Large-N and vacuum alignment in top-color models

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Large-N and Vacuum Alignment in Topcolor Models

R. Sekhar Chivukula[*]   Howard Georgi[†]
Boston University   Harvard University
Boston MA 02215   Cambridge, MA 02138

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Abstract

Topcolor and topcolor-assisted technicolor provide examples of dynamical electroweak symmetry breaking which include top-condensation, thereby naturally incorporating a heavy top quark. In this note we discuss the roles of the Nambu-Jona-Lasinio (NJL) and large-N approximations often used in phenomenological analyses of these models. We show that, in order to provide for top-condensation but not bottom-condensation, the top-color coupling must be adjusted to equal the critical value for chiral symmetry breaking up to $\mathcal{O}(1/N)$ in any theory in which the isospin-violating "tilting" interaction is a $U(1)$ gauge interaction. A consequence of these considerations is that the potentially dangerous "bottom-pions" are naturally light. We also show that the contributions to $\rho - 1$ previously estimated are of leading-order in N, are not included in the usual NJL analysis, and are the result of "vacuum-alignment".

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[*]sekhar@bu.edu
[†]georgi@physics.harvard.edu

1   Introduction

Topcolor and topcolor-assisted technicolor [1, 2] provide examples of dynamical electroweak symmetry breaking which include top-condensation [3, 4, 5, 6, 7, 8], thereby naturally incorporating a heavy top quark. In this note we discuss the roles of the Nambu–Jona-Lasinio (NJL) and large-N approximations often used in phenomenological analyses of these models.

We begin by reviewing the usual analysis of topcolor in the large-N NJL approximation. From this analysis one finds immediately [9] that, in order to provide for top-condensation but not bottom-condensation, the top-color coupling must be adjusted to equal the critical value for chiral symmetry breaking up to $\mathcal{O}(1/N)$. We show that this is a general result in any theory in which the isospin-violating "tilting" interaction is a $U(1)$ gauge interaction. A consequence of these considerations is that the potentially dangerous "bottom-pions" [9, 10] are naturally light. In the NJL approximation, the effects of the portion of the strong topcolor interactions coupling left-handed currents to right-handed currents are presumed to dominate, and to give rise to chiral symmetry breaking. We show that other portions of the topcolor interactions, in particular products of pairs of left-handed currents or right-handed currents, give rise to contributions to $\rho - 1$. These contributions, which have been estimated previously [11] by direct computation, are of leading-order in N and are the result of "vacuum-alignment" [12, 13, 14, 15].

2   Topcolor

In top-color models [1, 2] all or part of electroweak symmetry breaking is due to the presence of a top-condensate. In many models this condensate is driven by the combination of a strong isospin-symmetric top-color interaction and an additional isospin-breaking $U(1)$ gauge boson which couples only to the third generation of quarks. These additional interactions are strong, but are spontaneously broken at a scale $M \gtrsim 1$ TeV. In the simplest model [2], the couplings of the third generation of quarks to the new $U(1)$ interaction were taken to be proportional to weak hypercharge, and the masses of the heavy topcolor and $U(1)$ gauge bosons were taken to be comparable. At low energies, the top-color and hypercharge interactions of the third generation of quarks could then be approximated by the four-fermion operators

$$\mathcal{L}_{4f} = -\frac{4\pi\kappa_{tc}}{M^2}\left[\overline{\psi}\gamma_\mu\frac{\lambda^a}{2}\psi\right]^2 - \frac{4\pi\kappa_1}{M^2}\left[\frac{1}{3}\overline{\psi_L}\gamma_\mu\psi_L + \frac{4}{3}\overline{t_R}\gamma_\mu t_R - \frac{2}{3}\overline{b_R}\gamma_\mu b_R\right]^2 , \quad (2.1)$$

where ψ represents the top-bottom doublet, κ_{tc} and κ_1 are related respectively to the top-color and $U(1)$ gauge-couplings squared.

The analysis of the dynamics of this model usually proceeds in two steps. First, using a Fierz transformation, those (LR) terms in eqn. (2.1) above which couple

left-handed and right-handed currents and can be converted into products of color-singlet scalar/pseudoscalar bilinears are rewritten as

$$\mathcal{L}_{NJL} = +\frac{8\pi}{M^2}\left[\kappa_t(\overline{\psi}_L t_R)(\overline{t}_R \psi_L) + \kappa_b(\overline{\psi}_L b_R)(\overline{b}_R \psi_L)\right] \; , \qquad (2.2)$$

where

$$\kappa_t = \kappa_{tc} + \frac{8\kappa_1}{9N} \qquad \kappa_b = \kappa_{tc} - \frac{4\kappa_1}{9N} \; , \qquad (2.3)$$

and $N(=3)$ is the number of top-colors.

