An Outlier in Zipf's World? A Case Study of China's City Size and Urban Growth

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Abstract

This paper examines the population distribution and urban growth patterns of Chinese cities, motivated by two stylized facts – Zipf's law and Gibrat's law for cities. Our findings suggest that China deviates from both laws from 1991 to 2017. In particular, its population is distributed more equally than Zipf's law would otherwise predict, and Chinese cities have experienced a significant mean reversion, rather than a homogenous growth path.

We develop three hypotheses for explaining why large cities experience slower urban growth in China, namely, economic productivity slowdown, amenity deterioration, and direct government interventions. Our results indicate that in China, productivity and amenities promote population growth, and large cities enjoy higher productivity and better amenities. On the other hand, China's population control policies, the one-child policy and the household registration (hukou) system, are more strictly enforced in large cities than in small and medium-size cities. Therefore, large cities grow slower due to direct government interventions, despite their higher productivity and better amenities.

Executive Summary

Zipf's law is a well-known empirical rule which fits the city size distribution for countries across the world well generally, but we find that China seems to deviate substantially from it in the past three decades. In particular, the population is distributed more equally in China than Zipf's law would otherwise predict, and there are far fewer megacities despite its huge population. To explain this departure from the norm, we examine whether Gibrat's law, the underlying assumption of Gabaix's (1999) Zipf's law model, holds in China. Instead of the homogenous growth path suggested by Gibrat's law, we observe a significant mean reversion of city sizes from 1991 to 2017. This pattern of mean reversion grows weaker over time. Our paper then develops three main hypotheses for explaining why large cities experience slower urban growth in China, namely, economic productivity slowdown, amenity deterioration, and direct government interventions. We exploit city-level urban characteristics data and conduct analyses to test these three hypotheses. We use a linear regression model for studying how economic productivity and urban amenities connect to urban growth and population size. We find that although these two factors indeed promote population growth as suggested by spatial equilibrium models, large cities in China enjoy both higher productivity and better amenities. Thus, the two urban growth determinants that apply to most countries cannot explain China's growth convergence. As for our third hypothesis, we investigate whether the city-level enforcement intensity of China's two main population control policies, the one-child policy and the household registration (hukou) system, increases in population size. We implement a differences-in-differences approach to quantify the effects of the nationwide relaxation of the universal one-child policy in 2011 on the rate of natural increase (RNI) for cities. Empirical evidence shows that relative to small cities, large cities suffered from lower RNI under the universal one-child policy and experienced a rise in RNI after the policy relaxation. We run a simple linear regression using the ratio of unofficial migrants to hukou population as a proxy for the strictness of the hukou system and find that such ratio decreases in population size. Our results indicate that China's population control policies are more constraining for large cities than for small and medium-size cities. Therefore, even though large cities are more appealing in terms of higher productivity and better amenities, populations grow slower because the government wants so. One implication for Zipf's law is that as China continues to loosen its birth planning and migration control in the 2010s, future convergence to Zipf's law may be plausible.

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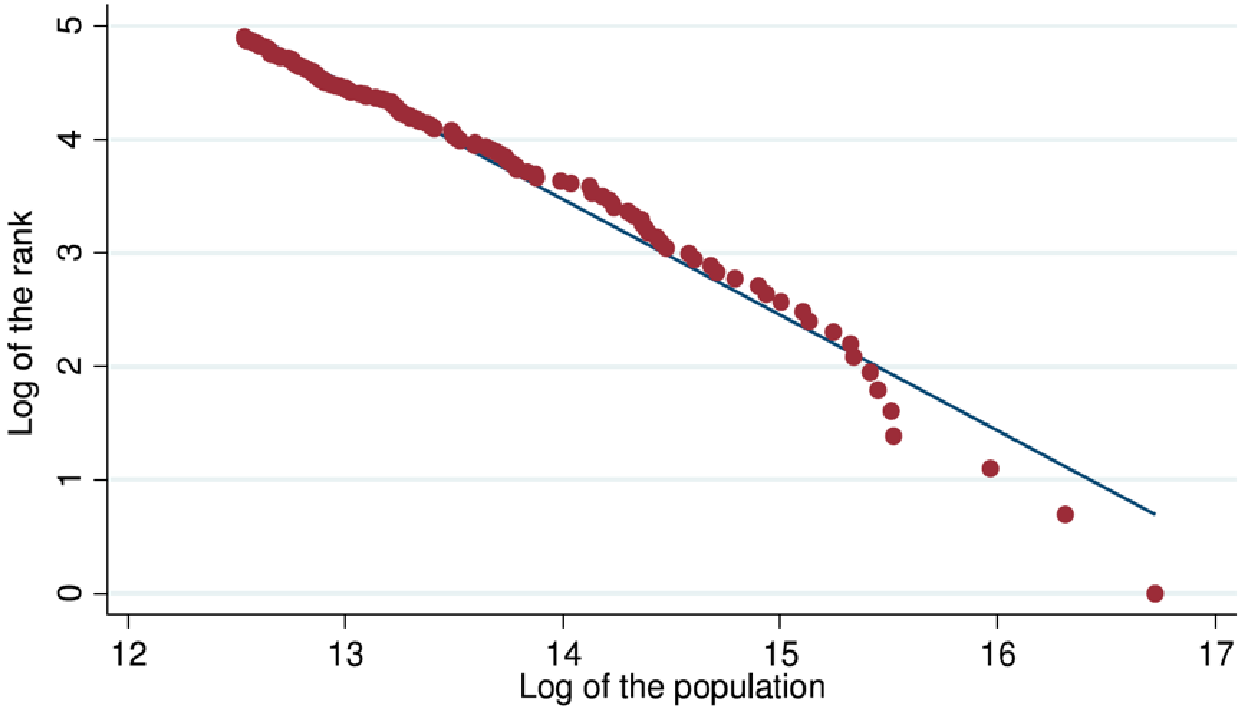
Contents

1 Introduction

Zipf's law strikes urban economists with its simplicity and empirical validity in capturing the distribution of city sizes. Most countries are believed to obey Zipf's law quite well, but the majority of studies on China suggest a deviation from it (Song and Zhang, 2002; Anderson and Ge, 2005; Luckstead and Devados, 2014). Some argue this exception is attributable to a violation of the homogenous growth process, or Gibrat's law, by Chinese cities (Chauvin et al., 2017). Others believe China's unique urbanization and public policies may explain its departure from Zipf's law, albeit with little empirical evidence (Song and Zhang, 2002). Our paper contributes to the debate by examining economic and policy factors that may cast light on why China's city size distribution deviates from Zipf's law, and in particular, why the large cities do not grow as large as Zipf's law predicts. We ultimately find that the stricter enforcement of government's population control policies in large cities, rather than the productivity slowdown or rising urban issues, is responsible for the slower population growth of large cities in China.

Zipf's law says that graphing the logarithm of city ranks in terms of population size against the logarithm of city populations, we obtain a straight line with -1 as the slope coefficient. Figure 1 below presents a visualization of Zipf's law taken from Arshad et al. (2018) using data of the largest 135 U.S. metropolitan areas in the census year 2010. The slope of the fitted line is -1.019. In terms of distribution, Zipf's law states that the probability of a city size being greater than some threshold S is proportional to $1/S$. This is a special case of the well-known Pareto distribution $\mathrm{Prob}(\mathrm{size} > \mathrm{S}) = \mathrm{aS}^{-\zeta}$, with the Pareto coefficient equal to one ($\zeta = 1$).

Figure 1. Log Size versus Log Rank of the Largest 135 U. S. Metropolitan Areas in 2010.



Source: Statistical Abstract of the United States (2010).

Empirical testing of Zipf's law has been conducted for different countries over different periods and suggests that China may be an exception to Zipf's law. The majority of studies on the United States agree that the upper tail of city size distribution conforms to Zipf's law (Krugman, 1996; Ioannides and Overman, 2003; Levy, 2009; Ioannides and Skouras, 2013). Other Western matured economies, like Germany (Giesen and Südekum, 2011) and Russia (Rastvortseva and Manaeva, 2019), have been shown to obey Zipf's law quite well. Some developing countries, like India (Luckstead and Devados, 2014) and Brazil (Moura and Ribeiro, 2013; Matlaba et al., 2013), also converge to Zipf's law after significant urbanization in recent decades. In contrast, findings on China's adherence have been mixed at best, and most studies suggest a rejection of Zipf's law (for example, Song and Zhang, 2002; Anderson and Ge, 2005; Luckstead and Devados, 2014).[1]

[1] See Section 2.1 for a detailed summary of previous studies on China.

Currently the most populous country in the world, China has long been treating the population and its distribution as a crucial "issue" to be dealt with. Various policies, including the family planning program and the household registration system, have been enforced by the central government since the founding of the People's Republic of China (PRC) in 1949. Over the past four decades, China has undergone rapid urbanization, but its city size distribution has yet to conform to Zipf's law. Compared with other well-studied countries that adhere to Zipf's law, China is unique for the central planning component in its economic and administrative policies.

To obtain a deeper understanding of how China's policies may affect urban growth and population distribution, we apply some well-established urban economics models to investigate China's case. Gabaix (1999) provides a baseline model for explaining why cities follow Zipf's law using a random walk with a lower barrier. One important assumption of his model is the homogeneity of growth processes, or Gibrat's law. That is, city growths have the same mean and the same variance, independent of their initial sizes. Gabaix argues that deviations from Zipf's law can be explained by deviations from Gibrat's law. Given China's well-recognized deviation from Zipf's law, we examine whether a deviation from Gibrat's law can also be observed and if so, what economic or policy factors may have facilitated or hindered the convergence of city growth. To propose potential economic factors, we consult Glaeser and Gottlieb's (2009) generalized Rosen-Roback spatial equilibrium framework, which shows that high productivity and high amenities are the two underlying factors of urban population growth. As for policy factors, we focus on China's unique population control policies, the one-child policy and the hukou system. We then design strategies to test whether these three hypotheses – namely, economic productivity, amenities, and government's population control policies - can explain China's urban growth patterns.

Our results suggest that China deviates from Zipf's law and Gibrat's law from 1991 to 2017. In particular, large cities in China have smaller population sizes than what Zipf's law would grant and grow slower than small and medium-size cities. Among the three hypotheses we propose, differences in economic productivity and amenities fail to justify why large cities have experienced slower growth. Instead, we find that the government's direct interventions through two population control policies have been effective in constraining population expansion in large cities, which explains why they grow slower and are not as large as Zipf's law prediction. In addition, our result (from incorporating new population data) shows that deviations from Zipf's law and Gibrat's law become weaker in 2011-2017 than previous decades and is in line with China's recent relaxation of its population control policies.

