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Dynamic Scoring:
A Back-of-the-Envelope Guide

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Abstract

This paper uses the neoclassical growth model to examine the extent to which a tax cut pays for itself through higher economic growth. The model yields simple expressions for the steady-state feedback effect of a tax cut. The feedback is surprisingly large: for standard parameter values, half of a capital tax cut is self-financing. The paper considers various generalizations of the basic model, including elastic labor supply, general production technologies, departures from infinite horizons, and non-neoclassical production settings. It also examines how the steady-state results are modified when one considers the transition path to the steady state.

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Introduction

To what extent does a tax cut pay for itself? This question arises regularly for economists working at government agencies in charge of estimating tax revenues. Traditional revenue estimation, called static scoring, assumes no feedback from taxes to national income. The other extreme, illustrated by the renowned Laffer curve, suggests that tax cuts can generate so much economic growth that they completely (or even more than completely) pay for themselves. Most economists are skeptical of both polar cases. They believe that taxes influence national income but doubt that the growth effects are large enough to make tax cuts self-financing. In other words, tax cuts pay for themselves in part, and the open question is the magnitude of the effect.

In 2002 the staff of the Joint Tax Committee, prompted by several members of Congress, started work on the difficult task of dynamic scoring of tax policy. That is, they started developing a set of economic models that might be used to estimate the feedback effects of tax proposals. Dynamic scoring also received prominent discussion in a 2003 report by the Congressional Budget Office and the 2004 Economic Report of the President. The task of dynamic scoring is formidable, because there is little agreement about how best to model long-run economic growth and the effect of taxes on the economy.\(^1\)

The purpose of this paper is to investigate what the neoclassical growth model can contribute to this endeavor. The neoclassical growth model, first introduced by Ramsey (1928), is the most widely taught model of capital accumulation and long-run growth and is the workhorse of modern growth theory. For example, see the popular graduate-level textbooks by Romer (2001) and Barro and Sala-i-Martin (1999). This model is also widely used for thinking about issues in public finance (Chamley 1986; Judd 1985). Here we use the neoclassical growth model to consider the revenue effects of changes in tax rates on capital and labor income. One virtue of the model is that it sheds light on the key parameters that govern these revenue effects. The model also yields simple formulas for how much the dynamic estimates of these revenue effects differ from the static estimates.\(^2\)

These formulas permit some illuminating back-of-the-envelope calculations. For conventional parameter values, the model implies substantial feedback effects in the steady state. For example, suppose that the initial tax rates on capital and labor are 25 percent, the production function is Cobb-Douglas, the

\(^1\)Auerbach (2005) provides a good introduction to the economic and policy issues involved in dynamic scoring.

\(^2\)In addition to the works already cited, our analysis is related to several strands of the literature on fiscal policy. One prominent example is Auerbach and Kotlikoff (1987), who analyze tax changes using computer-based simulations of overlapping-generations models. The subset of the literature closest to this paper has typically focused on Laffer effects: the possibility that tax cuts can be fully self-financing (e.g., Ireland 1994, Pecorino 1995, Agell and Persson 2001, and Nuñez and Ruiz 2002). McGrattan (1994) uses a framework similar to ours but is primarily concerned with the impact of changing taxes on explaining economic fluctuations; the feedback effect of tax rates on tax revenue is implicit in her analysis. Finally, a classic reference is Feldstein (1974), which uses an approach parallel to the one we employ here but is focused on tax incidence rather than revenue effects.
capital share is one-third, and labor supply is inelastic. Then, in the steady state, the dynamic effect of a cut in capital income taxes on government revenue is only 50 percent of the static effect. That is, one-half of a capital tax cut pays for itself.

There are various ways in which the benchmark Ramsey model can be generalized. One is to include elastic labor supply. We show that this generalization has only minor effects on the analysis of capital income taxes, but it has significant effects on the analysis of labor income taxes. We assume a form of preferences that yields no trend in hours worked, as the uncompensated elasticity of labor supply is zero. The compensated (constant-consumption) elasticity of labor supply, however, need not be zero. If this elasticity is one-half and the other parameters are as described above, then the steady-state feedback from a labor income tax cut rises from zero to 17 percent. The model shows that, regardless of the labor supply elasticity, if capital and labor tax rates start off at the same level, cuts in capital taxes have greater feedback effects in the steady state than cuts in labor taxes.

Another way to generalize the model is to consider production functions that are not Cobb-Douglas. We show that the elasticity of substitution between capital and labor has a crucial role in determining the dynamic feedback of a change in capital taxes. If the elasticity of substitution is raised from 1.0 to 1.5, the steady-state feedback from a capital tax cut rises from 50 percent to 71 percent. Conversely, if the elasticity is lowered from 1.0 to 0.75, the feedback falls from 50 percent to 44 percent. We discuss various reasons to believe that this crucial elasticity may differ from unity.

Many economists are skeptical of the Ramsey model because of its assumption of an infinite-horizon consumer. We therefore introduce finite horizons in two ways. We first add some rule-of-thumb households that consume their entire labor income in each period, but we find that this has no effect on the steady-state results. The infinite-horizon consumers dominate in the long run, as is suggested by the earlier work of Judd (1985), Smetters (1999), and Mankiw (2000). Alternatively, if all consumers have finite horizons, as in Blanchard (1985), the results change. Yet the changes are quantitatively modest for plausible parameter values. For example, if households have an expected horizon of 50 years, then the fraction of a capital tax cut paid for by growth falls from 50 percent to 45 percent.

We also consider two widely discussed departures from the neoclassical production setting. We first consider the impact of imperfect competition, for Judd (2002) has shown that market power can substantially change the analysis of optimal tax policy. We find that market power can raise the ability of tax cuts to be self-financing, but only if there are substantial economic profits not dissipated by the fixed costs associated with entry. We also examine the possibility that there are positive externalities to capital accumulation, as suggested by Romer (1987) and DeLong and Summers (1991). In this case, the dynamic effects of tax changes are much larger than they are in the standard model.

