# Imprints of Geodynamic Processes on the Paleoclimate Record

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Imprints of geodynamic processes on the paleoclimate record

A dissertation presented
by
Jacqueline Austermann

to
The Department of Earth and Planetary Sciences

in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy
in the subject of
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Imprints of geodynamic processes on the paleoclimate record

Abstract

In this thesis I investigate how solid Earth deformation associated with glacial isostatic adjustment and mantle convection impacted ice age climate. In particular, I discard approximations that treat the Earth’s internal properties as radially symmetric and demonstrate that lateral variations in viscosity and density within the Earth’s mantle play an important role in understanding and interpreting surface observations.

At the beginning of this thesis, I turn my attention to the Last Glacial Maximum, ~21 kyr ago. Estimates of the globally averaged sea level low stand, or equivalently maximum (excess) ice volume, have been a source of contention, ranging from ~120m to ~135m. These bounding values were obtained by correcting local sea level records from Barbados and northern Australia, respectively, for deformation due to glacial isostatic adjustment using 1-D viscoelastic Earth models. I demonstrate that including laterally varying mantle structure, and particularly the presence of a high viscosity slab consistent with seismic imaging and the tectonic history of the Caribbean region, leads to a significant reinterpretation of the Barbados sea level record. The revised analysis places the sea level low stand at close to ~130m, bringing it into accord with the inferred value from northern Australia within their relative uncertainties.

In the following three chapters I explore the effects of dynamic topography on sea level records during past warm periods. Dynamic topography is supported by viscous flow and buoyancy variations in the Earth’s mantle and lithosphere. I begin by developing a theoretical framework for computing gravitationally self-consistent sea level changes driven by dynamic topography and then combine this framework with models of mantle convective flow to investigate two important time
periods in the geologic past. First, I examine the Last Interglacial (LIG) period, approximately 125 kyrs ago, which is considered to be a recent analogue for our warming world. I show that changes in dynamic topography since the LIG are on the order of a few meters, making them a non negligible source of uncertainty in estimates of excess melting during this time period. Second, I turn to the mid-Pliocene warm period (MPWP), ca. 3 Ma ago, which is a more ancient analogue for climate of the near future since temperatures were elevated, on average by ~2°C. Dynamic topography has been shown to significantly deform the elevation of shoreline markers of mid-Pliocene age, particularly along the U.S. Atlantic coastal plain. It has also profoundly altered bedrock topography within the Antarctic over the last 3 Myr. I couple my dynamic topography calculations to an Antarctic Ice Sheet model to explore this previously unrecognized connection and find that changes in topography associated with mantle flow have a significant effect on ice sheet retreat in the marine-based Wilkes basin, suggesting levels of ancient instability that are consistent with offshore geological records from the region. This finding indicates that the degree to which the mid-Pliocene can be regarded as an analogue for future climate is complicated by large-scale dynamic changes in the solid Earth.

In the final section of this thesis, I move to the surface record of large igneous provinces (LIPs) – which are often cited as mantle flow induced drivers of critical events in Earth’s ancient climate – and examine whether the location of LIPs carries information about the stability of large-scale structures in the deep mantle that have been imaged by seismic tomography. In particular, I investigate the spatial correlation between LIPs, which are the surface expression of deep sourced mantle plumes, and large low shear wave velocity provinces (LLSVPs) at the core mantle boundary. A correlation between LIPs and margins of LLSVPs has been used to argue that LLSVPs are thermochemical piles that have been stationary over time scales exceeding many hundreds of millions of years. My statistical analysis indicates that there is a statistically significant correlation between LIPs and the overall geographic extent of LLSVPs, and this admits the possibility that LLSVPs may be more
transient, thermally dominated structures. I conclude that given the limited record of LIPs, one cannot distinguish between the two hypotheses that they are correlated with the edges or the areal extent of the LLSVPs.
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To my grandfathers, Prof. Gerd Hildebrandt and Günter Austermann.
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Since the beginning of the Pleistocene (~2.6 million years ago), ice sheets have periodically advanced and retreated as the Earth oscillated between warm interglacial periods and cold glacial conditions. Understanding the extent of this ice advance and retreat is crucial to unraveling the drivers and feedback mechanisms active in our climate system. In this thesis we approach this question through the interpretation of past sea level records. Any ancient marker of sea level not only records changes in ice volume but also perturbations in crustal elevation and the gravitational field. Therefore, taking full advantage of geological and geodetic measurements of sea level change requires a sophisticated understanding of solid Earth processes and structure. The research presented here lies, at least in part, at the interface between climate and geodynamic research and we address a series of key questions through geophysical modeling of the solid Earth and its cryosphere. In regard to the former, we advance earlier work by incorporating more realistic models of mantle structure and dynamics that include 3-D variations in viscosity and state-of-the-art models of mantle convective flow. The
chapters of this thesis are arranged in reverse chronological order, moving from the most recent time period to the geologic past.

The Last Glacial Maximum (LGM) occurred ~26-20 kyr ago and was accompanied by a sea level low stand that exposed most of the Earth’s continental shelves (Clark et al., 2009). Understanding the evolution of ice cover since the LGM, as reflected in the geological record of sea level, is an important element in any effort to elucidate both the complex coupling between ice sheets, oceans and atmosphere and the present level of isostatic disequilibrium in the Earth system. Sea level change driven by the growth or decay of grounded ice is spatially variable due to the direct gravitational effect of the evolving (ice-plus-ocean) surface mass load and the viscoelastic adjustment of the Earth in response to this load. Therefore, observations of local sea level change need to be corrected for glacial isostatic adjustment (GIA) to obtain a robust estimate of eustatic, or ice-equivalent, sea level change. In Chapter 2 we revisit the GIA correction of the coral record from Barbados (Peltier & Fairbanks, 2006), which has served as a canonical constraint on the magnitude of the globally averaged sea level low stand during the LGM. Our GIA correction incorporates three-dimensional viscoelastic mantle structure, a level of sophistication that is necessary to capture the tectonically complex region of the Caribbean subduction zone.

In addition to GIA, dynamic topography can also perturb the elevation of ancient sea level markers and deform the bedrock under ice sheets. Dynamic topography is supported by viscous flow and buoyancy variations in the mantle and lithosphere – the same process that drives plate tectonics – and it evolves as the mantle convects over geologic time. In this thesis, we use the mantle convection code ASPECT (Kronbichler et al., 2012) to model time dependent mantle convection and the resulting changes in dynamic topography. Our simulations are initialized using viscosity and density heterogeneity that are inferred from seismic tomographic models and that are carefully tuned to fit a wide range of present-day geophysical observables (e.g., long wavelength geoid variations, plate motions, core-mantle-boundary ellipticity, etc). In Chapter 3 we develop the first gravitationally
self-consistent theory for computing sea level changes driven by the combined effects of surface mass (ice, ocean and sediment) redistribution, dynamic topography and convection-induced gravity perturbations. This calculation is not straightforward because any change in crustal elevation causes the oceans to readjust to a new equipotential sea surface, and this redistribution (including shoreline migration) acts to load and deform the surface, which in turn drives further ocean redistribution, and so on. We use this theory, in combination with ASPECT, to estimate changes in dynamic topography and to explore the impact of these changes on past warm periods, as well as on the geological record that serves as a lens for studying these periods.

Past Interglacials provide an important natural laboratory for investigating the response of the cryosphere to moderate amounts of global warming. In this regard, the two interglacials that have been the focus of most research are the last interglacial (LIG), ~125 kyr ago, and MIS 11, ~400 kyr ago. Globally averaged sea level during the LIG is thought to have peaked ~6-9 m above the present-day (Kopp et al., 2009; Dutton & Lambeck, 2012; Hay et al., 2014), and the analogous peak during MIS 11 has been inferred to be of similar amplitude (Raymo & Mitrovica, 2012). While the effect of GIA for these interglacials has been investigated and included in previous studies, the sea level change associated with dynamic topography since these time periods is unknown. In Chapter 4 of this thesis we investigate this signal and the bias it has potentially introduced in estimates of excess ice melt during these past interglacials.

The Mid-Pliocene Warm Period (MPWP, ~3 Myr ago) also serves as an important analogue for future climate and provides an ideal testing ground for studying ice sheet dynamics and stability in response to prolonged warming. It is clear from recent work by Raymo et al. (2011) and Rowley et al. (2013) that the present elevation of Pliocene shoreline markers is significantly contaminated by the effects of GIA and dynamic topography. In the face of this complexity, ongoing efforts to estimate minimum ice volumes during the mid-Pliocene based on oxygen isotope reconstructions, sea level estimates and ice sheet modeling have yet to achieve consensus. In this thesis we consider a
heretofore-unrecognized issue in efforts to constrain ice sheet stability and peak sea level during the MPWP. Ice sheet models require, as input, bedrock elevation, which defines the slope of the crust at the grounding of marine-based ice cover. Previous modeling has assumed that the bedrock elevation beneath the Antarctic during the Pliocene was the same as the present-day elevation, with the exception of a simple crustal rebound component associated with the changing ice load. However, dynamic topography systematically perturbs the bedrock topography and has significantly altered bedrock topography in the region since the MPWP. In Chapter 5, we analyze the impact of dynamic topography on reconstructions of the Antarctic Ice Sheet during the MPWP. We demonstrate that incorporating this geophysical processes into the ice sheet modeling has a profound impact on the inferred stability of the ice sheet in the Wilkes subglacial basin.

While the Plio-Pleistocene is an ideal time period to explore geologically recent climate trends, given the wealth of observational constraints, one must delve deeper into the past to understand the long-term evolution of the mantle and its effect on the surface and atmosphere. In this regard, mantle plumes are of particular interest since they are active drivers of the mantle convective system and their eruption at the surface is thought to greatly impact the Earth’s ecosystem. The surface signatures of erupted plume heads are Large Igneous Provinces (LIPs) and their reconstructed location has been argued to carry information about the stability and hence composition of the two so-called “Large Low Shear Wave Velocity Provinces” (LLSVPs) in the deep mantle. In Chapter 6 we use statistical analyses to revisit the argument that LIPs rise from the margins of LLSVPs (Torsvik et al., 2006; Burke et al., 2008) and thereby test the robustness of related claims that LLSVPs are stationary thermochemical piles in the lowermost mantle that existed for billions of years.

In Chapter 7 we summarize the main findings described in this thesis, place them in context of ongoing research, and point to promising future research directions that this work has illuminated.
2 Barbados-Based Estimate of Ice Volume at Last Glacial Maximum Affected by Subducted Plate

2.1 Introduction

Estimates of ice volume at Last Glacial Maximum (LGM) provide a crucial boundary condition on models of ice age climate (Pinot et al., 1999) and glacial isostatic adjustment (GIA) (Lambeck et al., 2002). The oxygen isotope record of benthic foraminifera from deep-sea sedimentary cores serves as an important proxy for global ice volume (Chappell & Shackleton, 1986), however uncertainties in the mapping between $\delta^{18}O$ and ice volume associated with the effects of temperature (Waelbroeck

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et al., 2002) and the mean isotopic concentration of continental ice (Mix & Ruddiman, 1984) limit the accuracy of this proxy. Geophysical analyses of sea-level markers at sites in the far-field of the Late Pleistocene ice sheets (Fairbanks, 1989; Yokoyama et al., 2000) have also provided estimates of ice volume at LGM. In this case, the mapping between local sea-level histories and excess ice volume (or ice-equivalent eustatic sea-level change, ESL) is based on numerical corrections for the contaminating signal from GIA. An analysis of this type, based on the coral record at Barbados (Fairbanks, 1989; Bard et al., 1990; Peltier & Fairbanks, 2006), has yielded an estimate of excess ice volume at LGM of ~120 m ESL (Peltier & Fairbanks, 2006). However, this analysis, like all previous analyses of LGM-to-present far-field sea-level records (Lambeck et al., 2002), adopted 1-D (depth-varying) viscoelastic models of Earth structure. In this paper, we provide the first assessment of the bias introduced in estimates of ice volume at LGM based on the Barbados coral record by ignoring the complex tectonic setting of the Caribbean (Pindell et al., 1988).

The RSL history of Barbados (Figure 2.1a) is noteworthy because of the accurate U/Th dating of the coral-based record (Fairbanks, 1989; Bard et al., 1990; Peltier & Fairbanks, 2006). While the complete set of observations involves four coral species, Acropora palmata (Ap), Montastrea annularis, P. asteroids and Diploria, we focus here on the particularly important Ap record because this species resides within 5 m of sea level in the modern ocean (Peltier & Fairbanks, 2006). Various lines of evidence suggest that almost all ice sheets were at their maximum extent over the time span 26.5-20 ka (Clark et al., 2009), with no significant change in ice volume, and so we focus on RSL at the end of this time window. At Barbados, local relative sea level was approx. ~120 m at this time. Subsequent sea-level rise at the site was relatively gradual, with the exception of a pulse of rapid sea-level change during meltwater pulse 1A (mwp-1A) at ~14 ka.
Figure 2.1: Relative Sea Level at Barbados from LGM to present. a. Black line - RSL prediction at Barbados (reproduced from Peltier & Fairbanks, 2006) computed using the ICE-5G/VM2 GIA model (Peltier, 2004). Red line - eustatic sea-level variation associated with the ICE-5G model, also from Peltier & Fairbanks (2006). Blue line - analogous to black line, except that 3-D variations in viscoelastic structure are incorporated (see text). The green symbols are U/Th dated Acropora palmata (Ap) samples (Fairbanks, 1989; Bard et al., 1990; Peltier & Fairbanks, 2006) corrected for tectonic uplift (Peltier & Fairbanks, 2006). b. Difference between predicted RSL and ESL at 21 ka over the Caribbean region computed (Latychev et al., 2005) using the ICE-5G/VM2 GIA model (star denotes Barbados).
2.2 Glacial isostatic adjustment at Barbados

Numerical predictions of global scale, post-glacial sea-level change require models for the Earth’s viscoelastic structure and the history of ice cover. We begin by reproducing a numerical prediction of the post-LGM RSL history at Barbados (Peltier & Fairbanks, 2006) based on the ICE-5G ice model and the VM2 radial profile of mantle viscosity (Peltier, 2004) (black line, Figure 2.1a). VM2 has an elastic lithospheric thickness of 90 km and a viscosity which increases moderately from \(-5 \times 10^{20}\) Pa s in the upper mantle to \(-2.3 \times 10^{21}\) Pa s in the deep mantle. The prediction matches the coral record, which is expected given that the ICE-5G history was tuned to produce this match (Peltier & Fairbanks, 2006; Engelhart et al., 2009). The RSL prediction closely tracks the globally averaged, eustatic sea-level change associated with the ICE-5G history (red line, Figure 2.1a), which has a value of \(-123\) m at 21 ka; the maximum post-LGM discrepancy is \(-5\) m, and it is \(-4\) m at LGM (Figures 2.1a). This correspondence has led to suggestions that the net GIA-induced departure from eustasy at Barbados is close to zero (Peltier & Fairbanks, 2006), though this conclusion is dependent on the adopted viscoelastic Earth model (Milne & Mitrovica, 2008).

To explore this issue in detail, we have computed (Kendall et al., 2005) the difference between local and eustatic sea level at LGM across the Caribbean using the ICE-5G/VM2 combination (Figure 2.1b). While the difference is small at Barbados, deglaciation-induced ocean loading produces a strong departure from eustasy near the coastline of South America and at large islands stretching from Cuba to Puerto Rico in the northwest. In particular, the post-LGM meltwater load drives a subsidence (sea-level rise) of the oceanic crust and an uplift (sea-level fall) of these unloaded landmasses, in a process called continental levering (Nakada & Lambeck, 1989; Mitrovica & Milne, 2002). Moreover, the VM2 viscosity profile is relatively weak, and this places Barbados at the distant outer flank of the subsiding Laurentian peripheral bulge (Peltier & Fairbanks, 2006), thus muting the sea-level rise contributed by this process. The net effect of the continental levering and bulge
subsidence, together with other processes contributing to far-field sea-level rise (Milne & Mitrovica, 2008; Mitrovica & Milne, 2002), is a minor departure from eustasy at Barbados.

Barbados is proximal to west-dipping subduction of the South American Plate under the Caribbean Plate (Figure 2.2a). This suggests that the adoption of a 1-D viscosity profile in GIA predictions may be inadequate. The Cenozoic history of the Caribbean region is dominated by the eastward migration of the Caribbean Plate over the Americas and the development of a dextral transform fault at the northern margin of South America (Pindell et al., 1988). An analysis of seismic anisotropy using high-resolution, global mantle convection models (Miller & Becker, 2012) indicates that lateral variations in mantle viscosity in this region are defined by the juxtaposition of a high viscosity subducting slab and a thick continental craton in northern South America (Guyana Shield).

2.3 Methods

We have incorporated these structural elements into a finite-volume model (Latychev et al., 2005) of gravitationally self-consistent post-glacial sea-level change (Kendall et al., 2005) capable of treating 3-D viscosity variations (Figure 2.2). Our 3-D model of mantle structure is constructed as follows: First, global lateral variations in the thickness of an elastic lithosphere are taken from the model of Conrad & Lithgow-Bertelloni (2006). The model is characterized by a thickness of ~180 km in the Guyana Shield (Figure 2.2b). Second, we adopt a digital map of global plate boundaries (Bird, 2003) and model these boundaries as thin vertical zones in which the viscosity is set to $2 \times 10^{20}$ Pa s.

Lastly, large scale lateral variations in mantle viscosity were constructed by scaling the S40RTS seismic tomography model of global shear wave heterogeneity (Ritsema et al., 2011) using the method outlined in Latychev et al. (2005), which is similar to earlier approaches (Ivins & Sammis, 1995; Kaufmann et al., 2005). In particular, the 3-D viscosity field is prescribed through a sequence of steps that can be summarized by the following set of equations:
Figure 2.2: Tectonic setting of the Caribbean and 3-D Earth model. 

a. Crustal elevation (legend at bottom), with coastlines outlined in black. White dashed lines denote plate boundaries (Bird, 2003). Red to green lines parallel to the trench are contours of depth (inset) from surface down to the subducted slab (Gudmundsson & Sambridge, 1998). 

b-c. The 3-D viscoelastic Earth model adopted in GIA predictions. 

b. Thickness of elastic lithosphere (Conrad & Lithgow-Bertelloni, 2006). 

c. Depth dependence of average logarithm of viscosity below Barbados, relative to the 1-D VM2 profile, where the average is computed within a cone of diameter ranging from 300 km at surface to 3300 km at CMB (see inset).
\begin{align*}
\partial \ln \rho(r, \theta, \phi) &= \frac{\partial \ln \rho}{\partial \ln v_s} \cdot \partial \ln v_s(r, \theta, \phi) \\
\partial T(r, \theta, \phi) &= \frac{-1}{\alpha(r)} \cdot \partial \ln \rho(r, \theta, \phi) \\
\eta(r, \theta, \phi) &= \eta_0(r) \cdot \exp(-\epsilon \cdot \partial T(r, \theta, \phi))
\end{align*}

where \( r, \theta \) and \( \phi \) are the radius, colatitude and east longitude, respectively, \( v_s \) is the shear wave velocity, and \( \rho, T \) and \( \eta \) are the density, temperature and viscosity, respectively. The coefficient of thermal expansion, \( \alpha \), and the velocity-to-density scaling factor, \( \partial \ln \rho / \partial \ln v_s \), are both assumed to be functions of depth alone.

Relative variations in seismic wave speed given by the model S40RTS (Ritsema et al., 2011) are converted to relative perturbations in density (first equation) using a velocity-to-density scaling profile based on geodynamic modeling and constraints from mineral physics, and this density field is next converted to a temperature field (second equation) using a radial profile of the coefficient of thermal expansion (see Latychev et al., 2005, for complete details and choice of conversion factors). Finally, we assume that the viscosity field has an exponential dependence on temperature (third equation), where \( \eta_0(r) \) is the reference radial profile of mantle viscosity (i.e., \( \eta \) when \( \partial T = T - T_0 = 0 \)), which in our case is VM2 (Peltier, 2004). The free parameter \( \epsilon \) governs the strength of the temperature dependence of viscosity. We assume that this parameter is independent of depth. While this treatment is different to that used in previous studies (Ivins & Sammis, 1995; Kaufmann et al., 2005), by choosing a value of \( \epsilon = 0.04K^{-1} \) we obtain peak-to-peak variations in viscosity that reach \(~5\) orders of magnitude and that are comparable to these earlier studies. Note that we also include a radially dependent scaling on the right hand side of the third equation (not shown) to ensure that the spherically averaged logarithm of \( \eta(r, \theta, \phi) \) at any depth is the same as the reference model (VM2). Figure 2.3 shows the logarithmic perturbation in mantle viscosity, relative to the
Figure 2.3: Vertical cross-section showing the perturbation in the logarithm of viscosity (relative to the 1-D profile VM2; Peltier, 2004) derived, as discussed in section 2.3, from the tomographic model S40RTS (Ritsema et al., 2011). In the GIA modeling only the lateral variation in lower mantle viscosity is included in the 3-D Earth model. The upper mantle viscosity variation is replaced with a slab of high viscosity and a stiff lithosphere of variable thickness (see text).

1-D profile VM2, in a vertical cross-section perpendicular to the strike of the trench. Moreover, Figure 2.2c shows the depth-varying average viscosity below Barbados within a cone of diameter that increases from 300 km at the base of the lithosphere to 3300 km at the core-mantle-boundary.

To address the limited (~500-1000 km) spatial resolution of the S40RTS seismic model, and to avoid double counting heterogeneity, we replace the upper mantle viscosity field (including the transition zone) derived from the above methodology (as shown in Figure 2.3) with a South American slab geometry inferred from regional seismicity (Gudmundsson & Sambridge, 1998) (Figure 2.2a)
superimposed on the 1-D VM2 profile. (A similar procedure adopted by Miller & Becker, 2012, improved mantle flow predictions of seismic anisotropy in the Caribbean region.) The subduction zone is seismogenic to a depth of ~200 km, the deepest contour provided by Gudmundsson & Sambridge (1998). However, subduction below the Caribbean Plate has occurred since at least the Late Cretaceous (Pindell et al., 1988), and thus the slab must penetrate well below the seismogenic zone. We extend the slab down to the base of the upper mantle (Figure 2.2a) using the dip inferred from seismicity. This depth is also supported by seismic tomography (Ritsema et al., 2011) for most vertical cross-sections through the subduction zone (see Figure 2.3). We adopt a slab thickness of ~100 km, and a slab viscosity of $5 \times 10^{23}$ Pa s (Billen, 2008).

