Designing for and Understanding the Adult Learner in a Mathematics Course

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Designing for and Understanding the Adult Learner in a Mathematics Course

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A Thesis in the Field of Mathematics for Teaching
for the Degree of Master of Liberal Arts in Extension Studies

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Abstract

Currently, there is ample research on mathematics education in formal settings for K-16 populations, but very little research in the domain of adult education. There is very little to no extant research on how or why adults may pursue a personal interest course in mathematics. This thesis seeks to explore this gap as well as to investigate how adult learners engage in and make sense of mathematical ideas and to understand the characteristics of adults who pursue a personal interest class in mathematics. In the present study, a math course was designed for adult learners that drew upon the theories of andragogy and how people learn. The study investigated the learning experiences and the characteristics of the three adults enrolled in the course. To gain an in-depth understanding of their experiences, qualitative methods such as observations, interviews, and document analysis were utilized.

This study finds that adults leverage their experiences and prior knowledge when making meaning of mathematical ideas, and that they can value and enjoy mathematical thinking that is based on math’s practical as well as its recreational purposes. These findings can inform the design and implementation of mathematics classes for adult learners.
Acknowledgements

I am incredibly grateful to my thesis advisor, Andy Engelward, for his guidance, support, and discerning eye throughout this thesis project. I am also grateful for having had the opportunity to work with an amazing group of students during this project; it was a joy and a privilege to learn and to be curious with them.

I would be remiss if I didn’t convey a special thanks to my friend Sameer Sugwekar for cheering me on during this project. His jokes, encouragement, and words of wisdom provided me with both inspiration and levity.

Finally, I would like to express my heartfelt gratitude to my mother, Nazleen, my sister, Zainab, and my brother, Rahim, for their never ending encouragement and support in everything that I do. Thank you.
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Chapter 1 Introduction

Adult education is an important form of learning in the United States. According to a survey conducted by the *National Household Education Surveys: Participation in Adult Education and Lifelong Learning* (Kim, et al., 2004) by the National Center for Education Statistics, 46% or 92 million adults participated in some type of adult education activity in the United States in 2001. The survey defines seven types of adult education activities: English as Second Language, Adult Basic Education (basic math and reading skills), college or university programs, vocational or technical diploma, apprentice/on-the-job training, work-related, and personal interest courses. According to the survey, 21% of the adults who participated in an education activity did so through personal interest courses; this was second in ranking relative to the other adult education activities. The number one form of adult education was work-related courses, where 30% of the adults had participated in this type of education in 2001. Given that 1 in 5 adults who participated in an education activity, did so through a personal interest course, it is important to note that the pursuit of learning goes beyond career-related and formal education goals.

There are varying definitions of what is considered to be adult education. One definition of adult learning includes activities that are related to training or education for professional development purposes (Cervero, 1989). Another definition of adult learning includes activities that are voluntary in nature and that can occur throughout the lifespan (Belanger and Tuijnman, 1997). The National Center for Education Statistics describes adult education as including opportunities that encompass both voluntary and continuous
learning courses as well as those that are related to professional development and growth (Kim, et al., 2004). Thus adult education encompasses a broad range of learning opportunities that can occur throughout a person’s lifetime and that are voluntary in nature.

The NCSES survey describes the characteristics of the adults who participate in the various adult education activities. It includes demographic variables such as age, sex, race/ethnicity, prior educational status, marital status, and employment/occupation. This information can help us understand the segment of the population that is likely to pursue adult education. However, it is important to note that the survey does not describe the motivations or characteristics that drive an adult to pursue learning opportunities. The extant research on adult learning focuses on theories about why adults may pursue learning opportunities. The research has mostly focused on disciplinary specific knowledge in the workplace. For example, Fitzsimons (2005) discusses mathematical knowledge such as numeracy and how these concepts may manifest themselves in specific jobs. Another focus on the research has been to understand general characteristics of adult learners and theories of adult learning that are independent of the context (Knowles, 1972; Merriam, 2001).

Given the existing research in adult education, there is an opportunity to improve our understanding of adult learners and learning in a personal interest course in mathematics. The present study will be investigating two sets of questions: 1) how might adult learners engage in and make sense of the topics and activities in a math class? and 2) what are some characteristics of adult learners who pursue a personal interest course in mathematics?
Chapter 2 will describe the theories which served as a foundation for the design of the mathematics course. Specifically, it will draw upon one of the theories of adult learning, andragogy, and what is known about how people learn in classroom settings. The chapter will then describe how these theories and research informed the design of the class and its two key design principles: 1) leveraging everyday experiences to promote mathematical understanding, and 2) implementing sense-making opportunities in the classroom. The chapter will then lay out the conceptual framework and research questions addressed in this study.

Chapter 3 will go into more depth about the logistical and topical details of the course. Specifically, it will describe how the participants were recruited as well as discussing the design of the course. This chapter will also describe how the units and activities were organized and selected. Finally, it will exhibit the data that was collected throughout the mathematics course.

Chapter 4 will discuss the analyses that were performed with the data and then present the overall findings from the study. The findings will be presented in the form of themes that were observed across cases of the three individual students. Chapter 5 will then discuss these findings in more detail, with an objective of comparing and contrasting participants’ experiences in the math course.

The final chapter will put forth the instructor’s experiences with and reflections about the class. This will then lead to the instructor making recommendations about pedagogical approaches that one needs to take into account when designing a learning environment for adults.
Chapter 2  Theories of Adult Learning and its Implications for Designing a Mathematics Class for Adults

This chapter will provide an overview about what we know about adult learning in mathematics and how this can inform the design of a mathematics class for adults. First, it will describe one of the foundational theories of adult learning, andragogy. Second, it will address what is known about adult learning specifically in the domain of mathematics. Third, it will describe how andragogy and adult learning in mathematics informed the design of the mathematics class that was created as part of this study. Lastly, it will lay out the research questions and the conceptual framework for this study.

Adult Learning: Brief History and Assumptions

The history of adult education in the United States can be traced back to as early as the 1600s, when young adults engaged in apprenticeships to learn trades and crafts as well as reading, writing, and mathematics (Sticht, 2002). In the 1800s, more voluntary associations emerged that promoted adult learning. One of these associations was the Chautauqua Institution in western New York which sponsored education programs across the United States and in 1878 led to the “first integrated core program of adult education organized in this country on a national scale” (Knowles, 1977, p.37). The growth of adult education programs continued in the twentieth century, with more colleges, businesses, government agencies, and voluntary associations joining in the adult education effort.
Knowles (1977) characterizes this time period as the adult education movement in the United States. With this growth of adult education, there became a need to understand adult learning in a formal way. In 1972, Knowles put forth a theory of adult learning, andragogy, that highlighted the differences between adults and children as learners. Pedagogy is defined as the art and science of teaching children, whereas andragogy is the science of helping adults learn. He noted that people who have been working in the field of adult education, where often there is no degree and participation is voluntary, needed to use a new framework to understand adult learners that could accommodate their learning needs. This new framework could then be used to design courses appropriately for adult learners. In his theory of andragogy, Knowles (1972, 1973) and Merriam (2001) describe five main assumptions:

1. Changes in self-concept: The adult’s need to be self-directed in their learning experiences.
2. Experience: An adult has accumulated a reservoir of experiences that can be drawn upon for future learning.
3. Readiness to Learn: An adult’s approach to learning can be dependent upon their changing social roles (e.g., worker, spouse, organizational members, etc.).
4. Orientation to Learning: An adult views learning from a problem-centered approach as opposed to a subject-centered one. In other words, there is an immediacy in the adult’s application of their learning.
5. Motivation to Learn: An adult’s motivation is derived from internal instead of external factors.
Although andragogy serves as a foundation to understand adult learners, there are still criticisms about this theory. Merriam (2001) describes two of these: first, is the question of whether it is specifically a theory of adult learning and second, the issue of whether the assumptions apply to all adults at all times. In response to the first criticism, Knowles has acknowledged that andragogy is not a theory about learning; rather it is a “model of assumptions about learning or a conceptual framework that serves as a basis for an emergent theory” (1989, p. 112). Thus, andragogy can serve as a foundational framework for understanding the characteristics of adult learners. In response to the second criticism, Knowles began to represent pedagogy and andragogy on a continuum, where one end is more teacher-directed and the other end is student-directed (Knowles, 1984). Thus, adults can also be dependent upon a teacher for direction, depending on their level of knowledge and experience.

Andragogy can be a useful framework to understand the adult learner, however, it cannot fully help us understand why an adult may pursue a personal interest course in mathematics and what values they may hold about learning math. For this, one needs to understand what is known about adult learning in a mathematics context.

