What Measure of Inflation Should a Central Bank Target?

Citation	Mankiw, N. Gregory, and Ricardo Reis. 2003. What measure of inflation should a central bank target? Journal of the European Economic Association 1(5): 1058-1086.
Published Version	http://dx.doi.org/10.1162/154247603770383398
Citable link	http://nrs.harvard.edu/urn-3:HUL.InstRepos:3415322
Terms of Use	This article was downloaded from Harvard University's DASH repository, and is made available under the terms and conditions applicable to Other Posted Material, as set forth at http://nrs.harvard.edu/urn-3:HUL.InstRepos:dash.current.terms-of-use#LAA

WHAT MEASURE OF INFLATION SHOULD A CENTRAL BANK TARGET?

N. Gregory Mankiw
Harvard University

Ricardo Reis
Harvard University

Abstract
This paper assumes that a central bank commits itself to maintaining an inflation target and then asks what measure of the inflation rate the central bank should use if it wants to maximize economic stability. The paper first formalizes this problem and examines its microeconomic foundations. It then shows how the weight of a sector in the stability price index depends on the sector's characteristics, including size, cyclical sensitivity, sluggishness of price adjustment, and magnitude of sectoral shocks. When a numerical illustration of the problem is calibrated to U.S. data, one tentative conclusion is that a central bank that wants to achieve maximum stability of economic activity should use a price index that gives substantial weight to the level of nominal wages. (JEL: E42, E52, E58)

Over the past decade, many central banks around the world have adopted inflation targeting as a guide for the conduct of monetary policy. In such a regime, the price level becomes the economy's nominal anchor, much as a monetary aggregate would be under a monetarist policy rule. Inflation targeting is often viewed as a way to prevent the wild swings in monetary policy that were responsible for, or at least complicit in, many of the macroeconomic mistakes of the past. A central bank committed to inflation targeting would likely have avoided both the big deflation during the Great Depression of the 1930s and the accelerating inflation of the 1970s (and thus the deep disinflationary recession that followed).

This paper takes as its starting point that a central bank has adopted a regime of inflation targeting and asks what measure of the inflation rate it should target. Our question might at first strike some readers as odd. Measures of the overall price level, such as the consumer price index, are widely available and have been amply studied by index-number theorists. Yet a price index designed to measure the cost of living is not necessarily the best one to serve as a target for a monetary authority.

Acknowledgments: We are grateful to Ignazio Angeloni, William Dupor, Stanley Fischer, Yves Nosbusch, the editor Roberto Perotti, and anonymous referees for helpful comments. Reis is grateful to the Fundacao Ciencia e Tecnologia, Praxis XXI, for financial support.

E-mail addresses: Mankiw: ngmankiw@harvard.edu; Reis: reis@fas.harvard.edu

This issue is often implicit in discussions of monetary policy. Many economists pay close attention to "core inflation," defined as inflation excluding certain volatile prices, such as food and energy prices. Others suggest that commodity prices might be particularly good indicators because they are highly responsive to changing economic conditions. Similarly, during the U.S. stock market boom of the 1990s, some economists called for Fed tightening to dampen "asset price inflation," suggesting that the right index for monetary policy might include not only the prices of goods and services but asset prices as well. Various monetary proposals can be viewed as inflation targeting with a nonstandard price index: The gold standard uses only the price of gold, and a fixed exchange rate uses only the price of a foreign currency.

In this paper, we propose and explore an approach to choosing a price index for the central bank to target. We are interested in finding the price index that, if kept on an assigned target, would lead to the greatest stability in economic activity. This concept might be called the stability price index.

The key issue in the construction of any price index is the weights assigned to the prices from different sectors of the economy. When constructing a price index to measure the cost of living, the natural weights are the share of each good in the budget of typical consumer. When constructing a price index for the monetary authority to target, additional concerns come into play: the cyclical sensitivity of each sector, the proclivity of each sector to experience idiosyncratic shocks, and the speed with which the prices in each sector respond to changing conditions.

Our goal in this paper is to show how the weights in a stability price index should depend on these sectoral characteristics. Section 1 sets up the problem. Section 2 examines the microeconomic foundations for the problem set forth in Section 1. Section 3 presents and discusses the analytic solution for the special case with only two sectors. Section 4 presents a more realistic numerical illustration, which we calibrate with plausible parameter values for the U.S. economy. One tentative conclusion is that the stability price index should give a substantial weight to the level of nominal wages.

1. The Optimal Price Index: Statement of the Problem

Here we develop a framework to examine the optimal choice of a price index. To keep things simple, the model includes only a single period of time. The central bank is committed to inflation targeting in the following sense: Before the shocks are realized, the central bank must choose a price index and commit itself to keeping that index on target.

The model includes many sectoral prices, which differ according to four characteristics:

(1) Sectors differ in their budget share and thus the weight their prices receive in a standard price index;
(2) In some sectors equilibrium prices are highly sensitive to the business cycle, while in other sectors equilibrium prices are less cyclical;
(3) Some sectors experience large idiosyncratic shocks, while other sectors do not;
(4) Some prices are flexible, while others are sluggish in responding to changing economic conditions.

To formalize these sectoral differences, we borrow from the so-called "new Keynesian" literature on price adjustment. We begin with an equation for the equilibrium price in sector k:

$$p_k^* = p + \alpha_k x + \varepsilon_k \tag{1}$$

where, with all variables expressed in logs, p_k^* is the equilibrium price in sector k, p is the price level as conventionally measured (such as the CPI), α_k is the sensitivity of sector k's equilibrium price to the business cycle, x is the output gap (the deviation of output from its natural level), and ε_k is an idiosyncratic shock to sector k with variance σ_k^2. This equation says only that the equilibrium relative price in a sector depends on the state of the business cycle and some other shock. Sectors can differ in their sensitivities to the cycle and in the variances of their idiosyncratic shocks.

In Section 2 we examine some possible microeconomic foundations for this model, but readers may be familiar with the equation for the equilibrium price from the literature on price setting under monopolistic competition.[1] The index p represents the nominal variable that shifts both demand and costs, and thus the equilibrium prices, in all the sectors. This variable corresponds to a standard price index such as the CPI. That is, if there are K sectors,

$$p = \sum_{k=1}^{K} \theta_k p_k$$

where θ_k are the weights of different sectors in the typical consumer's budget. The output gap x affects the equilibrium price by its influence on marginal cost and on the pricing power of firms. One interpretation of the shocks ε_k is that they represent sectoral shocks to productivity. In addition, they include changes in the degree of competition in sector k. The formation of an oil cartel, for instance, would be represented by a positive value of ε_k in the oil sector.

1. For a textbook treatment, see Romer (2001, Equation 6.45).

Sectors may also have sluggish prices. We model the sluggish adjustment by assuming that some fraction of prices in a sector is predetermined. One rationale for this approach, following Fischer (1977), is that some prices are set in advance by nominal contracts. An alternative rationale, following Mankiw and Reis (2002), is that price setters are slow to update their plans because there are costs to acquiring or processing information. In either case, the key feature for the purpose at hand is that some prices in the economy are set based on old information and do not respond immediately to changing circumstances.

Let λ_k be the fraction of the price setters in sector k that set their prices based on updated information, while $1 - \lambda_k$ set prices based on old plans and outdated information. Thus, the price in period t is determined by

$$p_k = \lambda_k p_k^* + (1 - \lambda_k) E(p_k^*). \tag{2}$$

The parameter λ_k measures how sluggish prices are in sector k. The smaller is λ_k, the less responsive actual prices are to news about equilibrium prices. As λ_k approaches 1, the sector approaches the classical benchmark where actual and equilibrium prices are always the same.

The central bank is assumed to be committed to targeting inflation. That is, the central bank will keep a weighted average of sectoral prices at a given level, which we can set equal to zero without loss of generality. We can write this as

$$\sum_{k=1}^{K} \omega_k p_k = 0 \tag{3}$$

for some set of weights such that

$$\sum_{k=1}^{K} \omega_k = 1.$$

We will call $\{\omega_k\}$ the target weights and $\{\theta_k\}$ the consumption weights. The target weights are choice variables of the central bank. The sectoral characteristics (θ_k, α_k, λ_k, and σ_k^2) are taken as exogenous.

We assume that the central bank dislikes volatility in economic activity. That is, its goal is to minimize $Var(x)$. We abstract from the problem of monetary control by assuming that the central bank can hit precisely whatever nominal target it chooses. The central question of this paper is the choice of weights $\{\omega_k\}$ that will lead to greatest macroeconomic stability.

Putting everything together, the central bank's problem can now be stated as follows:

$$\min_{\{\omega_k\}} Var(x)$$

subject to:

$$\sum_{k=1}^{K} \omega_k p_k = 0$$

$$\sum_{k=1}^{K} \omega_k = 1$$

$$p_k = \lambda_k p_k^* + (1 - \lambda_k) E(p_k^*)$$

$$p_k^* = p + \alpha_k x + \varepsilon_k$$

$$p = \sum_{k=1}^{K} \theta_k p_k.$$

The central bank chooses the weights in its targeted price index in order to minimize volatility in the output gap, given the constraints the economy imposes on the evolution of prices over time. The solution to this problem will yield the set of weights ω_k in an optimal price index as a function of sector characteristics, which include θ_k, α_k, λ_k, and σ_k^2. We call the resulting measure the stability price index, because it is the price index that, if kept on target, would lead to the greatest possible stability in economic activity.