Finally, all our calculations are based on a gravitationally self-consistent sea-level theory that incorporates time-varying shorelines associated with either the local onlap or offlap of water or changes in the extent of grounded, marine-based ice sheets (Kendall et al., 2005).

2.4 Results and Discussion

Three-dimensional variations in viscoelastic structure have a significant impact on predictions of sea-level change in the Caribbean region (Figure 2.4). At LGM, a large positive perturbation to RSL is strongly correlated with the presence of the shallowest portion of the downgoing slab (Figure 2.4a). This signal peaks at ~13 m along the archipelago from Puerto Rico to Guadeloupe, and trends to lower values moving southward toward South America or eastward. This region of the Caribbean is subject to subsidence associated with meltwater loading, which contributes a component of sea-level rise to the net departure from eustasy. The presence of a high viscosity slab acts to dampen this subsidence relative to the 1-D Earth model calculation (see Fig. 2.5), reducing the total predicted sea-level rise from LGM to present at Barbados (Figure 2.1a, blue line relative to dashed black line). The opposite effect occurs in the thick continental craton of South America. In this region, the thick lithosphere dampens the ocean load-induced uplift of the crust, leading to an increase in the
Figure 2.4: Impact of lateral variations in mantle viscoelastic structure on predictions of RSL at Barbados. a. Perturbation in RSL at 21 ka in the Caribbean due to 3-D mantle viscoelastic structure (see Figures 2.2, 2.3). The numerical prediction (Latychev et al., 2005) adopts the ICE-5G model of ice geometry (Peltier, 2004). b. Blue line - perturbation in the predicted RSL at Barbados, as a function of time, associated with 3-D mantle viscoelastic structure. Green line: same as blue, except the high viscosity slab in the upper mantle, associated with subduction under the Caribbean plate, is removed from the viscosity model.

predicted post-LGM sea-level rise.

At LGM, the perturbation to the predicted RSL at Barbados due to 3-D mantle structure exceeds 7 m (Figures 2.1a, 2.4b). This perturbation is nearly twice the total GIA-induced departure from eustasy predicted using the 1-D VM2 Earth model (Figure 2.1b). To isolate the impact on the predictions of the high viscosity slab, we repeated the 3-D calculation for a case in which the slab was removed. The result (Figure 2.4b, green line) indicates that the effect of the slab dominates the signal associated with 3-D structure.

The presence of a high viscosity slab below the Caribbean reduces the predicted RSL rise at Barbados such that, when the ICE-5G model history is adopted, this prediction no longer fits the coral-based sea-level history (blue line, Figure 2.1a). To maintain a fit to the RSL record at Barbados using
Additional water due to deglaciation
Subsidence to reach isostatic equilibrium

Figure 2.5: Schematic illustration of the impact of a high viscosity slab on meltwater-driven crustal deformation in the far-field of the Late Pleistocene ice sheets. a. Low sea level at LGM in two vertical cross-sections, one involving a nominal upper mantle viscosity (right) and one in which a high viscosity slab is superimposed on this nominal viscosity field (left). b. Ongoing deglaciation adds meltwater to the oceans, driving c. crustal subsidence of the ocean floor and contributing to the predicted sea-level rise at far-field sites. The subsidence is a function of the mantle viscosity. In the presence of the high viscosity slab, the meltwater-induced crustal subsidence, and the sea-level rise it contributes, is muted relative to the scenario in which no slab is present (left versus right in frame c).

This 3-D Earth model, one must scale upward the excess ice volume at LGM. To illustrate this, Figure 2.6 (blue line) was constructed by scaling the blue line in Figure 2.1a, which adopted the ICE-5G ice history and accounts for the effects of 3-D structure, upwards to reestablish a fit to the observations. The excess ice volume of the scaled model is ~130 m ESL at 21 ka (Figure 2.6, red line). We note that scaling the ice model in this manner would also improve the fit to the RSL record at Sunda Shelf relative to a prediction based on ICE-5G/VM2 (Peltier & Fairbanks, 2006).

We performed a suite of sensitivity tests related to 3-D viscoelastic Earth structure. For example, we used an alternate model of lithospheric thickness (Watts, 2001) characterized by the absence of a thick root in the Guyana Shield, and the magnitude of the perturbation associated with 3-D structure increased by ~0.5 m. The effect of reducing the viscosity within plate boundary zones was smaller, a result that reflects the relatively uniform ocean load in the vicinity of Barbados. We also repeated the analysis using a spherically averaged mantle viscosity profile characterized by upper and
Figure 2.6: Prediction of RSL at Barbados based on a 3-D viscoelastic model. Blue line - RSL prediction at Barbados that accounts for 3-D viscoelastic Earth structure (Figure 2.1a, blue line), but scaled upward so as to maintain a fit to the observations. Red line: eustatic sea-level variation associated with the scaled ICE-5G model. Symbols are plotted as in Figure 2.1a.
lower mantle viscosities of $5 \times 10^{20}$ Pa s and $5 \times 10^{21}$ Pa s, respectively. Inferences of eustatic sea level at LGM based on such models (Lambeck et al., 2002) tend to be higher than inferences using VM2; the perturbation in the predicted RSL at Barbados at LGM due to 3-D viscoelastic structure increased by $\sim 1$ m (to 8 m) when the former was adopted as the background viscosity profile.

Subduction zones have complex rheological properties, including a transition from brittle to plastic behavior at the trench site (Scholz, 2002) and the possibility of a low-viscosity zone in the mantle wedge due to hydrous phases and melt. The latter was incorporated as a global layer in a study of seismic anisotropy in the Caribbean (Miller & Becker, 2012), though analyses of late-Holocene RSL data using 1-D Earth models suggest that a low-viscosity mantle wedge may not be a universal feature of subduction zones (Yokoyama et al., 2012). We performed several additional simulations to consider these issues. For example, increasing or decreasing the slab viscosity by an order of magnitude perturbed the RSL prediction at LGM by $\sim 0.5$ m. Increasing the thickness of the slab to 150 km increased the perturbation due to 3-D structure by $\sim 1$ m. Finally, we ran a simulation in which a mantle wedge of thickness 50 km, lateral extent of 100 km and viscosity of $3 \times 10^{19}$ Pa s was incorporated into the model. The perturbation due to 3-D structure was only marginally reduced at Barbados ($\sim 0.3$ m).

2.5 Conclusion

Incorporating lateral variations in mantle viscosity into GIA predictions of RSL change brings the estimate of excess ice volume at LGM based on data from Barbados ($\sim 130$ m ESL) into accord with independent estimates based on RSL constraints at Bonaparte Gulf (Yokoyama et al., 2000). This reconciliation raises an important enigma. A recent study of geodetic and geological records in the Antarctic (Whitehouse et al., 2012) suggest that LGM ice volume in the region was $\sim 10$ m ESL lower than widely adopted reconstructions (Fairbanks, 1989; Yokoyama et al., 2000). This result, together with our inference, suggests that a significant volume of ice in the northern hemisphere at LGM
remains unaccounted for. We conclude that analyses of RSL histories at sites close to subduction zones, whether they are focused on estimates of ice volume at LGM or other periods of paleoclimatological interest (e.g., the Last Interglacial, the late Holocene, etc.), or used to infer the sources of events such as meltwater pulse 1A, must incorporate slab structure into the numerical predictions of the GIA process.
3.

Calculating gravitationally self-consistent sea level changes driven by dynamic topography

3.1 Introduction

The earliest discussions of plate tectonics focused on horizontal motions at the Earth’s surface, reflecting a preoccupation with the concepts of continental drift and sea floor spreading. However, the driving force for these motions - thermochemical convection within the Earth’s mantle - is also responsible for vertical deflections of the Earth’s surface (McKenzie, 1977) that have come to be known as dynamic topography (e.g. Hager et al., 1985). The earliest simulations of time-varying dynamic

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topography connected the process to epeirogenic motions of continents and long-term regional and global scale sea level changes, and typically focused on subduction-controlled dynamics (e.g. Mitrovica et al., 1989; Gurnis, 1990, 1992, 1993; Holt & Stern, 1994; Coakley & Gurnis, 1995; Mitrovica et al., 1996; Lithgow-Bertelloni & Gurnis, 1997; Pysklywec & Mitrovica, 1998; Gurnis et al., 1998; Pysklywec & Mitrovica, 1999). However, several studies also demonstrated a clear link between upwelling mantle flow and large scale fluctuations in the topography of continents (e.g. Lithgow-Bertelloni & Silver, 1998; Gurnis et al., 2000; Forte et al., 2010).

Predictions of present-day, global scale dynamic topography have been derived from viscous flow modeling constrained by seismic inferences of mantle heterogeneity (e.g. Hager et al., 1985; Ricard & Wuming, 1991; Forte et al., 1993; Wen & Anderson, 1997; Steinberger, 2007). As the resolution of seismic imaging has progressively improved, viscous flow modeling of time-varying dynamic topography has shown concomitant advances in resolution, allowing applications to the detailed, regional-scale geological record of continents (e.g. Daradich et al., 2003; Moucha et al., 2009; Rowley et al., 2013; Liu et al., 2014). These studies have had fundamental implications for our understanding of long-term sea level change. For example, they have highlighted the fact that dynamic topography is present globally, not only at plate boundaries, and that it can significantly contaminate records of long-term sea level change at all locations, even passive continental margins (e.g. Moucha et al., 2008; Müller et al., 2008b; Conrad & Husson, 2009; Spasojevic & Gurnis, 2012; Rowley et al., 2013). Moreover, these analyses have shown that the globally averaged change in the shape of ocean basins is governed by the integrated effect of dynamic uplift and subsidence throughout these basins, not just changes in elevation linked to fluctuations in the rate of plate creation.

While the connection between dynamic topography and regional and global sea level changes has been established, a rigorous mathematical and numerical formalism for accurately mapping viscous flow predictions of time-dependent dynamic topography into geographically variable sea level change is lacking. Previous efforts to compute this mapping have several shortcomings:
(1) A change in dynamic topography will redistribute water and the loading (or unloading) associated with this redistribution will, in turn, drive displacements of the crust and hence cause sea level changes. This loading effect is generally treated in an approximate sense through isostatic amplification of the ‘air-loaded’ dynamic topography. In this regard, some studies (e.g. Rowley et al., 2013; Moucha et al., 2008) assume water cover everywhere and apply a global amplification to the air-loaded dynamic topography obtained from their viscous flow modeling. In contrast, Müller et al. (2008b) applied no amplification to the dynamic topography change for their predictions along the U.S. East coast. A hybrid approach only amplifies the dynamic topography at sites with ocean cover (Steinberger, 2007; Flament et al., 2013), however, it, and the other approaches neglect water loading within any region of shoreline migration. This assumption is problematic given that the error introduced into the prediction will be largest in the vicinity of an ancient sea level marker;

(2) The above studies also ignore the possibility of flexural effects in the response to water loading; that is, they assume local hydrostatic equilibrium in the crustal response. Flexure has a direct effect on the computed sea level change via its impact on crustal elevation (e.g., the crustal displacement will extend outside the region of loading). Moreover, it has indirect effects on sea level change because any departures from hydrostatic equilibrium will drive gravitational and rotational effects that feedback into sea level;

(3) Studies of dynamic topography are often only concerned with the change in crustal elevation due to mantle stresses. Sea level, however, is the difference between the elevation of both the gravitational equipotential defining the sea surface and the crust and therefore changes in the gravitational field that arise from mantle convection will perturb sea level, an effect only rarely taken into account (Spasojevic & Gurnis, 2012; Conrad & Husson, 2009);

(4) A further shortcoming involves the assumption of a local application of isostasy (e.g. Müller et al., 2008b; Conrad & Husson, 2009; Spasojevic & Gurnis, 2012). Any loading induced uplift (subsidence) of the crust must be matched by subsidence (uplift) elsewhere to conserve mass of the solid
Earth. Local treatments of isostasy (whether they incorporate flexure or not), ignore this global scale compensation;

Finally, predictions of long-term sea level change should also conserve the mass of the ocean (or ice plus ocean), but this constraint is not always applied in such calculations (e.g. Flament et al., 2013).

In this article, we present a gravitationally self-consistent mathematical treatment of global scale sea level change driven by time-varying dynamic topography, convection-driven geoid perturbations, ice mass changes and sediment redistribution. Our approach is based on a formalism developed in studies of ice age sea level change (Farrell & Clark, 1976; Mitrovica & Milne, 2003; Kendall et al., 2005; Dalca et al., 2013) and it conserves mass of both the surface (ice plus ocean) load and the solid Earth while taking precise account of the migration of shorelines. In its most general form, and under the assumption that the dynamic topography, convection-driven geoid perturbations, ice mass and sediment histories are known, the method is valid for viscoelastic Earth models of arbitrary complexity; one only needs to have a numerical method for computing the response of the Earth model to an evolving surface mass load. Our approach may be used to consider dynamic topography in isolation; however, we have included the possibility of contemporaneous ice mass changes and sediment fluxes to provide a framework that avoids errors that may be incurred by assuming that the total sea level change can be computed by treating each of the driving processes in isolation. Finally, we also outline the extension necessary when the time scale being considered is sufficiently long that plate motions must be taken into account.

In addition, we present successive, special cases that assume: (1) isostatic equilibrium is maintained throughout the evolution of the system; and (2) the Earth model varies with depth alone. The first assumption simplifies the numerical calculations by ignoring time-dependent viscous adjustments and this allows one to evolve the system backward in time starting from a known (present-day) topography. Taken together, the two assumptions allow us to make use of fluid Love num-
ber theory to compute the response of the Earth model to the surface mass loading. In this case, we adopt profiles of density and elastic moduli from the PREM seismic model (Dziewonski & Anderson, 1981) and the thickness of the elastic lithosphere is a free parameter in the modeling. To illustrate the new, gravitationally self-consistent treatment of sea level change driven by dynamic topography, we present a series of numerical predictions based on these special cases.

3.2 Calculating long-term sea level change

3.2.1 Calculating dynamic topography from mantle convection

A variety of approaches have been adopted to compute dynamic topography in mantle convection simulations. The most rigorous, but computationally expensive approach is to allow the surface to deform freely. These free surface calculations allow the upper boundary of the computational domain to respond dynamically to evolving stresses in the mantle (Zhong et al., 1996; Gurnis et al., 1996; Kaus et al., 2010; Kramer et al., 2012). An alternate, fixed surface approach introduces a top layer in the computational domain with a comparatively low viscosity and low density, which serves as a proxy for air and yields quasi-free surface deformation. This approach is often referred to as the ‘sticky-air’ method (Schmeling et al., 2008; Crameri et al., 2012). The third, and most common approach models mantle convection within a fixed domain and computes the normal stresses at the top of the domain. The dynamic topography ($DT$) is then calculated as the compensation height that balances these normal stresses (McKenzie, 1977; Hager et al., 1985; Zhong et al., 1993; Flament et al., 2013):

$$DT = \frac{\sigma_{rr}}{\Delta \rho \cdot g}$$  \hspace{1cm} (3.1)

where $\sigma_{rr}$ is the radial stress at the upper surface of the mantle flow model, $\Delta \rho$ is the density contrast between the mantle and the overlying material (e.g., air, water or sediment) and $g$ is the gravita-
tional acceleration. This approach can be improved by including self-gravitation associated with the disturbed topography \(\text{(Zhong et al., 2008)}\) or flexural effects of an elastic lithosphere in response to the internal deformation \(\text{(Golle et al., 2012)}\). The three distinct approaches yield comparable predictions for the dynamic topography \(\text{(Crameri et al., 2012)}\); the last approach is characterized by a slight shift backward in time, compared to the others, because it assumes an instantaneous isostatically adjusted response of the surface to internal loading.

As is clear from the basic principles of isostasy that underlie equation (3.1), the amplitude of predicted dynamic topography arising from viscous stresses associated with mantle flow increases as the density difference between the crust and overlying material decreases. Thus, the choice of an appropriate density contrast is a key issue in comparing predictions of dynamic topography with the geological record. Regardless of the method employed for modeling dynamic topography, one has to choose an appropriate density contrast between the mantle and the overlying material. As described in the introduction, this a shortcoming in existing models of dynamic topography, which can be overcome using the framework discussed below.

### 3.2.2 Sea level changes in the presence of dynamic topography

The derivation presented below is based on the ice age sea level theory developed by Mitrovica & Milne \(\text{(2003)}\), Kendall et al. \(\text{(2005)}\) and Dalca et al. \(\text{(2013)}\), following the canonical treatment of Farrell & Clark \(\text{(1976)}\). Farrell & Clark \(\text{(1976)}\) derived the so-called “sea-level equation” governing gravitationally self-consistent sea level changes on a viscoelastic Earth driven by global ice mass variations. Their derivation assumed that the position of shorelines was fixed in time, which is equivalent to assuming that all shorelines were characterized by steep vertical cliffs such that any local changes in sea level produce no transgression or regression (i.e., no shoreline migration). Their calculations were based on spherically symmetric, Maxwell viscoelastic and non-rotating Earth models, and in this case they adopted viscoelastic Love number theory \(\text{(Peltier, 1974; Wu & Peltier, 1982)}\) to compute
time-dependent perturbations in the elevation of the solid surface and gravitational field. Mitrovica & Milne (2003) extended the sea-level equation to accurately incorporate shoreline migration and Kendall et al. (2005) presented an algorithm for solving the equation in the case of 1-D and 3-D Earth models of arbitrary complexity. The Mitrovica & Milne (2003) treatment allows for the feedback of perturbations in Earth rotation into sea level, and in the case of a 1-D, rotating Earth model, Kendall et al. (2005) derived the necessary equations making use of viscoelastic Love number theory. The Dalca et al. (2013) study extended these treatments to incorporate sediment redistribution into the gravitationally self-consistent sea level calculation. Their form of the theory, which defined the topography as including ice and sediment, is advantageous to our treatment of dynamic topography, and so it forms the basis of our derivation.

We adopt the following symbolism:

\( R \) - Earth’s bedrock elevation;

\( H \) - Sediment thickness;

\( I \) - Thickness of land-based and grounded marine-based ice;

\( G \) - Height of the equipotential surface that defines the sea surface.

All these quantities have a dependence on position, expressed in terms of colatitude \( \theta \) and east longitude \( \varphi \), and time \( t \). We will henceforth suppress these explicit dependencies to simplify the expressions. In this regard, we abbreviate a generic function \( \chi(\theta, \varphi, t_j) \) by \( \chi \), or \( \chi_j \) when it is necessary to make the time dependence explicit.

Sea level is defined as the difference between the heights of two bounding surfaces: the sea surface equipotential and the solid surface (including the height of ice and/or sediments), where the latter is given by \( R + H + I \):

\[
SL = G - (R + H + I).
\]  
\[ (3.2) \]
In this case, topography is simply defined as the negative of sea level:

\[ T = -SL = (R + H + I) - G. \]  

(3.3)

Moreover, the ocean height is simply the projection of sea level onto the location of oceans:

\[ S = SL \cdot C, \]  

(3.4)

where C is the ocean function defined by:

\[ C = \begin{cases} 
1 & \text{if } SL > 0 \\
0 & \text{if } SL \leq 0 
\end{cases} \]  

(3.5)

The total surface mass load \( L \) is comprised of water (oceans), ice and sediments:

\[ L = \rho_W \cdot S + \rho_I \cdot I + \rho_H \cdot H, \]  

(3.6)

where, \( \rho_W \), \( \rho_I \) and \( \rho_H \) are the densities of water, ice and sediment, respectively.

Geological markers record local changes in sea level, as defined in equation (3.2), and thus we are interested in how changes in grounded ice, sediments, water and dynamic topography, perturb the elevation of the bedrock and sea surface height from some initial state. Any time-dependent field \( \chi \) at some time \( t_j \) can be expressed as a sum of the initial value at the onset of loading, at time \( t_0 \), and a perturbation from this state:

\[ \chi_j = \chi_0 + \Delta \chi_j. \]  

(3.7)

This form may be applied to any of the quantities \( R, H, I, G, SL, T, S, C \) and \( L \). As an example, the change in bedrock elevation may be written as:
\[ R_j = R_0 + \Delta R_j, \] (3.8)

where the term \( R_0 \) includes the background signal from dynamic topography. The perturbation of the bedrock elevation can furthermore be decomposed into a time-dependent perturbation in air-loaded dynamic topography plus a perturbation due to the response of the bedrock to the changing surface mass (ice plus ocean plus sediment) load. If we denote these terms by \( \Delta DT_A \) and \( \Delta R^L \), respectively, we have:

\[ \Delta R_j = \Delta DT_{A,j} + \Delta R^L_j. \] (3.9)

It is important to emphasize that it is air-loaded dynamic topography that should appear in this equation. The amplification from air-loaded to water-loaded dynamic topography at sites under water, as indicated by equation (3.1), is naturally incorporated into equation (3.9) (as well as the broader sea level calculation) via the term \( \Delta R^L_j \). That is, an increase in the air-loaded dynamic topography (i.e., an uplift) at sites within the oceans will displace water and this unloading will act to amplify the uplift, as required. Similarly a dynamic subsidence computed in the air-loaded case at such sites will increase the local water load and amplify the subsidence. To be consistent, the air-loaded dynamic topography change and the load-induced perturbation to the bedrock elevation (the two terms on the right-hand-side of equation (3.9)) should be computed using the same Earth structure model.

