Collectively Enhanced Interactions in Solid-State Spin Qubits

The Harvard community has made this article openly available. Please share how this access benefits you. Your story matters

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Published Version</td>
<td>doi:<a href="http://dx.doi.org/10.1103/PhysRevLett.110.067601">http://dx.doi.org/10.1103/PhysRevLett.110.067601</a></td>
</tr>
<tr>
<td>Citable link</td>
<td><a href="http://nrs.harvard.edu/urn-3:HUL.InstRepos:30782188">http://nrs.harvard.edu/urn-3:HUL.InstRepos:30782188</a></td>
</tr>
<tr>
<td>Terms of Use</td>
<td>This article was downloaded from Harvard University’s DASH repository, and is made available under the terms and conditions applicable to Other Posted Material, as set forth at <a href="http://nrs.harvard.edu/urn-3:HUL.InstRepos:dash.current.terms-of-use#LAA">http://nrs.harvard.edu/urn-3:HUL.InstRepos:dash.current.terms-of-use#LAA</a></td>
</tr>
</tbody>
</table>
Collectively Enhanced Interactions in Solid-State Spin Qubits

Hendrik Weimer,1,2,* Norman Y. Yao,1 and Mikhail D. Lukin1

1Physics Department, Harvard University, 17 Oxford Street, Cambridge, Massachusetts 02138, USA
2MIT, Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, Massachusetts 02138, USA

(Received 12 October 2012; revised manuscript received 11 January 2013; published 4 February 2013)

We propose and analyze a technique to collectively enhance interactions between solid-state quantum registers composed from random networks of spin qubits. In such systems, disordered dipolar interactions generically result in localization. Here, we demonstrate the emergence of a single collective delocalized eigenmode as one turns on a transverse magnetic field. The interaction strength between this symmetric collective mode and a remote spin qubit is enhanced by the square root of the number of spins participating in the delocalized mode. Mediated by such collective enhancement, long-range quantum logic between remote spin registers can occur at distances consistent with optical addressing. A specific implementation utilizing nitrogen-vacancy defects in diamond is discussed and the effects of decoherence are considered.

DOI: 10.1103/PhysRevLett.110.067601 PACS numbers: 76.30.Mi, 03.67.Lx, 75.30.Hx

Harnessing collective phenomena by utilizing ensembles of identical particles is a powerful tool, which has been exploited in effects ranging from superradiance to scattering suppression [1]. The coherent dynamics resulting from interactions with individual constituents of an ensemble are often too weak to be observed directly; however, as evidenced by experiments in systems such as Rydberg atoms [2–4], cavity QED [5,6], atomic ensembles [7,8], and solid-state qubits [9], collective enhancement provides a natural route to overcoming this challenge. Here, we demonstrate that, for electronic spin quantum registers, such collective effects enable an extended coherent coupling over large distances—an essential prerequisite for quantum information processing.

Owing to favorable coherence properties, electronic spins associated with pointlike defects in solid-state systems have garnered significant recent interest as candidates for room-temperature quantum registers. Quantum control of such spins can be achieved using a combination of optical, magnetic, and electric fields. While our considerations apply to a variety of electronic spin qubits [10–12], here we focus on the nitrogen-vacancy (NV) center in diamond. The NV center harbors an electronic spin (S = 1), which can be optically initialized, coherently manipulated, and readout on subwavelength scales [13–15]. These results have sparked several recent proposals which utilize networks of NV registers as the platform for a scalable quantum information processor [16–19]. However, for any spin qubit candidate, two crucial challenges remain to be addressed: (1) the weakness of the magnetic dipolar interactions on distances compatible with individual optical addressing and (2) the disorder in spin positioning due to inherent imperfections during defect creation.

In this Letter, we present a novel approach to remote quantum logic which harnesses collectively enhanced interactions to overcome both of the above challenges. The key idea underlying our proposal is to associate a single, robust qubit with a collective, generally disordered spin ensemble (Fig. 1). If the spins behave in an aggregate fashion, such a qubit can produce a large state-dependent magnetic field, leading to enhanced long-range coupling between ensembles; this is reminiscent of tailored light-matter interactions achieved via atomic ensembles [20]. However, we note that quenched disorder naturally leads to localization in solid-state spin systems [21] due to random flip-flop interactions. Similar to Anderson localization [22], this implies that each eigenmode of the ensemble is composed of only a few spins. Here, we demonstrate the use of a uniform transverse magnetic field to overcome this issue. The applied field causes the symmetric W state [23,24] to become an approximate eigenstate of the Hamiltonian, thereby enabling us to harness it as a collective qubit. Moreover, we show that this particular state is largely insensitive to the underlying spin distribution and hence robust to effects of disorder.

