



# New Phases and Dissociation-Recombination of Hydrogen Deuteride to 3.4 Mbar

## Citation

Dias, Ranga P., Ori Noked, and Isaac F. Silvera. 2016. "New Phases and Dissociation-Recombination of Hydrogen Deuteride to 3.4 Mbar." *Physical Review Letters* 116 (14) (April 8). doi:10.1103/physrevlett.116.145501.

## Published Version

doi:10.1103/PhysRevLett.116.145501

## Permanent link

<http://nrs.harvard.edu/urn-3:HUL.InstRepos:29374824>

## Terms of Use

This article was downloaded from Harvard University's DASH repository, and is made available under the terms and conditions applicable to Other Posted Material, as set forth at <http://nrs.harvard.edu/urn-3:HUL.InstRepos:dash.current.terms-of-use#LAA>

## Share Your Story

The Harvard community has made this article openly available.  
Please share how this access benefits you. [Submit a story](#).

[Accessibility](#)

## New Phases and Dissociation-Recombination of Hydrogen Deuteride to 3.4 Mbar

Ranga P. Dias, Ori Noked, and Isaac F. Silvera\*

*Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*

(Received 30 September 2015; published 8 April 2016)

We present infrared absorption studies of solid hydrogen deuteride to pressures as high as 340 GPa (100 GPa = 1 Mbar) in a diamond anvil cell and temperatures in the range 5–295 K. Above 198 GPa the HD sample transforms to a mixture of HD, H<sub>2</sub>, and D<sub>2</sub>, interpreted as a process of dissociation and recombination. Three new phase lines are observed, two of which differ remarkably from those of the high-pressure homonuclear species, but none are metallic. The time-dependent spectral changes are analyzed to determine the molecular concentrations as a function of time; the nucleon exchange achieves steady state concentrations in ~20 h at ~200 GPa.

DOI: [10.1103/PhysRevLett.116.145501](https://doi.org/10.1103/PhysRevLett.116.145501)

In 1935 Wigner and Huntington predicted that hydrogen would transform to an atomic metallic solid at high density [1]. There is a decades-long effort to metallize the hydrogens (H<sub>2</sub>, D<sub>2</sub>, and HD) in the solid state at high pressures and low temperatures or moderate pressures and high temperatures. We have studied HD to very high pressures. With increasing pressure we first observe the line separating phases I–III; then when entering a new phase at 198 GPa, HD transforms to HD, H<sub>2</sub>, and D<sub>2</sub>. We propose that the nucleon exchange is due to a process of dissociation followed by recombination that we call DISREC (see discussion below). At still higher pressures and low temperatures, yet another phase is entered. In contrast to pure isotopic systems, the new phase lines are almost vertical and appear to intersect the  $T = 0$  K axis. The observation of new phases is unexpected, with a significant departure from the behavior of the pure isotopic solids; DISREC confirms phenomena that have only been observed in theoretical simulations. None of the newly observed phases are metallic.

The hydrogens form remarkable solids whose properties are dominated by quantum effects: the ortho-para isomers due to the Pauli principle and large zero-point energies or motions in the crystal lattice. At low pressure (LP), ortho and para, fundamentally distinguished by their nuclear spin states, have distinct structures and properties. The ortho-para concentrations are metastable with slow conversion to equilibrium [2]. At megabar pressures there are predictions that hydrogen may be a liquid at  $T = 0$  K or low temperature due to the zero-point energy [3,4]. HD is a special case because the nucleons are distinguishable, and thus the ortho-para distinction is not a feature and molecular states rapidly thermalize. A distinction of HD is that the free molecules have a small permanent electric-dipole moment ( $5.85 \times 10^{-4}$  D) [5]. Our ultimate goal is to extend the phase diagram to higher static pressures and low or modest  $T$  for any of the isotopes to achieve the metallic phase. A first-order phase transition to liquid metallic hydrogen at

static pressures and high temperatures has recently been observed [6], and liquid metals of hydrogen and deuterium have been observed in high-temperature shock experiments [7,8]. There have been a number of very high static pressure studies of the solid homonuclear species at lower temperatures, but none have achieved metallization [9–14].

