1. INTRODUCTION

M33 X-7 is the first stellar-mass black hole discovered to be eclipsed by its companion (Pietsch et al. 2006). The X-ray eclipse and the precisely known distance of this system, $D = 840 \pm 20$ kpc, underpin the most accurate dynamical model that has been achieved for any of the 21 known black hole binaries (Orosz et al. 2007, hereafter O07). The two dynamical parameters of interest in this Letter are the black hole mass $M = 15.65 \pm 1.45 M_\odot$ and the orbital inclination angle $i = 74.6^\circ \pm 1.0^\circ$ (O07).

Our group has published spin estimates for three stellar-mass black holes using the X-ray continuum fitting method: GRO J1655−40, $a_* = 0.65–0.75$; 4U 1543−47, $a_* = 0.75–0.85$; and GRS 1915+105, $a_* = 0.98–1.0$ (Shafee et al. 2006; McClintock et al. 2006, hereafter M06). For LMC X-3, Davis et al. (2006) find $a_* < 0.26$. Meanwhile, the Fe line method has been used to obtain two additional estimates of black hole spin (Brenneman & Reynolds 2006; Miller et al. 2008). The dimensionless spin parameter $a_* \equiv a/M = cJ/GM^2$, where $M$ and $J$ are the mass and angular momentum of the black hole; $-1 \leq a_* \leq 1$ (Shapiro & Teukolsky 1986).

The continuum-fitting method, which was pioneered by Zhang et al. (1997; also, see Gierliński et al. 2001), is based on the existence of an innermost stable circular orbit (ISCO) for a particle orbiting a black hole, inside which the particle suddenly plunges into the hole. In the continuum-fitting method, one identifies the inner edge of the black hole’s accretion disk with the ISCO and estimates the radius $R_{\text{isco}}$ of this orbit by fitting the X-ray continuum spectrum. Since the dimensionless radius $r_{\text{isco}} \equiv R_{\text{isco}}/(GM/c^2)$ is solely a monotonic function of the black hole spin parameter (Shapiro & Teukolsky 1986), knowing its value allows one to immediately infer the black hole spin parameter $a_*$. Our estimates of spin are based on our fully relativistic accretion disk model (Li et al. 2005) and an advanced treatment of spectral hardening (Davis et al. 2005). We consider only rigorously selected thermal-state data (Remillard & McClintock 2006), which are largely free of the effects of Comptonization. Furthermore, we only accept data for which the bolometric disk luminosity is moderate, $L/L_{\text{Edd}} \lesssim 0.3$, in order to ensure that the standard geometrically thin thermal disk model is applicable (Shafee et al. 2008; M06).

For the continuum-fitting method to succeed, it is essential to have accurate values of the black hole mass, orbital inclination, and distance (M06), quantities that are known precisely in the case of M33 X-7. Other virtues of M33 X-7 for the determination of spin are the abundance of Chandra and XMM data, the remarkably thermal and featureless spectrum of the X-ray source, and its moderate luminosity (§3).

2. DATA SELECTION AND REDUCTION

There have been 17 Chandra ACIS observations and 12 XMM-Newton EPIC observations of M33 X-7. We analyzed Chandra observations (downloaded from the Chandra Data Archive) with CIAO 3.4 (Fruscione et al. 2006) and extracted the spectra from source ellipses enclosing 95% of the source photons as reported by wavdetect. The XMM observations were downloaded from the HEASARC archive and analyzed with SAS 7.0.0 (Gabriel et al. 2004). Using the standard procedures and excluding intervals of high background, we extracted separate spectra from the PN and two MOS chips using radii of 400 pixels (i.e., $20^\prime\prime$) and fitted them independently.