The neoclassical model yields particularly simple expressions for steady-state feedback effects, but it is also important to consider the transition path to the
steady state. We therefore consider a log-linearized version of the model for the special case of unitary intertemporal elasticity of substitution. For our canonical parameter values, we find that the immediate revenue feedback effects are quite similar for capital and labor taxes: slightly more than 10 percent of a tax cut immediately pays for itself through higher labor supply and national income. For both types of taxes, the feedbacks grow over time toward their steady-state values, with the feedback for a capital tax cut reaching halfway after about ten years.

In all experiments that we consider, the government budget constraint is satisfied, as it must be in any well-specified model. Throughout the paper, we assume that some form of lump-sum transfers (or taxes) adjusts in response to the changes in tax rates. We have in mind such spending programs as welfare, social security, and farm subsidies. The dynamic scoring question that we are proposing, then, is how much such transfer spending needs to fall to offset a cut in tax rates.

Implicit in our use of a model of long-run growth is that we ignore any short-term effects of tax cuts that arise from traditional Keynesian channels. Many government and private-sector analysts have instead emphasized the power of tax cuts to stimulate a weak economy. Although we abstract from these effects in this study, we do not mean to suggest that such effects are insignificant. Integrating a model of long-run growth with a model of short-run business cycles remains a challenge for future research on dynamic scoring.

The paper is organized as follows. Section 1 presents the basic model and previews results. Section 2 derives and solves a more general version of the model which includes elastic labor supply. Section 3 discusses how the results change if we relax the assumption of infinite horizons, and Section 4 investigates departures from the neoclassical production setting. Section 5 considers the transition path. Section 6 concludes.

1 The Basic Ramsey Model

Before delving into the details of a more general model, which we do in the next section, it will be useful for many readers to preview our results for a familiar special case—the steady state of the Ramsey growth model. We modify this model by including taxation at a rate $\tau_k$ on capital income and $\tau_n$ on all labor income. The population is normalized to one, and labor is supplied inelastically. Using conventional notation, we can write the steady state of the economy as follows:

\[ r = f'(k). \]  
\[ w = f(k) - kf'(k). \]  
\[ (1 - \tau_k)r = \rho + \gamma g. \]

For standard introductions to the Ramsey model, we refer the reader to Romer (2001), chapter 2, or to Barro and Sala-i-Martin (1999), chapter 2.
This system of four equations fully specifies the steady-state values of the four endogenous variables: $k$ is capital per efficiency unit of labor, $w$ is the wage rate, $r$ is the before-tax rate of return to capital, and $R$ is total tax revenue per efficiency unit. In addition, $f(k)$ is total output per efficiency unit, $\gamma$ is the curvature coefficient in our instantaneous utility function (the reciprocal of the intertemporal elasticity of substitution), $g$ is the rate of labor-augmenting technological change, and $\rho$ is the subjective discount rate. In this section, we assume that the production function is Cobb-Douglas:

$$y = f(k) = k^\alpha$$

where $y$ is output per efficiency unit and the parameter $\alpha$ is capital’s share of income. The next section will consider generalizations of this production function.

Our goal is to estimate the impact of a tax change on steady-state tax revenue $R$. A conventional scoring assuming no dynamic effects from the tax cut yields the following results:

$$\left. \frac{dR}{d\tau_k} \right|_{\text{static}} = rk = \alpha y.$$  

$$\left. \frac{dR}{d\tau_n} \right|_{\text{static}} = w = (1 - \alpha)y.$$  

These equations show the impact of a tax change on tax revenue, assuming that national income and other macroeconomic variables are held constant. Notice that each of these derivatives equals the tax base of the respective tax.

Dynamic scoring estimates the impact of a tax change, taking into account the tax change’s consequence for growth. By fully differentiating equations (1) through (4), we obtain the following results:

$$\left. \frac{dR}{d\tau_k} \right|_{\text{dynamic}} = \frac{1 - \alpha \tau_k + (1 - \alpha)\tau_n}{(1 - \tau_k)(1 - \alpha)} \left. \frac{dR}{d\tau_k} \right|_{\text{static}}.$$  

$$\left. \frac{dR}{d\tau_n} \right|_{\text{dynamic}} = \frac{dR}{d\tau_n} \bigg|_{\text{static}}.$$  

These equations show the impact of a tax change on tax revenue, including the feedback from taxes to national income.

The central goal of this paper is to compare these dynamic and static revenue estimates. In this conventional Ramsey model, with its assumption of inelastic labor supply, the revenue impact of a change in the labor income tax rate is the same under dynamic and static scoring. This explains equation (6). The more interesting analysis pertains to result (5), the impact of a change in the capital
tax rate $\tau_k$ on tax revenue. Consider the empirically plausible parameter values of $\tau_k = \tau_n = \frac{1}{4}$ and $\alpha = \frac{1}{3}$. Then, (5) yields

$$\left. \frac{dR}{d\tau_k} \right|_{\text{dynamic}} = \frac{1}{2} \left. \frac{dR}{d\tau_k} \right|_{\text{static}}.$$ 

A capital income tax cut has a long-run impact on revenue that is only half of its static impact. In other words, growth pays for 50 percent of a capital income tax cut in the steady state.\(^4\)

This simple example illustrates two lessons. First, dynamic and static revenue estimation can lead to very different results. Second, the steady state of the Ramsey model yields simple expressions that can provide useful benchmarks for the task of dynamic scoring. In the sections that follow, we develop more general models to examine the robustness of these conclusions.

## 2 A More General Ramsey Model

In this and the next three sections, we extend the basic Ramsey model along a number of dimensions. In this section we include elastic labor supply and a more general production technology. We also present a more detailed derivation of our results.

To allow for elastic labor supply, we use a form of preferences over consumption and labor proposed by King, Plosser, and Rebelo (1988). King-Plosser-Rebelo preferences have the property that the uncompensated elasticity of labor supply is zero. This feature has the appealing implication that long-run growth caused by technological progress does not lead to a trend in hours worked. The compensated (constant-consumption) elasticity of labor supply need not be zero, however. This parameter, which we will call $\sigma$, will have a significant role in some of our results.