To understand HD in the context of the hydrogens, we first discuss the known phases of the isotopes. Ideally the structures of phases are determined by x-ray or neutron diffraction techniques, but these are very challenging measurements for the high-pressure hydrogens as the samples are very small and the molecules are very weak scatterers. In practice the phase lines of the hydrogens have been determined by optical methods, studying Raman or IR active modes. At a phase transition, modes disappear, appear, or undergo discrete shifts, as the symmetry of the lattice changes; these changes pinpoint the phase lines. Samples with mixed ortho-para concentrations are disordered and lack translational symmetry but still demonstrate similar phase transitions as determined by optical measurements. The loss of translational symmetry is more distinct for isotopic mixtures due to the different masses (a much larger distinction than nuclear spin states), yet in this Letter we present clear evidence of phase transitions in these disordered systems. We believe that the observed transitions are structural, for example transitions to structures having graphenelike planes with translational disorder within the planes. There are a small number of ways that clusters of isotopes can be formed and these can lead to distinct spectra.

The phase diagrams of the solid hydrogens at high pressure are similar. We discuss these with the help of Fig. 1, where we show known phase diagrams, as well as our new data that we discuss later. Since the phase diagrams of hydrogen and deuterium are similar, for comparison to HD we shall focus on one of these, D<sub>2</sub>. For *pure* solid ortho-D<sub>2</sub>, a transition from the low-pressure hexagonal phase to an orientationally ordered structure called the

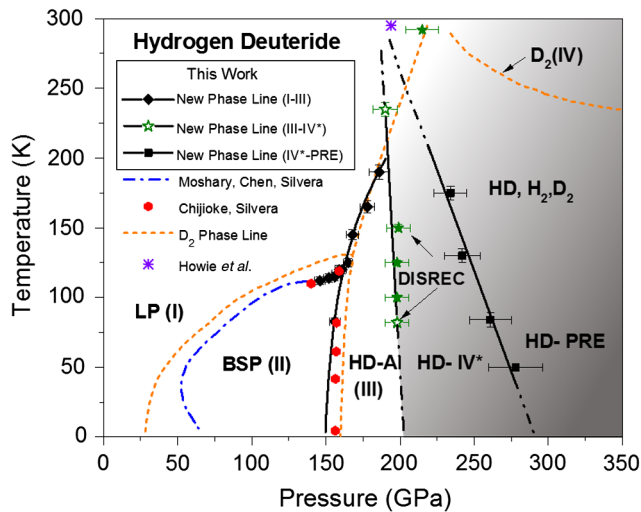


FIG. 1. The phase diagram of HD, extending the earlier measurements beyond the triple point to establish the I–III line. For comparison, we show the phase diagram of  $D_2$ . Solid lines (above  $\sim 200$  GPa) indicate new phase lines of mixtures of isotopes. For the III to HD-IV\* line, we only traversed the phase line at  $P \sim 198$  GPa,  $T = 82$ , and 240 K, indicated by open stars, while DISREC was observed all along the phase line (filled stars). The shaded area shows where hydrogen, deuterium, and hydrogen deuteride spectra were observed. We also show a DISREC point at room temperature and a point from Howie *et al.* [12] where they observed a phase transition.

broken symmetry phase (BSP) takes place at 28 GPa [15]; a second transition occurs at  $\sim 150$  GPa [16] to the D-A phase, which is also orientationally ordered. For *mixed* ortho-para concentrations, which lack translational symmetry, the LP, BSP, and D-A phases are called phases I, II, and III. Recently, at higher pressures ( $\sim 230$  GPa) and around room temperature, an insulating phase named IV has been observed for  $H_2$  and  $D_2$  [11,12,17], having a  $P, T$  phase line with a rather weak pressure dependence [17,18].