Next, this effective "NJL" model (eqn. (2.2)) [16, 3, 4, 5, 6, 7] is analyzed to leading order in N. This can be conveniently done by introducing a complex 2×2 matrix field Φ and writing the NJL interactions in the form

$$\mathcal{L}_{NJL} \to \left(\overline{\psi}_L \Phi \begin{pmatrix} t_R \\ b_R \end{pmatrix} + h.c.\right) - \frac{M^2}{8\pi}\text{Tr}\left[\Phi^\dagger \Phi \begin{pmatrix} \frac{1}{\kappa_t} & 0 \\ 0 & \frac{1}{\kappa_b} \end{pmatrix}\right] \; . \qquad (2.4)$$

Note that, written this way, the interaction of Φ with the fermions is $SU(2)_L \times SU(2)_R$ symmetric. To leading order in N, the theory is now solved by computing the trace of the fermion propagator in the presence of a background Φ field [7]. Computing the effective potential for the field Φ using a momentum-space cutoff of order M, we find

$$V^{eff}(\Phi) \approx \frac{M^2}{8\pi\kappa_c}\text{Tr}\left[\Phi^\dagger \Phi \begin{pmatrix} \frac{\kappa_c}{\kappa_t} - 1 & 0 \\ 0 & \frac{\kappa_c}{\kappa_b} - 1 \end{pmatrix}\right] + \frac{N}{16\pi^2}\text{Tr}\left[(\Phi^\dagger \Phi)^2 \log\left(\frac{M^2}{\Phi^\dagger \Phi}\right)\right] \; , \qquad (2.5)$$

where $\kappa_c = \pi/N$.

The essential features of this model in the large-N NJL approximation can now be determined from eqn. (2.5). For $\kappa_{t,b}$ close to κ_c, the field Φ yields four light complex scalar fields which have the quantum numbers of two independent 2-component "Higgs" fields (ϕ_t & ϕ_b) (with hypercharges ∓ 1, respectively). Choosing the values of κ_{tc} and κ_1 such that

$$\kappa_t > \kappa_c > \kappa_b \; , \qquad (2.6)$$

we obtain the phenomenologically desirable result that the doublet ϕ_t develops a vacuum expectation value,

$$\langle\phi_t\rangle = \begin{pmatrix} \frac{f_t}{\sqrt{2}} \\ 0 \end{pmatrix} \; , \qquad (2.7)$$

giving rise to a (potentially large) top-quark mass and leaving $\langle\phi_b\rangle \equiv 0$. Taking into account the necessary wavefunction renormalization for the scalar field [7] we find [17]

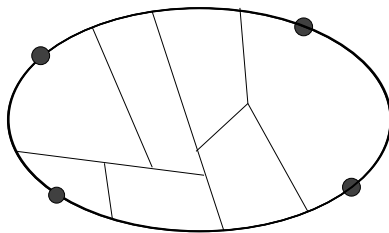$$f_t^2 \approx \frac{N}{8\pi^2}m_t^2 \log\left(\frac{M^2}{m_t^2}\right) \; . \qquad (2.8)$$

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Figure 1: A "typical" planar contribution, of leading order in $1/N$, to the effective potential for Φ. Fermions are represented by the thick solid lines, gluons by the thin solid lines, and insertions of Φ by the dots.

For $m_t \approx 175$ GeV and $M \approx 1$ TeV, this yields $f_t \approx 64$ GeV. From the equations of motion for ϕ_t derived from eqn. (2.4), we see that this expectation value can be interpreted as a top-quark condensate. To the extent that κ_t and κ_b are close to the "critical value" κ_c for chiral symmetry breaking, the fields ϕ_t and ϕ_b have masses (and expectation values) small compared to M and are composite Higgs fields [7, 18].

At this level of approximation, for fixed M, the effective mass-squareds of the Higgs fields change smoothly from positive to negative as the respective κ's vary from below to above the "critical value" κ_c. That is, in this approximation the chiral phase transition (for fixed M and viewed as a function of κ_{tc}) is second-order. For κ's close to κ_c the effective scalar lagrangian is a Landau-Ginzburg theory of the chiral phase transition with order parameter Φ.

