A depth-derived Pleistocene age model: Uncertainty estimates, sedimentation variability, and nonlinear climate change

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[1] A new chronology of glaciation, spanning the last 780,000 years, is estimated from 21 marine sediment cores using depth as a proxy for time. To avoid biasing this “depth-derived” age estimate, the depth scale is first corrected for the effects of sediment compaction. To provide age uncertainty estimates, the spatial and temporal variability of marine sediment accumulation rates are estimated and modeled as an autocorrelated stochastic process. Depth-derived ages are estimated to be accurate to within ±9000 years, and within this uncertainty are consistent with the orbitally tuned age estimates. Nonetheless, the remaining differences between the depth and orbitally tuned chronologies produce important differences in the spectral domain. From the δ18O record, using the depth-derived ages, we infer that there are weak nonlinearities involving the 100 kyr and obliquity frequency bands which generate interaction bands at sum and difference frequencies. If an orbitally tuned age model is instead applied, these interactions are suppressed, with the system appearing more nearly linear. INDEX TERMS: 1620 Global Change: Climate dynamics (3309); 1869 Hydrology: Stochastic processes; 3005 Marine Geology and Geophysics: Geomagnetism (1550); 3220 Mathematical Geophysics: Nonlinear dynamics; 4267 Oceanography: General: Paleoclimatology; KEYWORDS: tuning, sediment accumulation, nonlinear


1. Introduction

[2] Inference concerning past climate change relies heavily upon the assignment of ages to measurements and events recorded in marine and ice cores as well as to a variety of isolated markers in the geological record. Sedimentation and snow accumulation are analogous to strip-chart recorders, marking the past climate state in a large variety of physical variables. These records tend to be noisy and blurred by bioturbation and a variety of diffusive-like processes, [e.g., Pestiaux and Berger, 1984]. The major difficulty however, is that these strip-chart recorders run at irregular rates, stop completely, or even rewind and erase previous sections. If depth is taken as a simple proxy for time, irregularities in sedimentation stretch and squeeze the apparent timescale, and so distort the signals being sought. To the degree that the changes in rates are proportional to the signals themselves, one has a challenging signal demodulation problem. It is not an exaggeration to say that understanding and removing these age-depth (or age model) errors is one of the most important of all problems facing the paleoclimate community. Timing accuracy is crucial to understanding the nature of climate variability and the underlying cause and effect. Here we attempt to understand the nature of some of these age model errors, and to then apply that insight to construct a timescale for marine sediment cores spanning the last 780,000 years.

[3] The currently favored method for estimating Pleistocene age is orbital tuning [e.g., Imbrie et al., 1984; Martinson et al., 1987; Shackleton et al., 1990] wherein a constant phase relationship is assumed between paleoclimatic measurements and an insolation forcing based on Milankovitch theory [Milankovitch, 1941]. One of the well-known successes of orbital tuning was the Johnson [1982], and later Shackleton et al. [1990], prediction of a Brunhes-Matuyama magnetic reversal (B-M) age older than previously estimated, an inference which was subsequently confirmed by argon-argon dating [e.g., Singer and Pringle, 1996]. A number of radiometric dates for termination 2 also support the orbital age model [e.g., Broecker et al., 1968].

[4] Milankovitch theory, however, has come under question [e.g., Karner and Muller, 2000; Elkibbi and Rial, 2001; Wunsch, 2003a] and radiometric ages conflicting with the orbital ages have also been reported: for termination 2 by Henderson and Slowey [2000], and Gallup et al. [2002]; for terminations 3 by Karner and Marra [1998]; and for a variety of events by Winograd et al. [1992], among others. To understand long term climate change, it is necessary to resolve these conflicting age estimates. To avoid circular reasoning, an age model devoid of orbital assumptions is needed.

[5] As suggested by Shaw [1964], the age of geological events identifiable in multiple stratographies may be estimated using mean sediment accumulation rates, here termed “depth-derived” ages. The literature has numerous examples of depth-derived ages (e.g., Shackleton and Opdyke [1972], from 900 to 0 kyr BP; Hays et al. [1976], 500–0 kyr BP; Williams et al. [1988], 1900–0 kyr BP; Martinson et al. [1987], 300–0 kyr BP; and Raymo [1997], 800–0 kyr BP), but whose results have been inconclusive. The most comprehensive existing study, that by Raymo [1997], used...
11 marine sediment cores. Owing to her inference of systematic core extension during recovery, she could not distinguish between the conflicting orbital and radiometric termination age estimates.

This present study extends the depth-derived approach to 21 sediment cores, described below and, in what is a critical factor, accounts for the down-core trend in sediment compaction. An age uncertainty estimate for the depth-derived age model is provided, in part, by modeling accumulation rate variability as an autocorrelated stochastic process. Within the estimated uncertainty, the depth-derived and orbital age models are consistent with one another, but the depth-derived age model implies nonlinear relationships between earth’s orbital variations and the $\delta^{18}O$ climate proxy that are absent when the orbital age models are applied.

2. Data

An ensemble of 26 $\delta^{18}O$ records from 21 separate coring sites are used in this study. The core sites are shown in Figure 1 and can be divided into four geographical regions: the North Atlantic, eastern equatorial Pacific, equatorial Atlantic, and the Indian and western equatorial Pacific Oceans. Core site locations heavily favor the Northern Hemisphere. Four of the records are from piston cores (V22-174, V28-238, V28-239, and MD900963) while the remainder are composite records spliced together from multiple cores recovered by the Deep Sea Drilling Program (DSDP) or Ocean Drilling Program (ODP). For ODP and DSDP sites, the composite depth scale or, if available, the ODP revised composite depth scale, is used. Table 1 lists the pertinent statistics and gives references for each core.

All $\delta^{18}O$ records that were available to us, believed to be stratigraphically intact, and which extend through the B-M were included in this study. Use of planktic records, in addition to the benthic, allows for the inclusion of seven more sediment cores and decreases the uncertainty associated with the depth-derived age model. The depth of the B-M was reported in the literature as identifiable via magnetic stratigraphy in 12 of the 21 cores, and these cores are indicated by an “M” appended to the name in Table 1. For the $\delta^{18}O$ records associated with these 12 cores, the B-M invariably occurs within $\delta^{18}O$ stage 19. Where the B-M transition is not identifiable, the depth of event 19.1, the most negative $\delta^{18}O$ value in stage 19, is instead used, and in all cases an age of 780 kiloyears before present (kyr BP) (Singer and Pringle [1996], rounded to the nearest ten kyr) is assigned.

At the outset, it is convenient to correct for the effects of compaction on the depth scale. Sediment compaction typically increases with depth [e.g., Bahr et al., 2001] and thus systematically compresses a greater quantity of time into a given depth interval. Assuming that the estimated trends in porosity reflect inhomogeneities in relative compaction, we apply a correction based on conservation of dry sediment volume wherein the thickness of each sediment layer is adjusted so as to remove trends in porosity. Porosity trends are estimated by fitting a low-order polynomial to porosity observations; for cores without observed porosity profiles, comprising 13 of the 21 cores, the mean down-core porosity trend from the observed porosity profiles is instead used. While this method introduces an age model uncertainty of up to ±6 kyr, the alternative is an expected age model bias of up to 15 kyr. See Appendix A for more details. All subsequent depth references are to this decompacted scale. Note Huybers [2002] did not adequately account for the effects of compaction and thus arrived at older age estimates.
[10] It is helpful to define some terminology used in estimating the depth-derived age model. An “event” is a $\delta^{18}O$ feature whose depth can be uniquely identified within each $\delta^{18}O$ record. When an age is fixed to an event, it becomes an age control point (ACP). Two types of events are referred to, “stages” and “terminations.” Stages are defined as local minima or maxima in the $\delta^{18}O$ record [Prell et al., 1986] where the numbering system suggested by Imbrie et al. [1984] is used. All the stages referred to in this study have odd numbers after the decimal point, corresponding to low ice volume excursions in the $\delta^{18}O$ record. Terminations are defined as an abrupt shift from glacial to interglacial conditions [Broecker, 1984], where the assigned depth is the midpoint between the local $\delta^{18}O$ minimum and maximum. Figure 2 shows the eight terminations midpoints and nine stages which were visually identified in each $\delta^{18}O$ record. For comparison purposes, Figure 2 also shows these seventeen events identified on the SPECMAP $\delta^{18}O$ stack [Imbrie et al., 1984].

[11] A second, more objective, method of event correlation was also implemented using an automated cross-correlation maximization procedure. Within an expected error of 1 kyr, this algorithm, termed XCM, yielded event correlations which were identical to those determined from the visual procedure. This result gives some confidence in the ability to relate events in different $\delta^{18}O$ records. For more detail, see Appendix B and Huybers [2002].