Using the above expressions, the change in sea level is given by:

\[ \Delta SL_j = SL_j - SL_0 = \Delta G_j - (\Delta R^L_j + \Delta H_j + \Delta I_j + \Delta DT_{A,j}). \] (3.10)

The change in ocean height \( S \) cannot simply be written as a combination of perturbations in \( G, R, I \) and \( H \) because it involves the projection onto the oceanic function given by equation (3.4), which
is itself time-dependent. Using this equation, together with the general form (3.7) and the simple relationship between sea level and topography (3.3) yields:

\[ \Delta S_j = \Delta SL_j \cdot C_j - T_0 \cdot \Delta C_j. \] (3.11)

Equations (3.3), (3.5), (3.10) and (3.11) represent the generalized form of the sea level equation valid for any time scale of forcing and Earth models of arbitrary complexity. Physically, equation (3.11) indicates that the ocean height change at time \( t_j \) is equal to the change in sea level projected onto the ocean function, both at time \( t_j \), plus a correction term that accounts for shoreline migration (Mitrovica & Milne, 2003). The latter involves a projection of the initial topography onto a field (\( \Delta C_j \)) that is only non-zero in regions experiencing shoreline migration between times \( t_0 \) and \( t_j \) (i.e., \( \Delta C_j \) is only non-zero when \( C_0 \neq C_j \)).

It is also useful to have an expression for the change in the surface mass load from its initial state. Using equations (3.6) and (3.7), this change is given by

\[ \Delta L_j = \rho_W \Delta S_j + \rho_I \Delta I_j + \rho_H \Delta H_j. \] (3.12)

The change in the water load driven by dynamic topography is embedded in both fields on the right-hand-side of equation (3.4). Sea level, \( SL \), is dependent on dynamic topography via equation (3.2) (and equations (3.8) and (3.9)), and this dependence also alters the ocean function \( C \) via equation (3.5). The latter demonstrates that ocean mass redistribution driven by dynamic topography will, in general, lead to shoreline migration, even at shorelines which experience no local change in dynamic topography. The sea level equation incorporates this physics while maintaining gravitational self-consistency.

The sea surface equipotential is perturbed by both the surface mass loading and by mantle convection. We could have incorporated this second contribution into our definition for dynamic to-
pography; however, models of mantle flow-induced dynamic topography typically define dynamic topography as an absolute measure of bedrock height (or as being relative to the center of mass of the Earth), not as a height relative to the geoid (or sea surface equipotential). The perturbation in the height of the sea-surface may be expressed as:

$$
\Delta G_j = \Delta G^{mc}_j + \Delta G^L_j,
$$

(3.13)

where the first term on the right-hand-side denotes the perturbation due to mantle convection and the second term is the sea surface equipotential due to surface mass loading. The former is the perturbation to the geoid that mantle flow modelers often compute in tandem with the dynamic topography.

Equations (3.10) - (3.12) highlight the integral nature of the sea level equation (3.11). In particular, from equations (3.9), (3.10), (3.11) and (3.13), the change in ocean height, $\Delta S_j$, depends on surface load-induced perturbations to the height of the bedrock ($\Delta R_j^L$) and sea surface potential ($\Delta G_j^L$). However, both $\Delta R_j^L$ and $\Delta G_j^L$ are, in turn, dependent on the ocean height change since the latter forms a component of the total surface mass load given by equation (3.12).

It is traditional in gravitationally self-consistent ice age sea level theory to decompose the right-hand-side of equation (3.13) into geographic variable and globally uniform terms (Farrell & Clark, 1976). To this end, we can write:

$$
\Delta G_j = \Delta G^{mc}_j + \Delta G^L_j + \frac{\Delta \Phi_j}{g},
$$

(3.14)

where $\Delta G^{mc}$ and $\Delta G^L$ are the geographically variable components of $\Delta G^{mc}$ and $\Delta G^L$, respectively. While the sea surface must remain an equipotential surface, the specific equipotential that defines the sea surface need not remain constant over time (Farrell & Clark, 1976; Dahlen, 1976). Indeed, changes in the mass of the ocean in response to variations in global ice volumes, as well as
changes in the shape of the ocean basin associated with load-induced deformation or the redistri-
butution of sediments, will perturb the equipotential that defines the sea surface. In equation (3.14),
this time-dependent perturbation is denoted by $\Delta \Phi_j$, and dividing this value by the surface gravita-
tional acceleration, $g$, converts the perturbation into a geographically uniform perturbation in the
height of the initial sea surface equipotential.

Using equation (3.14) in equation (3.10) allows us to decompose the sea level change as follows:

$$\Delta S L_j = \Delta S L_j + \frac{\Delta \Phi_j}{g},$$

(3.15)

where

$$\Delta S L_j = \Delta G^L_j + \Delta G^{mc}_j - (\Delta R^L_j + \Delta H_j + \Delta I_j + \Delta DT_{A,j})$$

(3.16)

and thus the sea level equation (3.11) can be rewritten as

$$\Delta S_j = \Delta S L_j \cdot C_j + (\frac{\Delta \Phi_j}{g}) C_j - T_0 \cdot \Delta C_j,$$

(3.17)

where $\Delta S L_j$ is given by equation (3.16) and $C_j$ by equation (3.5).

The term $\Delta \Phi_j/g$ can be solved for by invoking conservation of mass of the ocean plus ice reser-
voirs, which requires that

$$\rho_I \int_\Omega \Delta I_j d\Omega + \rho_W \int_\Omega \Delta S_j d\Omega = 0,$$

(3.18)

where $\Omega$ spans the full surface area of the Earth. Integrating equation (3.17) over the Earth’s surface,
using equation (3.18) and solving for $\Delta \Phi_j/g$ yields:
\[
\frac{\Delta \Phi_j}{g} = -\frac{1}{A_j} \rho f \int_{\Omega} \Delta I_j d\Omega + \int_{\Omega} \Delta S L_j \cdot C_j d\Omega - \int_{\Omega} T_0 \cdot \Delta C_j d\Omega.
\] (3.19)

In this expression,

\[
A_j = \int_{\Omega} C_j d\Omega.
\] (3.20)

The sea level equation (3.17), together with the set of equations (3.3), (3.5), (3.16), (3.19) and (3.20) govern gravitationally self-consistent sea level changes in response to mantle convection-induced deformation of the Earth’s solid surface (dynamic topography) and gravitational field, ice mass variations and sediment redistribution.

The time-dependent fields \(\Delta DT_A\), \(\Delta G^{mc}\), \(\Delta I\), and \(\Delta H\) are provided on input, and the full solution yields the fields \(\Delta R_L\), \(\Delta G^L\) and \(\Delta C\), the (time-dependent) scalar \(\Delta \Phi\) and, both the global sea level change \(\Delta SL\) (via equation 3.15) and the ocean height change \(\Delta S\). The global sea level change and the ocean height change are related through the sea level equation (3.17), and the fact that the former is in general non-zero over land has an important physical interpretation, as elucidated by Dahlen (1976). In particular, at some specific site \((\theta, \varphi)\) on land at time \(t\), the value \(\Delta SL(\theta, \varphi, t)\) represents the change in the local depth of water that would be measured if one were to dig a deep, infinitesimally thin canal connecting the site to the ocean.

As we noted above, the sea level equation is an integral equation since the quantity being solved for, ocean height change, is a component of the surface mass loading. In practice, this complication is generally addressed numerically by solving the sea level equation (3.17) iteratively, whereby at each time step an initial guess to the ocean height change is input into the right-hand-side of the equation, which is then solved to yield an updated estimate of \(\Delta S_j\), and the process is repeated until convergence. One then moves to the next time step and the process is repeated. A reasonable first guess to the solution is to assume that the load-induced perturbation in the elevation of the bedrock
and sea-surface are equal to zero. In this case:

$$\Delta S L^i_j = \Delta G_j^{mc} - (\Delta H_j + \Delta I_j + \Delta D T_{A,j})$$  \hspace{1cm} (3.21)$$

and

$$\Delta S^i_j = \Delta S L^i_j : C^i_j + (\frac{\Delta \Phi^i_j}{g}) C^i_j - T_0 \cdot \Delta C^i_j.$$  \hspace{1cm} (3.22)$$

with

$$\frac{\Delta \Phi^i_j}{g} = -\frac{1}{A^i_j} \left[ \frac{\rho_l}{\rho_W} \int \int \Delta I_j d\Omega + \int \int \Delta S L^i_j : C^i_j d\Omega - \int \int T_0 \cdot \Delta C^i_j d\Omega \right]$$  \hspace{1cm} (3.23)$$

and

$$A^i_j = \int \int C^i_j d\Omega.$$  \hspace{1cm} (3.24)$$

In these equations the superscript $i$ denotes the iteration counter.

The fields $I_j$ and $\Delta I_j$ represent grounded ice height. In solving the sea level equation, a check is made at each time step to ensure that the ice thickness in marine sectors is sufficient to ensure that the ice is grounded, and if it is not, the ice model is revised to remove any floating ice (Kendall et al., 2005). If we denote the input model ice history as $I_j^{model}$, then this check is based on the following criterion:
\[ I_j = \begin{cases} 
I_j^{\text{model}} & \text{if } SL_j + I_j^{\text{model}} < 0 \\
I_j^{\text{model}} & \text{if } SL_j + I_j^{\text{model}} > 0 \text{ and } I_j^{\text{model}} \rho_I > (SL_j + I_j^{\text{model}}) \rho_W \\
0 & \text{otherwise.} 
\end{cases} \]  

(3.25)

The sea level equation (3.17) requires that the initial topography \( T_0 \) is specified. In traditional ice age applications, \( T_0 \) is computed through a second iterative procedure. In particular, one begins with a first guess for \( T_0 \) and this is used to compute sea level changes from the beginning of the simulation to the present-day. The difference between the computed present-day topography and the observed topography is then used to update the estimate of the \( T_0 \) field and the process is repeated until convergence, i.e., until the computed present-day topography matches the observed topography to within a specified tolerance. This “outer” iteration, which loops over the full time span of the simulation, should, in general, be adopted in the present application, however, we note an important exception below.

The set of equations we have derived here are valid for arbitrarily complex Earth models. Regardless of the level of complexity, the main requirement is a method for computing perturbations in the elevation of the Earth’s bedrock surface and sea surface, \( \Delta R^L \) and \( \Delta G^L \) respectively, driven by surface mass loading. If one adopts a spherically symmetric Earth model, then the equations can be cast in the spectral domain making use of viscoelastic Love number theory (Peltier, 1974; Wu & Peltier, 1982). The relevant equations are given in Dalca et al. (2013). For models with 3-D mantle structure, \( \Delta R^L \) and \( \Delta G^L \) can be computed using a variety of numerical techniques developed for problems in glacial isostatic adjustment (e.g. Wu & van der Wal, 2003; Zhong et al., 2003; Latychev et al., 2005).

For long timescale applications that are generally of interest when considering changes in dynamic topography, one might additionally assume that the Earth system reaches full isostatic equi-
librium across each time step in which the dynamic topography is updated. That is, one may assume
that these time steps are long enough that all viscous stresses in response to the surface loading have
relaxed and isostatic equilibrium has been reached. Any departure from hydrostatic equilibrium
would then only arise if the lithosphere retained elastic strength across this time scale. In the case of
a 1-D Earth model, the viscoelastic Love number theory can then be simplified to use the long-term,
fluid limit of the Love numbers. The relevant equations for this case are provided in appendix A.

The assumption of isostatic equilibrium further simplifies the algorithm by making the “outer”
iteration (to find $T_0$) unnecessary. In this case, since time-dependent viscous effects are ignored, the
simulation is reversible in time and $T_0$ can be interpreted as the present-day topography, which is
known. The simulation then steps backward in time through the history of ice, sediment, dynamic
topography and gravity fields. The assumption of isostatic equilibrium also allows one to evolve the
system directly from $t = 0$ to $t_j$ without calculating intermediate time steps.

The underlying assumption in these simplifications is that any computed dynamic topography
variation has a time scale sufficiently long that the adoption of fluid Love number theory is valid.
In the presence of internal density discontinuities that contribute buoyancy in response to the con-
vective flow, the assumption may require time scales of several million years (Piromallo et al., 1997).
Isostatic adjustment in response to a surface mass load is also impacted by these internal disconti-
uinuities, and thus reaching complete isostatic equilibrium would require a similarly long time scale.
However, the departure from the equilibrium state would be relatively small for shorter time scales
since the viscous modes associated with the internal discontinuities only contribute at the percent
level to the load-induced surface displacement.

While we make the assumption that the adoption of fluid Love number theory is valid in the
results presented below, we emphasize that a time-forward and iterative procedure is necessary in cal-
culations in which viscous effects cannot be ignored, as is the case for Pleistocene ice age calculations
where changes in ice and ocean loading occur on time scales such that glacial isostatic adjustment
must be modeled as a viscoelastic phenomenon.

3.2.3 Schematic illustration of sea level physics in the presence of dynamic topography

Figure 3.1 provides a schematic illustration of the physics of sea level change near a shoreline in the presence of the combined effects of a local perturbation in dynamic topography and a lowering of the sea-surface height (neglecting for this illustration the flexural effects of the elastic lithosphere). The latter may arise from a variety of processes, including, for example, the growth of ice cover at some location distant from the shoreline. Figs. 3.1a-c decompose the total sea level change into a two step evolution, the first associated with the perturbation in dynamic topography and the second from the lowering of the sea surface height. If the perturbation in dynamic topography and sea surface height occur in tandem (i.e., in one step), then the sea level change is given by the evolution from Fig. 3.1a to (directly) Fig. 3.1d.
In the first time step of the two-step evolution \((t_0 \text{ to } t_1 \text{ or frame a to b})\), a relatively constant perturbation to the air-loaded dynamic topography occurs producing an uplift of \(\Delta DT_{A,1}\) (Fig. 3.1b). In the region landward of the original shoreline, there is no amplification in the dynamic topography since the region begins and ends under air cover. As one moves from left to right between the shoreline position at \(t_0\) and \(t_1\), progressively more water is displaced by the dynamic topography and the unloading leads to a crustal uplift (or amplification from the air-loaded dynamic topography) that increases linearly \((\Delta R_{L}^{1})\) up to the new shoreline because at this point the crustal uplift has raised the bedrock to the sea-surface. To the right of this shoreline, the water unloading associated with the air-loaded dynamic topography is constant since the amount of water that is displaced is the same everywhere right of the shoreline at \(t_1\), and hence the response to this changing load is the same.

In the second time step \((t_1 \text{ to } t_2 \text{ or frame b to c})\), the sea surface height drops to the level \(G_2\), and the unloading produces crustal uplift at all locations to the right of the shoreline position at \(t_1\) (Fig. 3.1c). The uplift of the crust increases linearly as one moves from this shoreline to the location where the newly uplifted crust intersects the new sea surface height \((\Delta R_{L}^{2})\), and this intersection defines the shoreline at the end of the second time step. To the right of the shoreline at \(t_2\) the crustal uplift due to unloading is constant. The final position of the solid surface is given by the line \(R_2\).

The two step processes merges into one step in the evolution from Fig. 3.1a directly to Fig. 3.1d. The drop in the sea surface from \(G_0\) to \(G_1\) and the contemporaneous perturbation in air-loaded dynamic topography, \(DT_{A,1}\), leads to a crustal uplift from \(R_0\) to \(R_1\) and a migration of the shoreline from \(t_0\) in Fig. 3.1a to its position at \(t_1\) in Fig. 3.1d. To the left of the original shoreline the crust remains sub-areal throughout the evolution and there is no ocean unloading effect. To the right of the new shoreline position, the crust remains under ocean cover and the unloading-induced uplift, \(\Delta R_{L}^{1}\), is constant. Between the old and new shoreline this crustal uplift increases monotonically toward the latter.
This simple example illustrates the equivalence between the water unloading and the amplification of air-loaded dynamic topography as discussed in the context of equation (3.1). We emphasize, however, that as mentioned above, these schematics miss an important component of the sea level response, namely elastic flexure. If one includes an elastic crust/lithosphere layer in the physical response, as we do in the calculations described below, then the crustal uplift will extend laterally beyond the region of unloading, and this will impact both the amplitude of the sea level change and the predicted migration of the shoreline.

3.2.4 Including plate motion

We have thus far ignored the signal associated with the horizontal motion of tectonic plates, which would alter the geometry of the surface mass load and move any given geographic site through the field of mantle convection induced dynamic topography and geoid fluctuations. On time scales of $10^3 - 10^5$ years, this effect can generally be neglected. However, on longer timescales, the impact of plate motions needs to be considered.

Let us assume, in the following, that the Earth remains in a state of isostatic equilibrium in response to any change in the surface mass load associated with plate motions. In this case, the impact of changes in ice and sediment loads, dynamic topography and the gravity field on sea level can, for a given site, be most simply expressed in a reference frame fixed to the site (i.e., a Lagrangian frame of reference) rather than in a fixed mantle reference frame (e.g., hot spot reference frame).

As described in equation (3.7), a perturbation in a general time-dependent quantity $\chi$ from the onset of loading, at time $t_0$, to a time $t_j$ is simply the difference between the initial value of the quantity and its value at time $j$. If, across this time interval, a specific point on a given plate travels from $(\theta_0, \varphi_0)$ to $(\theta_j, \varphi_j)$ as a consequence of plate motion, then the response of the Earth at this site only depends on the net difference in the load and dynamic topography that the site experiences across this time interval, and not on the full time history from $t_0$ to $t_j$: 37
\[
\Delta \chi = \chi_j(\theta_j, \varphi_j) - \chi_0(\theta_0, \varphi_0)
\]  

(3.26)

If we consider, for example, the air-loaded dynamic topography, then \(\Delta DT_{A,j}\) expressed in equation (3.26) will change if the dynamic topography due to mantle convection evolves, or if the plate moves through a gradient in dynamic topography. The change in the parameter \(\chi\) can be positioned in a coordinate system fixed to either the initial geography (i.e. \(\Delta \chi = \Delta \chi(\theta_0, \varphi_0)\)) or the final geography (i.e. \(\Delta \chi = \Delta \chi(\theta_j, \varphi_j)\)). In either case, there will be regions for which \(\Delta \chi\) is undefined; areas of plate destruction (e.g. subduction zones) for the case \(\Delta \chi(\theta_0, \varphi_0)\), or areas of plate creation (e.g. mid-ocean ridges) for \(\Delta \chi(\theta_j, \varphi_j)\).

Within the sea level formalism, changes in parameters such as ice, sediments, the gravity field and dynamic topography must be defined globally to guarantee mass conservation. Therefore, any undefined regions have to be interpolated while preserving the conservation of water plus ice mass and solid Earth mass (or volume) between \(t_0\) and \(t_j\). This is a relatively easy procedure for changes in ice volume because ice sheets are located away from plate boundaries. For all other quantities, an appropriate interpolation scheme must be chosen.

### 3.3 Applications

In the following section we apply the gravitationally self-consistent sea level methodology derived above to a series of case studies that assume a spherically symmetric (i.e., depth varying) Earth that maintains isostatic equilibrium at all times (i.e., the case considered in appendix A). Our goal is to highlight the accuracy of the new approach relative to previous, approximate treatments of sea level changes driven by dynamic topography. We use the density and elastic structure of the seismic model PREM (Dziewonski & Anderson, 1981) in all calculations. Furthermore, while our standard results will adopt an elastic lithosphere of thickness 90 km, we will also consider a thickness
of 130 km to investigate the sensitivity of the results to this parameter. Finally, to emphasize the underlying physics of sea level changes, all dynamic topography fields discussed below are simple, hypothetical geometries that are imposed rather than computed from a realistic simulation of mantle convection.

3.3.1 Example 1: Isostatic amplification of dynamic topography

To illustrate the results of the new sea level methodology, we begin with a simple scenario in which an axisymmetric air-loaded dynamic topography perturbation centered near Norfolk, West Virginia, is applied along the U.S. East Coast. The air-loaded perturbation is characterized by a peak uplift of 160 m. Figs. 3.2 a-c show the sea level change that one would predict by simply scaling the input air-loaded dynamic topography using different density contrast models, as discussed in Section 3.2.1. Specifically, we consider three cases in which we apply: (a) no amplification (air cover); (b) an amplification assuming a water cover; and (c) no amplification in zones of present day land cover and water cover amplification in present-day ocean regions. Fig. 3.1d is the sea level change predicted using the full, gravitationally self-consistent treatment described in the last section. In each frame of the figure we superimpose the predicted shoreline prior to the perturbation in dynamic topography. Note that the shoreline after the perturbation is applied corresponds to the modern coastline and the shoreline prior to the perturbation is the ancient coastline. In practice, we calculate this by going backwards in time (as discussed in section 3.2.2), starting at the known present-day topography and calculating the unknown ancient topography by applying the negative change in dynamic topography and in the case of Fig. 3.2d, solving the sea level equation.

To explore the error in the three approximate solutions, Figs. 3.2a-c show differences between the prediction in Fig. 3.2d and each of the fields in Figs. 3.2a-c. The second column on Fig. 3.3 shows the same fields plotted in the first column, except that we zoom in on the U.S. East Coast. In all frames of Fig. 3.3, the white line is reproduced from the ancient shoreline position predicted from
Figure 3.2: Change in sea level near the U.S. East coast in response to a synthetic perturbation in dynamic topography. Frame d. is the sea level change based on the gravitationally self-consistent sea level methodology described in the text, while frames a-c are based on the following assumptions: a. air load everywhere; b. water load everywhere; and c. air load over continents and water load over oceans. The white lines are the predicted shorelines prior to the change in dynamic topography.
the gravitationally self-consistent calculation (Fig. 3.2d). The reason for this choice is that the frames of Fig. 3.3 represent errors in the prediction incurred by using an approximate treatment of sea level change, and our intent is to demonstrate how these errors will map onto the elevation of the “true” location of the ancient shoreline.

As illustrated in the schematic of Fig. 3.1b, the gravitationally self-consistent simulation accurately accounts for water mass unloading driven by dynamic topography. This unloading tends to increase as one moves from the ancient to modern shoreline and it remains relatively constant over the present-day ocean. The unloading associated with the simulation in Fig. 3.2d is more complex than in Fig. 3.1b because the initial bedrock topography ($R_0$) is more variable, and because the former incorporates elastic flexure.