3   Large-N and the Chiral Phase Transition

From eqn. (2.3), we see that the difference $\kappa_t - \kappa_b$ is of order κ_1/N. We will argue shortly that the ratio κ_1/κ_{tc} is independent of N. Therefore, in order to provide for top-condensation but not bottom-condensation, the top-color coupling must be adjusted to equal the critical value for chiral symmetry breaking up to $\mathcal{O}(1/N)$. In this section we show that this is a general property of the large-N limit, independent of the NJL approximation. We will also see that it is independent of the assumption that the topcolor and strong $U(1)$ gauge boson masses are equal — it persists even if these masses are very different.

Consider the topcolor theory in large-N [19]. As in QCD, in order to have a well-defined high-energy theory, we must choose a topcolor coupling

$$g_{tc} = \frac{\tilde{g}_{tc}}{\sqrt{N}} \ , \tag{3.1}$$

and hold $\tilde{g}_{tc}$ fixed as $N \to \infty$. The chiral symmetries of the topcolor theory are $SU(2)_L \times SU(2)_R$, under which the left-handed top-bottom doublet transforms as

a $(2,1)$ and the right-handed top and bottom transform together as a $(1,2)$. The behavior of the chiral symmetries is governed by the effective potential for an order parameter Φ, which transforms as a $(2,\tilde{2})$ under $SU(2)_L \times SU(2)_R$. To leading order in $1/N$, this potential comes from the sum of all *planar* diagrams involving one fermion loop as shown in figure 1, and is $\mathcal{O}(N)$. The flavor structure of this class of diagrams insures that the vectorial subgroup, $SU(2)_V$, remains unbroken[1] [21].

Because g_{tc} varies with N, we must be careful about what we mean by the scale of topcolor breaking. It will be most useful to define this scale as the Lagrangian mass of the topcolor gauge boson. This is appropriate because it is this mass which acts as the cut-off for the effective theory below the symmetry breaking scale. In fact we will soon see that the topcolor boson mass, M, remains fixed as $N \to \infty$, which means the the vacuum expectation value that is responsible for the breaking must actually grow like $\sqrt{N}$.

Let Λ_{tc} be the scale at which the topcolor interactions would become strong if topcolor remained unbroken, *i.e.* it is the analog of Λ_{QCD} for the ordinary strong interactions. Λ_{tc} is then independent [19] of N as $N \to \infty$. If topcolor is broken at scale M, we can analyze the theory in two limits [22]. First, if $M \gg \Lambda_{tc}$, we expect the low-energy theory to contain massless fermions which interact (ignoring the standard model interactions) only by the exchange of heavy, weakly-coupled, topgluons. In this limit, chiral symmetry is unbroken and $\langle \Phi \rangle = 0$. On the other hand, if $M \ll \Lambda_{tc}$ the topcolor interactions become strong and we may expect chiral symmetry to be broken in a manner similar to that in QCD. Here chiral symmetry is broken and $\langle \Phi \rangle \propto I \neq 0$. If the transition between these two regimes is continuous, *i.e.* if the chiral-symmetry breaking transition is of second order as $g_{tc}^2(M)$ (the value of the topcolor coupling at scale M) is varied, then the mass-squared of the effective Φ field goes continuously through zero at a critical coupling g_c^2.

Since Λ_{tc} is independent of N as $N \to \infty$, M must also be independent of N as $N \to \infty$. For this to be consistent with the condition of criticality, $g_{tc}^2(M) = g_c^2$, g_c^2 must be proportional to $1/N$, *i.e.*

$$g_c^2 = \frac{\tilde{g}_c^2}{N} \ , \tag{3.2}$$

with $\tilde{g}_c$ of $\mathcal{O}(1)$. This agrees with the NJL analysis in which $\kappa_c = \pi/N$ in (2.5) is proportional to g_c^2.

If the transition is second-order, the effective low-energy theory for $g_{tc}^2(M)$ close to g_c^2 is one with a light scalar Φ (with mass $\ll M$) coupled to fermions, just as in the NJL analysis [22]. Therefore, while the analysis of topcolor presented in the previous section depends on both the large-N and NJL approximations to the full top-color theory, the important properties may be expected to survive beyond these

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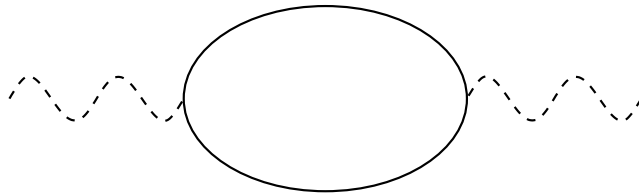[1]In fact, for any QCD-like vector gauge theory, this remains true exactly [20].

