A Study in the Use and Effectiveness of a Mathematical Modeling Project-Based Curriculum to Increase Students’ Interest, Enjoyment, and Success in a Precalculus Math Class

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A Study in the Use and Effectiveness of a Mathematical Modeling Project-Based Curriculum to Increase Students’ Interest, Enjoyment, and Success in a Precalculus Math Class

Mackenzie Ann Chaput

A Thesis in the Field of Mathematics for Teaching
for the Degree of Master of Liberal Arts in Extension Studies

Harvard University

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Abstract

This thesis describes the construction of a precalculus course that utilizes 21st century technologies, experimental observation, and data collection methods to facilitate a student’s ability to model real life situations mathematically. The course was centered on projects catering to students’ personal interests, tailored to each student by allowing him or her to solve current problems that were of particular interest to him or her; the exploration into all corners of society was encouraged.

People learn and perform best when they are genuinely interested in the material they are studying as shown in Oystein Sorebo and Reidar Haehre’s (2012) research on using educational gaming as a tool for learning. Sorebo and Haehre’s research is based on the Self-Determination Theory developed by psychologists Edward L. Deci and Richard M. Ryan (2000), which highlight humans’ basic need for autonomy, competence and relatedness. These three emotions are said to “foster the most volitional and high quality forms of motivation and engagement for activities” (Deci and Ryan, 2000, Overview para. 2).

In accordance with the foundational principles of the self-determination theory, it was determined in this thesis project that if you allow students to conduct research in areas of personal interest and develop their own mathematical models it enhances the retention of knowledge and promotes a deeper interest in mathematics. The results of this thesis provided more evidence that bridging student’s personal interests with mathematics, through a project-based curriculum where students desire to build
mathematical models is fueled by their desire to find a solution, both nurtures student’s growth in problem solving and increases happiness and satisfaction associated with mathematics.
Dedication

To Wes for his eternal support and optimism.

And to my friends for their encouragement.
Acknowledgements

Thank you to Andrew Engelward and Brendan Kelly for their support, guiding ideas and determination to see me through this thesis. Thank you to Nate Meleo for his openness to new ideas and support in executing new initiatives in the classroom.
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Chapter 1 Introduction

The precalculus course developed for this thesis was designed to benefit both students and teachers. For students, the hope was that this course would reveal math’s utility and spark an interest and curiosity in mathematics. For teachers, the benefits are two fold. First to provide a curriculum that intrinsically motivates students to learn mathematics because it is fun and interesting, increasing their engagement in math and happiness associated with math. Second as a result of the students exploring math through self-selected topics the teacher has the opportunity to gather information about the students’ interests fostering a more personal student-teacher relationship.

Sorebo and Haehre explain that “students’ interest in the core subject and their intrinsic motivation in learning through gaming are important for their perceptions of how relevant they believe the gaming session is for the discipline they are studying” (Sorebo and Haehre, 2012, p. 345). In my case, students’ interest in mathematics and perceptions of the relatedness of conducting research and utilizing mathematics to solve problems they identify impacts their learning and plays a significant role in student buy-in and in the student’s mindfulness of mathematics.

In this precalculus course students completed four projects, two each semester. The first semester they were charged with identifying a problem of interest to them, constructing models to mirror the situation, then present solutions that were grounded in mathematics. The medium for which they delivered their first project was a 10-minute video. The student’s second project was an iteration of the first project where they were
instructed to pare down the 10-minute video to a 5-minute video, requiring them to be organized, concise and intentional with every fragment of the video. They were instructed to build upon and develop their thought process as they learned more mathematical techniques; the goal was to have the projects drive the students’ need to discover mathematics. William Pinar supported the rationale for guiding students in this manner in his book *What is Curriculum Theory* where he noted that teachers should present “academic knowledge, but configured around faculty and student interests” (Pinar, 2004, p. 21). The rationale for Pinar’s previous thought is that teachers and students need to “connect academic knowledge to self-formation” (Pinar, 2004, p. 21) and that in this intertwinement there will exist a strongly rooted thought process that will remain with the student long after high school. The students expressed to me that after their work on the video projects, where math and their personal interests intersected, that they realized the utility and need for math. Where initially there was great hesitation and aversion from the unknown format of a project-based curriculum came enthusiasm and excitement towards mathematics. Many of the students professed that they wanted another shot at the video projects in the second semester proclaiming they had many ideas for topics they would like to study.

During the second semester two more projects were piloted, each with a different format and goal in mind. The first was a semester long topic review project with three deadlines for the students to meet. Each student drew straws to determine the order in which they could pick their topic from a list of topics that we studied during the year. The students were responsible for becoming an expert in this topic and charged with the task of preparing and executing a review class at the end of the year for their classmates.
Bearing the responsibility to deliver a satisfactory review for their peers and their desire to perform at a high level provided them motivation to succeed in this task. The rationale for the three deadlines and structure of the review project will be discussed in more detail in Chapter 4. The second project of the second semester was adopted from a Moody Math Challenge. Moody’s Mega Math challenges present a prescribed problem where students are asked to develop a solution in a tight time constraint of eight hours working in groups of three or four (SIAM, 2013). The contrast of being given a problem, not one that they chose, and having to work with others, as opposed to alone, was a great learning experience for them. They were permitted two weeks of both class time and homework time to propose a solution, grounded in mathematics, to alleviate the trash surplus produced by our society. The parameters, successes and conclusions of this project are considered in Chapter 5.

Following in the suit of creativity, challenging problem sets were distributed first on a weekly basis, then on a bi-weekly basis (to help balance their workload), to manufacture grit and perseverance. Drawing from different texts, as well as from the author’s personal educational experiences and magazines these problem sets were written so the students had difficulty googling the problems, thus they learned to use the internet’s vast body of knowledge in support of their thinking, not to replace their thinking. Problem sets can be found in Chapter 6 with more thoughts on their purpose and configuration.

Recognizing to the necessity to ensure mastery of procedural content, what is sometimes referred to as traditional math pedagogy, weekly assessments were administered. The students were given a set of topics to study for the assessment,
typically the topics that were covered during the previous week in class, and they were charged with the responsibility of preparing themselves for the assessment. For some, this meant doing very little, and for others, as they learned throughout the year, this meant doing a considerable amount; for some the self-actualization learning curve required a great deal of effort. The structure of these assessments and a discussion of testing in this format are discussed in Chapter 7.
Chapter 2  Course Outline

This chapter will highlight the structure of class time and the author’s reflections on what went well, and what could be changed for years to come. To give the reader a feel for the atmosphere of the learning environment in which this precalculus course was piloted it is necessary to note that this project was conducted at a boarding school in New England. It is useful to illuminate the students’ busy schedules to provide a vivid picture of what their lives are like as students. It is likely that this depiction holds true for most high achieving high school students. Students attend each of their classes four times per week for either 45 or 50-minute time blocks. The boarding school has half days on Wednesdays to allow time to travel to sporting events that can be up to 2 ½ hours away. About once a month the school has a half-day of class on Saturday to recoup lost class time. Similar to a public high school, students typically take five core courses, sometimes six, their grade-level requirements (a Foundations course, Human Development or Transition to College) and then additional electives such as art or music. Each core subject can assign up to 45 minutes of homework per class meeting and electives can assign up to 30 minutes. Students are required to participate in two active extracurricular seasons and must do some activity all three seasons. Consistent with most high school students they try to include in their schedule as many extracurricular activities as possible for enjoyment, and, potentially to increase their chance of getting into the college of their choice.
Having established that the students have busy lives, we next introduce the structure of the precalculus course and convey classroom tactics that aided in achieving the desired learning outcomes. There are four components that comprise a student’s grade: problem sets (25%), weekly assessments (30%), projects (30%) and a final exam (15%). In subsequent chapters the successes and failures of the problem sets, weekly assessments, and projects will be discussed. Now we describe how class time was spent and what teaching practices led to the best learning outcomes.

As has been found out by many teachers, it can be the case that one can be an expert in the content, have a comprehensive lesson plan and still conduct a class that is not engaging, meaningless, and quite possibly boring. David Urso states in his doctoral dissertation titled *Sustained partnerships: The establishment and development of meaningful student-faculty relationships* that to build an ideal atmosphere for learning teachers need to establish a classroom culture that is warm and welcoming (Urso, 2012, p. 201). As previously noted, Pinar (2004) found that students relate to material better when facts are presented around their interests and this proclamation is bolstered by Urso’s research. David Urso revealed many tactics that (successful) faculty members employed to connect with students including playing music that the students requested, showing up to class early and staying late to converse with students, and acknowledging student’s birthdays in class (Urso, 2012, p. 201). Urso found that “as a unit, these faculty members were credited with building a culture that was relaxed and fun and made students look forward to going to class and establishing personal relationships with the instructors” (Urso, 2012, p. 201). Getting students to invest in your class and put forth effort is critical to the success of executing a student-led project based curriculum.
Teachers ability to create buy-in and motivate the students to work hard hinges on their commitment to the students as individuals and the course objectives. It was necessary to believe in the curriculum and demonstrate that belief everyday; the level to which the students perceived my commitment directly impacted the classroom culture. For most of the students this was the first time that a class was centered on them; where their interests and opinions were the medium for which material was presented. For new international students in the class, this was, without exception, the first time they had experienced a class like this. For me overcoming the challenges of introducing a new way of learning was not always easy and I owe a lot of credit to the support I received from my peers in the math department and Nate Meleo, the math department chair.

Tactics I employed to achieve the goals of the curriculum and convey my commitment to both the students and to the format of class include, giving them my phone number to text me when they encountered difficulties, meeting them outside office hours to discuss math or other issues they were having, and playing music they requested (within reason) while they worked on difficult problems in class. One interesting idea a student gave me last year is giving out ‘brownie points’ when the students catch me making a mistake on the board; this builds excitement and humility for everyone in the class. The students work as a team to collect ‘brownie points’, they get one point per mistake, and when they collectively accumulated ten points I bake them brownies. Brownie points are a fun way to build camaraderie in the classroom and have the students work together to achieve a common goal.

Another way I built cohesiveness was at the start of each class I presented a city of the day, a picture of a city from around the world that the students would try to guess.
Although seemingly off-topic this daily activity strengthened my effort to build an inclusive class where students value each other’s input, ideas, thoughts, and perspectives. One reason that this was effective is that most classes at the boarding school are very diverse, often having students from many different countries, different ethnic backgrounds, different socioeconomic status’, and different upbringings, each possessing a unique perspective. Getting students to recognize this and value each other’s intellectual thoughts is integral in establishing a culture that is best fit for learning from one another. Often the students were shocked to discover who was able to come up with the answer to the city of the day and it frequently sparked a short conversation on the experience that led to their knowledge of that particular city. The city of the day activity touches on Urso’s discovery that “the most important factor in the faculty members’ ability to draw students into the lecture [in this manner] is their awareness of the life experiences of the students in the room” (Urso, 2012, p. 202). Furthermore, I learned that students from some cultures, for example many of the students in my class from China, hesitate to speak in class and are not accustomed to sharing their thought process or experiences. This is a result of their early education where they were taught to listen intently, only speak when spoken to, and most importantly, the teacher is assumed to be the only one with the correct answer; they are often taught one specific way to do each problem. There are many benefits of a having a multi-cultural class, but two in particular stand out to me, international students are able to learn how to articulate their thoughts and domestic students are given a chance to better learn how to listen to other students.

The process of building a student-led classroom is exactly that, a process. Most students were habituated to teacher-led classes, where the teacher provides all
information necessary to be successful and the teacher dictates the direction of classroom
discussion and the manner in which the students should work. It was surprising to
experience a fair amount of resistance from the students when I tried to give them more autonomy. They were unsure of themselves, and they were unsure of me as well, and it seemed as if some questioned my competency as a teacher. In September and October I frequently questioned what I was trying to do and the methods by which I was using to accomplish my goals, but thankfully I had the support of Nate Meleo, and many others who encouraged me to follow through with the teaching project, and it turned out successfully. The students became more responsive to the new teaching style; they learned to embrace autonomy; they became less averse to taking risks; In addition, they said that they loved the connectedness they felt towards their classmates and became more confident in their ability to break down difficult problems and work together to find solutions.