A number of years ago, Brown and Daniels [19] showed that when a mixture of  $H_2$  and  $D_2$  is pressurized (pressures between 1.4 and 60.5 GPa) at room temperature, a process takes place to form HD, leaving a  $H_2, D_2, HD$  mixture (this process has recently been confirmed with higher pressure studies by Howie *et al.* up to almost 300 GPa [20]). Subsequently, Moshary, Chen, and Silvera [21] studied a sample of HD to 110 GPa at 77 K, showing that  $2HD \rightarrow H_2 + D_2$  does not take place, while Chijioke and Silvera [22] saw no evidence of isotope changes to 159 GPa at 160 K. In this Letter we report on a study of HD to a pressure of  $\sim 340$  GPa. No changes of HD were observed after 14 h at 184 GPa and 82 K. Above 198 GPa and temperatures ranging from 5 to 295 K, we observe the formation of  $H_2$  and  $D_2$ , while HD remains a component. We extend the phase diagram for HD, studied earlier [22,23], to our highest pressures. The lattice is not metallic at the pressures studied and we conclude that the process  $2HD \leftrightarrow H_2 + D_2$  can take place at ultrahigh pressure.

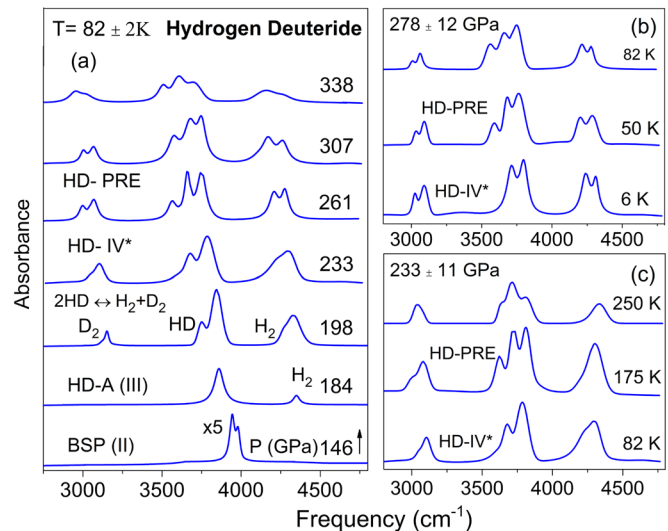


FIG. 2. (a) Infrared spectra as a function of pressure at a temperature of 82 K. At 146 GPa the impurity hydrogen vibron line is too weak to be observed in phase II. With increasing pressure one sees the onset of deuterium lines and growth of hydrogen lines, evidence of DISREC, as well as splitting of the spectral lines. For the spectrum at 198 GPa, the sample was in the process of transformation and not at final equilibrium concentrations. Right: IR spectra as a function of temperature for the indicated pressures. (b) As temperature is lowered, a transition from PRE to IV\* is observed; (c) shows the reversal, IV\* to PRE.

The samples of HD gas (our virgin sample of HD had  $\sim 4\%$   $H_2$  impurity) were cryogenically loaded in a diamond anvil cell. The diamond anvil cell was mounted in a cryostat similar to an apparatus described elsewhere [24]. Conic-shaped type II diamonds with 50- $\mu$ m diameter culets were coated with alumina that acts as a hydrogen diffusion barrier; a rhenium gasket was used. IR spectra were obtained in the temperature range 5–295 K with a Nicolet FTIR interferometer that uses an internal thermal light source. The experiment lasted 6 weeks and terminated with failure of the diamond anvils. At the highest pressures the diameter of the sample was about 14  $\mu$ m [see photo in Supplemental Material (SM) [25]]; interferograms were averaged for about 35 min to obtain adequate signal-to-noise ratio (see SM [25]). Although the sample contained a ruby particle for pressure determination, our studies extended far beyond the calibrated ruby scale [26]. As is common for ultrahigh-pressure experiments on the hydrogens, we used frequencies of the IR vibron modes for pressure determination [27,28], calibrated against the diamond phonon scale [29] (see SM [25]).