At this point, there are two questions that might intrigue readers of this paper. What are the microfoundations behind this problem? What is the solution to this problem? Those interested in the first question should continue on to Section 2. Those interested only in the second question should jump to Section 3.

2. Some Microeconomic Foundations

In this section we build a general equilibrium model that delivers, in reduced form, the problem presented in the previous section. We approach this task aiming for simplicity rather than generality. We suspect that the stability-price-index problem, or some variant of it, arises in settings more general than the one we examine here. Our goal now is to give one example and, at the same time, to relate the stability-price-index problem to the large new Keynesian literature on price adjustment.

2.1 The Economy Without Nominal Rigidities

The economy is populated by a continuum of yeoman farmers, indexed by their sector k and by i within this sector. They derive utility from consumption C and disutility from labor L_{ki}, according to the common utility function:

$$U(C, L_{ki}) = \frac{C^{1-\sigma}}{1-\sigma} - L_{ki}.$$

There are many types of consumption goods. Following Spence (1976) and Dixit and Stiglitz (1977), we model the household's demand for these goods using a constant elasticity of substitution (CES) aggregate. Final consumption C is a CES aggregate over the goods in the K sectors of the economy:

$$C = \left[\sum_{k=1}^{K} \theta_k^{1/\gamma} C_k^{(\gamma-1)/\gamma} \right]^{\gamma/(\gamma-1)}. \tag{4}$$

The parameter γ measures the elasticity of substitution across the K sectors. The weights θ_k sum to one and express the relative size of each sector.

Within each sector, there are many farmers, represented by a continuum over the unit interval. The sector's output is also a CES aggregate of the farmers' outputs:

$$C_k = \left[\int_0^1 C_{ki}^{(\gamma-1)/\gamma} di \right]^{\gamma/(\gamma-1)}. \tag{5}$$

Notice that, for simplicity, we have assumed that the elasticity of substitution is the same across sectors and across firms within a sector.[2]

Each farmer uses his labor to operate a production function, which takes the simple form:

$$Y_{ki} = (e^{-a_k}(1 + \psi) L_{ki})^{1/(1+\psi)}. \tag{6}$$

The a_k stand for random productivity shock and ψ is a parameter that determines the degree of returns to scale in production.

The household's budget constraint is, for the agent that supplies good k, i:

$$\sum_{k=1}^{K} \left(\int P_{ki} C_{ki} di \right) = B_{ki} + P_{ki} Y_{ki}.$$

The household obtains income from selling the good it produces in the market for the price P_{ki}, and spends its income on the consumption goods C_{ki}. There are complete markets in the economy that allow the household to insure itself against his idiosyncratic income risk due to the specialization in production. The state-contingent payment associated with such bonds is represented by B_{ki}.

2. As is usual, there are two ways to interpret these CES aggregators. The more common approach is to view them as representing consumers' taste for variety. Alternatively, one can view C as the single final good that consumers buy and the CES aggregators as representing production functions for producing that final good from intermediate goods.

From this household problem, we can derive the demand functions for each sector and each good. It is useful to begin by first defining these price indices:[3]

$$P = \left[\sum_{k=1}^{K} \theta_k P_k^{1-\gamma} \right]^{1/(1-\gamma)},$$

$$P_k = \left[\int_0^1 P_{ki}^{1-\gamma} di \right]^{1/(1-\gamma)}.$$

The demand function can then be expressed as:

$$\begin{aligned} C_k &= \left(\frac{P_k}{P} \right)^{-\gamma} \theta_k C, \qquad \text{and} \\ C_{ki} &= \left(\frac{P_{ki}}{P_k} \right)^{-\gamma} C_k \\ &= \left(\frac{P_{ki}}{P} \right)^{-\gamma} \theta_k C. \end{aligned} \tag{7}$$

The quantity demanded of the good produced by firm i in sector k is a function of its relative price, P_{ki}/P, with an elasticity of demand of γ. It also depends on the sector size θ_k and aggregate consumption C. Since there are complete markets ensuring that all farmers have the same disposable income and they have the same preferences, they will all choose the same level of consumption C.

Let's now turn to the supply side of the goods market. The real marginal cost of producing one unit of a good for every farmer equals the marginal rate of substitution between consumption and leisure (the shadow cost of labor supply) divided by the marginal product of labor:

$$MC(Y_{ki}) = C^{\sigma} e^{a_k} Y_{ki}^{\psi}. \tag{8}$$

We write the desired price of farmer i in sector k as:

$$\frac{P_{ki}^{*}}{P} = m_k MC(Y_{ki}). \tag{9}$$

The relative price of any good is a markup m_k times the real marginal cost of producing the good. The markup m_k can capture many possible market structures from standard monopoly (which here implies $m_k = \gamma/(\gamma - 1)$) to com-

3. For a derivation of these price indices, see either the original article by Dixit and Stiglitz (1977) or a textbook treatment, such as Obstfeld and Rogoff (1996, p. 664).

petition ($m_k = 1$). We allow m_k to be both stochastic and a function of the level of economic activity.[4] We express this as

$$m_k = Y^{\phi_k} e^{\mu_k}$$

where μ_k is a random variable capturing shocks to the markup. The parameter ϕ_k governs the cyclical sensitivity of the markups in sector k, and it can be either positive or negative.

We can now solve for the economy's equilibrium. Using the pricing equation (9), the demand function for variety i in sector k (7), and the market-clearing conditions that $C_{ki} = Y_{ki}$ and $C = Y$, we obtain the following equation for the log of the equilibrium price:[5]

$$p_k^* = p + \alpha_k y + \frac{\mu_k + a_k + \psi \log(\theta_k)}{1 + \gamma\psi}, \tag{10}$$

where $\alpha_k = (\sigma + \phi_k + \psi)/(1 + \gamma\psi)$ and $y = \log(Y)$. In this general equilibrium model, an increase in output influences equilibrium prices both because it raises marginal cost and because it influences the markup. Increases in markups or declines in productivity both lead to an increase in the price that firms desire to set.

It will prove convenient to have a log-linearized version of the aggregate price index. Letting $p = \log(P)$ and $p_k = \log(P_k)$, a first-order approximation to the price index around the point with equal sectoral prices yields:

$$p = \sum_{k=1}^{K} \theta_k p_k.$$

This equation corresponds to the problem stated in Section 1.

Using this linearized equation for the price level, and the expression for the equilibrium prices in each sector, we can solve for the natural output level as a function of the parameters and shocks. The natural level (or efficient level) of output is defined as the output level that would prevail if prices were fully flexible and the markup equalled one. If $p_k = p_k^*$ and $m_k = 1$, then output is:

$$y^N = \frac{-\sum_{k=1}^{K} \theta_k(a_k + \psi \log(\theta_k))}{\sigma + \psi}. \tag{11}$$

The natural level of output is a weighted average of productivity across all the sectors in the economy. The output gap x is then defined as the difference

4. Rotemberg and Woodford (1999) survey alternative theories of why markups may vary over the business cycle. Clarida, Gali, and Gertler (2002) and Steinsson (2003) consider how supply shocks might be modelled as exogenous fluctuations in the markup.

5. Since all firms in a sector are identical they all have the same desired price. The right-hand side of the equation is the same for all i. Therefore we replace p_{ki}^* by p_k^*.

between the actual output level y and the natural level y^N. Using the equation for the log of the equilibrium price, we find that:

$$p_k^* = p + \alpha_k x + \varepsilon_k, \tag{12}$$

where $\varepsilon_k = \alpha_k y^N + [\mu_k + a_k + \psi \log(\theta_k)]/(1 + \gamma\psi)$ is a random variable. The supply shock ε_k reflects stochastic fluctuations in the markup as well as shocks to productivity in sector k relative to the economy's productivity shock reflected in y^N. This is the equation for the desired price posited in the previous section.

Notice that the shocks ε_k reflect sectoral productivity shocks and markup shocks. In general, the problem imposes no structure on the variance-covariance matrix of the ε_k. However, in the special case where there are no markups, so $m_k = 1$, one can show that $\sum \theta_k \varepsilon_k = 0$. Later, we will discuss the implications of this special case.

2.2 The Economy With Nominal Rigidities

We now introduce nominal rigidities into the economy. We assume that although all firms in sector k have the same desired price p_k^*, only a fraction λ_k has updated information and is able to set its actual price equal to its desired price. The remaining $1 - \lambda_k$ firms must set their prices without current information and thus set their prices at $E(p_k^*)$. Using a log-linear approximation for the sectoral price level similar to the one used above for the overall price level, we obtain

$$p_k = \lambda_k p_k^* + (1 - \lambda_k) E(p_k^*).$$

The sectoral price is a weighted average of the actual desired price and the expected desired price. As we noted earlier, this kind of price rigidity can be justified on the basis of nominal contracts as in Fischer (1977) or information lags as in Mankiw and Reis (2002).