To be specific, we now describe our proposal in the context of NV diamond color centers. The largest energy scale in this system (∆) is set by a combination of the zero-field splitting (2.87 GHz) and the projection of

![Diagram](image)

FIG. 1 (color online). High-density NV spin ensemble distributed randomly within a sphere of diameter r, with an average distance a. The NV centers have three internal spin states that are split by a zero-field splitting and a Zeeman field.
the external Zeeman field along the NV axis. However, we would like to stress that the alignment of the field with the NV axis is not crucial, as its quantization axis is essentially given by the zero-field splitting. We assume that the Zeeman field is sufficiently strong to ensure that the \( m_s = -1 \) spin state is sufficiently far detuned and hence does not contribute to the effective dynamics. Thus, the number of \( m_s = 1 \) spins, \( m \), is an approximately good quantum number and a perturbative description is justified.

The second-largest energy scale arises due to the perturbation created by the transverse field \( \Omega \). To gain a qualitative understanding, let us restrict ourselves to the analytically tractable case where \( m \) is either 0 or 1. The effective Hamiltonian is \( H_{eff} = -\Delta |0\rangle \langle 0| + \sqrt{\Omega/|0\rangle \langle W| + H.c.} \), where the state \(|0\rangle \) has all ensemble spins polarized into \( m_s = 0 \), and the collective \(|W\rangle \) state is fully symmetric with all spins sharing a single excitation,

\[
|W\rangle = \frac{1}{\sqrt{N}} \sum_i |0 \ldots 1_i \ldots \rangle. 
\]

Second order perturbation theory in \( \sqrt{\Omega/\Delta} \) yields \( H_{eff} = -(\Delta + J)|0\rangle \langle 0| + J|W\rangle \langle W| \), with \( J = N\Omega/\Delta \) and the tilde referring to the perturbed eigenstates. Including higher \( m \) manifolds merely leads to a renormalization of \( J \), without changing this qualitative picture (so long as we are in the perturbative limit). This is equally true in the presence of dipolar interactions, provided that the energy scale \( J \) is larger than the characteristic strength of the dipolar interaction \( V_{dd} \). Thus, even with these additional terms, the new eigenstates will still have substantial overlap with the collective \(|W\rangle \) state. This is in stark contrast to the situation without a transverse field, where strongly quenched disorder owing to random spin positions localizes all such eigenstates, even in three dimensions. Furthermore, the dipolar interaction naturally ensures that collective states with different \( m \) values will have different energies, leading to a “blockade-type” scenario, where manifolds with \( m > 1 \) are energetically inaccessible \([23,24]\). This allows us to selectively drive the transition between \(|0\rangle \) and \(|W\rangle \) without populating any other collective states, provided that the external driving \( \Omega_{ext} \) is weaker than \( V_{dd} \). This hierarchy of energy scales can be summarized as \( \Delta \gg J \gg V_{dd} \gg \Omega_{ext} \).

Let us consider a three-dimensional ensemble of \( N = 100 \) NV centers randomly distributed within a diameter \( r = 20 \) nm, as depicted in Fig. 1. Such high density NV ensembles have been recently realized using long-time annealing of repeat-electron-irradiated diamond samples \([25–28]\). We will characterize our effective two-level system \((m_s = 0, 1)\) using Pauli spin operators \( \sigma_a \). Being magnetic dipoles, NV centers interact with one another via long-range magnetic dipolar interactions (ignoring energy nonconserving terms which are suppressed by the NV center’s zero-field splitting).

