Brownian Motion of Molecular Probes in Supercooled Liquids

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Joint Analysis of BICEP2/Keck Array and Planck Data

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(BICEP2/Keck and Planck Collaborations)

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We report the results of a joint analysis of data from BICEP2/Keck Array and Planck. BICEP2 and Keck Array have observed the same approximately 400 deg$^2$ patch of sky centered on RA 0 h, Dec. $-$57.5°. The combined maps reach a depth of 57 nK deg in Stokes Q and U in a band centered at 150 GHz. Planck has observed the full sky in polarization at seven frequencies from 30 to 353 GHz, but much less deeply in any given region (1.2 μK deg in Q and U at 143 GHz). We detect $150 \times 353$ cross-correlation in $B$ modes at high significance. We fit the single- and cross-frequency power spectra at frequencies $\geq 150$ GHz to a lensed-$\Lambda$CDM model that includes dust and a possible contribution from inflationary gravitational waves (as parametrized by the tensor-to-scalar ratio $r$), using a prior on the frequency spectral behavior of polarized dust emission from previous Planck analysis of other regions of the sky. We find strong evidence for dust and no statistically significant evidence for tensor modes. We probe various model variations and extensions, including adding a synchrotron component in combination with lower frequency data, and find that these make little difference to the $r$ constraint. Finally, we present an alternative analysis which is similar to a map-based cleaning of the dust contribution, and show that this gives similar constraints. The final result is expressed as a likelihood curve for $r$, and yields an upper limit $r_{0.05} < 0.12$ at 95% confidence. Marginalizing over dust and $r$, lensing $B$ modes are detected at 7.0σ significance.

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I. INTRODUCTION

The cosmic microwave background (CMB) [1] is an essential source of information about all epochs of the Universe. In the past several decades, characterization of the temperature and polarization anisotropies of the CMB has helped to establish the standard cosmological model (ΛCDM) and to measure its parameters to high precision (see, for example, Refs. [2,3]).

An extension to the standard big bang model, inflation, postulates a short period of exponential expansion in the very early Universe, naturally setting the initial conditions required by ΛCDM, as well as solving a number of additional problems in standard cosmology. Inflation’s basic predictions regarding the Universe’s large-scale geometry and structure have been borne out by cosmological measurements to date (see Ref. [4] for a review). Inflation makes an additional prediction, the existence of a background of gravitational waves, or tensor mode perturbations [5–8]. At the recombination epoch, the inflationary gravitational waves (IGW) contribute to the anisotropy of the CMB in both total intensity and linear polarization. The amplitude of tensors is conventionally parametrized by $r$, the tensor-to-scalar ratio at a fiducial scale. Theoretical predictions of the value of $r$ cover a very wide range. Conversely, a measurement of $r$ can discriminate between models of inflation.

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Tensor modes produce a small increment in the temperature anisotropy power spectrum over the standard ΛCDM scalar perturbations at multipoles $\ell \lesssim 60$; measuring this increment requires the large sky coverage traditionally achieved by space-based experiments, and an understanding of the other cosmological parameters. The effects of tensor perturbations on $B$-mode polarization is less ambiguous than on temperature or $E$-mode polarization over the range $\ell \lesssim 150$. The $B$-mode polarization signal produced by scalar perturbations is very small and is dominated by the weak lensing of $E$-mode polarization on small angular scales, making the detection of an IGW contribution possible [9–12].

Planck [13] was the third generation CMB space mission, which mapped the full sky in polarization in seven bands centered at frequencies from 30 to 353 GHz to a resolution of 33 to 5 arc min [14,15]. The Planck Collaboration has published the best limit to date on tensor modes using CMB data alone [3]: $r_{0.002} < 0.11$ (at 95% confidence) using a combination of Planck, SPT, and ACT temperature data, plus WMAP polarization, although the Planck $r$ limit is model dependent, with running of the scalar spectral index or additional relativistic degrees of freedom being well-known degeneracies which allow larger values of $r$.

Interstellar dust grains produce thermal emission, the brightness of which increases rapidly from the 100–150 GHz frequencies favored for CMB observations, becoming dominant at $\geq 350$ GHz even at high galactic latitude. The dust grains align with the Galactic magnetic
field to produce emission with a degree of linear polarization \cite{16}. The observed degree of polarization depends on the structure of the Galactic magnetic field along the line of sight, as well as the properties of the dust grains (see, for example, Refs. \cite{17,18}). This polarized dust emission results in both $E$ mode and $B$ mode, and acts as a potential contaminant to a measurement of $r$. Galactic dust polarization was detected by Archeops \cite{19} at 353 GHz and by WMAP \cite{20,21} at 90 GHz.

BICEP2 was a specialized, low angular resolution experiment, which operated from the South Pole from 2010 to 2012, concentrating 150 GHz sensitivity comparable to Planck on a roughly 1\% patch of sky at high Galactic latitude \cite{22}. The BICEP2 Collaboration published a highly significant detection of $B$-mode polarization in excess of the $r = 0$ lensed-$\Lambda$CDM expectation over the range $30 < \ell < 150$ in Ref. \cite{22} (hereafter BK-I). Modest evidence against a thermal Galactic dust component dominating the observed signal was presented based on the cross spectrum against 100 GHz maps from the previous BICEP1 experiment. The detected $B$-mode level was higher than that projected by several existing dust models \cite{23,24} although these did not claim any high degree of reliability.

