_{\mathcal{M}_{\rm CY}} {\rm i}\Omega \wedge \bar{\Omega} = 36\|\Omega\|^2 V_{\rm CY} \ , \tag{57}$$

with $V_{\rm CY}$ defined in (47). Here and in the following it is useful to introduce the rescaled $(3,0)$ form

$$\widehat{\Omega} = e^{\mathcal{K}/2}\Omega \ , \tag{58}$$

which thus satisfies $\int_{\mathcal{M}_{\rm CY}} \widehat{\Omega} \wedge \bar{\widehat{\Omega}} = -{\rm i}$. Inserting (57) into (56) we obtain

$$\frac{1}{2\sqrt{2}}\int_{\mathcal{C}^3} d^3\xi\sqrt{\det h_{ij}} = \int_{\mathcal{C}^3} d^3\xi \frac{|\phi|}{\|\Omega\|} \geq \frac{1}{\|\Omega\|}\left|\int_{\mathcal{C}^3} d^3\xi\phi\right| = \frac{1}{6\|\Omega\|}\left|\int_{\mathcal{C}^3} f^*\Omega\right| = V_{\rm CY}^{1/2}\left|\int_{\mathcal{C}^3} f^*\widehat{\Omega}\right| \ . \tag{59}$$

It is now clear that the bound is saturated whenever the phase of ϕ is constant over the 3-brane. In virtue of condition (53) this happens for the supersymmetric cycles. Therefore, for these latter cycles we obtain indeed (51), since the central charge can be expressed as

$$\sqrt{8\pi}k\, Z = \int_{\mathcal{C}^3} f^*\widehat{\Omega} \ . \tag{60}$$

Equation (60) is established through the following steps. First one recalls that the holomorphic symplectic section of special geometry is provided by the periods of the holomorphic 3-form along a homology basis:

$$\begin{pmatrix} X^I \\ F_J \end{pmatrix} = \begin{pmatrix} \int_{A^I} \widehat{\Omega} \\ \int_{B_J} \widehat{\Omega} \end{pmatrix} \ . \tag{61}$$

Then one interprets the electric and magnetic charges of the black hole as the components of the cycle $\mathcal{C}^3$ in the same homology basis:

$$\mathcal{C}^3 = \sqrt{8\pi}k\left(-q_I\, A^I + p^J\, B_J\right) \ . \tag{62}$$

The overall factor is introduced because p and q have arbitrary normalization. Considering (12), we see that the minimal value of q and p is $1/(\sqrt{8\pi}k)$, leading to the factor above. Comparing with (16), this leads immediately to (60).

In this way we have established that wrapping along a supersymmetric 3-cycle the kinetic term of the 3-brane we get

$$\begin{aligned} S_{3,\rm Kin} &\equiv -\frac{T_3}{\kappa_{10}}\int_{M_4} d^4\xi\sqrt{-\det h_{mn}} = -\frac{T_3}{\kappa_{10}}\int_{M_1} d\tau\sqrt{-h_{00}}\, 2\sqrt{2}\, V_{\rm CY}^{1/2}\, \sqrt{8\pi}k\,|Z| \\ &= -8\pi k\int_{M_1} |Z|\sqrt{-h_{00}}\, d\tau \ , \end{aligned} \tag{63}$$

which is the kinetic term of the 0-brane in the form we have discussed in the previous section.

Wrapping the Wess Zumino term. To obtain the WZ part of the 0-brane action, we consider the WZ action for the 3-brane action (44), written as an action on a five-dimensional manifold M_5, having the 3-brane world-volume as its border. The Ramond–Ramond field strength F_5^+ is real and self-dual. Using the results in (135), it contains upon compactification self-dual field strengths in 4 dimensions multiplied by $(3,0)$ and $(1,2)$ forms and antiself-dual field strengths with $(0,3)$ and $(2,1)$ forms. Considering Kähler weights and symplectic invariance we fix the expression of the WZ action in terms of the graviphoton and matter field strengths T and G^i defined in (14):

$$S_{3,\mathrm{WZ}} = \mu_3 b \int_{M_5} \left[\widehat{\Omega}^{(3,0)}(-\mathrm{i})T^+ + \widehat{\Omega}^{(1,2)}_{\bar{\imath}} 2G^{+\bar{\imath}} + c.c.\right] . \tag{64}$$

The factor b should be chosen such that b times the integral gives $\sqrt{2\pi}\,\mathbb{Z}$, according to the quantization condition (see e.g. (13.3.12) and (13.3.13) in [21]). Here, $\widehat{\Omega}^{(3,0)}$ is the $(3,0)$ form which we previously just denoted as $\widehat{\Omega}$, while $\widehat{\Omega}^{(1,2)}_{\bar{\imath}}$ are a basis of $(1,2)$ forms provided by Kähler covariant derivatives from its complex conjugate $\widehat{\Omega}^{(0,3)}$. This expression (64) is symplectic covariant. Indeed, due to the special Kähler identity

$$-\mathrm{i}X^I T^+ + 2\bar{f}^I_{\bar{\imath}} G^{+\bar{\imath}} = \mathcal{F}^{+I} , \tag{65}$$

it can be written as

$$S_{3,\mathrm{WZ}} = \mu_3 b \int_{M_5} \begin{pmatrix} \widehat{\Omega}^{(3,0)} & \widehat{\Omega}^{(1,2)}_{\bar{\imath}} \end{pmatrix} Y^{-1} \mathcal{F}^{I+} + \mathrm{c.c.} , \tag{66}$$

where Y is the invertible $(n+1)\times(n+1)$ matrix

$$Y = \begin{pmatrix} X^I & \bar{f}^I_{\bar{\imath}} \end{pmatrix} . \tag{67}$$

The integral is performed over M_5 which is $M_2 \times \mathcal{C}^3$, the former being the manifold whose boundary is the worldline M_1 as in (27). We perform the integral over the supersymmetric cycle $\mathcal{C}^3$, using (61), which can be written elegantly as

$$\begin{aligned} \int_{\mathcal{C}^3} \begin{pmatrix} \widehat{\Omega}^{(3,0)} & \widehat{\Omega}^{(1,2)}_{\bar{\imath}} \end{pmatrix} &= \sqrt{8\pi}k \begin{pmatrix} p^I & q_J \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} X^J & \bar{f}^J_{\bar{\imath}} \\ F_I & \bar{h}_{\bar{\imath}I} \end{pmatrix} \\ &= \sqrt{8\pi}k \begin{pmatrix} p^I & q_J \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} Y \\ \mathcal{N}Y \end{pmatrix} . \end{aligned} \tag{68}$$

This allows us to write the WZ term as

$$\begin{aligned} S_{3,\mathrm{WZ}} &= \mu_3 b \sqrt{8\pi}k \int_{M_2} \begin{pmatrix} p^I & q_J \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} Y \\ \mathcal{N}Y \end{pmatrix} Y^{-1}\mathcal{F}^{I+} + \mathrm{c.c.} \\ &= \mu_3 b \sqrt{8\pi}k \int_{M_2} (-q_I + p^J \mathcal{N}_{JI})\mathcal{F}^{I+} + \mathrm{c.c.} \\ &= \mu_3 b \sqrt{8\pi}k \int_{M_2} (p^J \mathcal{G}_J - q_I \mathcal{F}^I) . \end{aligned} \tag{69}$$

Using (11) and (12) one now establishes that $b(\sqrt{8\pi}k)4\pi k/(8\pi k^2) = \sqrt{2\pi}$, as we demanded before, thus $b=1$. With $\mu_3 = \sqrt{2\pi}$ we find the same result as in (43). Putting together the Wess–Zumino and the kinetic term we have shown that the wrapping of the 3-brane action over a supersymmetric CY cycle leads to the 0-brane action derived previously, (43). Let us note that in [22] it was shown that by wrapping a D3-brane on a cycle of a T^6/Z_3 orbifold, one obtains a Reissner–Nordström black hole solution of $\mathcal{N} = 2$ supergravity.