Adult Learning in Mathematics

Currently, our understanding of the adult learner in mathematics environments is limited. The studies that have been conducted in this area include topics such as what is appropriate mathematical training for vocational jobs, along with factors to consider when designing a mathematics learning environment for adults for this purpose. Gail Fitzsimmons, in her book “What Counts as Mathematics? Technologies of Power in Adult and Vocational Education,” studied vocational educational training in Australia.
Specifically, Fitzsimmons studied the intersection of mathematics, vocational education, and workplace issues from social, political, and educational perspectives, with the question of what counts as mathematics as the central theme of her book. Other studies have looked at the intersection of mathematics and adult education (Benn, 1997). Benn explores factors related to adults learning mathematics such as how a person’s level of confidence in and knowledge of mathematics can affect the individual and society, an analysis of the mathematical skills that people should develop, and the importance of mathematics for full participation in a democracy. Thus, there are many factors to take into consideration when designing a mathematics curriculum for adults:

Teaching adults mathematics in the classroom as if it is objective and context-free without unraveling these complexities of meaning or exposing the social structure of each individual’s mathematics is unlikely to lead to anything but superficial success. (Benn, p. 61)

**Design Framework for a Mathematics Class for Adults**

There are several perspectives to consider when designing learning environments in math for adult learners. Two main goals for the learning environment developed for this study, were to create it in such a way so that it valued learners’ experiences and knowledge, as well as enabling them to become active sense-makers of information. Thus, the environment was designed to be both learner-centered and knowledge-centered.

The National Research Council’s Report entitled *How People Learn* (Bransford, Brown, and Cocking, 1999) describes learner-centered environments as follows:

…environments that pay careful attention to the knowledge, skills, attitudes, and beliefs that learners bring to the educational setting… Teachers who are learner centered recognize the importance of building on the conceptual and cultural knowledge that students bring with them to the classroom. (Bransford, et al., 1999, p. 133-134)
A knowledge-centered environment complements a learner-centered environment, such that it considers both the knowledge the learners bring to bear on the learning situation and implements activities that allow them to understand the content (Bransford, et al., 1999). With this in mind, the learning environment was designed with two design principles:

Design Principle 1: Leveraging adults’ everyday experiences to promote mathematical understanding

Design Principle 2: Implementing sense-making opportunities in the classroom

These design principles were selected because there are theories and/or evidence to support their effectiveness for learners and specifically, in a mathematics learning context. The first design principle draws upon the theory of andragogy presented by Knowles. Andragogy, the art and science of helping adults learn, has several assumptions about how best to help adults learn. One of these principles is the role of experience. According to Knowles (1972, p. 35), “this assumption is that as an individual matures he accumulates an expanding reservoir of experience that causes him to become an increasingly rich resource for learning, and at the same time provides him with a broad base with which to relate new learnings.” Thus, by leveraging adults’ everyday experience, there is an opportunity for them to build and expand on what they already know. In the math class, several activities were designed to elicit adults’ everyday experiences. For example, adults were asked to give examples of when and why they perform mental math in their everyday life. In another example, adults were asked about where they see fractions in their everyday life and how they make sense of them. For a complete list of activities and resources, please see Appendix 1.
The second design principle draws upon several studies that show the importance of sense-making in a mathematics class. According to Carpenter, et al. (1983, p. 656),

…it students may not understand some of the problems they do solve. Most of the routine problems can be mechanically solved by applying a computational algorithm. In such problems the students may have no need to understand the problem situation, why the particular computation is appropriate, or whether the answer is reasonable.

The overemphasis of rote procedures can stifle the development of thinking, reasoning, and problem-solving skills. In a study of high school math classes, Schoenfeld (1988) found that for the classes that valued the use of rote procedures, the students in these classes gained procedural proficiency but lacked coherent subject knowledge. In other words, they failed to understand the conceptual knowledge underlying the procedural knowledge. Thus, an effective mathematical learning environment should frame the activity of mathematics as being a constructive activity that supports students’ mathematical interpretations and does not steer them towards predetermined solutions (Cobb, et al., 1991). Mathematics education research has supported the idea that learning environments that create a space for students to engage and reason with their math interpretations can lead to increased levels of engagement and promote a value of understanding in mathematics (Cobb, et al., 1991; Ames, 1992). In the math class, several activities were designed to draw on adult’s sense-making strategies for a problem. For example, adults were asked to give examples of how they could make sense of a very large number (e.g., the number of people in the United States). In other words, this asked them to consider what strategies they could use to make the number more meaningful and accessible to them. In another example, adults were asked to reason about the outcomes of different events, including events such as flipping a coin, rolling a dice, and winning a
lottery. For a complete list of activities and associated design principles, please see Appendix 1.

Given the design principles from above, the adult learning course that was created for this thesis project has a strong emphasis for the adult learners to use their experiences as a foundation that they can build on and for them to feel empowered by their own sense-making opportunities.

Research Questions and Conceptual Framework

The current study is interested in the following questions:

1. How might adult learners engage in and make sense of the topics and activities in a mathematics class?

2. What are some characteristics of adult learners who pursue a personal interest course in mathematics?

The first question will draw upon the andragogy framework as a theoretical lens for analyzing the data. This framework can serve as a foundation to understand how adult learners engage in and how they make sense of mathematical ideas. Since the class was designed to enable adults to make sense of mathematics using their everyday experiences, the principle of experience will help us understand how adults’ apply their prior knowledge in both their understanding and approach towards mathematics.

The second question will also draw upon the andragogy framework. According to Knowles (1972), an adult’s orientation towards learning shifts from subject centeredness to problem centeredness. One interpretation of this principle is that the adult is shifting from a “postponed application of knowledge” mindset to one in which there is an “immediacy in application of knowledge.” Thus, the adult values and seeks knowledge
that can be applied to a goal that is relevant to him or her. In order to fully understand the importance and relevance of mathematics to the adults, it is important to take into account what adult perceives to be as valuable in mathematics.

It is important to note that the andragogy framework, by itself, does not account for why an adult would pursue a personal interest course in mathematics. In other words, it does not capture the values and attitudes that adult learners have towards mathematics, and how this could affect their decision to pursue such a course. To assess the values and attitudes that adults may have towards mathematics, the definition of “Appreciation and Use of Mathematics” from the National Association of Education Progress (1970) will be used in the analysis of the data. The appreciation and use of mathematics is defined using two subcategories: 1) recognizing the importance and relevance of mathematics to the individual and to the society (value of mathematics), and 2) enjoyment of mathematics.

In order to operationalize what value means in a mathematics context, the definition from National Association of Education Progress (1970) was used. More recent versions of the NAEP have asked 13- and 17-year old students about their attitudes towards math, however, their surveys have focused on students’ self-perception of ability in math and their perceptions of math courses that they have taken (Campbell, et al., 1997). Since this survey was being adapted for adult learners, who may not have taken a math class in the recent past, it was critical to select a survey that evaluated adults’ attitudes about math more generally. The NAEP (1970) describes the value of mathematics as one of the categories in the “Appreciation and Use of Mathematics” section. Alken (1974) developed a survey instrument to assess the value of mathematics. In his instrument, he included the following items (p. 68):
• “Mathematics helps develop a person’s mind and teaches him to think”
• “Mathematics is important in everyday life”

Additionally, the NAEP (1970) describes the enjoyment of mathematics as one of the categories in the “Appreciation and Use of Mathematics” section. Alken (1974) developed a survey instrument to assess the enjoyment of mathematics. In his instrument, he included the following items (p. 68):

• “Student has an interest and willingness to use math outside school and on the job”
• “Student has an interest and willingness to acquire further knowledge in math”
• “Student goes beyond assigned work and tries to solve new math problems”
• Student describes mathematics as “‘enjoyable’, ‘stimulating’, and/or ‘interesting’”

These constructs will be used to understand the characteristics of the adult learners in the math class.
Chapter 3 The Design and Study of an Adult Course in Mathematics

This chapter will describe the design of the mathematics course for adults. It will cover the recruitment of the participants, the overall course design, the development of the units, and the day-to-day activities of the class. These four areas will be described from a logistical perspective, so that the reader can get a sense of how the course was implemented. It will also touch upon some of the challenges that were encountered in some of these areas, with a more in-depth discussion of these challenges in chapter 6. Finally, this chapter will describe the type of data that was collected over the course.

Recruitment of Participants

The course took place at a local community center in the Pacific Northwest. The community center developed a flyer to market the course, with guidance from the instructor-researcher. The course was entitled “Everyday Math.” with the following description:

Free Adult Math classes for individuals interested in developing their personal awareness of how everyday math effects their lives. Ever wonder what 0% APR financing means, or when the weather forecast says there is a 30% chance of rain? In this class, we will use the power of mathematics to make sense of these topics! This course is open to everyone regardless of their math background.
Since the class was designed for adults, the class was held once a week in the evenings from 6-7 PM. The instructor and the community center began marketing the class about six weeks before it began. This included posting fliers in the halls of the community center, distributing fliers at other community centers and nearby libraries, as well as through word-of-mouth advertising by both the instructor and the staff at the community center. Although the class had room for about fifteen students, only three female students signed up for the class, and each of them attended at least half of the classes. Given that the present research study is qualitative in nature, the small class size enabled the instructor-researcher to have more in-depth interactions with the participants, thus allowing for a deeper understanding of the participants and their experiences.