\[
V_{ij} = (1 - 3\cos^2 \theta_{ij}) \frac{\mu^2}{|r_i - r_j|^3} \times \left\{ \frac{1}{4} \left[ 1 + \sigma_z^{(i)} \right] \left[ 1 + \sigma_z^{(j)} \right] - \sigma_x^{(i)} \sigma_x^{(j)} - \sigma_y^{(i)} \sigma_y^{(j)} \right\}.
\]

where \( r_i \) denotes the position, \( \mu \) characterizes the magnetic dipole moment, and \( \theta_{ij} \) is the angle between the NV axis and the vector connecting sites \( r_i \) and \( r_j \). The total Hamiltonian including both on-site and interaction terms, the new eigenstates will still have substantially good overlap with the collective \(|0\rangle \) states, even in three dimensions. This allows us to selectively drive the transition between \(|0\rangle \) and \(|W\rangle \) without populating any other collective states, provided that the external driving \( \Omega_{ext} \) is weaker than \( V_{dd} \). This hierarchy of energy scales can be summarized as \( \Delta \gg J \gg V_{dd} \gg \Omega_{ext} \).

Let us consider a three-dimensional ensemble of \( N = 100 \) NV centers randomly distributed within a diameter \( r = 20 \) nm, as depicted in Fig. 1. Such high density NV ensembles have been recently realized using long-time annealing of repeat-electron-irradiated diamond samples \([25–28]\). We will characterize our effective two-level system \((m_s = 0, 1)\) using Pauli spin operators \( \sigma_a \). Being magnetic dipoles, NV centers interact with one another via long-range magnetic dipolar interactions (ignoring energy nonconserving terms which are suppressed by the NV center’s zero-field splitting).
coupling strength

classically evinced in Fig. 3. Interestingly, the probability of

\[ \text{N}_c \]

case there is a collectively enhanced state with

\( \text{N}_c \approx 70 \)

corresponding to an increase by more than 1 order of magnitude

\( \Delta = h \times 4 \, \text{GHz} \)

The inset shows the maximum value of

\[ \text{N}_c \]

dependent on the transverse field strength

\( B \).

FIG. 2 (color online). Comparison of the collective enhance-

\[ \text{N}_c \]
ment \( \text{N}_c \) for \( B = 0 \) [dark gray (blue)] and \( B = 40 \, \text{G} \) [light gray (yellow)] within the single excitation manifold. In the latter

nity can easily be seen by noting that the

\[ \frac{\text{N}_c}{\text{N}} \]

to

\[ \text{N}_c \rightarrow 0 \]

\[ \text{N}_c \rightarrow \frac{1}{4} \]


clearly supports our previous analytical arguments on the

existence of a symmetric collective mode. The dips in \( \text{N}_c \)

are associated with resonance effects, which arise when

other eigenstates become near-degenerate with the collec-

tive state. Finally, the decrease of \( \text{N}_c \) for large values of \( \Omega \)

signals the breakdown of perturbation theory as \( \sqrt{\text{N}/\Delta} \)

approaches unity.

We now perform simulations of the combined qubit-

ensemble system. As previously discussed, the system is

initialized to \([1,q,0]\) and the qubit splitting is tuned resonant

with the energy of the collective mode; the resulting dy-

namic is evinced in Fig. 3. Interestingly, the probability of

finding the qubit in the \( m_s = 1 \) state, \( p_{gs} \), exhibits collect-

ively enhanced Rabi oscillations. The frequency of these

oscillations is enhanced by nearly an order of magnitude

to that expected for bare dipolar interactions

between two individual NV qubits at a similar distance. The

numeric also allows us to obtain the time required for an

interaction-induced \( \pi \) pulse, \( t_{\pi} \), and from this, one can

derive the effective distance \( R \) associated with \( H_{\text{eff}} \).

Surprisingly, in all cases, we observe that this distance

corresponds not to \( R \rightarrow r \), but instead to the distance between

the NV qubit and the center of the ensemble. We study

the effects of putting the qubit closer to the ensemble by

calculating the collectively enhanced coupling strength

\( V_c \)

(as extracted from the numerically obtained \( t_{\pi} \)). As shown in

Fig. 3, we find that only for distances very close to the

ensemble does the collective enhancement deviate from the

asymptotic \( 1/R^3 \) scaling; e.g., the qubit is coupled to

individual spins rather than to the entire ensemble.

