Complementarity and stability conditions

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Complementarity and stability conditions

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1. Introduction

This note is an attempt to understand better the classic papers by Fradkin and Shenker [1], Banks and Rabinovici [2], 't Hooft [3] and Dimopoulos, Raby and Susskind [4,5] related to complementarity between the Higgs and confining phases in gauge theories. In model building, this is important because it sometimes happens that one takes a Higgsed theory that is perturbatively calculable for small couplings and pushes it into regions in which perturbation theory is questionable. If the Higgs phase and confining phase are complementary, that is if there is no phase transition separating the Higgs phase and confining phase, then one may hope that this will give a picture of the physics that is qualitatively correct even if it is not quantitatively reliable. But if the two phases are genuinely different, then you have no right to expect that this procedure will make any sense at all.

A recent example is an $SU(N+3) \times SU(3) \times U(1)$ model that was suggested as a possible explanation of the di-photon excess at 750 GeV [8]. The model has $(N+3,3)$ scalar field $\xi$ that is trying to break the symmetry down to $SU(N) \times SU(3) \times U(1)$. In the limit in which only one of the couplings gets strong, we can think of the strong non-Abelian group as the gauge symmetry and treat the other approximately as a global symmetry.

If $SU(3)$ gets strong and $SU(N+3)$ is global, the issue is easy. Here, I think that there is no hope of complementarity. Because in this case, in the Higgs phase, we have the $SU(N+3) \times U(1)$ global symmetry broken down to $SU(N) \times SU(3) \times U(1)$. There is a coset space

$$\frac{SU(N+3)}{SU(N) \times SU(3)}$$

(1.1)

describing an $(N,3)$ of massless Goldstone bosons in the Higgs phase and there is no unbroken gauge symmetry And even if the $SU(N+3)$ is weakly gauged, the heavy vectors are light and still present in the low energy theory.

In the confining $SU(3)$ theory, there is no reason for the global $SU(N+3)$ to break and no reason for anything to be light. So in this situation, the phases are distinguished by different symmetries and different massless particles in the low energy theory.

What happens if $SU(N+3)$ gets strong? Then presumably the $SU(3)$ is unbroken both in the confining phase and in the Higgs phase. So this could perhaps be complementary. In the Higgs phase we have massless $SU(N)$ gauge bosons, and the rest of the $SU(N+3)$ gauge bosons have mass of order $g$ and $\Lambda_N$ is of the same order of magnitude times the exponential factor that goes to 1 as the coupling gets large. Thus in the gauge invariant spectrum there are glueballs and bound states of heavy vectors. As the coupling increases, all of these things get heavy! Likewise, in the confining phase of the full $SU(N+3)$ theory, we expect that all the particle states will have mass of the order of the $SU(N+3)$ confinement scale or greater.

Thus in both the confining phase and the Higgs phase, the low energy theories are trivial. This is consistent with complementarity, and in this case, we believe that the phases are in fact complementary. However, in general, the equivalence of the effective low energy theories in the confining and Higgs phases [3] is not a suf-
ficient condition for complementarity. And we suggest another diagnostic for complementarity that can be useful.

It may be that even when the low energy particles and symmetries acting on them are identical, there are sectors describing heavy particles in the two phases with different properties that distinguish the two phases. The property that we will focus on is stability. In a sense, a heavy stable particle is part of the effective low energy theory because if something puts one in the low-energy world, it stays there and its interactions do not involve any high-energies. Stability conditions can be an easy and very physical way of identifying this situation.

It is important to note that stability for a particular set of parameters is not enough because complementarity is about how the physics changes as parameters change. We are interested in the situation in which stability is guaranteed independent of the phase space. An example of this is a theory with a conserved quantized charge. A conserved charge divides the space of physical states up into sectors with definite charge, separated by superselection rules. In a theory with a single conserved charge, the sector with the lowest non-zero positive charge must contain stable states — either a single particle with the minimum charge or a collection of stable particles with total charge equal to the minimum. There is stability here, but it is not a property of the particle. We can certainly imagine changing the parameters in the theory continuously to make some a different particle carrying the conserved charge (not necessarily the same value of the charge) the lightest particle. And indeed, no single particle with the lowest charge has to exist at all. But at least some particles carrying the charge will always be stable as long as the charge is conserved. We might say that each sector of charged states is unconditionally stable, because there is always some combination of particles that is the lightest state with the appropriate charge.