In regard to the approximate calculations of sea level change, Fig. 3.2a does not incorporate any water unloading and therefore the air-loaded dynamic topography is not amplified in this case. The error in the calculation of the sea level change (Figs. 3.3a,d) is largely limited to regions oceanward of the ancient shoreline (i.e., $\Delta R_L$ in Fig. 3.1b is not captured). This is not strictly true because elastic flexure extends the effect of the water unloading landward of the ancient shoreline in the accurate sea level calculation (Fig. 3.2d). In contrast to this case, Fig. 3.2b assumes that water unloading occurs everywhere. This leads to a significant error landward of the ancient shoreline and an error that decreases as one moves from the ancient to modern shoreline (Figs. 3.3b, e). The error in Figs. 3.3b and 3.3d remains non-zero over a localized region oceanward of the modern shoreline because flexural effects associated with water unloading are not accounted for. Finally, the calculation that assumes a water-amplified dynamic topography oceanward of the modern shoreline, and no amplification landward of this shoreline (Fig. 3.2c), neglects the effects of ocean unloading, and associated flexure, within the region of shoreline migration. As a consequence, the error is largest in the zone between the two shorelines, and it tapers to zero on either side of this region (Figs. 3.3c, f). The geometries of the errors associated with the approximate treatments is dependent on whether the
Figure 3.3: (a-c) Difference between the predicted sea level change calculated using a fixed amplification of an imposed, air-loaded dynamic topography (Figs. 3.2 a-c) and the sea level change calculated using our gravitationally self-consistent sea level methodology (Fig. 3.2d). (d-f) Close-up of the frames (a-c), respectively.
region is subject to a regression or transgression, but these plots nevertheless provide a sense of the scale and amplitude of these errors.

Fig. 3.4a shows the sea level change predicted for sites along the ancient shoreline in the gravitationally self-consistent calculation (blue; see also Fig. 3.2d). For the purpose of comparison, we also show predictions along the same profile for the three approximate treatments of sea level change in Figs. 3.2a-c. (The results for the case where no isostatic amplification is applied, Fig. 3.2a, and the hybrid case treated in Fig. 3.2c, are indistinguishable on Fig. 3.4 and therefore simply plot on top of each other.) All calculations predict a significant drop in sea level across the profile, as expected given the dynamic uplift of the region in this simulation (see Fig. 3.2). However, the air-loaded and hybrid calculations predict a sea level fall ~15% less than the gravitationally self-consistent calculation, while the water load approximation yields an amplitude ~25% greater than the calculation based on our new methodology.

The sign of the error associated with the air load and hybrid calculations can be understood with reference to Figure 3.1b, which illustrates the cross-sectional profile expected from a gravitationally self-consistent solution of sea level change. In contrast to this case, the approximate solutions neglect the water unloading in the region between the ancient and modern shoreline (and the flexural effects of this unloading) because they assume air loading for all sites landward of the modern shoreline (corresponding to the shoreline at $t_1$). Thus, within the region of shoreline migration the approximate solution does not accurately capture the uplift of the crust and thus it predicts a sea level fall of smaller amplitude than the gravitationally self-consistent solution in the vicinity of this region. In contrast, the water load approximation overestimates the change in load (and hence crustal uplift) in all regions landward of the modern shoreline and therefore predicts a sea level fall of greater amplitude than the self-consistent calculation.

Fig. 3.4b shows the computed sea level change perpendicular to the shoreline. The water load calculation overestimates the sea level change over continents while the air load calculation underesti-
Figure 3.4: Change in sea level calculated using an air load approximation (Fig. 3.2a), a water load approximation (Fig. 3.2b), a hybrid approximation (Fig. 3.2c) and the full gravitationally self-consistent methodology (Fig. 3.2d) with a lithospheric thickness of 90km (solid blue) and 130km (dashed blue). 

- **a.** Prediction along the location of the ancient shoreline (see inset) as predicted by the full methodology. The air load approximation and hybrid approximation are identical over the modern continents and therefore these two approximations yield the same values.
- **b.** Prediction perpendicular to the shoreline (see inset). The hybrid approximation changes from the air load approximation to the water load approximation at the position of the modern shoreline. A thicker lithosphere leads to more pronounced flexure that extends further into the continent and oceans.
mates the change over oceans. The self-consistent calculation is characterized by a smooth transition between the two regions across the zone of shoreline migration and slightly beyond. The modern shoreline is marked by the transition of the hybrid approximation from the air to water load prediction. Assuming a thicker lithosphere (dashed blue line) results in broader flexural effects and a wider zone of transition.

3.3.2 Example 2: Can independent signals be summed?

The gravitationally self-consistent sea level methodology described above can be used to consider the impact of dynamic topography in isolation (as in the example of the last section), however, it also admits the possibility of contemporaneous dynamic topography fluctuations, ice mass changes and sediment fluxes. To date, studies have computed these effects independently and summed them up (e.g. Müller et al., 2008b; Rowley et al., 2013). This assumes the processes are independent of one another, an assumption that does not necessarily hold. To consider the potential error introduced in the latter approach, we return to, and extend, the numerical example treated in the last section. In particular, we apply the same change in air-loaded dynamic topography as shown in Fig. 3.2a but in addition we assume a contemporaneous growth of an ice sheet in the far field of the U.S. East coast that leads to a fall in sea level over the oceans of approximately 30m. These tandem effects are analogous to the example illustrated schematically in Fig. 3.1.

To begin, we perform a single, gravitationally self-consistent calculation that considers both processes, a change in dynamic topography and an increase in ice volume, together. The change in sea level computed for this case using our new sea level methodology is given in Fig. 3.5a. Next, we perform two independent calculations. The first involves the perturbation in dynamic topography and no far-field change in ice volume, and the second includes the change in ice volume in the absence of a signal from dynamic topography. Both calculations are also based on our gravitationally self-consistent sea level methodology. We then sum the sea level changes obtained from these two
Figure 3.5: a. Change in sea level near the U.S. East coast in response to a perturbation in both dynamic topography and ice volume in the far field of the region, as computed using the gravitationally self-consistent sea level methodology described in the text. The white line shows the predicted location of the ancient shoreline. b. Difference between the calculation shown in frame (a) and an approximate approach in which the sea level change due to both perturbations is computed separately using the same sea level method and the total response is calculating by summing the two effects.

calculations. Fig. 3.5b shows the difference between the latter ‘superposition’ procedure and the full calculation shown in Fig. 3.5a.

The error incurred in the approximate solution is greatest in areas of shoreline migration, but it extends beyond this region as a consequence of flexural effects. We can understand this trend by once again considering Fig. 3.1. The full solution accurately accounts for the total migration of the shoreline as a consequence of the tandem effects of a dynamic topography change and far-field draw-down of the sea surface (i.e., Fig. 3.1a to d). However, the approximate solution yields a different initial shoreline for each process. In the example of Fig. 3.5, both processes act to move the shoreline oceanward, and thus the approximate solution errs by not accounting for the full lateral extent of the shoreline migration (i.e., the migration of the shoreline at \( t_0 \) in Fig. 3.1a to the position at \( t_1 \) in
Fig. 3.1d) and thus underestimates the water loading. The sea level change computed using the approximate treatment will predict a sea level fall of lower magnitude than the full solution, and this leads to the sign of the error in Fig. 3.5b.

The amplitude of the error in the sea level prediction along the predicted location of the ancient shoreline is about 10% of the sea level fall over the oceans due to far field ice growth (30 m). This is the level of error one would incur if the geological record were corrected for dynamic topography (using a gravitationally self-consistent calculation) and the residual used to infer the ice volume change. The error would of course be much larger if one had used the approximate treatments of sea level change due to dynamic topography (as illustrated in Fig. 3.4). Finally, differences in the location of the ancient shoreline, as predicted using the gravitationally self-consistent treatment of sea level change and the approximation in which the effects are computed independently, differ negligibly (compare the white lines in both frames of Fig. 3.5). We note, however, that this agreement largely reflects the relatively high topographic slope in the region of the paleoshoreline. A shallow slope would, of course, result in a greater discrepancy in the predicted shoreline migration, and this would in turn influence the amplitude of the error incurred in the approximate solution.

3.4 Summary

In this study we have outlined a gravitationally self-consistent formalism for including dynamic topography and perturbations in the gravity field due to mantle convection in calculations of sea level change. Our formalism is based on ice age sea level theory (Mitrovica & Milne, 2003; Kendall et al., 2005; Dalca et al., 2013), and indeed includes the possibility of ice mass changes and sediment redistribution. It further accurately accounts for shoreline migration, conservation of the mass of ice plus water and, of course, the mass of the solid Earth. (The latter is enforced by ensuring that the globally averaged radial deformation of the crust in response to the total surface mass load is zero.)

Our formulation is valid for arbitrary Earth models but we also consider the special case of a 1-D
Earth model that maintains isostatic equilibrium throughout the simulation. In this case, the relevant response equations are based on fluid Love number theory. Numerical examples based on this special case have allowed us to explore the level of error incurred in previous predictions of sea level change that: (1) assume a simplified amplification of air-loaded dynamic topography to account for the influence of water cover; and (2) consider the impact of ice mass changes and dynamic topography on sea level in isolation. These demonstrate that the error associated with the simplified treatment of the amplification of dynamic topography can vary from approx. 15% to approx. 25% of the predicted sea level change. Superimposing ice mass and dynamic topography effects introduces an additional error of 10% in the ice volume estimate. Rowley et al. (2013) have argued for a dynamic topography change in water-loaded dynamic topography of up to 50 m along the U.S. East Coast since the mid-Pliocene climate optimum. In this case, the two sources of error we have identified might lead to an error of ~10 m in any effort to use geological data from this region to infer sea level equivalent changes in ice mass flux over the last 3 Myr. This level of error is significant and comparable to the total eustatic sea level change associated with the collapse of the Greenland and West Antarctic Ice Sheets.

As interest in ice age sea level change and ice sheet stability broadens from the usual focus on Holocene and modern sea level to encompass records of early Pleistocene and Pliocene age, traditional glacial isostatic adjustment analyses must account for the sea level signal associated with mantle dynamics. The methodology presented here is an attempt to incorporate the latter signal, and in particular dynamic topography and associated geoid fluctuations, into the gravitationally self-consistent theory of ice age sea level change that dates to the pioneering work of Farrell & Clark (1976).
4

The role of mantle convection in past interglacial sea level highstands

4.1 Introduction

Estimating the magnitude of future sea level rise is an important, ongoing challenge for climate scientists. The instrumental record has been vital in improving our understanding of the climate system; however, due to limited sampling (in time and space) and a suite of uncertainties, the predictive capabilities of modeling based on this data are limited (Rahmstorf, 2007; Mengel et al., 2016). To gain further insight into ice sheet stability in the face of global warming, scientists have turned to studies of the geologic past, and in particular periods during which temperatures and carbon dioxide levels were elevated relative to the present-day, as analogues for future climate (Kemp et al., 2015).

This chapter is in preparation for submission.
However, such records are contaminated by a series of geodynamic processes, and identifying and properly correcting for their signals is an ongoing effort (Dutton et al., 2015). In this study we revisit this issue by analyzing how topographic variations driven by mantle convection, a process that is generally neglected on ice age timescales, may bias estimates of past interglacial ice volume.

Reconstructions of minimum ice volumes during past warm periods are commonly based on sea level indicators such as fossilized corals, indicative (trace) fossils or characteristic stratigraphic features. Other methods for inferring past sea levels exist, e.g., analyses of oxygen isotope variations, but uncertainties associated with them are generally too large for this purpose. (Isotope analyses have, however, been used to estimate sea level rates rather than absolute magnitudes; Rohling et al., 2008). Sea level indicators provide a marker of the local elevation of sea level during past interglacials. This elevation, or local sea level, has to be corrected for post depositional deformation in order to extract robust estimates of global mean sea level change. A well-studied example of such deformation is GIA (Farrell & Clark, 1976; Mitrovica & Milne, 2002), which represents the adjustment of the solid surface and sea surface elevation due to changes in the ice and ocean load over glacial cycles. This contamination is generally corrected for using numerical models of the process (Kopp et al., 2009; Dutton & Lambeck, 2012; O’Leary et al., 2013; Raymo & Mitrovica, 2012), though the precision of such corrections depends on uncertainties in ice history and mantle viscoelastic structure.

Another geodynamic mechanism that deflects the surface is mantle convection. The oceanic lithosphere is the top boundary layer of the convecting mantle and it subsides as it cools and moves across the Earth’s surface. This thermal subsidence affects oceanic islands and continental shelves and while it has been corrected for in studies of long term (10s of Myr) sea level change (e.g. Miller et al., 2005) it is rarely considered in analyses of interglacial highstands. Furthermore, convection within the Earth’s mantle has been known to drive vertical deflections of the lithosphere over similarly long timescales with amplitudes of up to several kilometers (McKenzie, 1977; Hager et al., 1985;
Mitrovica et al., 1989; Gurnis, 1990; Forte et al., 1993, 2015; Steinberger, 2016) and therefore even small, percent level changes in this mechanism can lead to meter scale changes in surface topography. While the impact of this process on relative sea level during the mid-Pliocene warm period has been investigated (Rowley et al., 2013; Rovere et al., 2014), it has never been considered in studies of global mean sea level during past interglacials. While the second of the above processes is universally termed dynamic topography, the first is also driven by mantle flow and some groups include it under the same term (Forte et al., 1993).

Of particular interest in the paleoclimate record are the last interglacial (LIG, or MIS 5e) and marine isotope stage 11 (MIS 11) as they are most recent and most similar (in terms of orbitally configuration) to the present interglacial. Sea level highstands during the LIG occurred from 129 to 116 ka (Govin et al., 2015). Atmospheric CO$_2$ concentrations were comparable to pre-industrial values (Rundgren & Bennike, 2002) and temperatures were consistent with simulations of 1-2$^\circ$ global warming (Clark & Huybers, 2009). Current estimates of excess ice melt range from 6-9 m, a range cited in both a probabilistic analysis of records from widely distributed locations (Kopp et al., 2009), and an independent GIA study of two sites (Western Australia, Seychelles) argued to be particularly reliable (i.e., tectonically stable) (Dutton & Lambeck, 2012).

Fig. 4.1a shows an updated database of the location of LIG sea level indicators from Rovere et al. (2016b). In this figure and the following analysis we exclude sites that are within 4$^\circ$ of a plate boundary since such sites may be contaminated by tectonic processes that are not considered in the models employed in this study. While estimates of the peak global mean sea level during MIS 5e are converging, there is a significant spread in local sea level highstands dated to this period (Fig. 4.1b). This variability reflects, at least in part, the signal from GIA. As we discuss in detail in section 4.4, we ran a large series of GIA simulations to explore this issue. Fig. 4.1c corrects the histogram in frame b by applying a correction using the GIA model that best fits the variability in the observed highstands. It is clear from the figure that other processes dominate this variability.
Figure 4.1: Last Interglacial sea level sites. a. Location of sea level indicators dating to the LIG, after Rovere et al. (2016b). This compilation represents a revision to published databases (Kopp et al., 2009; Pedoja et al., 2014; Ferranti et al., 2006). We do not show sites that are located within 4° of a plate boundary (white lines). For sites where more than one highstand value was reported we chose the highest. b. Elevations of sea level indicators relative to the mean tide level. We incorporate the reported uncertainty in the measured elevation at each site by assuming that it follows a normal distribution from which we randomly drew 1000 samples. c. As in panel b, but after a correction for a “best-fit” model of GIA (see text).

MIS 11 spanned two precession cycles and was the longest interglacial of the past 500 kyr (Loutre & Berger, 2003; Rohling et al., 2010). Sea level indicators for this time period are located as high as 20m above present-day sea level in Bermuda and the Bahamas and have been interpreted as highstands that reflect global mean sea level (Hearty et al., 1999; Olson & Hearty, 2009) or mega-tsunami deposits (van Hengstum et al., 2009). Raymo & Mitrovica (2012) pointed out that these two sites are both on the peripheral bulge of the Laurentide Ice Sheet and would thus be significantly affected by GIA. After correcting for this ~10 m signal, they estimated peak global mean sea level for MIS 11 to be ~6-13 m. A similar range has been inferred on the basis of GIA-corrected sea level markers from South Africa (Roberts et al., 2012; Chen et al., 2014).

In this study we calculate the rate of sea level change associated with dynamic topography and estimate the magnitude that this process can have on existing sea level based estimates of ice volumes during MIS 5e and MIS 11. In the discussion above we reviewed two processes associated with mantle flow; subsidence of the oceanic lithosphere as it cools with age, and vertical deflections of the
crust driven by viscous stresses coupled to convective flow. We will consider these two signals separately and use the term “dynamic topography” for the latter, though both are supported by mantle flow. Our distinction underlines an important issue, namely, that the contamination of a sea level marker by dynamic topography can have either sign, the same is not true of subsidence driven by cooling of the oceanic lithosphere.

4.2 Thermal subsidence of the ocean floor

As oceanic lithosphere moves away from the mid-ocean ridge it cools, thickens, and subsides. This behavior can be approximated with a half-space cooling model, which produces the canonical square root age laws that state that lithospheric thickness and depth below the ridge axis scale with the square root of the age of the oceanic lithosphere. Parsons & Sclater (1977) explored this relationship and noted that it was accurate for ocean floor younger than 70 Ma; the elevation of older oceanic crust tends to be shallower than the model prediction. Stein & Stein (1992) refined the parameters of the equations that describe ocean subsidence with age of the lithosphere utilizing updated bathymetric data (see Fig. 4.2a). While newer estimates of this relationship exist that highlight more detailed variability in bathymetry (Crosby, 2007, Fig. 4.2a), the predicted basement depths are close to identical (Müller et al., 2008a).

The main difference between the two plate models is an added sinusoidal perturbation between the ages of 70 and 120 Ma in Crosby (2007) based on observations in the North Pacific (Crosby et al., 2006). Since this result is mainly driven by observations in areas with few LIG sites, we will use the depth-age relationship by Stein & Stein (1992) for the analysis below. Fig. 4.2b shows the amount of subsidence that occurs for oceanic lithosphere of a given age over 125 kyr. Subsidence is highest for young lithosphere since the slope in the depth-age relationship is steepest. The elevation change for old oceanic lithosphere approaches zero. In this model, all oceanic lithosphere experiences subsidence as it cools. This differs from the model by Crosby (2007), which can lead to uplift.
for certain ages.

Fig. 4.2c shows the location of LIG sea level sites and the age of the closest oceanic lithosphere, as determined from the map of Müller et al. (2008a). We grouped sites into three different categories: (i) Sites that are located on oceanic lithosphere (defined as any site within the age grid of Müller et al., 2008a); (ii) sites that are on continental lithosphere but proximal (within 5°) to oceanic lithosphere and therefore might be affected by subsidence; and (iii) continental sites that are far (more than 5° from oceanic lithosphere) and are likely unaffected by subsidence. For sites that are not located on oceanic lithosphere (category (ii) and (iii)), we assign the age of the closest oceanic lithosphere. Finally, Fig. 4.2d shows the elevation correction due to crustal subsidence for all LIG sites grouped by their proximity to oceanic lithosphere. We note that most sites are close to, but not situated directly on, oceanic lithosphere.

At sites in the database, the impact of ocean cooling and subsidence is largest (several meters) in the Pacific, a region where the process has been recognized as an important driver in post depositional deformation (Woodroffe et al., 1991; Camoin et al., 2001; Dickinson, 2004). Nevertheless, few studies of interglacial sea levels have considered this correction (Hearty, 2002; Kopp et al., 2009; Dutton & Lambeck, 2012). In addition to the Pacific, there are sites with LIG records in the Atlantic and Indian Oceans that will be significantly impacted by thermal subsidence. For example, consider the Seychelles, a granitic island located on oceanic crust of age ~60 Myr, and one of the two sites considered by Dutton & Lambeck (2012). If the island followed the thermal subsidence pattern of the nearby ocean floor, it has subsided ~1.5 m since the LIG. This correction would increase the estimate of peak global mean sea level based on the GIA-corrected record from the site from 9 m to 10.5 m. A second important LIG record is located on the Yucatan peninsula (Blanchon et al., 2009). Due to its proximity to the Cayman Trough, we have assigned it an age of 40 Ma which would imply and upward correction of 3 m, raising the previously inferred 6 m highstand estimate to 9 m, and thus in better agreement with estimates from the Seychelles (Dutton & Lambeck, 2012) and
Figure 4.2: Analysis of oceanic subsidence since the LIG. a. Depth-age relationship from the plate model of Stein & Stein (1992) (black line) and Crosby (2007) (grey line). The color of the symbols reflects the age of the crust associated with the LIG sea level indicators, as computed from the Stein & Stein (1992) relationship. The shape of the symbol denotes the location of the site relative to the oceanic lithosphere (defined by the oceanic age grid derived Müller et al. (2008a), panel c): squares, on the oceanic lithosphere; circles, within 5° of oceanic lithosphere; and diamonds, more than 5° from oceanic lithosphere. When sites are not positioned on oceanic lithosphere, the age of the closest oceanic lithosphere is assumed. b. Amount of ocean thermal subsidence since the LIG (125 kyr) as a function of the sea floor age. The solid line is computed using the plate model by Stein & Stein (1992), the grey line is based on Crosby (2007). Symbols are defined as in panel b and their elevation change is calculated using the model by Stein & Stein (1992). c. Age of the oceanic lithosphere based on Müller et al. (2008a). Symbols mark the location of sites with LIG records, and the color is the age of the oceanic lithosphere closest to the site. d. Distribution of the elevation correction (as shown in panel b) associated with oceanic subsidence. Note that all corrections are negative since all sites are subsiding. Different colors denote the different proximity to oceanic lithosphere.
Western Australia (O’Leary et al., 2013). However, the site is also close to old oceanic lithosphere in the Gulf of Mexico, and it is therefore questionable whether our age and subsidence rate assignment is robust. Additional continental sites where the effect of thermal subsidence should be considered include southeastern Australia and Europe; however, local tectonic activity may be significant in these locations (Murray-Wallace & Belperio, 1991; Ferranti et al., 2006).

