Scaling dimensions of monopole operators in the CPNb!1 theory in 2 + 1 dimensions

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Erratum: Scaling dimensions of monopole operators in the $\mathbb{CP}^{N_b-1}$ theory in $2+1$ dimensions

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A missing factor of $i$ was found in the computation of the mixed kernel $F_{j_k}^{q,B}(\omega)$. This results in minor changes in our formulas, but a major change in our conclusions. First, we correct the formulas, then we present the corrected conclusions.

Corrected formulas. The correct expression replacing (4.47) in the original article is

$$F_{j_k}^{q,B}(\omega) = \frac{16q\pi^2}{j(j+1)} \sum_{j',j''=q} \frac{E_{q,j'} + E_{q,j''}}{2E_{q,j'}E_{q,j''}(\omega^2 + (E_{q,j'} + E_{q,j''})^2)} \mathcal{I}_H(j,j',j''). \quad (1)$$

Because $F_{j_k}^{q,B}(\omega)$ is pure imaginary the matrix of coefficients $M_j^q(\omega)$ is not Hermitian, and (4.12) is modified to

$$M_j^q(\omega) = \begin{pmatrix} D_j^q(\omega) & F_j^{q,B}(\omega) & F_j^{q,T}(\omega) & F_j^{q,E}(\omega) \\ -F_j^{q,B^*}(\omega) & K_j^{q,BB}(\omega) & K_j^{q,TB}(\omega) & K_j^{q,EB}(\omega) \\ -F_j^{q,T^*}(\omega) & K_j^{q,TB^*}(\omega) & K_j^{q,T^*T}(\omega) & K_j^{q,TT}(\omega) \\ -F_j^{q,E^*}(\omega) & K_j^{q,EB^*}(\omega) & K_j^{q,E^*T}(\omega) & K_j^{q,EE}(\omega) \end{pmatrix}. \quad (2)$$
Consequently, the eigenvalues of the matrix are changed, (4.16) and (4.19) are changed to

\[
\lambda_\pm^q = \frac{(D_j^q(\omega) + K_j^{q,BB}(\omega)) \pm \sqrt{(D_j^q(\omega) - K_j^{q,BB}(\omega))^2 - 4|F_j^{q,B}(\omega)|^2}}{2},
\]

\[
\lambda_E^q = \frac{j(j+1) + \omega^2}{j(j+1)} K_j^{q,\tau\tau},
\]

and

\[
\delta F_q = \frac{1}{2} \int \frac{d\omega}{2\pi} \left[ \log \frac{D_j^q(\omega)}{D_0^q(\omega)} + \sum_{i=1}^\infty (2j+1) \log \frac{K_j^{q,\tau\tau}(\omega) [D_j^q(\omega)K_j^{q,BB}(\omega) + |F_j^{q,B}(\omega)|^2]}{D_j^q(\omega)K_j^{0,\tau\tau}(\omega)K_j^{0,BB}(\omega)} \right]
\]

respectively.

All formulas in the appendices are fixed by inserting an \(i\) in the appropriate places. The only nontrivial replacement is in (C.49), which correctly reads

\[
L_j^q(\omega) = \frac{8\mu^2_q}{(j + \frac{1}{2})^2 + \omega^2} + \frac{12\mathcal{F}_q}{\pi} \frac{(j + \frac{1}{2})^2 - \omega^2}{[(j + \frac{1}{2})^2 + \omega^2]^{5/2}}
\]

\[
- \frac{(q^2 + 4\mu^2_q(8\mu^2_q - 1))(j + \frac{1}{2})^2 + 4(-q^2 + \mu^2_q(8\mu^2_q - 5))\omega^2}{2 [(j + \frac{1}{2})^2 + \omega^2]^3}
\]

\[
+ 144\mu^2_q (25 + 48\mu^2_q)(j + \frac{1}{2})^4 - 24(j + \frac{1}{2})^2\omega^2 + 8\omega^4
\]

\[
\frac{3\mathcal{F}_q}{2\pi} \frac{25 - 48\mu^2_q}{[(j + \frac{1}{2})^2 + \omega^2]^{9/2}}
\]

\[
+ \frac{3\mathcal{F}_q}{2\pi} \frac{25 - 48\mu^2_q}{[(j + \frac{1}{2})^2 + \omega^2]^{9/2}}
\]

\[
+ O\left(\frac{1}{[(j + \frac{1}{2})^2 + \omega^2]^3}\right).
\]