Spectra of the sample were obtained as a function of pressure and temperature. In Fig. 2 we show fits to the IR absorption spectra (also see SM [25]). Remarkably, the IR spectral lines that we observe from the three species remain rather sharp and resemble spectra in the pure isotopic samples, in spite of the conceptual picture that any

molecule is in a cluster of neighbors of mixed isotopic composition. This is probably because the intermolecular interactions average out for mixed samples and impurity modes (due to isotopes doped into an otherwise isotopically pure sample) are experimentally known to follow the IR mode pressure dependence, as discussed in Ref. [21]. Figure 2(a) shows IR spectra at a fixed temperature (82 K). At 198 GPa we see the appearance of a  $D_2$  vibron line (after  $\sim 11$  h), splitting of the HD vibron line, and growth of the  $H_2$  vibron line intensity; all spectral lines clearly show a splitting at 261 GPa. Figures 2(b) and 2(c) show spectra for the crossing of phase lines at fixed pressures and varying temperatures. The splittings, shifts, and changes for all of our measurements are summarized in Fig. 3, which shows the peak frequencies of the IR lines as a function of pressure at 82 K. It is not surprising that the splittings for  $H_2$  and  $D_2$  are different from splittings for the pure species [14,17]; the isotopic concentrations are very different. Furthermore, the phase lines in the case of the HD isotopic mixtures are almost vertical, indicating possible different structures.

Figure 1 shows the  $P, T$  data points for the phases of HD. First of all, our data confirm the triple point found in Ref. [22] and we determine the phase line between phase I and III. We observe two additional phase lines, very different from the phase line for pure deuterium or hydrogen. We call the lower-pressure phase HD-IV\*; the higher-pressure phase may precede the transition to metallic hydrogen deuteride and we have named it HD-PRE. Both phases have negative slopes with regard to pressure and extrapolate to 200 and 290 GPa at the  $T = 0$  K axis, respectively. These phase lines, which are almost vertical, are quite different from the behavior in the pure isotopic samples in which phase IV only appears at relatively high temperature with a weak dependence on pressure (see Fig. 1 for phase IV of  $D_2$ ).

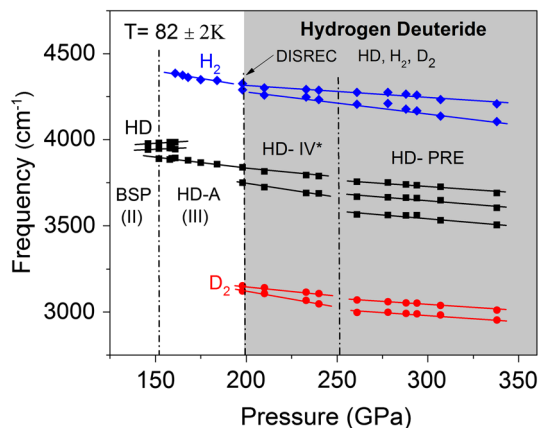


FIG. 3. Peak frequencies of the IR modes as a function of pressure, indicating phase transitions where splittings or new modes appear. The shaded area is where a mixed isotope sample, due to DISREC, was observed.

Simulations have shown that there are several structures, close in energy, that are composed of hexagonal graphene-like planes for the pure isotope solids [30–35]. We do not have an explanation for the difference between the phase lines for pure or mixed isotope samples exhibited in our data, as the solid structures are unknown. We comment that in the pure homonuclear species the transition is believed to be entropy driven [33] which may be mitigated by a mixed molecular sample. Raman spectroscopy supports a structure with alternating hexagonal layers in the pure isotopic solids. Two distinct Raman vibrons were observed, corresponding to the two proposed hexagonal planes. We note that only the structures of the lowest-pressure phases are known [2] and the structures of phases II–IV are at best calculated and compared to spectroscopic observations.