The equilibrium in this economy involves $K + 2$ key variables: all the sectoral prices p_k and the two aggregate variables p and y. The above equation for p_k provides K equations (once we substitute in for p_k^*). The equation for the aggregate price index provides another equation:

$$p = \sum_{k=1}^{K} \theta_k p_k.$$

The last equation comes from the policymaker's choice of a nominal anchor:

$$\sum_{k=1}^{K} \omega_k p_k = 0.$$

We do not model how this target is achieved. That is, we do not model the transmission mechanism between the instruments of monetary policy and the level of prices. Instead, our focus is on the choice of a particular policy target, which here is represented by the weights ω_k.

The choice of weights depends on the policymaker's objective function. In this economy, since all agents are ex ante identical, a natural welfare measure is the sum over all households' utility functions. Since $Y = C$ in equilibrium, we can express this utilitarian social welfare function as:

$$U = \frac{Y^{1-\sigma}}{1-\sigma} - \sum_{k=1}^{K} \int L_{ki} di.$$

In the Appendix, we take a second-order logarithmic approximation to this utility function around expected output to obtain that expected utility is proportional to:

$$E(U(\cdot)) \approx -\left(Var(x) + \frac{(\gamma^{-1} + \psi)}{(\sigma + \psi)} E[Var_k(x_k) + E_k(Var_i(x_{ki}))]\right) \qquad (13)$$

where $Var_k(\cdot)$ stands for the cross-sectional dispersion across sectors, $Var_i(\cdot)$ the dispersion across firms within a sector, and $E_k(\cdot)$ the cross-sectional average across sectors. Expected utility depends on the variance of the output gap and on the dispersion of the output gap across sectors and firms. The dispersion of output gets a smaller weight in the welfare function if consumers are more risk averse (so σ is larger) or if the goods are more substitutable (so γ is larger).

In Section 1 we assumed the central bank's objective function is $Var(x)$, the variance of the output gap. This is similar to Equation (13), but it omits the term involving the cross-sectional dispersion of output. In the remainder of this paper we continue with this simplifying assumption, for two reasons.[6]

First, this assumption connects our problem more closely to the issues facing real monetary policymakers. In our experience, central bankers are more concerned with stability in aggregate economic activity than they are with the distribution of output across firms. Academic discussions of monetary policy sometimes emphasize cross-sectional effects because these effects arise in canonical models. The practical importance of such effects, however, is open to debate.

Second, the simpler objective function allows us to establish some theoretical results that are intuitive and easy to interpret. Extending the results to the case where the central bank takes both terms into account in designing optimal policy would certainly be a useful exercise, but the extra complexity would likely preclude clean analytic results. In Section 5 we compare our results to

6. Of course, we are not the first study of optimal monetary policy to assume that the central bank's goal is to minimize the volatility of economic activity. For example, see Fischer's (1977) classic analysis.

those obtained in papers that numerically worked with welfare functions similar to that in Equation (13).

The bottom line from this analysis is that the stability-price-index problem stated in Section 1 is closely related to a reduced form of a model of price adjustment under monopolistic competition. The canonical models in this literature assume symmetry across sectors in order to keep the analysis simple (e.g., Blanchard and Kiyotaki 1987; Ball and Romer 1990). Yet sectoral differences are at the heart of our problem. Therefore, we have extended the analysis to allow for a rich set of sectoral characteristics, which are described by the parameters θ_k, α_k, λ_k, and σ_k^2.

3. The Two-Sector Solution

We are now interested in solving the central bank's problem. To recap, it is:

$$\min_{\{\omega_k\}} Var(x)$$

subject to

$$\sum_{k=1}^{K} \omega_k p_k = 0$$

$$\sum_{k=1}^{K} \omega_k = 1$$

$$p_k = \lambda_k p_k^* + (1 - \lambda_k) E(p_k^*)$$

$$p_k^* = p + \alpha_k x + \varepsilon_k$$

$$p = \sum_{k=1}^{K} \theta_k p_k.$$

The central bank chooses a target price index to minimize output volatility, given the constraints imposed by the price-setting process.

To illustrate the nature of the solution, we now make the simplifying assumptions that there are only two sectors ($K = 2$), which we call sector A and sector B, and that the shocks to each sector (ε_A and ε_B) are uncorrelated. We also assume that α_A and α_B are both nonnegative. Appendix 2 derives the solution to this special case. The conclusion is the following equation for the optimal weight on sector A:

$$\omega_A^* = \lambda_B \frac{\alpha_A \sigma_B^2 - \theta_A \lambda_A (\alpha_A \sigma_B^2 + \alpha_B \sigma_A^2)}{\alpha_B \lambda_A (1 - \lambda_B) \sigma_A^2 + \alpha_A \lambda_B (1 - \lambda_A) \sigma_B^2}.$$

Notice that the optimal target weight depends on all the sectoral characteristics and, in general, need not be between 0 and 1.

From this equation, we can derive several propositions that shed light on the nature of the solution. We begin with a very special case. Of course, if the two sectors are identical (same θ_k, α_k, λ_k, and σ_k^2), then the stability price index gives them equal weight ($\omega_k^* = 1/2$). This result is not surprising, as it merely reflects the symmetry of the two sectors.

More interesting results arise when the sectoral characteristics (θ_k, α_k, λ_k, and σ_k^2) vary. Let's start with the two characteristics that describe equilibrium prices:

PROPOSITION 1 *An increase in α_k raises the optimal ω_k. That is, the more responsive a sector is to the business cycle, the more weight that sector's price should receive in the stability price index.*

PROPOSITION 2 *An increase in σ_k^2 reduces the optimal ω_k. That is, the greater the magnitude of idiosyncratic shocks in a sector, the less weight that sector's price should receive in the stability price index.*

Both of these propositions can be viewed from a signal-extraction perspective: A sector's price is useful for a central bank when its signal about the output gap is high (as measured by α_k) and when its noise is low (as measured by σ_k^2). Propositions 1 and 2 both coincide with aspects of the conventional wisdom. When economists point to commodity prices as a useful economic indicator for monetary policy, they usually do so on the grounds that these prices are particularly responsive to the business cycle. The index of leading indicators, for instance, includes the change in "sensitive materials prices." Proposition 1 can be used to justify this approach. At the same time, when economists reduce the weight they give to certain sectors, as they do with food and energy sectors in the computation of the core CPI, they do so on the grounds that these sectors are subject to particularly large sector-specific shocks. Proposition 2 can be used to justify this approach.

Let's now consider the effects of price sluggishness on the optimal target weights:

PROPOSITION 3 *If the optimal weight for a sector is less than 100 percent ($\omega_k <$ 1), then an increase in λ_k reduces the optimal ω_k. That is, the more flexible a sector's price, the less weight that sector's price should receive in the stability price index.*

As earlier, some intuition for this result comes from thinking about the problem from a signal-extraction perspective. Price stickiness dampens the effect of the business cycle on a sector's price. Conversely, when prices are very sticky, a small price movement signals a large movement in the sector's desired price, which in turn reflects economic activity. An optimizing central bank offsets the effect of this dampening from price stickiness by giving a greater weight to stickier sectors.

A special case is noteworthy:

PROPOSITION 4 *If the two sectors are identical in all respects except one has full price flexibility* (*same* α_k, θ_k, *and* σ_k^2 *but* $\lambda_A = 1$, $\lambda_B < 1$), *then the monetary authority should target the price level in the sticky-price sector* ($\omega_B = 1$).

This result is parallel to that presented by Aoki (2001). But the very strong conclusion that the central bank should completely ignore the flexible-price sector does not generalize beyond the case of otherwise identical sectors. Even if a sector has fully flexible prices, the optimal target weight for that sector is in general nonzero.

The last sectoral characteristic to consider is θ_k, the weight that the sector receives in the consumer price index.

PROPOSITION 5 *An increase in* θ_k *reduces the optimal* ω_k. *That is, the more important a price is in the consumer price index, the less weight that sector's price should receive in the stability price index.*

This proposition is probably the least intuitive one. It illustrates that choosing a price index to aim for economic stability is very different than choosing a price index to measure the cost of living.

What is the intuition behind this surprising result? Under inflation targeting, undesirable fluctuations in output arise when there are shocks ε_k to equilibrium prices, which the central bank has to offset with monetary policy. The effect of a shock in sector k depends on the consumption weight θ_k. The greater is the consumption weight, the more the shock feeds into other prices in the economy, and the more disruptive it is. Thus, to minimize the disruptive effect of a shock, a central bank should accommodate shocks to large sectors. Under inflation targeting, such accommodation is possible by reducing the weight of the sector in the target index. Hence, holding all the other parameters constant, sectors with a larger weight in the consumption index should receive a smaller weight in the target index.[7]

To sum up, the ideal sectoral prices for a central bank to monitor are those that are highly sensitive to the economy (large α_k), experience few sectoral shocks (small σ_k^2), have very sluggish prices (low λ_k), and are relatively small in the aggregate price index (small θ_k). It is important to acknowledge, however, that these results depend on the assumption that the correlation between the ε_k

7. The idea of giving a large weight to a small sector may sound implausible at first, but that is precisely the policy that many nations adopted during the nineteenth century. Under a gold standard, the small gold sector receives a target weight of 100 percent.

is zero. With a general covariance matrix, only two propositions survive. Proposition 1 still holds since one can show that $\partial\omega_k/\partial\alpha_k \geq 0$, so the optimal target weight does not decrease as a sector's cyclical sensitivity increases. Moreover, if the sectors are identical in all respects except that in one sector prices are fully flexible, optimal policy targets the sticky price alone, as in Proposition 4. In the empirical application next, however, we find that the off-diagonal elements of the covariance matrix do not influence the most important conclusions. So perhaps the special case highlighted in Propositions 1 through 5 is empirically plausible.