The resultant count rate data were folded on the M33 X-7 X-ray eclipse ephemeris (Pietsch et al. 2006): HJD (2,453,639.119 $\pm$ 0.005) $\pm$ N(3,453,014 $\pm$ 0.00020). The folded light curve for all the Chandra data are shown in Figure 2a in O07. For the purpose of measuring the spin of M33 X-7, we excluded (1) spectra obtained in the phase range $-0.3$ to $0.2$, i.e., during the eclipse or the pre-eclipse period of erratic X-ray variability (which is presumably caused by the accretion stream; O07); (2) four ACIS spectra (ObsIDs 6384, 6385, 7170, 7171) that are severely affected by pileup; and (3) all spectra that contain less than 1000 counts. The 15 spectra so selected...
are listed in Table 1 and comprise eight *Chandra* ACIS spectra and seven EPIC PN/MOS spectra from five XMM observations. We refer throughout to the four ACIS spectra with \( \geq 5000 \) counts as the "gold" spectra and to the rest as the "silver" spectra.

### 3. ANALYSIS AND RESULTS

The procedures used here are precisely the same as those that are described fully in M06. Briefly, the relativistic accretion disk model *kerrbb2* has just two fit parameters, namely the black hole spin \( a_* \) and the mass accretion rate \( \dot{M} \) (or equivalently, \( a_* \) and the Eddington-scaled bolometric luminosity, \( L_{\text{bol}}(a_*, \dot{M})/L_{\text{Edd}} ; \text{M06} \)). In the case of M33 X-7, we also fit for a third parameter, \( N_{\text{H}} \), the hydrogen column density (phabs in XSPEC).

The spectral hardening factor \( f = T_{\text{obs}}/T_{\text{eff}} \) was computed as a function of \( l \) for the appropriate metallicity of M33 X-7 (\( Z = 0.1 \ Z_{\odot} ; \text{O07} \)) using the model of Davis & Hubeny (2006; bhspec in XSPEC). These values of \( f \) are contained in a pair of lookup tables, which correspond to two representative values of the viscosity parameter \( (\alpha = 0.01, 0.1 ; \text{M06}) \) for a wide range of the spin parameter (e.g., \( 0 < a_* < 0.99 \)). We (J. Liu et al. 2008, in preparation) find that our results are quite insensitive to the choice of \( \alpha \) or an increase in metallicity. We have also experimented with varying the input parameters \( M, D, i \), and \( N_\text{H} \), and we find that the values of \( f \) are scarcely affected.

All of the spectra were well fitted using a simple absorbed *kerrbb2* model [i.e., phabs (*kerrbb2*) in XSPEC]. Notably, neither Fe line/edge components nor an additional non-thermal component were required, as they were in our earlier work (Remillard & McClintock 2006; M06). We fitted each spectrum for \( a_* \), the mass accretion rate \( \dot{M} \), and the neutral hydrogen column density \( N_{\text{H}} \) with the input parameters fixed at their baseline values (see § 1; O07). The normalization was fixed at unity (as appropriate when \( M, i, D \)) and are held fixed). We included the effects of limb darkening ([flag = 1] and returning radiation effects ([flag = 1], and we set the torque at the inner boundary of the accretion disk to zero (\( \eta = 0 \)).

The fits obtained for all 15 spectra are quite acceptable with \( \chi^2 < 1.2 \); results for fits over the energy range 0.3–8 keV are summarized in Table 1. An inspection of the fitting residuals for all the spectra show them to be free of any systematic effects, as illustrated in Figure 1. Given the modest luminosities, \( 0.07 < l < 0.11 \) (Table 1), which are well below our selection limit of \( l = 0.3 \), the accretion disk in M33 X-7 is quite thin, \( H/R \leq 0.04 \) (see Fig. 17 in Shafee et al. 2008), and our assumption of zero torque at the inner boundary is likely to be valid.