The Planck survey released information on the structure of the dust polarization sky at intermediate latitudes \cite{25}, and the frequency dependence of the polarized dust emission at frequencies relevant to CMB studies \cite{26}. Other papers argued that the BICEP2 region is significantly contaminated by dust \cite{27,28}. Finally, Planck released information on dust polarization at high latitude \cite{29} (hereafter PIP-XXX), and in particular examined a field centered on the BICEP2 region (but somewhat larger than it) finding a level of polarized dust emission at 353 GHz sufficient to explain the 150 GHz excess observed by BICEP2, although with relatively low signal-to-noise.

Keck Array is a system of BICEP2-like receivers also located at the South Pole. During the 2012 and 2013 seasons Keck Array observed the same field as BICEP2 in the same 150 GHz frequency band \cite{30} (hereafter BK-V). Combining the BICEP2 and Keck Array maps yields $Q$ and $U$ maps with rms noise of 57 nK in nominal 1 deg$^2$ pixels — by far the deepest made to date.

In this Letter, we take cross spectra between the joint BICEP2/Keck maps and all the polarized bands of Planck. The structure is as follows. In Sec. II we describe the preparation of the input maps, the expectations for dust, and the power spectrum results. In Sec. III the main multi-frequency cross-spectrum likelihood method is introduced and applied to the data, and a number of variations from the selected fiducial analysis are explored. Section IV describes validation tests using simulations as well as an alternate likelihood. In Sec. V we investigate whether there could be decorrelation between the Planck and BICEP2/Keck maps due to the astrophysics of dust and/or instrumental effects. Finally we conclude in Sec. VI.

II. MAPS TO POWER SPECTRA

A. Maps and preparation

We primarily use the BICEP2/Keck combined maps, as described in BK-V. We also use the BICEP2-only and Keck-only maps as a cross-check. The Planck maps used for cross-correlation with BICEP2/Keck are the full-mission polarized maps from the PR2 Planck science release \cite{31,32}, a subset of which was presented in PIP-XXX. We compute Planck single-frequency spectra as the cross-power spectra of two data-split maps, in which the data are split into two subsets with independent noise. We consider three data split maps: (i) detector-set maps, where the detectors at a given frequency are divided into two groups, (ii) yearly maps, where the data from the first and second years of observations are used for the two maps, and (iii) half-ring maps, where the data from each pointing period is divided in halves. To evaluate uncertainties due to Planck instrumental noise, we use 500 noise simulations of each map; these are the standard set of time-ordered data noise simulations projected into sky maps (the full focal plane 8 (FFP8) simulations defined in Ref. \cite{33}).

While the Planck maps are filtered only by the instrument beam (the effective beam defined in Refs. \cite{34} and \cite{35}), the BICEP2/Keck maps are in addition filtered due to the observation strategy and analysis process. In particular, large angular scales are suppressed anisotropically in the BICEP2/Keck mapmaking process to avoid atmospheric and ground-fixed contamination; this suppression is corrected in the power spectrum estimate. In order to facilitate comparison, we therefore prepare “Planck as seen by BICEP2/Keck” maps. In the first step we use the ANAFAST, ALTERALM, and SYNFAST routines from the HEALPix \cite{36} package \cite{37} to resmooth the Planck maps with the BICEP2/Keck beam profile, assuming azimuthal symmetry of the beam. The coordinate rotation from Galactic to celestial coordinates of the $T$, $Q$, and $U$ maps is performed using the ALTERALM routine in the HEALPix package. The sign of the Stokes $U$ map is flipped to convert from the HEALPix to the IAU polarization convention. Next we pass these through the “observing” matrix $R$, described in Sect. VI.B of BK-I, to produce maps that include the filtering of modes occurring in the data processing pipeline (including polynomial filtering and scan-synchronous template removal, plus deprojection of beam systematics).

Figure 1 shows the resmoothed Planck 353 GHz $T$, $Q$, and $U$ maps before and after filtering. In both cases the BICEP2/Keck inverse variance apodization mask has been applied. This figure emphasizes the need to account for the filtering before any comparison of maps is attempted, either qualitative or quantitative.

B. Expected spatial and frequency spectra of dust

Before examining the power spectra it is useful to review expectations for the spatial and frequency spectra of dust.
Figure 2 of PIP-XXX shows that the dust BB (and EE) angular power spectra are well fit by a simple power law $D_\ell \propto \ell^{-0.42}$, where $D_\ell = C_\ell (\ell + 1)/2\pi$, when averaging over large regions of sky outside of the Galactic plane. Section VB of the same paper states that there is no evidence for departure from this behavior for 1% sky patches, although the signal-to-noise ratio is low for some regions. Presumably we expect greater fluctuation from the mean behavior than would be expected for a Gaussian random field.

The spectral energy distribution (SED) of dust polarization was measured in Ref. [26] for 400 patches with 10° radius at intermediate Galactic latitudes. Figure 10 of this reference shows empirically that the mean polarized dust SED is described by a simple modified blackbody spectrum with $T_d = 19.6$ K and $\beta_d = 1.59 \pm 0.17$ to within an accuracy of a few percent over the frequency range 100–353 GHz. Within this frequency range variations in the two parameters are highly degenerate and the choice is made to hold $T_d$ fixed at the value obtained from a fit to the SED. The power spectrum estimation proceeds exactly as in BK-I, including the matrix based purification operation to prevent $E$ to $B$ mixing. Figure 2 shows the results for BICEP2/Keck and Planck 353 GHz for $TT$, $TE$, $EE$, and $BB$. In all cases the error bars are the standard deviations of lensed-$\Lambda$CDM + noise simulations [38] and hence contain no sample variance on any other component. The results in the left column are autospectra, identical to those given in BK-I and BK-V—these spectra are consistent with lensed $\Lambda$CDM + noise except for the excess in $BB$ for $\ell < 200$.