There were certainly times during the year when it was necessary to move more quickly through particular curriculum content and practice procedural knowledge. For example, it was necessary to do this when the class needed to find the inverse of 2 by 2 and 3 by 3 matrices. When introducing a topic that we needed to work through more directly as opposed to a topic where the students could spend time discovering a concept out of necessity, it was crucial to discuss the who, what, when, where, and why of the topic. Such history provides context that demonstrates the need to learn about the topic, any time it is possible to tell a story or put into context what is being talked about will help to engage the students. When teaching a less-than exciting topic, it was found that if humorous stories were included, or if I told them a story about the people I was with
when I studied the topic for the first time, then it really changed the demeanor of the class. This tactic is supported by Olov Viirman a Swedish mathematician who studies teaching practices in a university setting, stating “if used well humor can be very effective in promoting student engagement and interest in the class and is an important part of teacher immediacy” (Viirman, 2015, p. 1174). Every chance one gets to articulate how the skills they are learning will translate to success in solving real world problems draws them forward. Students curiosity grew tremendously throughout the year, they went from not really caring “why” to demanding to know “why” every time. This is an example of what Viirman referred to as a motivational routine, demonstrating math’s intra- and extra-mathematical utility, which is the motivation to study the material at hand for both its usefulness in later math courses and for its practicality in other life endeavors (Viirman, 2015, p. 1173). For example, when we were studying vector projections and using the dot product, I had difficulty elaborating what the dot product is beyond the fact that it is a scalar product of two vectors, what I had been told. My students demanded more, so we spent some time investigating using the Internet and had a great discussion on what dot products are and why they are necessary and useful. Together, we discovered many interesting facts about the origin of dot products; dot products came into use at the turn of the 20th century from linear algebra work performed by J. Willard Gibbs, a professor at Yale, who is credited with creating the basis for modern linear algebra (McGill University, 2016). As a result of this conversation the students’ interest in dot products deepened and their experience learning about them had a more embodied context, making the geometric proof of the law of cosines more meaningful and hopefully created a lasting memory of the study of dot products. From the vector proof of the law of
cosines, the students concluded many other uses such as finding an angle, $\theta$, via $\cos(\theta) = \frac{u \cdot v}{\|u\| \|v\|}$. Both of the aforementioned proofs are succinctly summarized by Robert Winters, a professor of mathematics at MIT, and are below in figure 1a and 1b (Winters, 2016).

**II. The Law of Cosines:** Given any triangle with sides of length $A$ and $B$ adjacent to an angle $\theta$ and with the side opposite this angle of length $C$, then $C^2 = A^2 + B^2 - 2AB \cos \theta$.

**Proof of the Law of Cosines** – Referring to the variables in the diagram, this is a straightforward application of the Pythagorean Theorem and basic trigonometry. The case for an acute angle shown. The proof is similar for an obtuse angle.

![Diagram](image)

In order to relate this to the dot product, we need to use a few easy-to-show facts about the dot product, namely:

a) $u \cdot u = \|u\|^2$ where $\|u\|$ denotes the length of the vector $u$;

b) $v \cdot u = u \cdot v$ (commutative law); and

c) $u \cdot (v + w) = u \cdot v + u \cdot w$ and $(u + v) \cdot w = u \cdot w + v \cdot w$ (left and right distribution laws).

Using these facts, the left-hand side of our vectorized Law of Cosines reads:

$$\|u - v\|^2 = (u - v) \cdot (u - v) = u \cdot u - u \cdot v - v \cdot u + v \cdot v = \|u\|^2 + \|v\|^2 - 2u \cdot v.$$

**Figure 1a Proof of the Law of Cosines**

Comparing this to the original expression, we get the all-important property that $u \cdot v = \|u\|\|v\| \cos \theta$ where $\theta$ is the angle between the two vectors $u$ and $v$.

The significance of this property is that the left-hand side is purely algebraic and the right-hand side is purely geometric. This opens the possibility that we can use basic algebraic operations to calculate geometric quantities like lengths and angles. For example, we can rewrite this result as:

$$\cos \theta = \frac{u \cdot v}{\|u\| \|v\|}.$$

**Figure 1b Angle Measure**
Building relationships with one’s students and fostering relationships among students is essential to creating an environment where the students are motivated to work hard, take risks, and collaborate with one another. Removing the fear of judgment is also crucial to obtaining committed citizenship in a class. As a teacher, it is important to be humble and confident, and above all make sure that one’s students know that they are cared about both as people and as mathematicians. Creating and maintaining a supportive environment affords teachers the opportunity to have learning experiences that are student-driven, and this is not possible without a trusting and safe environment. Students spent the majority of class time working on difficult problems with minimal assistance from me; if they were stuck, I posed questions to them, a suggestion given to me by Nate Meleo, this allowed them to find their own path to a solution following no direction but their own. Over time they also adapted to using one another as resources, posing questions to each other, instead of working in their own silos, they worked as a community bringing one another along in their thinking. Asking questions instead of projecting information at students allowed them to assume more control of their learning and possess accountability, “the social pressure to justify one’s views, behavior, and decisions to others” (Fandt, 1991, p. 300).
Pre-Calculus Honors Syllabus

Course Overview
This calculus course has been designed to connect mathematical concepts with applications in physical sciences, engineering, social sciences, biological sciences and other real world occurrences. In doing so we will explore practical values of mathematics and illuminate common uses of the concepts and procedures we study. I am looking forward to journeying with you through the gateway to higher mathematics: Precalculus!

Required Text
Pre-Calculus, 7th Edition, Cohen, Lee, Stär

Course Materials
- Calculator
- Textbook
- 1" Binder
- Notebook
- Pencils & Pens

Helpful Resources
- Common Mistakes: save yourself some frustration glancing at this list of common algebra and calculus errors https://www.math.hmc.edu/calculus/tutorials/algebrareview/
- Famous Curves: play around with the parameters to see how the curves change http://www-groups.dcs.st-and.ac.uk/~history/Java/index.html
- Tad Wert’s YouTube channel, teacher at Harpeth Hall: https://www.youtube.com/playlist?list=PL3C5882D54030A3A01
- PatrickJMT’s YouTube channel: https://www.youtube.com/channel/UCFe6jenM1Bc54qlbJGRZQ
- Your classmates, friends, family members, teachers, textbooks, the library, the Academic Resource Center (ARC), the Internet and me!

Office Hours:
Monday/Tuesday: 7:30-8:00am | Wednesday: Block 7 | Thursday: Block 5 | By appointment, please email me if you would like to set up a time to meet.

Course Schedule Topics
Chapter 1: Review and Fundamentals
Chapter 2: Equations and Inequalities
Chapter 3: Functions
Chapter 10: Trigonometry
Chapter 11: Systems of Equations
Chapter 4: Polynomial and Rational Functions. Applications to Optimization
Chapter 5: Exponential and Logarithmic Functions
Chapter 14: Additional Topics in Algebra
Limits and Derivatives

2015-2016
Grading

Grades will be comprised of the following:

- Weekly Problem Sets & Practice Problems (25%)
- Weekly Assessments (30%)
- Projects (30%)
- Final Exam (15%)

Assigned Work Policy

Studies show that the best way to learn math is to do math, not just read over problems in the textbook or your notes. That said I strongly encourage you to make a solid effort on the practice problems. You will have the solution manual available to you for all practice problems. “A-level” students will attempt each problem with an earnest effort and consult the solutions when necessary. I ask that you show your first attempt in pencil and then your corrections in pen.

Problem Sets are due at the start of class on Thursdays. If you need to miss class please contact me within 24 hours of the missed class to arrange a plan to turn in your problem set. If you do not email, call, or see me in person to make alternate arrangements you will receive a zero for that assignment.

You will have weekly assessments on Thursdays. These will assess your understanding of the material covered that week and weeks prior.

You will have 4 major assessments, projects, which will assess your ability to apply mathematics to organic, real world problems. I want the content of your projects to be of personal interest to you and thus I expect you to invest yourself in producing work that you exude confidence in presenting to stakeholders whoever that may be. Your projects will include collecting data, analyzing data, creating mathematical models, formulating action plans and then articulating your thought process and ideas in a well-thought out presentation. The projects comprise a large portion of your grade and thus I expect they will consume a large portion of your time. They will be graded according to the project grading rubric posted on OnCampus.

All work should be written up legibly and carefully; working out problems on scratch paper and then transferring them to a fresh sheet might be a good idea. Please show all your work, credit cannot be given for answers without work. I will do my best to give you as many points as possible, and I expect you to do your best working out the problems. Unless otherwise noted please give exact answers instead of decimal approximations. For example, √7 instead of 2.645751311…

You may use your resources to complete the practice problems, problem sets and the projects, but all work that you hand in must be your own or properly cited if you use someone else’s idea. See Tabor’s policy on Academic Integrity or ask me if you have any questions.

You have many resources available to aid in your success: your textbook, your notes, your classmates, your friends, any teachers, the Internet, the resources listed above, and me! You can email, text or call me (before 10:00pm) if you have any questions or need assistance on an assignment, I am here to help you and excited to learn with you!

Good Luck!

Mrs. Chaput
Chapter 3  Video Projects

“Math textbooks [are] essentially recipe books. Now all those math recipes have been coded into devices, some of which we carry round in our pockets.” (Shapiro, 2014, p. 5) The emphasis on procedural knowledge in mathematics education is giving way to an emphasis on synthesizing mathematical knowledge as our lives becomes more and more intertwined with technology. We can program computers to compute routine exercises, simply write the program and prompt the user for inputs; our students interact with these interfaces almost non-stop, but do they know how they work? Do they have the logic and critical thinking skills to identify problems and employ technology to help them uncover and present solutions? To stay relevant and be desirable in the workforce our students will need to be more adept at identifying problems, collecting data, utilizing technology, and formulating solutions that are timely and presented in an accessible format. This has been true for years and for many generations; the difference now is the rate at which technology is advancing. In a discussion with a businessman in the agriculture industry he stated that “technology has impacted the way every industry conducts business and it will continue to do so at an increasing rate” (Sheldon, 2016).

Our students not only need to be technologically literate but have the ability to think critically in order to stay current with changing technologies; they need to be tech savvy, ready to take on the new technological challenges. “The skill that is in great demand today, and will continue to grow, is the ability to take a novel problem, possibly not well-defined, and likely not having a single “right” answer, and make progress on it, in some
cases (but not all!) ‘solving’ it” (Shapiro, 2014, p. 5-6). To address the need for people who can think critically and can take on open-ended projects in the real world students completed open-ended video projects during the first semester. The grading rubric for project 1 which was adapted from (Buck Institute for Education, 2013) is shown in Figures 3 and 4.

---

**Project 1 Grading Rubric**

**Overview**

Students will utilize their mathematical skills to solve real organic problems that are of interest to each individual or pair of students. Students will gain experience conducting research, collecting data, and constructing a coherent presentation grounded in mathematics that proposes a clear solution. Students will design an action plan for implementing their recommendations, as well as an impact analysis manifesting how their solutions would impact the target audience. i.e. the local community, the school, a business.

**Objectives**

Research-based projects will challenge you to be resourceful, think creatively, and use all the mathematical tools you have learned this year and in years past. Be sure to utilize a variety of sources to find all relevant information. Be sure your written summary and notebook pages containing your calculations meet the criteria as outlined on the rubric. A 5-10 minute video presentation articulating your thought process, trials and tribulations, solutions and impact analysis is required.

Get creative with this, I have intentionally provided a bare framework that allows you to take these projects in any direction that interests you and/or you and your partner. I am excited to see what you will choose to do! Above all else, please make sure that the work you submit is entirely your own. Ideas that you borrow from an outside source (anything other than your own brain) need to be cited following the American Psychological Association (APA) format.

**Project 1: Due October 29th**

Task: Think of a problem that has been nagging you recently, one that if resolved would add value to our community or larger society. Then use math to model what is occurring and develop a concrete action plan or product that is backed by your research. For example using vectors (Chapter 10 in our textbook) you can model the flow of people through spaces to improve customer experience in a spa or create a new marketing plan for a retailer. Please come see me sooner rather than later if you are having trouble deciding on a topic. Almost anything is possible!! Have fun!
This video project was distributed during the first week of school, when so many things were already new for students and potentially somewhat overwhelming. It is likely that the ambiguity of a project worth 30% of their grade added to their already elevated level of anxiety. The open-ended, limitless potential of this project caused anxiety for many students, with few constraints they gazed at me motionless and begging for...
direction. To which I responded with questions, asking where their interests lie, where there is a need for improvement and where mathematics might be helpful in alleviating problems or at least mitigating negative impacts. Again, I reiterate the importance of timing, that the new school year, with new classmates, and a new curriculum can cultivate a natural angst, and that this natural angst was compounded by the introduction of a lengthy challenging project. In the future, it would seem to be a better idea to wait until the second week of school to distribute such projects with the hope that students will have more time to devote thought and generate excitement with less anxiety. Adding to the turbulent start of the year was the fact that the students did not know me well at this point, some knew me informally, but none knew me as their teacher. I had not yet demonstrated my care and commitment to their education and to them as people, so when they were given a high stakes open-ended project they seemed skeptical of my intensions and my competency. As a result of the student’s uncertainty of both themselves and of me they did very little work on the project for the first three weeks of school; I think they were hoping I would say that the project would be changed and they did not have to complete it.