Figure 2: Fermion-loop contribution to vacuum polarization correction to the $U(1)$ gauge-boson propagator. The corresponding term in the β-function for the coupling grows as N. To have a well-defined large-N limit, therefore, the $U(1)$ coupling must scale as $N^{-1/2}$.
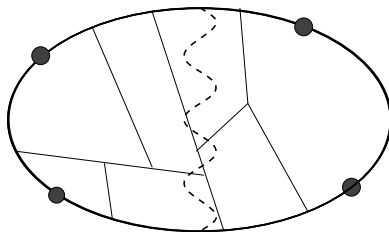

Figure 3: A typical contribution to the effective potential for Φ to leading order in $1/N$ and lowest order in the (symmetry-violating) $U(1)$ couplings. The $U(1)$ gauge boson propagator is shown in dashed lines.

approximations so long as the chiral phase transition is sufficiently second-order [22].

Now consider the contribution of the isospin-violating $U(1)$ interaction. In order to have a well-defined large-N limit, we must insure that the vacuum polarization diagrams (see figure 2) have a finite large-N limit. Hence, in analogy with eqn. (3.1), we must take a $U(1)$ coupling

$$ g_1 = \frac{\tilde{g}_1}{\sqrt{N}} \ , \tag{3.3} $$

and hold $\tilde{g}_1$ fixed as $N \to \infty$. This immediately implies that κ_1 in (2.1), is proportional to $1/N$ — so that κ_1/κ_{tc} in independent of N, as promised above.

We can now generalize the analysis to the general effective field theory description, beyond the NJL approximation. To leading order in $1/N$, the contribution of the $U(1)$ interaction to the effective potential for Φ comes from planar diagrams involving one fermion loop and one $U(1)$ gauge-boson exchange (figure 3). Regardless of the specific charges chosen, these contributions are $\mathcal{O}(1)$.

For the $U(1)$ interaction to "tilt" the vacuum and break $SU(2)_V$, the contribution of the $U(1)$ coupling to the mass-squared of the Φ field must compete with

5

the leading contribution from topcolor. Therefore, the contribution of the topcolor interactions to the mass-squared of Φ must be adjusted to be $\mathcal{O}(1)$. That is, the topcolor chiral phase transition must be a second order transition to subleading order[2] in N *and*

$$\frac{\Delta g_{tc}^2(M)}{g_c^2} = \frac{g_{tc}^2(M) - g_c^2}{g_c^2} = \mathcal{O}\left(\frac{1}{N}\right), \qquad (3.4)$$

i.e. $\tilde{g}_{tc}^2(M)$ must be "tuned" to equal $\tilde{g}_c^2$ to $\mathcal{O}(1/N)$.

These considerations have an immediate phenomenological consequence. Treating $\Phi = (\phi_t\,\phi_b)$ as a pair of Higgs fields, we see that the difference in the mass-squared of ϕ_t and ϕ_b

$$\frac{|m_{\phi_t}^2 - m_{\phi_b}^2|}{M^2} = \mathcal{O}\left(\frac{1}{N}\right) \qquad (3.5)$$

is subleading[3] in N [9, 26]. Independent of the NJL-approximation, therefore, we expect that the mass of the ϕ_b doublet cannot be significantly larger than the weak scale unless M is taken to be much larger than the weak scale. This extra light ϕ_b doublet could give rise to dangerous effects in B-$\bar{B}$ mixing and B-meson decays [9, 10].

Note that this argument does not depend on any of the details that lead to (2.1). We need not assume that the topcolor and $U(1)$ gauge boson masses are equal, or even of the same order of magnitude.

4   Beyond the NJL Approximation: $\Delta\rho$

The NJL approximation treats only the terms in eqn. (2.1) which couple left-handed and right-handed currents. Consider the effects of the topcolor interactions in eqn. (2.1) coupling *pairs* of left-handed (LL) or right-handed (RR) currents, which are not included in the NJL approximation. The LL terms may be Fierz transformed to color-singlet form

$$\left(\bar{\psi}_L\gamma^\mu\frac{\lambda^a}{2}\psi_L\right)^2 \to \frac{1}{2}(\bar{\psi}_L\gamma^\mu\psi_L)_y^x(\bar{\psi}_L\gamma_\mu\psi_L)_x^y - \frac{1}{2N}(\bar{\psi}_L\gamma^\mu\psi_L)(\bar{\psi}_L\gamma_\mu\psi_L) . \qquad (4.1)$$

Here x and y are $SU(2)_L$ (flavor) indices and an analogous expression holds for the terms involving a product of right-handed currents with $L \leftrightarrow R$.

