Preparation and Detection of Magnetic Quantum Phases in Optical Superlattices

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(Received 11 April 2007; revised manuscript received 31 July 2007; published 5 October 2007)

We describe a novel approach to prepare, detect, and characterize magnetic quantum phases in ultracold spinor atoms loaded in optical superlattices. Our technique makes use of singlet-triplet spin manipulations in an array of isolated double-well potentials in analogy to recently demonstrated control in quantum dots. We also discuss the many-body singlet-triplet spin dynamics arising from coherent coupling between nearest neighbor double wells and derive an effective description for such systems. We use it to study the generation of complex magnetic states by adiabatic and nonequilibrium dynamics.

DOI: 10.1103/PhysRevLett.99.140601

PACS numbers: 05.50.+q, 03.67.Mn, 05.30.Fk, 05.30.Jp

Recent advances in the manipulations of ultracold atoms in optical lattices have opened new possibilities for exploring many-body systems [1]. A particular topic of continuous interest is the study of quantum magnetism in spin systems [2–4]. By loading spinor atoms in optical lattices it is now possible to “simulate” spin models in controlled environments and to explore novel spin orders.

In this Letter we describe a new approach for preparation and probing of many-body magnetic quantum states that makes use of coherent manipulation of singlet-triplet pairs of ultracold atoms loaded in deep period-two optical superlattices. Our approach makes use of a spin dependent energy offset between the double-well minima to completely control and measure the spin state of two-atom pairs, in a way analogous to the recently demonstrated manipulations of coupled electrons in quantum double dots [5]. As an example, we show how this technique allows one to detect and analyze antiferromagnetic spin states prepared via equilibrium and nonequilibrium dynamics.

In the following we assume that the atoms are strongly interacting, $U \gg J$, and that effective vibrational energy of each well $\hbar \omega_0$ is the largest energy scale in the system $\hbar \omega_0 \gg U, \Delta, J$, i.e., deep wells.

Singlet $|s\rangle$ and triplet $|t\rangle$ states form the natural basis for the two-atom system. The relative energies of these states can be manipulated by controlling the energy bias $\Delta$ between the two wells. In the unbiased case ($U \gg 2\Delta$) the lowest energy state, when $\Delta = U \gg 2\Delta$.

(0,2)s ± (2,0)s

(a)

(1,1)t

(1,1)s, (1,1)t

U

(b)

(c)

FIG. 1 (color online). (a) Energy levels of fermionic atoms in a spin independent double well as $\Delta/U$ is varied. While in the regime $2\Delta \ll U$, $(1,1)|s\rangle$ is the lowest energy state, when $2\Delta \approx U$, $(0,2)|s\rangle$ becomes the state with lowest energy. (b) In spin dependent potentials the two species feel different lattice parameters. (c) Restricted to the $(1,1)$ subspace $Y$ acts as an effective magnetic field gradient and couples $|s\rangle$ and $|t\rangle$. 

The key idea of this work is illustrated by considering a pair of ultracold atoms with two relevant internal states, which we identify with spin up and down $\sigma = \uparrow, \downarrow$ in an isolated double-well (DW) potential as shown in Fig. 1. By dynamically changing the optical lattice parameters, it is possible to completely control this system and measure it in an arbitrary two-spin basis. For concreteness, we first focus on the fermionic case. The physics of this system is governed by three sets of energy scales: (i) the on site interaction energy $U = U_{\uparrow\downarrow}$ between the atoms, (ii) the tunneling energy of the $\sigma$ species $J_{\sigma}$ and (iii) the energy difference between the two DW minima $2\Delta_{\sigma}$ for each of the two species. The $\sigma$ index in $J$ and $\Delta$ is due to the fact that the lattice that the $\uparrow$ and $\downarrow$ atoms feel can be engineered to be different by choosing laser beams of appropriate polarizations, frequencies, phases, and intensities. In the following we assume that the atoms are strongly interacting, $U \gg J$, and that effective vibrational energy of each well $\hbar \omega_0$ is the largest energy scale in the system $\hbar \omega_0 \gg U, \Delta, J$, i.e., deep wells.

