Scaling dimensions of monopole operators in the CPNb!1 theory in 2 + 1 dimensions

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Erratum: Scaling dimensions of monopole operators in the $\mathbb{CP}^{N_b-1}$ theory in 2 + 1 dimensions

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Erratum to: JHEP06(2015)037

ArXiv ePrint: 1504.00368

A missing factor of $i$ was found in the computation of the mixed kernel $F_{j}^{q,B} (\omega)$. This results in minor changes in our formulas, but a major change in our conclusions. First, we correct the formulas, then we present the corrected conclusions.

Corrected formulas. The correct expression replacing (4.47) in the original article is

$$F_{j}^{q,B} (\omega) = \frac{16q^2 \pi^2}{(2j+1)\sqrt{j(j+1)}} \sum_{j',j''=q}^\infty \left[ E_{qj'j''}^{\omega} (\omega^2 + (E_{qj'j''})^2) \right] I_H (j,j',j'').$$

(1)

Because $F_{j}^{q,B} (\omega)$ is pure imaginary the matrix of coefficients $M^{q}_j (\omega)$ is not Hermitian, and (4.12) is modified to

$$M^{q}_j (\omega) = \begin{pmatrix}
D^q_j (\omega) & F_{j}^{q,B} (\omega) & F_{j}^{q,T} (\omega) & F_{j}^{q,E} (\omega) \\
-F_{j}^{q,B*} (\omega) & K_{j}^{q,BB*} (\omega) & K_{j}^{q,TB*} (\omega) & K_{j}^{q,EB*} (\omega) \\
-F_{j}^{q,T*} (\omega) & K_{j}^{q,TB*} (\omega) & K_{j}^{q,TT*} (\omega) & K_{j}^{q,ET*} (\omega) \\
-F_{j}^{q,E*} (\omega) & K_{j}^{q,EB*} (\omega) & K_{j}^{q,ET*} (\omega) & K_{j}^{q,EE*} (\omega)
\end{pmatrix}. $$

(2)
Consequently, the eigenvalues of the matrix are changed, (4.16) and (4.19) are changed to

\[ \lambda^q_{\pm} = \frac{D^q_j(\omega) + K^{q,\BB}_j(\omega) \pm \sqrt{(D^q_j(\omega) - K^{q,\BB}_j(\omega))^2 - 4F^{q,\BB}_j(\omega)^2}}{2}, \]

\[ \lambda^q_E = \frac{j(j+1) + \omega^2}{j(j+1)} K^{q,\tau \tau}_j, \]

and

\[ \delta F_q = \frac{1}{2} \int \frac{d\omega}{2\pi} \left[ \log \frac{D^q_j(\omega)}{D^0_q(\omega)} + \sum_{j=1}^{\infty} (2j+1) \log \frac{K^{q,\tau \tau}_j(\omega) \left[ D^q_j(\omega) K^{q,\BB}_j(\omega) + |F^{q,\BB}_j(\omega)|^2 \right]}{D^0_j(\omega) K^{0,\tau \tau}_j(\omega) K^{0,\BB}_j(\omega)} \right] \]

respectively.

All formulas in the appendices are fixed by inserting an \( i \) in the appropriate places. The only nontrivial replacement is in (C.49), which correctly reads

\[ L^q_j(\omega) = \frac{8\mu^2_q}{(j + \frac{1}{2})^2 + \omega^2} + \frac{12F^\infty_q}{\pi} \frac{(j + \frac{1}{2})^2 - \omega^2}{[(j + \frac{1}{2})^2 + \omega^2]^{5/2}} \]

\[ - \frac{(q^2 + 4\mu^2_q(8\mu^2_q - 1))(j + \frac{1}{2})^2 + 4(-q^2 + \mu^2_q(8\mu^2_q - 5))\omega^2}{2 [(j + \frac{1}{2})^2 + \omega^2]^3} \]

\[ + 144B_q \frac{3(j + \frac{1}{2})^4 - 24(j + \frac{1}{2})^2\omega^2 + 8\omega^4}{[(j + \frac{1}{2})^2 + \omega^2]^{9/2}} \]

\[ + \frac{3F^\infty_q}{2\pi} \frac{(25 - 48\mu^2_q)(j + \frac{1}{2})^4 + 3(64\mu^2_q - 55)(j + \frac{1}{2})^2\omega^2 + 20\omega^4}{[(j + \frac{1}{2})^2 + \omega^2]^{9/2}} \]

\[ + O \left( \frac{1}{[(j + \frac{1}{2})^2 + \omega^2]^3} \right). \]