As a very explicit (and fairly silly) example imagine a world with a conserved charge and three types of charged particle, A, B and C with charges 2, 3, and 5 respectively. The lowest positive charge is 1, and the stable states in the charge 1 sector could be AB, ACC or CBB, depending on the particle masses. Charge conservation guarantees that two of the particle types are stable, and which two are actually stable depends on the masses, but the charge 1 sector is stable independent of the details of the masses.

If in a phase transition, the lowest positive charge changes, then even if the light particles in the two phases are qualitatively similar, the possible structures of stable particles in the effective low energy theory must be different in the two phases. There is then no way to get continuously from one effective theory to the other, and the two phases cannot be complementary.

In the remainder of this note, we will give a series of examples based on familiar SU(N) groups. We hope they will convince the reader that this is an interesting approach.

2. SU(5) with a scalar 10

As a warm-up, and to get the reader used to the style of analysis, consider an SU(5) theory with a single 10 of scalars, \( \xi^{jk} = -\xi^{kj} \). The most general renormalizable Lagrangian again has a global U(1) symmetry, and for a range of parameters, \( \xi \) develops a VEV that can be put in the form

\[
\langle \xi \rangle = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & v \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & v & 0
\end{pmatrix}
\]

This breaks the SU(5) gauge symmetry down to SU(3) × SU(2), under which \( \xi \) transforms as

\[
\langle \xi \rangle = (\bar{3}, 1) + (3, 2) + (1, 1)
\]

with the VEV in the (1, 1). The (3, 2) and the imaginary part of the (1, 1) are eaten by the Higgs mechanism producing a (3, 2) and (1, 1) of massive vector bosons.

There is also a global U(1) symmetry that is a combination of the original global U(1) and the U(1) generator of the SU(5) that

\[
\langle \xi \rangle = (\bar{3}, 1) + (3, 2) + (1, 1)
\]

in some arbitrary normalization, and because the U(1) charge of the multiplet must be the average charge of the multiplet after symmetry breaking, we know that \( \xi \) is a 10/5. The condensate also breaks the global 5-ality of the SU(5) theory. down to triality × duality for the SU(3) × SU(2). In the Higgsed theory, the uneaten (3, 1) of scalars has triality 2 and charge 2, the (3, 2) massive gauge boson has triality 1, duality 1 and charge 1.

In both the Higgs phase and the confining phase, heavy particles carry a quantized conserved charge. Now we can examine the stable sectors in the Higgs phase and the confining phase. In this case, they match up perfectly. In the Higgs phase, all the triality and duality zero gauge singlet combinations like 3 (3,1)2 scalars or 6 (3,2)1 massive vector bosons all have U(1) charges which are multiples of 6. In the confining theory the 5-ality zero states are combinations of 5 10/5/3 scalars, which have the same property. The lowest positive charge is 6 in both cases.

Thus the stability conditions do not distinguish between this Higgs phase and the confining phase, and this is consistent with complementarity.

3. SU(5) with a scalar 15

Contrast the model discussed in section 2 with an SU(5) theory with a single 15 of scalars, \( \xi^{jk} = \xi^{kj} \). The most general renormalizable Lagrangian again has a global U(1) symmetry, and for a range of parameters, \( \xi \) develops a VEV that can be put in the form

\[
\langle \xi \rangle = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & v
\end{pmatrix}
\]

This breaks the SU(5) gauge symmetry down to an SU(4), under which \( \xi \) transforms as

\[
10 + 4 + 1
\]

with the VEV in the 1. The 4 and the imaginary part of the 1 are eaten by the Higgs mechanism producing a 4 and 1 of massive vector bosons.

There is also a global U(1) symmetry that is a combination of the original global U(1) and the U(1) generator of the SU(5) that
commutes with SU(4). The 1 in (3.6) must be neutral under the unbroken symmetry, so the charges must look like (again in an arbitrary normalization)

\[ 10_1 + 4_1 + 1_0 \]  
(3.7)

And because the U(1) charge of the multiplet must be the average charge of the multiplet after symmetry breaking, we know that \( \xi \) is a 15\( _{2/3} \).

This time the heavy stable particle sectors in the Higgs phase and the confining phase have different U(1) quantum numbers. In the Higgs phase, a bound state of 4 \( 4_1 \) massive vector bosons confined by the SU(4) has charge 4. The sector with charge 4 has the smallest non-zero value of the conserved U(1) charge, and thus it is unconditionally stable.

In the confining phase, there are no states with charge 4. The lowest nonzero charged state is a bound state of 5 15\( _{2/3} \) scalars, with charge 8, and the lightest charge 8 particle is unconditionally stable. Thus the Higgs and confining theories have different unconditionally stable sectors and cannot be complementary.