Singlet $|s\rangle$ and triplet $|t\rangle$ states form the natural basis for the two-atom system. The relative energies of these states can be manipulated by controlling the energy bias $\Delta$ between the two wells. In the unbiased case ($U \gg 2\Delta$)
only states with one atom per site (1,1) are populated, as
the large atomic repulsion energetically suppresses double
occupancy [here, labels (m, n) indicate the integer number of
atoms in the left and right sites of the DW]. For weak
tunneling and spin independent lattices \(J_1 = J_2 = J, \Delta_1 =
\Delta_2 = \Delta \) the states \((1,1)\{s\} \) and \((1,1)\{t\} \) are nearly
degenerated. The small energy splitting between them is
\(-4J^2/U\), with the singlet being the low energy state
[Fig. 1(a)]. As \(\Delta \) is increased the relative energy of doubly
occupied states \((0,2)\{s\} \) decreases. Therefore, states \((1,1)\{s\} \)
and \((0,2)\{s\} \) will hybridize. When \(2\Delta \approx U \) the
atomic repulsion is overwhelmed and consequently the \((0,2)\{s\}
becomes the ground state. At the same time, the Pauli
exclusion results in a large energy splitting \(h \omega_0 \) between
doubly occupied singlet and triplet states as the latter must
have an antisymmetric orbital wave function. Hence,
\((1,1)\{t\} \) does not hybridize with its doubly occupied coun-
terpart, and its relative energy becomes large as compared
to the singlet state. Thus the energy difference between
singlet and triplet states can be controlled using \(\Delta \).

Further control is provided by changing \(J_p \) and \(\Delta_p \) in
spin dependent lattices [see Fig. 1(b)]. Specifically, let us
now consider the regime \(2\Delta_p \ll U \) in which only (1,1)
subspace is populated. Within this manifold we define \[6\]
\(\{s\} = \beta^1 \{0\} = \frac{1}{\sqrt{2}} (\{1\} - \{2\}), \quad \{t\} = \beta^2 \{0\} = \frac{1}{\sqrt{2}} (\{1\} + \{2\}), \quad \{t_s\} = \beta^3 \{0\} = \frac{1}{\sqrt{2}} (\{1\} - \{2\}), \quad \{t_t\} = \beta^4 \{0\} = \frac{1}{\sqrt{2}} (\{1\} + \{2\}) \).

Here \(\beta^1 \) and \(\beta^2 \) are operators that create triplet and singlet
states from the vacuum \(\{0\} \) (state with no atoms). They
satisfy bosonic commutation relations and the constrain
\( \sum_{\alpha=x,y,z} \beta^\alpha \beta^\alpha = 1 \), due to the physical restriction
that the state in a double well is either a singlet or a triplet.
In the rest of the Letter we will omit the label (1,1) for the
singly occupied states.

When \(\Delta_p \) depends on spin, i.e., \(\Delta = \Delta_1 - \Delta_2 \neq 0 \), the
\((1,1)\{t\} \) component mixes with \((1,1)\{s\} \) [see Fig. 1(c)]. Note that, on
the other hand, \((1,1)\{s\} \) remain decoupled from \((1,1)\{t\} \)
and \((1,1)\{s\} \). As a result, the states \((1,1)\{s\} \) and \((1,1)\{t\} \) form an effective two-
level system whose dynamics is driven by the Hamiltonian:

\[ H^\prime_1 = \gamma (\hat{\sigma}^\dagger \hat{\sigma} - \hat{\tau}_t^\dagger \hat{\tau}_t) - Y \hat{S}^c \text{ const.} \tag{5} \]

Here \(\zeta \equiv 2J_1J_2/\bar{U} \) is the exchange coupling energy (with
\(\bar{U} = \frac{U^2 + (\Delta_1 + \Delta_2)^2}{\Delta_1 + \Delta_2} \)) and \(\hat{S}^c = \hat{\sigma}^\dagger \hat{\tau}_t + \hat{\tau}_t^\dagger \hat{\tau}_t \). If \(Y = 0 \), exchange
dominates and \((1,1)\{s\} \) and \((1,1)\{t\} \) become the ground and first
excited states, respectively. However, if \(Y \gg \zeta \), exchange
can be neglected and the ground state becomes either \([1]\) or
\([2]\) depending on the sign of \(Y \).