Our model for removing the impact of subsidence yields an upward correction for all sites and suggests a systematic underestimation of inferred global mean sea level highstands. This correction, if it is robust, will increase significantly when considering sea level markers of greater age. For example, Bahamas, Bermuda and South Africa are sites with some of the most reliable records of sea level highstands during MIS 11. Of these, only Bermuda will be subject to thermal subsidence. The age of the oceanic lithosphere in Bermuda is ~120 Myr, suggesting an upward correction of approximately 1 m for sea-level records of MIS 11 age.

4.3 Mantle driven changes in topography

In addition to the boundary layer, perturbations within the mantle play an important role in shaping the Earth’s topography. Convection within the mantle is driven by buoyancy variations and leads to vertical deflections of the crust, or dynamic topography (Mitrovica et al., 1989; Gurnis, 1990), that can reach amplitudes of 3-4 km (Flament et al., 2013). Numerical models have predicted uplift rates that range from 100 m/Myr for the South African superswell (Gurnis et al., 2000), to tens of meters per million years along the New Jersey margin, though the sign of the latter is debated (Moucha et al., 2008; Flament et al., 2013; Rowley et al., 2013). Uplift rates inferred from Pliocene shoreline markers suggest rates of 0-20 m/Myr (Rovere et al., 2014), or higher (Rovere et al., 2016a), whereas estimates from the geological record in Australia indicate rates up to 75 m/Myr (Czarnota et al., 2013). These observationally inferred signals may include topographic deflections due to active tectonics, volcanism and sediment transport, nevertheless the rates are sufficient to potentially bias
inferences of peak global mean sea level rise during past interglacials.

To calculate dynamic topography, we solve the continuity, momentum and transport equation for mantle convection assuming a simple equation of state that relates changes in density to changes in temperature through thermal expansion (e.g. Schubert et al., 2001). The convection code ASPECT (aspect.dealii.org; Kronbichler et al., 2012) solves the relevant equations for compressible flow within the Earth’s mantle. As input we adopt the density model TX2008, derived to match (in conjunction with a viscosity model) present-day geophysical observables such as plate velocities, the long wave-length present-day dynamic topography, the long wave-length geoid and the excess ellipticity of the core mantle boundary (Simmons et al., 2009). We also consider three different viscosity models (see Fig. 4.3) which have been inferred from simultaneous inversions of these convection-related data sets plus a suite of observations associated with GIA (V1 and V2; Mitrovica & Forte, 2004; Forte et al., 2010) or GIA data alone (Lau et al., 2016).

Our simulations do not include phase changes, thermal boundary layers, internal radiogenic heat production or the deflection of internal boundaries. The 1-D temperature and density profiles follow an adiabatic profile. Thermal conductivity, thermal expansivity and heat capacity vary with depth alone and are adopted from Glišović & Forte (2015). ASPECT does not calculate gravity self consistently and we therefore impose the radially varying gravity profile from Glišović & Forte (2015). We apply a free slip boundary condition at the core-mantle boundary. For the top boundary condition we vary between free slip, no slip and prescribed plate velocities. The depth resolution of our numerical grid is ~60 km in the upper 1000 km and varies from 130 km to 230 km below this depth. The lateral resolution is on the order of 140 km in the upper 1000 km and 350 km to 500 km below that depth. To calculate dynamic topography from the radial stress field output by the mantle convection model, we use the theory described in Chapter 2. The convection simulation is run forward in time to a steady state rate, which it reaches after ~500 kyr, and we then continue the simulation for 1 Myr to obtain the rate of change of dynamic topography. The combination of
the three viscosity models and three different boundary conditions applied to the top surface results in nine different predictions considered in our analysis.

Absolute dynamic topography and its rate of change are both dependent on the buoyancy and viscosity fields in the mantle. However, while dynamic topography is only a function of relative variations in viscosity, its rate of change depends on the absolute value of viscosity (Morgan, 1965). Therefore different convection simulations may yield very similar present-day dynamic topography (as the nine models we consider do), but predict significantly different rates of change.

Fig. 4.4a shows the average change in dynamic topography over 125 kyr predicted from the nine different simulations. We do not plot results in regions where the standard deviation between these nine simulations is larger than 2 m. It is clear that in most regions the models are predicting highly variable results. We note that the magnitude of the predictions that are plotted are consistent with observationally inferred long-term rates of dynamic topography change (~10s of m/Myr), leading to changes in elevation since the LIG of several meters.

Notable in these results is the predicted uplift along the U.S. East coast, which is consistent with the elevated Pliocene shoreline along this Atlantic coastal plain (Rovere et al., 2014). The Seychelles
Figure 4.4: Predicted changes in dynamic topography since the LIG. a. Mean change in dynamic topography since 125 ka computed by averaging the results of nine different convection simulations (see text). Areas in which the standard deviation between these model predictions exceeds 2m are not plotted. The location of LIG sites is marked, and the color corresponds to the magnitude of the mean prediction. b. Distribution of the predicted dynamic topography change since 125 ka derived from the nine different simulations. The color denotes whether a site was below or above the standard deviation threshold of 2m.

are predicted to have been subsiding (mean value of 4 m, with a standard deviation of 1.5 m). If this prediction is robust, then previous estimates of peak global mean sea level during MIS 5e based on GIA-corrected shoreline markers from this site (Dutton & Lambeck, 2012) would be biased low by ~4 m. In western Australia the models also predict an upward correction, but of smaller amplitude (1-2 m). As we have discussed, estimates of peak global mean sea level during the LIG based on GIA-corrected sea level markers from these sites range from 5.5 m (Dutton & Lambeck, 2012) to 9 m (O’Leary et al., 2013).

Since the predicted rate of change of dynamic topography is relatively constant across the million-year simulations, the above results, with suitable extrapolation, are also applicable to highstands during MIS 11. For example, the results in Fig. 4.4 suggest a net subsidence of ~5 m on the southern coast of Africa since MIS 11, and this would thus raise by the same amount previous estimates of global mean sea-level based on GIA-corrected data from the region (Chen et al., 2014). Further-
more, while Bermuda and the Bahamas are predicted to be uplifting, on average, there is significant variability in predictions from the nine model simulations.

4.4 Discussion and conclusion

Fig. 4.5 summarizes the impact of the different corrections we have discussed. Panel a shows a histogram of the observed elevations. The mean of these observations is 6.9 m with standard deviation of 29.6 m. While we excluded sites close to plate boundaries, we did not filter the data using any other criterion, and thus sites such as Hawaii, which involves both the highest (156 m) and lowest (−358 m) value in our database, contribute significantly to the considerable spread in the distribution.

GIA has been recognized as a significant contaminator of interglacial sea level highstands, and is generally (e.g. Kopp et al., 2009; Dutton & Lambeck, 2012), but not always (e.g. Hearty, 2002; Bowen, 2010), accounted for in studies aimed at estimating ice volumes during these periods. The GIA correction is sensitive to both the ice history and viscoelastic Earth structure and is characterized by considerable uncertainty. To explore this issue, we ran a suite of 70 GIA simulations in which we assumed a 1-D viscosity profile characterized by an elastic (essentially infinite viscosity) lithosphere of prescribed thickness, and constant upper and lower mantle viscosities. These three parameters were varied over the following ranges: 71-96 km, 2.5 × 10^{20} Pa s, and 3-50 × 10^{21} Pa s.

To construct a two-cycle ice history, we adopted the ICE-6G model (Argus et al., 2014; Peltier et al., 2015) for both the last and penultimate deglacial period, and the glaciation phase in each cycle was modeled to follow a eustatic curve inferred from Waelbroeck et al. (2002). Ice volumes during the LIG were assumed to be identical to the present day, so that the computed GIA corrections to LIG highstands at each site would not involve a change in the eustatic sea level. The calculations were based on the ice age sea level theory of Kendall et al. (2005).

Fig. 4.5 b and c show the distribution of the observed highstand data after their correction for

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GIA. The variance of the distribution is significantly reduced, which supports the notion that GIA is an important driver in post-depositional deformation since the LIG. The results in panel b have been corrected for the GIA simulation that produced the greatest variance reduction (the same model used in Fig. 4.1c), while the results in panel e were corrected by randomly selecting amongst the 70 different GIA models. For sites in the far field of the ice sheet, the predicted highstand occurs at the beginning of the interglacial, while it occurs at the end of the interglacial for near field (peripheral bulge) sites. This correction assumes that there was no melt pulse at the end of the interglacial. Correcting for GIA in this way therefore reduces, on average, the elevation of LIG sea level markers by ~2.5 m.

In the next step, we consider the bias introduced by not accounting for thermal subsidence of the oceanic lithosphere. Fig. 4.5c and f show the distribution of elevations corrected for GIA and oceanic subsidence. We varied the latter correction by using: (i) sites that are located on oceanic lithosphere; (ii) sites that are on, or within 5°, of oceanic lithosphere; or (iii) all sites. In panel c we use only the method that led to the greatest variance reduction. In panel f we show the distribution when we repeatedly and randomly sampled one of the three methods. As noted earlier, correcting for ocean subsidence raises the mean elevation of the distributions - in the results of Fig. 4.5c and f, this shift is 0.52 m and 0.62 m, respectively - and does not further reduce the variance of the data. Finally, we apply a correction for dynamic topography (Fig. 4.5d and g). In both panels, the mean highstand elevation is reduced by ~1 m when the correction is applied, reflecting the fact that sites with LIG records tend to be located in uplifting areas (see Fig. 4.4).

Changes in elevation associated with dynamic topography - including ocean subsidence - have been recognized to be important for longterm (Myr) sea level reconstructions. In this study we demonstrated that they should also be considered on shorter (ice age) timescales when trying to reconcile sea level highstand estimates on the meter level.
Figure 4.5: Bias in interglacial sea level estimates. a. Distribution of the elevation data (relative to mean tide level, Rovere et al., 2016b), after excluding sites that are within $4^\circ$ of a plate boundary. We account for the recorded uncertainty in the measurement elevation at each site by assuming that it follows a normal distribution from which we randomly drew 1000 samples. This sampling procedure is repeated for all histograms shown in the figure. The mean ($\bar{x}$) and standard deviation ($\sigma$) for each distribution are listed in each panel. b, e. The data after correction for GIA following the method described in the caption of Fig. 4.1. Panel b shows the distribution after correcting for the best fitting GIA model, which has a lithospheric thickness of 71 km, an upper mantle viscosity of $4 \times 10^{20}$ Pa s, and a lower mantle viscosity of $3 \times 10^{21}$ Pa s. Panel e shows the distribution when each of the sampled highstand records were randomly corrected using one of 70 different GIA simulations (see text). c, f. GIA plus an additional correction for thermal subsidence of the ocean floor is applied to the sampled highstand elevations. The subsidence correction was varied by including: (i) sites situated on oceanic lithosphere; (ii) sites either on or within $5^\circ$ of oceanic lithosphere; (iii) all sites. In panel c we choose method (ii), which produces the greatest reduction in variance. In panel f we randomly sampled from one of these correction methods. d, g. As in (c,f), except additional correction is applied for dynamic topography driven by mantle flow. For this purpose, we employ the nine different convection simulations described in the main text. In panel d we choose the convection simulation that yields the greatest variance reduction; viscosity profile V2 with prescribed plate velocities at the upper boundary. In panel g we randomly sampled from all nine models.
The impact of dynamic topography change on Antarctic ice sheet stability during the Mid-Pliocene Warm Period

5.1 Introduction

The Mid-Pliocene Warm Period (MPWP), from 3.264 Ma to 3.025 Ma (Dowsett et al., 2010), was characterized by atmospheric CO₂ levels similar to today (400 ppm; Pagani et al. (2010), and by global mean temperatures that were on average elevated by ~2-3°C relative to today (Dowsett et al., 2009). Accordingly, the period is considered an important case study for investigating po-

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lar ice sheet response in a modestly warmer world. There is general agreement that the Greenland Ice Sheet and the West Antarctic Ice Sheet experienced widespread deglaciation during the MPWP (Masson-Delmotte et al., 2013; Naish & Wilson, 2009; Dutton et al., 2015); however, the extent to which the East Antarctic Ice Sheet (EAIS) retreated during the same period remains contentious, with arguments ranging from a cold based and stable EAIS (e.g. Sugden et al., 1993) to a warmer and highly variable EAIS (e.g. Webb et al., 1984). Sediments recovered from cores in the Ross Sea (Naish & Wilson, 2009) and across the Antarctic margin (Cook et al., 2013; Patterson et al., 2014) point to a variable, obliquity-paced EAIS in the vicinity of the Transantarctic Mountains (TAM) and the Wilkes Basin (Fig. 5.1).

Simulations of Antarctic Ice Sheet evolution under MPWP conditions are generally characterized by a much smaller West Antarctic Ice Sheet, with little or no marine-based ice, but a stable EAIS with minor melting limited to the outer margins of some marine-based sectors (de Boer et al., 2015). Calculations by Pollard & DeConto (2009) suggest a net ice mass difference relative to present day equivalent to ~7 m of global mean sea-level rise. Combining this scenario with modeling of the Greenland Ice Sheet during MPWP yields a total global mean sea-level rise of ~14 m (Masson-
Delmotte et al., 2013). Pollard et al. (2015) have recently augmented their modeling to consider two additional ice sheet instability mechanisms, namely hydrofracturing due to surface melt drainage into crevasses and gravitational failure of ice cliffs. They point out that these processes are uncertain but, acting in tandem, could have significantly enhanced EAIS retreat during MPWP, yielding a total Antarctic Ice Sheet contribution of as much as ~17 m.

Ultimately, the stability of a grounded, marine-based ice sheet is largely governed by a balance between mass added to the ice sheet through precipitation and mass lost by outflow at the grounding line. The amount of outflow strongly depends on the ice thickness at the grounding line (Schoof, 2007) and therefore any changes in relative sea level, or equivalently bedrock elevation, at the grounding line will perturb the mass balance. Over the multi-million year time span from the mid-Pliocene to present, evolving viscous stresses and buoyancy associated with mantle convection (e.g. Mitrovica et al., 1989; Gurnis, 1990) could have driven large changes in bedrock elevation. These long-term mantle-driven changes in surface topography, termed dynamic topography, could either stabilize or destabilize the ice sheet depending on the sign of the crustal deflection. There are reasons to believe that mantle convective flow may have played a role in the evolution of Antarctic topography, including beneath some parts of the EAIS (Faccenna et al., 2008), and hence influenced ice mass loss across local grounding lines, however these effects have never been incorporated in ice sheet reconstructions.

In this study we reconstruct Antarctic bedrock elevation during the Mid-Pliocene using viscous flow modeling of dynamic topography constrained by a broad suite of seismic, mineral physics and geodynamic data sets (Forte et al., 2015). We then explore to what extent this change in bedrock elevation could have impacted EAIS stability during the MPWP.
We use the numerical code ASPECT (aspect.dealii.org; Kronbichler et al., 2012) to simulate compressible convective flow in the Earth’s mantle. As input, we adopt the density model TX2008 that has been derived on the basis of a joint inversion of a large database of shear-wave travel times and geodynamic data, with a scaling between seismic wave speeds and density based on constraints from mineral physics (Simmons et al., 2009). The geodynamic data include present-day observables such as long-wavelength free-air gravity anomalies, residual topography, plate divergence, and excess ellipticity of the core-mantle boundary. We further use a 3-D viscosity field that is based on the radial model V2 (Fig. 5.2). This radial profile was derived from a joint inversion of the geodynamic data set and a suite of observables related to glacial isostatic adjustment (Mitrovica & Forte, 2004; Forte et al., 2010). The simulations do not incorporate mantle phase changes, and we therefore assume adiabatic density and temperature profiles across the mantle. In order to produce realistic mantle temperatures, the top surface of the model domain is set to 1600 K and the core-mantle boundary temperature is then (following the assumption of adiabaticity) 2400 K, neglecting thermal boundary layers. The depth-varying thermal conductivity, thermal expansivity and heat capacity are adopted from Glišović & Forte (2015). ASPECT does not calculate gravity self consistently and we impose the radially varying gravity profile from Glišović & Forte (2015). We do not include internal heating from radioactive decay in the flow simulations.

We apply a free slip boundary condition at the core-mantle boundary and a no slip boundary condition at the top surface. Since the Antarctic plate is relatively stationary, this approach yields results comparable to those obtained by applying present-day plate velocities as a boundary condition on the top surface. The depth resolution of our numerical grid is ~60 km in the upper 1000 km and varies from 130 km to 230 km below this depth. The lateral resolution is on the order of 140 km in the upper 1000 km and 350 km to 500 km below this depth. The timesteps are of order 50 kyr, as
derived from a Courant-Friedrichs-Lewy (CFL) number of 0.2. We run ASPECT forward in time to calculate the rate of change of dynamic topography and have confirmed that the solution converges for the adopted tolerance of the linear solver and the spatial resolution. The dynamic topography change is then extrapolated back to 3 Ma. To calculate dynamic topography from the radial stresses of the mantle convection model we incorporate gravitationally self-consistent sea level variations driven by the effects of dynamic topography and associated ocean load changes as described in Chapter 2.

As a first benchmarking exercise, we compared results from an ASPECT simulation using the 1-D viscosity profile $V_2$ (i.e., no lateral variations in viscosity are imposed) to predictions from a global convection model used in Rowley et al. (2013) that are based on a spectral solution of the governing field equations (Forte & Peltier, 1991, 1994) and the same density and (radially varying) viscosity fields. There are several differences in the two approaches. Specifically, Rowley et al. (2013) use a distinct formulation of the top boundary condition (Forte & Peltier, 1994), a different spatial resolution, a self-consistent treatment of gravity, and a backward advection scheme for computing the change in dynamic topography since 3 Ma. Figure 5.3 presents a comparison of the dynamic topography change over Antarctica computed using the two approaches. The agreement is satisfactory. In the calculations we incorporate lateral variations in mantle viscosity linked to temperature perturbations. The three-dimensional viscosity field is calculated from the expression:

$$\eta(\phi, \theta, r) = \eta_0(r) \cdot \exp(-\epsilon \cdot (T(\phi, \theta, r) - T_0(r)))$$  \hspace{1cm} (5.1)$$

where $\eta_0$ is the depth varying viscosity profile $V_2$, $T_0$ is the adiabatic temperature profile and $\epsilon$ is an activation parameter (Ratcliff et al., 1996; Zhong et al., 2000). In our reference case we choose $\epsilon$ to be 0.02; in this case, 70% and 95% of the grid points in the mantle below the Antarctic plate are within ±1 and ±2 orders of magnitude relative to the depth average, respectively.
Figure 5.2: Radially varying viscosity profiles V1 and V2 derived from a joint inversion of convection and glacial isostatic adjustment observations (Mitrovica & Forte, 2004; Forte et al., 2010).

Figure 5.3: Computed change in dynamic topography since 3 Ma. Prediction from the Rowley et al. (2013) simulation (a), and from our calculation using the ASPECT convection code (b). Both predictions assume a water-loaded dynamic topography, the 1-D viscosity profile V2 and no plate motions.
5.3 Mantle flow beneath the Ross Shelf and Wilkes Basin

Figures 5.4a and 5.4b show lateral temperature variations (relative to the average at each depth) as well as mantle flow computed in two vertical cross-sections that are oriented to pass through the Wilkes Basin. The temperature fields are dominated by a hot (buoyant) anomaly that extends from the top of the lower mantle into the shallow upper mantle; the anomaly is located beneath the Ross Ice Shelf and the TAM and extends to the northwest around the thick East Antarctic craton, ultimately leading to shallow corner flow and upwelling at the edge of the craton. This same anomaly is a robust feature in a suite of different seismic tomography models (see Fig. 5.5). The computed flow beneath the Ross Shelf is characterized by two counter-rotating convection cells and associated divergent horizontal flow in the shallow mantle, consistent with arguments that an active mantle upwelling is the driving force for Cenozoic extension in the West Antarctic Rift System (Faccenna et al., 2008) as well as volcanism in several nearby locations (Faccenna et al., 2008; Gupta et al., 2009). Test calculations (not shown) indicate that density variations below the lithosphere drive uplift in the area of the TAM, however the shallow corner flow north of the Wilkes Basin is significantly reduced if density variations in the upper 200 km are removed from the calculation.

5.4 Dynamic topography and uplift

Changes in local dynamic topography (i.e., changes measured at a site fixed to a tectonic plate) are computed by combining the change in dynamic topography in the flow model domain with the motion of a plate through this topography field. Thus, to calculate local changes in dynamic topography we adopt the Euler pole by Sella et al. (2002) to describe the motion of the Antarctic plate relative to the underlying mantle. The computed change in local dynamic topography over the past 3 Myr. is shown in Figure 5.4c. The large, thermally buoyant anomaly evident in Figures 5.4a and 5.4b drives uplift of the TAM that extends into both the western and northern margin of the Wilkes
Figure 5.4: Mantle flow calculation based on the input density and viscosity fields described in the text. a, b. Present-day temperature fields (relative to the global average at each depth) and computed mantle flow vectors in two vertical cross sections (see c) passing through the Wilkes Basin, Antarctica. c. Change in dynamic topography over the past 3 m.y. computed by combining the flow field with Antarctic plate motion based on the reference frame by Sella et al. (2002). The direction of plate motion in the Wilkes Basin in this reference frame is indicated by the white arrow and has a magnitude of 1.6 cm/yr.