The Higgs phase and the confining phase are distinguished in spite of the fact that there is nothing in the low energy theory in either case, because the stable heavy particle sectors have different global U(1) charges. There is no complementarity.

It is interesting to compare this with a model with \( \xi \) being a single 5 of scalars, where we know that complementarity is preserved. In this case, again the gauged SU(5) is broken to SU(4) preserving a global, but now \( \xi \) breaks up into

\[ 4_1 + 1_0 \]  
(3.8)

and again the \( 4_1 \) is eaten by the Higgs mechanism to become the longitudinal component of the massive gauge boson. The Higgs phase in this case is missing the 10\( _2 \) of scalars, but otherwise looks remarkably similar to the 15 case. The 4-ality zero states have charges that are multiples of 4. But now the confining phase is not qualitatively different, because the 5 has global charge 4/5 (the average charge of the multiplet in (3.8)), so the 5-ality zero states also have charges that are multiples of 4.

One of the issues in the difference between \( \xi \) = 15 and \( \xi \) = 5 is that the charge structure of the Higgs phase is determined in part by the charges of the eaten Goldstone bosons which depend on the symmetry breaking but are independent of the details of the rest of the \( \xi \) multiplet. But in the composite phase, the full multiplet is involved in everything.

4. SU(5) with 3 scalar 10s

Next consider an SU(5) gauge group with three 10s of scalars. We can write the scalar fields as

\[ \xi^{ijk} = \xi^{[ijk]} \]  
(4.9)

where \( a \) is the SU(3) flavor index and \( j,k \) are SU(5) indices. We show below that we can find a potential with a global SU(3) \( \times \) U(1) symmetry that produces the vev\(^6\)

\[ \langle \xi^{ijk} \rangle = v \varepsilon^{ijk} \]  
(4.10)

where \( \varepsilon^{ijk} \) is the 3-dimensional Levi-Civita tensor.

The VEV (4.10) preserves a global SU(3) symmetry generated by the sum of the global SU(3)\(_C\) symmetry generator and the generator of an SU(3)\(_5\) subgroup of the gauged SU(5) acting on the first 3 of the SU(5) indices. And it preserves a gauged SU(2)(\(_C\)) acting on SU(5) indices 4 and 5. Under SU(3)\(_C\) \( \times \) SU(5)\(_g\) \( \rightarrow \) SU(3)\(_C\) \( \times \) SU(3)\(_g\) \( \times \) SU(2)\(_g\), the SU(5) generators break up into

\[ (1, 24) \rightarrow (1, 1, 3) + (1, 8, 1) + (1, 3, 2) + (1, \bar{3}, 2) + (1, 1, 1) \]  
\[ \rightarrow (1, 3) + (8, 1) + (3, 2) + (\bar{3}, 2) + (1, 1) \]  
(4.11)

and the \( \xi \) transforms like

\[ (3, 10) \rightarrow (3, \bar{3}, 1) + (3, 3, 2) + (3, 1, 1) \]  
\[ \rightarrow (8, 1) + (1, 1) + (6, 2) + (\bar{3}, 2) + (3, 1) \]  
(4.12)

The vev (4.10) is in the real part of the singlet. The imaginary part of the (8, 1) and (1, 1) in (4.12) and the (3, 2) in (4.12) are eaten by the Higgs mechanism giving massive gauge bosons, producing an SU(3) adjoint, a complex (3, 2) and a singlet. If the gauge coupling is small, their masses are in the ratio 1 : 1 : \( \sqrt{8}/5 \). But the details here don’t really matter if the coupling is strong. They just all get heavy.

There is also a global U(1) symmetry that is a combination of the original global U(1) and the U(1) generator of the SU(5) that commutes with SU(3) \( \times \) SU(2). The SU(3)\(_C\) \( \times \) SU(5)\(_g\) singlet in \( \xi \) must be neutral under the unbroken U(1), so the charges must look like

\[ (3, \bar{3}, 1)_0 + (3, 3, 2)_q + (3, 1, 1)_{2q} \]  
\[ \rightarrow (8, 1) + (1, 1) + (6, 2) + (\bar{3}, 2) + (3, 1)_{2q} \]  
(4.13)

for some \( q \). The global charge of the \( \xi \) field in the theory before symmetry breaking must then be 4\( q/5 \), because this is the average charge of the multiplet after symmetry.