Figure 2 shows plots of \( a_* \) for all 15 observations, which are ordered by the number of counts detected. Each of the four panels corresponds to a different choice for the energy interval used in fitting the data (e.g., 0.3–8 keV, 0.5–8 keV, etc.; a comparison of the results in the four panels shows that this choice is quite unimportant. The four gold spectra with \( \geq 5000 \) counts each (filled symbols) yield spin estimates that agree with their mean value (indicated by the dotted lines) typically to within their \( \approx 2\% \) statistical uncertainties. The stability of these four gold spectra is especially remarkable given that three of

### TABLE 1

**kerrbb2 Fit Results for M33 X-7 in 0.3–8 keV**

<table>
<thead>
<tr>
<th>Spectrum</th>
<th>Obs. Date</th>
<th>( T_{\text{inv}} )</th>
<th>Counts</th>
<th>( a_* )</th>
<th>( \dot{M} )</th>
<th>( N_\text{H} )</th>
<th>( f_{\text{fit}} )</th>
<th>\log ( l )</th>
<th>( \chi^2/\text{dof} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>acis6376</td>
<td>2006 Mar 03</td>
<td>93.1</td>
<td>9748</td>
<td>0.751 ± 0.026</td>
<td>1.88 ± 0.12</td>
<td>11.1 ± 1.1</td>
<td>1.78</td>
<td>−1.01</td>
<td>0.107/180</td>
</tr>
<tr>
<td>acis6387</td>
<td>2006 Jun 26</td>
<td>77.3</td>
<td>7271</td>
<td>0.782 ± 0.019</td>
<td>1.64 ± 0.10</td>
<td>11.4 ± 1.2</td>
<td>1.76</td>
<td>−1.05</td>
<td>0.93/157</td>
</tr>
<tr>
<td>acis6382</td>
<td>2005 Nov 23</td>
<td>72.3</td>
<td>6515</td>
<td>0.772 ± 0.030</td>
<td>1.72 ± 0.14</td>
<td>9.9 ± 1.4</td>
<td>1.78</td>
<td>−1.04</td>
<td>1.17/152</td>
</tr>
<tr>
<td>acis1730</td>
<td>2000 Jul 12</td>
<td>49.5</td>
<td>4855</td>
<td>0.800 ± 0.026</td>
<td>1.37 ± 0.11</td>
<td>6.1 ± 1.3</td>
<td>1.77</td>
<td>−1.12</td>
<td>1.15/126</td>
</tr>
<tr>
<td>acis7344</td>
<td>2006 Jul 01</td>
<td>21.5</td>
<td>1711</td>
<td>0.873 ± 0.031</td>
<td>1.05 ± 0.14</td>
<td>8.6 ± 2.8</td>
<td>1.75</td>
<td>−1.16</td>
<td>0.69/55</td>
</tr>
<tr>
<td>acis6386</td>
<td>2005 Oct 31</td>
<td>14.9</td>
<td>1491</td>
<td>0.786 ± 0.041</td>
<td>1.55 ± 0.21</td>
<td>12.8 ± 3.2</td>
<td>1.77</td>
<td>−1.07</td>
<td>0.95/49</td>
</tr>
<tr>
<td>acis7197</td>
<td>2005 Nov 03</td>
<td>12.7</td>
<td>1117</td>
<td>0.892 ± 0.043</td>
<td>0.97 ± 0.20</td>
<td>9.1 ± 4.1</td>
<td>1.75</td>
<td>−1.17</td>
<td>0.99/37</td>
</tr>
<tr>
<td>acis7208</td>
<td>2005 Nov 21</td>
<td>11.5</td>
<td>1014</td>
<td>0.678 ± 0.110</td>
<td>1.73 ± 0.39</td>
<td>13.9 ± 4.6</td>
<td>1.78</td>
<td>−1.10</td>
<td>0.81/33</td>
</tr>
<tr>
<td>PN01002642301</td>
<td>2002 Jan 27</td>
<td>10.0</td>
<td>2724</td>
<td>0.852 ± 0.031</td>
<td>1.34 ± 0.14</td>
<td>7.3 ± 1.0</td>
<td>1.75</td>
<td>−1.09</td>
<td>0.84/103</td>
</tr>
<tr>
<td>PN01002642101</td>
<td>2000 Aug 02</td>
<td>10.3</td>
<td>1836</td>
<td>0.618 ± 0.056</td>
<td>2.51 ± 0.29</td>
<td>12.0 ± 1.5</td>
<td>1.78</td>
<td>−0.97</td>
<td>0.87/69</td>
</tr>
<tr>
<td>PN0141980801</td>
<td>2003 Feb 12</td>
<td>8.4</td>
<td>1596</td>
<td>0.657 ± 0.074</td>
<td>2.50 ± 0.36</td>
<td>10.7 ± 1.5</td>
<td>1.78</td>
<td>−0.96</td>
<td>0.90/60</td>
</tr>
<tr>
<td>PN0141980601</td>
<td>2003 Jan 23</td>
<td>11.6</td>
<td>1545</td>
<td>0.656 ± 0.077</td>
<td>2.21 ± 0.32</td>
<td>12.5 ± 1.6</td>
<td>1.77</td>
<td>−1.00</td>
<td>1.00/59</td>
</tr>
<tr>
<td>M10106242301</td>
<td>2002 Jan 27</td>
<td>12.3</td>
<td>1199</td>
<td>0.841 ± 0.042</td>
<td>1.30 ± 0.20</td>
<td>6.7 ± 2.1</td>
<td>1.75</td>
<td>−1.10</td>
<td>0.89/40</td>
</tr>
<tr>
<td>PN01002640401</td>
<td>2000 Aug 02</td>
<td>9.2</td>
<td>1136</td>
<td>0.838 ± 0.039</td>
<td>1.42 ± 0.20</td>
<td>6.8 ± 1.8</td>
<td>1.75</td>
<td>−1.06</td>
<td>1.15/44</td>
</tr>
<tr>
<td>M20106242302</td>
<td>2002 Jan 27</td>
<td>12.3</td>
<td>1135</td>
<td>0.839 ± 0.043</td>
<td>1.25 ± 0.20</td>
<td>7.2 ± 2.2</td>
<td>1.75</td>
<td>−1.12</td>
<td>0.83/39</td>
</tr>
</tbody>
</table>