5 Generalization to higher $\mathcal{N}$ supergravity, in particular $\mathcal{N} = 8$

In this section we discuss how our result for the 0-brane action coupled to $\mathcal{N} = 2$ supergravity can be extended to the case where the 0-brane moves in a higher extended supergravity background. In particular, we focus on the case $\mathcal{N} = 8$, for which the BPS black holes have been constructed and classified in [23],[24],[25].

As we show below the structure of the $\mathcal{N} = 8$ 0-brane is very similar to that of the $\mathcal{N} = 2$ brane and it is almost its straightforward generalization. Yet it is clear that for $\mathcal{N} > 2$, there is the possibility of having BPS configurations that preserve different fractions of supersymmetry. In particular, the BPS black holes for the case $\mathcal{N} = 8$ fall into three different classes, depending on the fraction of supersymmetry

$$\frac{\nu}{8} \ , \quad \nu = 1, 2, 4 \ , \tag{70}$$

that they preserve. Similarly, we will discover that there are three classes of $\mathcal{N} = 8$ 0-brane actions where the worldline κ-supersymmetry has, respectively, 1/2, 1/4 or 1/8 of the $32 = 8 \times 4$ components possessed by $\mathcal{N} = 8$ spacetime supersymmetry. Indeed, as it was already the case in $\mathcal{N} = 2$ supergravity, the appropriate projection satisfied by the κ-supersymmetry parameter coincides with the projection equation satisfied by the black hole BPS Killing spinor. It follows from these introductory remarks that, in order to discuss the $\mathcal{N} = 8$ 0-brane actions we have to recollect some results and properties of the corresponding black hole solutions.

Table 1: The three classes of $\mathcal{N} = 8$ BPS black holes

ν	SUSY	Central Charge	$G_{\text{stab}} \subset$ SU(8)
1	1/8	$Z_1(\infty) \neq Z_2(\infty) \neq Z_3(\infty) \neq Z_4(\infty)$	$\text{USp}(2)^4$
2	1/4	$Z_1(\infty) = Z_2(\infty) \neq Z_3(\infty) = Z_4(\infty)$	$\text{USp}(4) \times \text{USp}(4)$
4	1/2	$Z_1(\infty) = Z_2(\infty) = Z_3(\infty) = Z_4(\infty)$	USp(8)

As discussed extensively in [24, 11], the three cases $\nu = 1, 2, 4$ are characterized by the structure of the central charge at infinity or, more intrinsically, by the covariance group of the corresponding Killing spinor equation. In $\mathcal{N} = 8$ supergravity the central charge Z_{AB} is an antisymmetric field tensor transforming in the **28** representation of $SU(8)$. By means of $SU(8)$ local transformations it can always be brought to normal form, namely skew diagonalized as follows:

$$Z_{AB} = \mathrm{diag}(Z_1\varepsilon, Z_2\varepsilon, Z_3\varepsilon, Z_4\varepsilon) \; ; \qquad \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \; , \tag{71}$$

and the complex numbers $Z_i(x)$ $(i = 1, \ldots, 4)$ are the four skew eigenvalues. The structure of these eigenvalues at spatial infinity $(r \to \infty)$ characterizes the three distinct orbits of $\mathcal{N} = 8$ black holes. If we consider the $\mathcal{N} = 8$ supersymmetry algebra generated by these Killing spinors, where we have put Z_{AB} in block-diagonal form (71), then the analysis of the BPS bound on a state of mass M reduces to the usual $\mathcal{N} = 2$ analysis for each block. Thus we must have

$$M \geq |Z_1| \geq |Z_2| \geq |Z_3| \geq |Z_4| \; , \tag{72}$$

having assumed in (71) a specific ordering of the eigenvalues. When all Z_i are different, the BPS bound can be saturated in the first block only: $M = \pm |Z_1|^2$. If this is the case, a combination of the supersymmetry charges with indices in the first block annihilates the state, and 1/8 of supersymmetry is preserved. When $Z_1 = Z_2$, the saturation of the BPS bound occurs simultaneously in the first two blocks: $M = |Z_1| = |Z_2|$, leading to 2 preserved combination of supercharges (1/4 of the total number), and so on. When all the eigenvalues of Z are equal, BPS saturation corresponds to conservation of 1/2 of supersymmetry. The result is shown in table 1.

The stability subgroup $G_{\mathrm{stab}} \subset SU(8)$ of the central charge is related to, but not identical to, the covariance group of the Killing spinor equation. By the definition of the BPS state the supersymmetry transformations vanish in the black hole background if they are taken along a special supersymmetry parameter ϵ_A, the Killing spinor, satisfying a suitable projection equation, which is the higher-$\mathcal{N}$ generalization of (15).

As formulated in [26, 24, 25], such a condition is the following:

$$\gamma_0 \, \epsilon_A = \varpi_{AB} \epsilon^B \;\; , \quad \gamma_0 \, \epsilon^A = \varpi^{AB} \epsilon_B \; , \tag{73}$$

where, in block notation,

$$\varpi_{AB} = \varpi^{AB} = \mathrm{diag} \left(\mathbb{C}_{2\nu \times 2\nu}, \mathbf{0}_{(8-2\nu)\times(8-2\nu)} \right) \, . \tag{74}$$

The real antisymmetric matrix $\mathbb{C}_{2\nu\times 2\nu}$ satisfies $\mathbb{C}^2_{2\nu\times 2\nu} = -\mathbb{1}_{2\nu\times 2\nu}$. Notice that (73) implies that the Killing spinors are projected to the upper $2\nu \times 2\nu$ block:

$$\epsilon_A = h_A{}^B \, \epsilon_B \;\; , \quad \epsilon^A = h^A{}_B \, \epsilon^B \; , \tag{75}$$

by the matrix

$$h_A{}^B \equiv -\varpi_{AC} \, \varpi^{CB} = \mathrm{diag} \left(\mathbb{1}_{2\nu\times 2\nu}, \mathbf{0}_{(8-2\nu)\times(8-2\nu)} \right) \, . \tag{76}$$

The three choices $\nu = 1, 2, 4$ correspond to 1/8, 1/4 and 1/2 of preserved supersymmetry, respectively.

The invariance group G_{inv} of the Killing spinor equation (73) is $G_{\mathrm{inv}} = \mathrm{USp}(2\nu) \times \mathrm{U}(8 - 2\nu)$, i.e. the $SU(8)$ subgroup that leaves the matrix ϖ_{AB} unchanged.

