Levitating atmospheres of Eddington-luminosity neutron stars

The Harvard community has made this article openly available. Please share how this access benefits you. Your story matters

Citation

Citable link
http://nrs.harvard.edu/urn-3:HUL.InstRepos:27770114

Terms of Use
This article was downloaded from Harvard University’s DASH repository, and is made available under the terms and conditions applicable to Open Access Policy Articles, as set forth at http://nrs.harvard.edu/urn-3:HUL.InstRepos:dash.current.terms-of-use#OAP
Levitating atmospheres of Eddington-luminosity neutron stars

Maciek Wielgus,1⋆ Aleksander Sądowski,2,3† Włodek Kluzniak,1‡
Marek Abramowicz1,4,5 and Ramesh Narayan3

1 Nicolaus Copernicus Astronomical Center, ul. Bartycka 18, 00-716, Warszawa, Poland
2 MIT Kavli Institute for Astrophysics and Space Research, 77 Massachusetts Ave, Cambridge, MA 02139, USA
3 Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA
4 Physics Department, Gothenburg University, 412-96 Göteborg, Sweden
5 Institute of Physics, Faculty of Philosophy and Science, Silesian University in Opava, Běrzanůvo nam. 13, 746-01 Opava, Czech Republic

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT
We construct models of static, spherically symmetric shells supported by the radiation flux of a luminous neutron star in the Schwarzschild metric. The atmospheres are disconnected from the star and levitate above its surface. Gas pressure and density inversion appear in the inner region of these atmospheres, which is a purely relativistic phenomenon. We account for the scattering opacity dependence on temperature green by using the Klein-Nishina formula. The relativistic $M_{\text{f}}$ closure scheme for the radiation tensor provides a GR-consistent treatment of the photon flux and radiation tensor anisotropy. In this way we are able to address atmospheres of both large and moderate/low optical depths with the same set of equations. We discuss properties of the levitating atmospheres and find that they may indeed be optically thick, with the distance between star surface and the photosphere expanding as luminosity increases. These results may be relevant for the photospheric radius expansion X-ray bursts.

Key words: gravitation – stars: neutron – stars: atmospheres – X-rays: bursts.

1 INTRODUCTION
This paper discusses the structure of spherically symmetric, static, shell-like atmospheres of extremely luminous, compact, non-rotating stars. The results are expected to be relevant to the astrophysics of accreting neutron stars.

Under certain conditions neutron stars may become so luminous that the forces associated with radiation may exceed the pull of gravity. Several systems with super-Eddington luminosity have been reported (McClintock & Remillard 2006), the “LMC transient” A0535-668 (Bradt & McClintock 1983) being a particularly clear example. Super-Eddington luminosities may be achieved in some X-ray bursts (Strohmayer & Bildsten 2006), as well as during accretion of matter in a semidetached binary, especially in a ULX (Bachetti et al. 2014), or a detached binary with a Be star as the companion. At least in the case of X-ray bursts the radiation field is nearly spherically symmetric. In most cases, extended periods of time may occur in which the radiation field and the gas can be taken to be in a quasi-steady state, i.e., not varying on the dynamical time-scale. For these reasons we study the problem of extremely luminous neutron stars under the simplifying assumptions of steady-state conditions and spherical symmetry in the Schwarzschild metric.

Contemporary theoretical studies of neutron star atmospheres in X-ray bursts involve sophisticated, spectrally resolved, treatment of the radiation (Suleimanov et al. 2011, 2012). However, effects of general relativity (GR) are often neglected for simplicity. In fact, these may be quite important. The atmospheric structure of luminous stars in GR has been studied by Paczynski & Anderson (1986), who found that the atmosphere becomes very extended in the Klein-Nishina regime of scattering opacity. However, in the Thomson regime, as well as in the Newtonian solutions for either of the scattering regimes, the atmosphere is geometrically thick. Paczyński and Anderson’s results show that in the case of very luminous neutron stars it would be inappropriate to expect, and simply speak of, relativistic “corrections” to Newtonian solutions. In fact, qualitatively new results may appear when GR effects are included.