### Fig. 1

- **Top:** This spectrum (ObsID 6376) is representative of the four gold spectra (see text). The histogram shows a model that has been fitted to the spectrum (0.3–8.0 keV), which is composed of only the thermal disk component (*kerrbb2*) and a low-energy absorption model (phabs). The fit parameters are summarized in Table 1.
- **Bottom:** The fit is good (\( \chi^2/\text{dof} = 1.074/180 \)) and the fit residuals show no systematic structure; in particular, there is no evidence for an Fe line, absorption edges, or a non-thermal power-law/Comptonization component of emission at higher energies.

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the observations were separated by 3 month intervals in 2005–2006, and one of them was obtained 5 years earlier in 2000 (Table 1). The dispersion for the 11 silver spectra has about 3000 counts (open symbols) is much larger. However, in each panel, the mean of these 11 spin values agrees with the mean determined using the gold spectra to within ±1%. As concluded in the caption of Figure 2, our adopted average spin for the four gold spectra is \( \bar{a}_g = 0.776 \pm 0.018 \). The average for the 11 silver spectra is almost identical, although the dispersion is much greater, \( \bar{a}_s = 0.772 \pm 0.098 \). (b–d) Same as (a) except the fit interval is as indicated rather than 0.3–8 keV. [See the electronic edition of the Journal for a color version of this figure.]