Course Design

The classes took place once a week for seven weeks. The topics covered in the classes were categorized into three units: 1) Numerical Thinking and Magnitudes, 2) Fractions and Percentages, and 3) Probabilistic Reasoning. Although the topics for each class varied from week to week, there was a general format to all of the classes. At the beginning of every class, the students were asked to make sense of a math problem, which served as an anchor for the topic that would be covered in the class. The students then shared their sense making strategies. The class then, depending on the topic, delved into a shared reading or meaning making activity. At the end of each class, students were asked to do a weekly exploration (i.e., “homework” about the topic covered in that class). The term “weekly exploration” was used to emphasize the inquiry-oriented and sense-making goals of the activity, this phrasing was chosen to try to avoid students’
preconceived notions/expectations of what homework may be in a traditional math class (e.g., rote problem-solving).

Unit Organization

The topics covered in the class were categorized into three major units: numerical thinking and magnitudes, fractions and percentages, and probabilistic reasoning. These three topics were selected based on their prevalence and importance in our society.

Since the goal of the class was for students to engage in reasoning about mathematical ideas that they encounter in their everyday lives, each topic spanned about two to three classes so that students would have in-class time to make sense of the ideas and have time to reflect in between classes. In selecting the topics, the instructor also drew upon the research in mathematics education and cognitive psychology. There is research showing that fractions knowledge is a predictor of high school mathematics achievement (Siegler, et al., 2012). Thus, an understanding of fractions is a foundational building block for more complex mathematical understandings. Additionally, there is research that shows that adults have systematic misconceptions in probabilistic reasoning (Tversky and Kahneman, 1974). The topic of probability is an area of reasoning in which our intuitions can lead us to answers that are not mathematically sound, thus, reasoning through and discussing such situations with students can help them uncover and understand their intuitions.

There were other topics that were considered such as Bayesian reasoning and algebraic thinking, but these topics were not covered due to a lack of time and their being more “advanced” in nature. Since the class was designed to be a foundational course so that adults of all backgrounds could engage in sense-making and make connections to
their everyday lives, the instructor made a decision to not include these topics in the course.

Class Activities

In the first unit, numerical thinking and magnitudes, one of the activities was for the students to think about when they use mental math in their daily lives. They were then asked to describe some of the strategies that they use when performing mental math. In another activity, they were asked to think about how they make sense of large numbers. A reading was provided to them How to Comprehend Incomprehensibly Large Numbers by George Dvorsky (2014). Here’s an example of a strategy that Dvorsky provides:

We're often confronted with excessively large figures that would be better expressed with a different unit, like switching from feet to miles (or meters to kilometers), from ounces to pounds, from seconds to years, and so on.

“For instance, if someone says that the Mariana Trench reaches to a maximum depth of 36,000 feet, that's tough to make sense of," says Greenberg. "It's much easier for our brains to understand that as 6.8 miles (11 km), which is a distance we already have a pretty good intuition for.” That said, be sure to avoid units for which you don't have an intuitive grasp. Most people, for example, have little intuitive sense of how much a ton of weight is. (Dvorsky, p. 4-5)

In the second unit, fractions and percentages, the activities were centered around having students think about where they see fractions and percentages in their lives and how they reason with these quantities. Additionally, students were given activities in which they were encouraged to think about percentages in terms of fractions. Students were also given an activity where they had to reason about the term 0% APR financing. To help the students make sense of this topic, students were introduced to the terminology and mathematical formulas related to calculating simple and compound interest rates. Students were also asked to reason about these two types of interest rates,
and why one would yield a higher amount.

In the third unit, probabilistic reasoning, students were asked to reason about the outcomes of flipping a coin. Students were also introduced to the idea of a gambler’s fallacy and asked to think about when and why they make have committed this fallacy in their life. Students were also given more challenging probability problems to reason about including the Birthday problem and computing the odds of winning the Powerball lottery.

In the final class, the students were asked to identify or bring a mathematical problem or idea of interest to them. Since the class size was small, the students were able to share with each other on the first day of class what were their motivations for the class. In this class, each student had identified a goal that required a deeper understanding of mathematics. The instructor, when possible, would make time before or after class to talk to the students about their goals and to check on their progress. On the final day, the students were asked to come to class prepared to talk about their goals in more detail and the progress that they had made. The final class had a workshop format in which students worked on their individual projects with feedback from the instructor and their peers. To see a complete list of lesson objectives, activities, and resources please see Appendix 1.

Data Collected

There were three sources of data for this study: field notes, student work, and participant surveys and interviews.

Field notes were based on the instructor-researcher’s interactions with the participants, and included observations about the questions and comments each
participant made about math, how they made sense of math problems, and the experiences they described about using math in their everyday lives. This observational frame was predetermined from the instructor’s research questions and conceptual framework. During class, the instructor made quick notes that captured direct quotations or the substance of what the participants said and did (Merriam, 2009) that was related to the three categories described above. Immediately after the class, the instructor would write up the observation in narrative form so as to capture as much detail about what occurred (Merriam, 2009). In her field notes, the instructor described her observations in a narrative form, using neutral language for actions (e.g., “she said,” “she walked to the board”), using quotation marks for direct quotes, and being “highly descriptive” which Merriam (2009, p. 130) describes as “enough detail should be given that readers feel as if they are there, seeing what the observer sees.”

The student work collected included the students’ weekly explorations (e.g. “homework”) and in-class assignments.

The survey and interview were administered at two time points, on the first and the last day of the class. On the first day, the participants were given individual surveys to complete. On the last day, the participants were administered a group interview. Both the questions and the format of the survey and interview varied from each other. In the initial research design, the researcher had planned to give the participants a pre- and post-survey. However, the researcher noticed that there was some confusion about some of the survey questions and that some of the questions were left blank or not completely filled out. Since the researcher observed that the class was both small and comfortable with sharing their ideas with her, she opted to do a group interview in lieu of another survey,
as a way to collect more focused information, and to provide a way to probe particular areas of interest in more depth. In the group interview, the students were asked open-ended questions about two key areas: their perception of math before and after the class, and their learning experiences in this class and other classes. A full list of questions is provided in Appendix 2. The instructor took detailed notes that captured the participants’ direction quotations or the substance of what they were saying, and when possible, their actions as well. The interview was then put into a narrative format as described earlier.
Chapter 4 Analyses and Findings

This chapter will describe the analysis of the data from the adult mathematics class. It will then present the findings for each of the research questions as themes. The themes in this study represent the patterns that were observed across the three participants in the course.

The researcher began the analysis of the results by reading through the field notes, student work, and interview notes to identify themes and patterns in the data. The data were coded using a coding scheme that was created through both deductive and inductive methods. The conceptual framework guided the creation of the deductive codes. Specifically, the theories of constructivism, andragogy, and value and enjoyment in mathematics (as defined by Alken 1974) guided the creation of codes. For example, codes included student sense-making, personal experience, and statements about math. Additionally, open coding was utilized to search for emerging patterns in the data. These codes were then used to construct categories or themes to show a recurring pattern that cut through all of the data (Glaser and Strauss, 1967).

Below, the research question, the themes, and the supporting data from the three students will be presented separately as cases. Chapter five will then summarize and discuss these findings.

Research question 1: How might adult learners engage in and make sense of the topics and activities in a mathematics class?

Themes:

1. Making math relatable and putting it in their own language
2. Applying mathematical ideas to real world experiences in situations in which they are both prompted and unprompted

3. Identifying alternate conceptions or incomplete understandings

**Theme 1: Making math relatable and putting it in their own language**

**Case of Anita**

In the fourth class, the instructor wrote down the formula for calculating compounded interest rates. This formula, \( FV = PV(1+r/n)^{nt} \), was going to be used to make sense of the interest rate problems on a worksheet that had been given to the students. Since Anita was the only student in the class that day, she and the instructor were able to work the problem on the board in the front of the class. The instructor told Anita that she should feel free to ask questions that came up as they worked on the problem. While working on the problem, Anita asked a couple of questions. First, she asked if she could use other symbols, besides the one that the instructor had, to represent the equation. The instructor used specific symbols to represent the variables in the interest rate equation (e.g., \( FV = \) future value). The instructor said that it was fine for Anita to write the equation with the symbols that she was familiar with. Second, Anita asked if she could write the equation vertically instead of horizontally. The instructor said that would de fine.