The right column of Fig. 2 shows cross spectra between two halves of the Planck 353 GHz data set, with three different splits shown. The Planck Collaboration prefers the use of cross spectra even at a single frequency to gain additional immunity to systematics and to avoid the need to noise debias autospectra. The $TT$ spectrum is higher than $\Lambda$CDM around $\ell = 200$—presumably due to a dust contribution. The $EE$ and $BB$ spectra are noisy, but both appear to show an excess over $\Lambda$CDM for $\ell < 150$—again presumably due to dust. We note that these spectra do not appear to follow the power-law expectation mentioned in Sec. II B, but we emphasize that the error bars
contain no sample variance on any dust component (Gaussian or otherwise).

The center column of Fig. 2 shows cross spectra between BICEP2/Keck and Planck maps. The BICEP2 spectra are identical to those in BK-I, while the Keck Array and combined are as given in BK-V. The center column shows cross-frequency spectra between BICEP2/Keck maps and Planck 353 GHz maps. The right column shows Planck 353 GHz data-split cross spectra. In all cases the error bars are the standard deviations of lensed-ΛCDM + noise simulations and hence contain no sample variance on any other component. For EE and BB the χ² and χ (sum of deviations) versus lensed ΛCDM for the nine band powers shown is marked at upper and lower left (for the combined BICEP2/Keck points and DS₁ × DS₂, respectively). In the bottom row (for BB) the center and right panels have a scaling applied such that signal from dust with the fiducial frequency spectrum would produce signal with the same apparent amplitude as in the 150 GHz panel on the left (as indicated by the right-side y axes). We see from the significant excess apparent in the bottom center panel that a substantial amount of the signal detected at 150 GHz by BICEP2 and Keck Array indeed appears to be due to dust.

contain no sample variance on any dust component (Gaussian or otherwise).

The center column of Fig. 2 shows cross spectra between BICEP2/Keck and Planck maps. For TE one can use the T modes from BICEP2 and the E modes from Planck or vice versa and both options are shown. Since the T modes are very similar between the two experiments, these TE spectra look similar to the single-experiment TE spectrum which shares the E modes. The EE and BB cross spectra are the most interesting—there appears to be a highly significant detection of correlated B-mode power between 150 and 353 GHz, with the pattern being much brighter at 353, consistent with the expectation from dust. We also see hints of detection in the EE spectrum—while dust E modes are subdominant to the cosmological signal at 150 GHz, the weak dust contribution enhances the BK150 × P353 cross spectrum at ℓ ≈ 100.

The polarized dust SED model mentioned in Sec. II B implies that dust emission is approximately 25 times brighter in the Planck 353 GHz band than it is in the BICEP2/Keck 150 GHz band (integrating appropriately over the instrumental bandpasses). The expectation for a
dust-dominated spectrum is thus that the BK150 × P353 cross spectrum should have an amplitude 25 times that of BK150 × BK150, and P353 × P353 should be 25 times higher again. The y-axis scaling in the bottom row of Fig. 2 has been adjusted so that a dust signal obeying this rule will have equal apparent amplitude in each panel. We see that a substantial amount of the BK150 × BK150 signal indeed appears to be due to dust.

To make a rough estimate of the significance of deviation from lensed ΛCDM, we calculate $\chi^2$ and $\chi$ (sum of deviations) for each of the EE and BB spectra and show these in Fig. 2. For the nine band powers used the expectation value (standard deviation) for $\chi^2$ and $\chi$ are 9 (4.2) and 0 (3), respectively. We see that BK150 × BK150 and BK150 × P353 are highly significant in BB, while P353 × P353 has modest significance in both EE and BB.

Figure 3 shows EE and BB cross spectra between BICEP2/Keck and all of the polarized frequencies of Planck (also including the BICEP2/Keck autospectra). For the five band powers shown the expectation value (standard deviation) for $\chi^2$ and $\chi$ are 5 (3.1) and 0 (2.2), respectively. As already noted, the BK150 × BK150 and BK150 × P353 BB spectra show highly significant excesses. Additionally, there is evidence for excess BB in BK150 × P217 spectrum, and for excess EE in BK150 × P353. The other spectra in Fig. 3 show no strong evidence for excess, although we note that only one of the $\chi$ values is negative. There is weak evidence for excess in the BK150 × P70 BB spectrum but none in BK150 × P30 so this is presumably just a noise fluctuation.

There are a large number of additional Planck-only spectra, which are not plotted here. The noise on these is large and all are consistent with ΛCDM, with the possible exception of P217 × P353, where modest evidence for an excess is seen in both EE and BB (see, e.g., Fig. 10 of PIP-XXX).

D. Consistency of BICEP2 and Keck Array spectra

The BB autospectra for BICEP2 and Keck Array in the lower left panel of Fig. 2 appear to differ by more than might be expected, given that the BICEP2 and Keck maps cover almost exactly the same region of sky. However, the error bars in this figure are the standard deviations of lensed-ΛCDM + noise simulations; while the signal is largely common between the two experiments the noise is not, and the signal-noise cross terms produce substantial additional fluctuation of the difference. The correct way to quantify this is to compare the difference of the real data to the pairwise differences of simulations, using common input skies that have power similar to that observed in the real data. This was done in Sec. 8 of BK-V and the BICEP2 and Keck maps were shown to be statistically compatible.