In this paper we report the presence of a new type of atmospheric solution for neutron stars radiating at nearly Eddington luminosities, which is qualitatively different from the ones obtained in Newtonian physics. We find that fluid atmospheres of luminous stars in general relativity may have the form of a shell suspended above the stellar surface, with the maximum density of the atmosphere attained on a surface separated from the star by a “gap” in which the atmospheric density and pressure drop precipitously as the stellar surface is approached. Such shells may have already been observed in some X-ray bursts (in’t Zand et al. 2011). A certain group of bursts indicate radiation-driven ejection of the neutron
star gaseous envelope, e.g., Tawara et al. (1984); White & Angelini (2001); Wolff et al. (2005). Our model is expected to be of particular interest for modelling these so called photospheric radius expansion (PRE) bursts (Lewin et al. 1993). Unlike most models of atmospheres of highly luminous sources, e.g., Kato & Hachisu (1994), the discussed solutions do not involve dynamical outflows. Our results also differ from the static extended atmospheres of Paczynski & Anderson (1986), which share with their more familiar Newtonian counterparts the property that the atmospheric density increases monotonically as the stellar surface is approached. The key property of our solutions is that neither the density nor the pressure is monotonic, nor is the atmospheric density nor the pressure is monotonic, nor is the atmosphere.

In this paper we consistently treat atmospheres of both large and moderate optical depths. The unusual shell-like structure of our atmospheric solutions can be readily understood as a consequence of the spatial characteristics of the radiation field and of gravity. In general relativity, unlike in Newtonian theory, the pull of gravity and the radiation flux have a different dependence on the radial distance from the star. One consequence of this is that the Eddington luminosity is not a distance independent concept—in fact, typically the flux of radiation has a stronger dependence on the radius than the effective gravity. Hence, for a sufficiently luminous star, the radiation force may balance gravity only at a particular radial distance (Phinney 1987). Effectively, the Eddington flux is attained only on a certain surface (Bini et al. 2009; Oh et al. 2010), which is spherical for a spherically symmetric star. We refer to this surface as the Eddington Capture Sphere, or ECS (Stahl et al. 2012; Wielgus et al. 2012). Inside this surface, radiation force on an ionized atom exceeds the pull of gravity, outside it gravity prevails. Thus, the ECS is a locus of stable equilibrium positions for test particles (Abramowicz et al. 1990; Stahl et al. 2012).

Clearly, an atmosphere may exist, which is centred on the ECS and thinning out in both directions, towards and away from the luminous star, with the gas pressure gradient balancing the difference between the pull of gravity and the radiation pressure. In the optically thin limit the radiation force is simply given by the flux of radiation coming from the central star times the opacity and analytic solutions may be found (Wielgus et al. 2015). Here, we turn our attention to shells of more general optical depth, which require a numerical treatment of the gas-radiation interaction. In our numerical scheme we follow Levermore (1984) and assume that the radiation tensor is isotropic not in the fluid frame, but in the “rest frame” of the radiation. This leads to the $M_1$ closure. A generalization of the $M_1$ scheme to GR has been given in Sadowski et al. (2013).

Convenience, we parametrize the luminosity of the star by the ratio of the luminosity observed at infinity to the Eddington luminosity,

$$\lambda = L_\infty/L_{\text{Edd}};$$

with the standard expression for the latter,

$$L_{\text{Edd}} = 4\pi GMm_p c/\sigma_T = 4\pi GMc/\kappa_T,$$

for photon mass $m_p$, Thomson cross-section $\sigma_T$, and Thomson scattering opacity $\kappa_T$. In the Schwarzschild metric, with $|g_{tt}(r)| = 1 - 2GMc^{-2}/r$, the stellar luminosity at radius $r$ is

$$L(r) = L_\infty \left(1 - \frac{2GM}{rc^2}\right)^{-1},$$

and a static balance between gravity and radiation force with Thompson scattering can only be achieved at one radius $r = R_{\text{ECS}}$, with

$$R_{\text{ECS}} = R_S \left[1 - (L_\infty/L_{\text{Edd}})^2 \right]^{-1},$$

Thus, in terms of the constant introduced in Eq. (1),

$$R_{\text{ECS}}/R_S = 1/(1 - \lambda^2).$$

Numerous authors have shown that test particles initially orbiting the star (at various radii) will settle on the spherical surface at $r = R_{\text{ECS}}$, provided that

$$(1 - R_S/R_\infty)^{1/2} \leq \lambda < 1,$$

their angular momentum having been removed by radiation drag (Bini et al. 2009; Sok Oh et al. 2011; Stahl et al. 2012). In fact, any point on the ECS is a position of stable equilibrium in the radial direction (and neutral equilibrium in directions tangent to the ECS surface, Stahl et al. 2012).

