Spin symmetry breaking in bilayer quantum Hall systems

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Based on the construction of generalized Halperin wave functions, we predict the possible existence of a large class of broken spin symmetry states in bilayer quantum Hall structures, generalizing the recently suggested canted antiferromagnetic phase to many fractional fillings. We develop the appropriate Chern-Simons theory, and establish explicitly that the low-lying neutral excitation is a Goldstone mode and that the charged excitations are bimerons with continuously tunable (through the canted antiferromagnetic order parameter) electric charge on the individual merons.

Recently, a canted antiferromagnetic (CAF) state has been predicted to exist in bilayer quantum Hall (QH) systems at the special filling factor \( \nu = 2/\alpha \) or more generally at \( \nu = 2/m \) where \( m \) is an odd integer. The original theoretical prediction based on a microscopic Hartree-Fock calculation has been followed up by a number of subsequent theoretical works using a quantum nonlinear \( \sigma \) model, a bosonic spin approach, and more detailed Hartree-Fock calculations. Fairly persuasive experimental support for the CAF phase in bilayer systems have the property of electric charge on the individual merons.

The electron Hamiltonian for a bilayer QH system can be written as

\[
\mathcal{H} = \int d^2x \left\{ \frac{1}{2m} \left| -i \sigma - eA^\alpha \right| \psi_{a\alpha} \right|^2 + (u^\alpha_\uparrow + u^\alpha_\downarrow) (\psi_{a\alpha} \psi_{a\alpha}^\dagger)^2 + \Delta_{z} \psi_{a\alpha} \sigma_{ab} \psi_{a\beta} \\
+ (u^\beta_\downarrow - u^\beta_\downarrow) (\psi_{a\alpha} \sigma_{ab} \psi_{a\beta})^2 + \Delta_{SAS} \bar{\psi}_{a\alpha} \bar{\sigma}_{ab} \psi_{b\alpha} \right\}.
\]

Here \( a, b \) and \( \alpha, \beta \) are layer ("isospin") and spin indices, respectively; \( u^\alpha_\uparrow \) is the intralayer and \( u^\beta_\downarrow \) is the interlayer Coulomb interaction; \( \Delta_{SAS} \) is the splitting between symmetric and antisymmetric states due to interlayer tunneling; \( \Delta_{z} \) is the Zeeman splitting. In the discussion below we choose to work with the symmetric/antisymmetric electron wave functions of definite \( S^z \). Using such single electron wave functions, however, does not rely on assuming large interlayer tunneling and Zeeman splitting. As discussed in Ref. 1, Coulomb interaction itself generates effective tunneling and Zeeman field that are much larger than bare \( \Delta_{SAS} \) and \( \Delta_{z} \). The appearance of such strong effective field is related to the tendency of electrons in the lowest Landau level to obey Hund's rule \( 12 \) so that the fully polarized ferromagnetic or spin singlet phases discussed in Refs. 1–8 correspond to the system obeying Hund's rule in spin or isospin, and the CAF phase is a nontrivial phase that achieves a compromise between the two.

Of the four possible single particle states, the symmetric spin up (\( S^z \)) state always has the lowest energy and antisymmetric spin down \( (A^\downarrow) \) has the highest energy. Since the symmetric spin down \( (S^\downarrow) \) and the antisymmetric spin up \( (A^\uparrow) \) states may be close in energy, it is important to consider mixing between them. We now construct a Halperin-
like wave function\textsuperscript{10} for our spinful bilayer system. Our wave function does not fix the number of electrons in the $S^\downarrow$ and $A^\uparrow$ states individually, but fixes their sum. If we label $S^\uparrow$ states by $z$, $S^\downarrow$ states by $u$, and $A^\uparrow$ states by $w$, we can easily write the Halperin wave function that fixes the number of electrons in the $S^\downarrow$ and $A^\uparrow$ states together, but not in each of them separately:

$$
\Psi(\{z\},\{u\},\{w\}) = \prod (z_i - z_j) \prod (z_i - w_j) \prod (z_i - u_j)'
\times \prod (w_i - w_j) \prod (w_i - u_j) \prod (u_i - u_j)
\times \exp \left[ -\frac{1}{4} \left( \sum |z_i|^2 + \sum |u_j|^2 + \sum |w_k|^2 \right) \right].
$$

