Josephson effects between multigap and single-gap superconductors

D. F. Agterberg,1,4 Eugene Demler,2 and B. Janko3,4

1Department of Physics, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53201
2Department of Physics, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53201
3Department of Physics, Harvard University, Cambridge, Massachusetts 01238
4Department of Physics, University of Notre Dame, Notre Dame, Indiana 46617

Materials Science Division, Argonne National Laboratory, 9700 South Cass Avenue, Argonne, Illinois 60439

(Received 28 August 2002; published 6 December 2002)

Multigap superconductors can exhibit qualitatively different phenomena due to the existence of multiple order parameters. Repulsive electronic interactions may give rise to a phase difference of \( \pi \) between the phases of the order parameters. Collective modes due to the oscillation of the relative phases of these order parameters are also possible. Here we show that both these phenomena are observable in Josephson junctions between a single-gap and a multi-gap superconductor. In particular, a nonmonotonic temperature dependence of the Josephson current through the junction reveals the existence of the \( \pi \) phase differences in the multi-gap superconductor. This mechanism may be relevant for understanding several experiments on the Josephson junctions with unconventional superconductors. We also discuss how the presence of the collective mode resonantly enhances the dc Josephson current when the voltage across the junction matches the mode frequency. We suggest that our results may apply to MgB\(_2\), 2H-NbSe\(_2\), spin ladder, and bilayer cuprates.

DOI: 10.1103/PhysRevB.66.214507 PACS number(s): 74.50.+r, 74.20.Mn

I. INTRODUCTION

Multiband superconductors have been the subject of theoretical investigation since the original work of Suhl, Matthis, and Walker.1 Experimentally, it has recently been shown that MgB\(_2\) (Ref. 2–7) and 2H-NbSe\(_2\) (Ref. 8) belong to this class. In this paper we examine some qualitatively different features associated with superconductivity in the multi-band materials. We will focus on examining Josephson junctions between a multiband superconductor and a single-band superconductor. We show that such a junction can reveal important information about the role of electronic interactions in the pairing mechanism and further be used to detect superconducting collective modes specific to multiband superconductors.

A phase difference of \( \pi \) between the gap on different bands occurs when repulsive electronic interactions between the different bands play an important role in creating the superconducting state.9–14 Such a mechanism has been argued to be relevant for the spin-ladder cuprate superconductors, with the gap being of opposite sign on the bonding and antibonding bands of the two legs of the ladder.11,12 It has also been suggested that interband repulsive electronic interactions may be relevant to MgB\(_2\).13,14 Below we show that the temperature dependence of the Josephson current between a multigap and a single-gap superconductor can reveal the existence of a \( \pi \) phase difference between the order parameter on the two bands participating in the superconducting state. Such a Josephson junction can also be used to detect a collective mode originally proposed by Leggett.15 This mode may exist regardless of the relative sign of the order parameters in the two bands and involves an oscillation of their relative phase. It has been proposed theoretically for bilayer cuprates16 and Sr\(_2\)RuO\(_4\),17 and observed experimentally in SmLa\(_{0.8}\)S\(_{0.2}\)CuO\(_4\)–\( \delta \).18

In the following, we focus on a two-band superconductor with bands labeled by \( \sigma \) and \( \pi \). In spin-ladder cuprates these would correspond to the antibonding and bonding bands of the two legs of the ladder. In MgB\(_2\), the \( \sigma \) band would correspond to the quasi-two-dimensional (2D) hole bands due to the \( \sigma \)-bonding \( p_{\sigma z} \) boron orbitals and the \( \pi \) band would correspond to the 3D electron and 3D hole band due to the \( \pi \) bonding \( p_z \) boron orbitals.19 To describe the superconducting state, we use a two-band BCS model in the clean limit. This model is parametrized by the interaction matrix that describes both the intraband \( (V_{\pi,\pi} \) and \( V_{\sigma,\sigma} \)) and the interband matrix \( (V_{\sigma,\pi}) \) pair scattering elements. In the calculations presented below we employ the weak-coupling self-consistency gap equation,

\[
\Delta_n = -\pi T \sum_\beta V_{\sigma,\beta} N_\beta \sum_{\omega_n} \Delta_\beta / \sqrt{\omega_n^2 + |\Delta_\beta|^2},
\]

where \( N_\beta \) is the density of states at the Fermi surface for band \( \beta \) and \( \omega_n = \pi T (2n + 1) \).