Basin. The western margin has contributions from both the geometry of the upwelling as well as plate motion over this upwelling. The predicted change in dynamic topography in the Dominion Range is ~50 m, which is consistent with the bound on uplift inferred from geomorphological analyses and surface exposure dating (Ackert & Kurz, 2004). The computed uplift in the Dry Valleys and Admiralty Mountains is significantly larger, on the order of 250 m. This value is consistent with Pliocene uplift estimates based on submarine and subaerially erupted volcanic rocks (Wilch et al., 1993; Mortimer et al., 2007). We have performed a series of analyses to test the sensitivity of our predictions of dynamic topography to a suite of input variables (see Fig. 5.6). These analyses indicate that the computed uplift is a robust feature of all simulations, although the magnitude of the change in dynamic topography is sensitive to the adopted viscosity structure. We note that our reconstruction of the bedrock elevation does not include changes associated with sediment transport (Hill et al., 2007; Wilson et al., 2012), which are likely small on the time scale we are considering.
Figure 5.5: Two vertical cross-sections (see inset for orientation) through three different shear wave tomography models: S40RTS (Ritsema et al., 2011), Savani (Auer et al., 2014) and GyPSuM (Simmons et al., 2010). These models all show a slow shear wave velocity anomaly in the mid mantle below the Transantarctic Mountains (panels a-c, transect a-a’). They also all show a transition from slow upper mantle seismic wave speeds north of the Wilkes Basin to fast wave speeds within the East Antarctic craton (panels d-f, transect b-b’). Note that the variable extent of the East Antarctic craton in sections d-f will impact the location of the predicted corner flow upwelling at the northern edge of the Wilkes Basin.
Figure 5.6: Sensitivity of dynamic topography predictions to various input parameters. The reference prediction is based on the tomography model TX2008, the radial viscosity profile V2, lateral variations in viscosity prescribed using $\epsilon = 0.02$, and the plate reference frame of Sella et al. (2002). The following frames show predictions of dynamic topography in which these input parameters are varied. Specifically: (a) smaller lateral variations in viscosity ($\epsilon = 0.01$); (b) radial viscosity profile V1 (see Fig. 5.2); (c) plate velocity reference frame from Quéré et al. (2007); (d) plate velocity reference frame from Müller et al. (1993); (e) same as d, but with $\epsilon = 0.01$; and (f) plate velocity reference frame from Doubrovine et al. (2012). In each case, the white arrow indicates the Antarctic plate velocity relative to the model domain. Note that estimates of the Antarctic plate motion based on hotspot reference frames vary from one another (Müller et al., 1993; Gripp & Gordon, 2002; Doubrovine et al., 2012), while present-day motions based on geodetic measurements and a no-net-rotation reference frame are relatively consistent (Larson et al., 1997; Sella et al., 2002; Jiang et al., 2009). The Antarctic plate is surrounded by oceanic ridges and is therefore relatively stationary.
5.5 Antarctic Ice Sheet Stability

In this section, we use an established Antarctic Ice Sheet model (Pollard & DeConto, 2009; DeConto et al., 2012) to investigate whether reconstructions of EAIS stability during the MPWP are sensitive to the incorporation of dynamic topography changes in bedrock elevation. This model tracks ice thickness and temperature distributions in Antarctica that result from gravitationally driven ice sheet deformation as well as mass addition and removal due to precipitation, basal melt and runoff, oceanic melt and calving of floating ice (for details, see Pollard & DeConto, 2012). Furthermore, the model includes a crustal rebound term associated with ice unloading that assumes an elastic lithosphere and a viscous relaxation time of 3000 yr, and it applies the parameterization for ice flux across the grounding line derived by Schoof (2007). We force the model with Pliocene conditions (see Pollard et al., 2015, and references therein), including the adoption of a warm atmospheric climate from a slightly modified version of the RegCM3 Regional Climate Model, atmospheric CO$_2$ concentration of 400 ppmv, ocean warming of 2°C, and fixed, orbitally driven changes in summer insolation (DeConto et al., 2012). The ice model is first spun up to modern conditions, which is followed by an instantaneous change in forcing to a warmer Pliocene climate. The model is then run for an additional 5000 yr to allow for an equilibrium state to establish.

We ran two simulations of Antarctic ice cover during the MPWP, one that does not include dynamic topography changes in reconstructing Pliocene bedrock elevation (Figs. 5.7a) and one that does (Figs. 5.7b). The western and northern margins of the Wilkes Basin have been uplifting due to changes in dynamic topography for at least the past 3 Myr (Fig. 5.4c), in contrast to other marine-based sectors of the EAIS, which have been subject to much smaller dynamic topography changes (e.g., Aurora Basin). Thus, the elevation correction for changes in dynamic topography lowers the estimated mid-Pliocene bedrock elevation on the margins of the Wilkes Basin and makes the local, marine-based ice sheet more susceptible to retreat. In particular, grounding line retreat in the sim-
Figure 5.7: Pliocene ice sheet model predictions. a, b. Antarctic ice sheet elevation computed using simulations which do not include, and do include, respectively, the impact of dynamic topography change on mid-Pliocene bedrock elevations. Pink regions mark areas of floating ice. c. Difference in ice sheet thickness in the Wilkes Basin region between the model reconstructions shown in a and b.

ulation that includes a dynamic topography correction extends ~400 km further inland into the Wilkes Basin than the simulation based on present-day bedrock elevation. The integrated difference in ice melting in the two simulations is equivalent to ~2 m of global mean sea-level change.

We ran ice sheet simulations for a suite of mantle flow calculations that were performed to test the sensitivity of our predictions of dynamic topography change to model inputs (see Fig. 5.8). In this set of simulations, the correction of the mid-Pliocene bedrock elevation for dynamic topography leads to grounding line retreat of 200-560 km at the margin of the Wilkes Basin and an ice volume change equivalent to a global mean sea-level difference of 1.1-2.8 m relative to a simulation with no change in dynamic topography (or 5.7 - 7.4 m relative to present-day Antarctic ice cover).

Pollard et al. (2015) have argued that the combined effects of ice cliff failure and hydrofracturing may have enhanced the retreat of the EAIS, including within both the Wilkes Basin and adjacent Aurora Basin, during the MPWP. We performed an additional series of simulations that include these mechanisms (Fig. 5.9). These calculations indicate that changes in dynamic topography will enhance the retreat of grounded ice within the Wilkes Basin even when the cliff failure and hydrofracturing processes are included. We note that if one were to model each of these three mechanisms in isolation, only the change in bedrock elevation due to dynamic topography would have

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Figure 5.8: Sensitivity of our simulation of ice sheet extent at 3 Ma to different predictions of the change in dynamic topography within a region in the vicinity of the Wilkes Basin. Panels a-f are ice sheet simulations associated with the dynamic topography changes shown in panels a-f of Figure 5.6. The predicted shoreline retreat in the Wilkes Basin, relative to an ice sheet simulation with no change in dynamic topography, is approximately 200 km (a), 480 km (b), 560 km (c), 430 km (d), 300 km (e), and 250 km (f). The difference in predicted ice volume relative to the simulation without dynamic topography is, in units of equivalent globally averaged sea-level change, 1.3 m (a), 2.3 m (b), 2.8 m (c), 2.1 m (d), 1.1 m (e), and 1.6 m (f).
been sufficient to drive significant ice sheet retreat in the Wilkes Basin during the MPWP. In contrast to these destabilizing mechanisms, gravitationally self-consistent sea-level changes at the margin of an evolving EAIS will act to stabilize the ice sheet (Gomez et al., 2013). This effect will likely be small in the ice sheet modeling described here, in which the system is assumed to be close to isostatic equilibrium, but it may play a role in determining ice sheet extent during time-evolving Pliocene ice age cycles.

5.6 Conclusion

We have demonstrated that incorporating bedrock elevation changes due to dynamic topography is important when trying to reconstruct the EAIS in the vicinity of the Wilkes Basin during the MPWP. Mantle flow modeling of dynamic topography predicts that bedrock elevation was ~100-200 m lower on the western and northern margins of the Wilkes Basin during the mid-Pliocene, and including this change in ice sheet model simulations introduces a destabilizing influence on the local ice margin causing additional 200-560 km of grounding line retreat into the Wilkes Basin. The amplitude of dynamic topography change in other marine-based sectors of the EAIS (e.g., Aurora Basin, Prydz Bay; Fig. 5.1) is relatively small in our simulations and hence has little impact on the ice sheet stability in these sectors during the MPWP.

Our prediction is consistent with geochemical analysis of the Integrated Ocean Drilling Program core U1361, which has been used to argue for a retreat of the ice margin of several hundred kilometers in the Wilkes Basin during the Pliocene (Cook et al., 2013). However, this argument assumes that the enhanced erosion inferred from the geochemical analysis was sourced from the ice margin. Direct geologic evidence of ice sheet retreat in the Wilkes Basin is lacking, making it an important target for future field research. Finally, the impact of dynamic topography changes on ice sheet stability is not limited to the Pliocene and has likely affected the evolution of the Antarctic Ice Sheet since its inception.
Figure 5.9: Ice sheet model predictions under MPWP conditions without (a and b) and with (c and d) dynamic topography. Frames a and c are reproduced from Figure 5.7a and 5.7b. Frames b and d are simulations identical to a and c, respectively, except that the ice models include the additional instability mechanisms of cliff failure at ice margins and hydrofracturing (Pollard et al., 2015).
A statistical analysis of the correlation between Large Igneous Provinces and Lower Mantle Seismic Structure

6.1 Introduction

Large Igneous Provinces (LIPs) are regions where basaltic lava formed during massive eruptions of relatively short duration (Coffin & Eldholm, 1994). They have ages ranging from 15 Ma to 300 Ma (Torsvik et al., 2006, 2008a) and are commonly associated with plumes originating from the core-mantle boundary (e.g. Richards et al., 1989), although a shallow origin has also been postulated (Anderson, 1982). It has been recognized that LIPs correlate spatially with low shear-wave velocity struc-

tures in the lower mantle (e.g. Burke & Torsvik, 2004). However, efforts to establish this correlation face several complications. First, one has to restore the LIPs to their location on the Earth’s surface at the time of their formation using models of plate motions. Second, one has to restore present-day shear-wave velocity anomalies in the lower mantle to the time of putative plume generation, which is presumably older than the time of LIP eruption. In practice, this latter restoration has been avoided by assuming that seismic velocity anomalies are stable (i.e., stationary) in the highly viscous deep mantle.

At their restored locations, LIPs appear to be close to the (surface projected) edges of the two Large Low Shear Velocity Provinces (LLSVPs) at 2800 km depth (Figure 6.1a). On this basis, Torsvik et al. (2006) chose a specific shear-wave velocity anomaly contour (e.g. –1% for SMEAN; Becker & Boschi, 2002) to describe margins of LLSVPs and postulated that deep mantle plumes are generated close to this contour. Burke et al. (2008) presented additional comparisons with D” models and a statistical analysis to support their argument of a spatial correlation between restored LIP locations and margins of LLSVPs and smaller Low Shear Velocity Provinces (LSVPs). They subsequently referred to those margins as the plume generation zone. Torsvik et al. (2006) have, furthermore, argued that LIPs correlate with regions of steep gradients in deep mantle velocity perturbations (Figure 6.1b).

The conclusion that LIPs are preferentially generated at margins of LLSVPs has been cited in studies that have constrained longitude in paleomagnetic reconstructions (Torsvik et al., 2008b), proposed locations for Kimberlite and diamond exploration (Evans, 2010; Torsvik et al., 2010), and explored the implications for mantle convection (e.g. Steinberger & Torsvik, 2012). Moreover, since there is evidence for LIPs as old as 2.5 Gy (Ernst & Buchan, 2003), Burke et al. (2008) have argued that LLSVPs have been relatively stable over such long timescales, which makes them candidates for geochemical reservoirs of primordial mantle (e.g. Mukhopadhyay, 2012).

The net buoyancy of LLSVPs is a matter of contention. Published studies have argued that ther-
Figure 6.1: LIP locations (white circles) restored to their original position using a global paleomagnetic reference frame. The abbreviation for each LIP is taken from Table 1 of Torsvik et al. (2006). a. The color scale shows the percent perturbation in shear wave velocity relative to a 1-D model (Vs) at 2800 km depth from the seismic tomography model S20RTS. These are perturbations relative to the Preliminary Reference Earth Model (PREM) (Dziewonski & Anderson, 1981). The thick black line highlights the -0.65% value used by Torsvik et al. (2006) to define the margin of LLSVPs, and the white line shows the 0% contour. Analogous figures based on other seismic tomography models are provided in Torsvik et al. (2006) and Burke et al. (2008). b. The color scale shows the smoothed magnitude of the horizontal gradient in shear wave velocity anomalies at 2800 km depth calculated using the S20RTS seismic tomography model. The reconstructed LIP locations and thick black line are reproduced from frame (a).
mal effects on density dominate chemical effects and that LLSVPs are positively buoyant (Forte & Mitrovica, 2001; Davies et al., 2012; Schuberth et al., 2012) or that the reverse is true and LLVSPs are negatively buoyant thermochemical piles (Bull et al., 2009; McNamara & Zhong, 2005). However, regardless of the net buoyancy of LLSVPs, the consensus that they represent hotter than average mantle structures suggests that the entire surface of LLSVPs may act as zones of plume generation. This suggestion is supported by the recent seismic imaging of a plume originating from top surface of the LLSP below Africa (Sun et al., 2010).

In this study we present a new statistical analysis of the spatial correlation between LIPs and: (1) the margins of LLSVPs, (2) the full areal extent of LLSVPs; and (3) deep mantle regions characterized by the highest seismic velocity gradients. In the discussion below, we will include LSVPs in the term LLSVPs. Furthermore, unless otherwise stated, whenever we refer to LIP locations we mean the restored location of the initial eruption site.

6.2 Large Igneous Provinces, Tomography Models and Reference Frames

Following the analysis of Torsvik et al. (2006), we adopt three seismic tomographic inferences of mantle shear wave velocity heterogeneity: NGRAND (Grand, 2002), S20RTS (Ritsema & van Heijst, 2000) and SB4L18 (Masters et al., 2000). We also consider the D” tomography model of Kuo et al. (2000) that was used by Burke et al. (2008). In contrast to this previous work, we do not use SMEAN (Becker & Boschi, 2002), which combines the shear wave velocity models NGRAND, S20RTS and SB4L18, but instead base our statistical analysis on the three individual tomography models. This procedure provides a measure of the uncertainty related to errors in the tomography models.

Torsvik et al. (2006) reconstructed the position of LIPs at their time of formation using four different reference frames: global paleomagnetic, Africa fixed hotspot, Africa moving hotspot and global moving hotspot. We do the same. Moreover, following Burke et al. (2008), we added the
Skagerrak Centered LIP (SK) proposed by Torsvik et al. (2008a) to the list of LIPs reconstructed using the global paleomagnetic reference frame.

Figure 6.1a shows the reconstructed locations of LIPs using a global paleomagnetic reference frame superimposed on the shear wave velocity map at the 2800 km depth slice of S20RTS. Names, symbols, ages and locations of LIPs can be found in Table 1 of Torsvik et al. (2006). The Figure also shows the −0.65% contour of velocity perturbation (Vs) in black, which Torsvik et al. (2006) define as the margin of the LLSVPs in the S20RTS tomography model. Following their argument, the margin-defining contours for the NGRAND, SB4L18 and Kuo et al. (2000) models are −1%, −0.6% and −0.77% respectively (Torsvik et al., 2006; Burke et al., 2008). We adopt these choices in our statistical analyses.

For the analysis of the horizontal gradient of seismic velocity, we apply a 5x5 degree smoothing to the raw gradients. This processing makes steep gradients visually more evident. Figure 6.1b shows the (smoothed) horizontal gradients in seismic velocity model S20RTS at 2800 km depth. The reconstructed LIP positions and the −0.65% contour of the velocity perturbation are reproduced from Figure 6.1a.

6.3 Statistical tests

6.3.1 Correlation Between LIPs and Lower Mantle Seismic Structure

The statistical analysis of Burke et al. (2008) was based on the following question: If LIPs are the result of plumes generated at random locations in the deep mantle, how likely is the observed proximity of the LIPs to the margins of LLSVPs? To answer this question, they first determined the number of LIPs that are located within a specific angular distance from the margin of LLSVPs: Using the tomography by Kuo et al. (2000) and the paleomagnetic reference frame for LIP reconstructions, 16 out of 24 LIPs have an angular distance of 5° or less from the margins of LLSVPs. They
next determined the probability that 16 or more randomly chosen points on the Earth’s surface were located within this distance of the margins of LLSVPs, which they found to be very low (less than 0.1%). Similar results were obtained for analyses based on the other tomographic models of shear wave heterogeneity. On this basis, they concluded that LIPs are not located randomly on the Earth’s surface, and that the margins of LLSVPs serve as zones of plume generation.

Since LLSVPs are hotter than the surrounding mantle, their entire surface may act as zones of plume generation. This raises the question: Are the observed locations of LIPs statistically correlated with slower-than-average shear wave velocity regions (i.e., LLSVPs) and, if so, is the previously identified correlation between LIP locations and the margins of LLSVPs statistically distinguishable from this new test of correlation? If the answer to the former question is yes and the latter is no, then the correlation of LIP locations with LLSVP margins may simply be part of a more general correlation between these locations with LLSVPs themselves. In this case, it would be premature to conclude that the margins of LLSVPs act as a plume generation zone.

To address these issues we first conducted a series of Monte Carlo based statistical tests that follow the approach of Burke et al. (2008). Specifically, we ran 1000 simulations in which we randomly positioned 24 points on a sphere. These points were drawn from a distribution that is uniform in space. In each case, we calculated: (1) the mean distance of the points from the (surface projected) LLSVP margins; and (2) the percentage of points that overlie slower-than-average shear wave velocity anomalies at the 2800 km depth slice of a tomographic model. We consider observed LIPs to be spatially correlated with margins of LLSVPs if their mean distance from these margins is smaller than 95% of the mean distances obtained in the Monte Carlo sampling. Similarly, we consider observed LIPs to be spatially correlated with slower-than-average shear wave velocities if the percentage of LIPs that overlie slower-than-average shear wave velocity anomalies is greater than or equal to 95% of the percentages obtained in the Monte Carlo sampling. That means all tests will be performed at the 95% significance level and we assume that LIPs are independent of one another,
i.e. there is no clustering beyond just being near LLSVPs.

Figure 6.2a and b show the observed values (red lines) and the distribution that we obtain from the Monte Carlo sampling (blue histograms) when the above tests are applied to the S20RTS tomography model and a LIP reconstruction is based on the paleomagnetic reference frame. The observed mean distance between the LIPs and the LLSVP margins is $8^\circ$ (Figure 6.2a) and the percentage of LIPs that overlie slower-than-average regions is $87.5\%$ (21 out of 24). The former is smaller than all of the mean distances obtained from the 1000 Monte Carlo samples, and we therefore conclude that the observed locations of LIPs are correlated with margins of LLSVPs. The latter percentage is greater than all of the percentages obtained from Monte Carlo sampling and we therefore conclude that the observed locations of LIPs are also correlated with slower-than-average shear wave velocities (i.e., LLSVPs). We have repeated the above tests for all possible combinations of tomography models and reference frames for LIP reconstruction and the same conclusions hold in each case.

The next Monte Carlo test is designed to address the following question: If LIPs are the result of plumes generated throughout regions of slower-than-average seismic velocity (LLSVPs), does this naturally lead to a correlation between the observed location of LIPs and the margins of LLSVPs? In this test we ran 1000 simulations in which we randomly positioned 24 points within (surface projected) zones of slower-than-average seismic velocity. In each case, we calculated the mean distance of these points from the margins of LLSVPs. A histogram of the results is shown (in red) in Figure 6.2c. This histogram indicates that in 99.9% of all samples, the mean distance to the margin of LLSVPs is smaller than 95% (significance level) of the mean distances obtained in the first test (i.e., the histogram in Figure 6.2c falls entirely within the blue shaded region defined by the histogram in Figure 6.2a). That is, in almost all cases when plumes are generated in regions of slower-than-average shear wave velocity, they are also closer to the margins of LLSVPs than if they were randomly distributed around the globe. We conclude that a seemingly significant relationship be-
Figure 6.2: Results of Monte Carlo (1000 samples) statistical testing based on the S20RTS seismic tomography model at 2800 km depth and the paleomagnetic reference frame for LIP reconstruction. 

a. The distribution of mean distances to the margins of LLSVPs (horizontal axis) when 24 points are randomly positioned on the globe. The red vertical line denotes the value obtained from the reconstructed LIP locations. The light blue region indicates the range of mean distances that are smaller than 95% of the mean distances obtained from Monte Carlo sampling. 

b. Same as frame (a), except for the distribution of the percentage of points that overlie regions of slower-than-average ('slow') seismic shear wave velocity at 2800 km depth in the mantle. In this case, the light blue indicates the range of percentages that are larger than 95% of the percentages obtained from Monte Carlo samples. 

c. The distribution of mean distances to the margins of LLSVPs when 24 points are randomly positioned in regions of slower-than-average seismic shear wave velocity at 2800 km depth. The blue shaded region is the same as in frame (a). 

d. The distribution of the percentage of points located in regions of slower-than-average seismic velocity at 2800 km depth when they are randomly positioned close to margins of LLSVPs (where 'close' is defined quantitatively in the text). The blue shaded region is the same as in frame (b).
tween the location of LIPs and margins of LLSVPs will generally arise if plumes are generated in slower-than-average shear wave velocity regions and evaluated against a null hypothesis of a globally uniform distribution. Further testing indicates that this conclusion holds regardless of the combinations of seismic tomography models and reference frames for LIP reconstruction. As a final point, it is interesting to note that the peak in the histogram in Figure 6.2c, that is, the most likely mean distance to the margin of LLSVPs when plumes are randomly generated from LLSVPs, matches the observed mean distance of LIPs from the margins of LLSVPs.

In the next Monte Carlo test we address the following variation on the last question: If LIPs are the result of plumes generated at the margins of LLSVPs, does this naturally lead to a correlation between the observed location of LIPs and regions of slower-than-average seismic velocity (LLSVPs)? In this case, we ran 1000 simulations in which 24 points were randomly positioned within a zone close to margins of LLSVPs, where ‘close’ means that the distance follows a half-normal distribution with standard deviation of 5°. In each case, we computed the percentage of points that overlie zones of slower-than-average seismic velocity; a histogram of the results is shown (in red) in Figure 6.2d. This histogram indicates that, in every one of the 1000 Monte Carlo samples, the percentage of points that overlie zones of slower-than-average seismic velocities is greater than 95% (significance level) of the percentages obtained in the Monte Carlo analysis summarized in Figure 6.2b. This result is in line with the geometric rational that the slower-than-average contour resides, on average, 7.7° outside of the contour that denotes the margin on LLSVPs and serves to highlight that distinguishing between a contour of high gradient and region of slow velocity is, in this case, geometrically and statistically difficult. We conclude that a statistically significant correlation between the location of LIPs and regions of slower-than-average seismic velocity (LLSVPs) is the expected consequence of plumes being generated near the margins of LLSVPs. We repeated the analysis of Figure 6.2d for all possible combinations of tomography models and reference frames for LIP reconstruction. In all cases at least 90% of the realizations analogous to those shown in Figure 6.2d were greater than 95%
(significance level) of the realizations shown in Figure 6.2b. This means that the above conclusion (that points which lie close to margins of LLSVPs also tend to lie over slower-than-average seismic velocity perturbations) is independent of the adopted tomography model. Note that the peak in the histogram in Figure 6.2d, that is the most likely percentage of points within slow regions when plumes are randomly generated at the margins of LLSVPs, matches the observed percentage of LIPs that lie above slower than average mantle at 2800 km depth.