### 2.1 Firms

We begin with production. Assume there are many identical firms in competitive input and output markets, producing output with constant returns to scale technology according to the production function

$$Y = F(K, N),$$

\(^4\)The feedback depends critically on the tax rate. If the capital tax rate were 0.40 instead of 0.25, and all other parameter values are the same, the feedback from a capital tax cut would be 75 percent rather than 50 percent.

The literature on taxation in the United States suggests that our choice of $\tau_k = \tau_n = 0.25$ is within the range of plausible estimates, although perhaps a bit conservative. Mendoza, Razin, and Tesar (1994) estimate a 40.7 percent capital tax rate (applied to corporate and non-corporate capital) for the United States in 1988, the last year of their series. This is above the estimate given by Gravelle (2004), who reports a rate of 33 percent for all capital in that year. Gravelle extends her estimates through 2003, by which point the capital tax rate had fallen to 23 percent. Mendoza et al. estimate a labor tax rate of 28.5 percent in 1988, together with a consumption tax of about 5 percent; these tax rates would combine to be equivalent to a tax on labor of about 31 percent.
where \( Y \) is the total amount of output, \( K \) is the total amount of capital, and \( N \) is the total labor input, including the adjustment for labor-augmenting technological change. That is, if \( n \) is the labor input supplied by the representative household and \( g \) is the rate of labor-augmenting technological change, then \( N = ne^{gt} \). With these conventions, we can write the production function as

\[
y = f(k, n).
\]

where \( y = Y/e^{gt} \) is output per efficiency unit and \( k = K/e^{gt} \) is capital per efficiency unit. Note that we no longer assume that the production function \( f(k, n) \) is Cobb-Douglas. We will let \( \alpha \) denote the capital share and \( \xi \) denote the elasticity of substitution between capital and labor.\(^5\)

Given competitive markets, firms earn zero profits and capital earns a before-tax rate of return \( r \) equal to its marginal product:

\[
r = f_k(k, n).
\]

Each efficiency unit of labor is paid a wage \( w \) equal to its marginal product,

\[
w = f_n(k, n).
\]

Below, in Section 4, we consider generalizations to non-competitive production settings.

### 2.2 Households

We use a conventional, infinitely-lived representative household. The household’s instantaneous utility function takes the isoelastic form with curvature parameter \( \gamma \). To incorporate elastic labor supply, we add labor \( n \) to the household’s utility function. This labor variable should be interpreted broadly to include both time and effort.

The household’s utility function is

\[
U = \int e^{-\rho t} \frac{(ae^{gt})^{1-\gamma} (1-\gamma)u(n) - 1}{1-\gamma} dt,
\]

where \( u(n) \) is a differentiable function of labor supply and all other variables are defined as before. This functional form was introduced by King, Plosser, and Rebelo (1988) and has been more recently explored by Kimball and Shapiro (2003).

We can write the household’s dynamic budget constraint in per efficiency unit terms:

\[
\dot{k} = (1 - \tau_n)wn + (1 - \tau_k)rk - c - gk + T,
\]

\[
\lim_{t \to \infty} ke^{-(r+g)t} = 0.
\]

\(^5\)These variables need not be constant, but they will take on particular values in any steady state. Specifically, \( \alpha = \frac{f_kk}{f(k,n)} \) and \( \xi = \frac{f_nf_k}{f(k,n)k_n} \).
where $\dot{k}$ is the time derivative of the capital stock per efficiency unit and $T$ represents lump-sum transfers from the government. The second equation is the standard transversality condition.

Household maximization yields the following first-order conditions:

$$v'(n) = \frac{-(1 - \tau_n) w}{c}. \quad (10)$$

$$r = \frac{1}{1 - \tau_k} \left[ \rho + \gamma \left( \frac{\dot{c}}{c} + g \right) + (1 - \gamma)v'(n) \cdot \dot{n} \right],$$

Equation (10) is the static condition determining the allocation of time between work and leisure. From this equation, one can derive an expression for the constant-consumption elasticity of labor supply, which we will denote $\sigma$:

$$\sigma = \frac{v'(n)}{v''(n) \cdot n}.$$ 

In the steady state, consumption per efficiency unit $c$ and the wage per efficiency unit $w$ are both constant. As a result, labor supply $n$ is constant as well. The intertemporal first-order condition therefore reduces to

$$r = \frac{\rho + \gamma g}{1 - \tau_k}. \quad (11)$$

This is the same as in Section 1.

In the steady state, $\dot{k} = 0$, and we can write the steady-state level of consumption as:

$$c = f(k, n) - gk. \quad (12)$$

Equations (7) through (12) fully determine the steady-state values of six variables: $y$, $k$, $n$, $r$, $w$, and $c$.

### 2.3 Government

Total tax revenue per efficiency unit, denoted $R$, is the sum of taxes paid on capital income and labor income:

$$R = \tau_k rk + \tau_n wn. \quad (13)$$

The first term on the right of (13) is the capital tax rate times capital income, and the second term is the labor tax rate times labor income. The government collects this revenue and distributes it in the form of lump-sum transfers to households. For most of our results, the timing of these rebates is irrelevant, as the consumer is infinitely-lived. (Later, when we consider models with finite horizons, we assume that rebates occur immediately upon receipt of the tax revenue.)
2.4 Dynamic and Static Steady-State Scoring

A conventional scoring assuming no dynamic effects from the tax cut yields the following results for this model:

\[
\frac{dR}{d\tau_k}_{\text{static}} = r_k = \alpha f(k, n).
\]

\[
\frac{dR}{d\tau_n}_{\text{static}} = w_n = (1 - \alpha) f(k, n).
\]

By contrast, to find the true impact of the tax change on steady-state revenue, one would use all of the steady-state conditions. This yields the following:

\[
\frac{dR}{d\tau_k}_{\text{dynamic}} = \left[ 1 - \frac{(\alpha + \xi - 1) \tau_k + (1 - \alpha) \tau_n}{(1 - \alpha)(1 - \tau_k)} \right] \frac{\alpha \tau_k + (1 - \alpha) \tau_n}{(\rho + \gamma g) - \alpha (1 - \tau_k) g} \left[ \frac{(\rho + \gamma g)(1 - \xi) + (1 - \tau_k) (\xi - \alpha) g}{(1 - \alpha)(1 - \tau_k)} \frac{\sigma}{1 + \sigma} \right].
\]

\[
\frac{dR}{d\tau_n}_{\text{dynamic}} = \left[ 1 - \frac{\alpha \tau_k + (1 - \alpha) \tau_n}{(1 - \alpha)(1 - \tau_n)} \frac{\sigma}{1 + \sigma} \right] \frac{dR}{d\tau_n}_{\text{static}}.
\]

Note that if the labor supply elasticity \( \sigma \) equals zero and the elasticity of substitution \( \xi \) equals unity, then these results reduce to equations (5) and (6) in the basic model. In general, however, these two parameters play a crucial role in determining the dynamic effects of a tax change.