The rest of the paper is structured as follows: Section 2 summarizes previous empirical studies on China and reviews China's urbanization. Section 3 presents theoretical frameworks for Zipf's law, Gibrat's law, and urban growth. Sections 4 and 5 provide some background on the data and methodology utilized in this paper to test Zipf's law and Gibrat's law and to explain the population growth of Chinese cities. Empirical results are presented in Section 6. Section 7 discusses the findings in light of China's economic and administrative policies.

2 Background

2.1 Relevant Literature

Previous empirical studies on whether Chinese cities obey Zipf's law produce mixed results using population data prior to 2010. Some show evidence of Chinese city sizes converging to Pareto's distribution and Zipf's law, whereas others disagree. For example, Gangopadhyay and Basu (2009) find that the largest Chinese cities follow a Pareto distribution with coefficient one using data from 1990 and 2000. In contrast, Song and Zhang (2002) use city-level data from 1991 and 1998 and argue that although a Pareto distribution fits Chinese cities well, Zipf's law is rejected with the Pareto coefficient being statistically greater than one; Luckstead and Devadoss (2014) obtain similar results using data from 2010. These disagreements might be attributable to the fact that the Pareto coefficient is highly sensitive to the choice of sample size. In particular, Peng (2010) observes that China's Pareto coefficient is monotonically decreasing when lower truncating points are chosen. In addition, Zipf's law results seem to depend on the choice of city definition. While most studies of China use the administrative definition, which corresponds to the official reporting units, Dingel et al. (2019) argue that the night-lights-based metropolitan areas they construct conform well to Zipf's law using census data from 2000 and 2010. Anderson and Ge (2005) and Li et al. (2016), on the other hand, favored a lognormal distribution over the Pareto distribution for Chinese cities. All relevant studies on Chinese city size distribution use data prior to 2010 for testing Zipf's law and Pareto distribution, and our paper extends the time period of study with new data from 2011 to 2017.

A few studies examine the underlying assumption of Zipf's law - whether Chinese city growth is size-independent (Gibrat's law) - in an attempt to explain China's deviation from Zipf's law; however, they do not provide insights on what factors may have caused Chinese city growth to be size-dependent. Gabaix (1999) identifies the homogeneity of city growths as an important assumption for explaining why Zipf's law holds and recognizes that any violation of it may lead to deviations from Zipf's law. Evidence is again mixed for whether China's city growth follows Gibrat's law. Cen (2015) and Li et al. (2016) find that Gibrat's law approximately holds for all Chinese cities from the 1980s to 2000s. On the contrary, Fang et al. (2017) and Chauvin et al. (2017) observe that Chinese city growth is size-convergent before 2000 and size-independent after 2000. Gangopadhyay and Basu (2012) argue that Chinese cities experience parallel growth where small and medium-size cities have grown faster than large cities. Likewise, cities with similar policy regimes and natural resource endowments grow parallel in the long run (Chen et al., 2013; Wu and He, 2017).

To explain China's deviation from Zipf's law and Gibrat's law, the existing literature identifies the role of China's unique administrative and economic policies, but rarely supports these hypotheses with empirical evidence. Song and Zhang (2002) and Xu and Zhu (2009) propose several economic and institutional factors of the Chinese urban system, including rural-urban migration restrictions imposed by the household registration system ("hukou"), China's open-door policy and its subsequent boosts in foreign direct investments (FDI), and government's development strategies that favor small and medium-size cities. In addition to the hukou system and the economic reforms, some scholars emphasize China's family planning program, based on the one-child policy, in explaining China's deviation from Zipf's law (Anderson and Ge, 2005; Luckstead and Devadoss, 2014). Yet, past literature discusses these policy factors qualitatively without empirical evidence, and our paper fills this gap by collecting urban characteristics data at the city level and designing research strategies to test the effect of each factor.

2.2 Overview of China's Urban Development and Policies

As a socialist economy, the People's Republic of China has a complex urban system and a unique path of development since its founding in 1949. China experienced rapid population growth from 1949 to 1958, during which the average growth rate of the non-agriculture population was over 10 percent, with the majority of growth happened in large cities (NSB, 2000). The number of cities also doubled within nine years. Following the initial growth was a period of stagnation from 1958 to 1978 due to the Great Leap Forward and the Cultural Revolution. During these two decades of political turmoil, the rural population was forced to stay in their birthplace under the collectivization of agriculture. Urban expansion thus slowed down with little growth in urban population size and the number of cities. The year 1978 marked a turning point of urban development when the central government introduced a series of economic reforms that attempted to liberalize the economy. Policies involved the de-collectivization of agriculture, the opening up to foreign investments, and the permission for entrepreneurship. As a result, China experienced rapid urban growth in the past four decades following 1978. The non-agricultural population in urban areas increased 126 percent from 172 million in 1978 to 389 million in 1999; the number of officially designated cities increased from 191 to 667, and the urban share of total population increased from 18 to 31 percent (NSB, 2000). In the last two decades, China continued its economic reforms albeit with slower urban population growth. Large-scale privatization of previously state-owned enterprises occurred in the late 1990s and early 2000s, leading to an increase in total factor productivity and gross regional output levels. Meanwhile, the Chinese government further brought down trade barriers and reduced tariffs. Such efforts peaked as China joined the World Trade Organization (WTO) in 2001. As a result, China enjoyed a significant increase in foreign direct investment inflows, especially to large cities, in the following years.

Despite the economic growth that took place in the cities, population growth dropped with the emergence of many urban issues including crowding, congestion, pollution, and crime.

The most distinct feature of China's urban development is its direct government interventions. Chinese governments, from central to local levels, are far more active in planning and containing city populations than any other country in the world. The two main policy tools they have been using are the household registration (or "hukou") system and the one-child policy. In 1951, the Ministry of Public Security issued the first regulation regarding migration and formally initiated the household registration system. By 1958, all rural and urban citizens had been registered with the state, and rigorous control over any transfer of the hukou status had been put into place. Since the reform and opening up in 1978, the state has loosened its restrictions on migration from rural areas to small cities but imposed greater limits on migration into big cities like Beijing and Shanghai. The hukou status specifies the location of residence and is, essentially, an official permit that allows a person to stay in the designated location. Hukou also divides people into agricultural and non-agricultural categories, where a non-agriculture hukou status grants superior welfare benefits. In addition to issuing permits, the government implements complementary policies that discriminate against unofficial migrants (those without permits) in areas of job allocation, housing, education, healthcare, and social security in the city (Song, 2014). Thus, although unofficial migrants can physically stay in the city, it is much harder for them to live their own lives.

The one-child policy is a birth planning program of one child per family, first introduced in 1982, to control the rapid growth of the Chinese population. Intended to be applied universally, the one-child policy was, however, not uniformly followed across the country. It is commonly believed that urban cities oversaw stricter enforcement with penalties on families with "unauthorized" children, whereas rural families managed to find loopholes and conceive a second child if their first one was a girl. In urban cities, 91 percent of the mothers had only one child; in sharp contrast, only 59 percent of rural mothers followed the one-child policy (Li, 1995). Starting from 2011, the one-child policy was somewhat relaxed across the country as China issued a new law allowing parents who are both the only child to have a second child; it was formally replaced by the current universal two-child policy in 2016. However, birth rates in urban areas did not rebound significantly due to the long-lasting impact the one-child policy had created, including the imbalanced sex-ratio and the low fertility rate.

3 Conceptual Framework

3.1 Zipf's Law Model

The history of Zipf's law dates back to Auerbach (1913) and Singer (1936), who first apply the Pareto law to city size distribution. A mathematical statement of the Pareto law for city size distribution is as follows

$$r_i = A \cdot S_i^{-\alpha} \tag{1}$$

where r_i is the number of cities with population S_i or more, or equivalently, the rank of city *i* when cities are ranked from 1 to n by their population size in descending order; S_i is the population of the city i, α is the Pareto coefficient, and A is a constant. Equation 1 implies a linear relationship between the logarithm of city rank and the logarithm of city size

$$log(r_i) = log(A) - \alpha \cdot log(S_i) \tag{2}$$

Zipf's law for cities (also referred to as "the rank-size rule") is a special case of the Pareto law with the Pareto coefficient $\alpha = 1$ (Zipf, 1949). It states that the rank of a city is inversely proportional to its size.

$$r_i = \frac{A}{S_i} \tag{3}$$

This implies that within a geographical region of cities, the largest city is about twice the size of the second-largest city, about three times the size of the third-largest city, and so on. The Zipf-form of Equation 1 is as follows

$$S_{(k)} = S_{(1)} \cdot k^{-q} \tag{4}$$

where q is equal to 1 under the special case, and $S_{(k)}$ is the size of the *k*th largest city. A simple mathematical derivation can show that $q = \frac{1}{\alpha}$ and $S_{(1)} = A^{\frac{1}{\alpha}}$. Zipf's law holds when $\alpha = 1$ and $q = 1$. If $\alpha \to \infty$, then $q \to 0$, and $S_{(k)} = S_{(1)}$, $\forall k$, suggesting a perfectly even distribution of population as all cities have the same size.

Gabaix (1999) provides a theoretical framework for studying why some countries converge to Zipf's law whereas others fail to. He shows that if different cities in a region have homogenous random growth processes, then their limit distribution will converge to Zipf's law. Let $S_{i,t}$ denote the size of city *i* normalized by the total urban population at time *t*. City size follows Zipf's law if the upper tail distribution of city sizes at time *t*, $G_t(S) \coloneqq P(S_t > S)$, converges to some steady-state distribution function $G(S) = a \cdot S^{-\zeta}$, where a is a constant and $\zeta = 1$. Gabaix proves that this statement is true if we assume all cities grow randomly with the same expected growth rate and the same variance (Gibrat's law). In other words, city growth rates are identically distributed and independent of city sizes.