Eq. (2) gives $\nu=2/m$, i.e., the CAF state discussed in Refs. 1 and 2. In the CAF phase, the electrons in the two layers have the same $z$ component of spin but opposite $x$-$y$ components. The direction of the Néel order parameter (defined as the difference in the spin expectation values in the two layers) comes from the spontaneous breaking of the $S^\downarrow$ spin symmetry. It should be mentioned that Halperin wave functions for spontaneously broken spin symmetry states may also be constructed for single layer QH systems, leading to the possibility (at least in principle) of exotic spin states in a single layer QH system.\textsuperscript{14}

Properties of the state (2) are conveniently discussed using a bosonic Chern-Simons theory.\textsuperscript{15,16} For simplicity, we again assume that the $A^\uparrow$ states are empty and consider only three kinds of electrons: $\Psi_1$ for $S^\uparrow$, $\Psi_2$ for $S^\downarrow$, and $\Psi_3$ for $A^\uparrow$. Equation (2) tells us that the electron $\Psi_1$ is seen as a vortex of strength $n$ by other $\Psi_1$ electrons and a vortex of strength $l$ by electrons $\Psi_2,\Psi_3$; electrons $\Psi_2,\Psi_3$ are seen as vortices of strength $m$ and $l$ by electrons $\Psi_2,\Psi_3$, respectively. We are therefore led to consider the following (bosonic) Chern-Simons Lagrangian:

$$
\mathcal{L} = \bar{\psi_1}(i\partial_\tau - i\alpha_0)\psi_1 + \sum_{a=2,3} \bar{\psi_a}(i\partial_\tau - i\bar{\alpha}_0)\psi_a + \frac{1}{2m}[(-i\partial - n\bar{a} - \bar{\alpha} - \bar{\alpha}^*)]\psi_1|\psi_1|^2
$$

$$
+ \frac{1}{2m} \sum_{a=2,3} \left[ (-i\partial - m\bar{a} - \bar{\alpha} - \bar{\alpha}^*)\psi_a|\psi_a|^2 - \Delta_{X_{\uparrow}} + \Delta_{X_{\downarrow}} \right] \psi_1 - \left( \Delta_{X_{\uparrow}} - \Delta_{X_{\downarrow}} \right) \psi_2^2 + \left( \Delta_{X_{\uparrow}} - \Delta_{X_{\downarrow}} \right) \psi_3^2
$$

$$
+ \nu_{LL}(x-y)(\rho_L(x) - \bar{\rho})(\rho_L(y) - \bar{\rho}) + \mathcal{L}_{CS}(\alpha) + \mathcal{L}_{CS}(\bar{\alpha}),
$$

where $\mathcal{L}_{CS}(\alpha) = (i/4\pi)e^{i\phi_{\nu\lambda}} a_{\mu} a_{\lambda}$, and $L$ is a layer index ("top" or "bottom") in the Coulomb interaction term. We decompose the $\psi_i$’s into an amplitude, a trivial phase, and a vortex part:\textsuperscript{16,17} $\psi_1 = \sqrt{\rho_1}e^{i\theta_1}\phi_{\nu e_1}$ and $\psi_a = \sqrt{\rho_2}e^{i\theta_2}\phi_{\nu e_2}\bar{z}_{a-1}$ for $a=2,3$, with the constraints $\phi_{\nu e_1}\phi_{\nu e_2} = \bar{z}_{a-1} = 1$. Then Eq. (3) can be written as

