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Stable, levitating, optically thin atmospheres of Eddington-luminosity neutron stars

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ABSTRACT
In general relativity static gaseous atmospheres may be in hydrostatic balance in the absence of a supporting stellar surface, provided that the luminosity is close to the Eddington value. We construct analytic models of optically thin, spherically symmetric shells supported by the radiation pressure of a luminous central body in the Schwarzschild metric. Opacity is assumed to be dominated by Thomson scattering. The inner parts of the atmospheres, where the luminosity locally has supercritical values, are characterized by a density and pressure inversion. The atmospheres are convectively and Rayleigh-Taylor stable, and there is no outflow of gas.

Key words: stars: neutron, Stars – gravitation, Physical Data and Processes – radiation: dynamics, Physical Data and Processes – stars: atmospheres, Stars

1 INTRODUCTION
Several systems with super-Eddington luminosity have been reported (e.g., McClintock & Remillard 2006), the “LMC transient” A0535-668 (Bradt & McClintock 1983) being oft discussed. Recently, interest in very luminous neutron stars has been revived with the discovery of a clear example of a neutron star with super-Eddington flux in the guise of the ultraluminous 1.37 s pulsar NuSTAR J095551+6940.8 in the nearby galaxy M82 (Bachetti et al. 2014). It is clear that for some accreting neutron stars, radiation pressure may exceed the pull of gravity close to the star (as is usually the case), but also in the radial direction towards the star.

This paper discusses the simplest case of optically thin, spherically symmetric atmospheres in the Schwarzschild metric, which admit of analytic solutions. Optically thick atmospheres require numerical treatment of radiative transfer, and these will be reported elsewhere (Wielgus et al. 2015).

We consider atmospheres consisting of pure ionized hydrogen. All the results are given for the Schwarzschild spacetime. The luminosity of the star will be parametrized by the ratio of the luminosity observed at infinity to the Eddington luminosity,

\[ \lambda = \frac{L_\infty}{L_{\text{Edd}}}. \]  

with the standard expression for the latter, \( L_{\text{Edd}} = \frac{4\pi GMm_p c}{\sigma T} \). The stellar radius will be denoted by \( R_\ast \), it is taken to correspond to a canonical neutron star, e.g., \( R_\ast \approx 5Gm c^2 \), with \( M \approx 1.4M_\odot \) being the stellar mass.

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2 BALANCE BETWEEN GRAVITY AND RADIATION PRESSURE

In the Schwarzschild metric, the stellar luminosity (which we take to be constant in time), as measured by a local observer at radius \( r \), declines with the radial coordinate distance \( r \) as

\[
L(r) = L_\infty \left(1 - \frac{2GM}{rc^2}\right)^{-1},
\]

and a static balance between gravity and radiation pressure owing to Thomson scattering can only be achieved at one radius, at which

\[
L(r) = L_{\text{Edd}}(1 - R_S/r)^{-1/2},
\]

where \( R_S = 2GM/rc^2 \) is the Schwarzschild radius (e.g., Phinney 1987; Stahl et al. 2013). That radius can be written as

\[
r = R_S / (1 - \lambda^2) \equiv R_{\text{ECS}}.
\]

The sphere at \( R_{\text{ECS}} \) has been called the Eddington capture sphere (ECS) by Stahl et al. (2012). These results have been derivèd rigorously within the mathematical formalism of Einstein’s general relativity, and they can be intuitively understood in terms of the different dependence on redshift of the luminosity and of the “gravitational” acceleration. Eq. (4) reflects the dependence of luminosity at infinity on two redshift factors \((1 + z)^{-1}\), one for the energy of the photons, and one for the rate of their arrival. However, the acceleration of a static observer in the Schwarzschild metric scales with only one factor of \((1 + z)\), where the redshift is defined by

\[
1 + z(r) = (1 - R_S/r)^{-1/2}.
\]

Hence, the condition for local balance between acceleration and the momentum flux of photons (multiplied by the Thomson opacity) is given by Eq. (6), and contains one factor of \((1 + z)\), which introduces into the balance condition a dependence on the radial coordinate. This is in contrast with Newtonian gravity (which can formally be recovered in the limit \( R_S \to 0 \), or \( r \to 0 \)), where the balance condition has no radial dependence, and therefore is either satisfied, or not satisfied, equally at all distances from the source.