Treating the operator in eqn. (4.1) as a perturbation, its contribution in the low-energy effective scalar theory may be computed to leading-order as shown in figure 4. As in QCD [27, 28], to leading-order in N, the "vacuum insertion" approximation may be used to evaluate this contribution. The left-handed and right-handed flavor currents can then be matched individually to the corresponding flavor currents in

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[2]It is of note that at next-to-leading order in N there are contributions which may make the phase transition weakly first order [23, 24] due to the Coleman-Weinberg mechanism [25].

[3]This can be seen in eqn. 2.5, for example, because $m_{\phi_t}^2 - m_{\phi_b}^2 \propto |\kappa_t - \kappa_b| = \mathcal{O}(1/N)$.
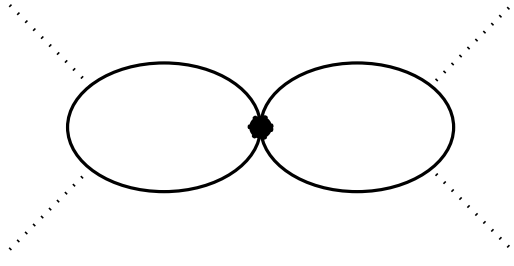
Figure 4: Leading contribution of product of left-handed currents in eqn. (2.1) to the low-energy effective scalar theory. Fermion lines are solid, scalar (Φ) lines are dotted, and the left-handed four-fermion operator is represented by the solid circle in the center.

the low-energy scalar theory. Since Φ transforms as a $(2, \bar{2})$ under $SU(2)_L \times SU(2)_R$, we find

$$
\begin{aligned}
(\bar{\psi}_L \gamma^\mu \psi_L)^x_y &\to -i(\Phi \overset{\leftrightarrow}{\partial^\mu} \Phi^\dagger)^x_y + \mathcal{O}\left(\frac{\Phi\Phi^\dagger}{M^2}, \frac{\partial^2}{M^2}\right) , \\
(\bar{\psi}_R \gamma^\mu \psi_R)^x_y &\to -i(\Phi^\dagger \overset{\leftrightarrow}{\partial^\mu} \Phi)^x_y + \mathcal{O}\left(\frac{\Phi\Phi^\dagger}{M^2}, \frac{\partial^2}{M^2}\right) .
\end{aligned}
\tag{4.2}
$$

The fermion currents above contains both $SU(2)_{L,R}$ and $U(1)_{L,R}$ pieces. However, in the *linear* sigma model, the lowest-order term in the effective Lagrangian which distinguishes between the $SU(2)$ triplet and singlet pieces of Φ is of dimension six. Therefore the relation of eqn. (4.2) holds for both parts up to corrections of $\mathcal{O}(1/M^2)$.

In the leading vacuum-insertion approximation we may then immediately write down the effective interaction arising from the left-handed and right-handed operators of the form of eqn. (4.1):

$$
\begin{aligned}
\delta\mathcal{L}_{LL} &= +\frac{2\pi\kappa_{tc}}{M^2}\mathrm{Tr}(\Phi \overset{\leftrightarrow}{\partial^\mu} \Phi^\dagger)(\Phi \overset{\leftrightarrow}{\partial_\mu} \Phi^\dagger) , \\
\delta\mathcal{L}_{RR} &= +\frac{2\pi\kappa_{tc}}{M^2}\mathrm{Tr}(\Phi^\dagger \overset{\leftrightarrow}{\partial^\mu} \Phi)(\Phi^\dagger \overset{\leftrightarrow}{\partial_\mu} \Phi) ,
\end{aligned}
\tag{4.3}
$$

plus corrections of order M^{-4} and $1/N$. Note that these interactions are $SU(2)_L \times SU(2)_R$ invariant. This is as it should be since the topcolor interactions are themselves $SU(2)_L \times SU(2)_R$ invariant.