In an analogous manner we can also ask if the B150 × P353 and K150 × P353 BB cross spectra shown in the bottom middle panel of Fig. 2 are compatible. Figure 4 shows the results. We calculate the $\chi^2$ and $\chi$ statistics on these.
We check the reliability of the power spectrum estimation with an alternative pipeline. The filtered and purified in Fig. 2 are transformed back into the HEALPix pixelization.

There is no evidence that these spectra are statistically incompatible.

E. Alternative power spectrum estimation

We check the reliability of the power spectrum estimation with an alternative pipeline. The filtered and purified Planck and BICEP2 maps used to make the spectra shown in Fig. 2 are transformed back into the HEALPix pixelization using cubic spline interpolation. The $B$-mode cross power is then computed with the XPOL [39] and PURECl [40] estimators. Figure 5 shows the difference between these alternative band powers and the standard band powers for the B150 $\times$ P353 $BB$ cross spectrum. As in Fig. 4 the error bars are the standard deviations of pairwise differences of simulations, which share common input skies and have power similar to that observed in the real data. The agreement is not expected to be exact due to the differing band-power window functions, but the differences of the real band powers are consistent with those of the simulations.

III. LIKELIHOOD ANALYSIS

A. Algorithm

While it is conventional in plots like Fig. 2 to present band powers with symmetric error bars, it is important to appreciate that this is an approximation. The likelihood of an observed band power for a given model expectation value is generally an asymmetric function, which can be computed given knowledge of the noise level(s). To compute the joint likelihood of an ensemble of measured band-power values it is, of course, necessary to consider their full covariance—this is especially important when using spectra taken at different frequencies on the same field, where the signal covariance can be very strong.

We compute the band-power covariance using full simulations of signal-cross-signal, noise-cross-noise, and signal-cross-noise. From these, we can construct the covariance matrix for a general model containing multiple signal components with any desired set of SEDs. When we do this we deliberately exclude terms whose expectation value is zero, in order to reduce noise in the resulting matrix due to the limited number of simulated realizations.

To compute the joint likelihood of the data for any given proposed model we use the Hamimeche-Lewis [41] approximation (HL; see Sec. 9.1 of Ref. [42] for mention details). Here we extend the method to deal with single- and cross-frequency spectra, and the covariances thereof, in an analogous manner to the treatment of, for example, $TT$, $TE$, and $EE$ in the standard HL method. The HL formulation requires that the band-power covariance matrix be determined for only a single "fiducial model." We compute multidimensional grids of models explicitly and/or use COSMOMC [43] to sample the parameter space.

B. Fiducial analysis

As an extension of the simplest lensed-$\Lambda$CDM paradigm, we initially consider a two component model of IGW with amplitude $r$, plus dust with amplitude $A_d$ (specified at
353 GHz and $\ell = 80$). [Here we assume that the spectral index of the tensor modes ($n_t$) is zero, and a scalar pivot scale of 0.05 Mpc$^{-1}$; all values of $r$ quoted in this Letter are $r_{0.05}$ unless noted otherwise.] Figure 6 shows the results of fitting such a model to $BB$ band powers taken between BICEP2/$Keck$ and the 217 and 353 GHz bands of $Planck$, using band powers $1-5$ ($20 < \ell < 200$). For the $Planck$ single-frequency case, the cross spectrum of detector sets (DS1 $\times$ DS2) is used, following PIP-XXX. The dust is modeled as a power law $D_{\ell} \propto \ell^{-0.42}$, with free amplitude $A_d$ and scaling with frequency according to the modified blackbody model.

As discussed in Sec. II B the simple modified blackbody model is shown empirically in Ref. [26] to describe the mean polarized dust SED at mid-Galactic latitudes to an accuracy of a few percent over the frequency range 100–353 GHz, with variation of the $\beta_d$ parameter being sufficient to characterize the patch-to-patch variation. Since it is not possible to constrain $\beta_d$ using the BICEP2/$Keck$ and $Planck$ cross-spectral band powers alone a tight Gaussian prior $\beta_d = 1.59 \pm 0.11$ is imposed, the uncertainty being scaled from the observed patch-to-patch variation at intermediate Galactic latitudes in Ref. [26], as explained in PIP-XXX. This prior assumes that the SED of dust polarization at intermediate latitudes [26] applies to the high latitude BICEP2/$Keck$ field. From dust astrophysics, we expect variations of the dust SED in intensity and polarization to be correlated [18]. We thus tested our assumption by measuring the $\beta_d$ of the dust total intensity in the BICEP2/$Keck$ field using the template fitting analysis described in Ref. [44], and find the same value.

In Fig. 6 we see that the BICEP2 data produce an $r$ likelihood that peaks higher than that for the $Keck$ Array data. This is because for $\ell < 120$ the autospectrum B150 $\times$ B150 is higher than for K150 $\times$ K150, while the cross spectrum B150 $\times$ P353 is lower than K150 $\times$ P353 (see Fig. 2). However, recall that both pairs of spectra B150 $\times$ B150/K150 $\times$ K150 and B150 $\times$ P353/K150 $\times$ P353 have been shown to be consistent within noise fluctuation (see Sec. II D). In Sec. IV A these likelihood results are also found to be compatible. Given the consistency between the two experiments, the combined result gives the best available measurement of the sky.