The manner in which a teacher distributes high stakes projects is critical in obtaining acceptance from students and gaining their trust as highlighted by Urso’s research on building relationships with one’s students. “The first impression the teachers make is significant” so if the first impression that are made with one’s students is a project that makes them uneasy, then this will be the impression they carry with them one works to change it (Urso, 2012, p. 203). This is not to say that I regret setting the tone that the course would be different and challenging, I simply wish to highlight the
importance of demonstrating care in the same breath that one discusses the nature of a course being taught. As Urso notes “in the earliest stages of the first class meeting, the students will develop critical opinions about the competence and capability of the instructors” this message is important to keep in mind when planning the distribution of assignments in the first week of school (Urso, 2012, p. 203).

The tone of the class changed six weeks into the semester, when I began to hear excited conversation surrounding the work that had been done on the projects. At this point, the students seemed eager to share their findings and wished they had more time to conduct research and compile their videos. It appeared as if the students wanted the time that it had taken them to accept the projects as beneficial to their learning and growth back as they neared the first deadline in October. Discussing ideas about math classes, Dan Meyer expressed it well in his TEDTalk titled, Math Class Needs a Makeover, when he said that it is more important to develop students’ problem solving and math skills through their curiosity and desire to have a more precise way to describe a real life scenario, than to give them stepping-stones from A-Z where they simply connect the dots. The transition from a traditional math course with contrived “smooth” problems that work out nicely and have specific answers, to a project-based course with “rough” and sometimes ill-defined problems, often with many answers can take some time. The time it takes is well spent, however, if one considers providing stepping-stones not to be the most effective form of teaching. As Dan put it, “it’s selling a product to a market that doesn’t want to buy it, but is forced by law to buy it” (Meyer, 2010). Adding to this sentiment, one can ask why work in a contrived environment when we live in a real environment with endless problems that need solving. Maybe it is not possible to solve
all the problems, but it is still useful to move closer to answers. This also provides the students with a more pragmatic view of reality in how difficult it can be to work with data and uncover viable solutions. Few real-world problems have one perfect answer, but rather many imperfect answers and this notion is highly beneficial to the students’ education.

When it was revealed to the students that they were going to have the opportunity to produce a second iteration of their video projects, there were mixed responses. Some were eager to keep working and were excited that they would have the opportunity to further pursue some of their research and re-work their presentations. Others were deflated that this was not the end of the project. The latter group felt that they had already done their best and there was no possible way that they could improve their work. At this point the students were introduced to a Harvard Business Review article titled Getting the Most out of your Product Development Process. This article illuminated the costs associated with improper allocation of resources and how this affects the timeliness of iterations of a product in development (Alder, 1996). For many students this was the first time they had thought about the process of developing a product; most thought about products only as physical objects, not as ideas or information. This provided a useful segue to the students’ second project and helped to motivate the necessity of multiple iterations when developing a product (video) for an audience.

The premise of second project was very similar to the first project, with one major change, the video needed to be pared down to a 4-6 minute clip from a 5-10 minute clip. This forced them to think about what information was integral to their research and proposed solution and also the most efficient and effective way to communicate this
information to their stakeholders. The rational for the shortened video was inspired by Anne Fisher who noted that “the average adult attention span has plummeted from 12 minutes a decade ago to just 5 minutes now”, according to research done by British bank Lloyds TSB (Fisher, 2013, p. 1). Beyond cutting the length of the video, students were charged with improving all aspects of their work, outlined below in Figures 5 and 6, project 2 grading rubric.

![Project 2 Grading Rubric](image)

### Project 2 Grading Rubric

**Overview**

You have identified a problem of personal interest to you, conducted research, collected data, and constructed presentation that is grounded in mathematics and proposes a clear solution to the problem. You will now improve or build upon the action plan you recommended as a solution, as well as conclude your presentation with a thorough impact analysis manifesting how your proposed solution will positively impact your target audience. i.e. the local community, the school, a business. Sell your idea!!

**Objectives**

- **Be resourceful:** Think what teachers or community members might be of help in either the composition of your presentation, providing data and knowledge or technical skills, or the implementation of your solution.
- **Think creatively:** What is going to grab the attention of your audience and propel them into action!
- **Be concise:** This time you will need to pare your video down to 4-6 minutes; that is a **maximum of 6 minutes**. According to an article written by Anne Fisher in Fortune Magazine, adults have an attention span of 5 minutes, down from 12 minutes just one decade ago (Fisher, 2013).
- **Cite your work:** It truly is important to give others credit for their knowledge and ideas. We are so fortunate to be surrounded by a vast body of knowledge and it is important to utilize the resources available. So remember, ideas that you borrow from another source (anything other than your own brain) need to be cited, please follow the guidelines set forth by the American Psychological Association (APA).

**Project 2: Due December 16th**

Task: Complete a second iteration of your project. What went well? What didn’t go so well? What do you wish you had more time to work on? What ideas came to you at the last moment that you weren’t able to work in? Take the feedback I gave you and the peer feedback you receive and put forth your best effort in creating a video that will really “wow” your audience. You all have great material to work with, whether you need to do a bit more research, work on your presentation, or make tweaks in a few areas you all have a great framework to build on. Please ask me questions and use all your resources, that includes your peers, classmates and friends! If you can sell a friend on an idea you can sell an audience, often our friends are our hardest critics. And remember to have fun!

---

 Figure 5 Project 2 Grading Rubric
The month of November was another significant turning point, the students’ showed minimal resistance to new ideas and in their discoveries many realized for the first time in their math careers that the body of mathematical knowledge is so vast that it is virtually impossible to know everything. For a group of high achieving honors students these projects and this precalculus class was likely their first encounter with the unknown and it seemed that they started to gain a certain amount of comfort with the unknown.

**Reference:**

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### Evaluation

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<thead>
<tr>
<th></th>
<th>Excellent</th>
<th>Competent</th>
<th>Needs work</th>
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<tbody>
<tr>
<td><strong>Knowledge/ Mathematical Understanding</strong></td>
<td>The project demonstrates profound mathematical understanding by using relevant and accurate points to support central ideas. Research is thorough and goes beyond the resources that were available in class, student finds unusual ways or places to get information (expert, community member, business or organization) (8-10)</td>
<td>The project uses knowledge which is generally accurate with only minor inaccuracies, and which is generally relevant to the student’s topic. Research is adequate but does not go much beyond what was available in class. (4-7)</td>
<td>The project uses little relevant or accurate information, not even that which was available in class. Little or no research is apparent. (1-3)</td>
</tr>
<tr>
<td><strong>Communication / Video Presentation</strong></td>
<td>The student(s) are articulate and effectively convey and justify their thought process. Use of visual aids successfully adds to the presentation. Compelling introduction and conclusion that leaves audience ready to take action. (8-10)</td>
<td>The student(s) conveys main ideas clearly, but lack robust explanations. Visual aids are somewhat unnecessary or distract from the presentation. (4-7)</td>
<td>The student(s) fails to capture the interest of the audience and/or is confusing in their delivery. Visual aids are unnecessary or distract from the presentation (1-3)</td>
</tr>
<tr>
<td><strong>Problem Solving</strong></td>
<td>Student(s) seeks out and uses feedback and critique to revise the project to better meet the needs of the intended audience. Student(s) further develop implementation and marketing techniques to get others on board with their idea. (4-5)</td>
<td>The student(s) showed an ability to overcome most obstacles, but with occasional difficulty and lack of focus. Student considers and may use some feedback and critique to revise the project, but does not seek it out. (2-3)</td>
<td>The student(s) got stuck frequently and lacked effort or determination to overcome these obstacles. Student does not consider or use feedback and critique to revise the project. (1)</td>
</tr>
<tr>
<td><strong>Wow factor!</strong></td>
<td>Student(s) produce work that uses ingenuity and imagination, going outside conventional boundaries. Project is new, unique and surprising; shows a personal touch. worthy of sharing with the faculty or greater community, whichever the target audience. (4-5)</td>
<td>Student(s) produce good, thoughtful work that shows some imagination, but stays within conventional boundaries. Student does not make many revisions from first submission. (2-3)</td>
<td>Student makes no revisions from first submission. (1)</td>
</tr>
</tbody>
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**Total:** ________/30 pts.

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**Figure 6 Project 2 Evaluation**
One issue that arose related to the student collaboration on projects, and the correct way to credit other students’ work. One student employed another student’s drawing and video editing skills after they heard how well the other student did on the first project. It was good to see that the students were working together on the projects, but the first student did not give appropriate credit to the other student for their work. For teachers considering project based work, it is important to be as explicit as possible as to what is the appropriate way to credit other students’ work and what constitutes plagiarism for the specific medium that they are being asked to use for the particular project.

Overall, the feedback that was received from the students was positive; they reported that they had enjoyed completing the projects, and that they were thankful to have had a chance to improve their work. Indeed, many students wanted to embark on a third iteration. Some asked if they were going to have a chance to take on a similar project in the second semester, proclaiming that they had so many “awesome” ideas to research. With these types of responses, the evidence suggested that the projects were a success, that allowing students to study math through their personal interests does indeed promote happiness and a genuine interest in mathematics. One other observation resulting from using the project approach to teacher was that students, and people alike, are often uncomfortable with change or new ways of learning, but once they see they can be successful and they are validated in their success by a teacher or other person in power they then become attracted to this new way of learning. This is the right moment to discuss with students that there are many useful avenues to study topics they are interested in.
Chapter 4 Review Project

During the second semester the students were given two more projects, one of which was as a class, collectively, conducting a full year review for the final exam. Each class section, as a whole, was granted the responsibility of preparing their own final exam review encompassing the 12 general topics that had been discussed during the year. Each individual was responsible for piloting one review class on one of the 12 general topics. The students’ preparedness and effectiveness in articulating mathematical concepts and conveying content through visual aids had an impact that was two fold. First it impacted their personal grade on the project, and second they were partially liable for their classmate’s readiness for the final exam. Following the common theme of this curriculum, modeling real world situations, this project mimics how the performance of one team member affects other team members’ ability to perform. Accountability in a business context according to businessdictionary.com is “the obligation of an individual or organization to account for its activities, accept responsibility for them, and to disclose the results in a transparent manner” (Definition of accountability, 2016). The majority of students have experience playing on athletic teams but few have experience dealing with the impact of a high or low performing team in an academic or professional setting. Their performance on this project got much more attention when they were given the task of conducting a 45-minute review class for their peers. The quality of their performance was immediately highlighted by their classmate’s reaction to the class, either with praise and affirmation or with criticism; I was somewhat surprised at how blunt and critical students
were of one another. Preceding the presentations it would be a useful exercise to discuss how to provide constructive feedback to one another, this exercise would be a useful in setting a positive tone.

Figures 7, 8 and 9 present the grading rubric and guidelines used for evaluating the review projects. The evaluation rubric was adopted from the Buck Institute (Buck, 2013).

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**Project 3 Grading Rubric**

**Overview**

Last semester you identified a problem of personal interest to you, conducted research, collected data, and constructed two iterations of a video presentation that proposed a solution to your problem that was grounded in mathematics. This semester you will have just one project in which you will be charged with the task of mastering a topic that we discussed/will discuss in class this year. There will be 12 topics to choose from and you will draw numbers to determine the order in which you choose your topic. There will be three components to your project, a research piece, a written analysis, and a presentation that will be delivered to your class in May. The presentations you prepare and deliver will be used as a course review for the final exam; thus your effectiveness and preparedness will affect both your personal grade on the project and your classmates preparation for the final exam.

**Objectives**

- **Be resourceful in your research.** You have so many resources at your fingertips, the Tabor Library, the Internet, our textbook, and people. Remember you have to cite EVERYTHING, including conversations with people. When using online resources remember that .edu or .org websites tend to be most reliable. If you have questions about the validity of a website please ask me.
- **Think creatively in designing your presentation.** What were your favorite classes this year, what was your least favorite? What was entertaining? When were you in “flow”, the feeling when you are so absorbed in a task that you lose track of time. What was the most effective way you learned? Visually? Verbally? In groups? Individually? When it is a Competition? Visual aids: make sure they add to your presentation, this includes videos.
- **Cite your work.** In accordance with Tabor Academy’s academic policies it is essential to give others credit for their ideas and knowledge. Ideas that you borrow from another source (anything other than your own brain) need to be cited please follow the guidelines set forth by the American Psychological Association (APA). Here is a link to a “how to” cite using APA format: [https://owl.english.purdue.edu/owl/resource/560/01/](https://owl.english.purdue.edu/owl/resource/560/01/). You can also use EasyBib with your Tabor login and password. You will be required to submit a bibliography at the end of your written submission and you will need to verbally cite during your presentation when necessary. A sample bibliography is provided for you on the last page of this document. If all you have is a group of URLs you fail this step. Please come and see me if you have any questions, it’s always better to ask when you are unsure.