Phase IV of  $H_2$  and  $D_2$  has been interpreted by Howie *et al.* [12] as being composed of distinct hexagonal layers, called a mixed molecular-atomic phase with molecular and atomic layers; however, our measurements, as well as a simulation [33], do not support this assertion. The existence of the two distinct vibron modes arising from two distinct sets of planes is incompatible with the existence of atomic layers; planes of atoms do not have vibron modes. If the model proposed by Howie *et al.* [12] were correct, that alternate layers are atomic, then when phase IV is entered, half of the molecules would dissociate and this should have a large impact on the intensity of the vibron modes.

For our high-pressure observations we believe that the mechanism to form hydrogen and deuterium molecules is a process involving fluctuating dissociation-recombination of molecules. There is a probability for recombination with atoms from dissociated neighbor molecules, which we call DISREC. This is the importance of studying HD, as the recombined atoms can form molecules of hydrogen and deuterium that can be spectroscopically detected. This is theoretically supported by the simulations [34,36] (see films in their SM), which showed that of the alternate hexagonal layers, layer I contained orientationally disordered molecules and layer II orientationally ordered molecules (Refs. [33,36] name these  $B$  and  $G$  layers, respectively). Layer I was stable against nucleon transfer with increasing pressure, while the molecules in layer II (which have expanded bonds) could dissociate and recombine. We believe that this process occurs in HD. For random processes, DISREC would lead to a 25%-25%-50% concentration of  $H_2$ ,  $D_2$ , and HD, respectively.

HD can be used as a diagnostic tool to understand very high pressure structures. Since we observe that HD molecules are stable until phase HD-IV\* is entered, one should see a growth of the  $H_2$  and  $D_2$  concentration due to the DISREC process. In transitioning from phase III to HD-IV\*, we observe the growth of  $H_2$  and  $D_2$  IR modes. When the molecular concentrations arrive at equilibrium, the composition of our sample is  $\sim 56\%$  HD,  $\sim 26\%$   $H_2$ , and  $\sim 18\%$   $D_2$  (our HD sample starts with 4%  $H_2$  impurity)

determined from the integrated intensities of the IR modes. This implies that for pressures above  $\sim 198$  GPa, the entire sample undergoes DISREC as the composition is  $\sim 0.5\text{HD} + 0.25(\text{H}_2 + \text{D}_2)$ , when our final concentrations are corrected for the initial  $\text{H}_2$  impurity.

Howie *et al.* [20] studied  $\text{H}_2$ ,  $\text{D}_2$ , HD mixtures at room temperature and observed splitting of the Raman vibron lines above about 195 GPa, suggesting a transformation to phase IV (called IV') correlated to the vibron frequencies arising from different layer types. However, they did not study the temperature dependence of the phase line. A linear extrapolation of the HD-IV\* – HD-PRE phase lines (our data) to room temperature (Fig. 1) shows that with increasing pressure at room temperature they may have been in the HD-PRE phase rather than the phase they identified as similar to IV and named HD-IV'. In our mixed sample the HD IR vibron line shown in Fig. 3 splits into three lines with the entry into the HD-PRE phase, while  $\text{H}_2$  and  $\text{D}_2$  IR vibron lines do not split, which is unexpected. Howie *et al.*, studying mixed samples [20], suggest that the transformation at 198 GPa is one involving double hexagonal layers. Apparently, they used the pressure dependence of the Raman vibrons from phase IV of the pure species for layers I (B) and II (G) to identify their structure. In the SM [25] we replot some of their data for a 50-50  $\text{H}_2$ - $\text{D}_2$  mixture that after transformation should be quite similar to our composition. For the pure species their B modes are approximately constant with increasing pressure while the G modes have a steep negative slope. This is the case for the modes that they associate with deuterium and hydrogen deuteride, but this is not the behavior for the hydrogen G mode, implying a problem with the mode identification. This is a complex system that should be reanalyzed to investigate possible structures and associated Raman and IR active modes.