In the Higgs phase, there are no states with charge q, so the sector with charge 2q must contain stable particles with total charge 2q. For weak coupling, there are both “fundamental” charge 2q states, like the scalar (3, 1)\(_{2q}\) in (4.13), and composite charge 2q states, like the bound states of two (\( \bar{3}, 2\))\(_q\) vector bosons, confined when the unbroken SU(2) gauge interaction gets strong.

In the confining phase, on the other hand, physical states confined by the strong SU(5) gauge interactions must have 5-ality 0. They therefore contain a multiple of 5 \( \xi \)s, and thus have charges which are a multiple of 4\( q \) and there is no stable sector with charge 2q. Thus the Higgs phase defined by (4.10) cannot be complementary to the confining phase.

Note that the global SU(3) symmetry here is almost certainly not necessary. It makes the analysis of the potential much easier, but if it is explicitly broken, a phase with the same unbroken U(1) and the same charges will very likely exist in some region of the parameter space.

5. SU(5) with 4 scalar \( \mathbf{T}_\mathbf{0} \)s

The examples in sections 3 and 4 have confining unbroken gauge symmetries in the Higgs phase. Again, this is not necessary. Here is an example similar to that in section 4, but slightly more complicated in which the non-Abelian gauge symmetry is completely broken. Consider an SU(5) gauge group with four \( \mathbf{T}_\mathbf{0} \)s of scalars. We can write the scalar fields as

\[ \xi^{ijkl} = \xi^{[ijkl]} \]  
(5.14)

where \( a \) is the SU(4) flavor index and \( j,k,\ell \) are SU(5) indices. Here we can find a potential with a global SU(4) \( \times \) U(1) symmetry that produces the vev\(^7\)

\[ \langle \xi^{ijkl} \rangle = v \varepsilon^{ijkl} \]  
(5.15)

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\(^6\) See section A.4.

\(^7\) See section A.5.
where $\epsilon_{abcde}$ is the 4-dimensional Levi-Civita tensor. As usual, we first discuss the non-Abelian structure and then go back and discuss the $U(1)$.

The VEV (5.15) preserves a global $SU(4)$ symmetry generated by the sum of the global $SU(4)_{G}$ symmetry generator and the generator of an $SU(4)_{G}$ subgroup of the gauged $SU(5)$ acting on the first 4 of the $SU(5)$ indices. Under $SU(4)_{G} \times SU(4)_{G} \to SU(4)_{G+g}$, the (complex) $\xi$ transform like

\begin{equation}
(4, 6) + (4, \bar{4}) \to 20 + \bar{4} + 15 + 1 \tag{5.16}
\end{equation}

When the singlet gets a vev corresponding to (4.10), the $SU(5)$ symmetry breaks completely and the $SU(5)$ generators break up into

\begin{equation}
24 \to 15 + 4 + \bar{4} + 1 \tag{5.17}
\end{equation}

all of which eat parts of the $\xi$ field giving rise to massive gauge bosons. At tree level, this gives mass to all the gauge bosons, producing an $SU(4)$ adjoint, a $4 + \bar{4}$ and 1 with masses in the ratio $1 : \sqrt{3}/2 : 3/\sqrt{5}$. Again the details here don’t really matter if the coupling is strong.

As in the example in section 4, there is also a global $U(1)$ symmetry that is a combination of the original global $U(1)$ and the $U(1)$ generator of the $SU(5)$. The $SU(4)_{G+g}$ singlet in $\xi$ must be neutral under the unbroken $U(1)$, so the charges must look like

\begin{equation}
(4, 6)_{0} + (4, \bar{4})_{0} \to 20_{0} + 3_{0} + 15 + 1_{0} \tag{5.18}
\end{equation}

for some $q$. The global charge of the $\xi$ field in the theory before symmetry breaking is the average charge of the multiplet which is $3q/5$.

Now the Higgs phase at small coupling has particles with charge are the $q$ for example the $4$ state. In the confining phase however, the physical states are all built out of multiples of 5 $\xi$s and thus have charges which are multiples of 3$q$.

So again, in this case, this Higgs phase and the confining phase are distinguished in spite of the fact that there is nothing in the low energy physics in either case, because there are different stable sectors of heavy particles. As in section 4, the $SU(4)$ global symmetry makes it easy to analyze the more general potential, but it is probably not necessary for the stability analysis, which depends only on the global $U(1)$.