Gabaix further examines the case where cities grow randomly with expected growth rates and variances dependent upon city sizes. The size of city *i* at time *t* follows the process (according to Equation 11 ibid., p. 756)

$$\frac{dS_t}{S_t} = \mu(S)dt + \sigma(S)dB_t \tag{5}$$

where $\mu(S)$ is the expected growth rate of the normalized city size S, $\sigma(S)$ is its standard deviation, and B_t is a reflected geometric Brownian motion. It follows that the limit distribution of city sizes will converge to Pareto's law with the following local Zipf coefficient (also, Pareto coefficient)

$$\zeta(S) = -\frac{S}{p(S)} \cdot \frac{dp(S)}{dS} \tag{6}$$

where $p(S)$ is the probability distribution of *S*. Then integrating the forward Kolmogorov equation (Equation 12 ibid., p. 757) into Equation 6, Gabaix derives the general form of Zipf coefficient, $\zeta(S)$, as a function of the mean and variance of city growth rates (according to Equation 13 ibid., p. 757)

$$\zeta(S) = 1 - 2 \cdot \frac{\mu(S)}{\sigma^2(S)} + \frac{S}{\sigma^2(S)} \cdot \frac{\partial \sigma^2(S)}{\partial S} \quad (7)$$

This general expression for the Zipf coefficient lays the foundations of our empirical approach to explain China's deviation from Zipf's law. As derived in Equation 7, deviations from Zipf's law ($\zeta(S) \neq 1$) can be explained either by deviations of the expected growth rates for a range of cities from the overall mean for all city sizes ($\mu(S) = \gamma(S) - \bar{\gamma} \neq 0$) or by the dependency of the variance of growth rates on city sizes ($\frac{\partial \sigma^2(S)}{\partial S} \neq 0$). When large cities exhibit lower growth rates ($\gamma(S) - \bar{\gamma} < 0$), their size distribution will decay faster than Zipf's law would predict and their local Zipf coefficient will be greater than one ($\zeta(S) > 1$). If Gibrat's law holds precisely, then the second and the third term in Equation 7 equal to zero, and the Zipf coefficient $\zeta(S) = 1$ regardless of city size *S*.

3.2 Gibrat's Law Model

Gibrat's law, also known as the law of proportional growth, is first proposed by Gibrat (1931) as an empirical regularity governing the dynamics of firm sizes and later applied to the field of urban economics to capture city growth processes and explain the resulting population distribution. Proportional growth states that the expected increment to a city's size in each period is proportional to its initial size. Let $S_{i,t}$ be the population size of city i at time t and δ_t be the proportional growth rate between period $t-1$ and t. The mathematical expression of proportional growth is $S_{i,t} - S_{i,t-1} = \delta_t \cdot S_{i,t-1}$, or

$$S_{i,t} = S_{i,t-1} \cdot (1 + \delta_t) \qquad (8)$$

$where\ \delta_t$ is an i.i.d. random variable with mean g and variance σ^2. Taking the logarithm of both sides in Equation 8 and moving terms around, we can obtain an equivalent formulation of Gibrat's law

$$log(S_{i,t}) - log(S_{i,t-1}) = log(1 + g) + u_{i,t} \qquad (9)$$

where $u_{i,t}$ represents the random shocks that the growth rate may suffer. Note that $E(u_{i,t}) = 0$ and $Var(u_{i,t}) = \sigma^2, \forall i, t.$

To capture deviations from Gibrat's law, we add the term $\beta \cdot log(S_{i,t-1})$ to model the possibility of population growth rate as a function of initial city size. Thus, the general expression of the growth equation is as follows

$$log(S_{i,t}) - log(S_{i,t-1}) = log(1 + g) + \beta \cdot log(S_{i,t-1}) + u_{i,t} \qquad (10)$$

In the case of a size-dependent growth path, β will be nontrivial.

3.3 Spatial Equilibrium Model

An abundance of static models attempts to characterize the spatial equilibrium across cities. Among them, Rosen (1979) and Roback (1982) provide a baseline for urban growth analysis. Based on the Rosen-Roback framework, Glaeser and Gottlieb (2009) construct a detailed three-sector general equilibrium model that is widely adopted for studying determinants of wages, housing prices, and population density. We follow their model to solve the three distinct equilibrium conditions for consumers, producers, and constructors and derive how productivity and amenities can lead to a larger population size.

We start with the representative consumer's problem. Consumers receive utility from three main parts: consumption of traded goods, denoted C, consumption of non-traded housing, denoted H, and location-specific amenities, denoted θ. Consumers supply one unit of labor inelastically, receive wage w, and spend all of their income on either consumer goods or housing. Let p_H be the per-unit cost of housing and the price of consumer goods be normalized to 1. It follows that consumers' budget constraint is $\mathrm{C} + p_H \cdot H = \mathrm{w}$. Further, assume consumers have Cobb-Douglas utility functions $\mathrm{U(C, H)} = \theta \cdot C^{1-\alpha} \cdot H^{\alpha}$, where α represents the share of labor income workers spend on housing. Thus, consumers' utility maximization problem is as follows

$$Max_{C,H}\ U(C,H) = Max_{C,H}\ \ \theta \cdot C^{1-\alpha} \cdot H^{\alpha} = Max_{H}\ \ \theta \cdot (w - p_H \cdot H)^{1-\alpha} \cdot H^{\alpha} \qquad (11)$$

The first order condition (FOC) with respect to H gives:

$$\frac{\partial U}{\partial H}:\ \ -p_H \cdot (1-\alpha) \cdot H + \alpha \cdot (w - p_H \cdot H) = 0 \qquad (12)$$

Intuitively, Equation 12 shows that the marginal utility from consumer goods must equal to the marginal utility from housing under optimized consumption behaviors. Plugging Equation 12 back into Equation 11 to get rid of H yields the indirect utility function V, as presented in Equation 13.

$$V = \alpha^{\alpha} \cdot (1-\alpha)^{(1-\alpha)} \cdot \theta \cdot w \cdot p_H^{-\alpha} \quad (13)$$

In the cross-city context, the standard assumption of free migration creates a spatial equilibrium where consumers' utility levels are equalized across all cities. Otherwise, if consumers are not indifferent between living in one city and living elsewhere, they would simply move to the location that provides greater utility. Thus, we require the indirect utility in Equation 13 equal to a reservation utility, denoted $\bar{V}$.

As for the production sector, firms take capital and labor as inputs and produce tradable goods. In the style of Mills (1967), there are two types of capital involved in production: tradable capital, denoted K, and non-tradable capital, denoted Z. Tradable capital can be purchased anywhere at a normalized price of 1, whereas the non-tradable capital comes from a fixed supply $\bar{Z}$ based on location. The cost of per unit labor is wage w, and let N denote the total number of workers. Firms operate under a city-level Cobb-Douglas production function $\mathrm{F(N, K, \bar{Z}) = A \cdot N^{\beta} \cdot K^{\gamma} \cdot \bar{Z}^{1-\beta-\gamma}}$, where A is the city-specific total-factor productivity. In equilibrium, firms choose tradable capital K and labor N to maximize their total profits Π.

$$Max_{N,K}\ F(N, K, \bar{Z}) - w \cdot N - K = Max_{N,K}\ \ A \cdot N^{\beta} \cdot K^{\gamma} \cdot \bar{Z}^{1-\beta-\gamma} - w \cdot N - K \quad (14)$$

The two FOCs are as follows

$$\frac{\partial \Pi}{\partial N} = \beta \cdot A \cdot N^{\beta-1} \cdot K^{\gamma} \cdot \bar{Z}^{1-\beta-\gamma} - w = 0 \quad (15)$$

$$\frac{\partial \Pi}{\partial K} = \gamma \cdot A \cdot N^{\beta} \cdot K^{\gamma-1} \cdot \bar{Z}^{1-\beta-\gamma} - 1 = 0 \quad (16)$$

Equations 15 and 16 give the standard conditions that the marginal productivity of labor or capital is equal to their marginal cost. We derive the inverse labor demand curve by substituting Equation 16 into Equation 15 to get rid of K.

$$w = \beta \cdot A^{\frac{1}{1-\gamma}} \cdot \gamma^{\frac{\gamma}{1-\gamma}} \cdot N^{\frac{\beta+\gamma-1}{1-\gamma}} \cdot \bar{Z}^{\frac{1-\beta-\gamma}{1-\gamma}} \tag{17}$$

Finally, for the construction sector, firms choose height, denoted h, and land, denoted L, which supply a total housing of $H = h \cdot L$, to maximize profits. Let p_H be the price for housing and p_L be the cost of land. The cost of producing $h \cdot L$ units of housing on top of L units of land is assumed to be $c_0 \cdot h^{\delta} \cdot L$ for some $\delta > 1$. Thus, the profit maximization problem of constructing firms is summarized in Equation 18.

$$Max_{h,L} \; p_H \cdot h \cdot L - c_0 \cdot h^{\delta} \cdot L - p_L \cdot L \tag{18}$$

The two FOCs are as follows

$$\frac{\partial \Pi}{\partial h} = p_H \cdot L - \delta \cdot c_0 \cdot h^{\delta-1} \cdot L = 0 \tag{19}$$

$$\frac{\partial \Pi}{\partial L} = p_H \cdot h - c_0 \cdot h^{\delta} - p_L = 0 \tag{20}$$

Assume there is a fixed quantity of land, denoted $\bar{L}$, available at each location. Since the housing market must clear in equilibrium, then the total supply equals total demand $\mathrm{h} \cdot \bar{L} = \frac{\alpha \cdot \mathrm{N} \cdot w}{p_H}$. (We can show from Equation 12 the units of housing demanded by each consumer is $\frac{\alpha \cdot w}{p_H}$, and there are N consumers in total.) Thus, substituting $\mathrm{h} = \frac{\alpha \cdot \mathrm{N} \cdot w}{p_H \cdot \bar{L}}$ into Equation 19, we obtain the housing price equation as a function of population N and income w.

$$p_H = \delta^{\frac{1}{\delta}} \cdot c_0^{\frac{1}{\delta}} \cdot \left(\frac{\alpha \cdot N \cdot w}{\bar{L}}\right)^{\frac{\delta-1}{\delta}} \tag{21}$$

Together, expressions of the indirect utility (Equation 13), the inverse labor demand (Equation 17), and the housing price (Equation 21) characterize the spatial equilibrium. The three endogenous variables are population N, wage w, and housing price p_H. Thus, we solve the system of equations and obtain the following expression for population size

$$log(N) = K_N + \frac{(\delta+\alpha-\alpha\delta)log(A)+(1-\gamma)\delta log(\theta)}{\delta(1-\beta-\gamma)+\alpha\beta(\delta-1)} \quad (22)$$

where K_N is a constant term that includes parameters other than A and θ. We can then take comparative statics of Equation 22 with respect to the logarithm of productivity A and amenities θ.

$$\frac{\partial log(N)}{\partial log(A)} = \frac{(1-\alpha)\delta+\alpha}{\delta(1-\beta-\gamma)+\alpha\beta(\delta-1)} > 0 \quad (23)$$

$$\frac{\partial log(N)}{\partial log(\theta)} = \frac{(1-\gamma)\delta}{\delta(1-\beta-\gamma)+\alpha\beta(\delta-1)} > 0 \quad (24)$$

As we can see, population size rises in productivity and amenities. Therefore, the spatial equilibrium framework provides us with two hypotheses - the economic productivity and the urban amenities - in explaining population growth.