Another noteworthy special case is the one in which $\sum \theta_k\varepsilon_k = 0$. In the microfoundations developed in Section 2, this case arises if there are productivity shocks but no markup shocks. In this special case, one can show that

$$\omega_A^* = \theta_A \frac{\lambda_B(1-\lambda_A)}{\lambda_A(1-\lambda_B) + \theta_A(\lambda_B - \lambda_A)}. \tag{14}$$

The optimal target weight rises with decreases in λ_k as before, but now it rises with increases in θ_A. These results are parallel to those in Benigno (2001). In addition, if one sector has flexible prices, then optimal policy targets the sticky-price sector. This case corresponds most closely to the one studied by Aoki (2001).

4. Toward Implementation: An Example

The two-sector example considered in the previous section is useful for guiding intuition, but if a central bank is to compute a stability price index, it will need to go beyond this simple case. In this section, we take a small step toward a more realistic implementation of the stability price index.

4.1 The Approach

We apply the model to annual data for the U.S. economy from 1957 to 2001. We examine four sectoral prices: the price of food, the price of energy, the price of other goods and services, and the level of nominal wages. The first three prices are categories of the consumer price index, while wages refer to compensation per hour in the business sector. As a proxy for the output gap, we use twice the deviation of unemployment from its trend value, where the trend is computed using the Hodrick-Prescott filter.[8] All series come from the Bureau of Labor Statistics.

8. The factor of two corrects for an Okun's law relationship and only affects the estimated α_k but not the target weights. We also tried estimating x using detrended output and obtained similar results.

A key question is how to assign parameters to the four sectors. We begin by noting the following equation holds in the model:

$$p_k - Ep_k = \lambda_k(p - Ep) + \alpha_k\lambda_k(x - Ex) + \lambda_k(\varepsilon_k - E\varepsilon_k). \tag{15}$$

That is, the price surprise in sector k is related to the overall price surprise, the output surprise, and the shock. To obtain these surprise variables, we regressed each of the variables p_k, p, and x on three of its own lags, a constant, and a time trend and took the residual. These surprise variables are the data used in all subsequent calculations.

In principle, one should be able to obtain the parameters by estimating Equation (15). In practice, the identification problem makes formal estimation difficult. Shocks (such as an energy price increase) will likely be correlated with the overall price level and the level of economic activity. Finding appropriate instruments is a task we leave for future work. Here, as a first pass, we adopt a cruder approach that is akin to a back-of-the-envelope calculation.

For the parameter λ_k, which governs the degree of price sluggishness, we rely on bald, but we hope realistic, assumptions. We assume the food and energy prices are completely flexible, so $\lambda_k = 1$. Other prices and wages are assumed to be equally sluggish. We set $\lambda_k = \frac{1}{2}$, indicating that half of price setters in these sectors base their prices based on expected, rather than actual, economic conditions.

Another key parameter is α_k, the sensitivity of desired prices to the level of economic activity. We estimate this parameter by assuming that the 1982 economic downturn—the so-called Volcker recession—was driven by monetary policy, rather than sectoral supply shocks. Thus, we pick α_k for each sector so that Equation (15) without any residual holds exactly for 1982. That is, we are using the price responses during the 1982 recession to measure the cyclical sensitivity of sectoral prices.

With α_k and λ_k, we can compute a time series of $\varepsilon_k - E\varepsilon_k$ and, thus, its variance-covariance matrix. Note that we do not assume that the shocks are uncorrelated across sectors. The previous section made this assumption to obtain easily interpretable theoretical results, but for a more realistic numerical exercise, it is better to use the actual covariances. Thus, if there is some shock that influences desired prices in all sectors (for a given p and y), this shock would show up in the variance-covariance matrix, including the off-diagonal elements.

The last parameter is the consumption weight θ_k. We take this parameter from the "relative importance" of each sector in the consumer price index as determined by the Bureau of Labor Statistics. For nominal wages, θ_k is equal to zero, because nominal wages do not appear in the consumer price index.

With all the parameters in hand, it is now a straightforward numerical exercise to find the set of target weights ω_k that solves the stability-price-index problem as set forth above. Appendix 3 describes the algorithm.

TABLE 1. RESULTS OF EMPIRICAL ILLUSTRATION

Sector	λ	α	$Var(\varepsilon)$	θ	ω^u	ω^c	ω^0
Energy	1.0	0.37	0.00279	0.07	0.10	0.03	0.01
Food	1.0	0.10	0.00025	0.15	0.37	0.21	0.10
Other goods	0.5	0.10	0.00016	0.78	−0.73	0	−0.07
Wages	0.5	0.29	0.00050	0	1.26	0.76	0.96

Correlation matrix of epsilon

	Energy	Food	Other goods	Wages
Energy	1.00	−0.27	0.19	−0.17
Food		1.00	−0.24	0.03
Other goods			1.00	0.30
Wages				1.00

4.2 The Results

Table 1 presents the results from this exercise. The column denoted ω^c imposes the constraint that all the sectoral weights in the stability price index be nonnegative. The column denoted ω^u, allows the possibility of negative weights. The substantive result is similar in the two cases: The price index that the central bank should use to maximize economic stability gives most of its weight to the level of nominal wages.

The intuition behind this result is easy to see. The value of α_k for nominal wages is 0.29, which is larger than the parameter for most other sectors. (This parameter value reflects the well-known fact that real wages are procyclical.[9]) The only other sector that exhibits such a large value of α_k is the energy sector. But the variance of shocks in the energy sector, measured by $Var(\varepsilon_k)$, is very large, making it an undesirable sector for the stability price index. The combination of high α_k and low $Var(\varepsilon_k)$ makes nominal wages a particularly useful addition to the stability price index.[10]

One might suspect that the zero value of θ_k for nominal wages in the consumer price index is largely responsible for the high value of ω_k in the stability price index. That turns out not to be the case. Table 2 performs the same empirical exercise as in Table 1, but it assumes that the economy's true price index gives half its weight to nominal wages (that is, $p = 0.5w + 0.5cpi$). Once

9. The estimate of the procyclicality of real wages we obtained here is similar to those found in other studies. Because α_k for nominal wages exceeds the α_k for other goods by 0.19, the desired real wage rises by 0.19 percent for every 1 percentage point increase in the output gap. If λ_k equals 0.5 for these two sectors, as we have assumed, then the actual real wage would rise by 0.095. For comparison, Solon, Barsky, and Parker (1994) estimate the elasticity of real wages with respect to output in aggregate data is 0.146.

10. Indeed, if a better index of wages were available, it would likely be more procyclical, reinforcing our conclusion. See Solon, Barsky, and Parker (1994) on how composition bias masks some of the procyclicality of real wages.

TABLE 2. RESULTS WITH ALTERNATIVE PRICE INDEX

Sector	λ	α	$Var(\varepsilon)$	θ	ω^u	ω^c	ω^0
Energy	1.0	0.33	0.00319	0.03	0.07	0.02	0.04
Food	1.0	0.06	0.00030	0.08	0.32	0.2	0.07
Other goods	0.5	0.07	0.00023	0.39	−0.55	0	0
Wages	0.5	0.26	0.00028	0.50	1.16	0.78	0.89

Correlation matrix of epsilon

	Energy	Food	Other goods	Wages
Energy	1.00	−0.01	0.42	−0.12
Food		1.00	0.03	−0.28
Other goods			1.00	0.02
Wages				1.00

again, the most important element of the stability price index is the level of nominal wages.[11]

Two other striking results in Table 1 are the large weight on the price of food and the large, negative weight on the price of goods other than food and energy. These results depend crucially on the pattern of correlations among the estimated shocks. The last column in Tables 1 and 2, denoted ω^0 sets these correlations to zero. The target weights for food and other goods are much closer to zero (while the target weight for nominal wages remains close to one).[12] In light of this sensitivity, this aspect of the results should be treated with caution. One clear lesson is that the variance-covariance matrix of the shocks is a key input into the optimal choice of a price index. The large weight on nominal wages, however, appears robust.

It is worth noting that the gain in economic stability from targeting the stability price index rather than the consumer price index is large. It is straightforward to calculate the variance of output under each of the two policy rules. According to this model, moving from a target for the consumer price index to a target for the stability price index reduces the output gap variance by 53 percent (or by 49 percent with a nonnegativity constraint on the weights). Thus, the central bank's choice of a price index to monitor inflation is an issue of substantial economic significance.