The vacuum expectation value of the field is

$$
\langle\Phi\rangle = \begin{pmatrix} \frac{f_t}{\sqrt{2}} & 0 \\ 0 & 0 \end{pmatrix} ,
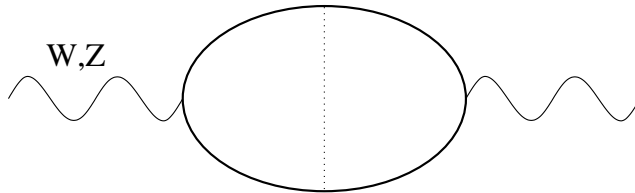\tag{4.4}
$$

Figure 5: Example of a potentially large two-loop contribution to $\Delta\rho$ arising from exchange of light composite scalars. Fermions (the top and bottom) are represented by solid lines and scalars by the dotted line. These contributions are subleading in $1/N$, but are not suppressed by $1/M^2$.

which breaks $SU(2)_L \times SU(2)_R \rightarrow U(1)_{em}$. Note that the potential "custodial" $SU(2)_V$ [29] symmetry is broken by the alignment of the vacuum [12, 13, 14, 15]. Because of the usual *accidental* symmetry, the terms of dimension four or less in the effective Lagrangian *cannot* give rise to a deviation of the rho parameter,

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W}, \tag{4.5}$$

from one. The leading contribution arises from the operator in eqn. (4.3). Promoting the partial derivatives to gauge covariant derivatives, we may calculate the gauge-boson masses and find the contribution to $\rho - 1$

$$\Delta\rho = \frac{2\pi e^2 \kappa_{tc}}{\sin^2 \theta_W \cos^2 \theta_W} \frac{f_t^4}{M_Z^2 M^2} . \tag{4.6}$$

Note that this contribution to $\Delta\rho$ is of leading-order in N and results from the LL and RR topcolor interactions not included in the NJL approximation. To leading order in N, this result agrees with the calculation given in [11].

The calculation of $\Delta\rho$ relies on treating the LL and RR portions of the topcolor interactions as perturbations to the NJL model. However, even beyond this approximation we expect the effective Lagrangian for the composite Φ will contain terms of the form shown in eqn. (4.3). We therefore generally expect contributions of this order of magnitude[4].

The corrections discussed above are of leading-order in $1/N$, but are suppressed by $1/M^2$. There are also corrections coming from the exchange of the light scalars in Φ (such as that shown in fig. 5). These contributions are subleading in $1/N$ but are *not* suppressed by $1/M^2$. They are also potentially large because the couplings to the top-quark are of order m_t/f_t, approximately four times *larger* than the corresponding contributions from a standard model Higgs boson. It is amusing to note

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[4]This may be viewed as a special case of the constraints on composite Higgs models discussed in [30].

that if these particles were light, this contribution could be *negative* [31, 32, 33, 34]. However, phenomenological constraints from $Z \to b\bar{b}$ [35] require that these particles be relatively heavy and these contributions are probably suppressed.

Finally, we comment on the situation in models [36, 26] which do *not* contain a $U(1)$ tilting interaction. Instead, the strong topcolor group is arranged to couple to the left-handed top-bottom doublet (and, in "topcolor II" [26], the charm-strange doublet) and the right-handed top, but not to the right-handed bottom. The large-N analysis of the first section then no longer applies — however, anomaly cancellation implies that the number of *flavors* to which topcolor couples must scale with the number of colors. This situation is reminiscent of the large-N & large-N_f analysis of [37]. Such a theory will have a large number of scalars and contributions of the form shown in fig. 5 would no longer be suppressed by $1/N$. Furthermore, as the topcolor interactions themselves violate custodial $SU(2)$, there will still generically be contributions analogous to eqn. (4.3) which yield contributions to $\Delta\rho$ of the same order of magnitude [30].

Topcolor and topcolor-assisted technicolor [1, 2] provide examples of dynamical electroweak symmetry breaking which include top-condensation [3, 4, 5, 6, 7, 8], thereby naturally incorporating a heavy top quark. In this note we have discussed the roles of the Nambu–Jona-Lasinio and large-N approximations used in phenomenological analyses and have discussed the dynamical behavior of topcolor theories beyond these approximations. We have shown that, in models with a $U(1)$ tilting interaction, in order to provide for top-condensation but not bottom-condensation the top-color coupling must be adjusted to equal the critical value for chiral symmetry breaking up to $\mathcal{O}(1/N)$. A consequence of these considerations is that the potentially dangerous "bottom-pions" [9, 10] are naturally light. We have also considered contributions beyond the NJL approximation and shown that the other portions of the topcolor interactions, in particular products of pairs of left-handed currents or right-handed currents, give rise to contributions to $\rho - 1$. These contributions, which have been estimated previously [11] by direct computation, are of leading-order in N and are the result of vacuum-alignment.

Acknowledgments

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