Numerous authors have shown that test particles initially orbiting the star (at various radii) may settle on the spherical surface at \( r = R_{\text{ECS}} > R_* \), provided that

\[
(1 - R_S/R_*)^{1/2} < \lambda < 1,
\]

their angular momentum having been removed by radiation drag (Bini et al. 2004; Sok Oh et al. 2011; Stahl et al. 2012; Mishra & Kluźniak 2014). In fact, any point on the ECS is a position of stable equilibrium in the radial direction (Abramowicz et al. 1990), and neutral equilibrium in directions tangent to the ECS surface (Stahl et al. 2012). Note that the redshift \( z(r) \) attains the following value on the ECS,

\[
z_{\text{ECS}} \equiv z(R_{\text{ECS}}) = 1/\lambda - 1.
\]

We now take the step of replacing test particles on the ECS with a fluid. For simplicity, let us assume\(^1\) that the fluid is optically thin. This will allow us to decouple the temperature of the fluid from the radiation field and to find analytic solutions of the atmospheric structure. It is clear that the condition of hydrostatic equilibrium will be achieved if the pressure gradient is in the radial direction and its magnitude compensates the imbalance between radiation pressure and gravity. In particular, the pressure must reach a maximum value at \( r = R_{\text{ECS}} \), and must decrease in both directions, towards and away from the star, as the distance to the ECS grows and so does with it the imbalance between radiation pressure and gravity.

For atmospheric temperatures of no more than a few keV, we can safely assume that the energy density of the fluid is given by its baryon rest mass energy density \( \rho c^2 \). In other words, one can neglect the contribution of pressure \( p \) and the internal energy \( \epsilon \) to the energy density, \( \rho c^2 \gg p + \epsilon \). The equation of hydrostatic equilibrium then becomes

\[
\frac{1}{\rho} \frac{d\rho}{dr} = -\frac{GM}{r^2 (1 - R_S/r)^{1/2}} \left[1 - \lambda \left(1 - \frac{R_S}{r}\right)^{-1/2}\right],
\]

where the last term reflects the radiation pressure owing to Thomson scattering, with the luminosity of the star scaled by the Eddington luminosity per Eq. (1). In the limit \( R_S/r \to 0 \) one recovers the classical Eddington balance at \( \lambda = 1 \), while in general the pressure gradient vanishes for \( r = R_{\text{ECS}} \), cf. Eq. (3).

Eq. (5) is readily solved for a polytropic or an isothermal atmosphere, as it can be integrated over the redshift to yield

\[
\int \frac{d\rho}{\rho c^2} = \ln(1 + z) - \lambda z.
\]

For an ideal gas equation of state, \( p = \rho k_B T / (\mu m_p) \) with \( k_B \) the Boltzmann constant, \( m_p \) the proton mass, and \( \mu = 1/2 \) the mean molecular weight, we obtain the following analytic solutions.

---

1 The optically thick case will be treated in a separate paper (Wielgus et al. 2015).

Figure 1. Density profiles for the isothermal solutions with \( T = 10^7 \) K. Each curve corresponds to a different value of the luminosity parameter: \( \lambda = 0.8, 0.85, 0.9, 0.95, 0.97, 0.99 \) from left to right (as labeled at the maxima).

---
3 ISOTHERMAL ATMOSPHERE

For an isothermal atmosphere at temperature $T$, the integral in Eq. (9) yields

$$\frac{k_B T}{\mu m_p c^2} \ln \left( \frac{\rho}{\rho_0} \right) = \ln(1 + z) - \lambda z. \quad (10)$$

with $z$ defined in Eq. (3). In the GR case, the maximum value of density is attained at the ECS, assuming a luminosity in the range given by Eq. (3), and the density can be taken to be proportional to this maximum value $\rho_{\text{max}} \equiv \rho(\pm \text{ECS})$,

$$\rho(z) = \rho_{\text{max}} \times \left[ \lambda (1 + z) \right]^{(\mu m_p c^2)/(k_B T)} \times \exp \left\{ \left[ 1 - \lambda (1 + z) \right] \left( \mu m_p c^2 \right) / (k_B T) \right\}. \quad (11)$$

Thus, as expected, the density decreases in both directions away from the ECS, giving the atmosphere a shell-like form.