The structure of the Killing spinor projection is related to the structure of these eigenvalues Z_i of the central charge. Indeed, by direct construction of the BPS black hole solutions three distinct possibilities, listed in table 1, have been found.

Comparing with our previous discussion we see that the two groups G_{inv} and G_{stab} admit the common factor $USp(2\nu)$. However, G_{inv} is bigger, as for its determination only the blocks of Z containing the highest eigenvalue(s) matter.

The analogy between the $\mathcal{N} = 8$ equation (73) and its $\mathcal{N} = 2$ counterpart (15) is complete only if, in the latter case, we get rid of the phase $Z/|Z|$ by imposing reality of the central charge $Z = |Z|$. In the $\mathcal{N} = 2$ theory this is a $U(1)$ gauge choice that can always be reached by means of suitable Kähler transformations. However, as our previous experience teaches, if we want a $U(1)$ gauge covariant form of the κ-supersymmetric action we had better keep the phase $Z/|Z|$ in place and write the projection on the κ-symmetry parameter as given in (34). So, although the black hole solutions of $\mathcal{N} = 8$ supergravity have been derived starting from equations (73), the previous observations suggest that such a construction is based on a $SU(8)$ gauge-choice that we had better undo in order to obtain an $SU(8)$ covariant form of the 0-brane κ-supersymmetric actions. Indeed, the role of $U(1)$ is now played by the group $SU(8)$ which is both the automorphism group of the $\mathcal{N} = 8$ supersymmetry algebra and the isotropy subgroup of the homogeneous scalar manifold.

To discuss the $SU(8)$ covariantization of the Killing spinor equation (73) we have to review the basic ingredients of the $\mathcal{N} = 8$ theory and fix our conventions.

5.1 Scalar fields, $E_{7(7)}$ structure and the central charge

The theory possesses 70 scalar fields ϕ^I that span the non-compact coset manifold $E_{7(7)}/SU(8)$. Using standard notations (see, for example, [11]) these fields are introduced through the coset representative $\mathbb{L}(\phi)$, that is an $E_{7(7)}$ matrix in the fundamental **56** representation. We have

$$\mathbb{L} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|c} f + \mathrm{i}h & \bar{f} + \mathrm{i}\bar{h} \\ \hline f - \mathrm{i}h & \bar{f} - \mathrm{i}\bar{h} \end{array} \right) , \tag{77}$$

where the 28×28 submatrices (h, f) are labeled by antisymmetric pairs (Λ, Σ) and (A, B), with $\Lambda, \Sigma = 1, \ldots, 8$ and $A, B = 1, \ldots, 8$, the first pair transforming under $E_{7(7)}$ and the second one under $SU(8)$:

$$(h, f) = \left(h_{\Lambda\Sigma|AB}, f^{\Lambda\Sigma}{}_{AB} \right) . \tag{78}$$

As expected from general arguments we have $\mathbb{L} \in USp\,(28, 28)$. Indeed, the theory contains 28 gauge bosons so that we have 28 electric field strengths and 28 magnetic ones which have to transform into one another through elements of the $USp(28, 28)$ group. The vielbein P_{ABCD} and the $SU(8)$ connection $\Omega_A{}^B$ of $E_{(7)7}/SU\,(8)$ are computed from the left-invariant 1-form $\mathbb{L}^{-1}d\mathbb{L}$:

$$\mathbb{L}^{-1}d\mathbb{L} = \left(\begin{array}{c|c} \delta^{[A}{}_{[C}\Omega^{B]}{}_{D]} & \bar{P}^{ABCD} \\ \hline P_{ABCD} & \delta_{[A}{}^{[C}\bar{\Omega}_{B]}{}^{D]} \end{array}\right) , \tag{79}$$

where $P_{ABCD} \equiv P_{ABCD,i}d\Phi^i$, with $(i = 1, \ldots, 70)$, is completely antisymmetric and satisfies the reality condition

$$P_{ABCD} = \frac{1}{24}\epsilon_{ABCDEFGH}\bar{P}^{EFGH} . \tag{80}$$

The bosonic Lagrangian of $\mathcal{N} = 8$ supergravity is [27]

$$\begin{aligned} \mathcal{L} &= \int \sqrt{-g}\, d^4x \left(-\tfrac{1}{2}R + \tfrac{1}{4}\mathrm{Im}\, \mathcal{N}_{\Lambda\Sigma|\Gamma\Delta}F_{\mu\nu}{}^{\Lambda\Sigma}F^{\Gamma\Delta|\mu\nu} - \tfrac{1}{6}P_{ABCD,i}\bar{P}^{ABCD}_j\partial_\mu\Phi^i\partial^\mu\Phi^j + \right. \\ &+ \left. \tfrac{1}{8}\mathrm{Re}\, \mathcal{N}_{\Lambda\Sigma|\Gamma\Delta}\frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}F_{\mu\nu}{}^{\Lambda\Sigma}F^{\Gamma\Delta}{}_{\rho\sigma}\right) , \end{aligned} \tag{81}$$

where the gauge kinetic matrix $\mathcal{N}_{\Lambda\Sigma|\Gamma\Delta}$ is defined by $\mathcal{N} = hf^{-1}$, i.e. explicitly by

$$\mathcal{N}_{\Lambda\Sigma|\Gamma\Delta} = h_{\Lambda\Sigma|AB}\,(f^{-1})^{AB}{}_{\Gamma\Delta} . \tag{82}$$

The same matrix relates the (anti)self-dual electric and magnetic 2-form field strengths:

$$G^-_{\Lambda\Sigma} = \bar{\mathcal{N}}_{\Lambda\Sigma|\Gamma\Delta}F^{-\ \Gamma\Delta} \ , \quad G^+_{\Lambda\Sigma} = \mathcal{N}_{\Lambda\Sigma|\Gamma\Delta}F^{+\ \Gamma\Delta} , \tag{83}$$

where the dual field strengths $G^\pm_{\Lambda\Sigma}$, are, as before, defined by $G^\pm_{\Lambda\Sigma} = \frac{i}{2}\frac{\delta\mathcal{L}}{\delta F^{\pm\ \Lambda\Sigma}}$. Note that the 56-dimensional (anti)self-dual vector $\left(F^{\pm\ \Lambda\Sigma}, G^\pm_{\Lambda\Sigma}\right)$ transforms covariantly under $Sp(56, \mathbb{R})$. The matrix transforming the coset representative $\mathbb{L}$ from the $USp(28, 28)$ basis, (77), to the real $Sp(56, \mathbb{R})$ basis is the Cayley matrix:

$$\mathbb{L}_{\mathrm{USp}} = \mathcal{C}\mathbb{L}_{Sp}\mathcal{C}^{-1} ; \qquad \mathcal{C} = \begin{pmatrix} \mathbb{1} & \mathrm{i}\mathbb{1} \\ \mathbb{1} & -\mathrm{i}\mathbb{1} \end{pmatrix} . \tag{84}$$

Having established our definitions and notations, let us now write the *dressed graviphoton* 2-form, defined according to the obvious generalization of (14):

$$T^{(-)}_{AB} = h_{\Lambda\Sigma AB}\,(\Phi)\,F^{-\Lambda\Sigma} - f^{\Lambda\Sigma}{}_{AB}\,(\Phi)\,G^-_{\Lambda\Sigma} . \tag{85}$$