This change has important consequences on our final results. All monopoles are stable, invalidating section 5.1. The large \( q \) analysis supports the stability of monopoles; (5.9), (5.10) and figure 4 are replaced by

\[ M^q_j(0) = \frac{\zeta \left( \frac{3}{2}, \frac{1}{2} + \chi_0 \right)}{8\pi \sqrt{2q}} \left( \begin{array}{ccc} \frac{1}{2} & i\sqrt{j(j+1)} \chi_0 & 0 \\ i\sqrt{j(j+1)} \chi_0 & 2j(j+1) \chi_1 & 0 \\ 0 & 0 & 4j(j+1)(\chi_0^2 + \chi_1) \end{array} \right), \]

\[ \lambda^q_\pm \approx \frac{-0.055251 \pm 0.023717i}{\sqrt{q}}, \quad \lambda^q_E \approx \frac{0.063044}{\sqrt{q}}. \]

and figure 1.
Figure 1. The numerical results for the three eigenvalues, $\lambda_E^q$, $\lambda_+^q$, and $\lambda_-^q$, are plotted against the analytic large $q$ value in black.

Table 1. Results of the large $N_b$ expansion of the monopole operator dimensions $\Delta_q$ obtained through calculating the ground state energy in the presence of $2q$ units of magnetic flux through $S^2$. In the last column of the table we listed our estimates for when the monopole operators are relevant.

<table>
<thead>
<tr>
<th>$q$</th>
<th>$\Delta_q$</th>
<th>$N_b$ for which $\Delta_q &lt; 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$&lt; \infty$</td>
</tr>
<tr>
<td>1/2</td>
<td>0.1245922 $N_b + 0.3815 + O(N_b^{-1})$</td>
<td>$\leq 21$</td>
</tr>
<tr>
<td>1</td>
<td>0.3110952 $N_b + 0.8745 + O(N_b^{-1})$</td>
<td>$\leq 6$</td>
</tr>
<tr>
<td>3/2</td>
<td>0.5440693 $N_b + 1.4646 + O(N_b^{-1})$</td>
<td>$\leq 2$</td>
</tr>
<tr>
<td>2</td>
<td>0.8157878 $N_b + 2.1388 + O(N_b^{-1})$</td>
<td>none</td>
</tr>
<tr>
<td>5/2</td>
<td>1.1214167 $N_b + 2.8879 + O(N_b^{-1})$</td>
<td>none</td>
</tr>
</tbody>
</table>

Results and conclusions. Because these corrections make all saddle points stable, we are able to compute the dimensions of monopole for many values of $q$. The results are listed in table 1. In particular our result for $\Delta_{1/2}$ is different, and compares favorably with the numbers obtained by quantum Monte Carlo simulations of refs. [1–3] even for small $N_b$, as shown in the replacement of figure 1 of the original article in figure 2. From comparing the scaling dimensions collected in table 1 to 3, we can also estimate the upper bound on $N_b$ below which the monopole operators are expected to be relevant; these bounds are also presented in table 1. There is inherently some uncertainty in these estimates, as they come from extrapolating the large $N_b$ expansion to small values of $N_b$. Nevertheless, our relevance bounds come close to what ref. [3] found from numerics, as can be seen from table I in [3].

Acknowledgments

We are grateful to Nathan Agmon, whose joint work with SSP found the factor of $i$ discussed in this Erratum in the original version of this paper.
Figure 2. The scaling dimension of the $q = 1/2$ monopole operator, $F_{1/2}$. The full line is the $N_b = \infty$ result (ref. [4]), and the dashed line is the leading $1/N_b$ correction computed in the present paper (see table 1). The quantum Monte Carlo results are for lattice antiferromagnets with global SU($N_b$) symmetry on the square (refs. [1, 2]), honeycomb (ref. [3]), and rectangular (ref. [3]) lattices.

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References