**Case of Mary**

In the fifth week of class, the class started with a set of probability questions. As a reminder, the instructor put down the following formula for computing probability (for the case that each outcome is equally likely) on the board:
Probability of event happening = (# of ways it can happen)
---------------------------------------------
(Total # of outcomes)

When the instructor gave out a worksheet with the problems, Mary asked, “do I need a gauge?” The instructor asked her what she meant by that. Mary gave examples by saying something as in “1 in 200 or 1 in 500”. The instructor told Mary that one does hear about probabilities like that in the real world. But in this case, the work would be done with a coin, where there are just two possible outcomes. The instructor then asked Mary if she understood that there are two possible outcomes when one flips a coin, and she said yes. The instructor then said that they would use that as a starting point to determine the probability for the situations with which they would work with.

Case of Teresa

In the third week of class, the class was discussing simple and compound interest rates. Later in the class, the instructor wrote the formula to calculate the future value of an investment based on the compound interest, $FV = PV(1+r/n)^{nt}$, where $FV$ stands for Future Value, $PV$ stands for Present Value, $r$ stands for interest rate per unit of time, $n$ is the number of times the interest is compounded, and $t$ is the amount of time. Teresa said that in her previous experiences, she saw the variable “$r$” written as an “$i$” instead. The instructor said that it was fine to use an “$i$” because it was a variable so it is possible to use any symbol or letter to represent it. When the instructor used the formula in an example later on, she wrote the expression: “$10(1+r)$.” At that point, Teresa said that she was confused by why there was no multiplication symbol between the 10 and the $(1+r)$. The instructor said that was another way to express that two quantities are being multiplied. The instructor thanked Teresa for bringing this up and said that she would try
to remember to write it in the format that Teresa brought up; the instructor wanted to ensure that she was using notation that was familiar to Teresa.

In the last class, Teresa shared a conversation that she had had earlier in the day with some “youngsters” on the bus ride to the class, and the topic was about mathematics. They were telling her that they find geometry less relatable than algebra, Teresa then told them that she agreed with that statement. The instructor asked her what she meant by relatable and Teresa said that geometry class (unlike algebra class) did not have examples that were related to the real world. She said that this was especially surprising to her because the same teacher that taught her algebra also taught her geometry. The instructor interpreted Teresa’s statement to mean that the teacher could have made the geometry class more relatable to the students, since she was able to do so with algebra.

**Theme 2: Applying mathematical ideas to real world experiences in situations in which they are both prompted and unprompted**

**Case of Anita**

In the first week of class, the instructor asked the students to describe a situation in which they do mental arithmetic and what strategy they used. In her response, Anita said, “I worked at Farmers Markets. We had scales but no cash register. I found it faster and better for me to use mental math or paper and pencil rather than a calculator – most of the time.” She then went on to say,

I often multiplied the tens first, then the ones place. For example 15 pounds at 60 cents per pound, I would think: 10 times 60, then 5 times 60 and add the two. I also would count the change back to customer, starting with the charges, then up to the amount I was handed.

In the third week of class, the students were using and reasoning with the compound interest rate formula. As Anita and the instructor were working through an
interest rate calculation, where the calculated interest was perceived to be high, Anita begin to describe her friends who have borrowed money at high interest rates and the total amount that they are paying back is a very large sum. She described the loan agency that lent them the money as being “crooks.” Later in the conversation, she described how her friend had two options for health insurance, one of which had very low monthly payments, but had a $6,000 deductible. Her friend had chosen this option, and she said could not understand why her friend would choose such an option.

Case of Mary

In the first week of class, the instructor asked the students to describe some activities that they do in their everyday life that involves doing math. In her response, Mary listed the following items: “phone calls, filing, planning, scheduling meetings and appointments, household budgets, shopping.” In another activity on the same day, the instructor asked the students to describe a situation in which they do mental arithmetic and what strategy they used. Mary wrote the following: “Remembering an appointment to call an office for confirmation between 3 to 5 PM – 1:15 busy 2:15 busy 4 PM called to confirm.” Furthermore, she identified the following strategies, “time awareness, checking the time, planning to call, making a conscious effort to call, getting the time right.”

In the fifth week of class, the instructor was discussing probability in the context of flipping a coin. The instructor talked about the gambler’s fallacy in the context of flipping a coin. Specifically, if a coin lands heads ten times in a row, then the probability of getting a heads on the next flip is still one-half. The instructor said that even though the probability of getting heads ten times in a row is very, very small, one could not use
this information to reason that the likelihood of getting a heads on the 11th flip. Mary then described a story that she had read in the genre of “oceanography horrors.” She said that a whale had burst through a structure, even though the probability of it happening was very low. She said that even though the probability was low, it could still happen.

In the last class, when asked about whether her approach to math in her everyday life has changed now that she has taken this class, she talked about her experience of getting charged extra for items at the grocery store. Instead of catching this error at home and then reporting it the next day, she said that she now feels more comfortable dealing with in the moment. Before the class, she thought that this action would be perceived as “quabbling” (the instructor assumed that Mary meant “quibbling”) but now she says that it is about “being accurate.”

Case of Teresa

In the third class, the students were having a conversation about how they make sense of percentages in the real world. Teresa said that whenever, she was at the local store and saw signs that said take an extra 15% off, she would do the math on the spot and then check to see if the price was in her budget. On the final day, during the interview, she said that the class session that covered the topic of percentages, particularly resonated with her. That same evening she went to Macy’s to buy a blanket sleeper. At Macy’s, the item was marked as being 30% off, and then one could get an additional 5 to 10% off with a coupon. She then bought the blanket at Macy’s. She then went to another store to see if she could get the blanket for a better price since they were having a sale. She also noted that just because there is a sale, this does not mean that one will save much, because it depends on the initial price. In this situation, she said that was
able to get the blanket for a better price. In fact, it was half the price at Macy’s, so she was able to save $27 to $28, so she could use that money to buy something else. As she told this story she said, this is the topic what we just covered – “gotta do my due diligence with this puppy.”

In the last day of class, during the interview, Teresa described a conversation that she was having with one of her friends a few weeks ago. She said that her friend’s husband bought a large screen TV, which her friend found to be too big for their living room. Teresa told her friend that she should have measured the dimensions of the living room before they bought the TV. She then commented to her friend that she should take the math class, and that this class would have helped her, but the scheduling did not work out.

Theme 3: Identifying alternate conceptions or incomplete understandings

Case of Anita

In the fourth week of class, the instructor began the class by writing the formula for calculating compounded interest rates. The instructor said that they were going to use this formula to make sense of the interest rate problems on the worksheet. Since Anita was the only student in the class that day, the instructor said that they could work the problem on the board and that Anita should feel free to ask questions as they came up. When substituting the value of the rate into the equation, Anita asked about the way to input 13.6%. She was not sure if it was supposed to be 1.36 or 0.136 She said that she always gets confused with this issue. The instructor discussed with her that 13.6% is actually the same thing as 0.136. This is because 13.6% means 13.6/100 as “percent”
actually means “per hundred”. When the instructor said this, Anita said that this explanation helped her to understand why 13.6% was written as 0.136 in the formula.

Case of Mary

In the fifth week of class, the class started with a set of probability questions. As a reminder, the instructor wrote down the same formula for computing probability as described earlier in this section. The instructor and Mary then talked about the scenario: of getting heads on the first flip and tails on the second flip, and the instructor asked what would be the probability of this happening. Mary did not respond. The instructor first talked about setting up all of the possible scenarios that one can get by flipping two coins in a row: heads followed by heads, heads-tails, tails-heads, and tails-tails She then said, based on this, only one of the scenarios matches the criteria that we are looking for out of the four possible outcomes. Mary said that this made sense to her. The instructor then said that there was another way to reason about this problem, and this approach would become helpful when they started to work with larger numbers of coin flips, when it is more cumbersome to write out all of the possible scenarios. One can take the probability of getting heads (1/2) on the first coin flip, and then multiply it with the probability of getting tails (1/2) on the second coin flip to get the probability of getting a heads followed by a tails. Mary said that she did not understand how the instructor got this result. The instructor told her that the coin flips are independent events (which Mary understood), and that there are two possible outcomes with each flip: heads or tails, which are both equally likely. Since there are two possible outcomes with each flip, one can multiply the possible outcomes on the first flip times the number of outcomes on the second flip to get the total number of possible outcomes. This total number of possible outcomes is
represented in the denominator, and the numerator represented the number of ways the desired outcome can happen, referencing the scenarios that were presented earlier. Mary nodded her head when asked by the instructor if she understood this concept. In the final week of the class, when asked about their learning experiences in the math class, Mary said that she was told that probability was about gambling, but she learned from this class that probability can be used in situations that do not have to do with gambling.