$$
\mathcal{L} = i\rho_1 \left( \frac{\partial_\tau \theta_1}{i} + \bar{\phi}_{\nu e_1} \frac{\partial_\tau}{i} \phi_{\nu e_1} - a_0 \right) + \rho_2 \left( \frac{\partial_\tau \theta_2}{i} + \bar{\phi}_{\nu e_2} \frac{\partial_\tau}{i} \phi_{\nu e_2} + \bar{z}_{a-1} \frac{\partial_\tau}{i} \bar{a}_0 \right)
$$

$$
+ i\bar{\theta}_1 \left( \bar{\phi}_{\nu e_1} - \frac{\partial_\tau}{i} \phi_{\nu e_1} - n\bar{a} - \bar{\alpha} - \bar{\alpha}^* \right) + \frac{K_1}{2}|\theta_1|^2 + \frac{K_2}{2}|\theta_2|^2 + \frac{1}{2K_2}((\partial_\tau \bar{z}_{a-1}^2 + (\bar{z}_{a-1})^2) - (\Delta_{X_{\uparrow}} + \Delta_{X_{\downarrow}})\rho_1
$$

$$
- (\Delta_{X_{\uparrow}} - \Delta_{X_{\downarrow}})\rho_2|z_1|^2 + (\Delta_{X_{\downarrow}} - \Delta_{X_{\uparrow}})\rho_2|z_2|^2 - \sum_{ab} \gamma_{ab}|z_a|^2|z_b|^2 + (\rho_1 + \rho_2 - \bar{\rho})(x)u(x-y)(\rho_1 + \rho_2 - \bar{\rho})(y)
$$

$$
+ \mathcal{L}_{CS}(\alpha) + \mathcal{L}_{CS}(\bar{\alpha}).
$$

Here $K_1 = m/\rho_1$, terms with $\gamma_{ab}$ come from the exchange part of the Coulomb interaction, and in the direct part of the Coulomb interaction we keep only the layer symmetric part of $u_{LL}$, which does not vanish in the limit of $d=0$. By integrating out $\theta_1$ and $\theta_2$ we find that $J_\mu = (\rho_1 \bar{J}_\mu)$ and $\bar{J}_\mu = (\rho_2 \bar{J}_\mu)$ are conserved. Therefore, we introduce dual gauge fields, $b_{\lambda}$ and $\bar{b}_{\lambda}$, such that $J_\mu = (1/2\pi)e^{i\nu\phi_0} \partial_\tau b_{\lambda}$ and $\bar{J}_\mu = (1/2\pi)e^{i\nu\phi_0} \partial_\tau \bar{b}_{\lambda}$. Then, we integrate out the statistical gauge fields, $a_{\mu}$ and $\bar{a}_{\mu}$, and the time component of the dual gauge fields, $b_0$ and $\bar{b}_0$. This gives (up to irrelevant constants)
where we have defined vortex and skyrmion currents as in\textsuperscript{17}
\[ J_\mu^v=(J_0^v,J_\alpha^v)=(1/2\pi)e^{\alpha\beta}\partial_\alpha\phi_\psi(x/\alpha,\phi_\psi), \]
\[ J_\mu^s=(J_0^s,J_\alpha^s)=(1/2\pi)e^{\alpha\beta}\partial_\alpha\phi_\psi(x/\alpha,\phi_\psi), \]
and \( J_\mu^a=(J_0^a,J_\alpha^a) \). The parameters \( r_1 \) and \( r_2 \) describe effective tunneling and Zeeman energy renormalized by the Coulomb interaction, and
\[ r(x)=2\pi J_\mu^v-x e^{\alpha\beta}\partial_\alpha b_\beta-\nu e^{\alpha\beta}\partial_\alpha b_\beta+e^{\alpha\beta}\partial_\alpha A_\mu^\beta, \]
\[ \tilde{r}(x)=2\pi J_\mu^s+2\pi J_\mu^a-\nu e^{\alpha\beta}\partial_\alpha b_\beta-\nu e^{\alpha\beta}\partial_\alpha b_\beta-\nu e^{\alpha\beta}\partial_\alpha A_\mu^\beta. \]