**Part 1:** Research & Outline **Due March 3**

**Part 2:** Written Summary & Lesson Plan **Due April 14**

**Part 3:** In Class Lesson **May 6-26**
## Evaluation: ** See page 3 for details and guidance

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<th>Excellent</th>
<th>Competent</th>
<th>Needs work</th>
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<td>Research &amp; Outline</td>
<td>Research is thorough and goes beyond the</td>
<td>Research is adequate but does not go much</td>
<td>No research or very minimal research is</td>
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<td>resources that were available in class; student</td>
<td>much beyond what was available from class and</td>
<td>apparent and does not go much beyond class</td>
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<td></td>
<td>finds unusual ways or places to get information</td>
<td>our textbook. Outline is complete but lacks</td>
<td>notes, or contains inaccuracies. Outline is</td>
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<td>(expert, community member, business or</td>
<td>ingenuity.</td>
<td>incomplete.</td>
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<td>organization). Outline is coherent and</td>
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<td></td>
<td>organized. Thoughtful and inventive ideas</td>
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<td>proposed. (7-9)</td>
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<td>Research: ____/9 pts.</td>
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<td>Bibliography: ___/1 pt.</td>
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<td>____/10 pts.</td>
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<tr>
<td>Written Summary &amp; Lesson</td>
<td>Student articulates and effectively conveys</td>
<td>Student conveys main ideas clearly, but lacks</td>
<td>Student fails to illustrate their understanding</td>
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<tr>
<td>Plan: ____/18 pts.</td>
<td>mastery of their topic. Includes history and</td>
<td>robust explanations. Student demonstrates that</td>
<td>of the topic and/or capture my interest. Vague</td>
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<td></td>
<td>utility of topic. Compelling introduction and</td>
<td>the presentation will be adequate. Average</td>
<td>lesson plan. (1-7)</td>
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<td>conclusion that leaves me excited to attend</td>
<td>lesson plan. (8-12)</td>
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<td>their class. Detailed lesson plan that is</td>
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<td>inventive and exciting. (13-16)</td>
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<td>In Class Lesson</td>
<td>Student seeks out and uses feedback and</td>
<td>Student considers and may use some feedback</td>
<td>Student had an average/ below average lesson</td>
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<td>____/15 pts.</td>
<td>critique to revise his/her lesson plan to</td>
<td>and critique to revise the lesson plan, but</td>
<td>plan and lacked effort or determination to</td>
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<td></td>
<td>better meet the needs of their classmates. Use</td>
<td>does not seek it out. Use of visual aids is</td>
<td>make changes. Student does not consider or use</td>
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<td>of visual aids successfully adds to the</td>
<td>minimal and stays within conventional means.</td>
<td>feedback and critique to revise the</td>
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<td>presentation, memorable. Thorough discussion</td>
<td>(7-11)</td>
<td>lesson plan. As a result presentation</td>
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<td>of topic and fields questions with confidence.</td>
<td></td>
<td>was boring and ineffective. (1-6)</td>
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<td>(12-15)</td>
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</tr>
<tr>
<td>Wow factor! ____/5 pts.</td>
<td>Student produces work that uses ingenuity and</td>
<td>Student produces good, thoughtful work that</td>
<td>Student does what is asked with nothing extra.</td>
</tr>
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<td>This is not going to be freely given, it must</td>
<td>shows some imagination, but stays within</td>
<td>(0-1)</td>
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<td></td>
<td>be earned</td>
<td>conventional boundaries. Project is new,</td>
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<td>unique and surprising; shows a personal</td>
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<td>touch. (4-5)</td>
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<td>Student produces good, thoughtful work that</td>
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<td>shows some imagination, but stays within</td>
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<td>conventional boundaries. (2-3)</td>
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### Example Bibliography:


### Topics:

- Law of Sine & Law of Cosine
- Vectors
- Parametric Equations
- Polar Coordinates & DeMoivre’s Theorem
- Matrices: Gaussian Elimination
- Matrix Inverses: Determinants & Cramer’s Rule
- Non-linear Systems & Systems of Inequalities
- Using Iteration to Model Population Growth & Setting up Equations That Define Functions
- Maximum and Minimum problems
- Polynomial & Rational Functions
- Exponential Functions: Compound Interest, Growth & Decay
- Logarithmic Functions: Properties, Inequalities & Equations

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Figure 8 Project 3 Evaluation
Research and Outline:
- Paragraph highlighting reasons why your classmates should learn about your topic
- Paragraph articulating your topics’ connection to the wider mathematical field—why is it useful/necessary
- The manifesto makes a good argument for why the topic ought to be studied in a thoughtful and thought provoking way that shows a good degree of knowledge on your part.
- Brainstorm a list of ideas on how best to “teach” or guide your classmates in a thoughtful discussion of your topic in a way that ensures understanding

Written Summary and Lesson Plan:
- Essential questions are posed: what are good questions to ask to get people thinking about your topic and its utility.
- Your 45 minute class is described in outline form: You clearly state your intentions and your process is clear, great enough detail that someone else could step in and teach the class for you and they would vividly know your intentions.
- A lesson plan that would earn and “A”: The lesson plans ask people to engage with mathematical questions, offer controversies to consider, ask people to think about material in new ways. “A” lesson plans have strong factual foundations, but they try to teach people to think/ analyze/ problem solve and debate in more sophisticated and relevant ways. “A” lesson plans do not underestimate Tabor students, nor do they talk at them for 45 minutes.
- A “C” lesson plans ask little of your peers beyond showing up and being entertained. “C” lesson plans are “on topic” but they act as reports about the information. (A lesson plan is not the same thing as a presentation to the class.) If you step back from your own project and think, “I am bored (the topic is poorly defined, there is close to no information, no point to learning the information presented) and I gained no skills (critical thinking, curiosity towards mathematics, mental challenges, or FUN), then the project is heading to a “C”.

Execution of Lesson Plan:
- Have fun, relax and be supportive of one another. The more you put into it, whether you are a student or leading the group, the more you will get out of it.
- Practice, practice, practice. Then once you’ve done that, practice some more.

Figure 9 Project 3 Guidelines

Public speaking was a core skill that was cultivated throughout the year. During each Saturday class meeting, which occurred either once, or twice per month, the students developed and exercised their public speaking skills through different means. In September and October, students were exposed to introductory activities, involving them to help to get to know each other better, and to help get them comfortable speaking in front of their peers about topics they were familiar with. One activity that was used for this is the object game, in which students were to bring in one object from their dorm room or home that was important to them and then speak about the importance or symbolic meaning of the object for one minute. After each speaking activity we would debrief as a class, discussing the positive engaging behaviors, and the negative distracting or disengaging behaviors we witnessed. As the year progressed the activities became
more challenging, such as drawing a card from an Apples to Apples deck and speaking
about the topic drawn for two minutes. They were asked to do these activities employing
all strategies that were previously discussed, eye contact, body language, being conscious
of filler words and so on. By December most students had a basic understanding of the
underpinnings of public speaking and had enough practice to comfortably, and with
moderate confidence speak in front of a group. Launching them into new activities where
they were asked to translate their newly acquired public speaking skills into a
mathematical context, gaining the vocabulary to better articulate and verbalize their
mathematical thinking. As discussed in Chapter 2 the students were encouraged to work
with one another on difficult problems instead of only looking to the teacher for help or
hints. The public speaking exercises dovetailed perfectly with the aim of forming group
cohesiveness among students and develop their problem solving tactics through
conversing with their peers. In research quoted by Patricia Fandt among members of
high-performing teams there existed significant interdependence and accountability to
one another (Fandt, 1991). In the precalculus class this interdependence resulted in a
more robust mathematical toolbox for all students, and hopefully nurtured a safe class
environment where students felt comfortable taking risks. Chapters 5 and 6 illuminate
how public speaking and communication skills were employed to complete problem sets
and execute the recycling project, the second project of the second semester.

The timeline of the review project spanned the entire second semester so three
deadlines were implemented to help students manage their workload and the time they
spent on each of the three sections of the project. There was a need to balance the two
opposing dynamics of providing enough of a framework to help prevent students from
procrastinating to the point of detriment versus allowing more autonomy to foster personal growth and development, as well as manufacturing personal accountability. During the first semester, with the video projects, a few students really struggled to pace themselves, and all students struggled to some degree. The three sections of the review project were: 1) the research and outline, 2) the written summary and lesson plan, and 3) the presentation. The research and outline section was designed to ignite their thinking and bring them to action in educating themself on their topic. From there they were instructed to write a summary that was to be a conclusive document on their topic, demonstrating their mastery of the content and providing them a framework from which to pare down and create a lesson plan. With the feedback that was provided to them at each step, they drew up a lesson plan for the review class. One observation made during this stage was the more autonomy one allows students, the more responsibility they take for themselves; if they are given the capacity to choose, whether it be a topic or timeline, they take more ownership of their learning.

The presentations were highly successful and supported Patricia Fandt’s findings that “high-accountability teams rely on more interdependent behaviors, experience greater satisfaction with their team, and expressed higher success than low-accountability teams” (Fandt, 1991, p. 300). The students were clearly proud of the work they did, and one could overhear many conversations during their studying for the exam expressing that “that was my topic I can help you.” Over the course of the year the students began to rely on each other to solve difficult problems, they began to take a true interest in mathematics and employ math to help them resolve problems pertaining to their personal interests.
Chapter 5  Recycling Project

Nate Meleo, the department chair and another teacher of precalculus honors at the boarding school, first suggested the idea of having the students work on a Moody’s Mega Math Challenge, which is “a mathematical modeling contest for high school juniors and seniors” (SIAM, 2016, para. 1). The intent of this contest is that “through participation, students [will] gain the experience of working in teams to tackle a real-world problem under time and resource constraints akin to those faced by industrial applied mathematicians” (SIAM, 2016, para. 1). The timing of Nate’s proposal coincided with a mid-year switch at the school to a single stream recycling system from a sorted recycling system. Conveniently the 2013 Moody’s Mega Math Challenge had a very similar premise and almost seamlessly paralleled the current scenario at the school. Adapting the guidelines of the 2013 M³ Challenge, we presented the students with an opportunity to design a recycling protocol that is economically, environmentally and logistically sound both for our own school and for another boarding school with different demographics. On the following pages are the guidelines and rubrics for the 2013 M³ Challenge as well as the adapted challenge that we presented to our students.
Waste Not, Want Not: Putting Recyclables in Their Place

Plastics are embedded in a myriad of modern-day products, from pens, cell phones, and storage containers to car parts, artificial limbs, and medical instruments; unfortunately, there are long-term costs associated with these advances. Plastics do not biodegrade easily. There is a region of the Northern Pacific Ocean, estimated to be roughly the size of Texas, where plastics collect to form an island and cause serious environmental impact. While this is an international problem, in the U.S. we also worry about plastics that end up in landfills and may stay there for hundreds of years. To gain some perspective on the severity of the problem, the first plastic bottle was introduced in 1975 and now, according to some sources, roughly 50 million plastic water bottles end up in U.S. landfills every day.

The United States Environmental Protection Agency (EPA) has asked your team to use mathematical modeling to investigate this problem.

**How big is the problem?** Create a model for the amount of plastic that ends up in landfills in the United States. Predict the production rate of plastic waste over time and predict the amount of plastic waste present in landfills 10 years from today.

**Making the right choice on a local scale.** Plastics aren’t the only problem. So many of the materials we dispose of can be recycled. Develop a mathematical model that a city can use to determine which recycling methods it should adopt. You may consider, but are not limited to:

- providing locations where one can drop off pre-sorted recyclables
- providing single-stream curbside recycling
- providing single-stream curbside recycling in addition to having residents pay for each container of garbage collected

Your model should be developed independent of current recycling practices in the city and should include some information about the city of interest and some information about the recycling method. Demonstrate how your model works by applying it to each of the following cities: Fargo, North Dakota; Price, Utah; Wichita, Kansas.