Case of Teresa

In the fifth week of class, the instructor was discussing probability in the context of flipping a coin. When working on the problem of trying to calculate the probability of getting 10 heads in a row, the instructor wrote the following formula: \((1/2)^{10} = 9.76 \times 10^{-4}\) or \(1/1028\). Teresa asked about what \(10^{-4}\) meant. The instructor explained this to her by fully writing out what the number was (i.e., 0.000976) and then told her that scientific notation is another way of expressing a number and that it was more convenient to write it the way the instructor had. In the exit ticket given at the end of class, when asked whether there was any confusion about any of the topics presented that day, Teresa wrote that “\(10^{-4}\) was explained to where I could understand it.”

Whereas the first research question investigated how adult learners engaged in and made sense of the topics and activities in a mathematics class, the second research question is investigating the characteristics of these adults. The themes and the supporting data from the three students will be presented separately as cases.

**Research question 2:** What are some characteristics of adult learners who pursue a personal interest course in mathematics?

**Themes:**
1. Orientation to mathematical understanding that is based on value for the discipline

2. Enjoyment of mathematics

**Theme 1: Orientation to mathematical understanding that is based on math’s intrinsic value**

**Case of Anita**

On the first day of class, Anita came into the class with an interest in understanding a particular problem scenario: if something is X% off, then what was the original price? Anita did not use the term “X%”, but used a range of values, indicating that she was interested in understanding how to solve this problem in various contexts. In the fourth class, the topic of the class was calculating and understanding interest rates. The instructor and Anita walked through an interest rate problem. Given that there was time remaining at the end of class, the instructor asked Anita if she would be interested in tackling the problem scenario that she had identified on the first day. Anita said that she was interested, and so the instructor created the following scenario: If something is 75% off so that the new price is $70, what was the original price? The instructor then talked about how she would set up the problem using ratios. In this case, instead of thinking of the new price, $70, as being 75% off the original price, it’s possible instead to think of the $70 as being equal to 25% of the original price. At this point the problem can be examined by using ratios by writing that 70 divided by x is equal to 25 divided by 100. Anita understood how the instructor set this problem up. The instructor then started talking about how to go about isolating “x”. Anita said she did not understand this process very well. She asked if the instructor could identify another way to solve the problem, not using algebra, because she felt uncomfortable with that process. Given the
class was about to end, the instructor said that she would look for other ways to make sense of this problem to help Anita.

Case of Mary

On the first day of class, Mary came in to the class wanting to understand the conditions of her loan, as she was quite perplexed that her loan amount did not seem to go down. On the seventh class, Mary had brought a copy of her loan statement to class. She said that there were certain terms that she did not understand such as “collateral.” During the group interview, when asked about her attitude towards math after taking this class, she said that by taking this class she better understood her loan statements. She now understands the term “APR”. She said that she had bought an expensive math book a long time ago to understand the math behind her loans, and she had picked it back up during this class to learn more about her loans.

Case of Teresa

Teresa said that she was interested in understanding how to do Sudoku puzzles and that she always liked math and found it stimulating and challenging. Throughout the course, she expressed her interest in the upcoming activities. For example, in week three, she said that she was interested in calculating the odds of winning a Powerball lottery and that she had told her father that they were going to do this activity in class. During the final interview, when she was asked about her experiences with the class, she said that she was grateful for this class and that it was a good exercise to “jog that side of the brain.” She also said that since she has taken the class, she goes to websites to learn more about math and to find challenging problems.
Theme 2: Enjoyment of mathematics

Case of Anita

Towards the end of the first day of class, students were given a problem called the magic squares problem. The students were encouraged to think about strategies for solving this problem. In the second class, Anita brought back the problem. She said that she and a co-worker were talking about how to solve the problem, but still could not figure out how it worked. The instructor talked to Anita about some of the ideas that she generated, since Anita was on the right track but still had not figured out the problem, the instructor encouraged her to continue thinking about the problem. In the third week, in addition to her continued work on the problem, Anita showed the instructor a book called “Painless Math Word Problems” by Marcie Abramson. She said that she checked out that book so that she can give her grandson (who is in 7th grade) challenging math problems. Anita said that she also looks in the book to give herself challenging problems.

Case of Mary

In the final class, the instructor conducted a group interview. She asked students about their perception about math. Mary responded that she “loved the challenge of not using the calculator” when solving math problems and that she found the subject of algebra to be interesting. Later when asked about their learning experiences in the class, Mary talked about how she really enjoyed the birthday problem (which was covered during the probability unit). She also said that she now understands that probability can be applied in non-gambling cases.

Case of Teresa

The topic for the second week of the class was making sense of large numbers. In that class, the instructor and students discussed strategies for making sense of large
numbers. In that class, Teresa asked how many zeros does a billion have. In the third week of class, she made a comment to the instructor that she became very curious about how many zeros large numbers (e.g., trillion, zillion) have so she looked them up online after the class. On the last class, during the group interview, Teresa was asked about what she enjoyed about the class. She said that she really enjoyed the “Birthday problem,” and that she had looked it up online as a result of learning about it in class. She said that she was fascinated by the number of sites that have math content; she told the instructor about two that she found: Wikipedia and mathway.com.

When asked about her attitude towards math after taking this class, Teresa said that she always liked math and found it stimulating and challenging. She said that since she was in business, she often had to deal with accounting and financing, which involves math. She said that she was never afraid of math, but that she was just rusty. Furthermore, Teresa said that now that she has taken the class, she goes to websites to learn more about math and to find challenging problems.
Chapter 5 Discussion of Findings

This chapter will discuss the findings for each of the research questions, with an emphasis on comparing and contrasting the findings across the three cases in order to reveal the implications that they have for adult learning in mathematics.

The first objective of this research was to understand how adult learners make sense of the topics and activities in a mathematics class. The analysis of the data provides evidence that the learning environment made it possible for them to apply mathematical ideas to real world examples, and to identify and work through alternate conceptions or incomplete understandings. While working through the math ideas, they demonstrated that it was important to making math relatable and that it was also important for them to put these concepts into their own language.

In specific learning situations, the students leveraged their prior knowledge to make sense of the mathematical ideas in the class. When using the equation for calculating compounded interest rates, \( FV = PV(1+r/n)^{nt} \), both Anita and Teresa recognized this equation from their previous experiences and used these experiences as a starting point to understand the equation. During class, they both noted that they had seen the equation before but with different symbols to represent the variables in the equation. Additionally, Anita asked the instructor for permission to use the equation with the variables that she was familiar with and to write the equation in a format with which she felt comfortable.
This observation shows that both Anita and Teresa were comfortable in using their prior knowledge in class and when there was a discrepancy between what they knew and what was presented to them, they asked questions and sought to reconcile these differences. If the instructor was not flexible and insisted that the equations be written with a specific set of symbols and a particular format, then this may have prevented Anita and Teresa from engaging with the content in a way that was accessible or relatable to them.

In the case of Mary, when she was presented with a probability scenario in the class, where she was asked to reason about the probability of an event occurring, she asked the instructor “Do I need a gauge?” and then went on to give a couple examples of what she meant. This situation showed that Mary had encountered probabilities before and that she was bringing these experiences to assist her in reasoning about the probability scenario. The instructor acknowledged and validated Mary’s ideas, but also encouraged her to think about probabilities in a different way. By doing so, she wanted to build on Mary’s understanding of probability. If the instructor had dismissed Mary’s ideas, then this may have prevented Mary from having an opportunity to build on her understanding with the new ideas.

These examples above that show Anita, Mary, and Teresa were relating the content in the math class to their prior knowledge. Knowles (1972) points out that an adult’s learning experience is “who he is”, whereas the experience of a child is something that happens to him or her. Thus, it is important to make use of these experiences as a resource for learning.
Throughout the class, the instructor encouraged the adults to apply and share how the mathematical ideas that were being presented in class applied to their lives. These ideas ranged from doing mental math, to knowing when to be exact versus approximate, to reasoning with percentages when shopping. Each adult had their own unique way of applying mathematical ideas to their lives. Anita had experience working in farmers’ markets without a cash register, thus she shared her strategies for doing mental math in this context. In her example, it was clear that she had a deep understanding of place value when she discussed her strategy. As she said, “I often multiplied the tens first, then the ones place. For example 15 pounds at 60 cents per pound, I would think: 10 times 60, then 5 times 60 and add the two.”