In the ground state there are no vortices or skyrmions, so the cancellation of the long range logarithmic interaction gives two conditions,
\[ \frac{1}{2\pi}\int d^2 x\left[ e^{\alpha\beta}\partial_\alpha A_\mu^\beta+n e^{\alpha\beta}\partial_\alpha b_\beta+e^{\alpha\beta}\partial_\alpha \tilde{b}_\beta \right]=0, \]
\[ \frac{1}{2\pi}\int d^2 x\left[ e^{\alpha\beta}\partial_\alpha A_\mu^\beta+m e^{\alpha\beta}\partial_\alpha b_\beta+e^{\alpha\beta}\partial_\alpha \tilde{b}_\beta \right]=0. \]

Recalling that \( 1/2\pi e^{\alpha\beta}\partial_\alpha b_\beta \) gives the density of \( \Psi_1 \) electrons and \( 1/2\pi e^{\alpha\beta}\partial_\alpha \tilde{b}_\beta \) gives the density of \( \Psi_2 \) and \( \Psi_3 \) electrons, we realize that Eq. (6) gives us the same filling fractions as the Halperin wave function (2).

From the last line of Eq. (5) it is obvious that as we change the strength of the Zeeman interaction and/or interlayer tunneling, we will stabilize various values of \( |z_1| \) and \( |z_2| \). Parameters \( \gamma_{ab} \) have been effectively computed in Hartree-Fock approximations for \( n=m=1 \) and \( l=0 \), i.e., \( \nu=2 \), in Refs. 1, 2, 7, and shown to obey \( \gamma_{12} \geq \gamma_{11} \geq \gamma_{22} \), which implies that in this case there is no direct transition from \( |z_1|=1 \) to \( |z_2|=1 \), but there is an intermediate phase where both \( |z_1|=\cos \theta_0 \) and \( |z_2|=\sin \theta_0 \) are finite.\textsuperscript{13} We will assume that the same holds for fractional fillings as well, although at this time we can offer no proof of this fact. The phase where both \( z \)'s are finite will correspond to the CAF phase. In this phase interactions fix the absolute values of \( z \)'s but not their relative phase. Therefore, when \( z_1z_2 \) develops an expectation value, we have a spontaneous breaking of the \( U(1) \) symmetry and the appearance of a Goldstone mode.

In the CAF phase, dynamics of the spin is determined by
\[ \mathcal{L}_z=i\tilde{\rho}_z \tilde{z} \partial_\alpha z_\alpha-1/2K_2 \left[ (\tilde{z} \partial_\alpha z_\alpha)^2+\gamma_{11}|z_1|^4+\gamma_{22}|z_2|^4 \right]. \]

In the CAF phase only \( (z_1z_2) \) develops a nonzero expectation value, but not \( (z_1^2) \) or \( (z_2^2) \). Therefore, we can write \( z_1=|z_1|e^{i\delta} \) and \( z_2=|z_2|e^{i\phi} \), the relative phase between \( z_1 \) and \( z_2 \), acquires an expectation value and gives rise to the Goldstone mode associated with the symmetry breaking. We also introduce \( q=|z_1|^2-|z_2|^2 \). Using that fluctuations of \( q \) are massive and their gradients may be neglected, we find from Eq. (7)
\[ \mathcal{L}_z=i\tilde{\rho}_z \tilde{z} \partial_\alpha z_\alpha-1/2K_2 \left[ (\tilde{z} \partial_\alpha z_\alpha)^2+\gamma_{11}|z_1|^4+\gamma_{22}|z_2|^4 \right]. \]