4 Data

4.1 Urban Population Data

Urban studies require at a minimum an appropriate definition of "city" and a consistent measure of the urban population, both of which are somewhat elusive in China's case. In China, "cities" are urban areas defined according to administrative divisions. There are three different administrative levels of cities in the Chinese urban system: province-level cities (or municipalities), prefecture-level cities, and county-level cities. As of January 2019, there are 4 province-level cities, namely Beijing, Chongqing, Shanghai, Tianjin, 293 prefecture-level cities, including Chengdu, Guangzhou, Baoding, Wuhan, and 375 county-level cities. We recognize this administrative definition of cities does not exactly correspond to the popular commuting-based definition, the Metropolitan Statistical Areas (MSAs), which merges administratively defined entities based on their social and economic ties. Yet, the ideal economic units for urban studies are still widely debated. Holmes and Lee (2010) point out that Zipf's law results and other empirical studies in urban economics are sensitive to the definition of city boundaries. They find that while the MSAs in the U.S. follow Zipf's law quite well, the six-by-six-mile squares they propose do not. Another alternative is the lights-based city definition. Dingel et al. (2019) construct night-lights-based metropolitan areas for China and India and find that they conform well to Zipf's law despite the deviation of administrative cities. Unfortunately, commuting data or night lights data are not readily available to us, so we stick with the administrative units, which are also widely adopted in urban studies of China, for example, Anderson and Ge (2005) and Chauvin et al. (2017). We realize the potential limitation of our study caused by our choice of city boundary definition. Nevertheless, a study using the administrative units has its advantaging in examining how government regulations may have affected urban growth and city size distribution, given many population control policies are implemented based on administrative units.

To examine China's city size distribution, we collect population data from the National Bureau of Statistics (NSB). Information on city populations from 1949 to 1999 is reported in *Fifty Years of Urban Development* (NSB, 2000). We use this source for population data from 1949 to 1978. Information on city populations from 1984 to 2017 is compiled from *Chinese Urban Statistical Yearbooks* (NSB, 1984-2017). In both sources, cities at all administrative levels are reported. For cities at prefecture level and above, populations of both urban areas ("Shiqu") and urban areas plus rural counties ("Diqu") are reported, where rural counties refer to the suburban and rural areas surrounding the urban areas. County-level cities and rural counties are very small in size and relatively underdeveloped, and their definitions vary from province to province. As such, we disregard these counties and focus on cities at prefecture level and above. Moreover, two main statistics are published officially as measures of the urban population in each city: the total city and town population and the non-agricultural population. Neither accurately reflect the actual urban population based on the residence principle according to international practice. Total city and town population counts all people within the administrative region of a city according to the household registration system; the non-agricultural population is a subset of the total city and town population consisting of those with non-agricultural hukou status. As discussed in Section 2.2, hukou is a part of Chinese government's planned economic system; it does not include migrant workers who do not have an official permit to stay in the city. Since our study mainly focuses on cities at prefecture level and above, where most agricultural population live in urban and surrounding suburban areas rather than actual rural areas, we choose the total city and town population as an estimate of the actual urban population. Yet, this measure may still underestimate city population due to unofficial migrations.

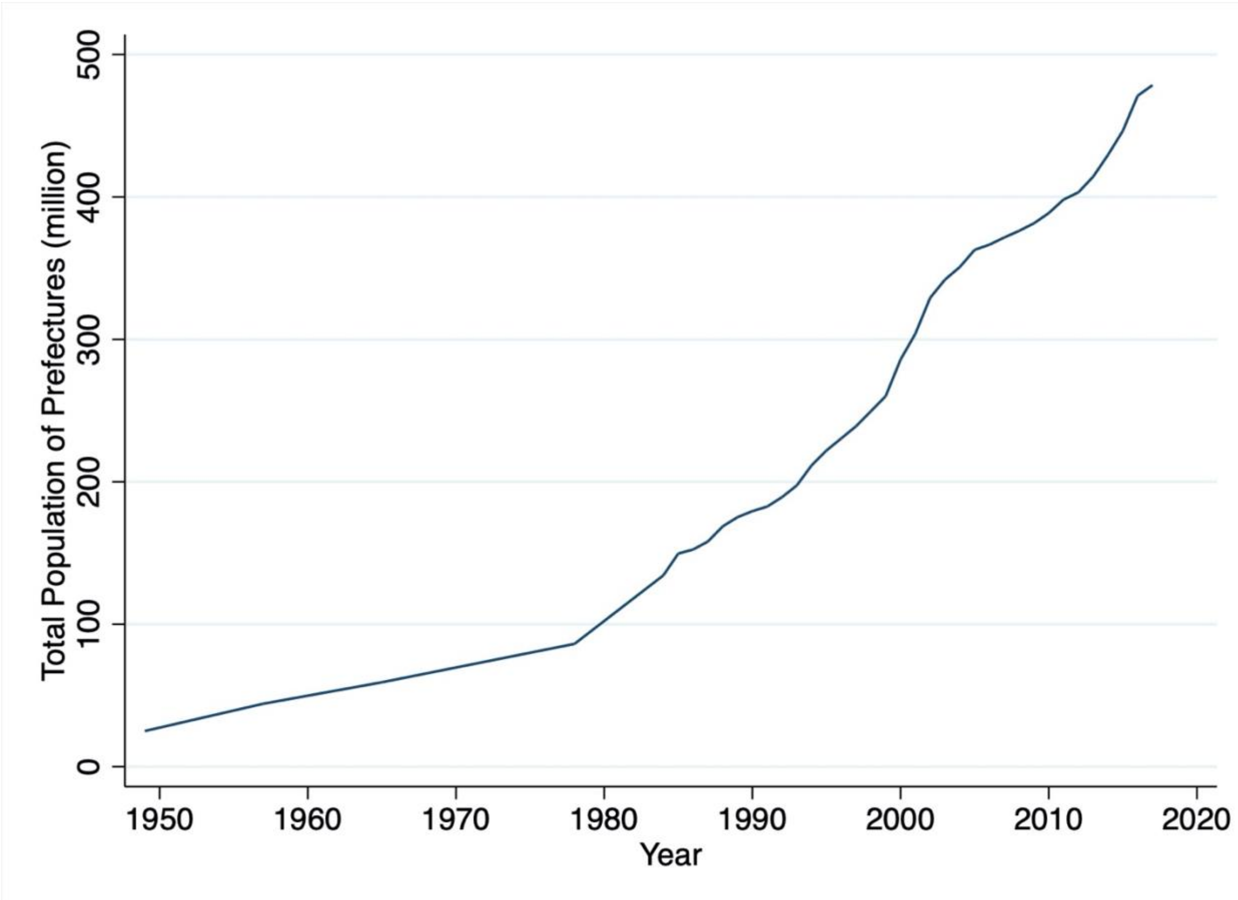
We choose 1949, the founding year of the People's Republic of China, as a natural starting point of our Zipf's law analysis. Yet, China did not systematically collect and report its urban population data until the economic reforms and the rapid urbanization that began in the early 1980s. In the three decades before 1984, China reported its urban population data in only four years, namely, 1949, 1957, 1965, and 1978; the number of cities designated and reported is also much lower compared to the post-reform period. As a result, our time series analysis of city growth and size distribution for the pre-reform period may be incomprehensive and biased due to these constraints. In addition, the administrative boundary of cities changes over time due to central and local governments' strategic planning, causing some populations to jump discontinuously from one year to another. Table 1 presents the summary statistics of city population data. Table 2 summarizes the average annual growth rates of city populations from 1991 to 2017 by rank groups. We calculate the average annual growth rate by taking the geometric mean of annual population growth rates and group cities based on their ranks in the initial year 1991. Finally, we graph the total population of all cities at prefecture level and above in Figure 2 to visualize China's overall urban growth.

Table 1. Summary Statistics for Provincial and Prefecture City Population, 1949-2017.

Year	Num. Obs.	Avg. Size (10,000s)	Std. Dev. (10,000s)	Min. Size (10,000s)	Max. Size (10,000s)
1949	51	49.056	69.558	5.46	418.94
1957	55	80.375	100.427	8.94	609.83
1965	58	102.168	121.335	10.07	643.07
1978	87	99.115	102.233	6.81	557.38
1984	147	91.417	99.279	8.76	688.13
1985	165	90.688	96.024	10.21	698.3
1986	169	90.212	97.425	10.27	710.16
1987	173	91.37	101.378	9.8	721.77
1988	184	91.692	100.154	9.86	732.65
1989	187	93.693	102.043	9.98	777.79
1990	188	95.42	102.841	10.19	783.48
1991	189	96.616	103.325	10.53	786.18
1992	193	98.031	103.019	10.93	792.75
1993	198	99.736	109.354	11.4	948.01
1994	207	102.21	108.339	11.97	953.04
1995	212	104.725	112.62	12.44	956.66
1996	221	104.289	112.139	12.91	961.02
1997	223	107.197	119.694	13.09	1018.59
1998	229	108.99	128.122	14.29	1070.62
1999	236	110.248	134.486	14.55	1127.22
2000	262	109.096	131.542	15.96	1136.82
2001	266	114.289	138.797	16.1	1262.41
2002	278	118.431	144.655	14.29	1270.22
2003	284	120.412	146.669	14.08	1278.23
2004	286	122.657	148.202	14.35	1289.13
2005	286	126.871	153.576	14.62	1290.14
2006	286	128.157	161.072	14.93	1510.99
2007	286	129.915	162.96	15.3	1526.02
2008	287	131.078	164.219	15.33	1534.5
2009	287	132.924	165.841	15.33	1542.77
2010	286	135.895	168.558	15.23	1542.77
2011	287	138.732	176.871	15.3	1770.6
2012	287	140.488	179.124	15.1	1779.1
2013	289	143.343	180.155	15.2	1787
2014	289	148.631	187.576	15.3	1943.9
2015	289	154.459	201.712	15.36	2129.09
2016	295	159.685	216.628	4	2449
2017	292	163.856	223.197	5	2451

Source: NSB (2000, 1984-2017).

Figure 2. Growth in Total Urban Population of Prefecture-Level Cities or Above, 1949-2017.



Source: Author's calculation using population data from NSB (2000, 1984-2017).

Table 2. Summary Statistics for Average Annual Population Growth Rates, 1991–2017.

Rank	Avg. (%)	Std. Dev. (%)	Min. (%)	Max. (%)
1-50	1.607	2.573	-7.497	8.4
51-100	1.667	1.845	-2.193	7.394
101-150	2.106	2.044	-2.541	6.815
151-200	3.093	2.258	.258	9.725
201-250	4.505	2.665	.627	9.788

Note: Average annual population growth rate is the geometric mean of annual growth rates, calculated using population data from NSB (1991-2017). Groups are based on ranks of city sizes in the initial year 1991.