11. How is the approximate irrelevance of θ_k here consistent with Proposition 5? The proposition examines what happens to ω_k when θ_k changes, holding constant other parameter values. But in this empirical exercise, if we change the weight given to some sector in the price index p, we also change the estimated values of α_k and the variance-covariance matrix of ε_k.

12. Although it is not easy to gain intuition for why the off-diagonal elements of the covariance matrix have the effect they do, here is our conjecture: The largest correlation in Table 1 is the 0.30 between the shock to wages and the shock to the prices of other goods. Thus, the stability price index, which gives a high weight to wages, tries to "purge" the shock to wages by giving a negative weight to the price of other goods. More generally, when there is correlation among sectors, the stability price index tries to choose the combination of prices such that shocks among the sectors are offsetting in the overall index.

A natural extension to this exercise would be to include asset prices, such as the price of equities. Although stock prices experience large idiosyncratic shocks (high σ_k^2), they are also very cyclically sensitive (high α_k). As a result, it is plausible that the stability price index should give some weight to such asset prices. When we added the S&P 500 price index to the sectoral prices used in Table 1, it received a target weight that was positive and around 0.2. The target weight on nominal wages remained large.

Finally, we should emphasize how tentative these calculations are. Our attempt at measuring the key sectoral parameters is certainly crude. Future work could aim at finding better econometric techniques to measure these parameters. Once credible estimation procedures are in hand, one could expand the list of candidate prices.

5. Relationship to the Previous Literature

The idea that a central bank should look beyond the consumer price index when monitoring inflation is not a new one. For example, in 1978 Phelps concluded, "the program envisioned here aims to stabilize wages on a level or a rising path, leaving the price level to be buffeted by supply shocks and exchange-rate disturbances."[13] In Mankiw and Reis (2003) we explored a model that supports Phelps's policy prescription. That model can be viewed as a special case of the stability-price index framework considered here, with some strong restrictions on the parameter values. If Sector A is the labor market and sector B is the goods market, then the earlier model can be written in a form such that $\theta_A = 1$, $\lambda_B = 1$, $\alpha_B = 0$, and $\sigma_A^2 = 0$. In this special case, the equation for the optimal target weight immediately implies that $\omega_A^* = 1$.

Erceg, Henderson, and Levin (2000) have recently also found that optimal monetary policy can be closely approximated by targeting the nominal wage. Their analysis differs substantially from ours. Whereas our calculations in Section 4 treat wages in exactly the same way as any other sectoral price, Erceg et al. focus instead on the specific features of the labor market, where nominal rigidities induce distortions in labor-leisure choices and shocks feed into the other sectors in the economy via wages and costs. Our argument for nominal wage targeting can be seen as complementary to theirs, further strengthening their conclusion.

The modern literature has also recently taken up the question of how should monetary policy be set if there are different sectors in the economy. Aoki (2001) studies optimal monetary policy in an economy with two sectors: one with perfectly flexible prices and the other with some nominal rigidity. He finds that the central bank should target the sticky-price sector only. We obtain this same

13. Phelps has told us that this idea dates back to Keynes, but we have not been able to find a reference.

result, but only in the special cases either where the two sectors were identical in all other respects, as stated in Proposition 4, or where there are only productivity shocks.

Benigno (2001) focuses instead on the problem facing a currency union with two regions. Even though his model has richer microfoundations than ours, we are able to reproduce two of his main conclusions within our simple framework. Benigno does not include markup shocks, focussing only on the presence of disturbances that correspond to our productivity shocks. He finds that the larger the weight of a sector in the economy is, the larger the weight it should receive in the stability price index, as we found in Section 3 when only productivity shocks were present. In addition, he shows that if the degree of nominal rigidity in the two sectors is the same, then the optimal policy is to target the CPI, regardless of any other differences between the sectors. If there are only productivity shocks, our model leads to this conclusion as well. Both Aoki and Benigno used models different from ours, notably by introducing nominal rigidities in the form of Calvo staggered pricing rather than predetermined prices as we do,[14] and by using a different objective function for the policymaker. Nonetheless, their conclusions carry over to our setting.

6. Conclusion

Economists have long recognized that price indices designed to measure the cost of living may not be the right ones for the purposes of conducting monetary policy. This intuitive insight is behind the many attempts to measure “core inflation.” Yet, as Wynne (1999) notes in his survey of the topic, the literature on core inflation has usually taken a statistical approach without much basis in monetary theory. As a result, measures of core inflation often seem like answers in search of well-posed questions.

The price index proposed in this paper can be viewed as an approach to measuring core inflation that is grounded in the monetary theory of the business cycle. The stability price index is the weighted average of prices that, if kept on target, leads to the greatest stability in economic activity. The weights used to construct such a price index depend on sectoral characteristics that differ markedly from those relevant for measuring the cost of living.

Calculating a stability price index is not an easy task. Measuring all the relevant sectoral characteristics is an econometric challenge. Moreover, there are surely important dynamics in the price-setting decision that we have omitted in our simple model. Yet, if the calculations performed in this paper are indicative, the topic is well worth pursuing. The potential improvement in

14. For a comparison of the different properties of “sticky price” and “sticky information” models of nominal rigidities, see Mankiw and Reis (2002).

macroeconomic stability from targeting the optimal price index, rather than the consumer price index, appears large.

Our results suggest that a central bank that wants to achieve maximum stability of economic activity should give substantial weight to the growth in nominal wages when monitoring inflation. This conclusion follows from the fact that wages are more cyclically sensitive than most other prices in the economy (which is another way of stating the well-known fact that the real wage is procyclical). Moreover, compared to other cyclically sensitive prices, wages are not subject to large idiosyncratic shocks. Thus, if nominal wages are falling relative to other prices, it indicates a cyclical downturn, which in turn calls for more aggressive monetary expansion. Conversely, when wages are rising faster than other prices, targeting the stability price index requires tighter monetary policy than does conventional inflation targeting.

An example of this phenomenon occurred in the United States during the second half of the 1990s. Here are the U.S. inflation rates as measured by the consumer price index and an index of compensation per hour:

Year	CPI	Wages
1995	2.8	2.1
1996	2.9	3.1
1997	2.3	3.0
1998	1.5	5.4
1999	2.2	4.4
2000	3.3	6.3
2001	2.8	5.8

Consider how a monetary policymaker in 1998 would have reacted to these data. Under conventional inflation targeting, inflation would have seemed very much in control, as the CPI inflation rate of 1.5 percent was the lowest in many years. By contrast, a policymaker trying to target a stability price index would have observed accelerating wage inflation. He would have reacted by slowing money growth and raising interest rates (a policy move that in fact occurred two years later). Would such attention to a stability price index have restrained the exuberance of the 1990s boom and avoided the recession that began the next decade? There is no way to know for sure, but the hypothesis is intriguing.

Appendix 1: Approximation of the Utility Function

In this appendix, following Woodford (2002), we derive the objective function of the policymaker as a Taylor second-order log-linear approximation of the utility function. This extends the multisector analysis of Benigno (2001) to the case where there are markup shocks in addition to productivity shocks.

The first issue to address is the choice of the point around which to linearize.

Following the literature, we linearize around the steady-state equilibrium of the economy with flexible prices and no real disturbances, so that all shocks are at their means. As Woodford discusses, it is important for the accuracy of the log-linearization that this is close enough to the efficient equilibrium of the economy. To ensure this is the case, we assume the average markup is one for all sectors: $E(m_k) = 1$. One way to make this consistent with the monopolistic competition model is to introduce a production subsidy to firms funded by lump-sum taxes on consumers.[15]

In order to interpret θ_k as the share of a sector in total output, units must be chosen appropriately so that all steady-state equilibrium sectoral prices $\bar{p}_k$ are the same. From Equation (10), this requires that units of measurement be such that average productivity respects the condition:

$$\bar{a}_k = -(\sigma + \psi)\bar{y} - \psi \log(\theta_k). \tag{A.1}$$

The $\bar{y}$ must be the same across sectors, and corresponds to the level of aggregate output around which we linearize y. From the demand functions in Equation (7), the equilibrium firm and sector output levels are $\bar{y}_{ki} = \bar{y}_k = \bar{y} + \log(\theta_k)$.

We can now turn to the linearization of the utility function:

$$U(\cdot) = \frac{Y^{1-\sigma}}{1-\sigma} - \sum_{k=1}^{K} \int_0^1 L_{ki} di, \tag{A.2}$$

which we do in a sequence of steps.

Step 1: Approximating $Y^{1-\sigma}/(1-\sigma)$

A second-order linear approximation of y around $\bar{y}$, letting $\hat{y} = y - \bar{y}$ yields

$$\begin{aligned}
\frac{Y^{1-\sigma}}{1-\sigma} &= \frac{e^{(1-\sigma)y}}{1-\sigma} \\
&\approx \frac{e^{(1-\sigma)\bar{y}}}{1-\sigma}\left(1 + (1-\sigma)\hat{y} + \frac{(1-\sigma)^2}{2}\hat{y}^2\right) \\
&\approx e^{(1-\sigma)\bar{y}}\left(\hat{y} + \frac{1-\sigma}{2}\hat{y}^2\right).
\end{aligned} \tag{A.3}$$

The approximation in the last line involves dropping a term that enters the expression additively and which the policymaker cannot affect. Therefore, it does not influence the results from the optimization and can be dropped.