In Mary’s example, she discusses the importance of being exact in the amount she pays when she is shopping. Her example came from a class session that asked the students to consider situations in which they had done mental arithmetic, and when would it be appropriate to use this technique and when it would not be (i.e., when it is important to be exact). Based on Mary’s experiences, she identified her trips to the grocery store as a situation in which it is important to insist that the exact amount be computed and paid. To her, “being exact” in shopping situations was important, and she wanted to ensure that she was being only charged for what she owed. For Teresa, she saw the opportunity to apply percentages in shopping contexts. She took these ideas to heart one night as she went shopping for a blanket one evening at two different stores so that she could compare the prices; this event occurred the same night that the class covered the topic of percentages, and when she had said that needed to be particularly careful to do this accurately (i.e., that she had to do “due diligence with this puppy”).
The application of math to their lives is not limited to the examples just given. In fact, there are many examples in which the students were discussing their friends’ decisions or what they had heard on the news. Such examples include Anita reflecting back on her friend’s decision to get a high interest loan, Mary talking about an improbable event that she had read about, or Teresa describing a situation in which her friend could have used geometry to figure out whether a TV was too big for their living room. These examples demonstrate that their reasoning about math extended beyond the classroom context, and that they were able to apply mathematical ideas and intuitions to their everyday experiences in a way that was relevant and important to them.

As a result of these observations, it is clear that this class reinforced the idea that doing math is not just a paper and pencil activity; it can also be conducted in the context of everyday life and practices. Unfortunately, this idea is not always common notion (Goldman and Booker, 2009). Goldman and Booker (2009) conducted an ethnographic study on families showing that they engage in mathematical problem solving in their everyday lives, such as setting budgets or interpreting a player’s batting average (in the context of baseball), but the families did not identify these activities as doing “math.” The authors claim that when the family is recognized as a unique and culturally legitimate setting in which mathematical learning occurs, then this setting can be more deliberately leveraged by parents to help their children learn mathematics. This idea promotes respect and value for the mathematics that occurs outside of a formal school setting.

In the class, the students had opportunities to work through their understanding of mathematical ideas, often on the board and one-on-one with the instructor. With these situations, there were more opportunities for the students to express their thinking and
reasoning and for the instructor to help the students work through alternate conceptions or incomplete understandings. In the case of Anita, while working with the equation for calculating compounded interest rates, she expressed her confusion about how one inputs the interest rate into the equation. In the case of Teresa, she had a question about what $10^{-4}$ meant in class. It is important to note that both Anita and Teresa felt comfortable in self-identifying their incomplete understandings. In a math class where the emphasis is on being correct, students may not feel comfortable in expressing that they do not understand a particular topic. Thus, a class that values meaning making can provide an environment that encourages students to share and clarify their ideas when they are confused.

The learning situations with Anita and Teresa may appear trivial on the surface. However, these alternate conceptions, if not addressed can become barriers to reasoning soundly and correctly with mathematics. In the case of Anita, the instructor attempted to help her think more about what a percent represents. In the case of Teresa, the instructor helped her to understand the convention of scientific notation for that particular example. To make the understanding more robust, the instructor could have leveraged other examples such as $10^{-4}$ to emphasize the meaning of a positive versus a negative exponent.

In the case of Mary, while working through a set of probability problems, it became clear that Mary had a specific conception of probability, as revealed by her question of “Do I need a gauge?” The instructor acknowledged and validated Mary’s understanding, but went on to describe how the situation they were working with required another way of representing the problem and thus needed a different “gauge”. By saying this, the instructor attempted to connect Mary’s understanding with the current context.
In the learning situations described above, the instructor focused on creating an environment in which the students felt open to share their ideas and questions about mathematics. This was done by listing the following norms explicitly in the class syllabus:

- Ask questions. When something doesn’t make sense, please feel free to ask the question in class. Someone else may have the same question as you. If you are uncomfortable asking in class, please do talk to me or e-mail me.

- Be open to taking risks, thinking in new ways, and making mistakes. “Give yourself permission to learn. It is impossible to get better and look good at the same time.” (Julie Cameron, personal communication)

The students were reminded of these norms during the classes. By adhering to these norms, this created a space for students to feel comfortable in expressing their questions about the math content that was puzzling to them. Additionally, the students were asked to reason about the problems in a public way, either on the board or verbally. This type of set-up allowed the instructor and the other students to hear each other’s thinking and reasoning. At times in the class, students had to reason about situations using equations (e.g., calculating interest rates) or schematic representations (e.g., writing out all of the possible outcomes when flipping a coin). In these situations, the instructor and the students made sense of the problem on the board together, thus giving students agency over the sense-making process. At other times in the class, the students had to reflect on examples from their own lives (e.g., when have they used the gambler’s fallacy in reasoning) or make sense of a problem (e.g., when have they encountered a large number and how did they make that number meaningful and accessible to themselves).
these situations, the instructor encouraged the students to share their ideas and solutions with each other. This promoted dialog, questioning, and group sense-making in the class. These examples demonstrate that the instructor made efforts to be responsive to the students’ learning needs and used different strategies that would be well-suited for the topic.

In the present study, there is evidence that adults leveraged their everyday experiences and engaged in sense-making experiences in the class. While doing so, there are several instances where the adults tried to relate the ideas they encountered in the math class with what they had learned in previous classes. In these cases, the adults tried to reconcile different ways of representing and thinking about math (e.g., symbols used to represent multiplication). Additionally, the class provided multiple opportunities for the adults to leverage their everyday experiences and to identify and work through alternate conceptions or incomplete understandings of a mathematical idea. In this class, the adults were able to draw upon their own experiences and to begin with their understanding of mathematics. Thus, this learner-centered environment prioritized and valued their knowledge and experiences, and gave them agency in the construction of their knowledge.

The second objective of this research was to understand the characteristics of adult learners who pursue a personal interest course in mathematics. The analysis of the data provides evidence that adults can have an orientation to mathematical understanding that is based on value for the discipline and that they can enjoy mathematics.

The cases of Anita, Mary, and Teresa demonstrate that they each brought salient learning goals to the math class. While they each had different learning goals, each
student wanted to gain a better understanding of mathematics so that they could make sense of a math problem or topic that was meaningful to them. In the case of Anita, she was interested in understanding how to solve a specific type of math problem that involved making sense of percentages. On the day that the class covered percentages, she and the instructor were able to work through a problem of this type together on the board. During this time, Anita expressed her discomfort with algebra and asked the instructor for help with a new approach to solve the problem. In the case of Mary, she was interested in understanding her loan statements. Although Mary and the instructor were not able to talk about her loan statements in the class, the instructor did cover the topic of interest rates in the class. Mary then took the initiative to supplement her learning outside of the class, by using a textbook to help her make sense of her loan statements using interest rates. In the case of Teresa, she was interested in pursuing more challenging and stimulating math problems, such as Sudoku. While the instructor was not able to cover the topic of Sudoku in the class, the instructor covered other challenging problems in the class, such as the birthday problem. Teresa expressed that she found the class to be stimulating and that she had been looking up challenging problems on other websites. The class created the space for the students to express their learning goals, and when possible, further opportunities were provided to the students within the class to make sense of their problem or topic.

In Knowles (1972) theory of andragogy, he states that an orientation to learning is a characteristic of adult learners, where they approach learning from a problem-centered approach as compared to a subject-centered approach. Furthermore, there is an immediacy in the adult’s application of their learning. Each of the three cases illustrates that the students each had learning goals that were problem-centered in mathematics. The
goals were of personal interest or relevance to their lives, and pursuing a deeper understanding of math would help them achieve their learning goals. These applications ranged from making calculations with percentages, to understanding interest rates for a loan, and to doing challenging math problems. The goals of Anita and Mary, which involved understanding how to do calculations with percentages and interest rates, respectively, are activities that can be present in one’s everyday life. Thus, there was recognition and value that mathematics could be useful as a tool to help them in achieving their learning goals.

It is important to note that in the case of Teresa, her learning goal can be construed as also being subject-centered because she did not have a specific problem she wanted to solve; rather she was interested in pursuing mathematics problems that were challenging and stimulating as an end goal. Thus, she was interested in the discipline of mathematics, itself, not a specific problem that involved math. Her value of math was based on the notion that it allowed her to think more critically and as she said it allowed her to “jog that side of the brain.”

Mathematics, as a discipline, is an enjoyable and rewarding activity in of itself. Alken (1974) characterized enjoyment of mathematics as including the following categories: 1) interest and willingness to use math outside school and work, 2) willingness to acquire further knowledge in math, 3) going beyond assigned work and trying new problems, and 4) describing math as enjoyable, stimulating, and/or interesting. There was at least one instance of each participant expressing enjoyment in the math class.
In the case of Anita, she demonstrated persistence in trying to solve the magic squares puzzle. After being introduced to the puzzle in the first week, she continued to wrestle with it in the subsequent weeks. Additionally, she sought other resources for challenging math problems for herself and her grandson. In the case of Mary, she stated that she enjoyed some of the topics in the class, such as the birthday problem. Additionally she stated that she “loved the challenge of not using the calculator” to solve math problems. In the case of Teresa, her curiosity about many of the topics covered in class, led her to find internet resources related to the topic. She said that she also found good resources for finding stimulating and challenging problems in math.