2.5 The Compensated Elasticity of Labor Supply

Let’s consider first the role of the labor supply elasticity \( \sigma \). In the case of a capital tax cut, the labor supply elasticity plays only a small role. If \( g = 0 \) and \( \xi = 1 \), then equation (14) is identical to equation (5) from the basic Ramsey model, and the labor supply elasticity is irrelevant for a change in capital taxes. In the case of a labor tax cut, however, the elasticity of labor supply plays a key role. The larger the elasticity of labor supply, the smaller the dynamic revenue impact of a labor tax cut. From equations (14) and (15), one can show that if the two tax rates are the same, a capital tax cut will always have a larger feedback effect than a labor tax cut.

To illustrate the effect of elastic labor supply, consider the following plausible parameter values: \( \tau_k = \frac{1}{4}, \tau_n = \frac{1}{4}, \alpha = \frac{1}{3}, \gamma = 1, g = .02, \rho = .05, \xi = 1, \) and \( \sigma = \frac{1}{2} \). These parameters yield:

\[
\frac{dR}{d\tau_k}_{\text{dynamic}} = 0.47 \frac{dR}{d\tau_k}_{\text{static}}.
\]
Under these assumptions, a capital tax cut has a long-run impact on revenue of only 47 percent of its static impact. That is, growth pays for 53 percent of the static revenue loss. A labor tax cut has a long-run impact on revenue of only 83 percent of its static impact, and growth pays for 17 percent of the tax cut.

These results show that the feedback effect for a labor tax cut depends crucially on the compensated elasticity of labor supply. Unfortunately, this is a parameter over which there is substantial uncertainty.

Kimball and Shapiro (2003) present a recent, extensive discussion of this parameter, including references to a broad literature. As they note, it is important to recognize that there are different notions of the compensated elasticity: a traditional constant-utility elasticity, a Frisch or constant-marginal utility elasticity, and a constant-consumption elasticity. Our parameter $\sigma$ represents the constant-consumption elasticity of labor supply, which Kimball and Shapiro show is generally larger than the traditional compensated elasticity. Using their results, one can derive that, for our standard parameter values $\gamma = 1$ and $\alpha = \frac{1}{3}$, the constant-consumption elasticity is about $\frac{2}{3}$ the traditional compensated elasticity. Thus, our assumption that $\sigma = 0.5$ is equivalent to assuming that the traditional compensated elasticity of labor supply is 0.3. Kimball and Shapiro estimate that the constant-consumption elasticity is about 1.0 to 1.5. If $\sigma$ is increased from 0.5 to 1.5 in our calculation, the revenue feedback effect of a labor tax cut rises from 17 percent to 30 percent.

Kimball and Shapiro point out there are economists with preferred values on both sides of their estimates. Labor economists analyzing micro data (e.g., Angrist, 1991 and Blundell, Duncan, and Meghir, 1998) tend to argue for smaller elasticities. A survey of labor economists conducted by Fuchs, Krueger, and Poterba (1998) found that the median labor economist believes the compensated elasticity of labor supply is 0.18 for men and 0.43 for women. By contrast, macroeconomists working in the real business cycle literature often choose parameterizations that imply larger values. Prescott (2004) examines cross-country data on hours worked and marginal tax rates and finds that these two variables are strongly correlated. He concludes that this international variation suggests a (constant-consumption) compensated elasticity of labor supply around 3.6. If we raise $\sigma$ to 3 in our calculation, the revenue feedback effect of a labor tax cut rises to 38 percent.

One possible way of reconciling these differing estimates is to generalize the neoclassical model to include a role for work norms, as suggested by Blomquist (1993) and Grodner and Knieser (2003). Suppose an individual’s disutility from supplying labor depends on how much other people are working. That is, working long hours when others are doing so is not as onerous as working long hours while others are enjoying substantial leisure. In this case, as Glaeser, Sacerdote, and Scheinkman (2002) point out, a "social multiplier" causes aggregate elasticities to exceed individual elasticities. The larger aggregate elasticity would be the relevant one for the purposes of dynamic scoring.
2.6 The Elasticity of Substitution

The elasticity of substitution $\xi$ between capital and labor plays a crucial role in determining how much of a capital tax cut is self-financing. For example, if the elasticity of substitution is 1.5 rather than 1.0, and the other parameters are as specified above, the dynamic feedback effect rises from 53 percent to 71 percent. If the elasticity of substitution is 0.75 rather than 1.0, the dynamic feedback effect falls from 53 percent to 44 percent.

Like the elasticity of labor supply, there is significant uncertainty about this parameter. As Ventura (1997) and Mankiw (1995) point out, international trade in goods can affect the degree of substitutability between capital and labor. In traditional Heckscher-Ohlin trade theory, a nation can move resources between industries with varying degrees of capital intensity. When a country’s stock of capital increases, it can export capital-intensive goods and import labor-intensive goods, avoiding changes in the returns to either capital or labor. In other words, international trade raises the effective elasticity of substitution in an economy. One corollary of this line of analysis is that international trade increases the extent to which capital tax cuts pay for themselves.