2 TREATMENT OF RADIATION

When treating the radiation as a fluid propagating through a possibly optically thick atmosphere, we need to employ a general formulation of the coupled energy-momentum conservation equations for the radiation ($R_E^\mu$) and gas ($T^{\mu}_\nu$) stress-energy tensors. In relativistic four-notation, the equations take the form

$$(R_E^\mu)_{;\mu} = -G_v,$$

$$(T^{\mu}_\nu)_{;\mu} = G_v,$$

where $G_v$ denotes the radiation four-force density (Mihalas & Mihalas 1984), a coupling term between gas and radiation. In the orthonormal fluid rest frame (hereafter denoted with a hat), under the spherical symmetry assumption, the only non-zero components of $G^\mu$ are $G^0$ and $G^3$, with

$$G^0 = k_\sigma (R^\mu - 4\sigma T^\mu) ,$$

$$G^3 = \chi p R^\theta.$$

Here $\chi = \kappa_\sigma + \kappa_s$ denotes the total opacity coefficient, $\kappa_\sigma$ is the frequency integrated absorption opacity, $\kappa_s$ is the scattering opacity, $\sigma$ is the Stefan-Boltzmann constant, and $T$ and $\rho$ are the temperature and rest-mass density of the gas. We neglect the transfer of energy by Compton scattering.

When a static, and spherically symmetric system is to be considered, only the radial derivatives are of interest. Eqs. (7)-(8) then become ordinary differential equations in the variable $r$. The “angular” components $R^{(\theta)(\theta)} = R^{(\theta)(\theta)}$ can be eliminated if one remembers that the radiation tensor has a vanishing trace, $R^\rho_{\mu} = 0$. We indicate components in the orthonormal Schwarzschild tetrad by indices in parentheses. Under our assumptions this tetrad coincides with the orthonormal fluid rest frame, hence $R^{(\theta)(\theta)} = R^{(\theta)(\theta)}$. This simplifies the calculations greatly, so that Eq. (7) yields the following
In order to solve for these quantities, it is necessary to make some additional assumptions, specifically to adopt an equation of state for the gas and a closure scheme for the radiation. In the limit of an isotropic radiation tensor, where $\rho_{\text{rad}} = -R_{\nu}^\mu = 3R_{\nu\gamma}^\gamma = 3R_{\nu\theta}^\theta = 3R_{\nu\phi}^\phi$, summing Eqs. (12) and (16), we recover the familiar equation

$$\frac{d\rho_{\text{tot}}}{dr} = -\frac{(\rho_{\text{tot}} + \rho_{\text{gas}})M}{r^2(1 - 2M/r)}$$

where $\rho_{\text{tot}}$ and $\rho_{\text{gas}}$ denote total pressure and total energy density of the gas and radiation mixture, given by

$$\rho_{\text{tot}} = \rho + \rho_{\text{rad}},$$

$$\rho_{\text{gas}} = \rho + \epsilon + \rho_{\text{rad}}.$$  

Equation (17) is the correct relativistic hydrostatic balance equation of an optically thick gas - radiation mixture.