The combined curves (BK $+$ P) in the left and center panels of Fig. 6 yield the following results: $r = 0.048^{+0.035}_{-0.032}$, $r < 0.12$ at 95% confidence, and $A_d = 3.3^{+0.9}_{-0.8}$. For the zero-to-peak likelihood ratio is 0.38. Taking $\frac{1}{2}[1-f(-2 \log L_0/L_{peak})]$, where $f$ is the $\chi^2$ cdf (for one degree of freedom), we estimate that the probability to get a number smaller than this is 8% if in fact $r = 0$. For $A_d$ the zero-to-peak likelihood ratio is 0.38. Taking $\frac{1}{2}[1-f(-2 \log L_0/L_{peak})]$, where $f$ is the $\chi^2$ cdf (for one degree of freedom), we estimate that the probability to get a number smaller than this is 8% if in fact $r = 0$. For $A_d$ the zero-to-peak likelihood ratio is 0.38. Taking $\frac{1}{2}[1-f(-2 \log L_0/L_{peak})]$, where $f$ is the $\chi^2$ cdf (for one degree of freedom), we estimate that the probability to get a number smaller than this is 8% if in fact $r = 0$. For $A_d$ the zero-to-peak likelihood ratio is 0.38. Taking $\frac{1}{2}[1-f(-2 \log L_0/L_{peak})]$, where $f$ is the $\chi^2$ cdf (for one degree of freedom), we estimate that the probability to get a number smaller than this is 8% if in fact $r = 0$. For $A_d$ the zero-to-peak likelihood ratio is 0.38. Taking $\frac{1}{2}[1-f(-2 \log L_0/L_{peak})]$, where $f$ is the $\chi^2$ cdf (for one degree of freedom), we estimate that the probability to get a number smaller than this is 8% if in fact $r = 0$. For $A_d$ the zero-to-peak likelihood ratio is 0.38. Taking $\frac{1}{2}[1-f(-2 \log L_0/L_{peak})]$, where $f$ is the $\chi^2$ cdf (for one degree of freedom), we estimate that the probability to get a number smaller than this is 8% if in fact $r = 0$. For $A_d$ the zero-to-peak likelihood ratio is 0.38.
Using only BK150 × BK150 and BK150 × P353: Also excluding the 353 GHz single-frequency spectrum from consideration makes little difference. The statistical weight of the BK150 × BK150 and BK150 × P353 spectra dominate (see Fig. 7).

Extending the band-power range: Going back to the base data set and extending the range of band powers considered to 1−9 (corresponding to 20 < \ell < 330) makes very little difference—the dominant statistical weight is with the lower band powers (see Fig. 7).

Including EE spectra: We can also include in the fits the EE spectra shown in Fig. 3. PIP-XXX (Figs. 5 and A.3) shows that the level of EE from Galactic dust is on average around twice the level of BB. However, there are substantial variations in this ratio from sky patch to sky patch. Setting EE/BB = 2 we find that the constraint on \( A_d \) narrows, while the \( r \) constraint changes little; this latter result is also shown in Fig. 7. The maximum likelihood model on the grid is unchanged and its \( \chi^2 \) PTE is acceptable.

Relaxing the \( \beta_d \) prior: Relaxing the prior on the dust spectral index to \( \beta_d = 1.59 \pm 0.33 \) pushes the peak of the \( r \) constraint up (see Fig. 7). However, it is not clear if this looser prior is self-consistent; if the frequency spectral index varied significantly across the sky it would invalidate cross-spectral analysis, but there is strong evidence against such variation at high latitude, as explained in Sec. VA. Nevertheless, it is important to appreciate that the \( r \) constraint curves shown in Fig. 6 shift left (right) when assuming a lower (higher) value of \( \beta_d \). For \( \beta_d = 1.3 \pm 0.11 \) the peak is at \( r = 0.021 \) and for \( \beta_d = 1.9 \pm 0.11 \) the peak is at \( r = 0.073 \).

Varying the dust power spectrum shape: In the fiducial analysis the dust spatial power spectrum is assumed to be a power law with \( D_\ell \propto \ell^{-0.42} \). Marginalizing over spectral indices in the range \(-0.8 \) to \( 0 \) we find little change in the \( r \) constraint (see also Sec. IV B for an alternate relaxation of the assumptions regarding the spatial properties of the dust pattern).

Using Gaussian determinant likelihood: The fiducial analysis uses the HL likelihood approximation, as described in Sec. III A. An alternative is to recompute the covariance matrix \( C \) at each point in parameter space and take \( L = \det(C)^{-1/2} \exp[-(d^T C^{-1} d)/2] \), where \( d \) is the deviation of the observed band powers from the model expectation values. This results in an \( r \) constraint which peaks slightly lower, as shown in Fig. 7. Running both methods on the simulated realizations described in Sec. IVA, indicates that such a difference is not unexpected and that there may be a small systematic downward bias in the Gaussian determinant method.

Varying the HL fiducial model: As mentioned in Sec. III A the HL likelihood formulation requires that the expectation values and band-power covariance matrix be provided for a single “fiducial model” (not to be confused with the “fiducial analysis” of Sec. III B). The results from HL are supposed to be rather insensitive to the choice of this model, although preferably it should be close to reality. Normally, we use the lensed-\( \Lambda \)CDM + dust simulations described in Sec. IVA below. Switching this to lensed-\( \Lambda \)CDM + \( r = 0.2 \) produces no change on average in the simulations, although it does cause any given realization to shift slightly—the change for the real data case is shown in Fig. 7.