We have used the integrated intensities of the IR lines to determine the concentrations of the various species as a function of time at a pressure of  $198 \pm 8$  GPa and temperature of  $82 \pm 2$  K. Observation of the sample for 14 h at  $P = 184$  GPa,  $T = 82$  K showed no DISREC; i.e., the HD concentration was stable. The change of concentration versus time for HD-IV\* is plotted in Fig. 4. To describe the spectral kinetics data (see Fig. 4), we used Avrami's equation [37–40]  $[x(t) - x_e]/(x_0 - x_e) = \exp(-bt^n)$ , where  $x$  is the concentration of a species at time  $t$ ,  $x_0$  and  $x_e$  are initial and equilibrium values, and  $b$  and  $n$  are the parameters relating to the rate of reaction and the processes that characterize the reaction (the Avrami exponent). The reaction rate  $k$  is equal to  $b^{1/n}$ . We obtained the best fit with the Avrami parameters (see inset in Fig. 4) of  $n_{\text{HD}} = 2.0 \pm 0.02$ ,  $n_{\text{H}_2} = 1.75 \pm 0.04$ , and  $n_{\text{D}_2} = 1.7 \pm 0.08$ , which indicates that the reaction is in the nucleation-dominated regime [41]. We believe that the nucleation process is one in which nuclei grow in the HD lattice, which transforms to hexagonal graphenelike planes

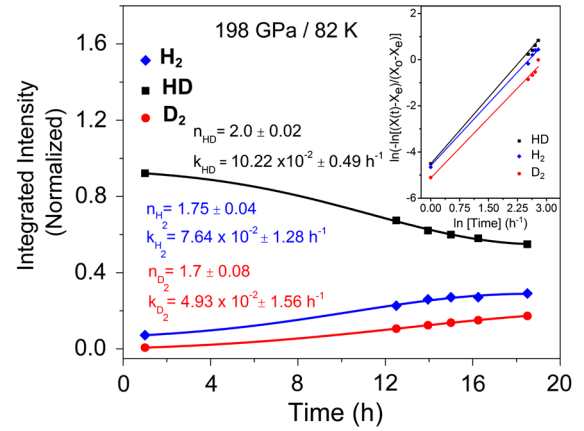


FIG. 4. The normalized IR integrated intensity changes of hydrogen isotopes plotted as a function of time, showing the kinetics associated with the DISREC at 198 GPa and 82 K. These time-dependent intensity changes are fitted to Avrami's equation (see the inset) to yield the rate constants and Avrami exponent marked on the plots.

such as the I, II (or B, G), studied in simulations, followed by DISREC. Our measurements are consistent with the transformation being homogeneous throughout the sample; our measurements cannot determine if the  $\text{H}_2$  impurities influence the nucleation.

In summary, we have studied hydrogen deuteride up to  $\sim 340$  GPa and compared its behavior with that of pure molecular hydrogen and deuterium. HD transforms to an isotopic mixture for pressures above  $\sim 200$  GPa. Three phase lines were determined: III/HD-IV\*, HD-IV\*/HD-PRE, and the I/III line. The new high-pressure phases reported here are evidently different from those of the homonuclear species, in particular, the pressure slopes of the phase lines; simulations may offer insight. In the goal of metallization of the hydrogens at lower temperatures, HD remains an important candidate, with the caveat that it would be challenging to transform from pure HD to atomic metallic H and D. When the phase HD-IV\* is entered, DISREC commences. Our view, based on results of simulations, is that in this region the structure is layered; a negligible density of atomic species exists and all atoms are bound as molecules, probably with bond lengths that depend on the planes they occupy. Atoms can exist due to bond-breaking fluctuations, but they rapidly recombine, so that the expectation value of free atoms in the lattice is very small. It is interesting to note that nucleon exchange probably takes place in the pure homonuclear samples at all pressures, but this is difficult to detect as there is no change in concentrations. The HD sample offered a unique opportunity to observe this exchange. At low pressure the process  $\text{H}_2 + \text{D}_2 \leftrightarrow \text{HD}$  is probably due to tunneling rather than DISREC. A reason why the reverse process does not take place for lower pressures may be energetics. In this case, the probability to tunnel back is blocked by energy

conservation; that is, back tunneling requires additional thermally populated excitations to conserve energy between the initial and final states. Low density HD is also of special astronomical interest with the recent detection of HD in interstellar clouds [42,43]. Deuterium is believed to have been created primordially in the big bang nucleosynthesis. Determination of the deuterium abundance, relative to hydrogen in the atmospheres of planetary systems and nebulae, provides important cosmological constraints on planetary and stellar formation [44]. In this regard, the dissociation and recombination process is quite important and abundances must also consider the HD concentrations.