15. Alternatively we could allow the markups to be of first or higher stochastic order.

Step 2: Approximating L_{ki}

Inverting the production function in (6) we obtain:

$$L_{ki} = \frac{e^{a_k}}{1+\psi} Y_{ki}^{1+\psi}.$$

A first-order approximation of this around $\bar{y}_{ki}$ and $\bar{a}_k$, letting $\hat{y}_{ki} = y_{ki} - \bar{y}_{ki}$ and $\hat{a}_k = a_k - \bar{a}_k$ leads to:

$$\begin{aligned} L_{ki} &= \frac{1}{1+\psi} e^{a_k + (1+\psi) y_{ki}} \\ &\approx \frac{e^{\bar{a}_k + (1+\psi)\bar{y}_{ki}}}{1+\psi}\left(1 + \hat{a}_k + \frac{1}{2}\hat{a}_k^2 + (1+\psi)\hat{y}_{ki} + \frac{(1+\psi)^2}{2}\hat{y}_{ki}^2 + (1+\psi)\hat{a}_k\hat{y}_{ki}\right) \\ &\approx e^{\bar{a}_k + (1+\psi)\bar{y}_{ki}}\left(\hat{y}_{ki} + \frac{1+\psi}{2}\hat{y}_{ki}^2 + \hat{a}_k\hat{y}_{ki}\right), \end{aligned}$$

where again in the last line, we drop additive constants that are independent of policy.

Step 3: Integrating to Obtain $\int L_{ki} di$

Integrating the previous expression over the farmers i in sector k, leads to:

$$\int L_{ki} di = \int e^{\bar{a}_k + (1+\psi)\bar{y}_{ki}}\left(\hat{y}_{ki} + \frac{1+\psi}{2}\hat{y}_{ki}^2 + \hat{a}_k\hat{y}_{ki}\right) di.$$

Since $\bar{y}_{ki} = \bar{y}_k$, and denoting by $E_i(\hat{y}_{ki}) = \int \hat{y}_{ki} di$ the cross-sectional average of output across firms in sector k, we obtain:

$$\int L_{ki} di = e^{\bar{a}_k + (1+\psi)\bar{y}_k}\left(E_i(\hat{y}_{ki}) + \frac{1+\psi}{2} E_i(\hat{y}_{ki}^2) + \hat{a}_k E_i(\hat{y}_{ki})\right).$$

From the definition of the cross-sectional variance, $Var_i(y_{ki}) = E_i(\hat{y}_{ki}^2) - E_i(\hat{y}_{ki})^2$, so:

$$\int L_{ki} di = e^{\bar{a}_k + (1+\psi)\bar{y}_k}\left(E_i(\hat{y}_{ki}) + \frac{1+\psi}{2}\left(Var_i(\hat{y}_{ki}) + E_i(\hat{y}_{ki})^2\right) + \hat{a}_k E_i(\hat{y}_{ki})\right). \tag{A.4}$$

Next, realize that a second-order approximation of the CES aggregator in Equation (5), around $\bar{y}_{ki} = \bar{y}_k$ yields:

$$E_i(\hat{y}_{ki}) \approx \hat{y}_k - \frac{1-\gamma^{-1}}{2} Var_i(\hat{y}_{ki}).$$

Using this expression to substitute for $E_i(\hat{y}_{ki})$ in Equation (A.4), rearranging and dropping third- or higher-order terms, we obtain:

$$\int L_{ki} di = e^{\bar{a}_k + (1+\psi)\bar{y}_k}\left(\hat{y}_k + \frac{1+\psi}{2}\hat{y}_k^2 + \frac{\gamma^{-1}+\psi}{2} Var_i(\hat{y}_{ki}) + \hat{a}_k \hat{y}_k\right).$$

Step 4: Adding to Obtain $\sum\int L_{ki} di$

Adding up the expression above over the k sectors, we obtain:

$$\sum_{k=1}^{K}\int L_{ki} di = \sum_{k=1}^{K} e^{\bar{a}_k + (1+\psi)\bar{y}_k}\left(\hat{y}_k + \frac{1+\psi}{2}\hat{y}_k^2 + \frac{\gamma^{-1}+\psi}{2} Var_i(\hat{y}_{ki}) + \hat{a}_k \hat{y}_k\right)$$

Since $\bar{y}_k = \bar{y} + \log(\theta_k)$, Equation ($A$.1) implies that $\bar{a}_k + (1+\psi)\bar{y}_k = (1-\sigma)\bar{y} + \log(\theta_k)$. Therefore:

$$\begin{aligned}\sum_{k=1}^{K}\int L_{ki} di &= e^{(1-\sigma)\bar{y}} \sum_{k=1}^{K} \theta_k\left(\hat{y}_k + \frac{1+\psi}{2}\hat{y}_k^2 + \frac{\gamma^{-1}+\psi}{2} Var_i(\hat{y}_{ki}) + \hat{a}_k \hat{y}_k\right)\\ &= e^{(1-\sigma)\bar{y}}\left(E_k(\hat{y}_k) + \frac{1+\psi}{2} E_k(\hat{y}_k^2)\right.\\ &\quad \left. + \frac{\gamma^{-1}+\psi}{2} E_k(Var_i(\hat{y}_{ki})) + E_k(\hat{a}_k\hat{y}_k)\right),\end{aligned}$$

where the cross-sectional average of output across sectors is denoted by: $E_k(\hat{y}_k) = \sum_{k=1}^{K} \theta_k \hat{y}_k$.

Approximating the terms in the CES aggregator in Equation (4) around $\bar{y}_k = \bar{y} + \log(\theta_k)$ we obtain:

$$E_k(\hat{y}_k) \approx \hat{y} - \frac{1-\gamma^{-1}}{2} Var_k(\hat{y}_k). \qquad (A.5)$$

Using this to replace for $E_k(\hat{y}_k)$ in the expression above and dropping third- or higher-order terms leads to:

$$\begin{aligned}\sum_{k=1}^{K}\int L_{ki} di \approx e^{(1-\sigma)\bar{y}}\Bigg(\hat{y} + \frac{1+\psi}{2}\hat{y}^2 + \frac{(\gamma^{-1}+\psi)}{2}\\ \times [Var_k(\hat{y}_k) + E_k(Var_i(\hat{y}_{ki}))] + E_k(\hat{a}_k\hat{y}_k)\Bigg).\end{aligned} \qquad (A.6)$$

Step 5: Combining all the Previous Steps

The second-order approximation of the utility function (A.2) is given by the sum subtracting the result in (A.6) from (A.3). Cancelling terms we obtain:

$$U \approx -e^{(1-\sigma)\bar{y}}\frac{(\sigma+\psi)}{2}\Bigg(\hat{y}^2 + 2\frac{E_k(\hat{a}_k\hat{y}_k)}{(\sigma+\psi)} + \frac{(\gamma^{-1}+\psi)}{(\sigma+\psi)}[Var_k(\hat{y}_k) + E_k(Var_i(\hat{y}_{ki}))]\Bigg).$$

Now focus on the term $E_k(\hat{a}_k\hat{y}_k)$. From (A.5), it is clear that $\hat{y}E_k(\hat{a}_k) \approx E_k(\hat{a}_k)E_k(\hat{y}_k)$, up to second-order terms. Therefore:

$$\begin{aligned} E_k(\hat{a}_k\hat{y}_k) &= \hat{y}E_k(\hat{a}_k) + E_k(\hat{a}_k\hat{y}_k) - \hat{y}E_k(\hat{a}_k) \\ &\approx \hat{y}E_k(\hat{a}_k) + E_k[(\hat{a}_k - E_k(\hat{a}_k))(\hat{y}_k - E_k(\hat{y}_k))] \end{aligned}$$

From the definition of the natural rate in Equation (11), we can replace $E_k(\hat{a}_k)$ in the expression above to obtain:

$$E_k(\hat{a}_k\hat{y}_k) = -\hat{y}(\sigma+\psi)\hat{y}^N + Cov_k(\hat{a}_k, \hat{y}_k),$$

where $Cov_k(\hat{a}_k, \hat{y}_k) = E_k[(\hat{a}_k - E_k(\hat{a}_k))(\hat{y}_k - E_k(\hat{y}_k))]$ stands for the cross-sectional covariance. Using this to replace for $E_k(\hat{a}_k\hat{y}_k)$ in our approximation of the utility function, and adding a term involving $\hat{y}^N$ (which is beyond the control of policy so leaves the maximization problem unchanged), leads to:

$$U \approx -e^{(1-\sigma)\bar{y}}\frac{(\sigma+\psi)}{2}\Bigg((\hat{y}-\hat{y}^N)^2 + \frac{(\gamma^{-1}+\psi)}{(\sigma+\psi)} \times \left[\frac{2\,Cov_k(\hat{a}_k, \hat{y}_k)}{(\gamma^{-1}+\psi)} + Var_k(\hat{y}_k) + E_k(Var_i(\hat{y}_{ki}))\right]\Bigg).$$