**How does this extend to the national scale?** Now that you have applied your model to cities of varying sizes and geographic locations, consider ways that your model can inform the EPA about the feasibility of recycling guidelines and/or standards to govern all states and townships in the U.S. What recommendations does your model support? Cite any data used to support your conclusions.

Submit your findings in the form of a report for the EPA.

The following references may help you get started:

- [http://www.epa.gov/epawaste/nonhaz/municipal/index.htm](http://www.epa.gov/epawaste/nonhaz/municipal/index.htm)
- [http://5gyres.org/what_is_the_issue/the_problem/](http://5gyres.org/what_is_the_issue/the_problem/)

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Figure 10 Moody’s Mega Math Challenge 2013
Overview

Plastics are embedded in a myriad of modern-day products, from pens, cell phones, and storage containers to car parts, artificial limbs, and medical instruments; unfortunately, there are long-term costs associated with these advances. Plastics do not biodegrade easily. There is a region of the Northern Pacific Ocean, estimated to be roughly the size of Texas, where plastics collect to form an island and cause serious environmental impact. While this is an international problem, in the U.S. we also worry about plastics that end up in landfills and may stay there for hundreds of years. To gain some perspective on the severity of the problem, the first plastic bottle was introduced in 1975 and now, according to some sources, roughly 50 million plastic water bottles end up in U.S. landfills every day. The United States Environmental Protection Agency (EPA) and Academy’s Administration have asked your team to use mathematical modeling to investigate this problem.

Objectives

1. Modeling Plastic Waste in the United States
   • How big is the problem? Create a model for the amount of plastic that ends up in landfills in the United States.
   • Predict the production rate of plastic waste over time and predict the amount of plastic waste present in landfills 10 years from today.

2. Modeling on a more local scale
   • Plastics aren’t the only problem. So many of the materials we dispose of in garbage, including plastics, can also be recycled. Develop a mathematical model that a boarding school can use to determine which recycling methods it should adopt. For your model, you might consider:
     o How much recyclable waste produces each year, month, week, day, etc.
     o Different options for collecting waste and/or separating trash from recyclables (cost-benefits)
     o Cost of single-stream vs. pre-sorted
     o Availability of local drop-off sites (landfill or town recyclable centers)
     o Cost effectiveness of paying a company to collect trash/recyclables vs. personal drop off
     o The most efficient mix of campus drop-off sites (dumpsters) vs. small individual recyclable bins (rooms)
       ● What mix creates the optimal % of recyclables being recycled and creating cultural buy-in?
     o Anything else you feel impacts whether people recycle and how cost efficient the recycling is...

Your model should be developed independent of current recycling practices at the school and should include some information about the school (demographics) and also some information about the each particular recycling method (ex. single stream). Demonstrate how your model works by applying it to both Academy and one other boarding school of a different size/region (ex: Milton). Does geographic region/campus demographics affect what recycling program you should have? How would this extend to boarding schools on the national scale?
3. Presenting Your Model and Making a Pitch

• Now that you have applied your model to Tabor and another school location, consider ways that your model can inform Administration about the most efficient recycling method for the school. What recommendations does your model support? Submit your findings in the form of an EPA report that explains your model, any assumptions you made, the data you used, and the overall conclusions. Your report should begin with a summarizing abstract.

Resources

Cite any data used to support your conclusions (see bibliography format below)

The following references may help you get started:

http://www.epa.gov/epawaste/nonhaz/municipal/index.htm

http://5gyres.org/what_is_the_issue/the_problem/

http://www.marionma.gov/Pages/index

- Plant Operations Office
- Business Office

You MUST explain any assumptions and all equations/math models you create.

Figure 12 Project 4 Grading Rubric Continued
The students knew that there would be four projects throughout the year and they had been aware that one would be done over a short, but intense time frame; in January...
they were told that this time period would be the two weeks before spring break. These
two weeks (in March) were entirely dedicated to completing the recycling project, both
class time and homework time was granted to them. The students noted that knowing the
time frame ahead of time helped them mentally prepare for the work; they also said it
helped them plan to get ahead in other classes. Notifying students about such a schedule
is always a good thing. However, even with advance notice, when the recycling project
was announced there was a similar reaction to the one the students exhibited in
September following the distribution of the video projects. Not knowing what an EPA
report was or consisted of, let alone being familiar with the format of an EPA report
contributed to some clear mental anxiety for many students. This coupled with a general
lack of understanding about trash removal systems, either at our school or in the broader
community added to their discomfort. Having experienced a similar feeling about the
video projects and subsequently achieving success with them, the student’s inability to
act was an abbreviated stint in comparison to September. After about a day of exhibiting
an inability to focus, they began to brainstorm, moving forward with the project much
more quickly than they previously had; they were more comfortable with the unknown at
this point, it was not necessarily the case that they liked it, but they no longer feared it as
they had at the beginning of the year. Their trust in me, that I present them with projects
that they have the ability to complete, their belief in themselves and their belief in each
other had been cultivated throughout the year and they now acted with greater autonomy
and with a greater sense of internal control. The students were significantly more
confident in March than when they arrived in September.
Having four members in each group the students quickly divvied up the tasks assuming different roles. During this time it was possible to overhear many of them identifying both strengths and weaknesses that each team member possessed. They understood that they would need to work together and use each member to his or her strengths to yield a reasonable and mathematically sound recommendation by the end of two weeks. One can attribute their ability to identify each other’s strengths (and weaknesses) to the communal nature of the class and the relationships they built and nurtured with one another. Prior to this project the students had some experience modeling real world situations, but this project took their learning to the next level. In Andrzej Sokolowski’s article titled How to Make Mathematics Problems Contextually Meaningful he notes that “if the modeling processes have not been well understood by the students, then problem solving in mathematics becomes a passive action of following a set of procedures (often algorithmic operations and formulas) without any thought on embodied reality” (Sokolowski, 2011, p. 295). This project was timely in that their recommendations on single-stream recycling could be implemented as the school worked to formulate new recycling protocols, giving the students a feeling of importance in the immediacy of their work. Another reality highlighted for the students was the experience of seeking information and compiling research from other boarding schools; they found that sometimes information was not readily accessible or easy to come by. The students found that people they were unfamiliar with or people who did not have a personal interest in their learning or in the project initiatives were more hesitant to help them out. This is also true in the real world, and posed another learning moment for them paralleling a situation they will likely find themselves in the future. Many chose a school
where they already knew a student, giving them a pathway to the information and data
they sought. This illuminated how difficult it can be to access information and draw
conclusive data to manufacture a sound recommendation given a time constraint. All of
these real-world experiences are instrumental to a student’s learning, and the more we can
facilitate this learning process the better. As Dan Meyer expressed it, the world is not
smooth, but rather full of rough bumps and unexpected turns (Meyer, 2010). The more
that a teacher can demonstrate this in class and have their students experience this, the
more prepared the students will be for the future.

The recycling project propagates Sokolowski’s consideration that mathematical
instruction “should focus on enhancing students’ conceptual understanding, problem
solving skills, analytical and transference skills” (Sokolowski, 2011, p. 295). Both the
quantity of time allotted and the time of the year, early March right before a long break,
worked well. The students were starting to become entrenched in the habitual flow of
school; this project mixed things up giving them a new challenge, utilizing their current
knowledge to find a viable solution to a real and pertinent problem facing their
community. This project also instigated their desire to learn new mathematics to help
solve the real problem at hand. A prime example of this was one group discovering a
need to learn about polar integration as a result of wanting to know the area of multiple,
repeated, circular shapes. They showed their drawings of where they intended to place
recycling receptacles to a student in a higher-level math class and asked the question of
how to find the area of these circular shapes described above. The older student
proclaimed that those shapes closely resembled those given by a particular set of
equations, and that if one integrate appropriately, then one can find the area, which was
the desired quantity the precalculus students wanted. The students described their interaction with their (slightly older) peers and asked me more questions about polar integration, _wanting_ to learn more; _curious_ about the topic because of the direct application it had to their solution. It was clear how much more invested they were in the learning process when it was directly tied to solving a problem they were invested in; a problem they had researched, devoted time and energy to and could proudly and confidently put forth a recommendation to the school community. It reignited their desire to learn and reinvigorated their curiosity of how math can improve the world.
Chapter 6 Problem Sets

The problem sets given to the students during this thesis project had many purposes, one of which was to help the students develop toughness and perseverance, which they did. It was satisfying to watch the students grow stronger and more determined as the year went on; as a result of the teaching approach, they became self-serving, independent learners who developed an awareness of the vast body of knowledge available to them. They learned how to employ the Internet, their friends, teachers and family members to facilitate their thinking; it was interesting to see how many times a student cited a family member. The students were given directions about properly citing all information sources, including that from family members. This newfound ability to persevere did not come easy. Consistent with their reaction to the projects, this new format of homework, challenging problem sets, caused some initial fear. In prior math classes the students had rarely been given a problem that they did not readily know how to do, or with a reasonable amount of effort navigate their way to an answer. Many of the problems they faced were not only difficult, but sometimes foreign looking, like the polar graph on problem set #1, figure 14.
Figure 14 Problem Set #1

The first week when the first project and the first problem set were distributed, the students thought that I was being deliberately cruel. This style of learning was new to them and they were new to me. As previously noted, developing a trusting, caring
relationship is integral to fostering a successful project-based, student-centered class. Nurturing their belief in one’s competency as a teacher and their belief in themselves as critical thinkers and mathematicians is essential to the success of such a class, and as described earlier these key features were not present during the first week. Again, it is critical for teachers to realize that this initial hesitation and anxiety will be replaced by a better more resilient reaction later on.

One recommendation that can be made is that when students are stuck and are giving you a blank, ‘I am so lost’ stare and there is a concern that they are becoming emotionally fragile, is to start asking them questions: ask what have they tried and point them in the right direction without telling them exactly where to look. It is critical for a teacher to get to know one’s students as soon as possible, to learn who they are as people, to learn when they can be pushed further and when are they are close to the point of shutting down; For those who coach, this is similar to when one is working with athletes who are new to a team. The purpose of the problem sets is to push students to their limit and to urge them to work through problems that are potentially more difficult than they can currently solve. The more one can push the limit without letting them repetitively fail, the better. It is sometimes okay for students to have a momentary failure; eventually they will have a successful result. It is important to stress the importance of relationships in these situations, the more students trust their teacher the better the learning experience, whether in times of joyful results or during periods of dismay. Evoking these emotions provides a mathematical experience that is memorable, often like no other they have experienced. The key is to ensure that this emotional experience is mostly positive and delightfully eye-opening.
It is essential to highlight the learning curve I experienced as a teacher in constructing the problem sets. It was difficult to formulate or find and adapt problems that were connected to what we were studying, and to determine an appropriate amount of time to complete a problem set. As teachers, it is usually possible to use summer time to try to get ahead and plan out assignments, but often the year does not follow the script created during this pre-planning time. Initially, the anticipation was that one problem set per week was a reasonable amount of work for the students, and for the teacher to oversee. Following strong feedback and self-reflection it was easy to see that allowing only one week to complete a problem set was not only extremely difficult for the students, forcing them to either lose sleep to finish the problem sets, or to not give their best effort and pursue inappropriate short-cuts. The result of this was to put the students in a difficult position. By the middle of October it was clear that the amount of work needed to be scaled back, and the decision was made to allow two weeks to complete each problem set. The majority of students were very relieved to find out about this (even though there were a few disappointed students as well). Related to the topic of scheduling and timeliness of assignments, a problem set was never assigned to be due the week that a project was due. This allowed them to focus their time on finishing the final aspects of their project, and this seemed to work well. The number of problems on each problem set averaged around five, with varying degrees of difficulty, if there was one particular problem that had many parts and had a high level of difficulty then it was balanced with an easier, less time consuming problem. This can be seen in problem set #7, figure 15, where one can compare the difficulty of problems number 1 and 5.
Problem Set #7 – Happy Holidays!! ☃️
Name: ____________________________
Precalculus Honors Due: 12/10

1. For each of the following state a polar equation that fulfills the condition given, graph the
equation using Desmos (print your graph), and include a table of values to demonstrate the
symmetries.
   a. Is symmetric about the polar axis
   b. Is symmetric about the line θ=π/2
   c. Is symmetric about the origin

2. To graph any imaginary number you use the real and imaginary components to determine the
unique point in the Cartesian plane. The magnitude of the complex number z = a + bi is the
length of the vector formed by connecting the origin to (a, b). The direction of this vector is
deﬁned to be the angle that z makes with the real axis, which is also the polar axis.
   a. Algebraically what is i(a + bi) equal to? Geometrically what affect does multiplying by
      i have?
   b. Please write z = a + bi in polar form.
   c. Why is it true that the geometric meaning of multiplication by the complex number
      z = r(cosθ + i sinθ) is a counterclockwise rotation by θ followed by stretching by the
      factor of r?
   d. Why is it true that the geometric meaning of division by the complex number
      z = r(cosθ + i sinθ) is a clockwise rotation by θ followed by stretching by the factor of
      \( \frac{1}{r} \)?