[12] The choice of seventeen ACPs reflects a minimalist strategy for constraining the $\delta^{18}O$ record, especially when compared with the SPECMAP $\delta^{18}O$ stack which uses five times as many ACPs in the same 770 kyr interval. We do not use more ACPs for three reasons: (1) only a small decrease in age model uncertainty would result (section 4.1); (2) while more high-frequency structure in the composite $\delta^{18}O$ record is expected to be retained, false structure could be built into the averaged record by aligning noisy features; and (3) more ACPs are not expected to aid in resolving the spectra of higher-frequency processes because of the spectral smearing due to age model uncertainty (section 5.1).

3. Time and Sediment Accumulation

3.1. A Random Walk Model

[13] To understand the relationships between age and depth, we need a model of sediment accumulation rates. Both are expected to have systematic and stochastic elements, the latter here modeled as a random walk. Let $d_n$ be the depth of a layer of sediment in a core at time step $n$. Then for a unit time step, $\Delta t$, $d_n$ increases as

$$d_{n+1} = d_n + \Delta t S + \Delta t S'_n + W_n,$$  \hspace{1cm} (1)$$

where $\bar{S}$ is the mean sediment accumulation rate, $S'_n$ is the zero-mean stochastic contribution, and $W_n$ is a systematic term. Dividing by $\bar{S}$ converts the change in depth for each increment to a true time increment plus two anomaly terms,

$$t'_{n+1} = t'_n + \Delta t S'/\bar{S} + W_n/\bar{S},$$  \hspace{1cm} (2)$$

where $t'_n = d_n/\bar{S}$ is the linear age estimate. $W_n$ is treated here primarily as the sediment compaction effect (see
Although long-term variation in bioproductivity, terrigenous discharge, dust transport, and coring artifacts are also implicated. We focus first on the random element.

The simplest case is when $S_n$ is a white noise process, $\langle S_n S_m \rangle = 0$, $n \neq m$ (brackets, $\langle \cdot \rangle$, denote an ensemble average) and the variance of the difference between the apparent and true time grows linearly on average [Feller, 1966],

$$\langle (r_n' - n \Delta t)^2 \rangle = n \Delta t \frac{\sigma^2}{S^2}.$$  \hspace{1cm} (3)
where \( \sigma^2 = \langle S_j^2 \rangle \). Following Moore and Thomson [1991] and Wunsch [2000], we term the variance growth rate the “jitter,”

\[
J = \left( \frac{\sigma}{S_j} \right)^2, \tag{4}
\]

an appropriate measure when only one \( \delta^{18}O \) event is constrained to a known age. If the duration of the temporal random walk is fixed by introducing a second ACP at \( t = N\Delta t \), the expected variance between the two fixed points behaves as a “Brownian bridge” process. Following Odell [1975] and Bhattacharya and Waymire [1990], the Brownian bridge analogue of equation (3) is

\[
\langle (t_n - n\Delta t)^2 \rangle = n\Delta t \left( 1 - \frac{n}{N} \right), \quad 0 \leq n \leq N, \tag{5}
\]

where \( N \) is the total number of time steps between the 2 ACPs. Age variance is then zero at the two end points, with a maximum at the midpoint. Integrating, and comparing equations (3) and (5), shows that the inclusion of a second ACP results in a three-fold reduction in mean age variance.

### 3.2. Determining the Stochastic Element

To estimate the character and degree of jitter in deep sea sediment cores, it is useful to construct some simple age models. Rather than using the mixed stage and termination notation, each event is assigned a number, \( 1 \leq k \leq 17 \), running in temporal sequence from termination 1 to stage 19.1. Mean accumulation rates in core \( j \) between events 17 (stage 19.1) and 13 (termination 7) can be estimated as

\[
\bar{S}_j^{(1)} = \frac{d_{j,17} - d_{j,13}}{160}, \tag{6}
\]

where \( d_{j,k} \) is the depth of event \( k \), and 160 kyr is roughly the duration between events 17 and 13. If event 1 is pinned to an age of 10.6 kyr before present (BP), the ages of events 1 through 13 are then estimated as

\[
A_{j,k}^{(1)} = \frac{d_{j,k} - d_{j,1}}{\bar{S}_j^{(1)}} + 10.6, \quad 1 \leq k \leq 13, \tag{7}
\]

where the superscript indicates the use of one ACP. If a second ACP at the B-M transition is incorporated, an age model may be expressed as

\[
\bar{S}_j^{(2)} = \frac{d_{j,17} - d_{j,1}}{780 - 10.6},
\]

\[
A_{j,k}^{(2)} = \frac{d_{j,k} - d_{j,1}}{\bar{S}_j^{(2)}} + 10.6, \quad 1 \leq k \leq 17, \tag{8}
\]

where 780 kyr BP is the age of B-M transition.

[16] Calculation of the variance in age estimates for each event permits comparison with the random walk models of sediment accumulation. First, the mean age of each event is determined by averaging over all cores,

\[
\bar{A}_k^{(i)} = \frac{1}{21} \sum_{j=1}^{21} A_{j,k}^{(i)}, \tag{9}
\]

for both the \( i = 1 \) and \( i = 2 \) ACP cases. When planktic and benthic \( \delta^{18}O \) records are available within the same core, only the benthic record is used. The age variance can then be estimated as

\[
\sigma_{k}^{(i)} = \frac{1}{20} \sum_{j=1}^{21} \left( A_{j,k}^{(i)} - \bar{A}_k^{(i)} \right)^2, \quad 1 \leq k \leq 17. \tag{10}
\]

[17] Figure 3 shows the calculated age variances, \( \sigma_{k}^{(i)} \) with \( i = \{1, 2\} \). As expected, \( \sigma_{k}^{(1)} \) increases with the elapsed time from event one, \( t \), and \( \sigma_{k}^{(2)} \) has a Brownian bridge character.
Also shown are the simple random walk and Brownian bridge models as determined from equations (3) and (5) with $J = 10$ in both cases. Were the model adequate, a single value of the jitter should be applicable to modeling both $v_k^{(1)}$ and $v_k^{(2)}$, but it is evident from Figure 3 that $J = 10$ underestimates the variance of $v_k^{(1)}$ and overestimates that of $v_k^{(2)}$. Equation (3) also predicts $v_k^{(1)}$ is proportional to $t$, but it appears more nearly proportional to $t^2$ and is thus inconsistent with the hypothesis of a simple random walk in sediment accumulation. Some other effect is required to explain the result.

3.3. Sediment Accumulation With Autocovariance

[18] A generalization of the simple random walk to a correlated random walk is capable of accounting for the observed quadratic growth in the $v_k^{(1)}$ age variance. This generalization is plausible because sediment accumulation rates are themselves climate variables and can be expected to have a structured frequency spectrum implying temporal autocorrelation. To proceed, it is first necessary to adopt an age model estimated independent of accumulation rates.

[19] The Devils Hole record is devoid of orbital assumptions [Winograd et al., 1992] and has a radiometric age model with uncertainties ranging from ±10 kyr at its oldest time, 519 kyr BP, to ±2 kyr at its youngest, 140 kyr BP. A complication, however, is that the Devils Hole record is, in places, offset from the marine δ18O by up to 10 to 15 kyr and is thus not suitable for directly dating the marine δ18O records [Winograd et al., 1997; Herbert et al., 2001]. In estimating marine sediment accumulation rates, only the duration between events needs to be equal, and we assume that the relative timing between the marine and Devils Hole δ18O records is constant during most intervals. Acknowledging that this fixed-lag assumption probably breaks down during glacial maxima and terminations, the marine $A_k^{(2)}$ age models are nonetheless adjusted to maximize the squared zero-lag cross-correlation between the marine and Devils Hole δ18O records using the XCM algorithm (see Appendix B). The derivative of depth relative the adjusted $A_k^{(2)}$ age models then provide estimates of accumulation rates.

[20] For the purpose of comparison, accumulation rates were also estimated from the orbitally derived age models provided by other authors, as indicated in Table 1. Figure 4 shows the power density spectral estimates of sediment accumulation rates using the multitaper method [Thomson, 1990] with both the Devils Hole and the published orbital age models. Both spectra may be characterized as

\[ \Phi(s, s_0) = \frac{1}{s^2 + s_0^2} \]  

where $\Phi$ is the power density and $s$ the frequency. Such a relationship is consistent with an autoregressive process of order 1 (AR(1)), and implies a minus two power law relationship for frequencies above $s_0$, with white noise at the lowest frequencies. The Devils Hole age model gives $s_0 \approx 1/40$ kyr, but the orbital age models are consistent with the result of Mix et al. [1995b] with $s_0 \approx 1/100$ kyr. This difference in shape is likely due to errors in one or both of the age models. The scope of the spectral damage owing to jitter is unclear, but as discussed later, either value of $s_0$ gives a parameterization of accumulation rate variations consistent with the observed $v_k^{(1)}$ and $v_k^{(2)}$ age variances.