Census data provides a better measure of the residence-based population but is only available in the census years 2000 and 2010. As highlighted before, annual population data are compiled from the household registration system and neglect the unofficial migrants who also live in the city. This problem can be solved if we use the census data from the household survey, which is conducted every ten years since 2000. So far, we only have two years of census data, which limit the scope of our study, but still provide us a more accurate measure of city size. Furthermore, by examining the numerical difference between the census data and the hukou data, we can obtain a measure of the enforcement level of the hukou system across different cities, which enables us to study the effect of administrative control on city growth. Table 3 below presents the summary statistics of the census population data. To get a better sense of Chinese cities and their sizes, we summarize the largest 10 cities along with their population according to the 2000 and 2010 census in Table 4.

Table 3. Summary Statistics for Census Population Data, 2000 & 2010.

Census Year	Num. Obs.	Avg. size (10,000s)	Std. Dev. (10,000s)	Min. Size (10,000s)	Max. Size (10,000s)
2000	337	135.051	153.154	1.287	1448.992
2010	337	197.392	224.358	2.191	2055.51

Source: Tabulation on the Population Census of China (NSB, 2000 & 2010).

Table 4. Population of the Largest Ten Cities, 2000 & 2010.

Rank	City Name	Population Size	Rank	City Name	Population Size
1	Shanghai	14,489,919	1	Shanghai	20,555,098
2	Beijing	10,522,464	2	Beijing	16,858,692
3	Chongqing	10,095,512	3	Chongqing	15,295,803
4	Guangzhou	8,090,976	4	Guangzhou	10,641,408
5	Tianjin	7,089,812	5	Shenzhen	10,358,381
6	Wuhan	6,787,482	6	Tianjin	10,277,893
7	Shenzhen	6,480,340	7	Chengdu	9,237,015
8	Chengdu	5,967,819	8	Wuhan	7,541,527
9	Harbin	5,370,174	9	Suzhou	7,329,514
10	Shenyang	5,066,072	10	Dongguan	7,271,322

Note: Residence-based urban population is reported in absolute values (2000 on the left, 2010 on the right).
Source: Tabulation on the Population Census of China (NSB, 2000 & 2010).

4.2 Urban Characteristics Data

To study the underlying factors for China's urban growth, we compile the following variables at the city level, as shown in Table 5, Table 6, and Table 7, from *Chinese Urban Statistical Yearbooks* (NSB, 1991-2017), *Tabulation on the Population Census of China* (NSB, 2000 & 2010), and *China Housing Price Data* (CREA, 2017).

Table 5. Summary Statistics for Urban Characteristics Data in 1991.

Variable	Unit	Obs	Mean	Std.Dev.	Min	Max
Population density	Per square km	252	1162.345	1185.1	3	10482
Employment rate	Percent	250	57.289	9.939	31.603	157.163
Gross regional product	CNY per capita	249	3486.955	2964.228	810.661	31358.48
Gross industrial output value	CNY per capita	250	6323.5	5649.1	414.671	54930.39
Amount of foreign capital utilized	USD per capita	184	36.555	113.799	.042	1186.114
Local government budget expenditure	CNY per capita	249	466.921	528.47	26.978	5914.534
Residential savings per capita	CNY per capita	250	1816.29	1340.648	152.333	14782.74
Average wage	CNY	250	2377.17	498.404	1416.2	5199.7
Number of hospital beds	Per 10,000 persons	250	52.979	22.442	6.306	113.766
Area of paved roads	Square m per capita	250	3.061	2.171	.1	14.7
Rate of natural increase	Percent	250	.8	.351	-.007	2.063
Wastewater	1,000 tons per square m	247	10.61	18.652	.018	176.909
Waste gas	1,000,000 mark per square m	245	.404	.697	.001	6.164
Dust	Tons per square m	241	21.463	91.376	.004	1322.711
Solid waste	1,000 tons per square m	245	.191	.385	0	3.237

Note: We calculate the employment rate by dividing total person employed by total population and record in percentage terms. All data are compiled from *Chinese Urban Statistical Yearbooks* (NSB, 1991).

Table 6. Summary Statistics for Urban Characteristics Data in 2017.

Variable	Unit	Obs	Mean	Std.Dev.	Min	Max
Population density	Per square km	253	846.687	721.513	5.024	5654.008
Employment rate	Percent	249	40.646	27.052	3.508	216.355
Gross regional product	CNY per capita	252	84086.03	56894.96	7998.31	517000
Gross industrial output value	CNY per capita	250	119000	103000	855.167	709000
Amount of foreign capital utilized	USD per capita	209	297.089	406.262	.107	2952.647
Local government budget expenditure	CNY per capita	253	13401.82	9119.554	925.278	106000
Residential savings per capita	CNY per capita	247	66493.96	34782.64	17197.1	249000
Average wage	CNY	245	67343.7	12702.69	40180	135000
Number of hospital beds	Per 10,000 persons	247	76.685	29.376	7.393	192.328
Area of paved roads	Square m per capita	234	14.174	7.002	2.354	51.454
Rate of natural increase	Percent	253	.264	.69	-1.677	2.933
Wastewater	1,000 tons per square m	214	11.985	27.881	.002	248.046
Waste gas	Tons per square m	216	36.641	59.065	.041	474.108
Dust	Tons per square m	215	36.098	57.115	.091	378.438
Public green space	Square m per capita	250	14.073	4.365	2.45	51.66
Average house price	CNY per square m	253	8148.032	7455.582	2214	62252

Note: We calculate the employment rate by dividing total person employed by total population and record in percentage terms. House price data are from CREA (2017) and the rest from *Chinese Urban Statistical Yearbooks* (NSB, 2017).

Table 7. Summary Statistics for Urban Characteristics Data in 2000 and 2010.

Census Year	Variable	Num. Obs.	Avg. size (10,000s)	Std. Dev. (10,000s)	Min. Size (10,000s)	Max. Size (10,000s)
2000	Unofficial migrant population	238	51.993	84.212	-489.852	523.114
	Population with BA degrees or higher	333	0.449	1.06	0.003	12.549
2010	Unofficial migrant population	245	92.353	111.873	-147.775	775.968
	Population with BA degrees or higher	334	1.451	3.198	0.031	37.998

Note: We calculate the number unofficial migrants by subtracting the hukou population reported in *Chinese Urban Statistical Yearbooks* from the residence-based population reported in *Tabulation on the Population Census of China* (NSB, 2000 & 2010).

We group these variables into three categories, namely, economic variables, measures of amenities and disamenities, and proxies for policy effects.

Similar to Glaeser et al. (1995), we look at urban characteristics such as the employment rate, gross regional product, amount of foreign capital utilized, residential savings, and average wage for measures of a city's overall economic performance. It is worth noting that the amount of foreign capital utilized is a plausible measure of the extent to which a city has "opened up" after the economic reforms. In addition, we use the share of population with a Bachelor of Arts (BA) degree or higher from the census data as a measure of labor skills and human capital within the city.

As for amenities, we consider the number of hospital beds as a measure of healthcare quality, the area of paved roads as a measure of infrastructure quality, and the area of public green space as a measure of leisure facilities. We use all variables in per capita terms, as opposed to in total terms, to better capture the individual utility gain from these urban amenities. Respecting disamenities, we collect data on the total amount of wastewater, waste gas, dust, and solid waste emitted per year and divide them by the total urban area to approximate the level of pollution within each city in per square meter terms. Ideally, we would want more data on air pollution and traffic congestion for estimating how close a city is to its carrying capacity, but such data are only available for the 30 provincial capitals starting in the year 2006. Population density can potentially be considered as a measure of how saturated a city is since crowding entails higher risks of epidemic, violence, crime, and psychological distress.

Lastly, we have the rate of natural increase and the unofficial migrant population as policy-pertaining variables. We use the rate of natural increase as a proxy for the effectiveness of the one-child policy. As for the migration control policy, we calculate the number unofficial migrants by subtracting the hukou population reported in *Chinese Urban Statistical Yearbooks* (NSB, 2000 & 2010) from the residence-based population reported in *Tabulation on the Population Census of China* (NSB, 2000 & 2010). Preferably, we would also want the quotas of new hukou status (or permits to live in a city) that local governments issue per year as a supplementary measure of how restraining the hukou system is; unfortunately, such information is not publicly available.

5 Methodology

We first conduct repeated cross-sectional ordinary least squares (OLS) regression for testing whether China's city size distribution obeys Zipf's law. As discussed in Section 3.1, the relationship between the logarithm of city rank and the logarithm of city size is linear as shown in Equation 2, where α is the Zipf (or Pareto) coefficient. We could thus run the following OLS regression and obtain a consistent estimate of the Zipf coefficient at time t, $\widehat{\beta_{1,t}}$.

$$log(rank_{i,t}) = \beta_{0,t} + \beta_{1,t} \cdot \ log(size_{i,t}) + u_{i,t} \tag{25}$$

However, Gabaix and Ibragimov (2011) argue that this procedure specified above is strongly biased in small samples. Alternatively, they propose a modified approach by subtracting $1/2$ from the rank, as presented in Equation 26 below.

$$log\left(rank_{i,t} - \frac{1}{2}\right) = \beta_{0,t} + \beta_{1,t} \cdot \ log(size_{i,t}) + u_{i,t} \tag{26}$$

Gabaix and Ibragimov (2011) prove the shift of $1/2$ is optimal for bias reduction and further show that the standard error on the Zipf coefficient α in this modified OLS regression is asymptotically $\left(\frac{2}{n}\right)^{\frac{1}{2}} \cdot \ \alpha$.

We implement the regression in Equation 26 with two different sample sizes to test the robustness of our results, since previous literature suggests that truncating points may affect the value of the Zipf coefficient (Eeckhout, 2004; Peng, 2010). We first include all cities (with observations) at prefecture level and above in the regression and then truncate the sample to the largest 100 cites. By looking at the largest 100 cities, we obtain a local linear relationship between the logarithm of city rank and the logarithm of city size and a local Zipf coefficient for large cities. Large cities are of particular interest to us because they are not as large as what Zipf's law predicts (as shown in Table 4 or Figure 3), and we want to find out what keeps them from growing large enough to grant convergence to Zipf's law.

In addition, we test Zipf's law (Equation 26) within each of the four economic regions of China to check for the robustness of our findings. Theoretically, as long as the basic assumptions of Gabaix (1999) are valid, Zipf's law will hold not only at national levels but also at regional levels or at the world level (Pasciuti, 2014). A regional study is important because if the overall pattern of city size distribution is salient even at regional levels, we can attribute China's deviation from Zipf's law to factors that are size-dependent instead of regional imbalances. We choose the four economic regions over other divisions, like the seven geographical regions or the administrative provinces, such that each region has a sufficient number of cities for possible statistical significance. Also, this choice is prudent because many urban development policies made by the central government target at a particular economic region as a whole, and we expect migrations across economic regions to be relatively low such that the population distribution within each region can be approximated using Gabaix's model.