Adding synchrotron: BK-I took the WMAP K-band (23 GHz) map, extrapolated it to 150 GHz according to \( \nu^{-3.3} \) (mean value within the BICEP2 field of the MCMC “Model f” spectral index map provided by WMAP [2]), and found a negligible predicted contribution (\( r_{\text{sync,150}} = 0.0008 \pm 0.0041 \)). Figure 3 does not offer strong motivation to reexamine this finding—the only significant detections of correlated BB power are in the BK150 × P353 and, to a lesser extent, BK150 × P217 spectra. However, here we proceed to a fit including all the polarized bands of Planck (as shown in Fig. 3) and adding a synchrotron component to the base lensed-\( \Lambda \)CDM + noise + \( r \) + dust model. We take synchrotron to have a power law spectrum \( D_\ell \propto \ell^{-0.6} \) [23], with free amplitude \( A_{\text{sync}} \), where \( A_{\text{sync}} \) is the amplitude at \( \ell = 80 \) and at 150 GHz, and scaling with frequency according to \( \nu^{-3.3} \). In such a scenario we can vary the degree of correlation that is assumed between the dust and synchrotron sky patterns. Figure 8 shows results for the uncorrelated and fully correlated cases. Marginalizing over \( r \) and \( A_d \) we find \( A_{\text{sync}} < 0.0003 \mu K^2 \) at 95% confidence for the uncorrelated case, and many times smaller for the correlated. This last is because once one has a detection of dust it effectively becomes a template for the synchrotron. This synchrotron limit is driven by the Planck 30 GHz band—we obtain almost identical results when adding only this band, and a much softer limit when not including it. If we instead assume synchrotron scaling of \( \nu^{-3.0} \) the limit on \( A_{\text{sync}} \) is...
approximately doubled for the uncorrelated case and reduced for the correlated. (Because the DS1 × DS2 data split is not available for the Planck LFI bands we switch to Y1 × Y2 for this variant analysis, and so we compare to this case in Fig. 8 rather than the usual fiducial case.)

Varying lensing amplitude: In the fiducial analysis the amplitude of the lensing effect is held fixed at the ΛCDM expectation ($A_L = 1$). Using their own and other data, the Planck Collaboration quote a limit on the amplitude of the lensing effect versus the ΛCDM expectation of $A_L = 0.99 ± 0.05$ [3]. Allowing $A_L$ to float freely, and using all nine band powers, we obtain the results shown in Fig. 9—there is only weak degeneracy between $A_L$ and both $r$ and $A_d$. Marginalizing over $r$ and $A_d$ we find $A_L = 1.13 ± 0.18$ with a likelihood ratio between zero and peak of $3 \times 10^{-11}$. Using the expression given in Sec. III B this corresponds to a smaller-than-probability of $2 \times 10^{-12}$, equivalent to a 7.0σ detection of lensing in the BB spectrum. We note this is the most significant to-date direct measurement of lensing in B-mode polarization.

IV. LIKELIHOOD VALIDATION

A. Validation with simulations

We run the algorithm used in Sec. III B on ensembles of simulated realizations to check its performance. We first consider a model where $r = 0$ and $A_d = 3.6$ μK², this latter being close to the value favored by the data in a dust-only scenario [45]. We generate Gaussian random realizations using the fiducial spatial power law $D_l \propto l^{-0.42}$, scale these to the various frequency bands using the modified black-body law with $T_d = 19.6$ K and $\beta_d = 1.59$, and add to the usual realizations of lensed-ΛCDM + noise. Figure 10 shows some of the resulting $r$ and $A_d$ constraint curves, with the result for the real data from Fig. 6 overplotted. As expected, approximately 50% of the $r$ likelihoods peak above zero. The median 95% upper limit is $r < 0.075$. We find that 8% of the realizations have a ratio $L_0/L_{\text{peak}}$ less than the 0.38 observed in the real data, in agreement with the estimate in Sec. III B. Running these dust-only realizations for BICEP2 only and Keck Array only, we find that the shift in the maximum likelihood value of $r$ seen in the real data in Fig. 6 is exceeded in about 10% of the simulations.

The above simulations assume that the dust component follows on average the fiducial $D_l \propto l^{-0.42}$ spatial power law, and fluctuates around it in a Gaussian manner. To obtain sample dust sky patterns that may deviate from this behavior in a way which better reflects reality, we take the prelaunch version of the Planck Sky Model (PSM; version 1.7.8 run in “simulation” mode) [24] evaluated in the Planck 353 GHz band and pull out the same 352 |b| > 35° partially overlapping regions used in PIP-XXX. We then scale these to the other bands and proceed as before. Some of the regions have dust power orders of magnitude higher than the real data and we cut them out (selecting 139 regions with peak $A_d < 20$ μK²). Figure 11 presents the results. The $r$ likelihoods will broaden as the level of $A_d$ increases, and we should therefore not be surprised if the fraction of realizations peaking at a value higher than the real data is increased compared to the simulations with
mean $A_d = 3.6 \mu K^2$. However, we still expect that on average 50% will peak above zero and approximately 8% will have a $L_0/L_{\text{peak}}$ ratio less than the 0.38 observed in the real data. In fact we find 57% and 7%, respectively, consistent with the expected values. There is one realization which has a nominal (false) detection of nonzero $r$ of $3.3\sigma$, although this turns out to also have one of the lowest $L_0/L_{\text{peak}}$ ratios in the Gaussian simulations shown in Fig. 10 (with which it shares the CMB and noise components), so this is apparently just a relatively unlikely fluctuation.