In a way similar to the $\mathcal{N} = 2$ case we also have the identities:

$$T^+_{AB} = 0 \to T^-_{AB} = T_{AB} ; \qquad \bar{T}^-_{AB} = 0 \to \bar{T}^{+\,AB} = \bar{T}^{AB} . \tag{86}$$

Thus we can define the central charge:

$$Z_{AB} = \frac{1}{\pi}\int_{S^2} T_{AB} = h_{\Lambda\Sigma|AB}p^{\Lambda\Sigma} - f^{\Lambda\Sigma}{}_{AB}q_{\Lambda\Sigma} , \tag{87}$$

which, as we already anticipated, is an antisymmetric tensor transforming in the **28** irreducible representation of $SU(8)$. In (87) the integral of the 2-form T_{AB} is evaluated on any static 2-sphere and the quantized charges ($p^{\Lambda\Sigma}$, $q_{\Lambda\Sigma}$) are defined, in analogy with (11) (choosing in this section the normalization with $k = 1/4$) by

$$p^{\Lambda\Sigma} = \frac{1}{\pi} \int_{S^2} F^{\Lambda\Sigma} \ , \quad q_{\Lambda\Sigma} = \frac{1}{\pi} \int_{S^2} G_{\Lambda\Sigma} \ . \tag{88}$$

Given these preliminaries let us now review the general form of the 1/2 and 1/4 BPS black hole solutions as constructed in [24] and compute the corresponding field-dependent central charge. This gives us the opportunity to see, through explicit formulae, how the Killing spinor equation (73) can be translated in terms of the central charge and in this way $SU(8)$ covariantized.

5.2 The $1/2$ Black-hole and its central charge

In the 1/2 supersymmetry-preserving case ($\nu = 4$), the Killing spinor projection (73) involves the matrix

$$\varpi = \mathbb{C}_{8\times 8} \ , \tag{89}$$

so that the projector $h_A{}^B$ is simply $\delta_A{}^B$. Recalling the results of [24], let us write the BPS black hole solution admitting the gauge-fixed Killing spinor (73), that contains a new parameter q. The metric is

$$ds^2 = -\left[H(x)\right]^{-1/2} dt^2 + \left[H(x)\right]^{1/2} d\vec{x}^2 \ . \tag{90}$$

The electric and magnetic field strengths are, respectively,

$$\begin{aligned} \mathcal{F}^{\Lambda\Sigma} &= -\, q\, \mathbb{C}^{\Lambda\Sigma} \left[H(x)\right]^{-2} \frac{1}{4r^3} dt \wedge \vec{x} \cdot d\vec{x} \ , \\ \mathcal{G}_{\Lambda\Sigma} &= -\, q\, \mathbb{C}_{\Lambda\Sigma} \left[H(x)\right]^{1/2} \frac{x^i}{8r^3}\, dx^j \wedge dx^k\, \epsilon_{ijk} \ . \end{aligned} \tag{91}$$

The scalar fields are described through the coset representative

$$\begin{aligned} f^{\Lambda\Sigma}_{AB} &= \frac{1}{8\sqrt{2}} \mathbb{C}^{\Lambda\Sigma} \mathbb{C}_{AB} \left[H(x)\right]^{-3/4} \ , \\ h_{\Lambda\Sigma|AB} &= -\frac{\mathrm{i}}{8\sqrt{2}} \mathbb{C}_{\Lambda\Sigma} \mathbb{C}_{AB} \left[H(x)\right]^{3/4} \ , \end{aligned} \tag{92}$$

where $H(x)$ denotes a harmonic function in the 3-dimensional transverse space with boundary condition $H(\infty) = 1$. Typically one has

$$H(x) = 1 + \frac{q}{r} \ , \tag{93}$$

where q is the same parameter appearing in (91).

Inserting (91) in the integrals (88) we obtain the values of the electric and magnetic charges for this solution:

$$p^{\Lambda\Sigma} = 0 \quad ; \qquad q_{\Lambda\Sigma} = -\, q\,\mathbb{C}_{\Lambda\Sigma} \ , \tag{94}$$

and using (94) and (92) into (87) we obtain the central charge:

$$Z_{AB} = \frac{q}{\sqrt{2}}\,[H(x)]^{-3/4}\,\mathbb{C}_{AB} \ . \tag{95}$$

From the explicit form (95) we see that the central charge at spatial infinity approaches an antisymmetric matrix with four coinciding skew eigenvalues:

$$Z_{AB} \stackrel{r\to\infty}{\longrightarrow} \frac{q}{\sqrt{2}}\mathbb{C}_{AB} \ , \tag{96}$$

as expected from table 1. On the other hand, near the horizon $r \longrightarrow 0$ the central charge goes to zero as $r^{3/4}$. This confirms what we also expect on general grounds, namely that the entropy of the black hole, proportional to the horizon value of $(Z_{AB}\,\bar{Z}^{AB})^{1/2}$ is zero in the 1/2 SUSY case.

As we stressed at the beginning of this section, by writing the Killing spinor equation in the form (73) we have worked in a fixed $SU(8)$ gauge. Yet it is now quite easy to relax this gauge choice by performing an arbitrary, local $SU(8)$ transformation on the expression we have obtained for the central charge. Let $U(x) \in SU(8)$ be such a gauge transformation. For the BPS 1/2 SUSY preserving black hole we obtain

$$Z_{AB}(x) \;\; = \;\; \lambda(x)\left[U(x)\,\mathbb{C}\,U^T(x)\right]_{AB} \ , \tag{97}$$

where

$$\lambda(x) \;\; = \;\; \bar{\lambda}(x) \equiv \frac{q}{\sqrt{2}}\,[H(x)]^{-3/4} \tag{98}$$

is a unique *real* skew eigenvalue characterizing the central charge at any point in space-time. Since by definition of the $USp(8)$ group the matrix $\mathbb{C}$ is invariant against such transformations, it follows that in (97) we have introduced $63 - 36 = 27$ new arbitrary functions parametrizing the coset manifold $SU(8)/USp(8)$. They are the $\mathcal{N} = 8$ analogue of the single phase $Z/|Z|$ appearing in the $\mathcal{N} = 2$ theory. Yet, what matters and is an intrinsic property of the 1/2 background is that: *in any spacetime point the four skew eigenvalues of the central charge Z_{AB} are real and coincide.*

From this property follows an identity which will be crucial in proving the κ-supersymmetry of the 0-brane action. Indeed, using the expression (97) for the central charge, we have

$$\begin{aligned}
\tfrac{1}{24}\,\epsilon^{ABCDEFGH}\,Z_{EF}\,Z_{GH} \;\; &= \;\; \tfrac{1}{24}\,\lambda(x)\,\epsilon^{ABCDEFGH}\,U_E{}^{E'}\,U_F{}^{F'}\,U_G{}^{G'}\,U_H{}^{H'}\,C_{E'F'}\,C_{G'H'} \\
&= \;\; \tfrac{1}{24}\,\lambda(x)\,\bar{U}^A{}_{A'}\,\bar{U}^B{}_{B'}\,\bar{U}^C{}_{C'}\,\bar{U}^D{}_{D'}\,\epsilon^{A'B'C'D'E'F'G'H'}\,C_{E'F'}\,C_{G'H'} \\
&= \;\; \lambda(x)\,\bar{U}^A{}_{A'}\,\bar{U}^B{}_{B'}\,\bar{U}^C{}_{C'}\,\bar{U}^D{}_{D'}\,C^{[A'B'}\,C^{C'D']} \\
&= \;\; \bar{Z}^{[AB}\,\bar{Z}^{CD]} \ .
\end{aligned} \tag{99}$$

Note that the last equality follows precisely from the reality of the skew eigenvalue $\lambda(x)$.