On the other hand, exponential depreciation of capital would tend to reduce the elasticity of substitution. For example, if gross output is produced according to a Cobb-Douglas production function $k^{\psi}n^{1-\psi}$ and capital depreciates exponentially at rate $\delta$, then the net production function is:

$$f(k, n) = k^{\psi}n^{1-\psi} - \delta k.$$ 

In this case, the elasticity of substitution $\xi$ can be written as:

$$\xi = \frac{(\frac{k}{n})^{\psi-1} \psi - \delta}{(\frac{k}{n})^{\psi-1} \psi - \delta \psi}.$$ 

For our canonical parameter values of $\tau_k = \frac{1}{4}$, $\tau_n = \frac{1}{4}$, $\psi = \frac{1}{3}$, $\gamma = 1$, $g = .02$, $\rho = .05$, and $\sigma = \frac{1}{2}$, if depreciation equals 0.03, then the elasticity of substitution is 0.82.\(^7\)

These results illustrate that future research on dynamic scoring will need to focus attention on the compensated elasticity of labor supply and the elasticity of substitution between capital and labor.

3 Finite Horizons

The results we have obtained so far rely on the neoclassical growth model with its assumption of a representative household who optimizes over an infinite planning horizon. This model is widely used and a natural benchmark. As

\(^7\)Notice that in the presence of depreciation, the gross capital share and the net capital share differ. In this example, the gross capital share $\psi = \frac{1}{3}$, while the net capital share $\alpha = 0.27$. The formulas above for the dynamic effects of tax changes are expressed in terms of the net capital share $\alpha$. 

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Barro (1974) famously noted, the infinite-horizon household can be viewed as the result of generations’ being linked via altruistic bequests. Nonetheless, some economists are skeptical of the model’s empirical realism. This raises the question: Would alternative models of household behavior lead to substantially different conclusions about dynamic scoring? It turns out that our results regarding steady-state feedback effects are surprisingly robust.

3.1 Rule-of-thumb consumers

A large part of the consumption literature has suggested that current income exerts a greater influence on consumer spending than is predicted by the model of the infinitely-lived consumer. Campbell and Mankiw (1989) suggested that about half of income goes to households who follow the rule of thumb of consuming their current income. A prominent role for current income has also been documented by Shea (1995), Parker (1999), and Souleles (1999).

To see the implications of such behavior for dynamic scoring, suppose the model is the same as the one presented in Section 1, except that a fraction of households always consume their current income. How would our previous results change? The answer is, not at all.

Here is the logic. Equations (1) through (4) pin down the steady state in the neoclassical growth model. These equations would continue to hold, even if some consumers spend their current income. The only equation that comes from household behavior is equation (3). This equation would still obtain: it would be derived from the intertemporal first-order condition for the subset of maximizing consumers. As Judd (1985), Smetters (1999), and Mankiw (2000) have previously noted, as long as some households behave according to the neoclassical growth model, the steady state is not at all affected by a subset of households who do not.

3.2 The Blanchard Model

Blanchard (1985) suggested another way to relax the Ramsey model’s assumption of infinite horizons. According to Blanchard’s model, all households face a constant probability $p$ of dying off every period and being replaced by a new household. Households respond to this risk by annuitizing all of their wealth. There are no bequests.\footnote{Annuity markets play a crucial role in the Blanchard model. We have worked out a version of the Blanchard model in which, instead of annuitizing, households leave accidental bequests, which we assume are distributed as lump-sum payments to the newly born households. This alternative model yields the same dynamic feedback effects as the Ramsey model. Most likely, reality lies somewhere between the model with no annuitization and the model with full annuitization.}

To keep things simple, we consider a special case similar to the one Blanchard emphasizes. In particular, we assume inelastic labor supply ($\sigma = 0$), log utility ($\gamma = 1$), Cobb-Douglas production ($\xi = 1$), and no technological progress ($g = 0$). In this case, the following equation determines the steady-state interest rate:
\[ r = \frac{1}{1 - \tau_k} \left[ \rho + p(\rho + p) \frac{k}{y} \right]. \] (16)

We skip the derivation of this equation, as it follows immediately from Blanchard’s equation (12). Note that for the special case of \( p = 0 \), the consumer faces an infinite horizon, and we obtain equation (5) from section 1.

The remainder of the Blanchard model is similar to the Ramsey model. The steady state of the economy is determined by equation (16) together with equations (1), (2), and (4). Because labor supply is inelastic, labor taxes do not yield interesting dynamic effects. Capital taxes, however, yield the following:

\[
\left. \frac{dR}{d\tau_k} \right|_{\text{dynamic}} = \left\{ 1 - \left( \frac{\alpha}{1 - \alpha} \right) \right.
\frac{[\alpha \tau_k + (1 - \alpha) \tau_n] 2p(\rho + p)}{ho^2 + 4p(\rho + p)\alpha(1 - \tau_k) - \rho \sqrt{\rho^2 + 4p(\rho + p)\alpha(1 - \tau_k)}}
\left. \frac{dR}{d\tau_k} \right|_{\text{static}}. \] (17)

In the limit as \( p \) approaches 0, this simplifies to equation (5).

This equation shows how finite horizons as modeled by Blanchard affect our results regarding dynamic feedback effects. Figure 1 illustrates how the feedback effect varies with the value of \( p \). Recall that for \( p = 0 \), we found that 50 percent of a capital tax cut pays for itself in the steady state. If \( p = .02 \), so the average time horizon is fifty years, the dynamic feedback effect falls from 50 to 45 percent. If \( p = .05 \), so the average time horizon is twenty years, the feedback effect falls to 39 percent.

The bottom line is that the Blanchard generalization of the Ramsey model does alter our results. For plausible parameter values, however, the changes are only modest in size.

## 4 Departures from Neoclassical Production

So far, we have assumed a neoclassical production setting. In this section, we explore the implications of two departures from this assumption: imperfect competition and positive externalities to capital investment.

### 4.1 Imperfect Competition

Many markets in the economy are imperfectly competitive. Because of patents, copyrights, and fixed costs, prices can remain above marginal costs for long periods. Over the past several decades, models of monopolistic competition have become increasingly central in the theories of international trade, economic growth, and the business cycle. Judd (2002) has recently proposed that
these models might also be important for the analysis of tax policy. Here we see whether adding imperfect competition to our generalized Ramsey model in section 2 alters our results about dynamic scoring.