**B. Subtraction of scaled spectra**

As previously mentioned, the modified blackbody model predicts that dust emission is 4% as bright in the BICEP2 band as it is in the $\text{Planck}$ $353\, \text{GHz}$ band. Therefore, taking the autospectra and cross spectra of the combined BICEP2/$\text{Keck}$ maps and the $\text{Planck}$ $353\, \text{GHz}$ maps, as shown in the bottom row of Fig. 2, and evaluating $(\text{BK} \times \text{BK} - \alpha \text{BK} \times \text{P})/(1 - \alpha)$, at $\alpha = \alpha_{\text{fid}}$ cleans out the dust contribution (where $\alpha_{\text{fid}} = 0.04$). The upper panel of Fig. 12 shows the result.

As an alternative to the full likelihood analysis presented in Sec. III B, we can instead work with the differenced spectra from above, a method we denote the “cleaning” approach. If $\alpha_{\text{fid}}$ were the true value, the expectation value of this combination over CMB and noise would have no dust contribution. However, dust would still contribute to its variance, but only through its 2-point function. In practice, we do not know $\alpha$ perfectly, and this uncertainty needs to be accounted for in a likelihood constructed from the differenced spectra. Our approach is to treat the differenced spectra as a form of data compression, and to compute the expectation value as a function of $r$, $A_d$, and $\beta_d$ at each point in parameter space [the dust dependence enters for $\alpha(\beta_d) \neq \alpha_{\text{fid}}$]. We use the method of Ref. [41], with a fiducial covariance matrix, to build a likelihood for the difference spectra, and marginalize over $A_d$ and $\beta_d$, and hence $\alpha$, adopting the prior $\beta_d = 1.59 \pm 0.11$. This alternative likelihood has the advantage of being less sensitive to non-Gaussianity of the dust, since only the 2-point function of the dust affects the covariance of the differenced spectra close to $\alpha_{\text{fid}}$, while the full analysis may, in principle, be affected by the non-Gaussianity of the dust through 4-point contributions to power spectra covariances. This cleaning approach does, however, ignore the (small amount of) additional information available at other frequencies. The lower panel of Fig. 12 compares the result to the fiducial analysis with the full multispectra likelihood. It is clear from the widths of the likelihood curves that compressing the spectra to form the cleaned difference results in very little loss of information on $r$. The difference in peak values arises from the different data.
treatments and is consistent with the scatter seen across simulations. Finally, we note that one could also form a combination \( (\text{BK} \times \text{BK} - 2\alpha \text{BK} \times P + \alpha^2 P \times P) / (1 - \alpha)^2 \) in which dust does not enter at all for \( \alpha = \alpha_{\text{fid}} \). However, the variance of this combination of spectra is large due to the Planck noise levels, and likelihoods built from this combination are considerably less constraining.

V. POSSIBLE CAUSES OF DECORRELATION

Any systematic error that suppresses the BK150 \( \times \) P353 cross-frequency spectrum with respect to the BK150 \( \times \) BK150 and P353 \( \times \) P353 single-frequency spectra would cause a systematic upward bias on the \( r \) constraint. Here we investigate a couple of possibilities.

A. Spatially varying dust frequency spectrum

If the frequency dependence of polarized dust emission varied from place to place on the sky, it would cause the 150 and 353 GHz dust sky patterns to decorrelate and suppress the BK150 \( \times \) P353 cross-frequency spectrum relative to the single-frequency spectra. The assumption made so far in this Letter is that such decorrelation is negligible. In fact, PIP-XXX implicitly tests for such variation in their Fig. 6, where the Planck single- and cross-frequency spectra are compared to the modified blackbody model (with the cross-frequency spectra plotted at the geometric mean of their respective frequencies). This plot is for an average over a large region of low foreground sky (24%); however, note that if there were spatial variation of the spectral behavior anywhere in this region it would cause suppression of the cross-frequency spectra with respect to the single-frequency spectra.

PIP-XXX also tests explicitly for evidence of decorrelation of the dust pattern across frequencies. Their Fig. E.1 shows the results for large and small sky patches. The signal-to-noise ratio is low in clean regions, but no evidence of decorrelation is found.

As a further check, we artificially suppress the amplitude of the BK150 \( \times \) P353 spectra in the Gaussian dust-only simulations (see Sec. IV.A) by a conservative 10% (PIP-XXX sets a 7% upper limit). We find that the maximum likelihood value for \( r \) shifts up by an average of 0.018, while \( A_s \) shifts down by an average of 0.43 \( \mu K \), with the size of the shift proportional to the magnitude of the dust power in each given realization. This behavior is readily understandable—since the BK150 \( \times \) BK150 and BK150 \( \times \) P353 spectra dominate the statistical weight, a decrease of the latter is interpreted as a reduction in dust power, which is compensated by an increase in \( r \). The bias on \( r \) will be linearly related to the assumed decorrelation factor.

B. Calibration, analysis, etc.

Figure 3 shows that the EE spectrum BK150 \( \times \) BK150 is extremely similar to that for BK150 \( \times \) P143. We can compare such spectra to set limits on possible decorrelation between the BICEP2/\textit{Keck} and Planck maps arising from any instrumental or analysis related effect, including differential pointing, polarization angle mischaracterization, etc. Taking the ratio of BK150 \( \times \) P143 to the geometric mean of BK150 \( \times \) BK150 and P143H1 \( \times \) P143H2, we find that for TT the decorrelation is approximately 0.1%. For EE the signal-to-noise ratio is lower, but decorrelation is limited to below 2%, and consistent with zero when compared to the fluctuation of signal + noise simulations.