Figure 4. The multitaper spectral estimate of sediment accumulation rates for each record (dots), the mean from all the spectral estimates (dashed line), and an approximate spectral fit (solid line). Left panel is the analysis using the Devils Hole age model, and the right panel is from the orbital age models. Vertical bars indicate the approximate 95% confidence interval for the estimates from individual records (dots).
To estimate the uncertainty in ages due to accumulation rate variability, it is simplest to generate ensemble members from the stochastic accumulation model and calculate derived statistics from them. A synthetic accumulation rate with specified jitter ($J$) and power density ($\Phi$) can be generated as

$$S(t) = \mathcal{S}^{-1}\left\{\hat{\mathcal{N}}\sqrt{J\Phi(s, 1/40)}\right\} + 1,$$

where $\mathcal{S}^{-1}$ is the inverse Fourier transform, $\hat{\mathcal{N}}$ is the Fourier transform of a white noise process, and $\Phi(s, s_0) = \Phi(s, s_0)/\sum\Phi(s, s_0)$ where the sum is over all frequencies. Summing the accumulation rate gives a depth profile, $d(t) = \sum S(t)$, with the specified autocorrelation and jitter. By generating a large number of synthetic depth profiles and converting each to age with equation (7), a least squares best fit was sought between the observed and modeled $v^{(i)}$, $1 \leq k \leq 13$, distribution by varying the jitter in equation (12). A best fit was achieved with $J = 0.5$, and the resulting modeled $v^{(1)}$ and $v^{(2)}$ are shown in Figure 3. The autocorrelated random walk model reproduces the quadratic growth in $v^{(1)}$ and a single value of the jitter fits both the calculated $v^{(1)}$ and $v^{(2)}$ age variances. Further tests (not shown) indicate the auto-correlated random walk is equally consistent when greater numbers of age control points are used, and we will assume the same value of $J$ is appropriate for our 17-ACP model. If $s_r = 1/100$ kyr, corresponding to the orbitally tuned accumulation estimates, the observations are fit equally well using a smaller value of $J$; with this method one cannot distinguish between the Devils Hole and orbital age model accumulation rate estimates in the marine cores.

### 4. Depth-Derived Age Model

An age model based on a single linear age-depth relationship will be stretched or squeezed by every variation in sediment accumulation and each coring artifact. We seek to mitigate these age model errors by using multiple age-depth relationships. Table 2 indicates the $A_k^v$ event ages for each record along with the averages, $\overline{A_k^v}$. An age model based on these mean event ages, using
For each record, accumulation simulation.

4.1. Uncertainty Analysis

Core ages.

Imposing simultaneity between benthic and planktic records can range out to 1000 years and longer [e.g., 2002]. For the mean accumulation rates of the cores sampled here, we estimate the depth uncertainty translates to approximately ±4 kyr. Larger errors are incurred if 

\[
A^{(17)} = \frac{A^2_k - A^2_j}{d_k - d_j} \quad 1 \leq j \leq 21, \quad 2 \leq k \leq 17.
\]

For each record, \( j \), age is linearly interpolated with depth, \( d_j \), between each pair of ACPs, \( k - 1 \) and \( k \), yielding a piecewise linear age model. \( A^{(17)} \) is our best estimate of the core ages.

4.1. Uncertainty Analysis

[23] There are at least five sources of error in the \( A^{(17)} \) age model: non-simultaneity between isotopic events, uncertainty in identifying the depth of each event, variations in accumulation rates, postdepositional processes, and uncertainty in the age of the B-M. Each source of error is considered in turn, and a Monte Carlo method is applied in conjunction with the stochastic sediment accumulation model to assess the overall uncertainty.

4.1.1. Simultaneity

[24] The ocean mixing times for the \( \delta^{18}O \) signal can range out to 1000 years and longer [Wunsch, 2003b]. Imposing simultaneity between \( \delta^{18}O \) events, if correct, deblurrs this mixing effect. To account for the ocean mixing time, a random variable with a ±1 kyr standard deviation is added to the \( \delta^{18}O \) event ages in the stochastic sediment accumulation simulation.

[25] This study incorporates benthic and planktic foraminiferal species over a wide geographic range. Foraminiferal \( \delta^{18}O \) responds to both the temperature and \( \delta^{18}O \) of their environments [e.g., Schrag et al., 1996], and these environmental values likely fluctuate asynchronously and spatially heterogeneously. The presence of systematic offsets between benthic and planktic records can be estimated by means of a lagged cross-correlation when both species are measured in the same core. Figure 5 shows that the five benthic and planktic \( \delta^{18}O \) pairs used here have a maximum cross-correlation at positions within a 1 kyr lag; a 1 kyr error is included in the stochastic accumulation model. More localized offsets between benthic and planktic records may occur in parts of the isotopic sequence, but we find no obvious pattern; if present, these localized offsets are apparently secondary to uncertainties associated with event identification.

4.1.2. Identification

[26] Owing to machine error in measuring \( \delta^{18}O \), finite sampling resolution, and bioturbational blurring, events are only identifiable to within a finite depth range [Pisias et al., 1984; Huybers, 2002]. For the mean accumulation rates of the cores sampled here, we estimate the depth uncertainty translates to approximately ±4 kyr. Larger errors are incurred if \( \delta^{18}O \) events are misidentified, but we do not account for this possibility.

4.1.3. Accumulation Rate Variations

[27] Core-site variations in accumulation rate will introduce errors in linear age-depth relationships, as discussed in section 3.3. Averaging multiple age-depth realizations, to the degree that they are independent, reduces this uncertainty. An empirical orthogonal functions (EOF, or “singular vector”) analysis [e.g., Wunsch, 1996; von Storch and Zwiers, 1999] of accumulation rate variability, as estimated using \( A^{(17)} \), indicates there are about 11 degrees of freedom in accumulation rate variations, and thus also in the age estimates.

[28] Trends in global mean accumulation rates, as monitored at these 21 core sites, could bias the depth-derived age model. Spectra from both Devils Hole and from orbitally tuned chronologies, however, show low frequency white noise behavior (Figure 4) precluding long period global variations in accumulation. In agreement with this inference, Lyle [2003] found no evidence for spatially coherent long-period trends in Pacific carbonate accumulation during the Pleistocene. Thus, no uncertainties due to trends in accumulation are incorporated into the model.

[29] Porosity is itself a climate variable and is known to change with other components of the climate system [Herbert and Mayer, 1991; Hagelberg et al., 1995]. While random variations in porosity are implicitly accounted for in (2) above, climatically induced quasiperiodic age errors could contribute to the nonlinear and/or non-Gaussian structure of the \( \delta^{18}O \) signal later discussed in section 5 [see also Herbert, 1994]. Changes in porosity are often linked with changes in organic and calcium carbonate deposition [Herbert and Mayer, 1991], and, it is likely that porosity-climate biases tend to cancel out when one aggregates cores from different ocean basins, owing to the opposite response of Pacific and Atlantic carbonate cycles. Furthermore, spectral estimates of sediment accumulation rate variations using the orbital age models (see Figure 4) show a smooth red noise trend both on a site-by-site basis and in the mean. This result indicates the absence of strong quasiperiodic variations in total accumulation rates, or alternatively that such variability is not resolved by orbital age estimates. In section 5.3 we further evaluate the potential influence of these quasiperiodic variations on our results.

4.1.4. Postdepositional Effects

[30] Appendix A compares \( A^{(17)} \) with a similar age model in which compaction is not accounted for. The latter
displays a bias with ages, on average, 10 kyr older than the compaction-corrected age model, but tapering to zero at the fixed end-points. Two sources of error exist in the compaction correction. First, scatter in the porosity measurements introduces uncertainty in determining the trend toward lower porosity with depth. Second, a larger source of uncertainty results from the absence of porosity measurements for 13 of the records, requiring an indirect compaction correction as discussed in Appendix A. The combined compaction correction uncertainty averages ±5 kyr. While large, this uncertainty is preferable to an age model bias introduced by differential compaction according to sediment composition. [31] 2. The effects of coring on a sediment column are a further source of uncertainty for the depth-derived age model. Most of the records used here are from the advanced piston corer of the Ocean Drilling Program (ODP-APC) which uses a rigid drill pipe and a stationary piston in extracting cores. This drilling method reduces age-depth uncertainties related to over-sampling, a common problem for conventional non-rigid piston-corers, and uncertainties related to under-sampling, a common problem for gravity-corers [Skinner and McCave, 2003]. A remaining problem, however, is that the depth scale of cores obtained with the ODP-APC are typically stretched due to elastic rebound of the sediment after the core is recovered [MacKillop et al., 1995; Moran, 1997]. The degree of rebound depends on sediment lithology and is likely to be heterogeneous. The high-frequency variations and down-core trends in age-depth relationships caused by sediment rebound are effectively folded into the previous estimates of accumulation rate variability and trends in sediment compaction. Because we seek only to estimate an age model, it is not necessary to disentangle these in situ and postcoring sources of uncertainty.