These considerations indicate that it is possible to per-
form arbitrary coherent manipulations and robust measure-
ment of atom pair spin states. The former can be
accomplished by combining time-dependant control over
\(\zeta, Y \) to obtain effective rotations on the spin-1/2 Bloch
sphere within \([1,2] \) state. In the parameter regime of
interest, \(\zeta, Y \), can be varied independently in experiments.
In addition, by applying pulses (uniform) magnetic fields it is
possible to rotate the basis, thereby changing the relative
population of the \(\{sz, sz\} \) states. Atom pair spin states can
be probed by adiabatically increasing \(\Delta \) until it becomes
larger than \(U/2 \), in which case atoms in \(\{s\} \) will
adiabatically follow to \((0,2)\{s\} \) while the atoms in \((1,1)\{t\}
will remain in \((1,1)\{t\} \) state [Fig. 1(a)]. A subsequent mea-
surement of the number of doubly occupied wells will
reveal the number of singlets in the initial state. Such a
measurement can be achieved by efficiently converting the
doubly occupied wells into molecules via photoassociation
or using other techniques such as microwave spectroscopy
and spin changing collisions \([7]\). Alternatively, one can
continue adiabatically tilting the DW until it merges to one
well. In such a way the \([s] \) will be projected to the \((2,0)\{s\},
while the triplets will map to \((2,0)\{t\} \). As \((2,0)\{t\} \) has one of
the atoms in the first vibrational state, by measuring the
population in excited bands one can detect the number of
initial \((1,1)\{t\} \). Hence the spin-triplet blockade \([5]\) allows
effective control and measurement of atom pairs.

Detection and diagnostics of many-body spin phases
such as antiferromagnetic \((AF) \) states is an example of
direct application of the singlet-triplet manipulation and
measurement technique. The procedure to measure the \(AF
state population is the following: after inhibiting tunneling
between the various DWs, one can abruptly increase \(Y \),
such that the initial state is projected into the new eigen-
states \([1]\) and \([2]\) at time \(\tau = \tau_0 \). For \(\tau > \tau_0 \) \(Y \) can then be
adiabatically decreased to zero, in which case the \([1]\) pairs
will be adiabatically converted into \([1,1]\) and \([1,2]\) pairs to \([1,2]\).
Finally, the singlet population can be measured using the
spin blockade. As a result, a measure of the doubly occu-
pied sites (or excited bands population) will detect the
number of \([1]\) pairs and thus probe antiferromagnetic
states of the type \([1,1,1,\ldots]\).

These ideas can be directly generalized to perform mea-
surements of the more complex magnetic states that can be
represented as products of atom pairs. For example, a pulse of
rf magnetic field can be used to orient all spins, thus
providing the ability to detect \(AF \) states aligned along an
arbitrary direction. Moreover, one can determine the rela-
tive phase between singlet and triplet pairs in \(AF \) states of
the form \( C \{s\} + e^{i\phi} \{t\} \) by performing Ramsey-type spec-
trscopy. After letting the system evolve freely (with \(Y = 0 \)
so that the \([s] \) and \([t] \) components accumulate an addi-
tional relative phase due to exchange, a readout pulse
controlled by pulsing \(Y \) will map the accumulated phase
onto the population of singlet and triplet pairs. To know \(\phi \)
is important as it determines the direction of the antiferro-
magnetic order. Furthermore, by combining the blockade with
noise correlation measurements \([8]\) it is possible to
obtain further information about the magnetic phases.
While the blockade probes local correlation in the DWs,
noise measurements probe nonlocal spin-spin correlations
and thus can reveal long range order.

Before proceeding we note that ideas similar to that
outlined above can be used for bosonic atoms if initially
no \((1,1)\{t\} \) states are populated. The latter can be done by
detuning the \(|t_{x,j}\rangle\) states by means of an external magnetic field. In the bosonic case the doubly occupied \(|t_{e}\rangle\) states will be the ones that have the lowest energy. They will be separated by an energy \(\hbar \omega_0\) from the doubly occupied singlets as the latter are the ones that have antisymmetric orbital wave function in bosons. Consequently, the role of \(|s\rangle\) in fermions will be replaced by \(|t_{e}\rangle\) in bosons. The readout procedure would then be identical to that described above, while the coherent dynamics will be given by the Hamiltonian Eq. (5) apart from the sign change \(\zeta \rightarrow -\zeta\).

Up to now we have ignored tunneling between nearest neighbor DWs, but in practice inter-DW tunneling \(t_{\sigma}\) can be allowed by tuning the lattice potential. When atoms can hop between DWs, the behavior of the system depends on the dimensionality. For simplicity we will restrict our analysis to a 1D array of \(N\) double wells, where \(t_{\sigma}\) corresponds to hopping energy of \(\sigma\)-type atoms between the right site of the \(j\)th DW and the left site of the \((j+1)\)th DW.