(5)

This change has important consequences on our final results. All monopoles are stable, invalidating section 5.1. The large \(q\) analysis supports the stability of monopoles; (5.9), (5.10) and figure 4 are replaced by

\[
M_j^q(0) = \frac{\zeta(\frac{3}{2}, \frac{1}{2} + \chi_0)}{8 \pi \sqrt{2q}} \begin{pmatrix}
\frac{1}{\frac{1}{2} \sqrt{j+1} \chi_0} & i\sqrt{j(j+1)} \chi_0 & 0 \\
i\sqrt{j(j+1)} \chi_0 & 2j(j+1) \chi_1 & 0 \\
0 & 4j(j+1)(\chi_0^2 + \chi_1) & 0 \\
\end{pmatrix},
\]

\[
\lambda_\pm^q \approx \frac{0.055251 \pm 0.023717i}{\sqrt{q}}, \quad \lambda_E^q \approx \frac{0.063044}{\sqrt{q}}.
\]

(6)

(7)


**Figure 1.** The numerical results for the three eigenvalues, $\lambda_\pm^q$, $\lambda_+^q$, and $\lambda_-^q$, are plotted against the analytic large $q$ value in black.

<table>
<thead>
<tr>
<th>$q$</th>
<th>$\Delta_q$</th>
<th>$N_b$ for which $\Delta_q \leq 3$</th>
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<tr>
<td>0</td>
<td>0.01245922 $N_b + 0.3815 + O(N_b^{-1})$</td>
<td>$&lt; \infty$</td>
</tr>
<tr>
<td>1/2</td>
<td>0.3110952 $N_b + 0.8745 + O(N_b^{-1})$</td>
<td>$\leq 21$</td>
</tr>
<tr>
<td>1</td>
<td>0.5440693 $N_b + 1.4646 + O(N_b^{-1})$</td>
<td>$\leq 6$</td>
</tr>
<tr>
<td>3/2</td>
<td>0.8157878 $N_b + 2.1388 + O(N_b^{-1})$</td>
<td>$\leq 2$</td>
</tr>
<tr>
<td>2</td>
<td>1.1214167 $N_b + 2.8879 + O(N_b^{-1})$</td>
<td>none</td>
</tr>
<tr>
<td>5/2</td>
<td>1.1214167 $N_b + 2.8879 + O(N_b^{-1})$</td>
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**Table 1.** Results of the large $N_b$ expansion of the monopole operator dimensions $\Delta_q$ obtained through calculating the ground state energy in the presence of $2q$ units of magnetic flux through $S^2$. In the last column of the table we listed our estimates for when the monopole operators are relevant.

**Results and conclusions.** Because these corrections make all saddle points stable, we are able to compute the dimensions of monopole for many values of $q$. The results are listed in table 1. In particular our result for $\Delta_{1/2}$ is different, and compares favorably with the numbers obtained by quantum Monte Carlo simulations of refs. [1–3] even for small $N_b$, as shown in the replacement of figure 1 of the original article in figure 2. From comparing the scaling dimensions collected in table 1 to 3, we can also estimate the upper bound on $N_b$ below which the monopole operators are expected to be relevant; these bounds are also presented in table 1. There is inherently some uncertainty in these estimates, as they come from extrapolating the large $N_b$ expansion to small values of $N_b$. Nevertheless, our relevance bounds come close to what ref. [3] found from numerics, as can be seen from table I in [3].

**Acknowledgments**

We are grateful to Nathan Agmon, whose joint work with SSP found the factor of $i$ discussed in this Erratum in the original version of this paper.
The scaling dimension of the $q = 1/2$ monopole operator, $F_{1/2}$. The full line is the $N_b = \infty$ result (ref. [4]), and the dashed line is the leading $1/N_b$ correction computed in the present paper (see table 1). The quantum Monte Carlo results are for lattice antiferromagnets with global $SU(N_b)$ symmetry on the square (refs. [1, 2]), honeycomb (ref. [3]), and rectangular (ref. [3]) lattices.

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**References**