4.1.5. Brunhes-Matuyama Reversal

[32] Singer and Pringle [1996] estimate that the age of the B-M is radiometrically constrained to within ±2 kyr. However, the depth of the reversal however, is not always clearly identifiable [Tauxe et al., 1996] and thus an additional uncertainty of ±4 kyr is added.

4.1.6. Monte Carlo Analysis

[33] The combined uncertainties associated with the depth-derived age model are incorporated into a stochastic age-depth model and estimated with a Monte Carlo analysis. All errors, except those associated with the compaction correction and accumulation rate variations, are modeled as independent realizations of a zero-mean Gaussian distribution. The expected squared error in the $A_k^{(17)}$ age estimates is then

$$\langle \sigma_k^2 \rangle = \frac{1}{21} A_1^2 + I_1^2 + I_2^2 + 2^2 \mid_{k=17}, \quad 1 \leq k \leq 17. \quad (14)$$

The first term on the right is the event-depth determination error, assumed to be independent in each core and hence divided by the number of cores, 21. The second and third terms are the benthic/planktic timing error and the ocean signal propagation times. The last term is the estimated radiometric age error applied only for $k = 17$, the Brunhes-Matuyama. Apart from the depth determination error, each error is likely to be correlated between cores, and thus not effectively reduced by averaging.

[34] The compaction correction uncertainty, denoted $e_k$, is strongly correlated between events, biasing the entire age model toward either younger or older ages. Realizations of $e_k$ are thus generated by multiplying the expected uncertainty structure (see Table 3) by a single value drawn from a zero-mean unit standard deviation Gaussian distribution.

[35] To account for the effects of jitter, a depth profile is generated according to equation (12) with $J = 0.5$ and $s_0 = 1/40$ kyr. This depth profile nominally spans events 1 (10.6 kyr BP) to 17 (780 kyr BP), and has a true age, $t$, associated with each depth. Seventeen depths are identified such that

$$t(d_k) = \hat{A}_k, \quad 1 \leq k \leq 17, \quad (15)$$

dewhere each $d_k$ represents the depth of a synthetic event and $\hat{A}_k$ are the fixed values estimated in equation (9). Applying equation (8), the depth profile is linearly converted to age yielding a jittered age estimate for each synthetic event. This process is repeated 11 times, corresponding to the approximately 11 degrees of freedom in accumulation rate estimates. Averaging over each of the synthetic records, $j$, yields a mean jittered age estimate,

$$\bar{A}_k' = \left( \frac{1}{11} \sum_{j=1}^{11} A'_{j,k} \right) + e_k + c_k, \quad 1 \leq k \leq 17, \quad (16)$$

to which the additional $e_k$ and $c_k$ error realizations are added. A prime is used to distinguish these synthetic realizations, $\bar{A}_k'$, from the real age estimates, $\hat{A}_k$.

[36] Applying equation (13) to equation (16) generates a single stochastic depth-derived age model realization. The root-mean square (rms) age deviation of numerous stochastic model realizations are used to estimate the expected $A_k^{(17)}$ age model uncertainty. Each event is a local minimum in uncertainty and events are spanned by short Brownian bridges. The event uncertainties are also tabulated in Table 2 and have a mean of ±9 kyr. As the magnitude of the short Brownian bridges is on the order of ±1 kyr and there are approximately 11 independent age-depth relationships, additional ACPs and independent age-depth relationships would only marginally reduce the uncertainty of this age model. Compared to the expected accuracy of most geochronological markers, particularly between the B-M and termination 2, the $A_k^{(17)}$ depth-derived age model has good age control.

[37] In section 5.1 the depth-derived age model is used in estimating the spectra of $\delta^{18}O$ records. It is expected that higher frequency processes will, in general, be more susceptible to age model jitter [Moore and Thomson, 1991; McMillan et al. 2002]. To gain a sense of jitter’s influence on spectral estimates, consider the harmonic process

$$H(t) = \cos(2\pi t/100) + \cos(2\pi t/41) + \cos(2\pi t/23). \quad (17)$$
Figure 6 shows successive periodograms of $H(t)$, where time, $t$, is stretched and squeezed to $\hat{t}$ using an increasingly large jitter. Jitter is modeled as realizations of equation (16) with $J$ increasing from zero to one and $s_0 = 1/40$ kyr (see equation (11)). For comparison, periodograms of $H(t)$ are also shown with the jitter expected for a single age-depth relationship, i.e., equation (16) with $e_k = 0$ and without the summation. For a single age-depth relationship, the 100 kyr variability is poorly resolved, and the higher frequency variability is smeared into a red noise background. The depth-derived age model does considerably better at resolving the 100 kyr and 41 kyr (obliquity) variability, but nonetheless has significant spectral smearing associated with the 23 kyr variability. Thus, if present, excess precession band variability is expected to be poorly resolved.

4.2. Comparison With Other Age Models

[38] The $A^{(17)}$ age model makes no assumptions about orbital control of climate, and thus provides independent age estimates to compare against the orbitally tuned chronologies. Figure 7 shows the difference between $A^{(17)}$ and the orbitally derived age models for the SPECMAP stack [Imbrie et al., 1984] and the ODP677 benthic $\delta^{18}O$ record [Shackleton et al., 1990]. The SPECMAP orbital age estimates extend beyond 625 kyr BP are generally considered too young, due to an incorrect B-M age [e.g., Shackleton et al., 1990; Singer and Pringle, 1996], and ages beyond termination 7 for SPECMAP are adopted from the orbitally tuned ODP677 chronology. There are up to 2 kyr differences between termination ages listed in Imbrie et al. [1984] and in Table 2 due to our use of the $\delta^{18}O$ midpoint in defining termination depths; also note there are typographical errors for the termination 5 and 7 ages in the Imbrie et al. [1984] table. Using the Table 2 ages, the root-mean square (rms) event age discrepancies between the depth and orbital age models are 3 kyr (SPECMAP) and 5 kyr (ODP677). Considering $A^{(17)}$ has an estimated uncertainty of $\pm 9$ kyr and SPECMAP one of $\pm 5$ kyr, the depth-derived chronology is consistent with the orbitally derived age estimates.

[39] The depth-derived estimate for termination 2 closely agrees with the orbitally derived age estimates (128 kyr BP), thus supporting the younger termination 2 radiometric age estimates [e.g., Broecker et al., 1968; Bard et al., 1990] over the older ages [e.g., Gallup et al., 2002; Henderson and Slowey, 2000]. Note, however, this conclusion is directly dependent upon the compaction correction which shifts the mean termination 2 age from 139 to 129 kyr BP (see Table 3; at termination 2 uncertainties in the compaction correction are about $\pm 4$ kyr.) Using a depth-tuning approach, but not correcting for compaction, Raymo [1997] estimated an age of 136 kyr BP for termination 2 and concluded this age was anomalously old due to sediment extension in the upper core. In general, however, the magnitude of sediment extension is expected to increase down-core because of the greater changes in effective stress [Moran, 1997; MacKillop et al., 1995]. Acting alone, greater extension with depth would give anomalously young ages. The anomalously old ages are more readily explained by a down-core increase in compaction which, because compaction is partially plastic [Moran, 1997], is not fully compensated for by postcoring sediment rebound. If uncorrected, this residual trend in compaction leads to the anomalously old termination 2 ages (see Table 3).

[40] Figure 7 also compares the Vostok deuterium ($\delta^D$) ages (Petit et al. [1999], GT-4 ice age) with the $A^{(17)}$ event ages. Clearly, the $\delta^D$ of Antarctic ice (Vostok) and $\delta^{18}O$ need not have a simple relationship with marine foraminiferal $\delta^{18}O$ records. Nonetheless, the RMS age deviation between GT-4 and $\bar{A}^{(17)}$ is only 6 kyr (events 1 through 8 only), and is within the expected uncertainty of the depth-derived age model. More striking in Figure 7 is the tendency of some of the Devils Hole event dates to differ markedly from those of the deep-sea cores, beyond the one-sigma error estimates of both data types. Devils Hole has an RMS age deviation with $\bar{A}^{(17)}$ (events 2 through 11) of 11 kyr where the depth-derived chronology is relatively younger between terminations 2 and 5, and older beyond termination 5. One should not infer from this result that either is incorrect: as noted, there is no necessity in the climate system for open ocean changes to be contemporaneous with those nearshore or over continents [Winograd et al., 1997].

[41] Of the available Pleistocene age models, $\bar{A}^{(17)}$ most closely accords with the orbitally tuned age estimates. Because the orbital and depth-derived age models were estimated using completely independent assumptions, their approximate accord encourages the belief that there is real skill in both of them. Nonetheless, as we will see, the differences between them have important consequences for the interpretation of the climate record.