We see that the spin wave velocity is \( v_s=(\gamma_{11}+\gamma_{22}-2\gamma_{12})2\sin^2(\theta_0)/(8\tilde{\rho}_z) \). By introducing an infinitesimal external Zeeman field and integrating out fluctuations in \( \phi \), one can also calculate the \( \tilde{S} \) correlation function which explicitly shows a Goldstone resonance
\[ \chi^{zz}(q,\omega)=(\omega^2/4\alpha)\omega^2-\omega^2-\omega^2/k^2, \]
\[ q^2/4\alpha \]
\[ \omega \]
\[ \tilde{S} \]
\[ Q \]
\[ n_c \times \cos^2 \theta_0 \]
\[ n_c \times \sin^2 \theta_0 \]
\[ S \text{ vortex} \]
\[ T \text{ vortex} \]
\[ 0 \]
$Q_{\text{meron}} = \frac{1}{2\pi} \left[ e^{i\theta} \partial_\theta b_\beta + e^{i\theta} \partial_\theta \tilde{b}_\beta \right]$

$$= \left[ (n-l)/(nm-l^2) \right] \times Q^i.$$ (12)

For $l=0$, which includes the $\nu=2/m$ states discussed in Ref. 2, we find $Q_{\text{meron}} = 1/m \times Q^i$. Note that if we were to create simple quasiparticles by squeezing a vortex into the ground state, $j_x^0 = \delta^2(x-x_0)$ or $j_y^0 = \delta^2(x-x_0)$, we could use the same arguments to find their charges: $q = -(m-1)/(nm-l^2)$ and $\tilde{q} = -(n-1)/(nm-l^2)$. So, as in the simple case of a meron in the $(m,m,m)$ state, two merons add up to a charge of 0 or the charge of a single quasiparticle, $\tilde{q}$.18

In the simplest case of $\nu=2$, one can give a simple picture of the meron excitation in the CAF phase using a generalization of the Berry’s phase argument in Ref. 11. As suggested in Ref. 3, the CAF phase can be described by combining pairs of electrons into hard core bosons and writing the wave function as $|\Psi\rangle = |S\rangle + e^{i\beta} |\tilde{S}\rangle$, where $|S\rangle$ and $|\tilde{S}\rangle$ denote singlet and triplet combinations of electrons with orbital momentum $m$, and $|\tilde{S}\rangle$ is the Fock vacuum. Using the definition of the Neel order parameter, $N^2_{\alpha\beta} = \langle |S\rangle |S\rangle |\tilde{S}\rangle |\tilde{S}\rangle \Psi \rangle$, we can easily prove that state $|\Psi\rangle$ has a uniform $\tilde{N}$ in the XY plane $N^2_{\alpha\beta} = 1/2 \cos \theta |\tilde{S}\rangle |\tilde{S}\rangle \Psi \rangle$. To have a meron we need a wave function where the direction of the Neel vector winds around as one goes around the center of the meron. This is achieved by considering the following wave function: $|\Psi\rangle = \Pi m_{\alpha\beta} \cos \theta |S\rangle |S\rangle |\tilde{S}\rangle |\tilde{S}\rangle \Psi \rangle$, where $\Pi m_{\alpha\beta} \cos \theta |S\rangle |S\rangle |\tilde{S}\rangle |\tilde{S}\rangle \Psi \rangle$ for $\nu=1$. For $\nu=2$, we find that $N^2_{\alpha\beta} = 1/2 \sum_{m} \Psi^\dagger_{m+1}(z) \cos \theta |\tilde{S}\rangle |\tilde{S}\rangle \Psi \rangle$. Since $\Psi^\dagger_{m+1}(z) \cos \theta |\tilde{S}\rangle |\tilde{S}\rangle \Psi \rangle$ for $\nu=2$, we have an $S$ vortex with charge $-\sin \theta$. In summary, we have developed an analytic theory for the bilayer QH CAF phase. Our theory is consistent with the original Hartree-Fock theory for $\nu=2$, but is general enough to predict a different class of fractional QH CAF phases as well as the correct excitation spectra.

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