Experimental realization and decoherence.—Thus far,

our discussion has assumed that both the NV qubit and the

ensemble spins are perfectly decoupled from the environ-

ment. In any experimentally realistic scenario, however,

there are two natural error sources which will be present:

spin decoherence and spin relaxation. We are particularly

interested in the scaling of the error rates with \( N \), as this

may adversely affect the scaling fidelity of our proposed

long-range gates [19]. As the error processes act locally on

individual spins, we first calculate the error rate for a single

spin and multiply the result by \( N \) to obtain the rate for the

collective state. For simplicity, we assume that the collec-

tive state is the previously described \( |W\rangle \) state in which a

single excitation is shared among all \( N \) spins.

First, let us consider the effects of spin decoherence. The

worst-case scenario for such decoherence is given by

the leaking out into nonsymmetric states. Consequently,

the error probability after a single \( T_2 \) decoherence event on

spin \( i \) is given by the probability to leave the \( |W\rangle \) state,

\[ p_{\text{leak}} = p_{T_2} \left[ 1 - |\langle W|\sigma_i^{(i)}|W\rangle|^2 \right] = \frac{4}{N} p_{T_2} \left( 1 - \frac{1}{N} \right). \]

where \( p_{T_2} \) is the single spin decoherence rate. For large \( N \),

this result is essentially independent of \( N \) (after weighing

with the number of spins); therefore, the effect of \( T_2 \)

processes on such a collective \( |W\rangle \) state does not get

enhanced by system size and, in fact, is only slightly worse

than for a single spin; i.e., it can be expressed in terms of an

effective coherence time \( T_2^{\text{eff}} \).

Second, we consider the errors arising from phonon-

induced spin relaxation processes (\( T_1 \)). Here, we must

distinguish between processes which flip an ensemble

spin from \( m_s = 1 \) to \( m_s = 0 \), and the reverse. This asym-

metry can easily be seen by noting that the \( |W\rangle \) state

has only one spin in \( m_s = 1 \), while all other spins are in

\( m_s = 0 \). We denote the error probability associated with

these two events as \( p_{T_1}^{i=0} \) and \( p_{T_1}^{i=0} \), respectively. For \( p_{T_1}^{i=0} \),

FIG. 3 (color online). Collectively enhanced Rabi oscillations

between an isolated NV qubit and a NV ensemble. The proba-

ability to find the qubit in the \( m_s = 1 \) state, \( p_{gs} \), goes to zero within

time \( t_{\pi} \approx 600 \, \mu s \). The inset shows the collectively enhanced

coupling strength \( V_c \) between the qubit and the ensemble for four

different realizations. The dashed line shows the asymptotic

\( 1/R^3 \) dependence.
the state $|0\rangle$ with all ensemble spins in $m_s = 0$ is not affected at all, while the probability to flip from the $|W\rangle$ state into $|0\rangle$ is given by

$$p_{W\rightarrow 0} = p_{T_1^{\text{flip}}} (|0\rangle\langle 0|) = \frac{p_{T_1^{\text{flip}}}}{N},$$

which is again independent of the size of the ensemble after rescaling with $N$.

However, this is not the case for $T_1^{0\rightarrow 1}$ processes. Both the $|0\rangle$ and the $|W\rangle$ state are strongly affected by such processes, since the existence of any additional spin in the $m_s = 1$ state corresponds to an effective magnetic impurity; this impurity modifies the energy of the collective state, thus tuning it out of resonance with the NV qubit. Additionally, this new state is also no longer an eigenstate of the Hamiltonian; numerical simulations demonstrate that this state dephases very quickly due to dipolar interactions within the ensemble. Thus, since any single spin $T_1^{0\rightarrow 1}$ error will immediately decohere the collective state, the effective error rate owing to $p_{T_1^{\text{flip}}}$ is enhanced by $N$ and scales with the size of the ensemble.