5. The $\delta^{18}O$ Signal and Nonlinear Climate Change

[42] We now turn our attention away from the age models and toward the $\delta^{18}O$ signal itself. To extract a well resolved and representative signal from the ensemble of 26 $\delta^{18}O$ records, the leading empirical orthogonal function (EOF1) is calculated from the five planktic and five benthic records with an accumulation rate of 3 cm/kyr or greater and the smallest available mean sampling interval (see Table 1). EOF1 explains 78% of the $\delta^{18}O$ variance and represents an almost uniformly weighted average of the ten best resolved $\delta^{18}O$ records, thus making it similar to a mean or “stacked” record. For comparison, the leading EOFs of the five best resolved benthic records (EOFb) and planktic records (EOFp) are also calculated (see Figure 8). EOFb and EOFp are very similar to EOF1: each has a squared cross-correlation with EOF1 of 0.97. The squared cross-correlation between EOFp and EOFb is 0.89, indicating the ubiquity of the oceanic $\delta^{18}O$ signal. Given these high correlations, it is not surprising that the spectral description of EOF1 presented in the following sections also holds for EOFb and EOFp.

[43] Figure 8 shows EOF1 from $A^{(17)}$ and the SPECMAP $\delta^{18}O$ stack [Imbrie et al., 1984] on its orbitally tuned age model. The SPECMAP age model was constructed by imposing a constant phase relationship between the obliquity and precession orbital parameters and the respective frequency bands in five separate $\delta^{18}O$ records [Imbrie et al., 1984].
The five orbitally tuned $^{18}$O records were then averaged to form the stack. The initial discussion here compares EOF1 with the SPECMAP stack; afterward, for purposes of comparison, an orbitally tuned version of EOF1 is also investigated.

The ten $^{18}$O records used in EOF1 are independent of the five SPECMAP stack records, yet the isotopic variations in the SPECMAP stack and EOF1 are strongly similar in timing, number, and amplitude. That there is only a 3 kyr RMS age model difference between the SPECMAP stack and EOF1 is rather remarkable. When pinned to their respective independent age models, the squared correlation between EOF1 and SPECMAP is 0.68. This is a higher correlation than between the exclusively planktic SPECMAP stack and EOFp, even when the single high-latitude planktic record from ODP982 is excluded from EOFp.

5.1. Spectral Description of the $^{18}$O Record

The spectral distribution of the SPECMAP stack, shown in Figure 9, has a power law relationship with frequency, $s^q$, $q \approx 2.7$ and spectral peaks lying above the approximate 95% level-of-no-significance in the 1/100, 1/41 (obliquity), 1/23 and 1/18 kyr (precession) bands relative to the background continuum. Bands are defined as the interval ±1/400 kyr about the central frequency. The SPECMAP distribution of energy has been widely accepted as accurately representing long-period $^{18}$O variability [e.g., Imbrie et al., 1993] with the spectral peaks in the obliquity and precession bands commonly interpreted as showing linear responses to the respective orbital variations [e.g., Hagelberg et al., 1991]. Of course, this obliquity and precession prominence is assumed in the orbital tuning. Note in particular that the energy fraction lying in the obliquity and precessional bands is a small fraction of the record total. The origins of the 100 kyr band variability are much more contentious owing to the paucity of insolation forcing in this band. Climatic resonance, nonlinear climatic response, and additional forcing mechanisms have all been postulated as explanations for the 100 kyr-band variability (for a review, see Elkibbi and Rial [2001]). Roe and Allen [1999] point out the difficulty in differentiating among these competing 100 kyr-band orbital theories, and there is some doubt whether an orbital relationship exists at all [Wunsch, 2003a].

The depth-derived age model provides a somewhat different perspective on $^{18}$O variability. The periodogram
of EOF1, shown in Figure 9, has a power law, like that of the SPECMAP stack, with $q / C^2 2.7$. But unlike the SPECMAP result six, rather than four, spectral bands are above the approximate 95% level-of-no-significance at 1/100, 1/70, 1/41, 1/29, 1/23, and 1/18 kyr. A simple relationship between the central frequencies, $s(n)$, of these bands is

$$s(n) = \frac{1}{41} + \frac{n}{100}, \quad -1 \leq n \leq 3. \quad (18)$$

$s(n)$ is written in terms of the 1/41 kyr band (obliquity) rather than the 1/23 or 1/18 bands (precession) because the 1/41 kyr band accounts for a greater fraction of the $\delta^{18}O$ variability.

[47] The energy in the 1/100, 1/41 ($n = 0$), 1/23 and 1/18 kyr ($n = 2, 3$) bands has been much discussed. Excess energy near 1/70 and 1/29 kyr has also been noted in the literature [e.g., Nobes et al., 1991; Yiou et al., 1991; Bolton and Maasch, 1995; Mix et al., 1995b]. The simple rule embodied in equation (18) is strongly suggestive of a spectral structure resulting from a weak nonlinear interaction of the obliquity band with the 100 kyr band. The conventional interpretation, referred to as the “pacemaker” hypothesis [Hays et al., 1976], requires that the timing of the very energetic quasi-100 kyr variability be controlled by the weaker high frequency elements. Here it appears that the most energetic bands (100 kyr, 41 kyr) interact to produce sum and difference frequencies, as is typical of a weakly nonlinear system. A complication of the conclusion is the possibility that the enhanced precession band energy is due, all or in part, to overtones of the obliquity band response.

5.2. Higher-Order Spectral Analysis

[48] A higher-order statistic, the autobicoherence, aids in distinguishing the behaviors of EOF1 and the SPECMAP stack age model. The 95% level-of-no-significance for autobicoherence, computed by Monte Carlo methods for Gaussian red noise with a power law of minus two, is 0.7 along the diagonal ($s_1 = s_2$) and 0.55 off the diagonal ($s_1 \neq s_2$). Appendix D discusses the autobicoherence test in more detail. Before examining autobicoherence in the $\delta^{18}O$ records, the nature of the possible forcing is investigated using a test signal,

$$T(t) = \theta(t) + p(t). \quad (19)$$

where $\theta$ is obliquity and $p$ is precession as calculated by Berger and Loutre [1992]. Both components of $T(t)$ are normalized to have unit standard deviation and zero mean. Because the origins of the 100 kyr band are so uncertain, no corresponding forcing term is included. The completely deterministic $T(t)$ displays a number of significant autobicoherencies (see Figure 10) related to the amplitude and frequency modulations inherent to these orbital parameters [e.g., Hinnov, 2000]. The strong autobicoherence at (1/41, 1/41) highlights the potentially ambiguous origins of the precession band; that is, the first harmonic of obliquity (2/41 kyr) and the precession band (1/23 to 1/18) overlap. Note that a rectification of the annual cycle is required for
long-term precessional variability to appear in a record [Rubincam, 1994; Huybers and Wunsch, 2003], and that such rectification is also expected to generate harmonics of the obliquity energy.

[49] Significant autobicoherence can indicate the presence of a nonlinearity in a record, or that the distribution is non-Gaussian, or both (nonlinear records are usually non-Gaussian). Here \( T(t) \) is non-Gaussian (it is deterministic). The distribution of the \( \delta^{18}O \) record, shown as a histogram of \( \delta^{18}O \) measurements from the 26 records shown in Table 1, appears in Figure 10. The \( \delta^{18}O \) signal has a skewness of \(-0.1\) and a kurtosis of \(2.5\), clearly indicating its non-Gaussian nature and as with \( T(t) \), interpretation of the autobicoherence must account for this fact.

[50] The autobicoherences of EOF1 and the SPECMAP stack are shown in Figure 11. The SPECMAP estimate displays significant autobicoherence at frequency pairs \((1/70, 1/70), (1/70, 1/41), \) and \((1/41, 1/29)\), a pattern which resembles that of \( T(t) \), on which the chronology of SPECMAP is based. The SPECMAP autobicoherencies which are most emphasized however, involve the \(1/70\) and \(1/29\) kyr bands, and unlike EOF1, these bands display no significant concentrations of energy. Hagelberg et al. [1991] also find evidence of a \((1/80, 1/41)\) autobicoherence in the orbitally tuned ODP 677 benthic and planktic \( \delta^{18}O \) records, which given the coarseness of their frequency resolution, is indistinguishable from the SPECMAP \((1/70, 1/41)\) pair.

[51] EOF1 displays a gridded pattern of autobicoherencies: all combinations of frequencies in equation (18) with integers \(-1 \leq n \leq 2\) are coincident with significant local maxima in autobicoherence except for \((1/29, 1/29)\) and \((1/23, 1/23)\). Whether the autobicoherence arises from non-Gaussian statistics in the forcing, or nonlinearity in the response, its distinct frequency structure supports the inference of weak interband interaction within the climate system. The absence of bicoherence at the strongest precession band \((1/23, 1/23)\) points to obliquity’s central role in this coupling.