### 3 Optically Thin Polytropic Atmospheres

While this paper is mainly concerned with atmospheres of arbitrary optical depth, we will first briefly discuss the optically thin solutions. In this regime the radiation stress-energy tensor is known a priori (Abramowicz et al. 1990), rendering the radiation transfer description trivial. This suffices to solve for hydrostatic equilibrium of a polytrope, as the model reduces to the following simple system of equations (one ordinary differential equation supplemented by algebraic ones)

$$\frac{dp}{dr} = -\frac{(\rho + p + \epsilon)M}{r^2(1 - 2M/r)} + \frac{\lambda(\chi/\kappa T)pM}{r^2(1 - 2M/r)^{3/2}},$$

where $K$ is the polytropic constant, the adiabatic index is taken to be $\Gamma = 5/3$, the mean molecular mass $\mu = 1/2$, and the Thomson scattering opacity $\kappa T = 0.4 \text{ cm}^2/\text{g}$ (these correspond to pure ionized hydrogen in the non-relativistic limit). We neglect absorption, but account for temperature dependence of the scattering coefficient, corresponding to the (averaged) Klein-Nishina scattering model, i.e., $\chi = \kappa_{\text{KN}}(T)$. The following approximate scattering opacity formula (Buchler & Yueh 1976; Paczynski 1983; Lewin et al. 1994)
The system of equations (20)-(21) can be readily solved numerically. From Eq. (20) one finds the location of the pressure maximum, 

\[ R_E = \frac{2M}{1 - \lambda^2 a_1^2 a_2^2}, \]  

with 

\[ a_1(T) = \frac{k_\text{a}(T)}{\kappa T}, \] 
\[ a_2(T) = \left[ 1 + \frac{k_{\text{b}} T}{\mu m_p c^2} \left( e + p \right) \right]^{-1} = \left[ 1 + \frac{k_{\text{b}} T M}{\mu m_p c^2} \frac{\Gamma}{\Gamma - 1} \right]^{-1}, \]  

where the temperature is taken at its maximum value, treated as an arbitrary constant parametrizing the family of solutions, \( T = T_M \equiv T(R_E) \). Because of the relativistic correction \((p + e)\) to density in Eq. (20) and the opacity temperature dependence, the expression for \( R_E \) differs from that for the test particle \( R_{\text{ECS}} \) by the presence of the correction factors \( a_1, a_2 \). The first factor gives no correction \((a_1 = 1)\) if the Klein-Nishina modification to the Thomson scattering is neglected \((k_\text{a} = \kappa T)\). As long as the temperature is much lower than 1 GeV, or \( T_M \ll 10^{13} \text{ K} \), the second correction, \( a_2 \), is insignificant, \( |1 - a_2| \ll 1 \).

For non-relativistic temperatures, i.e., when the temperature correction factors are equal to unity, \( a_1 = 1 = a_2 \), it is straightforward to find analytic solutions for optically thin atmospheres (Wielgus et al. 2015). In particular, for high luminosities the atmospheric shells are suspended above surface of the star, with the gas density falling off on both sides of the sphere on which it attains its maximum. The results obtained here are qualitatively similar in general, and virtually identical to the analytic solutions in the particular case \( a_1 = 1 \). Figure 1 compares atmospheres described in this section with the analytic results for polytropic optically thin Thomson scattering atmospheres, discussed in Wielgus et al. (2015), for the same set of maximum temperatures \( T_M \) and fixed luminosity \( \lambda = 0.95 \). Since the equations are homogeneous in density (and pressure, for a given temperature), we show density profiles normalized by \( \rho_{\text{M}} = \rho(R_E) \).

When the luminosity is close to Eddington the atmospheric shells are suspended much closer to the neutron star for Klein-Nishina solutions than would be the case for purely Thomson scattering, since the opacity decreases at high temperatures in the Klein-Nishina model. This is because at high luminosities the denominator of Eq. (23) is very small, and even a small correction to the luminosity parameter significantly changes the value of the denominator, and hence of the radius of equilibrium \( R_E \).

Furthermore, the radial atmospheric profile of the density becomes extremely asymmetric, the atmosphere falling off quite sharply towards the star, but being rather extended on the side away from the star (i.e., for \( r > R_E \)) owing to the rapid growth of the running value of \( a_1(T) \) as the scattering cross-section increases with decreasing temperature, the gradient of pressure in Eq. (20) being sensitive—at any radius \( r \)—to the difference \( 1 - a_1(T) \).