Let us now turn our attention to the 1/4 susy preserving black holes.

5.3 The $1/4$ black hole and its central charge

In the $1/4$ case ($\nu = 2$) we have:

$$\varpi_{AB} = -\varpi_{BA} = \mathrm{diag}\left(\mathbb{C}_{4\times 4}, \mathbf{0}_{4\times 4}\right) \ , \tag{100}$$

and following [24] it is convenient to introduce the further notations:

$$\begin{aligned} \Omega_{AB} = -\Omega_{BA} &= \mathrm{diag}\left(\mathbf{0}_{4\times 4}, \mathbb{C}_{4\times 4}\right) \ , \\ \tau^{\pm}_{AB} \equiv \tfrac{1}{2}\left(\varpi_{AB} \pm \Omega_{AB}\right) &= \tfrac{1}{2}\,\mathrm{diag}\left(\mathbb{C}_{4\times 4}, \pm\mathbb{C}_{4\times 4}\right) \ . \end{aligned} \tag{101}$$

Then the BPS black hole solution admitting the gauge-fixed Killing spinor (73) can be written as follows. The metric is

$$ds^2 = -\left[H_1(x)\,H_2(x)\right]^{-1/2}\,dt^2 + \left[H_1(x)\,H_2(x)\right]^{1/2}\,d\vec{x}^2 \ . \tag{102}$$

The electric and magnetic field strength are

$$\begin{aligned} \mathcal{F}^{\Lambda\Sigma} &= -\frac{1}{2\sqrt{2}}\left(\left[H_1(x)\right]^{-2}\,q_1\,\tau^{+}_{\Lambda\Sigma} + \left[H_2(x)\right]^{-2}\,q_2\,\tau^{-}_{\Lambda\Sigma}\right)\frac{1}{4r^3}dt\wedge\vec{x}\cdot d\vec{x} \ , \\ \mathcal{G}_{\Lambda\Sigma} &= -\frac{1}{8\sqrt{2}}\left(q_1\left[\frac{H_2(x)}{H_1(x)}\right]^2\tau^{+}_{\Lambda\Sigma} + q_2\left[\frac{H_1(x)}{H_2(x)}\right]^2\tau^{-}_{\Lambda\Sigma}\right)\frac{x^i}{8r^3}\,dx^j\wedge dx^k\,\epsilon_{ijk} \ . \end{aligned} \tag{103}$$

The coset representatives describing the scalar fields are

$$\begin{aligned} f^{\Lambda\Sigma}_{AB} &= \frac{1}{2\sqrt{2}}\left(\left[\frac{H_2(x)}{H_1^3(x)}\right]^{1/4}\tau^{+}_{\Lambda\Sigma}\,\tau^{+}_{AB} + \left[\frac{H_1(x)}{H_2^3(x)}\right]^{1/4}\tau^{-}_{\Lambda\Sigma}\,\tau^{-}_{AB}\right) \ , \\ h_{\Lambda\Sigma|AB} &= \frac{-\mathrm{i}}{2\sqrt{2}}\left(\left[\frac{H_1(x)}{H_2^3(x)}\right]^{-1/4}\tau^{+}_{\Lambda\Sigma}\,\tau^{+}_{AB} + \left[\frac{H_2(x)}{H_1^3(x)}\right]^{-1/4}\tau^{-}_{\Lambda\Sigma}\,\tau^{-}_{AB}\right) \ , \end{aligned} \tag{104}$$

where $H_{1,2}(x)$ are two harmonic functions in the 3-dimensional transverse space with boundary condition $H(\infty) = 1$:

$$H_1(x) = 1 + \frac{q_1}{r} \quad ; \qquad H_2(x) = 1 + \frac{q_2}{r} \tag{105}$$

where $q_{1,2}$ is the same parameter appearing in (103).

Inserting (103) in the integrals (88) we obtain the values of the electric and magnetic charges for this solution:

$$p^{\Lambda\Sigma} = 0 \quad ; \qquad q_{\Lambda\Sigma} = -\frac{1}{8\sqrt{2}}\left[q_1\,\tau^{+}_{\Lambda\Sigma} + q_2\,\tau^{-}_{\Lambda\Sigma}\right] \ , \tag{106}$$

and using (106) and (104) in (87) we obtain the central charge:

$$Z_{AB} = -\frac{1}{16}\left(q_1\left[\frac{H_2(x)}{H_1^3(x)}\right]^{1/4}\tau^{+}_{AB} + q_2\left[\frac{H_1(x)}{H_2^3(x)}\right]^{1/4}\tau^{-}_{AB}\right) \ . \tag{107}$$

From the explicit form (107) we see that at spatial infinity the central charge approaches the antisymmetric matrix

$$Z^{\infty}_{AB} = -\tfrac{1}{16}\left(q_1\,\tau^{+}_{AB} + q_2\,\tau^{-}_{AB}\right) = -\tfrac{1}{16}\,\mathrm{diag}\,\left((q_1+q_2)\,\mathbb{C}_{4\times 4}, (q_1-q_2)\,\mathbb{C}_{4\times 4}\ \right)\ , \qquad (108)$$

whose four skew eigenvalues are real and coincide in pairs as expected from table 1. On the other hand, near the horizon $r \to 0$ the central charge goes to zero as $r^{1/2}$. Also in this case the entropy is zero.

Once again, we can remove the $SU(8)$ gauge fixing utilized in deriving the 1/4 BPS solution by writing the analogue of (97), namely

$$Z_{AB} = \left[U(x)\left(\lambda_1(x)\,\tau^{+}_{AB} + \lambda_2(x)\,\tau^{-}_{AB}\right)\,U^T(x)\right]_{AB}\ , \qquad (109)$$

where

$$\lambda_1(x) = -\frac{q_1}{16}\left[\frac{H_2(x)}{H_1^3(x)}\right]^{1/4}\ ,\quad \lambda_2(x) = -\frac{q_2}{16}\left[\frac{H_1(x)}{H_2^3(x)}\right]^{1/4}\ , \qquad (110)$$

and $U(x)$ is an arbitrary $SU(8)$ gauge transformation.

5.4 The κ-supersymmetry projection and the 0-brane action for the cases of 1/2, 1/4 and 1/8 BPS backgrounds

Having clarified these preliminaries we can now proceed to write the appropriate $SU(8)$ covariant form of the projection on the κ-supersymmetry parameter κ_A, κ^A and the world-line action that is invariant against such κ transformations.