3. Using the results of the rules for multiplication and division of complex numbers and
   mathematical induction (page 983 in our text) we have DeMoivre’s Theorem, which states that
   for any integer n, and any real θ , \([r(cosθ + i sinθ)]^n = r^n(cosnθ + isinnθ)\). Prove this Theorem
   using mathematical induction. **You must explain each step to demonstrate your knowledge
   and understanding of mathematical induction.**

4. Show \((1 + i)^5 = 32i\).

5. Algebraically verify that if \(a \in C\) and \(r \in R\) (that is, a belongs to the complex numbers and r
   belongs to the real numbers) that the set of points \(a + re^i\), where \(0 \leq t \leq 2\pi\), describes a circle
   with center at \(a\) and radius \(r\).

**Practice Problems:** Please be sure to check your answers in the solution manual! Seek help if you are
unclear on how to do these problems after reviewing the solution manual. Please turn them in with your
problem set solutions.
10.7: 1, 10, 14, 31, 37, 40
10.8: 10, 13, 31, 48, 55, 61

Figure 15 Problem Set #7

At the end of each problem set there was a selection of ‘practice problems’ from
our textbook that the students were to complete. The students were provided access to the
textbook solution manual, there was one copy in the classroom and a copy in the school
library that they could borrow to check their answers. If after looking at the solutions they
needed an additional explanation, they were instructed to attend office hours. The idea of
providing the solution manual came from Nate Meleo, but similar ideas have been
implemented in other contexts, such as in James Tanton’s thoughts on assessments. It is
probably the case that all, or most, math teachers would agree that “[we are] not so
interested in the numerical answer but the method towards obtaining it” (Tanton, 2012, p.
7). The majority of students had few questions on the problems from the textbook, as
these were the types of problems they were accustomed to solving in prior math courses.
In our textbook, there are A, B, and C level problems that vary in difficulty, the A level
being easier problems and the C level more challenging. The practice problems came
from the A and B levels; these mainly consisted of procedural problems and direct
application of content covered in that section of the textbook. The weekly assessments,
which will be discussed in greater detail in Chapter 7, consisted of problems that
paralleled the difficulty of the A and B level problems as well. Knowing that these
problems would help prepare them for the weekly assessment provided the students’ with
motivation to complete them, concurrently the students expressed that they found the
textbook problems boring and that they preferred to spend time working on the problem
set problems or their projects. This was encouraging, after experiencing their initial
dissent for both the problem sets and the projects.

The problems came from a variety of sources, and it was often necessary to
reference many different sources to formulate a problem. One frequently used reference
was the National Council of Teachers of Mathematics magazines, as well as the book
Mathematical Connections by Al Cuoco, and The Art and Craft of Problem Solving by
Paul Zeitz were helpful sources. Figures 16-25 are the remainder of the 12 problem sets that were given throughout the year.

<table>
<thead>
<tr>
<th>Problem Set #2</th>
<th>Name:</th>
<th>Due:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precalculus Honors</td>
<td></td>
<td>10/1</td>
</tr>
</tbody>
</table>

1. Is it possible to find the average rate of change of a discontinuous function over an interval? Please show your work and explain your thought process.
   a. Can the number of users on Facebook be expressed as a continuous function of time or is it a discontinuous function of time?
   b. Below is a graph of the number of Facebook users from the 4th quarter of 2008 through the 2nd quarter of 2015. Please calculate the average rate of change over this interval.
   c. Please describe a situation that changes discretely.
   d. Please describe a situation that changes continuously.

2. Please skim this article on Julia and Mandelbrot Sets: [http://www.juliasets.dk/Pictures_of_Julia_and_Mandelbrot_sets.htm](http://www.juliasets.dk/Pictures_of_Julia_and_Mandelbrot_sets.htm). Reaction: whoa way over my head! Or maybe not! Either way, it’s some pretty high level math, and really neat images arise out of what seems to be mathematical chaos. Please write a reflection on one of the images. I know I am going to get the question “how long?” so here’s my answer: as long as you feel appropriate, make it interesting, but please don’t waste your time (and my time) filling space on a page.

3. If \( f(x) = x^3 + 3x \) and \( g(x) = x + 2 \) please find the average rate of change (AROC) of \( f(g(x)) \) on \([-2,2]\) and \( g(f(x)) \) on \([-2,2]\).

4. If \( f(x) = \sin(x) \) please write the equation of \( g(x) \) if \( g(x) \) is \( f(x) \) translated up 5 units, then vertically stretched by a factor of 3, then horizontally compressed by a factor of 4, and finally reflected over the x-axis. Parentheses are necessary, so be sure to place them correctly. Hint: [www.desmos.com](http://www.desmos.com) will be helpful.

5. According to our text, “A function \( f \) is a one-to-one function if each element in the range of \( f \) comes from a unique element in the domain of \( f \).” Generally speaking, are even-degree polynomials one-to-one? Are odd-degree polynomials one-to-one? Does the degree of a polynomial affect whether the polynomial is invertible? Explain your thought process and provide examples.

Practice Problems: Please be sure to check your answers in the solution manual!
2.2: 9, 33
2.3: 25, 39
3.1: 7, 22, 43
3.2: 19
3.3: 14, 24
3.4: 1, 36
3.5: 7
3.6: 6

Figure 16 Problem Set #2
Problem Set #3
Precalculus Honors

1. Please state (and cite) an academic definition of an injective function:
   a) Given an example of an injective function then prove that it is injective. (Be sure to indicate the domain and codomain)

2. Please state (and cite) an academic definition of a surjective function:
   a) Give an example of a surjective function then prove that it is surjective. (Be sure to indicate the domain and codomain)

3. Given that the domains of $f$ and $f^{-1}$ are both $(-\infty, \infty)$ and that $f^{-1}(-3)=1$, solve the equation $5+f(4t-3)=2$ for $t$.

4. Demonstrate that the difference of consecutive squares is an odd number. Write a mathematical formula to model this for all consecutive squares. Explain in 3-5 sentences why your model works. Then find the consecutive positive integers whose squares differ by 2015 and the consecutive positive integers whose squares differ by 222,449.

5. Resolve how many integers fulfill the following condition: When twice the integer is subtracted from twice the square of the integer, the result is at most 12.

Practice Problems: Please be sure to check your answers in the solution manual! Seek help if you are unclear on how to do these problems after reviewing the solution manual. Please turn them in with your problem set solutions.

3.2: 10, 16, 34
3.3: 6, 30
3.4: 46
3.5: 10, 25
3.6: 37

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Problem Set #4
Precalculus Honors

1. Using an iterated function approximate the square root of 5 accurate to 20 decimal places. State the function you used, and show your work for each iteration.

2. The picture below has a regular octagon in green and an equilateral hexadecagon, the outer shape with purple and yellow (isosceles) triangles attached to each side of the green octagon. Please find the ratio of the area of the hexadecagon to the area of the octagon.

3. Given triangle DEF, the bisector of angle D intersects EF at G. If DE=16, DF=30, and the measure of angle D=60°, find EG.

4. The angle bisector of angle A in triangle ABC divides side BC into two segments that are lengths 4 units and 6 units. If the length of the segment bisecting angle A is 8 units, what is the product of the side lengths AB and AC, in other words what is $(AB)(AC)$?

5. Please outline your project for me. What ideas do you have, what math equations have you used, what is your end goal, who is your target audience, what is the potential positive impact will your research have on this group of stakeholders?

Practice Problems: Please be sure to check your answers in the solution manual! Seek help if you are unclear on how to do these problems after reviewing the solution manual. Please turn them in with your problem set solutions.

10.1: 8, 9, 15, 20, 35, 46
10.2: 2, 10, 13, 31, 35
1. How do you prove that two vectors are perpendicular?

2. Given two vectors \( \mathbf{u} \) and \( \mathbf{v} \), where \( \mathbf{u} = \{15, -3\} \) and \( \mathbf{v} = \{4, y\} \) find the value of \( y \) such that \( \mathbf{u} \) and \( \mathbf{v} \) are perpendicular.


4. What is the dot product of two vectors \( \mathbf{a} \) and \( \mathbf{b} \) that are defined by the following: \( \mathbf{a} = \{2, 4, 6\} \) and \( \mathbf{b} = \{1, 7, 8\} \)? What dimension are vectors \( \mathbf{a} \) and \( \mathbf{b} \) in? Please find a vector that is perpendicular to both \( \mathbf{a} \) and \( \mathbf{b} \).

5. Prove that vector addition is commutative.

6. Update me on your project, what research have you conducted, what data have you gathered, what mathematical models have you created and/or used? How are you going to formulate an action plan for your audience?

Practice Problems: Please be sure to check your answers in the solution manual! Seek help if you are unclear on how to do these problems after reviewing the solution manual. Please turn them in with your problem set solutions.

- **10.3:** 12, 32, 36, 48
- **10.4:** 8, 31, 40, 47, 61

Figure 19 Problem Set #5
1. Find an equation whose roots are the squares of the roots of \( f(x) = x^4 - 5x^3 + x^2 - 3x - 4 \). And if you don’t succeed initially, that’s ok, try again, and use facts that you know:
   a) An \( n \)th degree polynomial has \( n \) roots.
   b) We are looking for a new function and all functions need names, so call it \( g(x) \) or \( h(x) \) or \( p(x) \) it doesn’t matter what you call it just give it a name 😊
   c) I’ll look at \( f(x)f(-x) \), this polynomial will be helpful.
   d) Finally once you find a function that has the structure you want, that is that the roots are the squares of the roots of \( f(x) = x^4 - 5x^3 + x^2 - 3x - 4 \) you will need to use a property of parametric equations to massage it into the correct form.

2. What is \(-6\)? Explain how you know this? What is \(-6\)? Explain how you know this?

3. Given that complex numbers are written as \( a+bi \) where \( a \) and \( b \) are real numbers.
   a) What is the conjugate of a real number? For example, if \( z = -2 \), what is \( \overline{z} \)? Where \( \overline{z} \) is the conjugate of \( z \). Explain your answer.
   b) What is the conjugate of a "pure imaginary" number? For example if \( z = 4i \) what is \( \overline{z} \)? Explain your answer.
   c) What is the conjugate of \( \overline{z} \)? Explain your answer.
   d) Characterize all complex numbers \( z \) so that \( \overline{z} = z \).
   e) Find a complex number \( z \) so that \( z^2 = z \).

4. Graph \( z = 3+4i \) in Cartesian form in the complex plane.
   a) Find \( |z| \)
   b) Project \( z \) onto the real axis, call this vector \( w \) and state the components of vector \( w \) and then write \( w \) as a complex number.
   c) Add \( z \) and \( w \). Graph the resultant. Explain why \( 0, z, w, \) and \( z+w \) are the vertices of a parallelogram.

5. Graph \( z = 3+4i \) in polar form in the complex plane.
   a) Show that \( |z| = |\overline{z}| = |z^2| = |z| \).
   b) Describe geometrically all the points with the same absolute value as \( z \).

Practice Problems: Please be sure to check your answers in the solution manual! Seek help if you are unclear on how to do these problems after reviewing the solution manual. Please turn them in with your problem set solutions.

10.5: 2, 8, 14, 21, 28a
10.6: 2, 4, 6, 12, 22, 29, 38, 40
1. Argue by contradiction to show that \( c^2 + c + 1 = d^2 \) has no positive integer solutions.

2. Take a square of any size and inscribe a circle inside it, then inside that circle inscribe another square. What is the ratio of the area of the larger square to the area of the small square?

3. Encode a message by means of a matrix (see page 802 of your textbook), and then decode your message by using a matrix inverse. Your message must be 3 to 5 words. Your calculator will be helpful in this exercise.

4. Recall the orthogonal projection onto a line passing through the origin in \( \mathbb{R}^n \) (a subspace with \( n \) dimensions) is the vector projection of a vector \( \mathbf{v} \) in the direction of a unit vector \( \mathbf{u} \). This is given by \( \text{Proj}_\mathbf{u}\mathbf{v} = (\mathbf{v} \cdot \mathbf{u}) \mathbf{u} \). Find the 2x2 matrix for the orthogonal projection in \( \mathbb{R}^2 \) onto the 60° line.