The results discussed above may be sensitive to several assumptions that were made in the analyses. First, consider our adoption of a 95% significance level. If we had chosen a more stringent significance level of 99%, then the shaded blue regions in all of the frames in Figure 6.2 would decrease. Our conclusions based on a comparison of the observed values with the histograms in Figures 6.2a and 6.2b would not have been altered in this case, but the conclusions from our test comparing the histograms in Figure 6.2c and 6.2d with the histograms in Figures 6.2a and 6.2b, respectively, would have been marginally weakened. That means, the percentage of realizations that falls within the significance level of 99% drops to 99.1% for the test shown in Figure 6.2c, but stays at 100% for the test shown in Figure 6.2d. Second, in regard to Figure 6.2d, one might question our adoption of a half-normal distribution with a standard deviation of 5° to define ‘closeness’. Increasing this standard deviation would move the (red) histogram in Figure 6.2d to lower values. As an example, increasing the standard deviation to 10° (which would produce a mean distance of 8°) moves the peak of the histogram to ~70%. Third, in regard to Figure 6.2c, one might argue that not all LIPs lie over slower-than-average regions, so we should only locate some arbitrarily large fraction of the 24 points in slower-than-average regions and the rest in colder-than-average regions. Using a fraction less than the 100% adopted in the test summarized in Figure 6.2c would move the histogram to higher mean distances. As an example, repeating the test in Figure 6.2c for a case in which

In defense of the choices we made concerning the second and third assumptions listed above, we note again that the histograms in Figure 6.2c and 6.2d are approximately centered on the observed
values (red lines in Figures 6.2a and 6.2b). This suggests that our choices led to a reasonable distribution of results in the Monte Carlo tests.

6.3.2 Correlation Between LIPs and High Seismic Velocity Gradients in the Deep Mantle

Previous studies have argued for a correlation between regions of high horizontal gradient in seismic shear wave velocity in the deep mantle and various surface expressions of plumes. For example, Thorne et al. (2004) found that such regions are correlated to present-day hotspot locations. However, independent analyses (Torsvik et al., 2006; Courtillot & Renne, 2003; Montelli et al., 2006) suggest that only 11 of the 44 hotspots that were used in the Thorne et al. (2004) analysis can be identified as having a deep origin. As we noted in the introduction, Torsvik et al. (2006) argued that the margin of LLSVPs (the region they defined as the plume generation zone) was correlated with zones of high seismic velocity gradients. Indeed, a zone defined by some specific range in velocity gradients in the deep mantle might be a better candidate for a plume generation zone than a specific seismic velocity contour because instabilities in the boundary layer, which are thought to initiate plumes, are driven by gradients in temperature rather than absolute temperature. Figure 6.1b, which juxtaposes these zones with the locations of LIPs, suggests that it would be worthwhile to rigorously test the robustness of the correlation between these two.

Consider, once again, the Monte Carlo simulation based on repeatedly locating a set of 24 points randomly on the Earth’s surface (as in the results summarized in Figures 6.2a and 6.2b). For each of the 1000 samples, we compute the mean value of the (magnitude of) horizontal gradients in the seismic velocity that underlie these 24 points. We consider LIPs to be spatially correlated with high horizontal gradients in seismic velocity if the mean gradient in velocity perturbations that LIPs overlie is greater than 95% of the mean gradients determined in the Monte Carlo procedure.

Figure 6.3 summarizes the results of our testing for all combinations of seismic tomography
model and reference frames for LIP reconstruction. For each of these combinations, the y-axis represents the percentage of the set of 1000 mean gradients that is exceeded by the mean gradient that the LIPs overlie. The results in Figure 6.3 vary across a wide range. Except for the model using the African moving reference frame, all combinations that adopt the NGRAND seismic tomography model show that a statistically significant percentage of LIPs are in proximity to high seismic velocity gradients. In contrast, none of the tests based on combinations adopting either the SB4L18 or Kuo et al. (2000) tomography models show a statistically significant (to 95% level) correlation. Finally, of the tests based on the S20RTS tomography model, the LIP reconstructions based on the paleomagnetic and the African fixed reference frame lead to a significant correlation. The average value of the suite of results in Figure 6.3 is ~70%. (This average is 85% if the SB4L18 tomography model is not considered, although we have no a priori reason to discount this model.) We thus conclude that the proximity of LIPs to high seismic shear wave velocity gradients in the deep mantle is not necessarily statistically significant, given current uncertainties in deep mantle seismic tomography and the appropriate choice of reference frames, nor is it necessarily diagnostic of a physical relationship because proximity to high gradients is readily conflated with proximity to LLSVPs.

Despite this conclusion, it will be instructive to repeat the analysis in Figure 6.2 replacing the distance-to-margin metric with a metric based on the magnitude of the velocity gradient. As in Figure 6.2, we will adopt, for the purpose of illustration, the S20RTS tomography model and the paleomagnetic reference frame. From Figure 6.3, the mean horizontal gradient in seismic velocity underlying the 24 LIPs in this case is greater than 95% of the mean gradients of the 1000 simulations in which the 24 points were randomly located on the Earth’s surface (see Figure 6.4a). In the case of a Monte Carlo simulation in which the 24 random points were restricted to lie above slower than average mantle at 2800 km depth (Figure 6.4c), the mean seismic velocity gradient of ~40% of the simulations exceed 95% of the gradients obtained in Figure 6.4a. The peak value of this distribution does not coincide with the observed value. To continue, Figure 6.4b, reproduced from Fig-
Figure 6.3: Results of Monte Carlo testing of the correlation between the location of LIPs and regions of high horizontal gradient in seismic velocity perturbations at 2800 km depth. 1000 sets of 24 points were randomly placed on the Earth's surface and the mean gradient in deep mantle seismic velocity that these points overlie is computed in each case. The ordinate axis represents the percentage of this set of 1000 mean gradients that is exceeded by the observed mean gradient that the LIPs overlie. The symbols represent results of the Monte Carlo tests based on different seismic velocity models (as labeled in the inset) and, in each case, the abscissa bins results according to the reference frame adopted in the reconstruction of LIP locations (PM, paleomagnetic; AF, Africa fixed; AM, Africa moving; and GM, global moving). Orange squares denote average values obtained for each reference frame.
ure 6.2b, shows the percentage of points that overlie slower than average mantle at 2800 km depth when these points are placed randomly on the Earth’s surface. In Figure 6.4d, we repeat the Monte Carlo test by placing the 24 points within regions of high gradient, that is, at gradients that exceed 0.04%/° (the observed values in gradients that LIPs overlie a range from approximately 0.04%/° to 0.11%/°). We find that the positioning of plumes in this manner does not necessarily lead to a high percentage of the plumes being located above slower than average mantle at 2800 km depth, as is the case for the observed location of LIPs (Figure 6.4b). Indeed, only 13% of the simulations in Figure 6.4d have a percentage of points in slow regions that exceed 95% of the simulations in Figure 6.4b. These results indicate that LLSVPs might be a better geographic description for the preferred locations of plumes than high gradients in velocity.

6.3.3 Correlation between the velocity and gradient field at LIP locations

We performed an additional Monte Carlo test to explore this further. We used a Monte Carlo approach to test whether one can distinguish between deep mantle plume sources that are located within regions of low seismic velocity or a zone defined by a range of velocity gradients. For this test we adopt the S20RTS tomography model, which provides perturbations in shear wave velocity at a depth of 2800 km (Figure 6.1a), and the smooth gradient of this field (Figure 6.1b). We normalize these scalar fields so that if we repeatedly take the average of 24 randomly located values we obtain distributions with the same mean (of zero) and standard deviation (of one) for both fields. Let $V_i$ and $G_i$ be the shear wave velocity and magnitude of the velocity gradient that the ith plume overlies. We define the test statistic, $E$, as the mean of all $E_i = G_i + V_i$. We calculate the test statistic for the observed data, which results in $E_{obs} = -0.2$.

In the first Monte Carlo test, we compute $E$ for 24 points that are distributed such that 21 are positioned within slow regions and 3 within fast regions as defined by the S20RTS model at 2800 km depth. This positioning matches the distribution of reconstructed LIP locations relative to
Figure 6.4: Results of Monte Carlo statistical testing (1000 samples) based on the S2ORTS seismic tomography model at 2800 km depth and the paleomagnetic reference frame for LIP reconstruction. a. Distribution of the mean seismic velocity gradient (horizontal axis) for 24 points that are randomly positioned on the globe. The red vertical line denotes the value obtained from the reconstructed LIP locations. The light blue region indicates the range of mean gradients that are greater than 95% of the mean gradients obtained from Monte Carlo sampling. b. Same as frame (a), except for the distribution of the percentage of points that overlie regions of slower-than-average ('slow') seismic shear wave velocity at 2800 km depth in the mantle. This frame is reproduced from Figure 6.2b. The light blue region indicates the range of percentages that are larger than 95% of the percentages obtained from Monte Carlo samples. c. The distribution of the mean seismic velocity gradient when 24 points are randomly positioned in regions of slower-than-average seismic shear wave velocity at 2800 km depth. The blue shaded region is the same as in frame (a). d. The distribution of the percentage of points located in regions of slower-than-average seismic velocity at 2800 km depth when they are randomly positioned within regions of high seismic velocity gradient (defined by gradients that exceed 0.04%/°). The blue shaded region is the same as in frame (b).
the same tomographic model. We calculate a distribution for the statistic \( \overline{E} \) from 10,000 repetitions (Figure 6.5a). Using this distribution, \( \overline{E}_{\text{obs}} \) is in the 91\(^{th}\) percentile, i.e. 91\% of the Monte Carlo outcomes are smaller than the observed value. We conclude that if we prescribe the relative distribution of plumes/LIPs within slow and fast seismic velocity regions, we predict the observed range in velocity gradients that the LIPs overlie reasonably well (using a significance level of 95\%).

In the second Monte Carlo test we repeatedly compute \( \overline{E} \) for a set of 24 points that we locate within a specific range of velocity gradients. The observed distribution of velocity gradients sampled by LIPs is relatively uniform between approximately 0.04\%/° and 0.11\%/°, and so we assume a uniform distribution within this range for the 24 points in the Monte Carlo analysis. We calculate a distribution of \( \overline{E} \) from 10,000 repetitions (Figure 6.5b) and find that \( \overline{E}_{\text{obs}} \) is in the 0.6\(^{th}\) percentile. Thus, if we prescribe a range of velocity gradients consistent with the observed distribution of LIPs, we poorly predict (at a 95\% confidence level) the velocity range sampled by the LIPs.

We conclude from these two tests that being located above seismically slow deep mantle regions (i.e., in the region defined by LLSVPs) is a stronger geographic requirement for plume generation than being located in a specific range of high seismic velocity gradients at 2800 km depth.

It is important to note that this result does not imply that high temperature gradients are unimportant in plume generation. This is because first, gradients in shear velocity near the CMB may be strongly controlled by chemical heterogeneity and second, limiting the analysis to velocity gradients at sites at 2800 km depth ignores the 3-D geometry of surfaces of high seismic velocity gradient.

6.4 Discussion

We conclude, on the basis of current constraints on LIP locations, seismic shear wave velocity heterogeneities in the deep mantle and plate motions, that the reconstructed locations of LIPs are correlated with both LLSVPs and the margins of these structures. However, we also conclude, given these current constraints, that these two correlations cannot be statistically distinguished at a 95\%
Figure 6.5: Results of Monte Carlo statistical testing (10,000 samples) based on the S20RTS seismic tomography model at 2800 km depth and the paleomagnetic reference frame for LIP reconstruction. a. The distribution of $E$ (mean sum of normalized velocities and normalized gradients in velocity) when 21 of 24 points are randomly positioned in regions of slower-than-average seismic shear wave velocity at 2800 km depth. The vertical red line denotes the value associated with the reconstructed LIP locations. b. Same as frame (a), except that the 24 points are randomly located within the range of velocity gradients sampled by the reconstructed LIP locations.

confidence level. Our conclusions indicate that it is therefore premature to argue, as in some previous studies (Torsvik et al., 2006; Burke et al., 2008), that the margins of LLSVPs represent plume generation zones.

This conclusion is strengthened by noting that there are other substantial uncertainties that have not been quantified in these statistical analyses, including the lateral deflection of plumes during their ascent, incomplete knowledge of LIP locations with respect to the corresponding plume impact site on the base of the lithosphere, and the appropriate depth at which the geographic extent of LLSVPs are defined. Torsvik et al. (2006) estimate the combined uncertainty of the first two of these effects to be ~500 km. However, Steinberger (2000) estimated that the plume conduit can be deflected by as much as 800 km in the trajectory of a plume rising from the D” region to the asthenosphere. (This deflection may be smaller if plumes are anchored at high points along the interface between LLSVPs and the overlying mantle.) Additionally, given the evident clustering, it is
not clear whether each LIP observation provides an independent constraint. This issue has direct implications for the analysis since fewer effective independent observations lead to less statistically significant results.

Our analysis has not considered hotspot data, including for example the tabulation by Steinberger (2000). As discussed above, not all (Torsvik et al., 2006; Courtillot & Renne, 2003; Steinberger, 2000), if any (Anderson, 1982), of these hotspots can be sourced to the deep mantle. Moreover, some of these hotspots are thought to originate from the same plumes associated with LIPs, and they therefore do not provide an entirely independent data set.

The mechanisms for plume formation remain poorly constrained and controversial. The correlation we have identified between the location of LIPs and the areal extent of LLSVPs is consistent with the recent seismic imaging of a plume-like structure rising from the upper surface of the LLSVP beneath Africa (Sun et al., 2010), and, more generally, with the suggestion by Kellogg (1999) that plumes would tend to arise from local high spots along the surface of LLSVPs.

In mantle convection models, the location of plume formation related to LLSVPs depends strongly on the buoyancy and geometry of these structures (e.g. Tan et al., 2011). Plumes are generally sourced from warm mantle material that rises along the slope of LLSVPs when the latter are modeled as having high density. In this case, if LLSVPs have steep slopes, plumes will tend to originate at their margins (when the LLSVPs are mapped in plan view), whereas shallow slopes lead to material rising from crests along the upper surface of LLSVPs. Furthermore, to be stable over long time scales, LLSVPs need to have a minimum integrated negative buoyancy. Tan et al. (2011) used convection models to explore LLSVP geometry and the location of plume generation. They found that LLSVPs modeled as having high bulk modulus and density where characterized by a dome-like geometry with plumes emerging from steep margins. However, the lateral position of these structures was stable for periods longer than a few hundred million years only if subduction geometry was relatively stationary in time. Steinberger & Torsvik (2012) proposed a strong coupling be-
tween deep mantle processes and plate tectonics to explain the formation of plumes along margins of LLSVPs. They introduced sinking slabs (constrained by plate reconstructions) that pushed heavy chemical material into piles with steep edges. Moreover, these slabs deformed the thermal boundary layer at the CMB initiating upward motion of hot material along the steep edges of LLSVPs and plume generation.

Clearly, the argument that plume formation occurs at the margins of LLSVPs has been an important motivation for recent studies of mantle convection. The implication of our statistical analysis is that the conclusions from these studies regarding the thermochemical structure of the LLSVPs, and their long-term stability, may not be robust. An improved understanding of this thermochemical structure, and its relationship to plume generation, will come from advances in lower mantle composition and mineral physics (e.g. Mattern et al., 2005; Irifune et al., 2010), in combination with geodynamic modeling of global geophysical observables (Forte & Mitrovica, 2001), seismic imaging of the steepness of LLSVPs (Ni et al., 2002; To et al., 2005; He & Wen, 2009) and plume conduits (Montelli et al., 2006; Boschi et al., 2007), geochemistry of LIPs and ocean island basalts, (Sobolev et al., 2011; Chabaux, 1994; Bourdon et al., 2006) and the incorporation of plate motions into thermochemical convection models (McNamara & Zhong, 2005; Zhang et al., 2010; Steinberger & Torsvik, 2012; Bower et al., 2013).
This thesis has highlighted the interactions between solid Earth dynamics and the paleoclimate record over a wide range of time scales and emphasizes the importance of treating the Earth’s internal structure in its full complexity.

7.1 From LGM to present

In Chapter 2, we showed results that establish a consistency between estimates of excess ice volume at LGM based on far field sea level records. This work raises two important issues. First, to what extent do lateral variations in mantle viscosity associated with subducted slabs impact the predictions of other important observables related to GIA? To answer this question, we are currently working to improve the treatment of complex slab structures in our finite volume GIA simulations using local grid refinement. This effort will allow for more accurate modeling of globally distributed slab geometries as inferred from earthquake locations and seismic tomography (see Fig. 7.1). Second, the
Figure 7.1: Improved treatment of subducting slabs in our GIA analysis. a. Locations of the Sunda Shelf and Bonaparte Gulf sites, where records exist that constrain sea level lowstands at LGM (Yokoyama et al., 2000; Hanebuth et al., 2009). The red zone denotes the downgoing slab within the mantle associated with the Sumatra subduction zone. b. An improved numerical meshing scheme that uses local refinement to more accurately capture the morphology of the slab.

Consensus that global mean sea level at LGM was approx. ~130 m introduces a fundamental ‘missing ice’ problem since recent inferences of the excess volume of the Antarctic Ice Sheet at LGM have reduced it from ~20 m ESL to less than 10 m ESL (Whitehouse et al., 2012). This reduction, coupled with the work described in Chapter 2, indicates that current models of global ice inventories at the LGM fall short by ~30 m ESL. Where is this missing ice? We hope to answer this question by constructing an ice history and Earth model combination that yields a GIA signal that satisfies the existing, global database of sea level records while remaining consistent with geological constraints on individual ice sheets and glaciers. This effort is not only important for our understanding of ice age climate, but also for accurately estimating the ongoing GIA signal in modern datasets - for example tide gauge and satellite altimetric records of recent sea-level change - that record the Earth’s response to global warming.

A different research direction aimed at constraining the evolution of individual ice sheets since
LGM involves examining the evolution of continental drainage patterns driven by the GIA process from the LGM to present-day. As an example, recent work by Wickert et al. (2013) has shown that GIA led to major changes in the Mississippi drainage basin since the LGM. However, no previous GIA study of this kind has incorporated a self-consistent treatment of proglacial lakes into post-glacial sea level calculations. We have developed a theory for this extension that incorporates the growth and discharge of proglacial lakes self-consistently as the ice sheet retreats and the crust deforms. We are currently working on combining these model simulations with shoreline data from Lake Agassiz and Lake Superior (e.g. Breckenridge, 2015). In future work, we plan to couple this model to a hydrology model in order to investigate the evolution of drainage patterns and lakes that are distant from the ice margin and to constrain these simulations using isotopic records at drainage outlets.

One of the lakes that formed downstream of the drainage networks associated with the Laurentide Ice Sheet was Lake Bonneville, which reached its maximum extent at ~18.5 ka and was approximately the size of present-day Lake Michigan (Oviatt, 2015). Preserved shorelines reflect a deformation associated with the rebound due to water discharge, and they are also tilted due to the evolution of the peripheral bulge of the Laurentide Ice Sheet (e.g. Bills & May, 1987). This down to the northeast tilt indicates that the lake formed on the trailing edge of the peripheral bulge and provides an important constraint on both shallow and deep mantle viscosity. We will combine updated measurements of the lake level elevations (Chen & Maloof, 2016) with gravitationally self-consistent sea level modeling to estimate the lithospheric thickness and viscosity structure in this area. Preliminary calculations indicate that fitting the paleo lake level record requires an upper mantle that is significantly weaker than a global average, a finding consistent with earlier work (Bills & May, 1987) and the geological setting of the lake in the Basin and Range region.
7.2 The Last Interglacial

As we have discussed, the LIG is a key time period for efforts to improve our understanding of ice sheet stability during an extended warm interval. Estimates of peak eustatic sea level during the LIG range from 6-9 m (Kopp et al., 2009; Dutton & Lambeck, 2012; Hay et al., 2014), but the relative timing and sources of melt contributions are issues of active debate. In Chapter 4 we showed that dynamic topography may have perturbed the elevation of LIG sea level markers by several meters and this signal represents a heretofore neglected source of uncertainty in inferences of ice volumes. While better observational constraints on dynamic topography will be important to further address this issue, improving models of GIA is also crucial. GIA corrections are generally based on simulations over the glacial cycle and, in the case of LIG studies, the last two glacial cycles. While it is widely recognized the latter predictions are sensitive to the adopted Earth structure, they are also sensitive to the uncertain ice history prior to the LIG, and in particular the penultimate deglacial (Lambeck et al., 2006). We are currently working on exploring this sensitivity and its implication for inferences of ice volumes.

As an example, we tested the impact of changing the ESL history on predictions of sea level highstands during the LIG. The histories we considered all adopt the ICE-6G ice model history for the LGM-present (Argus et al., 2014; Peltier et al., 2015). Prior to the LGM, the ice models are scaled to an oxygen isotope curve so that the ice sheet configuration for a given $\delta^{18}O$ value prior to the LGM is identical to the configuration when the same $\delta^{18}O$ value is obtained in the post-LGM period. We construct three ice histories in the following manner: (i) From LIG to LGM we use the oxygen curve of Waelbroeck et al. (2002) and then repeat this glacial cycle over the three prior cycles; (ii) from 400 ka to the LGM we use the oxygen isotope curve of Waelbroeck et al. (2002); and (iii) from 400 ka to the LGM we use the oxygen isotope curve of Shakun et al. (2015). All models are constructed so that the LIG extends from 129 - 116 ka (Dutton et al., 2015) and eustatic sea level
Figure 7.2: LIG relative sea level predictions for different ESL curves. a. Eustatic sea level curves for three different ice models described in the text: blue line, model (i); green line, model (ii); yellow line, model (iii). The model was run for four glacial cycles (back to approx. 400 ka), not only the two glacial cycles shown here. b. Prediction of relative sea level for (b) Bermuda and (c) the Cape Range, western Australia, based on the three different ice models and an Earth model with a lithospheric thickness of 96 km, an upper mantle viscosity of $0.5 \times 10^{21}$ Pa s and a lower mantle viscosity of $5 \times 10^{21}$ Pa s. The grey region in all frames marks the LIG.

during this period is identical to the present day (Fig. 7.2a).