II. TEMPERATURE DEPENDENCE OF THE JOSEPHSON CURRENT

A sign difference between \( \Delta_\sigma \) and \( \Delta_\pi \) can only be detected through a phase sensitive experiment. In Ref. 20, it was shown that, under the right circumstances, a corner junction experiment can reveal a sign difference in the gap between two bands in a conventional superconductor. Here, we show that simply the temperature dependence of the Josephson current can reveal this sign difference. We examine the Josephson current through a junction between a multiband superconductor and a single-band superconductor. Prior to a detailed explanation, we present our main result (which can be seen in Figs. 1 and 2). The Josephson current shows a finite temperature maximum when the two gaps are of opposite sign. This maximum occurs due to thermal effects. At high temperatures, the thermally excited quasiparticles easily deplete the Josephson current arising from the overlap of the
order parameter with the smaller amplitude in the multiband superconductor, and the order parameter of the conventional, single-band superconductor. However, as the temperature is lowered, the contribution from the band with the smaller (opposite sign) gap becomes more important, which leads to a downturn in the total Josephson current. Given that this behavior stems from gaps of opposite sign, it will also exist when strong-coupling effects are incorporated. Now we turn to a detailed examination of this effect. Note that the Josephson current between a single-band and a multiband superconductor in which the gaps are the same sign has been examined in Ref. 21.

The Josephson current through a junction can be found once the boundary conditions for the quasiclassical equations have been specified. This has been done by Zaitsev22 (see also Ref. 23) and generalized by Mazin et al.23 to multiband superconductors. The resulting current through the junction with multiband superconductors on both the right side $\mathcal{R}$ and the left side $\mathcal{L}$ is

$$I_S = \frac{\pi T}{e} \sum_{i,j} \frac{1}{R_{N,ij}} \sin(\phi_{L_i} - \phi_{R_j})$$

$$\times \sum_{\omega_n > 0} \frac{|\Delta_{L_i}(T)||\Delta_{R_j}(T)|}{\sqrt{|\Delta_{L_i}(T)|^2 + \omega_n^2 \sqrt{|\Delta_{R_j}(T)|^2 + \omega_n^2}},$$

(2)

where $\phi_{R(L),i}$ is the phase of $\Delta_{R(L),i}$, $R_{N,ij} = \min(R_{N,1}, R_{N,2})$ with $(A R_{L_i})^{-1} = (2e^2/\hbar) \int_{\omega_n > 0} \mu_n \xi_{L_i} d^2 S_{L_i}$, and $D_{L_i}$ is the probability for a quasiparticle to tunnel from band $i$ in $\mathcal{L}$ to band $j$ in $\mathcal{R}$. The total junction resistance is given by $R_{N,ij} = \sum_{\omega_n} R_{N,1,2}$. The functions $\Delta_{L_i}(T)$ take on the bulk values, as is justified for $s$-wave superconductors near nonmagnetic insulating surfaces.

Here we consider the simplest case of a Josephson junction between a conventional single-band superconductor $\Delta_{\mathcal{R}}$ and a two-band superconductor ($\Delta_{\mathcal{L},i} = \{\pi, \sigma\}$). We consider a geometry for which both bands contribute to the transport and assume that the conductance through the junction is limited by the two-band superconductor. In our calculations we take $V_{\pi,\sigma} = 0$ and $\{|V_{\pi,\sigma}| = 0.35|V_{\pi,\sigma}\}$ with a density of states ratio $N_{\pi}/N_{\sigma} = 1.35$ (Ref. 21). We consider two possible values for $\epsilon = |R_{N,\pi}/R_{N,\sigma}| = 2$ and $\epsilon = 1$ (with results shown on Figs. 1 and 2). The most important feature for $\epsilon = 1$ is that the maximum in the Josephson current occurs at finite temperature, not at zero temperature.

The behavior for $\epsilon = 2$ is even more striking. In this case the Josephson current becomes zero at some temperature. This remarkable behavior occurs because the $\pi$ band is assumed to have the smaller gap but the larger conductivity for $\epsilon = |R_{N,\pi}/R_{N,\sigma}| = 2$. This can be done by comparing the sign of $I_S$ given by Eq. (1) at $T = 0$ and at $T = T_c$. If the sign changes then there must be a zero in $I_c$. For $T = T_c$,

$$I_S = \frac{3\pi F(|\Delta_{\mathcal{R}}(T)|)}{4T_c} \sin(\phi_{L,\pi} - \phi_{R,\sigma}) \left( \frac{|\Delta_{L,\sigma}(T)|}{R_{N,\sigma}} - \frac{|\Delta_{L,\pi}(T)|}{R_{N,\pi}} \right),$$

where

$$F(|\Delta|) = \frac{3}{\pi^2} \sum_{n=0}^{\infty} \frac{|\Delta|}{(2n+1) \sqrt{(2n+1)^2 + |\Delta|^2} \sqrt{2\pi T^2}}$$

(3)