5. Let \( \mathbf{A} \) and \( \mathbf{B} \) be different \( n \times n \) matrices with real entries. If \( \mathbf{A}^3 = \mathbf{B}^3 \) and \( \mathbf{A}^2 \mathbf{B} = \mathbf{B}^2 \mathbf{A} \), can \( \mathbf{A}^2 + \mathbf{B}^2 \) be invertible?

Helpful Notes from Robert Winters: A matrix is invertible, or nonsingular, if it is both one-to-one (injective) and onto the codomain (surjective). This is the same notion of invertibility we have for functions elsewhere.

Also: If \( \mathbf{A} \) is an \( n \times n \) (square) matrix then the following statements are equivalent:

\[
[1.] \ \mathbf{A} \ \text{is invertible} \\
[2.] \ \text{The system} \ \mathbf{A} \mathbf{x} = \mathbf{b} \ \text{has a unique solution} \ \mathbf{x} \ \text{for all} \ \mathbf{b} \in \mathbb{R}^n \\
[3.] \ \text{ref} (\mathbf{A}) = \mathbf{I}_n \\
[4.] \ \text{rank} (\mathbf{A}) = n \ (\text{full rank}) \\
[5.] \ \text{im} (\mathbf{A}) = \mathbb{R}^n \\
[6.] \ \ker (\mathbf{A}) = \{0\} \ (\text{the zero subspace})
\]

If you do not know what all these things mean, and I expect you don’t do some research, there is a lot of information at your fingertips. Please remember to cite others ideas, it’s very important that you give credit.

Practice Problems: Please be sure to check your answers in the solution manual! Seek help if you are unclear on how to do these problems after reviewing the solution manual. Please turn them in with your problem set solutions.

11.2: 22, 29
11.3: 4, 57
11.4: 6, 8, 17, 20, 33
11.5: 13, 27, 46
Problem Set #9
Precalculus Honors

Name:  
Due: 2/4/2016

1. Prove that the product of any four consecutive integers is always 1 less than a perfect square. (I know the solution is online, if you look at someone else’s work, you must cite it. Also, it does you no good to simply copy someone’s work, take the time to understand it or you are wasting your time.)

2. p, q and r are three two-digit prime numbers that have the following relationship: $r + \frac{1}{2} = q$ and $q + \frac{1}{2} = p$ What are the values of p, q and r?

3. “A rooster is worth five coins, a hen three coins, and 3 chicks one coin. With 100 coins we buy 100 of them. How many roosters, hens, and chicks can we buy?” (From the Mathematical Manual by Zhang Qiujian, Chapter 3, Problem 38, 5th century A.D.)

4. The image to the right represents one-way streets in a town in England. The numbers represent the volume of vehicles that passed by during one hour. Suppose that the vehicles leaving the area during this hour were exactly the same as those entering it. Can you determine exactly how many vehicles passed by each side of the block in the center, where the 4 question marks are? If not, using a system of linear equations to describe one possible scenario. For each of the four locations, find the highest and lowest possible traffic volume.

5. You are given a circle with a radius of 10. This circle has a square inscribed in it. P is a point on the circumference of the circle, but not touching the square. Find the sum of the distance from P to each of the vertices of the square.

Please remember to cite others ideas, it’s very important that you give credit.

Practice Problems: Please be sure to check your answers in the solution manual! Seek help if you are unclear on how to do these problems after reviewing the solution manual. Please turn them in with your problem set solutions.
11.5: 49, 51, 52, 57, 58, 67, 70
11.6: 12, 16, 18, 22
11.7: 2, 16, 28

Figure 22 Problem Set #9
1. You’re making waffle cones for your creamery by cutting a wedge from an 8-inch radius waffle and mending the edges together to form a cone. What is the maximum possible volume? Hint: \( V = \frac{1}{3}\pi r^2 h \)

2. You have two turtles, Jim and Tom, and you want to build a box shaped home for them. The home you build will have two square ends and no top. The bottom is made out of wood and the sides are glass so the turtles can see out. The material for the bottom costs $3/ft.\(^2\) and the material for the sides costs $12/ft.\(^2\). If you have a total of $288 to spend, what dimensions maximize volume for Jim and Tom?

3. Your product costs $12/kg. to produce after a fixed monthly cost of $100. The price at which you can sell it is a linear function of the quantity produced each month. You can charge $20/kg. if you make 100 each month, and $18/kg. if you make 200 each month.
   a. How many kilograms should you make each month to maximize your profit?
   b. What is the maximum monthly profit?

4. A newly drilled oil well is 7 kilometers offshore and 10 kilometers east of an oil refinery that is already in place on the shoreline. If connecting pipe costs $5 million per kilometer in the water and $3 million per kilometer on the shore, what path will minimize the total cost of connecting the new well to the refinery? Be sure to show how you know you have found the absolute minimum. Please include a sketch that shows the amount of pipe in the water and the amount on the shoreline.


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Practice Problems: Please be sure to check your answers in the solution manual! Seek help if you are unclear on how to do these problems after reviewing the solution manual. Please turn them in with your problem set solutions.

- **4.2:** 9, 19, 39, 42
- **4.4:** 9, 20, 21, 22
- **4.5:** 2, 5, 11, 23
The pigeonhole principal, states that if you have more pigeons than pigeonholes, and you try to stuff the pigeons into the holes, then at least one hole must contain at least two pigeons. Use this principal and your brilliant mind to answer questions 1-3.

1. If every point on a plane is colored either yellow or green, prove that no matter how the coloring is done, there must exist two points, exactly a mile apart, that are the same color.

2. Given a unit square, show that if five points are placed anywhere inside or on the square, then two of them must be at most \(\frac{\sqrt{2}}{2}\) units apart.

3. Ascertain that from a set of ten distinct two-digit numbers, it is possible to select two disjoint subsets whose members have the same sum.

4. The logistic function \(P(t) = \frac{L}{1+ae^{-bt}}\) represents the human population in Marion measured in thousands. It passes through (0,1) and (1,2) and has an asymptote at \(P(t) = 5\). Using this information find the values for \(L\), \(a\), and \(b\). Graph this function; label all axes, the \(y\)-intercept, and the horizontal asymptotes.

5. Let \(r(s) = \ln(s + \sqrt{s^2 + 1})\). Find \(r^{-1}(s)\).

Please remember to cite others ideas, it’s very important that you give credit.

Practice Problems: Please be sure to check your answers in the solution manual! Seek help if you are unclear on how to do these problems after reviewing the solution manual. Please turn them in with your problem set solutions.
5.1: 46, 48
5.2: 24, 36
5.3: 11, 26
5.4: 19, 26
5.5: 14, 64
5.6: 10, 12
5.7: 10, 13
The students went from being frustrated by the problem sets to a significant portion actually enjoying them, asking for more problems, and voicing that they appreciated the creative challenge. Once students developed a self-efficacy and believed they had the skill set and resources available to solve the problems they valued...
the challenge and the joy of solving difficult problems. Credit should be given to the problem sets for building the student’s overall mathematical confidence.
The intention of the weekly assessments was to evaluate the student’s procedural knowledge, ensuring that they could successfully complete traditional problems, in a traditional context. Every Thursday the students were given the entire class period, 45 minutes, to complete a set of problems that were generated from the textbook test generating software. As referenced in Chapter 6, there are A level, B level, and C level problems in the textbook, the weekly assessments consisted of A and B level problems. Overall the students performed very well on the assessments; they perceived them to be relatively easy and straightforward. The head of the Math Department remarked that this student reaction differed from the previous year when he had given similar weekly assessments in a similar format. This year the students viewed the weekly assessments as a means to ensure they were making appropriate gains, whereas during the previous year the weekly assessments were viewed as tests. This is not to say there was no anxiety surrounding these assessments; like many honors students they applied unnecessary pressure upon themselves leading up to the assessments. The weekly class averages ranged from a low of 85% to a high of 97%; surprisingly the week the students achieved a 97% class average there was an unusually high amount of anxiety. One thing that was learned through this thesis project was when one works with high achieving students there will always be a certain amount of angst and nervousness surrounding assessments, or anything that provides a grade. Sometimes it seemed that what compounded their stress was the very nature of the class; it was so different from anything they had
experienced before and it appeared that they were always somewhat anxious about anticipating the next unknown challenge. This situation has advantages and disadvantages; on the other hand, it is probably a more valid representation of the world that we live and work in.

The averages on the weekly assessments were higher than were anticipated. Given this, though, there were two main reasons that it was decided to preserve the level of difficulty. First, to generate more acceptance with the new curriculum, both from the students and from their parents; if the students were continually given unfamiliar problems and were not given a sense that they would perform well at the next level then this would have risked their losing faith as well as their parents faith. Another reason the assessments were not made more challenging was that these assessments were a gauge from which it was possibly to judge how well the students would perform on the final exam and in the end be successful in AP BC Calculus, which is the path for 95% of the students taking precalculus honors. The weekly assessments were, in some sense, a useful control variable.

As was hypothesized, the students performed very well on a challenging final exam, earning an overall average of 88%. It was this result that made it evident that one could appropriately make the claim that this curriculum was a success. As previously highlighted there were many encouraging moments during the year, but when one is trying something new and it has an impact on how well students might be learning then there will always be a certain level of pressure and uncertainty. The weekly assessments served as a consistent reinforcement that the students were learning what they “needed”
to learn and that they possessed the procedural skills to be successful in their future math careers.

Again, the weekly assessments were constructed using the test generating software that came with Cohen’s precalculus textbook (Cohen, 2012). Weekly assessment #1 as pictured below in figures 26 and 27 consists mostly of routine questions with one challenging question; the challenge question mirrored a similar problem that we completed in class. The assessments contained no shocks or surprises, the content topics were revealed to them, so as long as they were (mentally) present in class and spent a sufficient amount of time preparing they generally did very well on the weekly assessments.

![Multiple Choice (2 pts each)]

Identify the choice that best completes the statement or answers the question.

1. Which of the following statements is true?
   a. $\sqrt{9} = -3, 0$
   b. $\sqrt{16} = 8$
   c. $5 \leq -5$
   d. $\sqrt{10} - 7 \leq 3$
   e. $\sqrt{25} - 6 \leq -5$

2. State whether the following number is a whole number, a natural number, an integer, a rational number and/or an irrational number.
   $\sqrt{\frac{9}{27}}$
   a. irrational number
   b. rational number
   c. whole number
   d. integer
   e. natural number

3. State whether the following number is a whole number, a natural number, an integer, a rational number and/or an irrational number.
   $\frac{13}{13} = 1$
   a. rational number
   b. whole number
   c. natural number
   d. integer
   e. irrational number

Figure 26 Weekly Assessment #1 Page 1
Multiple Response (2 pts each)

Identify one or more choices that best complete the statement or answer the question.

4. Which of the following statements are true?
   - $\frac{15}{12} > \frac{13}{14}$
   - $54 > 18x$
   - $54 > 15x$
   - $-18 > -15x$
   - $-54 > -56x$

5. Which of the following statements are true?
   - $\pi < -64$
   - $0\frac{3}{8} < 0.3$
   - $\frac{9}{3} < 2\frac{5}{1}$
   - $\sqrt{x} < 3$
   - $\pi < 54$

**CHALLENGE (10 pts)**

6. Using indirect proof, prove that $\sqrt{2}$ is an irrational number.

For this proof, you may assume:

1. If $x$ is an even number, then $x = 2k$ for some integer $k$.
2. Any rational number can be written in the form $a/b$, where $a$ and $b$ are integers with no common factors other than $\pm 1$.
3. If $x$ is a natural number and $x^2$ is even, then $x$ is even.

Assume $\sqrt{2}$ is rational. Then $\sqrt{2} = \frac{a}{b}$ where $a$ and $b$ are integers with no common factors other than $\pm 1$.

$\sqrt{2} = \frac{a}{b}$

$2b^2 = a^2$ since $a^2$ is even, then $a$ is even and $a = 2k$ for some integer $k$. $2b^2 = Qx^2 = 4k^2 \Rightarrow 2b^2 = Qx^2$ thus $b^2$ is even so $b$ is even. Wait! If $a$ and $b$ are even then they have common factors other than $\pm 1$, namely $2$. Contradiction! Thus $\sqrt{2}$ is irrational.
Figures 28-34 are examples of what could be considered to be routine, traditional, test questions, useful in having the students demonstrate their content knowledge, but exhibiting little to no signs of critical thinking or problem solving.
Precalculus Honors Assessment (Calculator)  Name: __________

16 pts. total

*For parts a and b, be sure to explain both why the particular functions are used, and the purpose of each constant.

a) Amplitude : 20

Period: \( T = \frac{2\pi}{\omega} \)  \( \omega = \frac{\pi}{5} \)

When \( t = 0 \) Jane is at \( \theta = \frac{\pi}{6} \).