6. Conclusion

The examples in this note should convince the reader that in constructing an effective theory, it is important to consider heavy stable particles as well as light particles. This can contain important information about the structure of the quantum field theory. In particular, we have shown that discontinuous changes in the structure of heavy stable particles can signal a phase transition. While this can show conclusively that two phases are not continuously related, we do not know of any way to sharpen these arguments to determine conclusively that two phases are complementary. For this we still need “theorems” like those of reference [1] and [2].

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Appendix A. Potentials and VEVs

A.2. $SU(5)$ with a scalar 10

For an $SU(5)$ theory with a single 10 of scalars, $\xi^{jk} = -\xi^{kj}$, we want to show that the most general renormalizable Lagrangian has a global $U(1)$ symmetry, and for a range of parameters, $\xi$ develops a VEV that can be put in the form

\begin{equation}
\langle \xi \rangle = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -v & 0 \\
0 & 0 & v & 0 & 0
\end{pmatrix} \tag{A.19}
\end{equation}

which breaks the symmetry down to $SU(3) \times SU(2)$. This is easy because we can treat the $\xi$ field as a $2 \times 2$ matrix and write the most general renormalizable potential as

\begin{equation}
\lambda_{1} \left( \left( \text{Tr}(\xi\xi^{\dagger}) \right)^{2} - 4v^{2} \text{Tr}(\xi\xi^{\dagger}) \right) - \lambda_{2} \left( \text{Tr}(\xi\xi^{\dagger}\xi\xi^{\dagger}) - 2v^{2} \text{Tr}(\xi\xi^{\dagger}) \right) \tag{A.20}
\end{equation}

This evidently has a global $U(1)$ and it is extremized for the VEV (A.19). If

\begin{equation}2\lambda_{1} > \lambda_{2} > 0 \tag{A.21}\end{equation}

then (A.19) is a local minimum. The massive scalars are a $(1,1)$ with mass squared $8(2\lambda_{1} - \lambda_{2})v^{2}$ and a complex $(3,1)$ with mass squared $4\lambda_{2}v^{2}$.

A.3. $SU(5)$ with a scalar 15

For an $SU(5)$ theory with a single 15 of scalars, $\xi^{jk} = \xi^{kj}$, we want to show that the most general renormalizable Lagrangian has a global $U(1)$ symmetry, and for a range of parameters, $\xi$ develops a VEV that can be put in the form

\begin{equation}
\langle \xi \rangle = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & v \\
0 & 0 & 0 & v & 0
\end{pmatrix} \tag{A.22}
\end{equation}

which breaks the symmetry down to $SU(4)$. Again we can treat the $\xi$ field as a $2 \times 2$ matrix and this time we will write the most general renormalizable potential as

\begin{equation}
\lambda_{1} \left( \left( \text{Tr}(\xi\xi^{\dagger}) \right)^{2} - 2v^{2} \text{Tr}(\xi\xi^{\dagger}) \right) - \lambda_{2} \left( \text{Tr}(\xi\xi^{\dagger}\xi\xi^{\dagger}) - 2v^{2} \text{Tr}(\xi\xi^{\dagger}) \right) \tag{A.23}
\end{equation}

This again has a global $U(1)$ and it is extremized for the VEV (A.22). If

\begin{equation}\lambda_{1} > \lambda_{2} > 0 \tag{A.24}\end{equation}

then (A.22) is a local minimum. The massive scalars are a real singlet with mass squared $8(\lambda_{1} - \lambda_{2})v^{2}$ and a complex 10 with mass squared $4\lambda_{2}v^{2}$.
A.4. SU(5) with 3 scalar 10s

Here we are interested an SU(5) gauge group with three 10s of scalars which we write as

$$\xi^{ajk} = -\xi^{bijk}$$  \hspace{1cm} (A.25)

where $a$ is the SU(3) flavor index and $j, k$ are SU(5) indices. We show below that we can find a potential with a global SU(3) x U(1) symmetry that produces the vev

$$\langle \xi^{ajk} \rangle = v \, \varepsilon^{ajk}$$  \hspace{1cm} (A.26)

where $\varepsilon^{ijk}$ is the 3-dimensional Levi-Civita tensor.

The VEV (A.26) preserves a global SU(3) symmetry generated by the sum of the global SU(3)g symmetry generator and the generator of an SU(3)g subgroup of the gauged SU(5) acting on the first 3 of the SU(5) indices. And it preserves a gauged SU(2) acting on SU(5) indices 4 and 5.