To incorporate imperfect competition, it is useful to imagine an economy that produces in two stages. In the first stage, a competitive sector produces an intermediate good using capital and labor inputs and a Cobb-Douglas production function. Competition ensures that price equals marginal cost. In the second stage of production, firms use the intermediate good to produce a final good that can be used for investment or consumption. Firms in this second stage produce one unit of the final good from one unit of intermediate good and sell the final good at a markup over marginal cost. They may also face fixed costs of entry. We let $\mu$ equal the ratio of price to marginal cost in the final good industry.

With this market structure, the price of the final good, $P$, is

$$P = \mu P_M = \mu MC$$

where $P_M$ is the price of the intermediate good, and $MC$ is the marginal cost of producing the intermediate good. Hereafter, we let the final good be the numeraire, so $P = 1$.

Because the intermediate good is produced with both capital and labor, its marginal cost can be computed from the marginal product of either factor. That is,

$$MC = \frac{w}{f_n} = \frac{r}{f_k}$$

The two equations above yield equilibrium factor prices:

$$r = \frac{f_k}{\mu}.$$  \hspace{1cm} (18)

$$w = \frac{f_n}{\mu}.$$  \hspace{1cm} (19)

These two equations replace (8) and (9) from section 2.

The existence of a markup raises the possibility of economic profit. The final goods producers buy a quantity $f(k, n)$ of the intermediate good and then earn operating profits of $\left(\frac{\mu - 1}{\mu}\right) f(k, n)$. We let $\theta$ be the fraction of operating profits that accrue to the owners of the firms as pure economic profits. That is, economic profits are

$$\pi = \theta \left(\frac{\mu - 1}{\mu}\right) f(k, n).$$

This formulation allows for the possibility that all of the operating profits accrue to firm owners ($\theta = 1$), that all of the operating profits are dissipated through the fixed costs associated with entry ($\theta = 0$), and a range of intermediate cases.
Because households own firms, economic profits enter the household budget constraint. As a result, the steady-state equation (12) becomes:

\[ c = \frac{f(k, n)}{\mu} + \pi - gk. \]  

(20)

Economic profits are assumed to be taxed at a rate \( \tau_\pi \), so equation (13) becomes:

\[ R = \tau_k rk + \tau_n wn + \tau_\pi \pi. \]

(21)

In the perfect competition case, the steady state of the economy was described by equations (7) to (13); the comparable system now includes equations (7), (18), (19), (10), (11), (20), and (21).

Analysis as before yields the following results, assuming Cobb-Douglas production (\( \xi = 1 \)):

\[
\frac{dR}{d\tau_k}\bigg|_{\text{dynamic}} = \left[ 1 - \frac{\alpha \tau_k + (1 - \alpha) \tau_n + \theta (\mu - 1) \tau_\pi}{(1 - \alpha) (1 - \tau_k)} \right] \frac{dR}{d\tau_k}\bigg|_{\text{static}}
\]

\[
\frac{dR}{d\tau_n}\bigg|_{\text{dynamic}} = \left[ 1 - \frac{\alpha \tau_k + (1 - \alpha) \tau_n + \theta (\mu - 1) \tau_\pi}{(1 - \alpha) (1 - \tau_n)} \right] \sigma \frac{dR}{d\tau_n}\bigg|_{\text{static}}
\]

(22)

(23)

These equations are the counterparts to equations (14) and (15).

To see how imperfect competition affects the analysis of dynamic scoring of capital taxes, we can compare equations (22) and (14). Of course, if \( \mu = 1 \), this more general model collapses to the earlier one. Note, however, that the two models become identical also if there are no economic profits (\( \theta = 0 \)). Thus, for imperfect competition to have important implications for dynamic scoring, it is crucial that not all profits be dissipated by the fixed costs associated with entry.

To get some sense of the magnitude of the effects that imperfect competition might generate, consider our standard parameter values \( \alpha = \frac{1}{4}, \gamma = \frac{1}{4}, \rho = .05, \gamma = .02, \sigma = \frac{1}{2}, \tau_\pi = \frac{1}{2} \). For a 25 percent markup (\( \mu = \frac{5}{4} \)) and no profit dissipation (\( \theta = 1 \)), the feedback effect from a capital tax cut is now 65 percent, compared to 53 percent under perfect competition.

Similar conclusions hold for labor tax changes. By comparing equation (23) with equation (15), we can see that the two cases become identical if there are...
no markups \((\mu = 1)\), if there are no economic profits \((\theta = 0)\), or if economic
profits are not taxed \((\pi = 0)\). On the other hand, if \(\theta = 1\), \(\mu = \frac{2}{3}\), and \(\pi = \frac{1}{4}\),
the feedback effect of a labor tax cut is 21 percent, compared to 16.7 percent in
the benchmark case.

Thus, imperfect competition raises the ability of tax cuts to be self-financing
only if it generates pure economic profits. If the fixed costs associated with
entry dissipate all profits, then imperfect competition as modeled in this section
acts like an adverse shift in the production function, lowering capital accumula-
tion, consumption, and welfare, but having no effect on dynamic scoring of tax
changes.

4.2 Externalities to Capital

In the model examined in Section 2, capital earns its marginal product. Some
economists, however, have suggested that the social marginal product of capital
that "the social [rate of] return to equipment investment in well-functioning
market economies is on the order of 30 percent per year," which is more than
twice the private rate of return. Romer (1987, pp. 165-166) suggests "The
correct weight on the growth of capital in a growth accounting exercise may be
closer to 1 than to 0.25. The true elasticity of output with respect to changes
in capital may be greater than the share of capital in total income because of
positive externalities associated with investment." In this section, we modify
the model in section 2 to include such externalities to capital.

Suppose that each firm’s production \(y_i\) is Cobb-Douglas but is a function
not only of its own capital \(k_i\), but also of the general pool of knowledge, \(\kappa\):

\[ y_i = \kappa k_i^{\alpha} n_i^{1-\alpha}. \]

Each firm takes \(\kappa\) as given. However, \(\kappa\) is assumed to be an increasing function
of the average firm’s level of capital, \(k\):

\[ \kappa = k^\beta. \] (24)

The parameters \(\alpha\) and \(\beta\) measure the direct (private) and indirect (social) benefits of capital. That is, \(\alpha\) determines the distribution of income between capital and labor, but \(\alpha + \beta\) determines the rate at which diminishing returns set in for economy-wide capital accumulation.