We thank M. Zaghoo, A. Salamat, G. Ackland, G. Borstad, and R. Husband for discussions of the properties of the hydrogens. The NSF, Grant No. DMR-1308641, and the DOE Stockpile Stewardship Academic Alliance Program, Grant No. DE-NA0001990, supported this research. Preparation of diamond surfaces was performed in part at the Center for Nanoscale Systems (CNS), a member of the National Nanotechnology Infrastructure Network (NNIN), which is supported by the National Science Foundation under NSF Award No. ECS-0335765. CNS is part of Harvard University.

\*Corresponding author.

silvera@physics.harvard.edu

- [1] E. Wigner and H. B. Huntington, *J. Chem. Phys.* **3**, 764 (1935).  
 [2] I. F. Silvera, *Rev. Mod. Phys.* **52**, 393 (1980).  
 [3] E. G. Brovman, Y. Kagan, and A. Kholas, *Sov. Phys. JETP* **35** 783 (1972).  
 [4] J. Chen, X.-Z. Li, Q. Zhang, M. I. J. Probert, C. J. Pickard, R. J. Needs, A. Michaelides, and E. Wang, *Nat. Commun.* **4**, 3064 (2013).  
 [5] M. Trefler and H. P. Gush, *Phys. Rev. Lett.* **20**, 703 (1968).  
 [6] M. Zaghoo, A. Salamat, and I. F. Silvera, *arXiv:1504.00259*.  
 [7] S. T. Weir, A. C. Mitchell, and W. J. Nellis, *Phys. Rev. Lett.* **76**, 1860 (1996).  
 [8] M. D. Knudson, M. P. Desjarlais, A. Becker, R. W. Lemke, K. R. Cochrane, M. E. Savage, D. E. Bliss, T. R. Mattsson, and R. Redmer, *Science* **348**, 1455 (2015).  
 [9] C. Narayana, H. Luo, J. Orloff, and A. L. Ruoff, *Nature (London)* **393**, 46 (1998).  
 [10] P. Loubeyre, F. Occelli, and R. LeToullec, *Nature (London)* **416**, 613 (2002).  
 [11] M. I. Eremets and I. A. Troyan, *Nat. Mater.* **10**, 927 (2011).  
 [12] R. T. Howie, C. L. Guillaume, T. Scheler, A. F. Goncharov, and E. Gregoryanz, *Phys. Rev. Lett.* **108**, 125501 (2012).  
 [13] C.-S. Zha, Z. Liu, and R. J. Hemley, *Phys. Rev. Lett.* **108**, 146402 (2012).  
 [14] P. Loubeyre, F. Occelli, and P. Dumas, *Phys. Rev. B* **87**, 134101 (2013).  
 [15] I. F. Silvera and R. J. Wijngaarden, *Phys. Rev. Lett.* **47**, 39 (1981).  
 [16] L. Cui, N. H. Chen, S. J. Jeon, and I. F. Silvera, *Phys. Rev. Lett.* **72**, 3048 (1994).  
 [17] C. S. Zha, Z. Liu, M. Ahart, R. Boehler, and R. J. Hemley, *Phys. Rev. Lett.* **110**, 217402 (2013).  
 [18] R. T. Howie, T. Scheler, C. L. Guillaume, and E. Gregoryanz, *Phys. Rev. B* **86**, 214104 (2012).  
 [19] D. M. Brown and W. B. Daniels, *Phys. Rev. A* **45**, 6429 (1992).  
 [20] R. T. Howie, I. B. Magdau, A. F. Goncharov, G. J. Ackland, and E. Gregoryanz, *Phys. Rev. Lett.* **113**, 175501 (2014).  