Recalling equation (76) we introduce an 8×8 projection matrix:

$$\mathcal{P}^{(2\nu)B}_{A}(x) \equiv \left[U^\dagger(x)\,h^{(2\nu)}\,U(x)\right]_{A}^{\ \ B}\ , \qquad (111)$$

where $U(x) \in SU(8)$ is the point-dependent $SU(8)$ gauge transformation that skew-diagonalizes the central charge. Such a transformation was already introduced in (97) and (109). The parameter ν takes the values $\nu = 4, 2, 1$ and distinguishes among the three cases of $\nu/8$ preserved supersymmetries. According to the conventions introduced in equation (73), 2ν is the rank of the projector $\mathcal{P}^{(2\nu)A}_{B}(x)$. Next we introduce the projected central charge tensor:

$$\widehat{Z}_{AB} = \mathcal{P}^{(2\nu)C}_{A}(x)\,\mathcal{P}^{(2\nu)D}_{B}(x)\,Z_{CD}\ . \qquad (112)$$

Using such a notation the projection on either the Killing spinor of the BPS background (73) or the κ-supersymmetry parameter can be rewritten in a manifestly $SU(8)$ covariant fashion as follows:

$$\begin{aligned}\Pi^a\gamma_a\kappa_A &= \sqrt{2\nu}\left(\widehat{Z}_{CD}\,\widehat{\overline{Z}}^{CD}\right)^{-1/2}\,\widehat{Z}_{AB}\,\kappa^B\ ,\\ \Pi^a\gamma_a\kappa^A &= \sqrt{2\nu}\left(\widehat{Z}_{CD}\,\widehat{\overline{Z}}^{CD}\right)^{-1/2}\,\widehat{\overline{Z}}^{AB}\,\kappa_B\ .\end{aligned} \qquad (113)$$

The generalization of the condition (75) is

$$\kappa_A = \mathcal{P}_A{}^B \, \kappa_B \ , \quad \kappa^A = \bar{\mathcal{P}}^A{}_B \, \kappa^B \ . \tag{114}$$

The κ-supersymmetric action of a 0-brane moving in the background of a $\nu/8$ supersymmetry preserving $\mathcal{N}=8$ on-shell configuration can now be written. It is an almost direct generalization of (27) and reads as follows:

$$\begin{aligned} \frac{1}{\pi} S^{\mathcal{N}=8}_{(2\nu)} &= \int_{M_1} -\alpha_{2\nu} \left(\widehat{Z}_{AB} \, \widehat{\bar{Z}}^{AB} \right)^{1/2} \left[- \left(\Pi^a \, V^b + \tfrac{1}{2} \, \Pi^a \, \Pi^b \, e \right) \eta_{ab} + \tfrac{1}{2} \, e \right] \\ &\quad + \int_{M_2} \left(p^{\Lambda\Sigma} \, \mathcal{G}_{\Lambda\Sigma} - q_{\Gamma\Delta} \, \mathcal{F}^{\Gamma\Delta} \right) \ , \end{aligned} \tag{115}$$

the parameter $\alpha_{2\nu}$ being fixed by κ-invariance:

$$\alpha_{2\nu} = \sqrt{\frac{2}{\nu}} \ . \tag{116}$$

The proof of κ-supersymmetry can be done along the same lines as in the $\mathcal{N}=2$ case; there are, however, a few subtleties that have to be taken into account, and that are different in the three cases $\nu = 4, 2, 1$. First of all we need to recall the supersymmetry transformation rules of $\mathcal{N}=8$ supergravity. They read as follows (see [27] and for current notations [23, 24, 25] and [11]):

$$\delta \chi_{ABC} = 4 \ P_{ABCD|i} \partial_\mu \Phi^i \gamma^\mu \epsilon^D + 3 T^{(-)}_{[AB|\rho\sigma} \gamma^{\rho\sigma} \epsilon_{C]} \ , \tag{117}$$

$$\delta \psi_{A\mu} = \nabla_\mu \epsilon_A \ + \frac{\mathrm{i}}{4} \, T^{(-)}_{AB|\rho\sigma} \gamma^{\rho\sigma} \gamma_\mu \epsilon^B \ , \tag{118}$$

$$\begin{aligned} \delta A^{\Lambda\Sigma}_\mu &= 2 \, \bar{f}^{\Lambda\Sigma|AB} \bar{\psi}_{A\mu} \epsilon_B \epsilon^{AB} \ + \ 2 \, f^{\Lambda\Sigma}_{AB} \bar{\psi}^A_\mu \epsilon^B \epsilon_{AB} \\ &\quad + \tfrac{1}{4} \left(\bar{f}^{\Lambda|AB} \, \bar{\chi}_{ABC} \, \gamma_a \, \epsilon^C + f^{\Lambda}_{AB} \bar{\chi}^{ABC} \, \gamma_a \, \epsilon_C \right) \, V^a_\mu \ , \end{aligned} \tag{119}$$

$$P^{ABCD}_i \delta \Phi^i = \bar{\chi}^{[ABC} \, \epsilon^{D]} \ + \ \tfrac{1}{24} \, \varepsilon^{ABCDPQRS} \, \bar{\chi}_{PQR} \, \epsilon_S \ , \tag{120}$$

$$P_{ABCD|i} \delta \Phi^i = \bar{\chi}_{[ABC} \, \epsilon_{D]} \ + \ \tfrac{1}{24} \, \varepsilon_{ABCDPQRS} \, \bar{\chi}^{PQR} \, \epsilon^S \ , \tag{121}$$

$$\delta V^a_\mu = \bar{\epsilon}^A \, \gamma^a \, \psi_{A\mu} + \bar{\epsilon}_A \, \gamma^a \, \psi^{A\mu} \ . \tag{122}$$

As before the κ-supersymmetry is an ordinary supersymmetry transformation of the background fields with a supersymmetry parameter satisfying a suitable projection, given in this case by equation (113). The variation of the 0-brane action is done as in (36), (41) and (40) and we have to check the separate cancellation of the gravitino and dilatino terms.

Just as before let us begin with the gravitino terms. At this level we obtain the variation:

$$\begin{aligned} \frac{1}{\pi} \delta S^{\mathcal{N}=8}_{(2\nu)} &= \int_{M_1} \alpha_{2\nu} \left(\widehat{Z}_{AB} \, \widehat{\bar{Z}}^{AB} \right)^{1/2} \left[\overline{\psi}_A \, \gamma^a \, \kappa^A + \overline{\psi}^A \, \gamma^a \, \kappa_A \right] \, \Pi_a \\ &\quad + 2 \, p^{\Lambda\Sigma} \int_{M_1} \left(\overline{h}_{\Lambda\Sigma}{}^{AB} \, \overline{\psi}_A \kappa_B + h_{\Lambda\Sigma|AB} \, \overline{\psi}^A \kappa^B \right) \\ &\quad - 2 \, q_{\Lambda\Sigma} \int_{M_1} \left(\overline{f}^{\Lambda\Sigma|AB} \, \overline{\psi}_A \kappa_B + f^{\Lambda\Sigma}{}_{AB} \, \overline{\psi}^A \kappa^B \right) \ . \end{aligned} \tag{123}$$

The second and third lines in equation (123) reconstruct the definition of the central charge tensors $Z_{AB}, \bar{Z}^{AB}$. Yet, because of (114), we can safely replace the central charges with their hatted counterparts:

$$\begin{aligned} & 2\, p^{\Lambda\Sigma} \left(\overline{h}_{\Lambda\Sigma}{}^{AB}\, \overline{\psi}_A \kappa_B + h_{\Lambda\Sigma|AB}\, \overline{\psi}^A \kappa^B \right) - 2\, q_{\Lambda\Sigma} \left(\overline{f}^{\Lambda\Sigma|AB}\, \overline{\psi}_A \kappa_B + f^{\Lambda\Sigma}{}_{AB}\, \overline{\psi}^A \kappa^B \right) \\ = \;& 2\widehat{\overline{Z}}^{AB}\, \overline{\psi}_A \kappa_B \,+\, 2\widehat{Z}_{AB}\, \overline{\psi}^A \kappa^B \;. \end{aligned} \tag{124}$$

In the first line of (123) we can reduce the terms involving the gamma matrix γ^a by use of the projection (113) and we obtain cancellation of the gravitino terms if the parameter $\alpha_{2\nu}$ satisfies the condition (116). Just as in the $\mathcal{N} = 2$ case, the non-trivial check is to verify that the same coefficient also guarantees the cancellation of the dilatino terms. This indeed happens in a different subtle way for the three values $\nu = 4, 2, 1$.