The same result, given as a function of the radius, is

$$\rho(r) = \rho_{\text{max}} \times \left[ \lambda (1 - R_{\text{ECS}}/r)^{-1/2} \right]^{(\mu m_p c^2)/(k_B T)} \times \exp \left\{ \left[ 1 - \lambda (1 - R_{\text{ECS}}/r)^{-1/2} \right] \left( \mu m_p c^2 \right) / (k_B T) \right\}. \quad (12)$$

However, far from the star this solution is unphysical, as the mass in an isothermal atmosphere would be infinite—note that at $z = 0$, i.e., as $r \to \infty$, the density tends to a finite value, $\rho \to \rho_{\infty} \equiv \rho_{\text{max}} \times \left[ \lambda \exp(1 - \lambda) \right]^{(\mu m_p c^2)/(k_B T)}$.

The Newtonian Eddington limit is recovered with a different normalization from the one in Eqs. (11)-(12). To first order in $z$, from any of the three Eqs. (10)-(12) one gets, $\rho(r) = \rho_{\infty} \times \exp\left\{ \mu m_p G M (1 - \lambda) / (k_B T) r \right\}$. The classical Eddington limit, $dp/dr = 0$, or $\rho = \text{const}$ for an isothermal atmosphere, is now obtained for $\lambda = 1$, while the Newtonian limit of an exponentially decaying atmosphere holds for $|r - r_0| \ll r_0$

$$\rho(r) \approx \rho_0 \exp \left\{ - \frac{\mu m_p G M (1 - \lambda)}{k_B T r_0} (r - r_0) \right\}, \quad (13)$$

with arbitrary constants $\rho_0$ and $r_0$ of dimension density and length, respectively.

Although we have assumed Thomson scattering, this is not strictly necessary. In fact, all the above considerations in Section 3 as well as Eq. (3), remain valid if we replace the Thomson scattering cross section by a more general cross-section $\sigma(T)$, e.g., the Klein-Nishina formula, provided that we make the substitution $\lambda \rightarrow \lambda_1 = \lambda a_1$, with $a_1 = \sigma(T)/\sigma_T$.

We stress once again that unlike the decaying atmosphere of the Newtonian limit in Eq. (11), the GR solution in Eqs. (10)-(12) is not monotonically decreasing with radius everywhere, but instead is monotonically increasing for $r < R_{\text{ECS}}$ up to the maximum on the ECS, and is only monotonically decreasing for $r > R_{\text{ECS}}$, as illustrated in Fig. 1.

4 POLYTROPIC ATMOSPHERE

For a polytrope, $p = K \rho^\Gamma$, the specific enthalpy $w = (p/\rho) \Gamma / (\Gamma - 1)$ following from Eq. (8) or Eq. (9) is

$$w(z) = w_{\text{ECS}} + c^2 \left\{ \ln \left[ \lambda (1 + z) \right] - \lambda (1 + z) + 1 \right\} \quad (14)$$

and it attains its maximum value, $w_{\text{ECS}} = w(z_{\text{ECS}})$, at $r = R_{\text{ECS}}$, see Eq. (7).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2}
\caption{Temperature profiles for the polytropic solutions. Each curve corresponds to a different value of the luminosity parameter: $\lambda = 0.8, 0.85, 0.9, 0.95, 0.97, 0.99$ from left to right. Note that the temperature difference on the y axis is in units of 10$^8$ K.}
\end{figure}

Making use of the ideal gas equation of state we obtain a corresponding solution for the temperature

$$T(r) - T_{\text{max}} = \left( \frac{\Gamma - 1}{\Gamma} \right) \left[ \frac{\lambda}{(1 - R_{\text{ECS}}/r)^{\Gamma/2}} - \ln \left( \frac{\lambda}{(1 - R_{\text{ECS}}/r)^{\Gamma/2}} \right) - 1 \right], \quad (15)$$

also attaining its maximum value on the ECS. Note that for any given value of the luminosity, $\lambda$, the temperature falls off on both sides of the ECS in a universal way (Fig. 2) independently of the value of the maximum temperature $T_{\text{max}}$, which is just an additive constant in Eq. (15). Of course the temperature cannot go to negative values, so the plot must be cut off at $T - T_{\text{max}} = -T_{\text{max}}$. Fig. 2 corresponds to $T_{\text{max}} = 10^9$ K, i.e., a hot $\sim 100$ keV corona.