In addition to (24), the steady-state conditions for this economy are as follows:

\[ y = \kappa k^n n^{1-\alpha}. \] (25)
\[ r = \alpha nk^{\alpha-1} n^{1-\alpha}. \] (26)
\[ w = (1 - \alpha)nk^{\alpha} n^{-\alpha}. \] (27)
\[ \psi'(n) = -(1 - \tau_n) w. \] (28)
Equations (24) through (30) fully determine the steady-state values of seven variables: $\kappa$, $y$, $k$, $n$, $r$, $w$, and $c$. The equation for tax revenue remains the same as equation (13):

$$R = \tau_k r k + \tau_n w n.$$  
Equations (29) and (30) yield

$$r = \frac{\rho + \gamma g}{1 - \tau_k},$$
$$c = \kappa k^\alpha n^{1-\alpha} - g k.$$  

In this setting, the dynamic feedback effects are as follows:

$$\left. \frac{dR}{d\tau_k} \right|_{dynamic} = \left[ 1 - \frac{\alpha \tau_k + (1 - \alpha) \tau_n + \alpha + \beta}{(1 - \alpha - \beta) (1 - \tau_k)} \right] \left[ \frac{\alpha \tau_k + (1 - \alpha) \tau_n}{(\rho + \gamma g) - \alpha (1 - \tau_k) g} \right] \frac{dR}{d\tau_k}_{static}.$$  

$$\left. \frac{dR}{d\tau_n} \right|_{dynamic} = \left[ 1 - \frac{\alpha \tau_k + (1 - \alpha) \tau_n}{(1 - \alpha - \beta) (1 - \tau_n)} \right] \frac{dR}{d\tau_n}_{static}.$$  

These results are analogous to equations (14) and (15).

The quantitative effects of externalities are potentially large. As before, consider the canonical values $\tau_k = \frac{1}{4}$, $\tau_n = \frac{1}{4}$, $\alpha = \frac{1}{3}$, $\gamma = 1$, $g = .02$, $\rho = .05$, and $\sigma = \frac{1}{2}$. Recall that in our Ramsey model of Section 2, 53 percent of a capital tax cut and 17 percent of a labor tax cut are self-financing. Suppose $\beta = \frac{1}{12}$, so that the externality from capital raises the return to capital by one-quarter (much smaller than DeLong and Summers 1991 estimate). Equations (32) and (33) yield

$$\left. \frac{dR}{d\tau_k} \right|_{dynamic} = .26 \left. \frac{dR}{d\tau_k} \right|_{static}.$$  

$$\left. \frac{dR}{d\tau_n} \right|_{dynamic} = .81 \left. \frac{dR}{d\tau_n} \right|_{static}.$$  

In this case, growth pays for 74 percent of a capital tax cut and 19 percent of a labor tax cut. These calculations indicate that modest externalities to capital slightly raise the dynamic feedbacks associated with labor income taxes and significantly raise the feedbacks associated with capital income taxes.

At this point, we should acknowledge that the existence and magnitude of these externalities are both speculative and controversial. Our analysis suggests that measuring their magnitude is crucial for the task of dynamic scoring.
5 Transitional Dynamics

The results presented so far in this paper consider only the economy's steady state. This section examines how our steady-state results from Section 2 are affected by considering the transition paths of labor supply and capital. After a tax cut, the capital stock, which is initially fixed, will gradually increase to its new steady-state level. Labor supply will immediately jump and then approach its new steady state.

We derive our results from a log-linearized version of a system of differential equations that describe the model dynamics. Readers who wish to see the derivation of the results of this section are referred to Mankiw and Weinzierl (2004). To keep things simple, we assume log utility ($\gamma = 1$), Cobb-Douglas production ($\beta = 1$) and no technological change ($g = 0$).

To compute the path of tax revenues, we need the transition paths of $n$ and $k$ and the values of $n$ and $k$ at three points in time: prior to the tax cut, immediately after the tax cut, and in the long-run steady state after the tax cut. Denote the levels of $n$ and $k$ at these three points as $n_0$, $n_x$, and $n^*$, and $k_0$, $k_x$, and $k^*$. Similarly, let $R_0$, $R_x$, and $R^*$ denote tax revenues per period prior to, immediately after, and in the long run after a tax cut. Using this notation, the transition paths can be written as:

$$\ln n_t - \ln n^* = (\ln n_x - \ln n^*) e^{\lambda t},$$

$$\ln k_t - \ln k^* = (\ln k_x - \ln k^*) e^{\lambda t}.$$  

(34)  

(35)

where $\lambda$ is equal to the negative eigenvalue of the characteristic matrix of the system of differential equations. Both $n$ and $k$, and thus $R$, transition from their jump values to their steady-state values at this rate.

Tax revenues at any time $t$ can be written as:

$$R_t = [\alpha \tau_k + (1 - \alpha) \tau_n] k_t^\alpha n_t^{1-\alpha}.$$  

(36)

This allows us to compute tax revenue at any point in time. With equations (34)-(36) and the model’s steady-state conditions from Section 2, we can calculate how much of the static impact of a tax cut is paid for over any given period. Table 1 shows these calculations for selected points along the transition path. We continue to assume our canonical values for the other parameters: $\tau_k = \frac{1}{4}$, $\tau_n = \frac{1}{4}$, $\alpha = \frac{1}{4}$, $\sigma = \frac{1}{4}$.

Recall that, for a capital tax cut, the feedback effect in the steady state is 50 percent. By contrast, the immediate feedback is 10.6 percent, as labor supply jumps up in response to the tax cut. The feedback is 21.3 percent by the fifth year, 29.1 percent by the tenth year, and 42 percent by the twenty-fifth year.

For a labor tax cut, the feedback effect in the steady state is 16.7 percent. The immediate feedback is 12.3 percent. The feedback is 13.5 percent by the fifth year, 14.3 percent by the tenth year, and 15.8 percent by the twenty-fifth year.