At the level of the dilatino terms the κ-supersymmetry variation of the 0-brane action reads as follows:

$$\begin{aligned} \frac{1}{\pi}\delta S^{\mathcal{N}=8}_{(2\nu)} \;=\;& \int_{M_1} -\frac{\alpha_{2\nu}}{2}\left(\widehat{Z}_{CD}\widehat{\overline{Z}}^{CD}\right)^{-1/2}\left(\widehat{\overline{Z}}^{CD} P_{CDRS,i}\delta_\kappa\Phi^i\overline{Z}^{RS} + \widehat{Z}_{CD}\overline{P}_i^{CDRS}\delta_\kappa\Phi^i Z_{RS}\right) \\ +\;& \tfrac{1}{4}\, p^{\Lambda\Sigma}\int_{M_1}\left(\overline{h}_{\Lambda\Sigma}{}^{PQ}\,\overline{\chi}_{PQC}\gamma_a\kappa^C + h_{\Lambda\Sigma|PQ}\,\overline{\chi}^{PQC}\gamma_a\kappa_C\right) V^a \\ -\;& \tfrac{1}{4}\, q_{\Lambda\Sigma}\int_{M_1}\left(\overline{f}^{\Lambda\Sigma|PQ}\,\overline{\chi}_{PQC}\gamma_a\kappa^C + f^{\Lambda\Sigma}{}_{PQ}\,\overline{\chi}^{PQC}\gamma_a\kappa_C\right) V^a \;. \end{aligned} \tag{125}$$

In deriving the above equation we have used the following differential relation satisfied by the central charge [28] and therefore by its projected version:

$$\nabla Z_{AB} \;=\; P_{ABCD}\overline{Z}^{CD} \;\;\longrightarrow\;\; \nabla\widehat{Z}_{AB} \;=\; \mathcal{P}^{A'}_{A}\,\mathcal{P}^{B'}_{B}\,P_{A'B'CD}\,\overline{Z}^{CD} \;, \tag{126}$$

the symbol ∇ denoting the $SU(8)$ covariant derivative.

Substituting the explicit form of the scalar field supersymmetry transformation (120) and utilizing the projection (113) we obtain:

$$\begin{aligned} \frac{1}{\pi}\delta S^{\mathcal{N}=8}_{(2\nu)} \;=\;& \int_{M_1} -\frac{\alpha_{2\nu}}{2}\left(\widehat{Z}_{CD}\widehat{\overline{Z}}^{CD}\right)^{-1/2}\left(\widehat{\overline{Z}}^{CD}\overline{Z}^{RS} + \tfrac{1}{24}\widehat{Z}_{IJ}Z_{KL}\epsilon^{IJKLCDRS}\right)\overline{\chi}_{[CDR}\,\kappa_{S]} \\ +\;& \frac{\sqrt{2\nu}}{4}\left(\widehat{Z}_{CD}\widehat{\overline{Z}}^{CD}\right)^{-1/2}\overline{Z}^{CD}\,\widehat{\overline{Z}}^{RS}\,\overline{\chi}_{CDR}\kappa_S + \text{Hermitian conjugate}\;, \end{aligned} \tag{127}$$

where special attention must be paid to some details. In the first line of (127) the term $\overline{\chi}_{[CDR}\,\kappa_{S]}$ is completely antisymmetrized in the indices $CDRS$, while the same term in the second line of equation (127) is not antisymmetrized. Moreover, some of the central charge tensors appearing in equation (127) wear a hat, namely are projected, and some do not.

We can now discuss the cancellation of the dilatino terms for the three values of ν.

<u>Case $2\nu = 8$</u>. Here the projection operator $\mathcal{P}^A_B$ is the identity operator and we have $Z_{AB} = \widehat{Z}_{AB}$. Furthermore, in view of the identity (99) we see that the two terms on the first line of (127) are identical and sum together. On the other hand, we also have $\overline{Z}^{[CD}\overline{Z}^{R]S} = \overline{Z}^{[CD}\overline{Z}^{RS]}$ so that the cancellation of the dilatino terms occurs if:

$$\alpha_8 = \tfrac{1}{2}\sqrt{2} \ , \tag{128}$$

which for $\nu = 4$ coincides with (116).

Case $\underline{2\nu = 4}$. Here there is a difference between $\widehat{Z}_{AB}$ and Z_{AB}. A little analysis is needed to show that the first and second term in the first line together reconstruct indeed all the possible cases present in the second line of (127), upon use of the extension of the identity (99) to the case that one of the central charges is projected. One eventually finds that, in agreement with (116), the dilatino terms cancel if

$$\alpha_4 = 1 \ . \tag{129}$$

Case $\underline{2\nu = 2}$. In this last case the second term in the first line of equation (127) vanishes identically since there is antisymmetrization on the indices IJ and S that all lie in a two-dimensional subspace. As for the first term on the same line, in order to single out the non-vanishing contribution that can cancel with the similar term in the second line one has to develop the antisymmetrization. We have

$$\widehat{\overline{Z}}^{CD}\,\overline{Z}^{RS}\,\overline{\chi}_{[CDR}\,\kappa_{S]} = \tfrac{1}{2}\widehat{\overline{Z}}^{CD}\,\overline{Z}^{RS}\,\overline{\chi}_{CDR}\,\kappa_S + \tfrac{1}{2}\overline{Z}^{CD}\,\widehat{\overline{Z}}^{RS}\,\overline{\chi}_{CDR}\,\kappa_S \ , \tag{130}$$

and the first of the two terms appearing on the right hand side of (130), vanishes for the same reason as before. We cannot antisymmetrize in CDR when all of the three indices lie in the two-dimensional subspace singled out by the projection. On the other hand, the second term on the right-hand side of (130) can cancel with the second line of equation (127). For this case, therefore, the dilatino terms cancels if

$$\alpha_2 = \sqrt{2} \ , \tag{131}$$

which is once again consistent with the $\nu = 2$ case of (116).

In this way we have completed the proof of κ-supersymmetry and shown that (115) is the correct 0-brane action for all $\mathcal{N} = 8$ supergravity backgrounds.