The density scales with its arbitrary value at the ECS, $\rho_{\text{max}} \equiv \rho(\pm \text{ECS})$, which is proportional to the total mass of the atmosphere:

$$\rho(r) = \rho_0 \left[ \ln(1 - R_{\text{ECS}}/r)^{-1/2} - \lambda (1 - R_{\text{ECS}}/r)^{-1/2} + \lambda \right]^{1/\Gamma}, \quad (16)$$

with $\rho(\pm \text{ECS}) = \rho_0 [\lambda - \ln(1 - 1)^{1/\Gamma(1)}$.

This being a polytrope, the atmosphere has a sharp edge, the enthalpy, temperature and density vanishing at the top of the atmosphere. In fact, when the condition of Eq. (10) is satisfied, there are two “tops” of the atmosphere (Fig. 3), one at $r_+ > R_{\text{ECS}}$, with $z_+ \equiv z(r_+) < z_{\text{ECS}}$, and one at $r_- < R_{\text{ECS}}$, with $z_- \equiv z(r_-) > z_{\text{ECS}}$. The redshift at the top of the atmosphere is given by the condition

$$\ln \left[ \lambda (1 + z_{\pm}) \right] - \lambda (1 + z_{\pm}) + 1 = -w_{\text{ECS}}/c^2. \quad (17)$$

For maximum temperatures not much larger than a few keV, the atmospheric profile is quite symmetric with respect to the sign of $r - R_{\text{ECS}}$ (Fig. 3), and the height of the atmosphere is proportional to the speed of sound at its “base,” $c_s = \sqrt{(\Gamma - 1) w_{\text{ECS}}}$. Indeed, for $H \equiv |r_\pm - R_{\text{ECS}}| \ll R_{\text{ECS}},$

$$\frac{H}{R_{\text{ECS}}} = \sqrt{\frac{2}{\Gamma - 1}} \frac{c_s c}{v_K}, \quad (18)$$

where $v_K = \sqrt{GM/R_{\text{ECS}}}$ is the Keplerian orbital velocity at the ECS. This is reminiscent of the result for accretion
disks, where $H/r \approx c_s/v_K$ (Shakura & Sunyaev 1973), however the ECS atmospheres are more extended, $H$ here being enhanced by a factor of $c/v_K$. In terms of the temperature at the base of the atmospheric shell, this reads

$$\frac{H}{R_{\text{ECS}}} = \frac{2\lambda^2}{1 - \lambda^2} \left( \frac{2\Gamma}{\Gamma - 1} \right)^{1/2} \left( \frac{k_B T_{\text{max}} \mu}{\mu m_p c^2} \right)^{1/2}. \tag{19}$$

For $k_B T_{\text{max}} = 2\text{keV}$, and $L_{\infty} \approx L_{\text{Edd}}$ the shell thickness works out to be $H \approx 2 \cdot 10^{-3}(1 - \lambda)^{-2} R_s$. For a neutron star this gives $H \approx 10(1 - \lambda)^{-2}\text{m}$, i.e., $H = 1\text{km}$ for $\lambda = 0.9$, and $H = 4\text{km}$ for $\lambda = 0.95$. One can see that as the luminosity increases, the thickness of the levitating atmospheric shell increases more quickly than the radius of the ECS.

5 DISCUSSION

We have shown that luminous compact stars (e.g., neutron stars) in Einstein’s general relativity may have atmospheres which are detached from the surface. This may have applications to astrophysical phenomena of those compact objects in which the luminosity is close to its Eddington value, either on a quasi-permanent basis or in a transient state. The former case includes highly luminous neutron stars accreting matter from low-mass stellar companions, such as the Z-sources (which include the brightest X-ray source in the sky, Sco X-1). One would expect quasi-spherical coronae in such sources to be described by our solutions with implications for the hard X-ray spectra in these sources, and possibly also for their quasi-periodic modulations. The latter include X-ray bursts, some of which exhibit the phenomenon of radius expansion (e.g., Strohmayer & Bildsten 2000). However, here we have only considered optically thin atmospheric shells, whereas the atmospheres of X-ray bursters are optically thick. For this reason we are postponing a detailed discussion of bursters to another paper, where optically thick solutions will be reported (Wielgus et al. 2013).