 [21] F. Moshary, N. H. Chen, and I. F. Silvera, *Phys. Rev. B* **48**, 12613 (1993).  
 [22] A. Chijioke and I. F. Silvera, *Phys. Rev. Lett.* **97**, 255701 (2006).  
 [23] F. Moshary, N. H. Chen, and I. F. Silvera, *Phys. Rev. Lett.* **71**, 3814 (1993).  
 [24] I. F. Silvera and R. J. Wijngaarden, *Rev. Sci. Instrum.* **56**, 121 (1985).  
 [25] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.116.145501> for pressure determination, provides a discussion of data analysis and fitting procedures, as well as photos of the sample at various pressures. We also show the thermodynamic pathways that were followed. A replotting of the data from Ref. [20] is also presented.  
 [26] A. Chijioke, W. J. Nellis, A. Soldatov, and I. F. Silvera, *J. Appl. Phys.* **98**, 114905 (2005).  
 [27] B. J. Baer, M. E. Chang, and W. J. Evans, *J. Appl. Phys.* **104**, 034504 (2008).  
 [28] R. T. Howie, E. Gregoryanz, and A. F. Goncharov, *J. Appl. Phys.* **114**, 073505 (2013).  
 [29] Y. Akahama and H. Kawamura, *J. Phys. Conf. Ser.* **215**, 012195 (2010).  
 [30] C. Pickard and R. J. Needs, *Nat. Phys.* **3**, 473 (2007).  
 [31] C. J. Pickard, M. Martinez-Canales, and R. J. Needs, *Phys. Rev. B* **85**, 214114 (2012).  
 [32] C. J. Pickard, M. Martinez-Canales, and R. J. Needs, *Phys. Rev. B* **86**, 059902(E) (2012).  
 [33] I. B. Magdau and G. J. Ackland, *Phys. Rev. B* **87**, 174110 (2013).  
 [34] H. Liu and Y. Ma, *Phys. Rev. Lett.* **110**, 025903 (2013).  
 [35] N. D. Drummond, B. Monserrat, J. H. Lloyd-Williams, P. L. Rios, C. J. Pickard, and R. J. Needs, *Nat. Commun.* **6**, 7794 (2015).  
 [36] H. Liu, J. Tse, and Y. Ma, *J. Chem. Phys. C* **118**, 11902 (2014).  
 [37] M. Borstad and C. S. Yoo, *J. Chem. Phys.* **135**, 174508 (2011).  
 [38] M. Avrami, *J. Chem. Phys.* **7**, 1103 (1939).  
 [39] M. Avrami, *J. Chem. Phys.* **8**, 212 (1940).  
 [40] M. Avrami, *J. Chem. Phys.* **9**, 177 (1941).  
 [41] S. F. Hulbert, *J. Brit. Ceram. Soc.* **6**, 1 (1969).  
 [42] D. A. Neufeld, J. D. Green, D. J. Hollenbach, P. Sonnentrucker, G. J. Melnick, E. A. Bergin, R. L. Snell, W. J. Forrest, D. M. Watson, and M. J. Kaufman, *Astrophys. J.* **647**, L33 (2006).  
 [43] E. A. Bergin, L. I. Cleaves, U. Gorti, K. Zhang, G. A. Blake, J. D. Green, S. M. Andrews, N. J. Evans, II, T. Henning, K. Öberg, K. Pontoppidan, C. Qi, C. Salyk, and E. F. v. Dishoeck, *Nature (London)* **493**, 644 (2013).  
 [44] J. K. Webb, R. F. Carswell, K. M. Lanzetta, R. Ferletk, M. Lemoine, A. Vidal-Madjark, and D. V. Bowen, *Nature (London)* **388**, 250 (1997).