As shown in Figure 4 (and Figure a. 3), China deviates from Zipf's law with a coefficient significantly greater than one since 1980, regardless of the sample size we choose; we then intend to understand if this result derives from China violating the homogenous growth assumption of Zipf's law. We first test whether the initial city sizes are correlated with cities' subsequent growth based on the general growth equation specified in Equation 10. We run the following OLS regression

$$log(S_{i,2017}) - log(S_{i,1991}) = \gamma_0 + \gamma_1 \cdot log(S_{i,1991}) + u_i \tag{27}$$

where $log(S_{i,2017}) - log(S_{i,1991}) = log(S_{i,2017}/S_{i,1991}) = log(1+g)$ is the logarithm of gross population growth rate of city i from 1991 to 2017 and $S_{i,1991}$ is the initial population size in 1991.[2] Coefficient γ_1 is the predicted effect of initial size on subsequent growth. Gibrat's law implies that this coefficient should be indistinguishable from zero. If China does not obey Gibrat's law, we would expect γ_1 to be significantly different from zero. In particular, $\gamma_1 > 0$ implies divergent growth as city growth depends positively on initial size, whereas $\gamma_1 < 0$ implies convergent growth. Secondly, we test whether the variance of growth rates is independent of the initial size. As suggested by Gabaix in his Footnote 10 (1999, p. 742), we divide cities into groups based on their ranks in the initial year to calculate a variance of growth rates for each group. For the optimal balance between the number of cities within each group and the number of groups, we use decile ranks to split up cities, and each group has about 25 observations. We then implement the following OLS regression to test for any difference in variances of growth rates across decile groups.

$$Var(g_i) = \gamma_0 + \gamma_1 \cdot Decile(S_{i,1991}) + u_i \tag{28}$$

Note that the choice of 1991 as the starting point of our analysis is largely due to limitations in data availability, as most Chinese cities started collecting urban characteristics data since that year. Yet, we believe this choice also has its advantage despite a limited scope of study: China has enjoyed relatively stable economic growths and carried out consistent policies since 1991; our Zipf coefficient plots (Figure 4) also show that China has consistently experienced a more even city size distribution than Zipf's law prediction. Thus, a study of China's urban growth from 1991 onwards excludes the political turmoil in the 60s and 70s and the radical economic reforms in the 80s and focuses instead on steady growth.

2 We also run the population growth and initial size regression specified in Equation 27 using census data, where 2000 is the starting year and 2010 the end year, and obtain similar mean reversion results.

After observing significant mean reversion for Chinese cities (Figure 6 and Table 8), we analyze the potential reasons as to why large cities have experienced slower population growth. There are three main hypotheses: productivity slowdown, rising disamenities, and government's direct population control policies. We start our analysis by testing the two hypotheses that are universal to urban studies of any country, as derived in Equations 23 and 24. To explain the size-dependent growth process, we first examine how the growth experiences of Chinese cities relate to initial urban measures of economic productivity and local amenities. We then identify whether these initial urban characteristics are correlated with the initial city size.

To understand how economic productivity and amenities affect urban growth in China, we conduct OLS regressions of city growth on initial growth conditions in the manner of Glaeser et al. (1995). Our dependent variable is the logarithm of population growth from 1991 to 2017, and explanatory variables are initial population and urban characteristics that capture the degree of economic productivity, amenities, and disamenities at the city level. We use average wage and per capita gross regional product as measures of productivity. As shown in Equation 15, average wage indicates the marginal productivity of labor; the per capita gross regional product, on the other hand, measures the average labor productivity if we assume labor is a constant share of the total urban population. Note that this assumption may be weakened if the labor force participation rate and age structure differ notably across cities. We propose per capita local government budget expenditure as a proxy for the overall level of urban amenities. Since a significant portion of city-level government spending goes to infrastructure maintenance, public healthcare and education subsidy, and the provision of public open space, all of which are important aspects of urban amenities, we expect cities with more government spending to provide better amenities. For an empirical justification, we run the following OLS regression to examine the connection between government spending and specific measures of urban amenities

$$log(Y_{i,t}) = \vartheta_t + \sum_j \theta_{j,t} \cdot log(Z^j_{i,t}) + u_{i,t} \tag{29}$$

where our dependent variable $Y_{i,t}$ is the per capita local government budget expenditure and our explanatory variables $Z^j_{i,t}$ include the number of hospital beds, paved road area, and public green space area (all in per capita terms). We can further add geographical dummies (one for each economic region) to control for regional differences in government spending. For a persuasive argument of using government spending as a proxy for amenities, we would want $\theta_{j,t}$ to be significantly positive for all variable Z^j consistently over time t. We first implement the regression for the initial year $t = 1991$ and then check for the robustness of our results using data from 2005, which is roughly the midpoint of the period from 1991 to 2017. We recognize that using government spending as a proxy for urban amenities has a significant drawback as it fails to indicate any natural amenities, such as temperature and humidity. Yet, we believe that the majority of migrations in China are not driven by the attraction of natural resources, so we do not incorporate data on landscape or climate in our analysis. Nevertheless, a more comprehensive amenity index is desirable to account for a wide range of variables discussed by Roback (1982), Gyourko and Tracy (1991), and Glaeser et al. (2001). Similarly, we propose population density as a proxy for the disamenities or urban issues, such as overcrowding, pollution, traffic, and crime. We test the validity of this proxy using the same Equation 29 specified before. Our dependent variable $Y_{i,t}$ is the population density and our explanatory variables $Z^j_{i,t}$ include the amount of wastewater, waste gas, dust, and solid waste emitted per square meter. Once again, despite being highly correlated, population density is limited in reflecting the exact severity of urban issues, but it is the second best we can obtain due to a lack of data on many disamenities measures at the city level, such as traffic and crime. All in all, the city growth regression model is set up as follows:

$$log(S_{i,2017}) - log(S_{i,1991}) = \gamma_0 + \gamma_1 \cdot log(S_{i,1991}) + \sum_j \beta_j \cdot log(X^j_{i,1991}) + \sum_r \phi_r \cdot 1\{i \in r\} + u_i \quad (30)$$

where explanatory variables $X^j_{i,1991}$ include measures of economic productivity (average wage or per capita GRP), amenities (per capita local government budget expenditure), and disamenities (population density) and indicator variables $1\{i \in r\}$ are geographical dummies to control for region-specific effects. The coefficients of interest in this long regression are the β_j's that represent the partial effect of each explanatory variable on urban growth.

Next, we test what urban characteristics $Z_{i,t}$ correlate with initial population size $S_{i,t}$ using the following simple linear regression model.

$$Z_{i,t} = \beta_{0,t} + \beta_{1,t} \cdot log(S_{i,t}) + u_{i,t} \quad (31)$$

The coefficient of interest $\beta_{1,t}$ has the form of the covariance of the log of initial size and the initial urban condition divided by the variance of the log of initial size. If β_1 is indistinguishable from zero, then the corresponding urban condition Z is independent of the urban population size. Otherwise, a significantly positive β_1 suggests that Z increases in population size, and vice versa. Apart from the explanatory variables in Equation 30 and measures of urban amenities and disamenities already discussed, we also test the relationship between urban size and other relevant urban characteristics that capture the economic well-being of cities, such as employment rate, industrial output, residential savings, and foreign direct investment. Additionally, since plenty of studies (Moretti, 2003; Bacolod et al., 2009) emphasize the close connection between human capital, total factor productivity, and urban success, we look at the relationship between urban size and the share of population with a Bachelor of Arts (BA) degree or higher, which we regard as an indicator of labor skills. We repeatedly implement these regressions as specified in Equation 31 using data from different years to check for the robustness of these correlations.

One additional exercise on the amenity and urban size connection is to use house prices to form an "amenity index," as proposed by Glaeser et al. (2001, p. 36). House price can be seen as a rough measure of the present value of the rental cost of housing. For each city, we regress the logarithm of average house price on the logarithm of average wage, as specified in Equation 32 below, and regard the residuals of this simple linear regression u_i as reflecting the demand for local amenities. A larger residual suggests that there is a higher level of amenities in the city to compensate for the greater cost of living relative to income under the spatial equilibrium. We then regress the house price residual on the logarithm of city size using Equation 33 (in the same vein as Equation 31). If the coefficient ρ_1 is significantly positive, then amenities rise in city size.

$$log(P_{i,t}) = \varphi + \alpha \cdot log(W_{i,t}) + u_{i,t} \tag{32}$$

$$u_{i,t} = \rho_{0,t} + \rho_{1,t} \cdot log(S_{i,t}) + \varepsilon_{i,t} \tag{33}$$

The advantage of this alternative approach over our previous proposal of using the per capita local government budget expenditure lies in the fact that the house price residual is a more comprehensive measure that reflects both social and natural amenities. Unfortunately, our urban growth analysis using this house price residual is restricted due to a lack of Chinese housing data at the city level prior to 2005. Since we do not have data on housing prices for the initial year 1991, we test the relationship between house price residual and city size in 2017.

If the two main hypotheses that the productivity slowdown and urban disamenities account for large cities' lower population growth hold, we would expect the following empirical results: The coefficient β_j in Equation 30 would be significantly positive for measures of economic productivity (average wage or per capita GRP) and amenities (per capita local government budget expenditure) and significantly negative for disamenities (population density), in order to support the popular view that higher productivity and better amenities promote urban growth. Moreover, the coefficient $\beta_{1,t}$ in equation 31 and $\rho_{1,\mathrm{t}}$ in Equation 33 would be significantly negative for measures of economic well-being and amenities and positive for disamenities. We could then conclude that as cities become larger, productivity slows down, amenities are diluted, and urban issues pile up such that population growth is deterred, and therefore our hypotheses on productivity and amenities may help explain the mean reversion of Chinese cities.

Now that we have tested the first two hypotheses, we turn to the third hypothesis that China's unique population control policies, mainly the one-child policy and the hukou system, are responsible for the slower growth of large cities. These intervention strategies target containing the population size, so the degree of enforcement directly reflects their effects on urban growth. In particular, the one-child policy aims at reducing the birth rate, or the rate of natural increase, and the hukou system intends to lower the net migration rate, where the sum of these two rates equals the total population growth rate. In order for the hypothesis to hold, we need to prove that these two population control policies are not uniformly enforced across the country, and in particular, the intensity of enforcement increase in population size. If that is the case, we could argue that Chinese government's direct population control constrains the growth of large cities much more than that of small and medium-size cities.