5.3. Importance of Age Models

[52] There are important differences between EOF1 and the SPECMAP \( \delta^{18}O \) stack: SPECMAP has more than three
times the energy concentrated within the precession band but no discernible concentration of energy at 1/70 and 1/27 kyr; furthermore the autobicoherent features are significantly different. The small amount of precession band energy in EOF1 may be a result of age model jitter (see Figure 6). We attribute the remaining differences to the orbital tuning of the SPECMAP age model; support for this hypothesis is provided by considering the effects of jittering SPECMAP and orbitally tuning EOF1.

Monte Carlo simulations indicate that random age model errors tend to diminish both concentrations of spectral energy and autobicoherence, making such errors an unlikely explanation for the structure in EOF1. Quasiperiodic age model errors, however, can create spurious structure in spectral estimates (see section 4.1; Herbert [1994]). To examine this possibility, the spectral and autobicoherence structures of SPECMAP were examined after distorting the age model using periodic and quasiperiodic functions. The most relevant results occur for 100 kyr periodic distortions of the SPECMAP age model, yielding significant concentrations of energy at the 1/70 and 1/27 kyr bands. Similarly, distorting SPECMAP ages in proportion to the $\delta^{18}O$ signal yields a concentration of energy at 1/70 kyr. All of these age model errors, however, tend to decrease

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**Figure 9.** Periodograms are (a) SPECMAP, (b) EOF1, and (c) the orbitally tuned EOF1. For presentation purposes, each periodogram, after the first, is shifted downward by two orders of magnitude. Dashed lines are third-order polynomials fit to the noise background of each periodogram. Numbers are the percentage of energy contained within each band (1/100, 1/70, 1/41, 1/29, 1/23, and 1/18 kyr) above the background noise level; the 1/100 kyr band accounts for almost half of the total energy within each spectrum. The approximate 95% confidence interval (from $\chi^2$ with two degrees of freedom) is indicated by the vertical bar in the upper right hand corner; the open circle represents the expected background level.

**Figure 10.** Histogram of $\delta^{18}O$ measurements between 10 to 780 kyr BP from the 26 records listed in Table 1.
autobicoherence without making the pattern appear more like that of EOF1.

On the other hand, in Appendix C, we show that the orbital tuning of EOF1 makes its spectra and autobicoherence pattern appear more similar to the SPECMAP stack. It is further demonstrated, using synthetic signals, that orbital tuning tends to suppress evidence of weak nonlinearity in a record, by shifting energy out of overtone and interaction bands and into the Milankovitch bands. We thus conclude that orbital tuning tends to suppress evidence of real nonlinearity in the $\delta^{18}O$ record.

6. Conclusions

Age models assigned to paleoclimatic records strongly influence the inferences drawn about past climate behavior. Variations in sediment accumulation rate cause errors in linear age-depth models, so that a simple linear age-depth relationship is often not sufficiently accurate to yield meaningful results. Use of orbital tuning to remove these age model errors, however, suppresses evidence of nonlinearity at low frequencies in the system.

An alternative to orbital tuning is to estimate sediment core age using spatial mean sediment accumulation rates, and in conjunction with an important compaction correction, this alternative is used at 21 core sites to construct a depth-derived age model spanning the last 780 kyr. The observed error in linear age-depth relationships is modeled as an autocorrelated stochastic process, and the $A^{(17)}$ age model is estimated to be accurate to within ±9 kyr. The depth-derived ages make no assumptions regarding orbital control, but agree with the orbitally tuned age models to within ±5 kyr, and thus within the error limits are consistent with one another. The remaining discrepancies, however, have important consequences.

Spectral analysis of EOF1, using the $A^{(17)}$ age model, indicates significant spectral energy at combination tones of the 1/100 kyr and obliquity bands. There is also significant autobicoherence between each of these bands in EOF1, all of which indicates a weakly nonlinear climatic response to obliquity forcing interacting with the quasi-100 kyr variability. These results may aid in differentiating between the various mechanisms proposed to explain glacial interglacial climate variability.

Appendix A: Compaction Correction

Sediment compaction is, to first order, a function of pressure and lithology [e.g., Athy, 1930; Baldwind and Butler, 1985]; factors such as time, temperature, and pore...
water chemistry are generally secondary. Because pressure increases with depth, systematic down-core compaction is expected, and this phenomenon is observed in a wide variety of marine cores [e.g., Baldwin and Butler, 1985; Bahr et al., 2001]. Postcoring sediment rebound partially compensates for in situ compaction, but because compaction is more plastic at higher pressure [e.g., Moran, 1997], residual down-core trends toward greater compaction are retained. Variations in lithology can also modify the compaction profile, for instance clay deposited above limestone can lead to reduced compaction with depth [e.g., Schwarzacher, 1975], but there is no reason to expect such structures to be systematically present in the global array of cores studied here. Climatically driven quasiperiodic changes in compaction are addressed in sections 4.1 and 5.3.

[59] The effect of compaction on linear age-depth relationships is discussed qualitatively by Hays et al. [1976], Williams et al. [1988], and Raymo [1997]. Here a quantitative age correction function is developed for gross trends in compaction and then applied to the depth scale of each core. Athy [1930] first showed an increasing load on porous sediment results in pore water draining from the sediment matrix and an exponentially decreasing porosity. Porosity, $\phi$, is the fraction of sediment volume occupied by water,

$$\phi = 1 - \frac{\rho}{\rho_d},$$

(A1)

where $\rho$ is the bulk density, and $\rho_d$ is the dry density.

[60] Given a functional relationship between depth and porosity, it is possible to estimate the effects of compaction on a linear age model. Take $h = 0$ and $t = 0$ as the sediment height and date of the B-M magnetic reversal. Sediment accumulates at a rate $S$ so that

$$h = \int_0^t S(t) dt,$$

(A2)

and without compaction, the final height would be, $H = \bar{S}T$. $\bar{S}$ is the mean accumulation rate, $H$ and $T$ are the final-time values of $h$ and $t$.

[61] If compaction is assumed to result solely in the upward expulsion of pore water [e.g., Berner, 1990], the compacted and un-compacted sediment column heights are related by

$$h = \int_0^{h'} \frac{1 - \phi'}{1 - \phi} dh',$$

(A3)

where primes indicate the compacted quantity. For the moment, assume postdepositional compaction is present, but accumulation rates are constant. Then, if age is taken to be linear with depth between $h = 0$ and $h = H$, an error is incurred as

$$\delta t = t' - t = T\left(\frac{h}{H} - \frac{h'}{H'}\right).$$

(A4)

The age error is zero at the top, $h = H, h' = H'$, and bottom, $h = h' = 0$, but between these fixed points errors occur to the degree that $h'$ is a nonlinear function of time. If compaction increases with depth, as expected, a layer of sediment between the top and bottom has $h/H > h'/H'$, $\delta t > 0$, and compacted age estimates which are erroneously old.

[62] To illustrate the possible effects of compaction on an age model, assume that $\phi$ is constant and that compaction occurs at a linear rate with depth, $c$, such that $\phi' = \phi - c(H' - h')$. Inserting this porosity relationship into equation (A3) and integrating yields

$$h = h' + \frac{ch'}{(1 - \phi)}\left(H' - h'\frac{H'}{2}\right).$$

(A5)

Substituting equation (A5) into equation (A4) and writing $h' = H'/T'$ gives

$$\delta t = t'\left(\frac{1 - \phi + cH'(1 - t'/2T)}{1 - \phi + cH'/2} - 1\right).$$

(A6)

Plausible values for equation (A6) are $\phi = 0.7$, $c = .01$ m/m, $H' = 30$ m, and $T = 800$ kyr, yielding a maximum age offset, $\delta t = 9$ kyr at 400 kyr BP. Equation (A6) shows that offsets toward older ages will increase with greater porosity, compaction, and accumulation rate.

[63] Figure 12 shows the porosity profile plotted against depth for eight ODP cores located in the eastern equatorial Pacific (ODP846, 849, and 851; Leg 138 ODP Initial Reports CD-ROM), Ceara Rise (ODP925 and 927; Leg 154 Log and Core Data CD-ROM, Borehole Research Group, LDO), and the N. Atlantic regions (ODP980, 982, and 983; Leg 162 Log and Core Data CD-ROM, Borehole Research Group, LDO) measured using gravimetric techniques [Boyce, 1976]. The eastern equatorial Pacific group shows a general trend of decreasing porosity with depth superimposed on a large degree of scatter where the scatter is in-part attributable to variations in lithology, coring effects, and measurement error.