[for $T_c = T_c^L$ where $F(|\Delta_{\mathcal{R}}(T)|) = |\Delta_{\mathcal{R}}(T)|$. For $T = 0$,23 and $|\Delta_{\mathcal{R}}(0)| > |\Delta_{\mathcal{R}}(0)|$,]
where $K(x)$ is a complete elliptic integral of the first kind. For $|\Delta_{L_0}(0)| < |\Delta_{R_0}(0)|$, $|\Delta_{L_0}(0)|$ and $|\Delta_{R_0}(0)|$ should be interchanged in the term proportional to $|\Delta_{R_0}(0)|$ in Eq. 4. For example, if $|\Delta_{R_0}| = 2|\Delta_{L_0}| = 3|\Delta_{a}|$, then the zero in $I_s$ exists for $3.0 > \varepsilon > 1.7$. Note that in the temperature region where the Josephson current vanishes, higher-order terms in the Josephson coupling should be included in the theory. This will not be done here.

Figure 2 was determined for only one choice of the parameters $V_{\alpha,\beta}$. We have explored a much wider parameter range and have found that there are three criteria for the observation of this finite temperature maximum or the vanishing of the Josephson current: (i) the gaps are of opposite sign; (ii) the smaller gap is smaller than the that of the single-band superconductor (note the larger gap can be larger or smaller than the gap of the single band superconductor); and (iii) both bands must contribute to the conductance through the junction.

It is useful to point out that there have been at least three experiments where the Josephson current exhibits a peak at finite temperatures (similar to Fig. 2 for $\varepsilon = 1$): in a UBe$_{13}$/Ta junction, a YBCO/Pb junction, and a Pb/Sr$_2$RuO$_4$/Pb junction. In all three cases, the suspected unconventional nature of UBe$_{13}$, YBCO, and Sr$_2$RuO$_4$ have been argued to be responsible for this behavior. However, all these materials have multiple bands and perhaps the explanation given above is relevant. One can also give simple generalizations of the discussion above to the cases of more complicated junction geometries in which different $k$ points on the Fermi surface play the role of separate bands.

A. Ladder cuprates

Since superconductivity in spin ladder cuprates represents a likely testing ground for the predicted behavior, it is worthwhile discussing the gap structure more carefully. In this case the relative phase difference in $\pi$ arises from repulsive interactions between the bonding ($\sigma$) and antibonding ($\pi$) bands of the two legs of the ladder. In general, the density of states is not the same for the $\pi$ and $\sigma$ bands. If a large on-site Coulomb repulsion exists, then the gap structure is easily determined by the constraint that the on-site pairing amplitude is zero. In particular, $\Sigma_{\pi,\sigma} = 0$, which implies $N_{\sigma}\Delta_{\sigma} = -N_{\pi}\Delta_{\pi}$ for all temperatures (e.g., the Fermi surface with the bigger density of states has the smaller gap). We have confirmed that the finite temperature maximum in $I_s$ (as seen in Fig. 2) occurs for this gap structure when $\varepsilon = 1$. 

III. COLLECTIVE-MODE-ASSISTED TUNNELING

Here we show that Leggett’s collective mode resonantly couples to the dc Josephson current of a junction between the two-band superconductor and a single-band superconductor. The gaps need not be of opposite sign as in the previous section. Consider a superconductor that has two superconducting order parameters $\Delta_1$ and $\Delta_2$, coming from two bands, with a Josephson coupling between them. Let $\phi_1$ and $\phi_2$ be their respective phases. First, let us derive Leggett’s mode from the microscopic equations of motion. We define $\phi = (\phi_1 - \phi_2)/2$, the chemical potential difference between the two bands $\Delta_\mu = \mu_1 - \mu_2$, and charge imbalance between the two bands $L = \Delta Q_1 - \Delta Q_2$. They are related through an appropriate compressibility $\kappa$ and the relation $L = \kappa \Delta \mu$. When the two bands are out of equilibrium, there is internal Josephson current $J_s \sin(2\Delta \phi)$ and some internal dissipative current $\alpha \Delta \mu$. From charge conservation and using the relation $[L,(\phi)] = -2i\epsilon$ we have

$$\frac{\partial L}{\partial t} = -J_s \sin(2\phi) - \alpha \Delta \mu,$$

$$\frac{\kappa \hbar}{e} \frac{\partial^2 \phi}{\partial t^2} = -J_s \sin(2\phi) - \frac{\alpha \kappa \hbar}{e} \frac{\partial \phi}{\partial t}.$$
When there is a constant voltage between the two superconductors we have $\theta = 2eVt/\hbar = \Omega_{e} t$. Assuming that $\phi$ is small, we can solve the last equation for $\phi$,

$$
\phi(t) = \frac{e}{\kappa \hbar} \lim_{t \to 1} \left( \frac{(J_1 - J_2)e^{\Omega_{e} t}}{(-\Omega_{e}^{2} + \omega_{0}^{2} + i\sigma \Omega_{e})} \right).
$$