These findings demonstrate that the students were able to enjoy mathematics, as an activity in and of itself. This enjoyment ranged from expressing interest in the topics presented in class to the pursuit of additional challenging problems outside of the class (in the cases of Anita and Teresa). Although each of the students came in with their own specific learning goals, they also became interested and engaged in other math topics and problems, such as the magic squares activity and the birthday problem. Thus, their engagement in the class was not limited to achieving their aforementioned learning goals.

In 2010, Steven Strogatz, a mathematics professor at Cornell University wrote a series of articles for the New York Times about various topics in math. He thought of this endeavor as “sharing the pleasures of math with an audience beyond my inquisitive friend” (Strogatz, p. x). He states that as a result of these articles, he has received numerous messages from people and stated,

[this] experience convinced me that there’s a profound but little recognized hunger for math among the general public. Despite everything we hear about math phobia, many people want to understand the subject a little better. And once they do, they find it addictive. (Strogatz, p. xi)
Thus, the enjoyment of mathematics is not limited to mathematicians; rather there is a general fascination with mathematics in the public. The students in the class each expressed an interest in the various puzzles and challenges in the math class, demonstrating that mathematics can be an inherently pleasing activity in of itself. It is important to highlight that the adults in this class appreciated mathematics even when it was challenging and involved new ways of thinking and reasoning. As Teresa pointed out, she always liked math and was never afraid of it; it was just that she was rusty at it.
Chapter 6  Conclusions and Implications

This chapter presents a synthesis of the findings from the study and then draws upon them as well as the instructor’s reflections to propose recommendations for the design of a mathematics course for adults.

The present study was interested in investigating how adult learners may make sense of mathematics and to understand some of the characteristics of these learners. In this course, there were examples of the adults making math relatable to them, applying mathematical ideas to real world examples, and working through alternate conceptions or incomplete understandings in math. Thus, the adults in this class leveraged their prior knowledge, skills, attitudes, and beliefs to make sense of mathematical ideas.

Furthermore, there were examples of the adults in the being oriented to specific mathematical topics or problems that were of interest to them. There were examples of the adults valuing and enjoying the discipline of mathematics based on their personal goals, experiences, and conversations with the instructor. These examples indicated that mathematics can be of interest to adults from both a practical and recreational perspective.

This research provides evidence that mathematical learning environments can be designed that emphasize math as making sense of everyday experiences as well as opportunities to tackle new and challenging mathematical ideas. Furthermore, adults who take such a math class may be likely to have math-specific goals and have a value and
appreciation of math as both an everyday activity and a recreational activity, and that includes challenging problems and puzzles.

When designing a personal interest course in mathematics for adults, there are several pedagogical tools and approaches that an instructor needs to take into account. Based on the instructor’s experiences and reflections with the class, the following recommendations can be made regarding constructing meaningful adult learning environments for adults interested in mathematics:

Recommendation 1: Being flexible with how students respond to math assignments outside of the class. In lieu of homework, the instructor gave the students a weekly exploration task that was meant to allow students to make sense of and to further explore a problem. The term “weekly exploration” was used to move away from preconceived notions of mathematics homework as just being rote problem solving. The instructor selected weekly explorations that involved readings, reflection questions, or students conducting and observations of their environment. The instructor expected the students to bring written responses to their weekly explorations the following week. Although students did not consistently provide written responses, when the instructor went over the content of the explorations in class, the students were responsive and engaged with the material, so it is likely that they had thought about the concepts outside of the class. From this observation, the instructor modified the class, so that the students received the weekly exploration at the end of the class and had time to think about the ideas between the classes. She began the next class using the weekly exploration, giving students time to share their reflections and questions with the class.
In this situation, it was critical for the instructor to adapt the way in which the weekly exploration activity was covered. By beginning the class with a review of the homework, it set the tone that the weekly explorations are apart of the learning process and would be covered in class. Since the instructor also chose problems that were about students’ experiences and sense-making, it allowed for more successful participation by all students.

Recommendation 2: Being responsive to student-driven inquiries. In the first two weeks of class, the instructor discovered that each of the students had personal goals for taking the math class. These ranged from understanding loan statements, solving a specific type of math problem, or being able to do challenge problems such as Sudoku. Given this, the instructor was able to adapt some of the upcoming instruction so that the content could overlap with the students’ learning goals. Throughout the course and especially on the last day of class, the instructor made time to check in with the students to see if they were making progress on their goals, and when possible, would provide guidance or additional resources. Empowering the students to work on personal goals throughout the course can provide meaning and coherence to the students’ learning experiences.

Opportunities for student-led investigations in mathematics can be designed into a course. At the beginning, an instructor can get a sense of the students’ interests and then make time to check on students’ progress throughout the course (either during or outside of class). For example, the last ten minutes of a class can be used for students to share their working ideas with the instructor and/or their peers, which can hold students accountable to their goals.
Recommendation 3: Providing multiple opportunities for student thinking. The instructor provided multiple opportunities for students to engage in the class. This included students going to the board to demonstrate their thinking and reasoning, and providing opportunities for students to talk with one another in the class. At first, the instructor perceived some hesitation when she asked students to come to the board to work on the problems with her. Another student mentioned that this was not a typical feature of the math classes that she had taken. This hesitation disappeared in the latter part of the course, as the students became more comfortable with the idea of working through problems on the board together. When promoting these activities in class, the instructor positioned it as a group activity and that the goal was to make sense together. Additionally, the instructor noted that some of the students preferred talking in class, as opposed to reasoning on paper. Thus, the instructor provided opportunities that involved a mix of activities that involved writing and verbal sharing, so that a diverse range of students’ ideas could be expressed. It is important to create multiple opportunities for students to express themselves, but to also be sensitive of their preferences.

Recommendation 4: Considering logistical factors. Two of the three students commented a few times to the instructor that they were able to take the course because the price was “just right” and that the timing worked well for them. The course was free and was held in the evening from 6-7 PM at a community center. Thus, timing and price factors are important considerations to take into account when trying to design a course that can reach students with varying schedules and budgets. Working with a local community center can help one understand the needs and interests of the local population.
Recommendation 5: Being flexible in how one approaches adult learners. The students in the class had unique ways of reasoning about mathematics. The instructor wanted to create an inclusive atmosphere, so she emphasized that there are multiple ways to make sense in mathematics. She set these norms early on in the class, so that it would become apart of the classroom culture. When a student engaged in a form of reasoning that was not clear, she tried, when possible, to ask students follow-up questions such as “can you say more.” The instructor wanted to maintain a respect for the students’ ideas as well as promote critical thinking. When designing a course, it is important to set norms early on so that it becomes a part of the classroom culture. Additionally, it is important to develop respectful and probing questions that encourage the students to think more critically about their ideas.

Personal interest courses are an important place where learning can occur. In an article published in the American Scientist (Falk and Dierking, 2010) entitled The 95 Percent Solution School is not where most Americans learn most of their Science, the authors claim that the average American spends less than 5 percent in school and based on this information, there should be more investments in free-choice learning resources. Although this article is primarily about science education, there are parallel ideas that can be applied to mathematics learning. Given that the average American spends less than 5 percent of their time in school learning math, there is an increasing number of Americans participating in adult education activities (Kim, et al., 2004), and examples of adults using math both as a tool for reasoning in multiple contexts as well as finding it an inherently valuable and enjoyable activity in of itself (from the present research study),
then there should be more opportunities for people to pursue self-enriching learning experiences in math.

The present study has explored how adults engage in and make sense of mathematical ideas in a mathematics class and some of the characteristics that these adults bring with them. When engaging in meaning-making in math, adults can leverage their everyday experiences in ways that are prompted (e.g., class activities) and unprompted (e.g., class discussions) and they are comfortable with thinking about and doing math in ways that are familiar to them (e.g., use of symbols, doing arithmetic). Furthermore, adults can have an intrinsic value for and enjoyment in doing mathematics. This is the case when mathematics is seen as not only an activity that can help in one’s everyday life, but also as a recreational activity in and of itself.

Given that the current study involved three participants, it would not be appropriate to generalize these findings to all adults who seek out a personal interest course in mathematics. However, it does provide evidence of how adults can learn as well as revealing some of their attitudes toward math, and it can serve as a resource for those who are interested in designing a mathematics classes for adults and for understanding adult learners in a mathematics context. Further research should be done to explore what factors influence adults to pursue learning opportunities in math and the impacts that they may have on their future learning in math, as well as attitudes toward math.
Appendix 1  Class-by-Class Topics, Objectives, Activities, and Resources

Unit 1: Numerical Thinking, Magnitudes, and Scale

Class 1
Topic: Mental math
Objectives: The goal of the class is for students to discuss the strategies that they use to do approximate math (i.e., without a calculator), and to do a challenge exercise that involves manipulating numbers (magic squares activity).