[64] For the eight cores in which data are available, porosity trends are estimated from 400 m below the seafloor to the core top. For the eastern equatorial Pacific cores, a line is fit to each porosity profile, and for the Ceara Rise and North Atlantic cores a second-order exponential is used. ODP980 was alone in showing no discernible trend. Assuming that the estimated trends in porosity reflect inhomogeneities in relative compaction, we apply a compaction correction based on conservation of dry sediment volume [e.g., Berner, 1990],

$$h(1 - \phi) = h'(1 - \phi').$$

(A7)

Here the thickness of a compacted sediment layer, $h'$, is adjusted to thickness, $h$, by adjusting the down-core trend in porosity, $\phi(h')$, to a constant value, $\phi$. Note, the depth-derived ages are insensitive to the choice of reference porosity, $\phi$, because they are pinned to a constant age at termination 1 and the B-M.
Although it is highly likely that trends in compaction exist at all, or most, of the core sites [e.g., Bahr et al., 2001], the compaction correction has considerable uncertainty for the thirteen sites at which porosity measurements are not available. Standard decompaction formula are only applicable at depths well below that of the B-M [Baldwin and Butler, 1985; Bahr et al., 2001] (greater than 200 m), and we choose to use the mean of the seven identified porosity trends (see Figure 12) as the basis for decompacting the remaining fourteen cores according to equation (A7), to include ODP980. To estimate the associated uncertainty, each of the fourteen cores are also decompacted using the individual porosity-depth trends, yielding seven estimates of decompacted depth. Age is then estimated from each realization of the decompacted depth scale according to equation (13), and the standard deviation of these ages is taken as the estimated uncertainty.

Table 3. Decompacted Age Corrections in Kiloyears Applied to Each Core Where All Age Corrections Produce a Relatively Younger Age Model*  

| Event (Associated Stage/Termination) | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |  
|-------------------------------------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| ddsdp502P                           | 0  | 2  | 4  | 5  | 8  | 9  | 10 | 11 | 12 | 12 | 12 | 11 | 10 | 10 | 8  | 6  | 3  | 0  |  
| ddsdp552B                           | 0  | 2  | 4  | 5  | 8  | 10 | 10 | 11 | 12 | 12 | 12 | 11 | 11 | 9  | 7  | 4  | 0  |  
| ddsdp6607B                          | 0  | 3  | 5  | 7  | 11 | 14 | 15 | 16 | 17 | 17 | 17 | 15 | 14 | 11 | 8  | 0  |  
| md900963P                           | 0  | 2  | 3  | 5  | 8  | 11 | 13 | 16 | 17 | 18 | 19 | 18 | 17 | 14 | 8  | 0  |  
| odp659B                             | 0  | 3  | 4  | 6  | 9  | 11 | 12 | 13 | 14 | 15 | 14 | 13 | 12 | 9  | 5  | 0  |  
| odp663P                             | 0  | 3  | 5  | 7  | 10 | 12 | 14 | 15 | 17 | 17 | 17 | 16 | 15 | 11 | 8  | 0  |  
| odp664B                             | 0  | 3  | 6  | 7  | 11 | 12 | 14 | 15 | 16 | 17 | 17 | 15 | 13 | 10 | 7  | 0  |  
| odp677B                             | 0  | 3  | 5  | 6  | 7  | 12 | 14 | 15 | 17 | 17 | 17 | 16 | 14 | 12 | 9  | 0  |  
| odp677P                             | 0  | 4  | 5  | 7  | 10 | 13 | 14 | 15 | 17 | 17 | 17 | 16 | 14 | 12 | 9  | 0  |  
| odp758B                             | 0  | 2  | 3  | 4  | 6  | 7  | 8  | 9  | 10 | 10 | 10 | 11 | 10 | 9  | 8  | 0  |  
| odp758P                             | 0  | 2  | 2  | 4  | 5  | 7  | 8  | 9  | 10 | 10 | 10 | 10 | 10 | 9  | 8  | 0  |  
| odp806B                             | 0  | 1  | 2  | 4  | 7  | 9  | 10 | 11 | 12 | 13 | 13 | 12 | 11 | 10 | 7  | 5  | 0  |  
| odp806P                             | 0  | 1  | 3  | 4  | 7  | 9  | 10 | 11 | 12 | 12 | 12 | 11 | 9  | 7  | 4  | 0  |  
| odp846B                             | 0  | 2  | 3  | 5  | 5  | 6  | 7  | 7  | 7  | 7  | 7  | 7  | 6  | 4  | 3  | 0  |  
| odp849B                             | 0  | 0  | 1  | 1  | 1  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 1  | 1  | 0  |  
| odp851P                             | 0  | 0  | 1  | 2  | 3  | 3  | 4  | 4  | 4  | 4  | 4  | 4  | 3  | 3  | 2  | 0  |  
| odp925B                             | 0  | 5  | 7  | 9  | 15 | 20 | 22 | 24 | 29 | 30 | 30 | 28 | 26 | 20 | 13 | 0  |  
| odp927B                             | 0  | 5  | 7  | 10 | 14 | 17 | 20 | 22 | 25 | 26 | 27 | 27 | 26 | 24 | 18 | 14 | 0  |  
| odp927P                             | 0  | 5  | 7  | 9  | 15 | 17 | 20 | 22 | 26 | 26 | 27 | 27 | 26 | 24 | 18 | 14 | 0  |  
| odp980B                             | 0  | 3  | 4  | 5  | 12 | 15 | 17 | 19 | 22 | 25 | 25 | 24 | 22 | 22 | 18 | 14 | 0  |  
| odp982B                             | 0  | 2  | 3  | 5  | 8  | 9  | 10 | 12 | 13 | 12 | 12 | 11 | 10 | 8  | 6  | 5  | 0  |  
| odp982P                             | 0  | 1  | 3  | 4  | 7  | 9  | 10 | 11 | 13 | 12 | 12 | 11 | 10 | 8  | 6  | 5  | 0  |  
| odp983B                             | 0  | 5  | 6  | 10 | 16 | 21 | 25 | 28 | 30 | 30 | 31 | 30 | 28 | 24 | 19 | 15 | 0  |  
| v22-174P                            | 0  | 2  | 3  | 5  | 7  | 8  | 10 | 11 | 11 | 11 | 11 | 11 | 10 | 10 | 8  | 6  | 0  |  
| v28-238P                            | 0  | 2  | 3  | 4  | 5  | 6  | 8  | 9  | 10 | 10 | 10 | 10 | 9  | 6  | 4  | 0  |  
| v28-239P                            | 0  | 1  | 2  | 3  | 5  | 6  | 6  | 7  | 7  | 7  | 7  | 6  | 6  | 4  | 2  | 0  |  
| mean                                | 0  | 3  | 4  | 5  | 8  | 10 | 12 | 13 | 15 | 15 | 15 | 14 | 13 | 10 | 7  | 0  |  
| σ                                    | 0  | 1  | 2  | 2  | 3  | 4  | 4  | 5  | 5  | 6  | 6  | 5  | 5  | 4  | 3  | 0  |  

*Event numbers are listed at top along with parenthetical associated stage (Arabic) and termination (Roman) numbers. At bottom are the mean correction and the associated uncertainty. The applied corrections are a continuous function of depth, but are listed only at the 17 selected events.
which maximizes the cross-correlation. To prevent unrealistic changes in implied accumulation rates, XCM may be constrained to not stretch or squeeze time beyond a specified factor. The final control-point arrangement provides a piecewise linear approximation to \( \mu(t) \). It should be noted that XCM may significantly increase the cross-correlation between two records without there being any true relationship (see Appendix C).

Most tuning algorithms employ narrow band-pass filtering to isolate the Milankovitch band of interest. A difficulty with this approach is that even slight errors in the preliminary age model can smear spectral energy across the entire frequency range [e.g., Martińson et al., 1987]. This mistiming results in a form of aliasing of the spectral power, and like all aliasing, no filter can undo it. Thus we have chosen not to filter records prior to tuning, and instead use

Figure 12. Changing porosity with depth in ODP cores from the eastern equatorial Pacific, Ceara Rise, and North Atlantic. An exponential curve or straight line (whichever is better) was fit to each porosity profile, except for ODP980, which showed no distinct pattern. The vertical dotted lines bound the change in porosity between termination 1 and the B-M.
what is termed the direct response approach [Martinson et al., 1987].

Appendix C: Impact of Orbital Tuning

If climate linearly responds to insolation variations, one would expect the modulation structure of the forcing to be at least qualitatively mimicked in the response. If one seeks to tune to precession, this assumption is immediately complicated by the requirement for a rectifier to be present [Rubincam, 1994; Huybers and Wunsch, 2003]. Nonetheless, assuming some climatic response to insolation forcing, a multitude of methods have been used to orbitally tune paleoclimatic records. The criteria generally used to assess the accuracy of an orbitally tuned timescale [e.g., Imbrie et al., 1984; Bruggerman, 1992; Shackleton et al., 1995] are that geochronological data should be respected within their estimated accuracies, sedimentation rates remain plausible, variance should become concentrated at the Milankovitch frequencies with a high coherency between the orbital signal and the data, and (what is often referred to as the clinching argument) similar amplitude modulation should appear in the Milankovitch derived insolation functions and in the orbitally tuned result.