The atmospheres presented here can be thought of as an (extreme) example of density and pressure inversion. Atmospheric density inversion owing to variations of opacity has been discussed previously in various contexts. In stellar structure theory discussion of density inversion goes back to Chandrasekhar (1936), see also Sen (1941). In contemporary papers such layers are often invoked in the context of wind loss and stellar pulsations (e.g., Townsend & MacDonald 2006).

The possibility of gas pressure inversion was pointed out by Erika Böhm-Vitense, and explained as a result of a change of sign in the effective gravitational acceleration, $g_{\text{eff}}$, with $g_{\text{eff}}$ being the difference between the true acceleration of gravity and the acceleration owing to radiative forces, which are proportional to the product of opacity and the radiative flux (Böhm-Vitense 1958). This is indeed the cause of the pressure inversion in the atmospheres presented in our paper. However, there are several important differences between the results obtained previously and those reported in this work.

First, the inversion layers familiar from stellar structure theory are caused by variations of opacity, while our results are of a different origin—the (Thomson) opacity in our solutions is uniform, and the density (and pressure) inversion arises solely owing to effects of general relativity. More generally, its presence is related to purely geometrical effects that allow the radiative flux to have a different functional dependence on the distance from the source than the acceleration of gravity.

Second, we are presenting optically thin atmospheres, whereas the stellar discussion occurred for optical depths greater than unity.

Third, in the optically thick case considered by Joss et al. (1973) density inversion occurs in regions where the luminosity is near-Eddington (although still subcritical), the pressure is dominated by radiation, and the temperature gradient is superadiabatic. While the atmospheres discussed in our paper do require near-Eddington luminosities, none of the other conditions is met. Density inversion occurs only in the region of supercritical luminosity, the radiation pressure is zero, as the radiation freely streams through the optically

\footnote{In LBV and hypergiant stars “the most striking proper[y] is the strong density inversion in the outer layers, where a thin gaseous layer floats upon a radiatively supported zone. This is due to the peak in the opacity which forces supra-Eddington luminosities in some layers.” (Chiosi 1998)}
thin gas, and the temperature gradient is subadiabatic for the isothermal atmosphere, of course, as well as for polytropic atmospheres with $\Gamma < 5/3$. Gas-pressure inversion occurs at supercritical luminosities. The region of pressure inversion coincides with that of density inversion in the optically thin atmospheres presented here.

Fourth, Joss et al. (1973), in their discussion of optically thick inversion layers in hydrostatic equilibrium, referring to the work of Lucy & Solomon (1970) observe that their own “results do not alter the well known conclusion that mass-outflow from the surface must result if $L > L_{\text{crit}}$, where $L$ and $L_{\text{crit}}$ are the photospheric values” of luminosity and its critical value. We note that our atmospheric solutions do “alter the well known conclusion,” as the levitating atmospheric shell is in hydrostatic equilibrium even though $L > L_{\text{Edd}}$ in (a part of) the optically thin shell, i.e., the luminosity is supercritical above the photosphere of the underlying luminous star. This is possible because Lucy & Solomon (1970) used Newtonian gravity while our solutions are valid in GR.

We conclude with a brief discussion of stability. The atmospheres presented in this paper are convectively stable, as shown by the appropriate Schwarzschild criterion, in the isothermal case and for polytropes with $\Gamma < 5/3$. This result can be intuitively understood by realizing that by the law of Archimedes a hot parcel of gas is initially accelerated in the direction opposite to the gradient of pressure, hence the pressure-inversion layer is as stable as the layer of radially decreasing pressure. A polytropic atmosphere with $\Gamma = 5/3$ is marginally stable to convection. The atmospheres are also Rayleigh-Taylor stable as the denser layers are always “below” (in the sense of the direction of effective gravity) the less dense layers. Investigation of the normal modes of the atmospheres will be presented elsewhere. Here, we only note that (as shown by Abarca & Kluzniak 2015) the radiation drag efficiently damps oscillatory motion in a class of fundamental modes, just as it does the radial and azimuthal motion of test particles (Stahl et al. 2012).

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