To see that this Higgs phase actually exists, consider the most general potential. The potential must involve 2 $\xi$s and 2 $\xi$'s. Bose symmetry implies that the 2 $\xi$ transform like

$$(3, 10) \times (3, 10)^{symmetric} = (6, 5) + (6, 50) + (\overline{3}, 45)$$  \hspace{1cm} (A.27)

so there are three independent quartic terms in the potential which we can take to be

$$\kappa_1 = \kappa_0^2$$  \hspace{1cm} (A.28)

$$\kappa_2 = \xi^{ajk_1 k_2} \xi_{ajk_1 k_2}$$  \hspace{1cm} (A.29)

$$\kappa_3 = \xi^{ajk_1 k_2} \xi_{ajk_1 k_2}$$  \hspace{1cm} (A.30)

If we then write the most general potential as

$$V = \lambda_1(k_1 - 12v^2k_0) + \lambda_2(k_2 - 4v^2k_0) - \lambda_3(k_3 - 4v^2k_0)$$  \hspace{1cm} (A.31)

$V$ is extremized for the vev (4.10), and if the $\lambda$s satisfy

$$3\lambda_1 + \lambda_2 > \lambda_3 , \quad 4\lambda_2 > \lambda_3 , \quad \lambda_3 > 0$$  \hspace{1cm} (A.32)

then (A.26) is a local minimum so the example works. The squared masses of the massive scalars are

- a real (1, 1) \hspace{1cm} 16v^2(3\lambda_1 + \lambda_2 - \lambda_3)
- a real (8, 1) \hspace{1cm} 4v^2(4\lambda_2 - \lambda_3)
- a complex (6, 2) \hspace{1cm} 4v^2\lambda_3
- a complex (3, 1) \hspace{1cm} 8v^2\lambda_3

A.5. SU(5) with 4 scalar 10s

The examples in sections 3 and 4 have confining unbroken gauge symmetries in the Higgs phase. Again, this is not necessary. Here is an example similar to that in section 4, but slightly more complicated example in which the non-Abelian gauge symmetry is completely broken. Again consider an SU(5) gauge group with four 10s of scalars. We can write the scalar fields as

$$\xi^{ajk\ell} = \xi^{a[a]jk\ell}$$  \hspace{1cm} (A.34)

where $a$ is the SU(4) flavor index and $j, k, \ell$ are SU(5) indices. Here we can find a potential with a global SU(4) x U(1) symmetry that produces the vev

$$\langle \xi^{ajk\ell} \rangle = v \, \varepsilon^{ajk\ell}$$  \hspace{1cm} (A.35)

where $\varepsilon^{ijk\ell}$ is the 4-dimensional Levi-Civita tensor.

The VEV (A.35) preserves a global SU(4) symmetry generated by the sum of the global SU(4)g symmetry generator and the generator of an SU(4)g subgroup of the gauged SU(5) acting on the first 4 of the SU(5) indices. Under SU(4)g \times SU(4)g \rightarrow SU(4)g + g, the (complex) $\xi$s transform like

$$\langle (4, 10) \times (4, 10)^{symmetric} = (10, 5) + (10, 50) + (\overline{3}, 45)$$  \hspace{1cm} (A.36)

so there are three independent quartic terms in the potential which we can take to be

$$\kappa_1 = \kappa_0$$  \hspace{1cm} (A.37)

$$\kappa_2 = \xi^{ajk_1 k_2} \xi_{ajk_1 k_2}$$  \hspace{1cm} (A.38)

$$\kappa_3 = \xi^{ajk_1 k_2} \xi_{ajk_1 k_2}$$  \hspace{1cm} (A.39)

We could write down a 4th along the same lines,

$$\xi^{ajk_1 k_2} \xi_{ajk_1 k_2}$$  \hspace{1cm} (A.40)

but we know from (A.36) that it is not independent. If we then write the most general potential as

$$V = \lambda_1(k_1 - 48v^2k_0) + \lambda_2(k_2 - 12v^2k_0) - \lambda_3(k_3 - 12v^2k_0)$$  \hspace{1cm} (A.41)

Then $V$ is extremized for the vev (A.35), and if the $\lambda$s satisfy

$$4\lambda_1 + 2\lambda_2 > \lambda_3 , \quad 9\lambda_2 > \lambda_3 , \quad \lambda_3 > 0$$  \hspace{1cm} (A.42)

then (A.35) is a local minimum. The squared masses of the massive scalars are

- a real singlet \hspace{1cm} 48v^2(4\lambda_1 + \lambda_2 - \lambda_3)
- a real \hspace{1cm} 12v^2(9\lambda_2 - \lambda_3)
- a complex \hspace{1cm} 8v^2\lambda_3

References