In the regime \(J_{x}\), \(t_{y}\), \(\Delta_{v} \ll U\), multiply occupied wells are energetically suppressed and the effective Hamiltonian is given by \(\hat{H}_{\text{eff}} = \hat{H}_{j} + \hat{H}_{t}\). Here the first term corresponds to the sum over \(N\) independent \(H_{j}\) Hamiltonians [see Eq. (5)], \(\hat{H}_{j} = \sum_{j=1}^{N} \tilde{H}_{j}\), each of which acts on its respective \(j\)th DW. On the other hand \(\hat{H}_{t}\) is nonlocal as it couples different DWs and quartics as it consists of terms with four singlet-triplet operators [9]. The coupled DWs system is, in general, complex and the quantum spin dynamics can be studied only numerically. However, there are specific parameter regimes where an exact solution can be found. For this discussion we will set \(\Delta_{v} = 0\). If \(t_{t}/t_{1} \rightarrow 0\), and at time \(\tau = 0\), no \(|t_{e}\rangle\), \(|t_{y}\rangle\) triplet states are populated, their population will remain always zero. Consequently, in this limit, the relevant Hilbert space reduces to that of an effective spin one-half system with \(|s\rangle\) and \(|t_{e}\rangle\) representing the effective \(\pm 1/2\) states, which we denote as \(|\uparrow\rangle\) and \(|\downarrow\rangle\). \(\hat{H}_{j}\) couples such effective spin states. In the restricted Hilbert space \(\hat{H}_{\text{eff}}\) maps exactly to an Ising chain in a magnetic field:

\[
\hat{H}_{\text{eff}} = -\sum_{j} \lambda_{c} \sum_{j} \hat{\sigma}^{j}_{x} \hat{\sigma}^{j}_{x} + \lambda_{s} \sum_{j} \hat{\sigma}^{j}_{x} \hat{\sigma}^{j}_{y} + \lambda_{t} \sum_{j} \hat{\sigma}^{j}_{x} \hat{\sigma}^{j}_{z} + \lambda_{s} \sum_{j} \hat{\sigma}^{j}_{x} \hat{\sigma}^{j}_{y} + \lambda_{t} \sum_{j} \hat{\sigma}^{j}_{x} \hat{\sigma}^{j}_{z}
\]

where \(\hat{\sigma}^{j}_{x}\) are the usual Pauli matrices which act of the effective \(|\uparrow\rangle\) and \(|\downarrow\rangle\) spins. In terms of singlet-triplet operators they are given by \(\hat{\sigma}^{j}_{x} = (\hat{\sigma}^{j}_{x} \hat{\sigma}^{j}_{x} - \hat{\sigma}^{j}_{x} \hat{\sigma}^{j}_{x})/2\), \(\hat{\sigma}^{j}_{y} = \hat{\sigma}^{j}_{y} + \hat{\sigma}^{j}_{y}\) and \(\hat{\sigma}^{j}_{z} = (\hat{\sigma}^{j}_{z} \hat{\sigma}^{j}_{z} - \hat{\sigma}^{j}_{z} \hat{\sigma}^{j}_{z})/i\). Here \(\lambda_{c} = \frac{t_{y}^{2}}{2U_{v}}\) and \(\lambda_{s} = \frac{t_{y}}{2U_{v}}\) and the upper and lower signs are for fermions and bosons, respectively. For fermions in the lowest vibrational level the on-site interaction energy between the same type of atoms \(U_{\uparrow}\), \(U_{\downarrow}\) \(\rightarrow\) \(\infty\) due to the Pauli exclusion principle.

The 1D quantum Ising model exhibits a second order quantum phase transition at the critical value \(|g| = 1\). For fermions (upper sign) when \(g \ll 1\) the ground state corresponds to all effective spins pointing up, i.e., \(|G\rangle = |\uparrow \ldots \uparrow\rangle = \Pi_{j}|s_{j}\rangle\). On the other hand when \(g \gg 1\), there are two degenerate ground states which are, in the effective spin basis, macroscopic superpositions of oppositely polarized states along \(x\). In terms of the original fermionic spin states this superposition correspond to the states \(|AF^{+}\rangle = \frac{1}{\sqrt{2}}(|\uparrow \downarrow \ldots \uparrow\rangle \pm |\downarrow \uparrow \ldots \downarrow\rangle)|. Therefore, by adiabatic passage one could start with \(|G\rangle\) and convert it into AF state(s). Because of the vanishing energy gap at the quantum critical point \(g = 1\), adiabaticity is difficult to maintain as \(N \rightarrow \infty\) [10–13]. In that respect, our projection scheme is useful to test adiabatic following. It can be done either by measuring the number of \(|\uparrow\rangle\) pairs in the final state or by adiabatically ramping down \(g\) back to zero and measuring the number of singlet-triplet pairs. The remaining number of triplets will determine the number of excitations created in the process.