We find that the ESL curves for models (ii) and (iii) exhibit a faster deglaciation from the penultimate glacial maximum (PGM) to the LIG, which leads to a delayed adjustment and significantly impacts relative sea level predictions during the interglacial (see Fig 7.2b and c). Predictions for Bermuda (Fig 7.2b) experience a sea level rise during the LIG since the site is located on the subsiding peripheral bulge of the Laurentide ice sheet. Ice model (i) yields a higher relative sea level due to its longer deglaciation phase, which allows for more pre-LIG isostatic adjustment. Sites in the far field, like the Cape Range in Australia, experience a sea level fall due to equatorial syphoning and continental levering. Similar to the near field case, model (i) exhibits a lower relative sea level prediction that reflects a more extensive level of adjustment compared to models (ii) and (iii).

Next, we investigated the sensitive of the predictions to a change in the ice distribution prior to the LIG. It has been suggested, for example, that the Fennoscandian Ice Sheet was more extensive at
the PGM (Svendsen et al., 2004). In this case, we considered three different ice sheet configurations at the PGM: (i) a geometry identical to the LGM configuration in ICE-6G; (ii) a Fennoscandian Ice Sheet with volume comparable to the Laurentide Ice Sheet, as modeled by Lambeck et al. (2006); and (iii) a Fennoscandian Ice Sheet with a volume greater than the Laurentide Ice Sheet, as modeled by Colleoni et al. (2016) (Fig. 7.3a). All models have the same total ice volume at the PGM. We find that for sites in the near field of the ice sheet, such as Bermuda, the ice sheet configuration has a major effect on the GIA prediction for the LIG (see Fig. 7.3b). A larger Laurentide Ice Sheet predicts that Bermuda will have a greater level of isostatic disequilibrium and a more rapid sea level rise during the LIG (green line in Fig. 7.3b). Far field sites show less sensitivity (Fig. 7.3c) because their deformation is largely driven by the changing ocean load which is nearly identical in the three simulations. In future work, we will explore a broader range of ice and Earth models and combine these results with relative sea level data to improve estimates of excess melt during the LIG.

The solid Earth adds another complication to the analysis of sea level records that has yet to be addressed in a rigorous manner. Many datasets that constrain sea level across the last glacial cycle (and, indeed, earlier cycles) are located close to tectonic plate boundaries (e.g., Barbados, the Huon Peninsula, etc). This is advantageous because many such sites have been uplifted over the past hundreds of thousands of years and, as a consequence, the geological record of sea level change has been well preserved. However, sites near a plate boundary are subject to long-term tectonic uplift, which must be accounted for. In recent work (Creveling et al., 2015), we have shown that nearly all previous estimates of tectonic uplift at a specific site, based on the current elevation of the LIG highstand at the same site, introduce significant errors in ice volume reconstructions. One promising avenue for future research will be to estimate tectonic uplift by modeling subduction zone dynamics at locations that have a coherent sea level record and good constraints on the geometry of subduction, including earthquake slip distributions and paleoseismicity. Disentangling the tectonic signal from the local sea level record would represent a major advance in ice age research.
Figure 7.3: Relative sea level predictions across the LIG for the three different models of ice sheet configuration during the PGM described in the text. a. green, model (i); red, model (ii); purple, model (iii). The numbers provide the sea level equivalent ice volume in the Fennoscandian and Laurentide Ice Sheets in each case. b. Prediction of relative sea level in Bermuda based on the three different ice sheet configurations during the PGM and an ESL curve (black line) based on Waelbroeck et al. (2002) (the model associated with the green curve in this frame is equivalent to the model shown by the green curve in Fig. 7.2). The Earth model used in the simulations has a lithospheric thickness of 96 km, an upper mantle viscosity of $0.5 \times 10^{21}$ Pa s, and a lower mantle viscosity of $5 \times 10^{21}$ Pa s (as in Fig. 7.2). c. Prediction of relative sea level in the Cape Range in western Australia based on the ice sheet configurations and Earth model used in panel b.
7.3 The mid-Pliocene warm period

The mid-Pliocene warm period is, as discussed earlier in the thesis, a time in the Earth’s history during which atmospheric CO$_2$ as well as temperatures were elevated relative to today. In Chapter 5 we showed that on time scales of $10^5$ to $10^6$ yrs dynamic topography may have significant impact on the climate system. In particular, dynamic topography can significantly perturb the elevation of paleoshorelines (Rowley et al., 2013; Rovere et al., 2014) and bedrock beneath marine-based ice sheets (e.g., the Wilkes Basin in East Antarctica). In our model runs we explored the sensitivity of our results to various model inputs and demonstrated that while our conclusions in regards to the ice cover in the Wilkes Basin appears to be robust, the exact magnitude of the dynamic perturbation to the bedrock elevation remains uncertain.

In future work, we plan to improve these models in several important ways. First, we will incorporate higher resolution (and local) shear wave tomography models for Antarctica. Second, the viscosity variability in our models is derived from global inversions of geodynamic data and a simplified temperature scaling. A comparison of our existing results with additional simulations that adopt viscosity models that have focused on the Antarctic region (Kaufmann et al., 2005; van der Wal et al., 2015) may narrow this source of uncertainty. Furthermore, there are additional data sets that we can use to validate our model results; these include shear wave splitting data that provide information on the direction of mantle flow, and heat flow measurements, which reflect variability in radial temperature gradients. With these model improvements, we hope to extend our work to explore the impact of dynamic topography on bedrock elevation and ice sheet dynamics at the time of the Antarctic inception and its subsequent evolution.

The glaciation of the northern hemisphere initiated around 3 Ma and was characterized by extensive geographic cover and significant ice age cycles by ~2.4 Ma (Raymo, 1994; Shackleton et al., 1984). Inception of the Laurentide Ice Sheet is likely to have occurred in Baffin Island (Dyke, 2009).
While variations in bedrock topography can, as we have shown, impact ice sheet stability, such variations can also lead to favorable conditions for glacial inception through uplift. Steinberger et al. (2015) have shown that plate motion, true polar wander and dynamic topography changes in Greenland lead to favorable conditions for the inception of ice cover in this region. Daradich et al. (2016) investigated the role of plate motion and true polar wander in the inception of the Laurentide Ice Sheet by quantifying glacial mass balance through positive degree days. In a companion to this latter study, we are exploring the role of dynamic uplift in enabling glacial inception in Baffin Island. Preliminary calculations show that the northern part of Baffin Island has been uplifting over the last few million years, supporting the idea that dynamic topography changes have favored northern hemisphere glacial inception. Further analysis of the uncertainties and a quantification of the resulting mass balance will be necessary to strengthen this argument.

Rowley et al. (2013) have demonstrated that dynamic topography has played an important role in warping the Orangeburg and correlative scarps along the Atlantic coastal plain of the US. However, we have shown that their prediction does not fully explain the present-day elevation of the paleoshoreline (Rovere et al., 2015). The residual misfit may be due to mechanisms that are not captured in existing simulations, e.g., deformation associated with sediment transport, but it is also likely that the coarse resolution and uncertainties associated with mantle buoyancy and viscosity fields adopted in the convection simulations may play a significant role. To explore the latter issue, we are conducting numerical simulations of dynamic topography that span the full range of uncertainty in the input parameters. In addition, an increasing number of Pliocene shorelines are being mapped (Rovere et al., 2014, 2016a), and these will provide a larger, global catalogue of data to constrain our simulations.
Large igneous provinces and mass extinctions

The Plio-Pleistocene sea level record provides invaluable constraints on the Earth’s internal structure and climatic history. However, to improve our understanding of the long-term evolution of the mantle and the Earth’s response to extreme climatic events, we must look to earlier times in Earth’s history. In Chapter 6 we assessed the claim that the location of Large Igneous Provinces (LIPs), which have served as a proxy for the location of plume trajectories, argue for stable thermochemical piles in the lowermost mantle. We argued that the sampling by existing LIPs is too sparse to draw such a conclusion.

In addition to carrying information about the internal structure of our planet, LIPs have also been suggested as a cause of widespread environmental change, including mass extinctions. While recent improvements in dating of LIPs indicate that extinctions post date LIP eruptions, a complete causal chain of events including kill mechanism is not yet established. In an ongoing study, we are investigating the role of sea level changes in the Triassic-Jurassic mass extinction, which was coincident to the emplacement of the Central Atlantic Magmatic Province (Blackburn et al., 2013). Sea level excursions at this time are particularly well documented and often indicate a rapid sea level fall followed by a sea level rise (Hallam & Wignall, 1999). In our study we are calculating sea level changes associated with three processes: (i) Uplift due to the ascending plume associated with the CAMP eruption; (ii) loading and flexure of the lithosphere associated with the emplaced magma; and (iii) sea level changes due to contemporaneous glaciation. All mechanisms are modeled on a reconstructed paleo-topography and combine a gravitationally self-consistent sea level calculation that accounts for proper loading (or unloading) of the displaced water, shoreline migration and rotational effects. Thus far, the research has consisted of a validation of field evidence for sea level changes and a comparison of these data with our numerical predictions. In future work we intend to broaden the study by investigating the periodicity of major extinctions through the Phanerozoic.
and, in particular, exploring whether these events correlate with the time scale of internal variability and plume formation within the convecting mantle of the Earth.
A

Expressions for $\Delta S_L$ in the case of spherically symmetric Earth models

This appendix derives the relevant equations, based on Love number theory, for a prediction of the change in global sea level due to the surface mass loading of a spherically symmetric Earth model. The derivation expands the theory described in Kendall et al. (2005) and Dalca et al. (2013) to include dynamic topography and prescribed perturbations in the geoid. However, we only treat the isostatic equilibrium response, i.e., we assume that the viscous response has fully decayed. We begin with the case of a non-rotating Earth.

A.1 Non-rotating Earth model

The deformational and gravitational response of a spherically symmetric Earth model to a surface mass load can be written as a space-time convolution of the appropriate Green’s function with the
surface load. In particular, the response $\Delta \chi$ can be expressed as:

$$\Delta \chi = \int_{-\infty}^{t} \int_{\Omega} \Delta L(\theta', \varphi', t') \cdot GF(\gamma, t - t') d\Omega' dt',$$

(A.1)

where $GF$ denotes the Green’s function, $\theta$ and $\varphi$ represent the colatitude and east longitude, and $\gamma$ is the angle between the observation point ($\theta$, $\varphi$) and the load point ($\theta'$, $\varphi'$). This angle can be calculated as:

$$\cos(\gamma) = \cos(\theta) \cos(\theta') + \sin(\theta) \sin(\theta') \cos(\varphi - \varphi').$$

(A.2)

If we denote the Green’s function for the radial displacement of the solid surface and the gravitational potential perturbation by $\Delta G$ and $\Delta \phi$, respectively, then these responses can be expressed as:

$$\Delta G = \int_{-\infty}^{t} \int_{\Omega} \Delta L(\theta', \varphi', t') \cdot \frac{\phi(\gamma, t - t')}{g} d\Omega' dt',$$

(A.3)

$$\Delta R = \int_{-\infty}^{t} \int_{\Omega} \Delta L(\theta', \varphi', t') \cdot \Gamma(\gamma, t - t') d\Omega' dt',$$

(A.4)

where $\Delta L$ is given by equation (3.12). Using these expressions in equation (3.16) gives

$$\Delta S L_j = \int_{-\infty}^{t} \int_{\Omega} \Delta L(\theta', \varphi', t') \cdot \left[ \frac{\phi(\gamma, t - t')}{g} - \Gamma(\gamma, t - t') \right] d\Omega' dt'$$

$$- \Delta H_j - \Delta DT_{A,j} - \Delta I_j + \Delta \phi_j^{inc}.$$  

(A.5)

The Green’s functions in the above equations can be written in terms of the viscoelastic $k$ and $h$ Love numbers at spherical harmonic degree $l$ (Peltier, 1974; Tromp & Mitrovica, 1999):
\[ k_l(t) = k_l^E \delta(t) + \sum_{k=1}^{K} r_k^l e^{-s_k^l t} \]  \hspace{1cm} (A.6)

\[ h_l(t) = h_l^E \delta(t) + \sum_{k=1}^{K} r_k^l e^{-s_k^l t}, \]  \hspace{1cm} (A.7)

where the response is decomposed into an instantaneous elastic contribution and a discrete set of exponential decays.

As we have noted, our calculations will assume that the viscous response has fully decayed. Our expression for the perturbation in global sea level can be simplified by expressing the surface mass load as having a single Heaviside step function time dependence, and performing the time convolution between the step function and the viscoelastic Green’s functions analytically (where the latter is expressed in terms of the viscoelastic Love numbers). At infinite time, i.e., for \( t \gg 1/s_k^l \), this convolution yields the following time-independent Green’s functions:

\[ \phi(\gamma) = \frac{a g}{M_e} \sum_{l=0}^{\infty} \left[ 1 + k_l^f \right] P_l(\cos \gamma) \]  \hspace{1cm} (A.8)

\[ \Gamma(\gamma) = \frac{a}{M_e} \sum_{l=0}^{\infty} h_l^f P_l(\cos \gamma), \]  \hspace{1cm} (A.9)

where \( a \) and \( M_e \) are the radius and mass of the Earth, respectively, and \( P_l \) is the Legendre polynomial at spherical harmonic degree \( l \). In these expressions, \( h_l^f \) and \( k_l^f \) are the so-called fluid Love numbers that have the form (Peltier, 1974, 1976):

\[ k_l^f = k_l^E + \sum_{k=1}^{K} r_k^l e^{-s_k^l t} \]  \hspace{1cm} (A.10)
Using these equilibrium forms, and the expression (3.12) for the change in surface mass load, yields the following version of equation (A.5) for the change in global sea level:

\[
\Delta SL = a \frac{E_l}{M_e} \sum_{l=0}^{\infty} E_l \int \int_{\Omega} \left[ \rho_W \Delta S(\theta', \varphi') + \rho_I \Delta I(\theta', \varphi') + \rho_H \Delta H(\theta', \varphi') \right] \cdot P_l(\cos \gamma) d\Omega' - \Delta H - \Delta I - \Delta DT_A + \Delta \mathcal{G}^{mc}
\]

where

\[
E_l = 1 + k_l^f - h_l^f.
\]

Note that under the assumption of isostatic equilibrium, computing the perturbation in sea level between any two time steps requires only the total difference in the load across the interval and thus, with no loss of generality, we consider only a single time step and drop the subscript \(j\).

We can simplify the spatial convolution in equation (A.12) by introducing spherical harmonic basis functions. A general spherical harmonic decomposition of a scalar field \(\chi\) can be written as:

\[
\chi(\theta, \varphi) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} \chi_{lm} Y_{lm}(\theta, \varphi)
\]

where the basis functions are normalized in the following way

\[
\int \int_{\Omega} Y_{l' m'}(\theta, \varphi) Y_{lm}^* (\theta, \varphi) \sin \theta d\theta d\varphi = 4\pi \delta_{l'l} \delta_{m'm}.
\]
following relationship holds (Mitrovica & Peltier, 1991)

\[
\int\int_\Omega \chi(\theta', \varphi') P_l(\cos \gamma) d\Omega' = \frac{4\pi a}{2l+1} \sum_{m=-l}^l \chi_{lm} Y_{lm}(\theta, \varphi)
\]  \hspace{0.5cm} (A.16)

We may now use the spherical harmonic formulation from (A.14) and (A.16) to rewrite (A.12):

\[
\Delta S L = \sum_{lm} T_l E_l [\rho_W \Delta S_{l,m} + \rho_I \Delta I_{l,m} + \rho_H \Delta H_{l,m}] \cdot Y_{l,m}(\theta, \varphi) - \sum_{lm} \Delta H_{l,m} Y_{l,m}(\theta, \varphi) \\
- \sum_{lm} \Delta I_{l,m} Y_{l,m}(\theta, \varphi) - \sum_{lm} \Delta DT_{l;l,m} Y_{l,m}(\theta, \varphi) + \sum_{lm} \Delta G_{l;m}^{mc} Y_{l,m}(\theta, \varphi)
\]  \hspace{0.5cm} (A.17)

where the subscript \(l, m\) refers to a spherical harmonic coefficient of degree \(l\) and order \(m\),

\[
T_l = \frac{4\pi a^3}{(2l+1)M_e},
\]  \hspace{0.5cm} (A.18)

and we adopt the short form:

\[
\sum_{lm} = \sum_{l=0}^{\infty} \sum_{m=-l}^l.
\]  \hspace{0.5cm} (A.19)

The fully spectral form of equation (A.17) is

\[
\Delta S L_{l,m} = T_l E_l [\rho_W \Delta S_{l,m} + \rho_I \Delta I_{l,m} + \rho_H \Delta H_{l,m}] \\
- \Delta H_{l,m} - \Delta I_{l,m} - \Delta DT_{l;l,m} + \Delta G_{l;m}^{mc}.
\]  \hspace{0.5cm} (A.20)

This equation describes the spatially variable component to sea level change. The spatially invariant component that follows from volume conservation (3.19) can also be written in terms of a spherical
harmonic expansion:

$$\frac{\Delta \Phi}{g} = \frac{1}{C_{0,0}} \left( -\frac{\rho_l}{\rho_W} \Delta I_{0,0} - RO_{0,0} + TO_{0,0} \right). \quad (A.21)$$

Here we have made use of the following property

$$\iint_{\Omega} \chi(\theta, \varphi) d\Omega = 4\pi \chi_{00}. \quad (A.22)$$

We also used the following projections

$$RO = \Delta SL \cdot C = \sum_{lm} RO_{l,m} Y_{l,m}(\theta, \varphi) \quad (A.23)$$

and

$$TO = T_0[C - C_0] = \sum_{lm} TO_{l,m} Y_{l,m}(\theta, \varphi), \quad (A.24)$$

where $C$ represents the final ocean function.

### A.2 Rotating Earth model

Changes in the Earth’s surface mass (ice, ocean, sediment) load and convective motions in the interior perturb the inertia tensor of the Earth, and thus the magnitude and orientation of the Earth’s rotation axis. A change in the rotation vector, and the perturbation in the centrifugal potential that this change implies, impacts global sea level (Han & Wahr, 1989; Milne & Mitrovica, 1996, 1998; Mound & Mitrovica, 1998). In this section we review how this rotational feedback into global sea level can be incorporated into the Love number theory outline in section A.1.

Convective perturbations to Earth rotation can be computed from the same viscous flow models of mantle convection used to compute dynamic topography and geoid anomalies, and a var-
ety of approximate methods have been derived for this purpose (Ricard et al., 1993; Steinberger & O’Connell, 2002; Chan et al., 2011). In ice age research, considerations of rotational stability deepen the integral nature of the sea level calculation since perturbations in the rotation vector are impacted by sea level changes and act, in turn, to perturb sea level. This rotational feedback can be included by augmenting equation (A.20) to include a term associated with the sea level response to the perturbed centrifugal potential (Milne & Mitrovica, 1998). In the present case, this additional term will include a signal associated with convection, which is prescribed at the outset, and a surface mass loading term that will be successively improved through the iteration procedure described above. The latter requires a method for predicting the perturbation to the rotation vector associated with a surface mass loading (Mitrovica et al., 2005). To be consistent with the assumption applied in section A.1, the ice age component of this calculation would treat the case of isostatic equilibrium.

The time-varying centrifugal potential $\Lambda(\theta, \varphi, t)$ can be decomposed into an initial value prior to the onset of loading and a perturbation from this state due to loading:

$$\Lambda(\theta, \varphi, t) = \Lambda(\theta, \varphi, t_0) + \Delta \Lambda(\theta, \varphi, t).$$  \hfill (A.25)

The perturbation to the centrifugal potential, also known as the rotational driving potential, may be described as the difference between two ellipsoidal forms with distinct amplitude and orientation, and it will involve only spherical harmonic coefficients of degree zero and two. This term can therefore be written as (e.g. Milne & Mitrovica, 1996):

$$\Delta \Lambda(\theta, \varphi, t) = \Delta \Lambda_{0,0} Y_{0,0}(\theta, \varphi) + \sum_{m=-2}^{2} \Delta \Lambda_{2,m} Y_{2,m}(\theta, \varphi).$$  \hfill (A.26)

In analogy to the case of surface mass loading, fluid Love numbers exist that describe the deformation of the solid surface and the gravitational equipotential surface due to a changing rotational driving potential. These have the form (Peltier, 1974, 1976):
\[ k_i^{f.T} = k_i^{T.E} + \sum_{k=1}^{K} r_{k.i}^{T.E} / s_k \]

\[ h_i^{f.T} = h_i^{T.E} + \sum_{k=1}^{K} r_{k.i}^{T.E} / s_k. \]

Using these expressions, (A.20) is revised in the following manner to include rotational feedback:

\[
\Delta S L_{l,m} = T_l E_l [\rho_W \Delta S_{l,m} + \rho_I \Delta I_{l,m} + \rho_H \Delta H_{l,m}] + \frac{1}{g} E_i^T \Delta \Lambda_{l,m} \]

\[-\Delta H_{l,m} - \Delta I_{l,m} - \Delta D T_{\Lambda_{l,m}} + \Delta \rho_{m,\epsilon} \]

\[ (A.29) \]

where

\[ E_i^T = 1 + k_i^{f.T} - h_i^{f.T} \]

\[ (A.30) \]

All other equations described in section A.1 remain unchanged.
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