In order to determine the error in \( a_\ast \) due to the combined uncertainties in \( M, i, \) and \( D \) (§ 1), we performed Monte Carlo simulations assuming that the uncertainties in these parameters are normally and independently distributed. The results for 3000 simulation runs are plotted in Figure 3. The histogram of \( \bar{a}_\ast - \bar{a}_0 \) shows that the 1σ error in the spin due to the combined uncertainties of the three input parameters is about 0.05. The error is dominated by the uncertainty in \( M \); the uncertainties in \( i \) and \( D \) are relatively unimportant. This error is based on a readily available table that was computed for solar metallicity. Despite this limitation, we believe that our error estimate is accurate because the effects of going from \( Z = 0.1 \) to \( Z_{\odot} \) are very small at the luminosities in question, \( l \approx 0.1 \) (J. Liu et al. 2008, in preparation).

4. DISCUSSION

The largest error in our spin estimate arises from the uncertainties in the validity of the disk model we employ. For example, the spin depends on accurate model determinations of the hardening factor \( f \); this problem is quite tractable and vigorous theoretical efforts are underway (Davis et al. 2005, 2006; Blaes et al. 2006). Possibly more problematic is our assumption that the viscous torque vanishes at the ISCO and that there is no significant emission from the gas inside the ISCO.

Hydrodynamic models of the accretion disk indicate that the viscous torque at the ISCO as well as emission from inside the ISCO should both be negligible for the geometrically thin disks and low luminosities (\( l \leq 0.3 \)) that we restrict ourselves to (Afshordi & Paczyński 2003; Shafee et al. 2008). The emission from inside the ISCO causes rather modest errors in spin estimates; in the case of M33 X-7, the estimated error is \( \Delta a_\ast \leq 0.01 \) since \( l \leq 0.1 \) and hence \( R/H \leq 0.04 \) (M06; Shafee et al. 2008). On the other hand, MHD simulations of accretion flows around black holes (Hawley & Krolik 2002; Beckwith et al. 2008) find a large torque at the ISCO and substantial dissipation inside the ISCO. We note, however, that these simulations carried out so far are for geometrically thick systems, with \( H/R \sim 0.2 \); these flows are nearly an order of magnitude thicker than the disk in M33 X-7. In the hydrodynamic models of Shafee et al. (2008) the stress at the ISCO increases rapidly with increasing disk thickness, so it is conceivable that there is no serious disagreement between the hydrodynamic and MHD results. Numerical MHD simulations of truly thin disks are necessary to resolve this issue. We note that a recent MHD simulation of a geometrically thin accretion disk for a pseudo-Newtonian potential does show a dramatic drop in the midplane density and vertical column density over a narrow range of radii close to the ISCO (Reynolds & Fabian 2008).

Although there is theoretical uncertainty about conditions near the ISCO, there is a long history of evidence suggesting that filling the X-ray continuum is a promising approach to measuring black hole spin. This history begins in the mid-
1980s with the simple nonrelativistic multicolor disk model (Mitsuda et al. 1984), which returns the color temperature $T_m$ at the inner-disk radius $R_m$. Tanaka & Lewin (1995) summarize examples of the steady decay (by factors of 10–100) of the thermal flux of transient sources during which $R_m$ remains quite constant (see their Fig. 3.14). More recently, this evidence for a constant inner radius in the thermal state has been presented for a number of sources in several papers via plots showing that the bolometric luminosity of the thermal component is approximately proportional to $T^4$ (McClintock et al. 2008 and references therein). Obviously, this nonrelativistic analysis cannot provide a secure value for the radius of the ISCO nor even establish that this stable radius is the ISCO. Nevertheless, the presence of a fixed radius indicates that the continuum-fitting method is a well-founded approach to measuring black hole spin.