Next, we simplify the term in the square brackets above. Since $Var_k(\hat{a}_k)/(\gamma^{-1}+\psi)^2$ is beyond the control of the monetary policy, we can add it to the term in brackets in the utility function to obtain:

$$\frac{2\,Cov_k(\hat{a}_k, \hat{y}_k)}{(\gamma^{-1}+\psi)} + Var_k(\hat{y}_k) \approx Var_k\left\{\hat{y}_k + \frac{\hat{a}_k}{(\gamma^{-1}+\psi)}\right\}. \tag{A.7}$$

Now, we calculate the natural rate of output in each sector. From the demand functions in (7), taking logs, at the natural rate equilibrium:

$$y_k^N = -\gamma(p_k^N - p^N) + \log(\theta_k) + y^N.$$

Subtracting $\bar{y}_k = \log(\theta_k) + \bar{y}$ we obtain:

$$\hat{y}_k^N = -\gamma(p_k^N - p^N) + \hat{y}^N. \tag{A.8}$$

From the pricing condition (10) and since at the natural rate equilibrium prices are flexible, and markups are 1:

$$(1 + \gamma\psi)(p_k^N - p^N) = (\sigma + \psi)y^N + a_k + \psi \log(\theta_k).$$

At the point of linearization, the condition above also holds but with the stochastic variable a_k replaced by its mean $\bar{a}_k$. In terms of deviations from the equilibrium around which we linearize, the expression above becomes:

$$(1 + \gamma\psi)(p_k^N - p^N) = (\sigma + \psi)\hat{y}^N + \hat{a}_k. \tag{A.9}$$

Combining (A.8) and (A.9) substituting out for relative prices, we obtain:

$$-\frac{\hat{a}_k}{\gamma^{-1} + \psi} = \hat{y}_k^N + \frac{\sigma - \gamma^{-1}}{\gamma^{-1} + \psi}\hat{y}^N.$$

The expression in (A.7) can therefore be rewritten as:

$$Var_k(\hat{y}_k - \hat{y}_k^N)$$

Going back to the utility function we then have:

$$U \approx -e^{(1-\sigma)\bar{y}}\frac{(\sigma + \psi)}{2}\left((y - y^N)^2 + \frac{(\gamma^{-1} + \psi)}{(\sigma + \psi)}[Var_k(y_k - y_k^N) + E_k(Var_i(\hat{y}_{ki}))]\right).$$

We drop the hats from $y - y^N$ and $y_k - y_k^N$ since the conditions defining the equilibrium around which we linearize include the conditions defining the natural rate equilibrium. Finally, using the assumption that $E(m_k) = 1$ made in the beginning of the appendix, the model in section 1 implies that $E(y) \approx E(y^N)$. This holds only up to second-order terms, since we use first-order approximations to obtain the price index of the economy and the result $E(\log(m_k)) \approx 0$. Taking expectations of the equation above, and dropping the proportionality factor that is outside the influence of the policymaker, we can write the objective of the policymaker setting his rule before observing the shocks as:

$$E(U) \approx -\left(Var(y - y^N) + \frac{(\gamma^{-1} + \psi)}{(\sigma + \psi)}E[Var_k(y_k - y_k^N) + E_k(Var_i(\hat{y}_{ki}))]\right).$$

Finally, note that $\bar{y}_{ki} = \bar{y}_k$ for all i, so we can add it to the last cross-sectional variance term. Moreover $y_{ki}^N = y_k^N$ since with perfect price flexibility all firms within a sector are identical and so have the same natural rate of output. We can therefore replace all output variables by gap variables in the expression above to obtain Equation (13) in the text.

Appendix 2: Results for the Two-Sector Case

In this appendix, we prove the results and propositions presented in Section 3 of the text.

The Optimal Weights in the Stability Price Index

First, express all variables as deviations from their expected value. Letting a tilde over a variable denote its deviations from its expected value ($\tilde{x} = x - E(x)$), the model can be written as:

$$\tilde{p}_k^* = \tilde{p} + \alpha_k \tilde{x} + \tilde{\varepsilon}_k$$

$$\tilde{p}_k = \lambda_k \tilde{p}_k^* + (1 - \lambda_k) E(\tilde{p}_k^*)$$

$$\tilde{p} = \theta_A \tilde{p}_A + \theta_B \tilde{p}_B$$

$$0 = \omega_A \tilde{p}_A + \omega_B \tilde{p}_B.$$

Next, we use the facts that (1) there are only 2 sectors in this application ($k = A, B$), (2) the expected value of any variable with a tilde over it is zero, and (3) the weights must sum to one, to re-express the system as:

$$\tilde{p}_A = \lambda_A(\tilde{p} + \alpha_A \tilde{x} + \tilde{\varepsilon}_A)$$

$$\tilde{p}_B = \lambda_B(\tilde{p} + \alpha_B \tilde{x} + \tilde{\varepsilon}_B)$$

$$\tilde{p} = \theta_A \tilde{p}_A + (1 - \theta_A)\tilde{p}_B$$

$$0 = \omega_A \tilde{p}_A + (1 - \omega_A)\tilde{p}_B.$$

This is a system of four equations in four variables ($\tilde{p}_A, \tilde{p}_B, \tilde{p}, \tilde{x}$). Solving for the variable of interest $\tilde{x}$, we obtain:

$$\tilde{x} = -\frac{[\omega_A + \lambda_B(\theta_A - \omega_A)]\lambda_A\tilde{\varepsilon}_A + [(1 - \omega_A) - \lambda_A(\theta_A - \omega_A)]\lambda_B\tilde{\varepsilon}_B}{\alpha_B\lambda_B + \omega_A(\alpha_A\lambda_A - \alpha_B\lambda_B) + \lambda_A\lambda_B(\omega_A - \theta_A)(\alpha_B - \alpha_A)} \qquad (A.10)$$

The policymaker will then choose the weight ω_A in order to minimize the variance of the previous expression. Using the first-order condition and rearranging we find the optimal ω_A^* given by:

$$\omega_A^* = \lambda_B \frac{\alpha_A\sigma_B^2 - \theta_A\lambda_A(\alpha_A\sigma_B^2 + \alpha_B\sigma_A^2)}{\alpha_B\lambda_A(1 - \lambda_B)\sigma_A^2 + \alpha_A\lambda_B(1 - \lambda_A)\sigma_B^2} \qquad (A.11)$$

The optimal ω_B^* is just given by $\omega_B^* = 1 - \omega_A^*$.

Proof of the Propositions

Special Case: Using the values $\alpha_A = \alpha_B$, $\sigma_A^2 = \sigma_B^2$, $\lambda_A = \lambda_B$, $\theta_A = \theta_B = 1/2$ in the formula for ω_A^* above we find that $\omega_A^* = 1/2$.

PROPOSITION 1: Taking derivatives of (A.11) with respect to α_A, we find that

$$\frac{\partial \omega_A^*}{\partial \alpha_A} = \frac{\alpha_B \lambda_A \lambda_B \sigma_A^2 \sigma_B^2 (1 - \theta_A \lambda_A - (1 - \theta_A)\lambda_B)}{(\alpha_B \lambda_A (1 - \lambda_B)\sigma_A^2 + \alpha_A (1 - \lambda_A)\lambda_B \sigma_B^2)^2}$$

The denominator is clearly nonnegative, and so is the numerator since $\lambda_k \leq 1$ and $\theta_k \leq 1$, so we can sign $\partial\omega_A^*/\partial\alpha_A \geq 0$. By symmetry $\partial\omega_B^*/\partial\alpha_B \geq 0$.

PROPOSITION 2: Taking derivatives of the solution (A.11):

$$\frac{\partial \omega_A^*}{\partial \sigma_A^2} = -\frac{\alpha_A \alpha_B \lambda_A \lambda_B \sigma_B^2 (1 - \theta_A \lambda_A - (1 - \theta_A)\lambda_B)}{(\alpha_B \lambda_A (1 - \lambda_B)\sigma_A^2 + \alpha_A (1 - \lambda_A)\lambda_B \sigma_B^2)^2},$$

which by the same argument as in the previous proposition, implies $\partial\omega_A^*/\partial\sigma_A^2 \leq 0$ (and $\partial\omega_B^*/\partial\sigma_B^2 \leq 0$ symmetrically).

PROPOSITION 3: Taking derivatives of ω_A^* with respect to λ_A:

$$\frac{\partial \omega_A^*}{\partial \lambda_A} = -\frac{\alpha_A \lambda_B \sigma_B^2 [\alpha_B \sigma_A^2 - (1 - \theta_A)\lambda_B (\alpha_B \sigma_A^2 + \alpha_A \sigma_B^2)]}{(\alpha_B \lambda_A (1 - \lambda_B)\sigma_A^2 + \alpha_A (1 - \lambda_A)\lambda_B \sigma_B^2)^2}.$$

From the solution for ω_A^*:

$$\omega_A^* < 1 \Leftrightarrow$$
$$\alpha_B \sigma_A^2 > (1 - \theta_A)\lambda_B (\alpha_B \sigma_A^2 + \alpha_A \sigma_B^2).$$

Therefore, as long as $\omega_A^* < 1$, then $\partial\omega_A^*/\partial\lambda_A < 0$. By symmetry it follows that $\partial\omega_B^*/\partial\lambda_B < 0$.