6 Outlook

In this paper we have constructed 0-brane actions for superparticles moving in generic $D = 4$ supergravity backgrounds that preserve a residual fraction of supersymmetry. Typically such backgrounds are BPS black holes. The main issue in writing these actions is the coupling of the 0-brane, not only to the metric and the gauge fields of the background, but also to its scalar fields.

The main application of our result appears to be its possible use as an instrument to investigate the structure and the properties of $(1 + 0)$-dimensional conformal field theories (superconformal quantum mechanics) living on the boundary of two-dimensional anti-de Sitter space. Indeed, in those cases where the black hole entropy is finite, the near-horizon geometry of the black hole is $adS_2 \times S^2$ and we can formulate the Kaluza–Klein programme for $D = 4$ supergravity as its compactification on S^2. In complete analogy with [6] we can then study the conformal field theory on the adS_2 boundary starting from our superparticle action. Other applications are possible but this is the main one that motivated us to undertake the present study.

In deriving our result, one main point that had to be cleared, was the correct formulation of the projection equation satisfied by the κ-supersymmetry parameter. This projection is identical to the equation satisfied by the BPS Killing spinor admitted by the corresponding supergravity background. In previous literature this Killing spinor equation was written in fixed $U(\mathcal{N})$ gauges. In the present paper we have restored its complete $U(\mathcal{N})$ covariance, $SU(8)$ covariance for $\mathcal{N} = 8$.

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A Notations and self-duality of antisymmetric tensors

In table 2 we give a list of indices, listing its range, and comparing with notations in some of the references. Our spacetime metric is thus mostly +, which is opposite to

Table 2: Comparison of indices and normalizations

Here	[15, 19, 29]	[16, 11, 23, 24, 25]	Meaning	Range
$i,\bar{\imath}$	$\alpha,\bar\alpha$	i,i^*	Moduli	$1,\ldots,n$
I	I	Λ	Symplectic	$0,\ldots,n$
A	i	A	Extended susy	$1,2$
$\alpha,\bar\alpha$		$\alpha,\bar\alpha$	CY coordinates	$1,2,3$
μ	μ	μ	Local $D=4$	$0,\ldots,3$
a	a	a	flat $D=4$	$0,\ldots,3$
i			Space	$1,2,3$
M			$D=10$ local	$0,\ldots,9$
m			$D=4$ world-volume	$0,\ldots,3$
$g_{\mu\nu}$	$g_{\mu\nu}$	$-g_{\mu\nu}$	Metric	
$\mathcal{F}_{\mu\nu}$	$-\mathcal{F}_{\mu\nu}$	$2\mathcal{F}_{\mu\nu}$	Field strengths	
A_μ	$-W_\mu$	A_μ	Vectors	
$\epsilon_{\mu\nu\rho\sigma}$	$-\mathrm{i}\epsilon_{\mu\nu\rho\sigma}$	$\epsilon_{\mu\nu\rho\sigma}$	Levi-Civita tensor	
γ^a	γ^a	$\mathrm{i}\gamma^a$		
X^I	X^I	L^Λ	Upper cov. hol. section	
F_I	F_I	M_Λ	Lower cov. hol. section	
	Z^I	X^Λ	Upper hol. section	
		F_Λ	Lower hol. section	
$T_{\mu\nu}$	$\frac{i}{4}T^{-ij}_{\mu\nu}\epsilon_{ij}$	$2T_{\mu\nu}$	Gravitino field strength	
ϵ_A	$\sqrt{2}\epsilon^i$	ϵ_A	susy parameter	
ψ^A_μ	$\frac{1}{\sqrt2}\psi_{\mu\, i}$	ψ^A_μ	gravitino	

[16, 11, 23, 24, 25]. There are more small differences in normalization between some of the articles in the same group. In the table we use the most recent ones. A more detailed comparison can be found in the appendix B of [29]. For $\mathcal{N}=8$, of course, $A=1,\ldots,8$, and we use also $\Lambda,\Sigma=1,\ldots,8$, related to E_7.

We adopt the notation where in any dimension $\epsilon_{01\ldots(d-1)}=1$, and the dual $\tilde F$ of a tensor F is defined such that it squares to the identity: $\tilde{\tilde F}=F$. With complex variables in the CY (say y^α and $\bar y^{\bar\alpha}$), we have

$$\epsilon_{\mu\nu\rho\sigma\alpha\beta\gamma\bar\alpha\bar\beta\bar\gamma}=\mathrm{i}\epsilon_{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\gamma}\epsilon_{\bar\alpha\bar\beta\bar\gamma}\ , \tag{132}$$

where the i now appears by going from real indices $5,\ldots,9$ to holomorphic and anti-holomorphic indices $\alpha,\bar\alpha$. Duality in 10-dimensional Minkowski space should be defined by

$$\tilde{F}_{MNPQR} = \frac{1}{5!}\epsilon_{MNPQRSTUVW}F^{STUVW} \ , \tag{133}$$

in order that $\tilde{\tilde{F}} = F$. Taking a 5-form with only non-zero components $F_{\mu\nu\alpha\beta\gamma} = F_{\mu\nu}\Omega_{\alpha\beta\gamma}$, and taking the metric with $g^{\alpha\bar{\alpha}} = 1$, the tensor with upper indices has non-zero components $F^{\mu\nu\bar{\alpha}\bar{\beta}\bar{\gamma}} = F^{\mu\nu}\Omega_{\alpha\beta\gamma}$. The factor i in (132) is absorbed in taking the dual in 4 dimensions, such that we have

$$F_{\mu\nu\alpha\beta\gamma} = F_{\mu\nu}\Omega^{(3,0)}_{\alpha\beta\gamma} \quad \rightarrow \quad \tilde{F}_{\mu\nu\alpha\beta\gamma} = \tilde{F}_{\mu\nu}\Omega^{(3,0)}_{\alpha\beta\gamma} \ . \tag{134}$$

For the other 3-forms in the CY some re-ordering of indices has to be made in the Levi-Civita tensor, such that

$$\begin{aligned} F_{\mu\nu\alpha\beta\bar{\gamma}} &= F_{\mu\nu}\Omega^{(2,1)}_{\alpha\beta\bar{\gamma}} \quad \rightarrow \quad \tilde{F}_{\mu\nu\alpha\beta\bar{\gamma}} = -\tilde{F}_{\mu\nu}\Omega^{(2,1)}_{\alpha\beta\bar{\gamma}} \\ F_{\mu\nu\alpha\bar{\beta}\bar{\gamma}} &= F_{\mu\nu}\Omega^{(1,2)}_{i\bar{\beta}\bar{\gamma}} \quad \rightarrow \quad \tilde{F}_{\mu\nu\alpha\bar{\beta}\bar{\gamma}} = \tilde{F}_{\mu\nu}\Omega^{(1,2)}_{\alpha\bar{\beta}\bar{\gamma}} \\ F_{\mu\nu\bar{\alpha}\bar{\beta}\bar{\gamma}} &= F_{\mu\nu}\Omega^{(0,3)}_{\bar{\alpha}\bar{\beta}\bar{\gamma}} \quad \rightarrow \quad \tilde{F}_{\mu\nu\bar{\alpha}\bar{\beta}\bar{\gamma}} = -\tilde{F}_{\mu\nu}\Omega^{(0,3)}_{\bar{\alpha}\bar{\beta}\bar{\gamma}} \ . \end{aligned} \tag{135}$$

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