It is reasonable to assume that the inner X-ray-emitting portion of the disk is aligned with the spin axis of the black hole by the Bardeen-Petterson effect (Lodato & Pringle 2007). Throughout, in making use of the orbital inclination angle, we have assumed that the black hole spin is aligned with the angular momentum vector of the binary system. As Figure 3 indicates, if any misalignment is $\lesssim 3^\circ$, then it will contribute an error in $a_*$ that is no larger than our total observational error of $\Delta a_* = 0.05$. There is no evidence for significant misalignments despite the often-cited examples of GRO J1655-40 and SAX J1819.3-2525 (see § 2.2 in Narayan & McClintock 2005; but see Maccarone 2002). The clear-cut way to assess the degree of alignment is via X-ray polarimetric observations of black hole systems in the thermal state (L.-X. Li et al. 2008, in preparation).

What is the origin of the spin of M33 X-7? Was the black hole born with its present spin, or was it torqued up gradually via the accretion flow supplied by its companion? In order to achieve a spin of $a_* = 0.77$ via disk accretion, an initially nonspinning black hole must accrete 4.9 $M_\odot$ from its donor (King & Kolb 1999) in becoming the that we observe today (O07). However, to transfer this much mass even in the case of Eddington-limited accretion ($M_{\text{tidal}} \equiv L_{\text{tidal}}/c^2 \approx 4 \times 10^{-8} M_\odot$ yr$^{-1}$) requires $\sim 120$ million years, whereas the age of the system is only 2–3 million years (O07). Thus, it appears that the spin of M33 X-7 must be natal, which is the same conclusion that has been reached for two other stellar black holes (Shafee et al. 2006; M06; but see Bethe et al. 2003 on the possibility of hypercritical accretion).

M33 X-7’s secure dynamical data and distance, the X-ray source’s clean thermal-state spectrum and moderate luminosity, and an abundance of Chandra and XMM data have provided arguably the most secure estimate of black hole spin that has been achieved to date: $a_* = 0.77 \pm 0.05$, where the error estimate includes all sources of observational error. Since an astrophysical black hole can be described by just the two parameters that specify its mass and spin (Shapiro & Teukolsky 1986), we now have a complete description of an asteroid-size object that is situated at a distance of about 1 Mpc.

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REFERENCES


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In the published version of the Letter, the reported value of the black hole spin parameter \( a_* = 0.77 \pm 0.05 \) has an error. The correct value is larger by 0.068 and is \( a_* = 0.84 \pm 0.05 \). The error is the result of a bug in the XSPEC accretion-disk model \( \text{kerrbb} \).¹ Prior to 2008 December 1, the model’s two parameter flags that switch limb darkening and self-irradiation of the disk on/off were reversed (e.g., “par8” incorrectly controlled limb darkening rather than self-irradiation). In computing tables of the spectral hardening factor \( f \), we use both \( \text{kerrbb} \) and the disk atmosphere model \( \text{bhspec} \) (McClintock et al. 2006). Because the latter model does not include the effect of self-irradiation, we switch this feature off in \( \text{kerrbb} \) when computing the \( f \)-tables. In this instance, because of the bug we switched off limb darkening instead of self-irradiation, which corrupted our results. Meanwhile, our earlier spin results for GRS 1915+105 (McClintock et al. 2006) and for 4U 1543−47 and GRO J1655−40 (Shafee et al. 2006) are unaffected by the bug.

The figures and tabular data in the Letter are essentially unaffected, apart from the increase in \( a_* \) and corresponding decreases in \( f \) and the Eddington-scaled luminosity \( L \) (8.7% and 4.5%, respectively, for the four gold spectra). The higher spin somewhat increases our estimate of how much mass (4.9 \( M_\odot \)) and time (∼120 million years) would be required to spin up an initially nonspinning black hole to the present spin of M33 X-7. In order to achieve \( a_* = 0.84 \), the black hole must accrete 5.7 \( M_\odot \), which would require ∼140 million years. Because the age of the binary system is only 2–3 million years, this change does not at all affect our conclusion that the spin of M33 X-7 is natal.

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¹ http://heasarc.gsfc.nasa.gov/docs/xanadu/xspec/issues/archive/issues.12.5.0an.html (patch 12.5.0a).