VI. CONCLUSIONS

BK-I reported a highly significant detection of \( B \)-mode polarization, at 150 GHz, in excess of the lensed-\( \Lambda \)CDM expectation over the range \( 30 < \ell < 150 \). This excess has been confirmed by additional data on the same field from the successor experiment \textit{Keck Array}. PIP-XXX found that the level of dust power in a field centered on the BICEP2/\textit{Keck} region (but somewhat larger than it) is of the same magnitude as the reported excess, but noted that, “the present uncertainties are large,” and that a joint analysis was required.

In this Letter we have performed this joint analysis, using the combined BICEP2/\textit{Keck} maps. Cross-correlating these maps against all of the polarized frequency bands of \textit{Planck} we find a highly significant \( B \)-mode detection only in the cross spectrum with 353 GHz. We emphasize that this 150 \( \times \) 353 GHz cross spectrum has a much higher signal-to-noise ratio than the 353 GHz single-frequency spectrum that PIP-XXX analyzed.

We have analyzed the data using a multifrequency, multicomponent fit. In this fit it is necessary to impose a prior on the variation of the brightness of the polarized dust emission with observing frequency, since the available data are unable to constrain this alone, due to the relatively low signal-to-noise ratio in \( B \)-mode polarization at 353 GHz. However, based on the available information from \textit{Planck} on the frequency dependence of polarized dust emission across the mid- and high-Galactic latitude sky, and the patch-to-patch stability thereof, this prior appears to be justified and conservative.

We have shown that the final constraint on the tensor-to-scalar ratio \( r \) is very stable when varying the frequency bands used, as well as the model priors. The result does differ when using the BICEP2 and \textit{Keck Array} data alone rather than in combination, but the difference is compatible with noise fluctuation. Expanding the model to include synchrotron emission, while also including lower \textit{Planck} frequencies, does not change the result.

Allowing the amplitude of lensing to be free, we obtain \( A_L = 1.13 \pm 0.18 \), with a significance of detection of 7.0\( \sigma \). This is the most significant direct detection to date of lensing in \( B \)-mode polarization, even compared to experiments with higher angular resolution. The POLARBEAR experiment has reported a detection of \( B \)-mode lensing.
on smaller angular scales (500 < \( \ell \) < 2100), rejecting the \( A_L = 0 \) hypothesis at 97.2% confidence [46]. Additionally, ACT [47] and SPT [48] have reported lensing detections in polarization in cross-correlation with some other tracer of the dark matter distribution on the sky.

We have validated the main likelihood analysis on simulations of a dust-only model and performed a simple subtraction of scaled spectra, which approximates a map-based dust cleaning (obtaining an \( r \) constraint curve that peaks somewhat lower). Finally, we investigated the possibility of astrophysical or instrumental decorrelation of the sky patterns between experiments or frequencies and find no evidence for relevant bias.

The final result is expressed as a likelihood curve for \( r \), and yields an upper limit \( r < 0.12 \) at 95% confidence. The median limit in the lensed-\( \Lambda \)CDM + noise + dust simulations is \( r < 0.075 \). It is interesting to compare this latter to dust-free simulations using only BICEP2/Keck where the median limit is \( r < 0.03 \)—the difference represents the limitation due to noise in the Planck maps, when marginalizing over dust. The \( r \) constraint curve peaks at \( r = 0.05 \) but disfavors zero only by a factor of 2.5. This is expected by chance 8% of the time, as confirmed in simulations of a dust-only model. We emphasize that this significance is too low to be interpreted as a detection of primordial B modes. Transforming the Planck temperature-only 95% confidence limit of \( r_{0.02} < 0.11 \) [3] to the pivot scale used in this Letter yields \( r_{0.05} < 0.12 \), compatible with the present result.

A COSMOMC module containing the band powers for all cross spectra between the combined BICEP2/Keck maps and all of the frequencies of Planck is available for download in Ref. [49].

In order to further constrain or detect IGW, additional data are required. The Planck Collaboration may be able to make progress alone using the large angular scale “reionization bump,” if systematics can be appropriately controlled [50]. To take small patch “recombination bump” studies of type pursued here to the next level, data with signal-to-noise comparable to that achieved by BICEP2/Keck at 150 GHz are required at more than one frequency. Figure 13 summarizes the situation. The BICEP2/Keck noise is much lower in the BICEP2/Keck field than the Planck noise. However, since dust emission is dramatically brighter at 353 GHz, it is detected in the cross spectrum between BICEP2/Keck and Planck 353 GHz. Synchrotron is not detected and the crossover frequency with dust is \( \lesssim 100 \) GHz. Planck’s PR2 data release [51] shows that for the cleanest 73% of the sky, at 40 arc min scales, the polarized foreground minimum is at \( \sim 80–90 \) GHz. During the 2014 season, two of the Keck Array receivers observed in the 95 GHz band and these data are under active analysis. BICEP3 will add substantial additional sensitivity at 95 GHz in the 2015, and especially 2016, seasons. Meanwhile, many other ground-based and suborbital experiments are making measurements at a variety of frequencies and sky coverage fractions.

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[13] Planck (http://www.esa.int/planck) is a project of the European Space Agency (ESA) with instruments provided by two scientific consortia funded by ESA member states (in particular the lead countries, France and Italy) with contributions from NASA (USA), and telescope reflectors provided in a collaboration between ESA and a scientific consortium led and funded by Denmark.