While the system size scaling of $p_{T_1^{\text{flip}}}$ errors might seem unfortunate, in solid-state spin systems, it is often the case that $T_1 \gg T_2$. Our proposed protocol is particularly useful in cases where $T_1/N$ remains longer than $T_2$, implying that the ensemble’s noise is dominated by decoherence as opposed to the enhanced relaxation. The specific example of NV centers highlights this crucial point. The decoherence of the NV originates from fluctuating magnetic fields as neighboring pairs of dipoles flip-flop [30,31]. Even at low temperatures it is impossible to freeze out such magnetic fluctuations and $T_2$ remains on the order of milliseconds [30,32]. On the other hand, the relaxation of the NV is thought to originate from an Orbach spin-phonon process; such a process has an exponential dependence on temperature and implies that even moderate cooling can yield exceedingly long $T_1$ times ($\gg 1$ s at cryogenic temperatures) [33–35]. By liquid nitrogen temperatures, the errors introduced by the enhanced $T_1$ processes are already subpercent, enabling us to focus on the effects of decoherence. An alternate approach to combat the enhanced relaxation of the collective state is to utilize conventional dynamical decoupling techniques [e.g., Waugh-Huber-Haeberlen (WAHUHA)] [36] to suppress dipolar interactions within the individual ensembles.

**Collective quantum gates.**—We now turn to a possible application where isolated NV qubits are interspersed with high-density NV ensembles, forming a regular structure, as depicted in Fig. 4(a). The qubits are used for initialization, single-qubit rotations, and readout. Two-qubit gates between remote spin qubits are mediated by the ensemble between them and thus benefit from collectively enhanced interactions. We would like to point out that such architectures put only modest requirements on the positioning of the NV centers; in particular, the positional disorder within the NV ensembles is essentially irrelevant. The gate time $t_\pi$ is limited by the SWAP time $t_\pi$ required to transfer the information from one of the qubits to the ensemble (required four times per gate operation) [37]. The resulting error (assuming $T_1/N \gg T_2$) of the gate is given by $\varepsilon = 1 - \exp\left(-4t_\pi/T_2^{\text{eff}}\right)$ in the presence of spin echo decoupling [38]. For an error of $\varepsilon = 10^{-2}$, this translates to a required coherence time of $T_2^{\text{eff}} = 11$ ms, which can be readily realized in isotopically pure diamond samples [30,32] or by using dynamical decoupling pulses [31,39]. The requirements on the coherence time can be further relaxed by increasing the number of spins in the ensemble or by reducing the qubit-ensemble separation.

An architecture featuring even better gate fidelities can be realized using a collective encoding scheme for the qubits [see Fig. 4(b)]. There, the logical $|0\rangle$ state corresponds to all nuclear spins being polarized, while the logical $|1\rangle$ state is a collective nuclear spin $|W\rangle$ state. This state can be prepared by applying a microwave pulse to map the electronic $|W\rangle$ state onto a nuclear spin $|W\rangle$ state [24]. The time scale for such a single-qubit operation is limited to approximately 100 kHz by the hyperfine splitting of the NV centers in the $m_s = 1$ state ($A_0 = -2.14$ MHz for $^{14}$N) [40]. Operation at cryogenic temperatures allows for resonant readout of the $|W\rangle$ state via the zero phonon line without significant background fluorescence [41]. In this collective qubit architecture, two-qubit gates between ensembles are enhanced by a factor of $N$ instead of $\sqrt{N}$, thus leading to a SWAP time of $t_\pi = 70$ $\mu$s. Thus, we find that a gate error of $\varepsilon = 10^{-2}$ requires a coherence time of $T_2^{\text{eff}} = 700$ $\mu$s, while for $\varepsilon = 10^{-4}$, a coherence time of $T_2^{\text{eff}} = 3$ ms is needed [30,32,42,43].

In summary, we have shown that collectively enhanced interactions can be realized between a NV qubit and a mesoscopic NV ensemble. Our proposed approach relies upon a transverse magnetic field to inhibit the localization
of a symmetric $W$ eigenstate. Our work enables the realization of collectively enhanced quantum gates with high fidelity and provides an important step towards the realization of scalable quantum information architectures involving solid-state electronic spins.

We acknowledge fruitful discussions with C. Laumann and S. Bennett. This work was supported by the National Science Foundation through a grant for the Institute for Theoretical Atomic, Molecular and Optical Physics at Harvard University and Smithsonian Astrophysical Observatory, a fellowship within the Postdoc Program of the German Academic Exchange Service (DAAD), the DOE (FG02-97ER25308), CUA, NSF, DARPA, AFOSR, MURI, and the Packard Foundation.

*hweimer@cfa.harvard.edu