To comply with the criteria for a successful orbital tuning result, the XCM algorithm is constrained to not stretch or squeeze a record by more than a factor of four, thus keeping accumulation rates within plausible levels. Considering the difficulty of determining geochronological dates in the interval between termination two (approximately 130 kyr BP) and the Brunhes-Matuyama (B-M) boundary (approximately 780 kyr BP), it seems unlikely the available geochronological constraints would conflict with most tuning results. Three signals are selected to demonstrate the impact of orbital tuning: EOF1, white noise, and a weakly nonlinear signal.

C1. EOF1

The selected target curve for orbitally tuning EOF1 is

\[ \tau(t) = \sqrt{2\theta'(t)} + \sqrt{8p'(t)}. \]  

(C1)

The primes indicate the phases of obliquity and precession are each phase-lagged assuming a linear response with a time constant of 17 kyr, consistent with the orbital target curves of Imbrie et al. [1984]. Rather than iteratively tuning to precession and obliquity respectively, as done by the

Figure 13. Results from the orbital tuning of white noise. (a) Time series of white noise. (b) Precession curve (thick line) and same white noise process tuned to precession (thin line). (c) Band-pass filtered, tuned white noise (thin line), and the precession curve. Note that the band-pass filtered white noise shows an amplitude modulation similar to the precession curve. Right panel displays the power density spectra of the original white noise (top), of the tuned white noise (middle), and of the band-pass filtered tuned white noise (bottom). These spectra are displaced in the vertical by a factor of $10^4$ for visual clarity, and the vertical dotted lines delineate the precession band, 1/23 to 1/18 kyr.
SPECMAP group, the two parameters are combined into a single target curve, $\tau(t)$, with precession accounting for 80% of the total variance. ACPs are assigned to the $A^{(17)}$ age model every eight kyr, and XCM was used to maximize the cross-correlation between EOF1 and $\tau(t)$.

[72] The difference between the $A^{(17)}$ and the fully orbitally tuned EOF1 age model is shown in Figure 7. Not surprisingly, orbital tuning brings $A^{(17)}$ into close agreement with the SPECMAP and orbital ODP677 age models. The periodogram (Figure 9c) and autobicoher-

Figure 14. (a) The orbital signal $\psi(t)$ from equation (C2) (left) and its associated periodogram (right). The linear components of $\psi(t)$ give spectral peaks at 1/100; 1/55, a sideband of obliquity; and 1/41 kyr, the main obliquity band. The nonlinear components give spectral peaks at 1/70, the $1/100 - 1/41$ combination tone; 1/50, the $2/100$ overtone; 1/29, the $1/100 + 1/41$ combination tone; 1/23, an interaction tone; 1/21, the $2/41$ overtone; and 1/17 kyr, another interaction tone. (b) After a small degree of orbital tuning, assuming a linear response to obliquity and precession (bottom curve), the signal is visually similar but the periodogram has concentrations of energy primarily at the 100 kyr, obliquity, and precession bands. The approximate 95% confidence interval for red noise is indicated by the vertical bar.  

Figure 15. The autobicoherence of $\psi(t)$ before (left) and after (right) orbital tuning. Significant autobicoherence is indicated by light shading for the off-diagonal and dark shading for the on-diagonal.
ence (Figure 11c) of the orbitally tuned EOF1 now resemble those from SPECMAP. In particular, orbital tuning enhances the obliquity and precession peaks in EOF1 while diminishing the 1/29 and 1/70 kyr spectral peaks and making the (1/70, 1/29) and (1/41, 1/41 kyr) autocohereence appear insignificant. The spectrum of EOF1 is sensitive to the process of orbital tuning, and assuming a linear response to obliquity and precession imposes a behavior consistent with the SPECMAP analysis.

C2. Noise

[73] It is also useful to investigate signals with known statistical properties. We begin with a white noise Gaussian distributed process, \( \psi'(t) \), and tune it to the precession parameter [Berger and Loutre, 1992] over a 800 kyr period. A typical realization of XCM tuning is presented in Figure 13 where the squared cross-correlation is increased from zero to 0.19. Consistent with the results of Neeman [1993], a concentration of variance at the triplet of precessional peaks occurs, coherence in the precession band is greater than 0.9 (0.65 is the approximate 95% level-of-no-significance), and both amplitude and frequency modulation similar to the precession parameter appears, completely spuriously. When band-pass filtered, the imposed frequency modulations produce amplitude modulations in the tuned signal [see Huybers, 2002]. Similar results hold when red noise, rather than white noise, is orbitally tuned. Thus precession-like amplitude modulation in an orbitally tuned record does not guarantee the accuracy of an age model.

C3. Nonlinear Signal

[74] Finally, the observations regarding EOF1 in section 5 motivate investigation of another signal,

\[
\psi(t) = 2\cos(2\pi t/100) + \theta(t) + 0.5(\cos(2\pi t/100) + \theta(t))^2,
\]

involving linear and nonlinear contributions from a 100 kyr harmonic and zero-mean unit variance obliquity variability. The relative amplitudes are selected to reflect the distribution of variance observed in EOF1, and for statistical stability, a small amount of white noise is added. As evident from the periodogram in Figure 14, the nonlinearity generates variability at a number of combination and over-tones. A potentially confusing result is that energy appears at the first overtone of the main obliquity band 1/21 kyr, and, because of the frequency and amplitude modulation inherent to obliquity, at interaction bands of 1/23 and 1/17 kyr. Without knowing the form of \( \psi(t) \), a triplet of spectral peaks at these frequencies could readily be mistaken for evidence of precession variability.

[75] Figure 14 also shows \( \psi'(t) \) after orbital tuning to the target curve, \( \tau(t) \), given in equation (C1). Typical results increase the squared cross-correlation between the target curve and \( \psi' \) from 0.1 to 0.25. After tuning, the nonlinear spectral peaks are suppressed while precession period variability is enhanced. Similarly, Figure 15 shows that the autocohereence structure of \( \psi(t) \) is almost totally obscured by the orbital tuning, all of which indicates that orbital tuning will suppress evidence of real nonlinearity.

Appendix D: Autobicoherence

[76] A test for quadratic coupling was presented by Hasselmann et al. [1963] and used to evaluate weak nonlinearities in shallow water wave propagation. When two harmonics are coupled so as to modulate one another, a third harmonic with a particular frequency and phase is expected,

\[
S(t) = e^{2\pi ij(t+\phi_0)}e^{2\pi ik(t+\phi_1)} = e^{2\pi i(f_1+f_2)t+\phi_0+\phi_1}.
\]

[77] To test for this relationship, we define the bispectrum as

\[
B_{k,l} = \langle \hat{S}_k \hat{S}_l \hat{S}_{k+l}^* \rangle,
\]

where \( \hat{S}_k \) is the discrete Fourier transform of \( S(t) \) at frequency \( k \), \( S^* \) is the conjugate (\( S_{k+l}^* = S_{-k-l} \)), and \( \langle \rangle \) indicates the expected value. Unless \( \phi_0 + \phi_1 = \phi_0 + \phi_2 \), \( B(k, l) \) will be complex. The magnitude of \( B(k, l) \) depends on both the magnitude of the complex Fourier coefficients, \( |\hat{S}_k||\hat{S}_l||\hat{S}_{k+l}| \), and the stability of the phase relationship between the coefficients i.e., for random phasing, \( \langle \hat{S}_k \hat{S}_l \hat{S}_{k+l} \rangle = 0 \). The autobicoherence is defined as

\[
C_{k,l} = \frac{\langle \hat{S}_k \hat{S}_l \hat{S}_{k+l} \rangle}{|\hat{S}_k||\hat{S}_l||\hat{S}_{k+l}|},
\]

where the denominator represents \( B(k, l) \) for the case of perfect phase coherence, and \( 0 \leq C_{k,l} \leq 1 \). The expected value of the autobicoherence is estimated here by adapting the bispectral routine presented by Muller and MacDonald.
The algorithm consists of subtracting the mean value of $\hat{S}(f)$, applying a Hanning window, and estimating the autobicoherence as

$$C_{k,j} = \frac{\sum_{k-2}^{k+2} \sum_{j=2}^{j+2} a_{k,j} \hat{S}_k \hat{S}_j \hat{S}_{k-j}}{\sum_{k-2}^{k+2} \sum_{j=2}^{j+2} a_{k,j} \hat{S}_k \hat{S}_j \hat{S}_{k-j}}$$

where $a_{k,j}$ is a weighting coefficient. A Monte Carlo method was used to estimate uncertainty levels for autobicoherence computed according to the above algorithm. Figure 16 shows the approximate 95% level-of-no-significance to reject the null hypothesis of Gaussian distributed red noise; levels are roughly 0.55 for $k \neq l$ and 0.7 for $k = l$. A significant autobicoherence can also indicate the presence of a non-Gaussian signal, thus care is required in interpreting the result.

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