PROPOSITION 4: Follows from evaluating the optimal solution ω_A^* at the point: $\alpha_A = \alpha_B$, $\sigma_A^2 = \sigma_B^2$, $\theta_A = \theta_B = 0.5$, $\lambda_A = 1$, $\lambda_B < 1$, to obtain $\omega_A^* = 0$.

PROPOSITION 5: Taking derivatives of ω_A^* with respect to θ_A, we obtain:

$$\frac{\partial \omega_A^*}{\partial \theta_A} = -\frac{\lambda_A \lambda_B (\alpha_B \sigma_A^2 + \alpha_A \sigma_B^2)}{\alpha_B \lambda_A \sigma_A^2 (1 - \lambda_B) + \alpha_A \lambda_B \sigma_B^2 (1 - \lambda_A)},$$

which is negative. Clearly $\partial\omega_B^*/\partial\theta_B$ is also negative.

The Stability Price Index with an Unrestricted Shock Covariance Matrix

Minimizing the variance of Equation (A.10) we obtain the optimal weight on sector A:

$$\omega_A^* = \lambda_B \frac{\alpha_A[\sigma_B^2 + \theta_A\lambda_A(\sigma_{AB} - \sigma_B^2)] - \alpha_B[\theta_A\lambda_A(\sigma_A^2 - \sigma_{AB}) + \sigma_{AB}]}{\alpha_A[(1-\lambda_A)\lambda_B\sigma_B^2 - \lambda_A(1-\lambda_B)\sigma_{AB}] + \alpha_B[(1-\lambda_B)\lambda_A\sigma_A^2 - \lambda_B(1-\lambda_A)\sigma_{AB}]}, \tag{A.12}$$

where σ_{AB} denotes the covariance between $\tilde{\varepsilon}_A$ and $\tilde{\varepsilon}_B$. Taking derivatives of (A.12) with respect to α_A we find:

$$\frac{\partial \omega_A^*}{\partial \alpha_A} = \frac{\alpha_B\lambda_A\lambda_B(1-\theta_A\lambda_A-(1-\theta_A)\lambda_B)(\sigma_A^2\sigma_B^2-\sigma_{AB}^2)}{(\alpha_A[(1-\lambda_A)\lambda_B\sigma_B^2 - \lambda_A(1-\lambda_B)\sigma_{AB}] + \alpha_B[(1-\lambda_B)\lambda_A\sigma_A^2 - \lambda_B(1-\lambda_A)\sigma_{AB}])^2}.$$

Clearly $\partial\omega_A^*/\partial\alpha_A \geq 0$, and by symmetry $\partial\omega_B^*/\partial\alpha_B \geq 0$, so Proposition 1 still holds.

Evaluating the optimal solution ω_A^* in Equation (A.10) at the point: $\alpha_A = \alpha_B$, $\sigma_A^2 = \sigma_B^2$, $\theta_A = \theta_B = 0.5$, $\lambda_A = 1$, $\lambda_B < 1$, we obtain $\omega_A^* = 0$, so Proposition 4 still holds.

Appendix 3: Multisector Problems

In this appendix, we describe how to find the optimal price index in a K sector problem as in Section 4 of the text. The algorithm has three steps. First, we solve for the equilibrium output in the economy, by solving the set of $K + 2$ equations:

$$\tilde{p}_k = \lambda_k(\tilde{p} + \alpha_k\tilde{x} + \tilde{\varepsilon}_k), \qquad k = 1, \ldots, K$$

$$\tilde{p} = \sum_{k=1}^{K} \theta_k\tilde{p}_k$$

$$0 = \sum_{k=1}^{K} \omega_k\tilde{p}_k.$$

in $K + 2$ variables ($\tilde{x}$, $\tilde{p}$, and the $\tilde{p}_k$), for the variable $\tilde{x}$, in terms of the parameters and the innovations $\tilde{\varepsilon}_k$. Second, we take the unconditional expectation of the square of $\tilde{x}$, to obtain the variance of output as a function of α_k, θ_k, λ_k, ω_k and the variances $\sigma_k^2 = E(\tilde{\varepsilon}_k^2)$ and covariances $\sigma_{kj} = E(\tilde{\varepsilon}_k\tilde{\varepsilon}_j)$:

$$Var(\tilde{x}) = f(\alpha_k, \theta_k, \lambda_k, \omega_k, \sigma_k^2, \sigma_{kj}).$$

Given values for (α_k, θ_k, λ_k, σ_k^2, σ_{kj}) the third step is to numerically minimize

$f(\cdot)$ with respect to the ω_k, subject to the constraint that $\sum_k \omega_k = 1$, and possibly additional nonnegativity constraints: $\omega_k \geq 0$.

References

Aoki, Kosuke (2001). "Optimal Monetary Policy Responses to Relative-Price Changes." *Journal of Monetary Economics,* 48, pp. 55–80.

Ball, Laurence and David Romer (1990). "Real Rigidities and the Nonneutrality of Money." *Review of Economic Studies,* 57, pp. 183–203.

Benigno, Pierpaolo (2001). "Optimal Monetary Policy in a Currency Area," Centre for Economic Policy Research (CEPR) Discussion Paper, 2755.

Blanchard, Olivier J. and Nobuhiro Kiyotaki (1987). "Monopolistic Competition and the Effects of Aggregate Demand." *American Economic Review,* 77, pp. 647–666.

Clarida, Richard, Jordi Gali, and Mark Gertler (2003). "A Simple Framework for International Monetary Policy Analysis," NBER Working Paper, 8870. *Journal of Monetary Economics,* forthcoming.

Dixit, Avinash K. and Joseph E. Stiglitz (1977). "Monopolistic Competition and Optimum Product Diversity." *American Economic Review,* 67, pp. 297–308.

Erceg, Christopher J., Dale W. Henderson, and Andrew T. Levin (2000). "Optimal Monetary Policy with Staggered Wage and Price Contracts." *Journal of Monetary Economics,* 46, pp. 281–313.

Fischer, Stanley (1977). "Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule." *Journal of Political Economy,* 85, pp. 191–205.

Mankiw, N. Gregory and Ricardo Reis (2002). "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve." *Quarterly Journal of Economics,* 117(4), pp. 1295–1328.

Mankiw, N. Gregory and Ricardo Reis (2003). "Sticky Information: A Model of Monetary Nonneutrality and Structural Slumps." In *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps,* edited by Philippe Aghion, Romain Frydman, Joseph Stiglitz and Michael Woodford. Princeton, New Jersey: Princeton University Press.

Obstfeld, Maurice and Kenneth Rogoff (1996). *Foundations of International Macroeconomics.* Cambridge, Massachusetts: MIT Press.

Phelps, Edmund S. (1978). "Disinflation Without Recession: Adaptive Guideposts and Monetary Policy." *Weltwirtschaftsliches Archiv,* 114(4), pp. 783–809.

Romer, David (2001). *Advanced Macroeconomics.* Second edition. New York: McGraw-Hill.

Rotemberg, Julio J. and Michael Woodford (1999). "The Cyclical Behavior of Prices and Costs." In *Handbook of Macroeconomics,* Vol. 1A, edited by John B. Taylor and Michael Woodford. Amsterdam: Elsevier.

Solon, Gary, Robert B. Barsky, and Jonathan Parker (1994). "Measuring the Cyclical Behavior of Real Wages: How Important is Composition Bias?" *Quarterly Journal of Economics,* 110(2), pp. 321–352.

Spence, Michael (1977). "Product Selection, Fixed Costs, and Monopolistic Competition." *Review of Economic Studies,* 43(2), pp. 217–235.

Steinsson, Jon (2003). "Optimal Monetary Policy in an Economy with Inflation Persistence." *Journal of Monetary Economics,* forthcoming.

Woodford, Michael (2002). "Inflation Stabilization and Welfare." *Contributions to Macroeconomics, The B.E. Journals in Macroeconomics,* 2(1), electronic journal.

Wynne, Mark A. (1999). "Core Inflation: A Review of Some Conceptual Issues," European Central Bank (ECB) Working Paper, 5, Frankfurt, Germany.

This article has been cited by:

1. BRAD E. STRUM. 2009. Monetary Policy in a Forward-Looking Input-Output Economy. *Journal of Money, Credit and Banking* **41**:4, 619-650. [CrossRef]
2. Mick Silver. 2007. Core Inflation: Measurement and Statistical Issues in Choosing Among Alternative Measures. *IMF Staff Papers* **54**:1, 163-190. [CrossRef]
3. Wojciech W Charemza, Svetlana Makarova. 2006. Ex ante Dynamics of Real Effects of Monetary Policy: Theory and Evidence for Poland and Russia, 2001–2003. *Comparative Economic Studies* **48**:3, 458-479. [CrossRef]