1 We omit from the denominator of Eq. (22) an additional factor of \((1 + 2.7 \cdot 10^8 p/T^2)\), which is appropriate in the limit of degenerate matter.
scattering opacity we use the (direction and frequency averaged) Klein-Nishina opacity, \( \kappa_s = \kappa_{KN}(T) \), as given by Eq. (22). Thus, \( \kappa_s \) is a decreasing function of the local gas temperature and equals the Thomson scattering opacity \( \kappa_T \) in the low temperature limit.

The system of equations describing our model is then as follows:

\[
\frac{dp}{dr} = -\left( \rho + p + \varepsilon \right) \frac{M}{r^2} + G_r ,
\]

\[
\frac{d}{dr} R_r' = \frac{1}{r} \left[ (1 - 3M/r)R_r' + (3 - 5M/r)R_r'' - G_r \right].
\]

\[
R_r''(r) = \frac{L_{\infty}}{4\pi r^2 (1 - 2M/r)} ,
\]

\[
p = \frac{k_B \mu m_p}{\kappa_T} \rho T = \frac{2}{3} \varepsilon ,
\]

\[
T = \left( \frac{R_r'}{4\kappa r} \right)^{1/4} ,
\]

\[
R_r'' = M_f (R_r', R_r''),
\]

\[
\chi = \kappa_s(T, \rho) + \kappa_{KN}(T) .
\]

The system of Eqs. (31)-(38) can be solved uniquely for the six unknowns, \( \rho, p, \varepsilon, T, R_r', \) and \( R_r'' \), as functions of radius. Equating the right hand side of Eq. (31) to zero we find that the radius at which the gas pressure attains a maximum is

\[
R_0 = \frac{2M}{1 - \rho_0^2 T_0^2 \beta^2} .
\]

where \( a_2 \) is a temperature correction similar to the one present for the polytropic optically thin model, given by Eq. (25) with \( T_M \) replaced by \( T_0 \equiv T(R_0) \), and \( a_3 \) is a new correction factor, reflecting the more general radiation transfer model assumed,

\[
a_3 = \chi(R_0) / \kappa_T .
\]

As already remarked, to high precision \( a_2 = 1 \) at temperatures prevalent in astrophysical neutron stars. However, the value of the \( a_3 \) parameter has a crucial influence on the position of an optically thick levitating atmosphere. For high temperatures, the opacity assumes low values because of the Klein-Nishina cross-section reduction and a much larger flux is required to balance gravity. On the other hand, dense gas at relatively low temperatures is characterized by large opacities, because of large absorption. In Fig. 2 a contour plot of the \( a_3 \) correction factor as a function of the model parameters is shown.

For the optically thick models considered here, in general, the radial positions of the density and pressure peaks do not coincide. The problem is no longer homogeneous in \( \rho \) and does not admit barytropic solutions. Thus, we need to specify two thermodynamic quantities as boundary values specifying the problem in order to perform the numerical integration. We parametrize the solutions with \( \{ T_0, \rho_0, \lambda \} \), i.e., with the temperature at the pressure maximum, \( T_0 \), density at the pressure maximum, \( \rho_0 \equiv \rho(R_0) \), and the luminosity parameter \( \lambda \) of Eq. (1). The initial integration conditions \( [\rho(R_0), R_r'(R_0), R_r''(R_0)] \) are calculated from the specified parameters using the ideal gas equation of state (pressure) and the LTE condition with \( M_1 \) closure (from \( T_0 \) we find \( R_0 \) using the LTE condition and we evaluate \( R_r'' \) via the \( M_1 \) closure). This procedure fixes the values of all physical quantities at \( R_0 \), including the temperature and opacity corrections \( a_2, a_3 \), from which \( R_0 \) itself can be determined, whereupon the system of Eqs. (31)-(38) can be integrated in both directions, starting at \( r = R_0 \).
to infinity. This determines the value of $T_0$ uniquely for any given \(\rho_0\) (and fixed values of the basic parameters such as \(M\) and \(\lambda\)), as illustrated in Fig. 3, where the thick curve corresponds to the minimum value of \(T_0\) among those that yield a finite solution.

To effectively find the correct solution we use a numerical relaxation routine assuming \(R''/R^3 = 1\) at the outer boundary, which corresponds to the radiation tensor of a point source in vacuum. While this condition is an approximation (it is only rigorously fulfilled at infinity), we find that the details of the outer boundary condition have negligible influence on the solution in the region of significant gas density. See the Appendix for some additional comments on the outer boundary condition.