To study the enforcement difference of the one-child policy on cities of different sizes, we adopt a differences-in-differences (DID) estimation approach. We consider the birth rate as a proxy of how strict the one-child policy is enforced. Unfortunately, China does not consistently provide such data at the city level, so instead, we use the rate of natural increase (RNI), which equals birth rate minus death rate. If we believe death rates are relatively constant either across time or among cities, which is a plausible assumption, then RNI can substitute for birth rate in our DID analysis. One might also argue that people's fertility decisions depend on many factors other than the policy, like parents' education backgrounds, so birth rate does not exclusively reflect the enforcement of the one-child policy. Yet, we believe most of those potential factors are city fixed effects that remain relatively stable through time, so they will not pose a major threat to our identification strategy. As discussed in Section 2.2, the one-child policy was officially introduced in 1982 and relaxed in 2011 to allow parents who are both the only child in their families to have two children. Unfortunately, we do not have city-level RNI data prior to 1991, so we cannot study the effect of introducing the one-child policy on impact; instead, we focus on the effect of relaxing the universal one-child policy in 2011 by comparing the differences in RNI between large cities and small cities before 2011 to after 2011. If the RNI was lower for large cities prior to 2011 and the relaxation of one-child policy indeed raised the RNI for large cities relative to small cities afterward, then we may infer that large cities experienced stricter enforcement of the universal one-child policy from 1991 to 2011.

To test whether such conjecture is the case, we implement the following DID regressions

$$RNI_{i,t} = \beta_0 + \beta_1 \cdot Post2011 + \beta_2 \cdot Large + \beta_3 \cdot (Post2011 \cdot Large) + \gamma_i + \psi_t + u_{i,t} \tag{34}$$

$$RNI_{i,t} = \beta_0 + \beta_1 \cdot Post2011 + \beta_2 \cdot Large + \beta_3 \cdot (Post2011 \cdot Large) + \beta_4 \cdot X_{i,t} + \gamma_i + \psi_t + u_{i,t} \tag{35}$$

where $RNI_{i,t}$ is the annual rate of natural increase (in percentage points) for city i in year t, $Post2011$ is a dummy variable indicating whether year t is before or after the nationwide relaxation of the one-child policy in 2011, $Large$ is a dummy variable that takes on value one if the city ranked from 1 to 50 and zero if ranked from 151 to 200 by population size in the year 1991, $X_{i,t}$ is a continuous variable measuring the quality of healthcare, γ_i is the city fixed effect, and ψ_t is the time fixed effect. We add the number of hospital beds per person as a control variable for healthcare quality to check for the robustness of our results. We generally expect cities with better healthcare to experience higher natural population growth. The city fixed effect controls for bias that may arise from city-specific characteristics that do not vary across time, such as family values and parents' education levels; time fixed effect controls for bias that vary from year to year, such as nationwide economic shocks that may affect RNI uniformly across all cities. Because observations may be more highly correlated within each city than across cities, we cluster standard errors by city. Our control group is a group of small cities and treatment group the large cities;[3] our treatment is the policy relaxation that happened in 2011. This method assumes homogeneous policy effects among large cities and among small cities, which may not be the case if we believe the policy effect to be a continuous function of initial city size. Also, it requires a parallel trend assumption in the absence of treatment. We test that the pre-trends are indeed parallel, as shown in Figure 11.[4] Also, given that there are no other birth planning policies or natural disaster shocks that would directly affect RNI asymmetrically, and our treatment (large cities) and control (small cities) groups have similar geographical distributions, we argue the parallel trends should apply. β_3 in Equations 34-35 is the coefficient of interest that estimates the difference of policy relaxation effects on the rates of natural increase for large cities versus small cities. β_1, on the other hand, estimates the difference of RNI between large cities and small cities under the universal one-child policy. A significantly positive β_3 and negative β_1 would support our hypothesis that large cities grew slower because they were more constrained by the one-child policy during 1991-2011.

[3] Note that our selection of the two groups, large cities as those ranked from 1 to 50 and small cities as those ranked from 151 to 200, takes into account the need for the two groups to be relatively distinct in city sizes and to have enough observations at the same time. We check the robustness of our results using other grouping choices and obtain similar findings.

[4] We run the following OLS regression: $\mathrm{RNI}_{\mathrm{i,t}} = \beta_0 \cdot Large + \sum_{t=1991}^{2010} \beta_{1,t} \cdot 1\{\mathrm{year} = \mathrm{t}\} + \sum_{t=1991}^{2010} \beta_{2,t} \cdot Large \cdot 1\{\mathrm{year} = \mathrm{t}\} + \gamma_i + \mathrm{u}_{\mathrm{i,t}}$, where $1\{\mathrm{year} = \mathrm{t}\}$ is a dummy variable for each year before the policy relaxation in 2011 and γ_i is the city fixed effect. We observe that the coefficient $\beta_{2,t}$ is indistinguishable from zero for all years 1991-2010.

Finally, to examine the enforcement difference of the hukou system across cities, we use the ratio of unofficial migrants to the hukou population as a proxy of the degree to which the hukou system is imposed and affects migration. Recall from Section 2.2 that the hukou system is an umbrella term referring to not only the permits government issues to register the official residential status but also the supportive policies that discriminate against the unofficial migrants in cities. As such, we expect that the stricter enforcement of the hukou system, the fewer unofficial migrants a city has relative to its total population. We calculate the number unofficial migrants by subtracting the hukou population reported in *Chinese Urban Statistical Yearbooks* (NSB, 2000 & 2010) from the residence-based population reported in *Tabulation on the Population Census of China* (NSB, 2000 & 2010). To see whether the population share of unofficial migrants correlates with city size, we run a similar OLS regression as specified in Equation 31:

$$R_{i,t} = \beta_{0,t} + \beta_{1,t} \cdot log\left(S_{i,t}\right) + u_{i,t} \tag{36}$$

where our dependent variable $R_{i,t}$ is the ratio of unofficial migrants to the hukou population and our independent variable is the logarithm of the hukou population. If $\beta_{1,t}$ is significantly negative, we may conclude that the hukou system contributes to the mean reversion of city sizes. Since China only reports the residence-based population in census years 2000 and 2010, we repeat this regression with data from these two years to check for the robustness of our results.

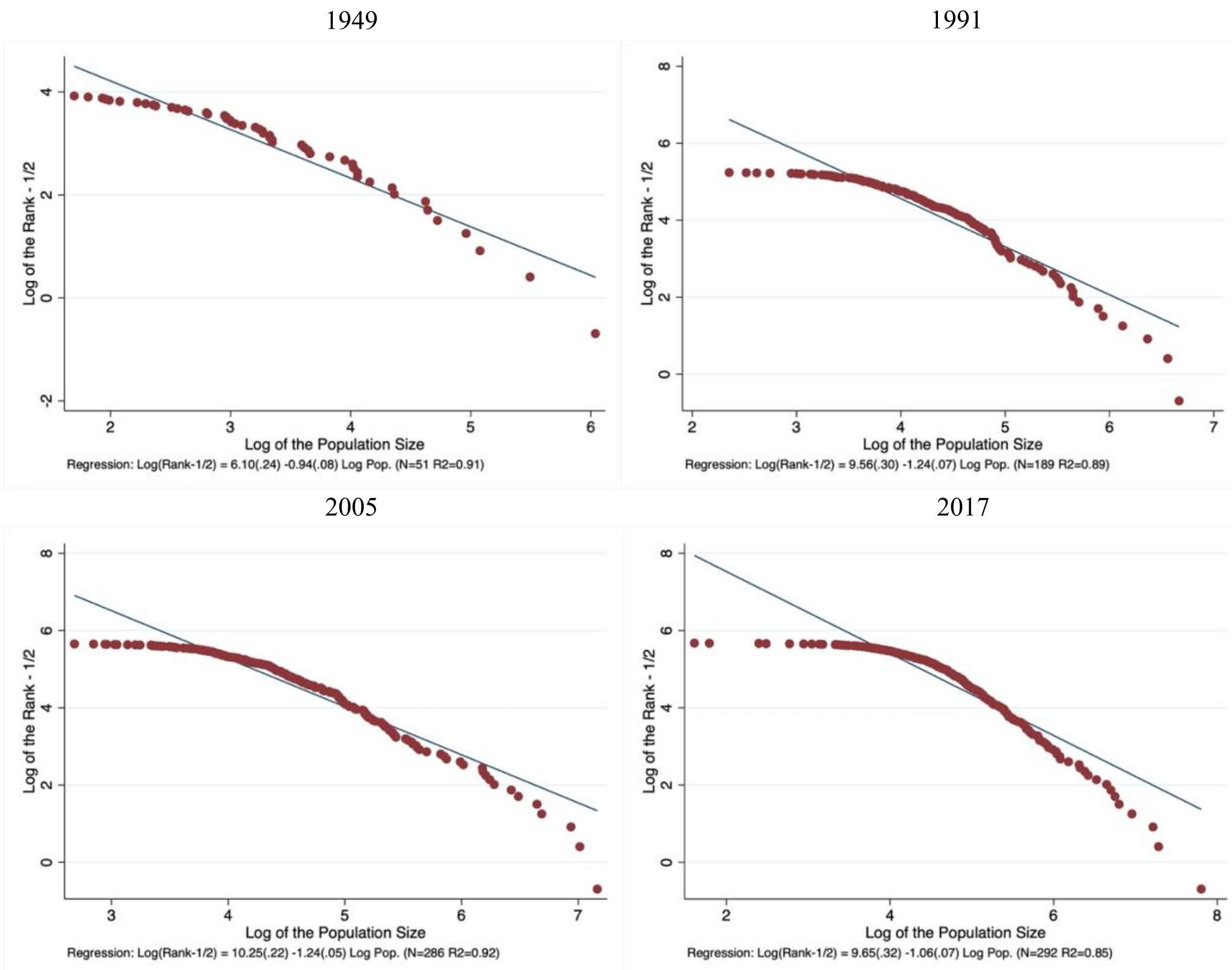
6 Results

6.1 Repeated Cross-Sectional OLS Regression for Zipf Coefficients

Figure 3 below shows the results of Zipf's law regression specified in Equation 26 for all cities at prefecture level and above in the year 1949, 1991, 2005, and 2017. We choose these four years to provide a visualization of the relationship between the log of population size and the log of population rank throughout our period of analysis. The rank-size scatter plots exhibit some concavity, which suggests that the regression function may be better approximated using quadratic terms of the log size to capture the non-linear relationship. This pattern becomes more obvious over time, with the 2017 plot demonstrating the most concavity and the lowest R-squared for the best linear fit line. Thus, a Pareto distribution may not describe the upper-tail distribution of city sizes very well for the entire sample of prefecture cities and above, in contrast to the United States, for example. The fact that the log of the population rank is concave down in the log of population size suggests large cities are smaller in size than what a Pareto distribution would predict.