We now turn to nonadiabatic dynamics. We will discuss the situation where initially the system is prepared in a product of singlet states (\(\lambda_{c} = 0\) ground state) and then one lets it evolve for \(\tau > 0\) with a fixed \(|\lambda_{s}| > 0\). Generically the coupling between DWs results in oscillations between singlet and triplet pairs with additional decay on a slower time scale. We present two important special cases:

(i) Singlet-triplet cluster state generation.—If the value of \(\lambda_{c}\) is set to be \(|\lambda_{s}| \gg |\zeta|\), then the Hamiltonian reduces to a pure Ising Hamiltonian and thus at particular times \(\tau_{c}\), given by \(\lambda_{c} \tau_{c}/\hbar = \pi/4 \pm \pi/2\), the evolving state becomes a \(d = 1\) cluster state \(|\zeta\rangle\) in the effective spin basis [14]. Up to single spin rotations \(|\zeta\rangle = \frac{1}{\sqrt{2^N}} \bigotimes_{j=1}^{N} (|\uparrow\rangle \hat{\sigma}_{j}^{x} + |\downarrow\rangle)\rangle\). Cluster states are of interest for the realization of one-way quantum computation proposals where starting from the state \(|\zeta\rangle\) computation can be done via measurements only. Preparation of cluster states encoded in the logical \(\uparrow\), \(\downarrow\) qubits may have significant practical advantages since the \(\uparrow\), \(\downarrow\) states have zero net spin along the quantization axis and hence are not affected by global magnetic field fluctuations. Additionally, the use of such singlet-triplet states for encoding might allow for the generation of decoherence free subspaces insensitive to collective and local errors [15] and for alternative schemes for measured-based quantum computation [16].

(ii) Nonequilibrium generation and probing of AF correlations.—The second situation is when the value of \(\lambda_{c}\) is set to the critical value, \(|\lambda_{s}| = |\zeta|\) (or \(g = 1\)). We will first focus on the fermionic system \(\lambda_{c} > 0\). To discuss it, we remind that the dynamics driven by \(\hat{H}_{\text{eff}}\) is exactly solvable as \(\hat{H}_{\text{eff}}\) can be mapped via the Jordan-Wigner transformation into a quadratic Hamiltonian of fermionic operators which can be diagonalized by a canonical transformation [13,17]. Using such transformation it is possible to show that at specific times, the shortest of them denoted by \(\tau_{m} = \hbar N_{m}/4g\), long range AF correlations build up and for small atom number the state approaches \(|AF^{+}\rangle\). To quantify the resulting state in Fig. 2 (inset) we plot the fidelity, defined as \(F(\tau_{m}) = |\langle AF^{+}\psi(\tau_{m})|g-1\rangle|^{2}\), as a function of \(N\). The
Before concluding we briefly mention that spin dependent superlattices of the form

$$V = \sum_{j=1/2} (A_j + B_j \sigma_z) \cos^2[kz/j + \theta_j]$$  \hspace{1cm} (8)

can be experimentally realized by superimposing two independent lattices, generated by elliptically polarized light, one with twice the periodicity of the other [19–21]. Complete control over the DW parameters is achieved by controlling the phases (which determine Δ), intensities (which determine U, J, and t) and polarization of the laser beams (which allows for spin dependent control).

In summary we have described a technique to prepare, detect and manipulate spin configurations in ultracold atomic systems loaded in spin dependent period-two superlattices. By studying the many-body dynamics that arises when tunneling between DWS is allowed, we discussed how to dynamically generate singlet-triplet cluster states and AF cat states, which are of interest for quantum information science, and how to probe AF correlations in dynamics that are far from equilibrium. Even though in this Letter we restrict our analysis to 1D systems the ideas developed here can be extended to higher dimensions and more general kinds of interactions.

We acknowledge discussions with G. Morigi. This work was supported by ITAMP, NSF (Career Program), Harvard-MIT CUA, AFSR, Swiss NF, the Sloan Foundation, and the David and Lucille Packard Foundation.

[18] In this case a different sign in the definition of kink density $\hat{\sigma}_j^\pm \hat{\sigma}_{j+1}^\mp \rightarrow -\hat{\sigma}_j^\pm \hat{\sigma}_{j+1}^\mp$ is required.