For any (fixed) value of the luminosity, \(\lambda < 1\), we obtain a family of physical solutions, differing by the density parameter \(\rho_0\), related to the total mass of the shell.

### 4.4 Properties of the optically thick solutions

We find that for a given luminosity \(\lambda\), levitating atmospheres of optical depth \(\tau_{\text{sc}} > 1\) only exist in a limited range of the \(\rho_0\) parameter (density at the pressure maximum). Values of the density \(\rho_0\) that are too low yield optically thin solutions, values of \(\rho_0\) that are too large yield solutions in which the density decreases monotonically with the radius (the atmosphere is supported by the surface of the star).

Examples of levitating atmosphere density profiles, found for \(\lambda = 0.95\) and \(\lambda = 0.99\), are shown in Fig. 4. In the direction towards the star, the atmosphere thins out rather rapidly, so that there is a clear and large gap between the stellar surface and the levitating atmosphere. Away from the star, the atmosphere may be quite extended, the thinning out being slower than exponential, until it becomes optically thin to scattering. In the optically thin outer region the density of the atmosphere decreases rapidly. For a given star (fixed \(M, L_\infty\)), the base of the levitating atmosphere can be located over a wide range of radii outside the star, with its position being determined by the total mass of the atmosphere, related to \(\rho_0\). In the region of monotonic decrease of \(\rho(r)\) all solutions coincide with a common envelope. The envelope, shown in Figs. 4-6 with thick black dashed lines, corresponds to the limit of a monotonic (non-levitating) solution, with no density inversion (formally \(K_{\text{eff}} \leq K_{\odot}\)).

Figure 5 illustrates the temperature as a function of radius for \(\lambda = 0.99\) solutions. The red dash-dotted line corresponds to an effective temperature, \(T_{\text{eff}}(r) = [R^2(r)/\sigma T^4]^{1/4}\). The photosphere location, as found from the back integration of Eq. (41), is consistent with the surface at which the gas temperature is equal to the local effective temperature \(T_{\text{eff}}(r)\).

We find that the solutions are strongly dominated by the scattering opacity, as \(\kappa_s/\kappa_g > 10^3\) for all radii. Radiation pressure strongly dominates over the gas pressure, with \(p/p_{\text{rad}} < 10^{-3}\) throughout the domain. The solutions vary from optically thin to scattering optical depths of the order of \(10^3\). Optical depths of solutions for \(\lambda = 0.99\), calculated according to Eq. (41), are shown in Fig. 6. An obvious property of the levitating atmospheres is that there exists also an optically thin region in the inner part of the shell. It is located below a transition surface \(r = R_{\text{tran}}\), where the optical depth integrated from the inside is equal to unity, \(\tau_{\text{sc}}(R_{\odot}) = \tau_{\text{sc}}(R) - \tau_{\text{sc}}(R_{\text{tran}}) = 1\). These locations are indicated in Fig. 6 with blue dots.

The set of universal density profile envelopes, parametrized with \(\lambda\), is shown in Fig. 7. Since the properties of these envelopes do not depend on \(\rho_0\), the location of the photosphere is common to all the

---

**Figure 4.** Comparison of the density profiles for luminosity \(\lambda = 0.95\) (family of less extended red curves) and \(\lambda = 0.99\) (family of more extended blue curves). The common envelopes of these two families of curves are indicated with dashed black lines. Corresponding locations of photospheres for each family are indicated with the vertical dashed lines. Test particle ECS for such luminosities are located at 20.5\(M\) and 100.5\(M\), respectively. This significant reduction of the equilibrium radius is due to the Klein-Nishina effect.

**Figure 5.** Temperature as a function of radius for LTE atmospheric shells with \(\lambda = 0.99\). The common envelope indicated with dashed black line, red dash-dotted line corresponds to the effective temperature. Photosphere location indicated with a vertical dashed line.