The immediate jump in labor supply plays a vital role in the timing of the feedback effects. The elasticity of labor supply determines the size of this initial
jump. If $\sigma = 3$, the instantaneous feedback of a labor tax cut is 32.6 percent, compared to a steady-state feedback of 38 percent. For the case of a capital tax cut, the instantaneous feedback is 30.4 percent, while the steady-state feedback remains 50 percent.

One way to summarize the transition to the steady state is by calculating the present value of the reduced tax revenue, using the after-tax return to capital along the transition path to discount the revenue streams. By comparing the present value of the dynamic and static estimates, we obtain the “present-value” feedback effect, which is, in essence, a weighted average of the feedback effects along the entire path. For the canonical parameter values we have been using, the present-value feedback of a capital tax cut is 32.4 percent, compared to a steady-state feedback of 50 percent. The present-value feedback of a labor tax cut is 14.7 percent, compared to a steady-state feedback of 16.7 percent.

The foregoing analysis of transitional dynamics is based on a linearization of the model. To check the accuracy of this linearization, we have also conducted simulations of the nonlinear model using Matlab. The results from the linearized model are quite accurate, although of course not perfectly so. For example, Table 2 shows the present-value feedback effect for various sized capital and labor tax changes and compares the results to what the linearized model predicts. Even for tax changes of five percentage points, the linearized model estimates feedback effects that differ from the nonlinear estimates by less than three percentage points. Note that the linearization overestimates the feedback effects for sizeable tax cuts but underestimates them for sizeable tax increases. This asymmetry can be explained by the positive relationship between the feedback effects of tax changes and the initial tax rate, as discussed in footnote 4. For example, following a sizeable tax increase, the economy is in a more distorted position, magnifying the feedback effects of tax changes.

Overall, the analysis of transitional dynamics demonstrates that the task of dynamic scoring is particularly important over longer time horizons. The time horizon is critical when analyzing capital taxes. In practical discussions of budget policy, scoring windows are only five or ten years. The generalized Ramsey model shows that many significant effects occur outside of this window.

6 Conclusion

This paper has examined the issue of dynamic scoring using the textbook neoclassical growth model and some generalizations of it. Our goal has been to provide theoretical guidance for economists interested in estimating the revenue effects of tax changes. The simple formulas we have derived permit back-of-the-envelope calculations that illustrate the degree to which tax cuts are self-financing.

In all of the models considered here, the dynamic response of the economy to tax changes is too large to be ignored. In almost all cases, tax cuts are partly self-financing. This is especially true for cuts in capital income taxes.

Not surprisingly, the results of this exercise depend on a number of key pa-
rameters. Because the values of some of these parameters are open to debate, reasonable people can disagree about the magnitude of the feedback effects. Three crucial parameters are the compensated elasticity of labor supply, the elasticity of substitution between capital and labor, and the externality to capital accumulation. Unfortunately, the empirical literature does not give clear guidance about their magnitudes. The degree of imperfect competition may also be important, but only to the extent that market power leads to pure economic profits. Finally, the time horizon of consumers, although important for many questions in economics, appears not to be crucial for the task of dynamic scoring.

Although we have explored several variations of the basic Ramsey model to evaluate the robustness our conclusions, there are surely issues still to be addressed. As we noted earlier, some economists have emphasized the short-run Keynesian effects of tax policy, and these effects may be important for dynamic scoring. In addition, much of the literature on economic growth has stressed the role of human capital, which is absent from the models considered here. How tax policy affects human capital accumulation and how human capital affects economic growth are hard questions, but they may be crucial for revenue estimation, especially over longer time periods. Finally, examining alternative financing regimes may also prove fruitful; our assumption that lump-sum transfers adjust immediately to revenue changes has usefully simplified the problem but may be empirically unrealistic. In light of all the open questions, the results presented in this paper should be viewed only as first steps.

Policy economists will need to focus the next steps on evaluating which generalizations of the basic model are most salient and then estimating the key parameters. The task is pressing. In 2003, the U.S. House of Representatives adopted a rule that requires the staff of the Joint Committee on Taxation to analyze the macroeconomic impact of any major tax bill before the House can consider the bill. One conclusion is impossible to escape: difficult as it may be, the subject of dynamic scoring should remain a high priority for those economists advising lawmakers on issues of tax policy.

References


Figure 1: Dynamic Feedback in the Blanchard Model

Table 1: Dynamic Feedback Effects along the Transition Path

Percent of static revenue impact offset by higher growth

<table>
<thead>
<tr>
<th>Time</th>
<th>Capital tax cut</th>
<th>Labor tax cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>Immediate impact</td>
<td>10.6</td>
<td>12.3</td>
</tr>
<tr>
<td>1 year</td>
<td>13.0</td>
<td>12.6</td>
</tr>
<tr>
<td>3 years</td>
<td>17.4</td>
<td>13.0</td>
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<td>5 years</td>
<td>21.3</td>
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<tr>
<td>10 years</td>
<td>29.1</td>
<td>14.3</td>
</tr>
<tr>
<td>25 years</td>
<td>42.0</td>
<td>15.8</td>
</tr>
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<td>50 years</td>
<td>48.4</td>
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</tr>
<tr>
<td>Steady-state impact</td>
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</tr>
<tr>
<td>Type of analysis</td>
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<td>Labor taxes</td>
</tr>
<tr>
<td>------------------</td>
<td>---------------</td>
<td>-------------</td>
</tr>
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<td></td>
<td>32.4</td>
<td>14.7</td>
</tr>
<tr>
<td></td>
<td>+0.05</td>
<td>35.2</td>
</tr>
<tr>
<td></td>
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<td>33.5</td>
</tr>
<tr>
<td></td>
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<td>33.0</td>
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<tr>
<td>Simulated tax rate change (from starting tax rates of 0.25)</td>
<td>33.5</td>
<td>15.6</td>
</tr>
<tr>
<td>+0.001</td>
<td>32.5</td>
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</table>

Table 2: Present Value of Dynamic Feedback from Tax Changes

Percent of static revenue impact offset by change in growth