(10)

If we average the total current $I_{tot} = J_1 \sin(\phi_1 - \phi_3) + J_2 \sin(\phi_2 - \phi_4)$ over time we find

$$
I_{tot} = \frac{e}{2\kappa \hbar} \frac{2\sigma eV}{h}(J_1 - J_2)^2 \left( \frac{\omega_{0}^{2} - 4\sigma^{2}V^2}{h^2} + \frac{2\sigma eV}{h} \right)^2 .
$$

(11)

We therefore have a resonance enhancement of the dc current when voltage matches the energy of the Leggett mode. If the experiments are done at finite current this will show up as Fiske steps. There is a simple physical picture of the result discussed in this paper. Classical Josephson junctions made of conventional superconductors have coherence between Cooper pair condensates on the two sides of the junction and support dc current in the absence of voltage difference across the junction. If we neglect quasiparticle tunneling, for any finite voltage there is the ac Josephson effect and no dc current: Cooper pairs oscillate between the electrodes with zero average current. When one of the superconductors has two bands, there is an additional possibility of Cooper pairs exciting Leggett excitations after traveling across the junction. If the energy gained by a Cooper pair during the junction crossing $(2eV)$ matches the collective mode energy $(\omega_{0})$ we get a dc current as described by Eq. (11). Similar interpretation may be given to the phonon assisted tunneling in Josephson junctions (see, e.g., Ref. 29), with an extra ingredient being the emission of a phonon at the final stage (for experiments see Ref. 30 and references therein). It is useful to point out that sharp peaks in $I(V)$ for Josephson junctions (or Fiske steps, when the experiments are done at a fixed current) may also arise from the Swihart waves.\(^{31}\) One should be able to separate the two mechanisms since the energy of the resonances coming from Swihart waves depend on the geometry of the junction, whereas the energy of the Leggett’s mode depends on the material properties only.

The interaction used in the previous section is such that the Leggett’s collective mode is absent. However, this interaction gives rise to the situation where superconductivity in one of the bands is induced entirely by the other band. Clearly, this need not always be the case, as the Cooper instability may be present in both bands independently. We note that an interlayer exciton has been recently observed in the $c$-axis optical conductivity experiments on SmLa$_{0.8}$Sr$_{0.2}$CuO$_{4-\delta}$ by D. Dulic \textit{et al.}\(^{18}\) This mode may be understood as a particular realization of the Leggett’s exciton where the two bands correspond to the individual layers in a bilayer. Leggett’s mode was also argued to exist in MgB$_{2}$ and in Sr$_{2}$RuO$_{4}$.\(^{17}\) All of these materials are promising candidates for observing the interesting Josephson phenomena discussed in this paper.

**IV. CONCLUSIONS**

Inspired by a series of recent experiments strongly supporting multiband superconductivity in a variety of compounds, we have examined the Josephson effect between such multiband superconductors and single-band superconductors. We have shown that this can be a rather powerful probe of physics associated with multiple phases in the multiband superconductors. In particular, such junctions can be used to detect the $\pi$ phase difference between the multiple gaps of the multiband superconductor, and consequently provide experimental support for the presence of repulsive interband interactions. This would add considerable support to the notion that an adequate description of the different multiband superconductors must go beyond models considering electron-phonon interaction alone. We have also pointed out that such hybrid Josephson junctions are capable of detecting collective modes arising from fluctuations in the relative phase between these gaps. Such an experiment, if successful, would indicate that Leggett’s mode, proposed more than three decades ago, is in fact exhibited by these multigap superconductors.

**ACKNOWLEDGMENTS**

We would like to thank Professor A. A. Abrikosov, Professor V. Ambegaokar, Professor G. W. Crabtree, Dr. K. E. Gray, Dr. D. Hinks, Dr. I. I. Mazin, Dr. W. K. Kwok, Dr. G. Karapetrov, Professor J. F. Zasadzinksi, and H. Schmidt for useful discussions. We are especially grateful to Dr. G. W. Crabtree and Dr. W. K. Kwok for their hospitality during our stay at the Materials Sciences Division of Argonne National Laboratory, where this research program was initiated. This research was supported in part by the U.S. DOE, Office of Science, under Contract No. W-31-109-ENG-38. D.F.A. was also supported by an award from Research Corporation. E.D. was supported by NSF Grant No. DMR-0132874 and by the Sloan Foundation. B.J. was also supported by NSF Grant No. NIRT-021019.

---

5. X.K. Chen, M.J. Konstantinovi, J.C. Irwin, D.D. Lawrie, and J.P.
See, for example P.W. Anderson, Rev. Mod. Phys. 38, 289 (1966); P. Carruthers and M.M. Nieto, ibid. 40, 411 (1968), and references therein.