**4.3 Outer boundary condition**

Although the system of Eqs. (31)-(38) can be solved for any choice of initial parameters \(\rho_0, T_0\), as described in the previous subsection, most of the solutions thus obtained would be unphysical, as the luminosity at infinity would bear no relation to the temperature of the photosphere. We need to impose an additional condition, that at scattering optical depth unity, \(\tau_{\text{sc}} = 1\), the LTE temperature is equal to the effective temperature. Here,

\[
\tau_{\text{sc}}(r) = \int_r^\infty \rho(r') \kappa_{\text{KN}}(r')(1 - 2M/r')^{-1/2} \, dr'.
\]  

(41)

In practice, it is enough to demand that the pressure is well behaved, it neither goes to infinity nor vanishes at finite radii. Thus, for any given initial value of density \(\rho_0\), we can reject all trial values of initial temperature which yield solutions that cannot be extended

---

M. Wielgus et al.
indicates marginal stability, }\) can be readily ob-

The condition (Stahl et al.). The condition (Fig. 7). We use (2015). Strohmayer & Bildsten 2006)- (18)

formally indicates convective

The situation in the inner region of the atmospheres is some-
what more complicated. While Fig. 8 formally indicates convective instability, \( S(r) < 0 \), near the transition radius \( r = R_{\text{tran}} \) the large optical depth condition is not met and hence the criterion itself is not strictly valid. Moreover, near-Eddington radiation flux is ex-
pected to have a strong stabilizing influence, damping the motion of the optically thin fluid, Stahl et al. (2013), and thus hindering the development of instabilities. For instance, Abarca & Kluzniak (in prep.) find that the fundamental radial mode of atmospheric oscil-
lations is overdamped.

There clearly exists a necessity for a more general convective stability criterion, that would remain valid regardless of the optical depth. This, however, is beyond the scope of this work and will be a subject of future investigations.

5 DISCUSSION: PHOTOSPERIC RADIUS EXPANSION BURSTS

The main result of this paper is that the atmospheres of luminous neutron stars may form static shells, suspended above the neutron star surface by the force of radiation. These shells may be optically thick or thin, depending on the amount of matter forming the shell, presumably ejected from the neutron surface in a luminous burst of thermonuclear origin.

Even if enough matter is ejected to initially form an optically thick atmosphere, it may easily become optically thin as luminosity increases and the envelope expands (if an envelope of fixed mass \( M_e \) is expanding, its optical depth goes down approximately with the inverse square of radius). This further justifies the necessity of a model capable of addressing properly the regime of optically thick gas and possible transition to the optically thin regime. An in-
teresting feature of our model is that if the photosphere is formed, its properties (location, temperature) only depend on the luminosity parameter \( \lambda \) and not on the mass of the envelope.

A certain group of X-ray bursts exhibit particularly strong peak luminosities, approaching the Eddington limit (Lewin et al. 1993; Strohmayer & Bildsten 2006). A strong radiative force may push the gaseous envelope of the neutron star away from the stellar surface. In some luminous bursts it is observed that the emitting
surface, inferred from the effective temperature and the luminosity, increases during the early stages of the bursts and then decays to its initial value during the so called touch-down phase. This group of bursts is referred to as PRE bursts. It was argued that the burst luminosity almost exactly reaches the Eddington luminosity during the expansion phase. (Kato & Hachisu 1994). The main argument to support this strong claim is that any excess energy from super-Eddington flux would be efficiently converted to kinetic energy, resulting in dynamical outflows. Sub-Eddington flux could not, on the other hand, explain the photospheric expansion, inferred from the observed spectra. Observations of PRE bursts can reveal important knowledge about the dependence between neutron star mass and radius (Damen et al. 1990; Özel 2006; Özel et al. 2015) and it is reasonable to expect that better understanding of the relations between luminosity, photosphere location and temperature in PRE bursts should provide more detailed insight. For instance, in the basic models, the location of the photosphere is assumed (Damen et al. 1990) to coincide with what was later recognized to be the test-particle Eddington capture sphere (Stahl et al. 2012). We find that this is a significant overestimation, since the photospheric radius of the levitating atmospheres is typically situated much closer to the stellar surface than the test particle ECS. (Fig. 9). We also observe that it is not necessary for the flux to be of Eddington value for the photosphere to start expanding. Figure 9 indicates that the expansion begins at luminosity about 0.85$\lambda E_{\text{edd}}$ and progresses with the increase of luminosity. For luminosity equal to 0.99$\lambda E_{\text{edd}}$ the photospheric radius expands by a factor of about 4, while simultaneously cooling down by a factor of 2, Fig. 10. This seems to be in agreement with a typical expansion magnitude inferred from the observational data, see, e.g., in’t Zand et al. (2015). In Figs. 9 and 10 the horizontal axes are labelled with $\lambda$ as used throughout this paper, as well as and with luminosity normalized by the factor of $L_{\text{Ed}}(1 - 2M/R_*)^{1/2}$, corresponding to the equilibrium luminosity at the stellar surface $= R_\ast$. We note that the discussed luminosities are mildly super-Eddington if the latter is adopted as an “Eddington luminosity” unit.