Thus, max x-direction = \( 20 \cos \left( \frac{\pi}{5} t \right) \) middle y-direction heading up.

b) \( x_2 = 10 \cos \left( \frac{\pi}{5} t \right) \)

\( \frac{\pi}{5} t = \frac{\pi}{6} \)

\( x_2 = -30 \pm 17.5 \)

\( y_2 = -10t^2 + 100 \sin \left( \frac{\pi}{5} t \right) \)

\( y_2 = -10t^2 + 50 \sqrt{2} \)

\( \frac{\pi}{5} t = \frac{\pi}{6} \)  \( t = \frac{5}{6} \)

\( y_2 = y_2 @ t = 2.14 \) so yes!

c) \( d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)

\( d(t) = \sqrt{\left( -30t - 17.5 \right)^2 + \left( \left( -10t^2 + 50 \sqrt{2} \right) - 0 \right)^2} \)

\( \frac{5}{6} \left( \frac{\pi}{5} \right)^2 \)

Figure 30 Weekly Assessment #6 Page 1: Classic Ferris Wheel
2. No Calculator

a) Use row operations to find the inverse of $A = \begin{bmatrix} 6 & 2 \\ 13 & 4 \end{bmatrix}$.

\[
\frac{13 + 13}{3} + \frac{13}{9} = \frac{13}{3}
\]

\[
\text{Inversion of } A = \begin{bmatrix} 6 & 2 \\ 13 & 4 \end{bmatrix}
\]

\[
-13R_1 + R_2 \Rightarrow \begin{bmatrix} 1 & 13 & 16 & 0 \\ 0 & -13 & \frac{1}{2} & -3 \end{bmatrix}
\]

\[
-13R_2 + R_1 \Rightarrow \begin{bmatrix} 1 & 13 & 16 & 0 \\ 0 & 1 & \frac{13}{2} & -3 \end{bmatrix}
\]

\[
A^{-1} = \begin{bmatrix} -2 & 1 \\ 13 & -3 \end{bmatrix}
\]

b) Use the inverse to solve the system $(3)$.

\[
\begin{align*}
6x + 2y &= -1 \\
13x + 4y &= 3
\end{align*}
\]

\[
A^{-1} = \begin{bmatrix} -2 & 1 \\ 13 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} -1 \\ 3 \end{bmatrix}
\]

\[
X = \begin{bmatrix} -2 & 1 \\ 13 & -3 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ -31/2 \end{bmatrix}
\]

\[
x = 5, \quad y = -\frac{31}{2} = 13.5
\]

Figure 31 Weekly Assessment #9 Page 2
#3 Solve the following system using a matrix algebra system, that is: $AX = B$.

15. $x + y + z = 6$
2$y + 5z = -4$

$A = \begin{bmatrix}
1 & 1 & 1 \\
0 & 2 & 5 \\
3 & 5 & -1
\end{bmatrix}$

$A^{-1} = \begin{bmatrix}
\frac{9}{11} & \frac{-21}{11} & \frac{-4}{11} \\
\frac{-10}{21} & \frac{11}{21} & \frac{5}{11} \\
\frac{-2}{11} & \frac{11}{21} & \frac{-4}{11}
\end{bmatrix}$

$X = \begin{bmatrix}
x \\
y \\
z
\end{bmatrix}$

$X = A^{-1}B$

$B = \begin{bmatrix}
6 \\
-4 \\
37
\end{bmatrix}$

$x = 5$

$y = 3$

$z = -2$
1. Carry out the expansion demonstrating the binomial theorem.

\[(2x+y)^4 = \binom{4}{0} (2x)^4 y^0 + \binom{4}{1} (2x)^3 y^1 + \binom{4}{2} (2x)^2 y^2 + \binom{4}{3} (2x)^1 y^3 + \binom{4}{4} (2x)^0 y^4\]

\[= 16x^4 + 4(8)x^3 y^1 + 6(4)x^2 y^2 + 4(2)x y^3 + y^4\]

2. Evaluate and simplify.

\[\frac{m[(n-2)!]}{(m+1)!} = \frac{m(m-2)!}{(m+1)!} = \frac{1}{n!n(n-1)(n-2)!} = \frac{1}{m^2}\]

3. Find the fourth term in the expansion of \((a-b)^3\).

4th term : \(\binom{3}{2}a^2(-b)^1 = \frac{3!}{2!1!}a^2(-b)^1\)

\[a^2(-b)^1 = -3000a^2b\]

4. Find the 73rd term in the expansion of \([x-\left(\frac{1}{x}\right)]^{75}\).

\[\binom{75}{72} x^3 (-x^{-2})^{72} = \frac{75!}{75!75!} x^3 (-x^{-1})^{72}\]

\[= \frac{15 \cdot 55}{3 \cdot 2 \cdot 1} x^{-1} = -6750 x^{-54}\]
Figures 35-37 are examples of more challenging problems that were used where the students are asked to synthesize different techniques. These require more problem solving skills, yet are not novel or unfamiliar problems.
10) By converting the polar equation \( r = a \cos \theta + b \sin \theta \) to rectangular form, show that the graph is a circle, and find the center and radius.

\[
\begin{align*}
(\frac{\sqrt{a^2+b^2}}{2})^2 &= \frac{a^2}{2} + \frac{b^2}{2} \\
R^2 &= a^2 + b^2 \\
R &= \frac{\sqrt{a^2+b^2}}{2} \\
(x - \frac{a}{2})^2 + (y - \frac{b}{2})^2 &= \frac{a^2 + b^2}{4}
\end{align*}
\]

center = \( (\frac{a}{2}, \frac{b}{2}) \)

---

4. A poster of height 33 in. is mounted on a wall so that its lower edge is 15 in. above the eye level of an observer. How far from the wall should the observer stand so that the viewing angle \( \theta \) subtended at his eye by the poster is as large as possible (see figure - not drawn to scale)?

\[
\begin{align*}
\text{Goal: maximize } \theta &= \alpha - (\alpha - \theta) \\
\tan(\alpha - \theta) &= \frac{15}{x} \\
\tan(\alpha) &= \frac{33}{x} \\
(\alpha - \theta) &= \tan^{-1}\left(\frac{15}{x}\right) \\
\alpha &= \tan^{-1}\left(\frac{33}{x}\right)
\end{align*}
\]

so \( \theta = \alpha - (\alpha - \theta) = \tan^{-1}\left(\frac{33}{x}\right) - \tan^{-1}\left(\frac{15}{x}\right) \)

Graph to find max: \( \theta = \tan^{-1}\left(\frac{33}{x}\right) - \tan^{-1}\left(\frac{15}{x}\right) \)

\( \theta \) is maximized when \( x = 27 \) inches

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Figure 35 Weekly Assessment #7 Page 2

Figure 36 Weekly Assessment #14 Page 2
4. Using induction, show that \( n^3 > (n+1)^2 \) for all natural numbers \( n \geq 3 \).

**Base Case**

\( n = 3 \)

\[ 3^3 > (3+1)^2 \]

\[ 27 > 4^2 \]

\[ 27 > 16 \]


**Inductive Hypothesis**

Assume \( n = k \geq 3 \)

\[ k^3 > (k+1)^2 \]


**Inductive Step**

Prove holds \( n = k+1 \)

\[ (k+1)^3 > (k+1)^2 \]

\[ (k+1)(k+1)(k+1) > (k+1)(k+1) \]

\[ (k^2 + 2k + 1)(k+1) > k^2 + 2k + 1 \]

\[ k^3 + 3k^2 + 3k + 1 > k^2 + 2k + 1 \]

\[ k^3 + 3k^2 + 3k + 1 > k^2 + 1 + 3 \]

\[ k^3 > k^2 + 2k + 3 \]
Chapter 8  Summary and Conclusions

Each project and aspect of the course had its purpose, and the successes and failures that accompanied each of these have already been described in the previous chapters. It is always important to create a thoughtful teaching plan ahead of time, but it is also important to be flexible, and open to changing one’s plan as well. Sometimes as teachers we become entrenched in our teaching approaches and firmly attached to lessons and ideas that worked in the past, but perhaps are not engaging our students any longer. It can be difficult to let go of a lesson plan that one has spent many hours constructing and believed was going to lead to an excellent class when one realizes the students are just not engaged with the class. Change is difficult for most people, just as was described as how difficult change was for the students piloting this new precalculus curriculum as part of this thesis project. When things were going well, the students thought the approach was great, and when things were going poorly, the students thought teaching was incompetent or inadequate. As John F. Kennedy stated after the failed attack at the Bay of Pigs in Cuba “victory has a thousand fathers, but defeat is an orphan,” taking full responsibility for the U.S. military and their actions that day, even though later it was shown that the catastrophic loss occurred as a result of poor military intelligence that Kennedy had acted upon (Peters, Woolley, 1961). The same holds true when one tries something new in a classroom; if it succeeds the victory is shared by all, the students, the teachers, the department, and the institution, but if it fails, the loss typically falls on the teacher alone.
Each project as part of this study had a target and through adjustments, both small and large, the learning objectives were met, and the students grew as people, developing a more robust character. As part of the results of this thesis project, presented here is a series of ten recommendations for teachers:

[1] Maintain flexibility. It is normal to make adjustments to one’s aims and it is integral to gracefully receive feedback, to ensure students and other constituents continue to help you work towards the betterment of the curriculum. It is then important to implement those adjustments accordingly and in a timely fashion (p. 39).

Just as it is important, yet sometimes difficult, for teachers to maintain flexibility, students have a similar rigidity and distaste for change, which leads me to the next recommendation:

[2] Remember that students and people alike are uncomfortable with change. Once students see they can be successful and they are validated in their success by a teacher or person of power then they become drawn to this new way of learning (p. 21).

Project one had many objectives, two of which were learning to deal with ambiguity, and learning to see math through their own personal lens. This leads me to recommendation number [3].

[3] Display math’s utility as often as possible (p. 2). Project two highlighted the necessity of completing multiple iterations of a project, the importance of re-working material and continually striving to improve, even when you think you’ve given it your best shot. Teachers should take a similar approach to their careers as educators and accept
that most of the time it is not possible to get it right the first time. Teachers should embrace the challenge to get better as we expect our students to do.

[4] Implementing a new curriculum is a process and it will get better (p. 8) it is likely that you will have to rework your plans and then rework them again.

[5] Work to build relationships and create buy-in (p. 6).

[6] Remember that timing is important, try to not overwhelm the students with work, yet know when it is possible to push them further (p. 16). If you make a mistake recognize it, and rectify it, if you have fostered strong relationships with your students they will forgive your error.

[7] Keep positive knowing that even if things are not seeming to be working out as well as hoped at the start. It will get better, remember that every assignment might not be an immediate success, but it could be setting things up for significant improvements in the following assignments and projects. This was illuminated in the student’s initial aversion to the problem sets, which eventually led their attraction to creative problem solving versus completing routine textbook problems (p. 41).

Project three set forth to fortify the students’ sense of accountability. The students’ charge to deliver a satisfactory final exam review for their peers coupled with their desire to succeed provided them with the motivation to produce high quality work (p. 3). The following recommendation addresses this issue:

[8] The more autonomy one allows for one’s students, the more personal responsibility they assume for themselves. Analogously:
[9] Personal responsibility fosters accountability; autonomy leads to personal responsibility, which then leads to accountability.

Lastly, project four compelled the students to consider time constraints, resource allocation, team dynamics, and the delivery of information in a medium most were unfamiliar with. These issues all parallel a potential professional climate that the students will experience later on. Thus the last recommendation is to:

[10] Present students with as many real world experience as possible, mimicking the sometimes uncertain ways in which people in various fields and industries work to solve problems; this contrasts the smoother approach of a typical math education (p. 35).

In conclusion the ten recommendations are:

1) Maintain flexibility
2) Remember people are uncomfortable with change
3) Display math’s utility
4) It is a process, it will get better
5) Build relationships to create buy-in
6) Timing is important
7) Patience is a virtue
8) Autonomy leads to personal responsibility
9) Personal responsibility fosters accountability
10) Provide as many real world experiences as possible
It is the hope that with the knowledge from this thesis project it will inspire other teachers to take on a project based curriculum. Incorporate one or two alternative teaching approaches each year to better prepare students for the real world and to develop critical thinkers who can take on problems that will face society in the years to come.
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