<table>
<thead>
<tr>
<th>Activity</th>
<th>Resources</th>
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<tbody>
<tr>
<td>Entry task: Can you describe a situation in which you have had to (or can) do mental arithmetic? What strategy did you use? Is it important to be exactly right in these cases? Why?</td>
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<tr>
<td>3x3 Magic Squares Activity</td>
<td>Information about magic squares: <a href="https://en.wikipedia.org/wiki/Magic_square">https://en.wikipedia.org/wiki/Magic_square</a></td>
</tr>
<tr>
<td>Weekly reflection: Sometimes, there are really large numbers where it is hard to have an intuitive sense of that number. Give an example of a large number that you see and/or use in your everyday life. What are some strategies that you can use to make sense of large numbers?</td>
<td>Reading about how to make sense of large numbers: <a href="http://io9.com/how-to-comprehend-incomprehensibly-large-numbers-1531604757">http://io9.com/how-to-comprehend-incomprehensibly-large-numbers-1531604757</a></td>
</tr>
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Class 2
Topic: Number sense and making sense of large numbers
Objectives: The goal of the class is for students to understand that they all have an approximate and intuitive number sense as well as to explore strategies that they can use to have large numbers make sense to them.

<table>
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<tr>
<th>Activity</th>
<th>Resources</th>
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<tbody>
<tr>
<td>Entry task: Can one have a concept of number</td>
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without being able to count? Do you think that you can know approximately how many items there are without counting?

Present students with a video clip of a tribe that does not have a formal counting system, but has the ability to keep track of quantities in an approximate way. Then have students revisit the entry task question above. Have their responses changed? Why or why not?

Weekly reflection: Where do you find the use of fractions and percentages in the world around you? Please bring an example to class next week where you see fractions or percentages being used. It can be a flier, a news article, a description of an experience you had, or something else. Also, please describe how you made sense of the example.

Unit 2: Fractions and Percentages

Class 3
Topic: Making sense of fractions and percentages
Objectives: The goal of the class is for students to describe the sense-making strategies they use when they encounter fractions and percentages.

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<tr>
<th>Activity</th>
<th>Resources</th>
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<tbody>
<tr>
<td>Entry task: Where do you find the use of fractions and percentages in the world around you? Please bring an example to class next week where you see fractions or percentages being used. It can be a flier, a news article, a description of an experience you had, or something else. Also, please describe how you made sense of the example.</td>
<td>Video about the tribe: <a href="https://www.youtube.com/watch?v=NDM8G5tuHF8">https://www.youtube.com/watch?v=NDM8G5tuHF8</a></td>
</tr>
<tr>
<td>Ask students to make sense of a few problems about percentages and fractions that are conceptual in nature. Here are some examples: 1. How would you represent 40% as a fraction?</td>
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</table>
2. Which option would you choose to save the most money:
   a) 10% off a $200 item
   b) $15 off a $200 item
   c) both options above will gave the same savings

Weekly reflection: Please think about the following scenarios:
1. You invest your money into a savings account with 1% interest. Would you earn more if you got simple interest or compounded interest? Why?
2. What is the relationship between annual percentage rate and interest rate? Which one is larger?

Class 4
Topic: Understanding and making sense of interest rates
Objectives: The goal of the class is for students to understand simple and compounded interest rates, and to use the interest rate formulas to make sense of problems.

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<tr>
<th>Activity</th>
<th>Resources</th>
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<tbody>
<tr>
<td>Define the following terms:</td>
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<tr>
<td>• Simple interest rate – do not get interest</td>
<td></td>
</tr>
<tr>
<td>on the interest</td>
<td></td>
</tr>
<tr>
<td>• Compound interest rate – you get interest</td>
<td></td>
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<tr>
<td>on the interest</td>
<td></td>
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<tr>
<td>• APR/APY – yearly interest rate</td>
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<tr>
<td>• Formula for calculating interest rate</td>
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<td>Future Value = Present Value * (1 + r/n)^t</td>
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Entry task: Please think about the following scenarios:
1. You invest your money into a savings account with 1% interest. Would you earn more if you got simple interest or compounded interest? Why?
2. What is the relationship between annual percentage rate and interest rate? Which one is larger?
Students are asked to make sense of problems using the simple and compounded interest rate formulas.

Simple interest problems and tutorial: http://www.mesacc.edu/~marfv02121/readings/applications/interest.htm

Compounded interest problems: http://www.mathwarehouse.com/compound-interest/formula-calculate.php

Weekly reflection: Where do you find the use of probabilities in the world around you? Please bring an example to class next week where you see probabilities being used. It can be a flier, a news article, a description of an experience you had, or something else. Also, please describe how you made sense of the example.

Unit 3: Probabilistic Reasoning

Class 5
Topic: Basics of probability
Objectives: The goal of the class is for students to understand and make sense of simple independent events (e.g., flipping a coin) and to understand the gambler’s fallacy and what it looks like in one’s everyday life.

<table>
<thead>
<tr>
<th>Activity</th>
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<tbody>
<tr>
<td>Entry task: Students are asked to reason about the following outcomes for flipping a coin.</td>
<td></td>
</tr>
<tr>
<td>-Probability of getting Heads</td>
<td></td>
</tr>
<tr>
<td>-Probability of Heads or Tails</td>
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</tr>
<tr>
<td>-Probability of Heads on the first flip and Tails on the second flip</td>
<td></td>
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<tr>
<td>-After two flips, the probability that you will have one Heads and one Tails</td>
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<tr>
<td>-Probability that you will have 10 Heads in a row</td>
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<tr>
<td>Students will read an article about the gambler’s fallacy and have a discussion about how they have seen the gambler’s fallacy wreak havoc in their everyday life.</td>
<td></td>
</tr>
<tr>
<td>Reading about gambler’s fallacy: <a href="http://bigthink.com/praxis/how-the-gamblers-fallacy-wreaks-havoc-in-everyday-life">http://bigthink.com/praxis/how-the-gamblers-fallacy-wreaks-havoc-in-everyday-life</a></td>
<td></td>
</tr>
<tr>
<td>Students are asked to reason about outcomes for problems that involve dice and playing cards.</td>
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</tbody>
</table>
Weekly reflection: Students are asked to think about how they would go about solving the Birthday problem.

There are 10 people in a room. What is the probability that at two people in there have the same birthday?

Reading about how to think about the Birthday problem from a conceptual perspective:
https://betterexplained.com/articles/understanding-the-birthday-paradox/

Class 6
Topic: Using probability to make sense of problems
Objectives: The goal of the class is for students to understand and make sense of more challenging problems such as the Birthday problem and to calculate the odds of winning the lottery.

<table>
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<tr>
<th>Activity</th>
<th>Resources</th>
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<tbody>
<tr>
<td>Entry task: Students are asked to describe the strategies that they used to make sense of the Birthday problem.</td>
<td>Reading about how to think about the Birthday problem from a conceptual perspective: <a href="https://betterexplained.com/articles/understanding-the-birthday-paradox/">https://betterexplained.com/articles/understanding-the-birthday-paradox/</a></td>
</tr>
<tr>
<td>Students are then paired into groups to make sense of the following problem: what is the probability that you will win the Powerball lottery?</td>
<td>Reading about how to think about lottery problems from a mathematical perspective <a href="https://en.wikipedia.org/wiki/Lottery_mathematics">https://en.wikipedia.org/wiki/Lottery_mathematics</a></td>
</tr>
<tr>
<td>Weekly reflection: Come to class next week ready to describe your individual project and the progress that you have made on it. Also, please identify what questions you still have.</td>
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</table>

Weekly reflection: Students are asked to think about how they would go about solving the Birthday problem.

There are 10 people in a room. What is the probability that at two people in there have the same birthday?

Reading about how to think about the Birthday problem from a conceptual perspective:
https://betterexplained.com/articles/understanding-the-birthday-paradox/

Class 7
Topic: Individual Projects
Objectives: The goal of the class is for students to have a chance to work on their individual projects and for the instructor to provide feedback and resources, as needed.

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<th>Activity</th>
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<tr>
<td>This class is set up as a workshop, where students share their projects and the progress that they have made. In this class, the instructor and students also provide guidance and resources for moving forward.</td>
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Appendix 2  Interview Questions

I. What is Math

Prior to this class, what was your definition/perception of math? What were some activities that involve math?

After this class, what is your definition/perception of math? What are some activities that involve math?

II. About Learning Math

What kinds of activities did you do in the previous class?

How did they affect/influence your understanding of the math?

How did the approach affect your ability to make meaning/sense of the math concepts? Did you feel comfortable asking questions? Challenging assumptions? Using your own strategies?

III. About this Class

How did the activities affect/influence your understanding of the math?

How did the approach affect your ability to make meaning/sense of the math concepts? Did you feel comfortable asking questions? Challenging assumptions? Using your own strategies?

IV. Approach/Attitudes towards Math

Prior to this class, what were your perceptions about math?

After this class, what are your perceptions about math?


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