In conclusion, we suggest that detailed modelling of near-Eddington photospheric expansion bursts should take into account the effects described in this work.

ACKNOWLEDGEMENTS
We thank Omer Blaes, Feryal Özel, and David Abarca for interesting comments on the subject of this paper. This research was partly supported by Polish NCN grants UMO-2011/01/B/ST9/05439 and UMO-2013/08/A/ST9/00795 as well as the Czech ASCRM100031242 CZ.1.07//2.3.00/20.0071 “Synergy” (Opava) project. MW also acknowledges the support of the Foundation for Polish Science within the START programme.

REFERENCES
Bini D., Jantzen R. T., Stella L., 2009, Classical and Quantum Gravity, 26, 055009
Mihalas D., Mihalas B. W., 1984, Foundations of radiation hydrodynamics
Ozel F., 2006, Nature, 441, 1115
Sok Oh J., Kim H., Mok Lee H., 2011, New Astronomy, 16, 183
Levitating atmospheres of luminous neutron stars

APPENDIX A: COMMENTS ON THE $M_1$ CLOSURE

In the limit of $M/r \to 0$ an analytic expression for the $M_1$ closure can be given

$$\frac{R''}{R'} = \frac{2 - a^2 + a(4a^2 - 3)^{1/2}}{a + (4a^2 - 3)^{1/2}},$$

(A1)

where $a = R''/R'$. The corresponding curve is shown in Fig. A1. With Eq. (32) being a differential equation for the $R''$ component, in this work we are actually interested in the “inverse $M_1$” problem, i.e., in finding the $R''$ component, given $R''$ and $R'$ at every step of the numerical integration of Eq. (32). It is worth noticing that while the $M_1$ closure scheme is a unique procedure, it is not an injective function of $(R_{tt}, R_{tr})$, meaning that a given $R''$ may correspond to more than one pair $R_{tt}/R_{tr}$, see Fig. A1. Closer inspection of the formula (A1) reveals that the minimum of the $M_1$ curve corresponds to $\beta = u_{rt}/u_{tr} = c/\sqrt{3}$ and separates the right “subsonic photon gas” optically thick branch from the left “supersonic photon gas” optically thin branch. The numerical relaxation procedure that we utilize is necessary for the solution to pass through that “sonic point”, and allow for a continuous transition from the optically thick regime of radiation trapped in the gas to the optically thin regime of freely streaming photons, forced by the outer boundary condition.

We note that in vacuum for an isotropic radiation stress tensor, i.e., for $G_r = 0$ and $R'_r = -3R''$, Eq. (12) takes the form

$$\frac{d}{dr}R'_r = -\frac{4M}{r^2(1 - 2M/r)}R''_r,$$

(A2)

with the solution $R''_r = p_0/(1 - 2M/r)^2$, where $p_0$ is an integration constant of dimension pressure. The energy density of this isotropic radiation field scales correctly with the fourth power of the redshift factor $(1 + z)$:

$$R^{(\text{iso})} = 3R^{(\text{iso})}_r = 3p_0\left(1 - \frac{2M}{r}\right)^{-2}.$$  \hspace{1cm} (A3)

We find that in the region between the stellar surface and the atmosphere our numerical solutions follow Eq. A3 quite closely.

This paper has been typeset from a \TeX/\LaTeX file prepared by the author.