Figure a. 2 in the Appendix truncates the city sample to the largest 100 cities ranked by population size in each year. As we reduce our sample size, we observe that a straight line becomes a better linear fit for the scatter plot of log size versus log rank, and the R-squared rises substantially (for instance, from 0.85 to 0.99 in 2017).

Figure 3. Zipf's Law: Urban Population and Urban Population Ranks, 1949, 1991, 2005 & 2017.



Note: Regression specifications and standard errors are based on Gabaix and Ibragimov (2011). Samples restrict to cities at prefecture level and above. Population data are from NSB (various years) measured in 10,000s.

Figure 4 displays the value of the Zipf coefficient in each year, which we obtain by taking the absolute value of the coefficient $\beta_{1,t}$ in the repeated OLS regression specified in Equation 26. (Note that it is also the absolute value of the slope of the best linear fit line in the rank-size plot, as shown in Figure 3). A detailed report of the numerical values of Zipf coefficient, along with the standard error and R-squared can be found in Table a. 1 in the Appendix. The Zipf coefficient is consistently greater than 1 with statistical significance from 1984 to 2015. Thus, even during years when convergence to Pareto's law is plausible (for example, around 2005), the logarithm of population rank and the logarithm of population size do not demonstrate a perfect Zipf's law relationship.

Overall, we conclude that China has deviated from Zipf's law in the past three decades. In particular, China's Zipf coefficient has been consistently higher than one, which indicates a more equal distribution of city size than Zipf's law would grant. This agrees with the intuition we obtain from the summary statistics for the largest ten cities (Table 4). For instance, in the census year 2010, the largest city in China, Shanghai, has approximately 1.2 times the size of the second-largest city, Beijing, which is much lower than the scale factor of two predicted by Zipf's law. Likewise, we observe that the fourth-, fifth-, and sixth-largest cities have roughly the same size of ten million people, suggesting that the population is distributed fairly evenly across the top large cities in China. Simply put, large cities are not "sufficiently large" for Zipf's law to hold. This result accords with Chauvin et al.'s (2017) finding that China has far fewer ultra-large cities than Zipf's law would suggest.

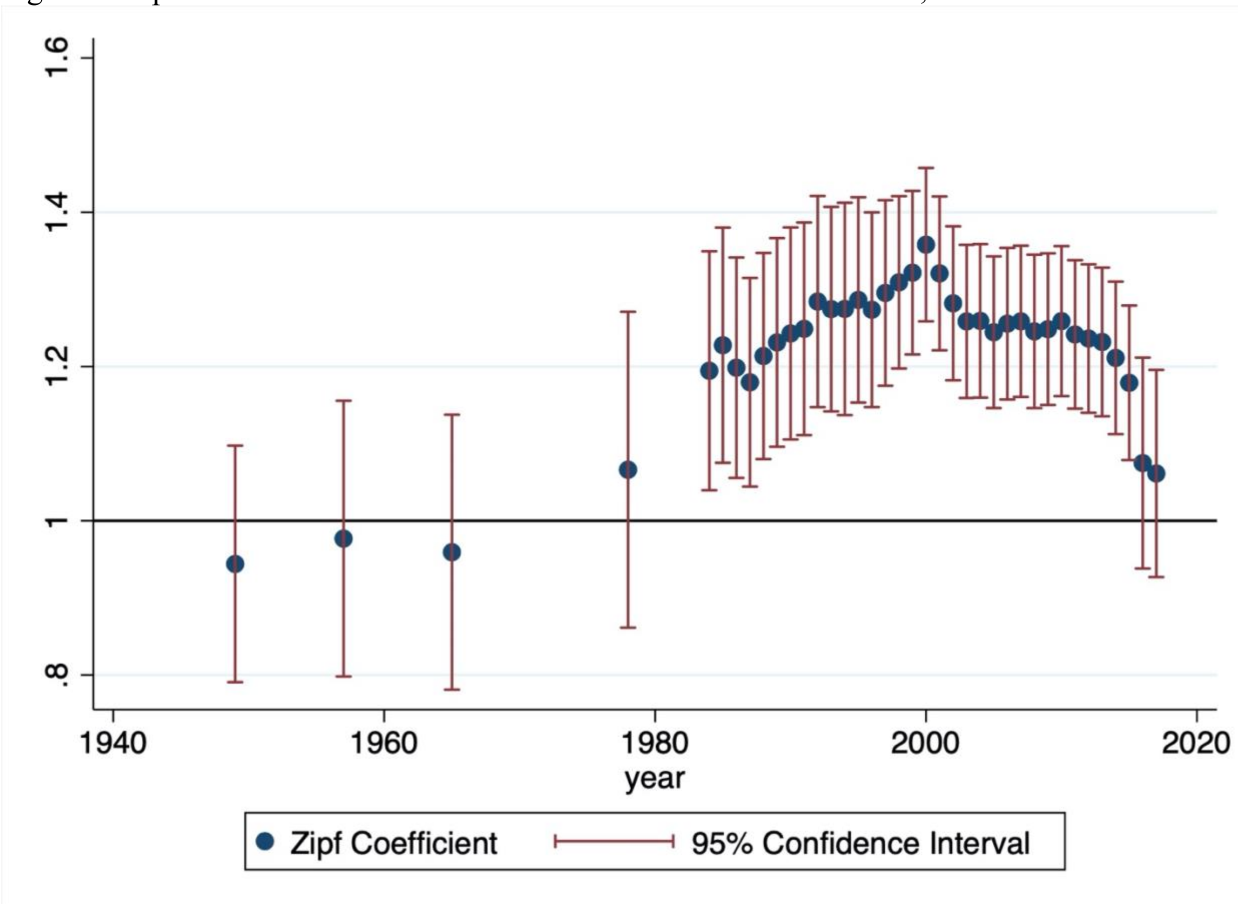
Such departure from Zipf's law is robust even at a regional level. If we restrict the sample to cities within each of the four economic regions, the Zipf coefficient for each region has also been consistently greater than 1 with the mere exception of the Western economic zone (See Figure

a. 3 in Appendix). One possible reason for the Western economic zone being an outlier is its lack of population data, which gives rise to huge standard errors and wide confidence intervals. As for the rest of the three regions, the Central, Northeastern, and Eastern economic zones have demonstrated significant deviations from Zipf's law at local levels since 1980. Therefore, cities are more evenly distributed across the country regardless of their geographical locations.

There are several other patterns we can draw from the graphs. In the 1950s and 1960s, the Zipf coefficient for all cities at prefecture level or above has been slightly lower than one; yet, because of the large standard errors, there is not sufficient evidence to conclude convergence to Zipf's law prior to 1984. Instead, one might propose a Pareto distribution that is less even than the special case with Zipf's law. The big difference in Zipf coefficients before and after 1980 might be attributable to the rapid urbanization that occurred with the economic reforms starting in the 1980s, or simply to the lack of urban population data from 1949 to 1984. Unfortunately, we do not have sufficient data to conclude on the cause of the "big jump" of the Zipf coefficient around 1980. Another observation in the time trend of the Zipf coefficient is the turning point that occurred in 2001 in Figure 4. Zipf coefficient increases consistently over time until 2000 but reverses its course in 2001, implying that the distribution of prefectures has been gradually converging to Zipf's law since then. The intriguing question of whether the coefficient will continue to converge to one remains uncertain, but our Gibrat's law results and one-child policy discussion in the following subsections attempt to shed light on this matter. Finally, if we compare the Zipf coefficients across different sample sizes, we obtain larger Zipf coefficients for higher truncate points. This observation accords with previous literature (Peng, 2010).

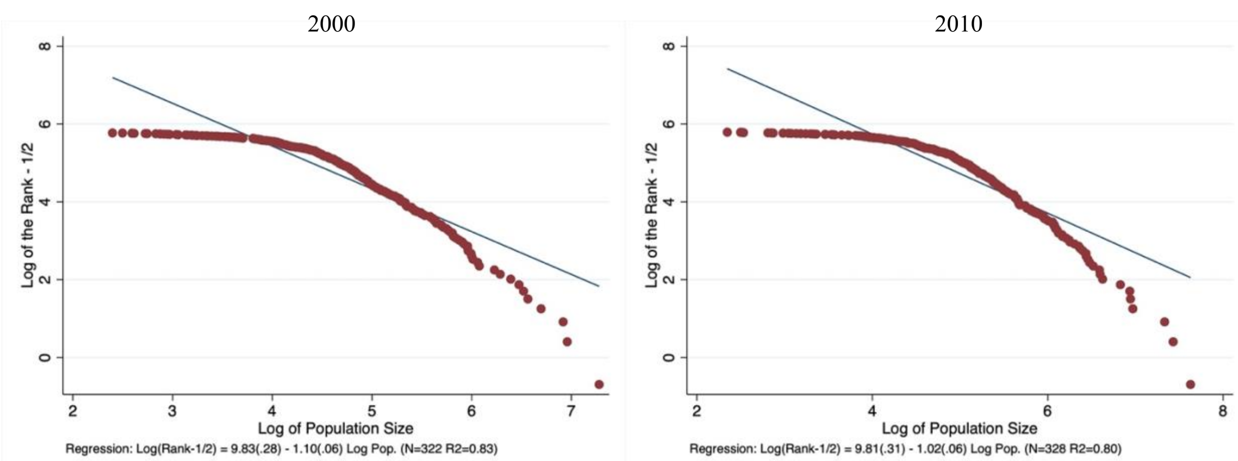
Figure 5 below shows the results from the Zipf's law regression specified in Equation 26 using residence-based population data from the census in 2000 and 2010. Similar to before, the upper-tail distribution of city size fails to converge to both Pareto's law and Zipf's law since we observe suspect linear fits with the absolute values of the slopes greater than one (1.10 for 2000 and 1.02 for 2010). Moreover, the value of Zipf coefficient drops over time from 2000 to 2010, mirroring the pattern of the hukou-based Zipf coefficient. Nevertheless, the census Zipf coefficients are lower in both years than the ones calculated using hukou data, as shown in Table a. 1 (1.358 for 2000 and 1.259 for 2010). Therefore, our previous findings on China's deviation from Zipf's law hold regardless of the population measure we choose, even though the residence-based population distributes less equally than the hukou-based population.

Figure 4. Zipf Coefficients for All Cities at Prefecture Level and Above, 1949-2017.



Note: Regression specifications and standard errors are based on Gabaix and Ibragimov (2011). Samples restrict to cities at prefecture level and above in each year. Population data are from NSB (various years) measured in 10,000s. The horizontal line at 1 indicates convergence to Zipf's law.

Figure 5. Zipf's Law: Urban Population and Urban Population Ranks, 2000 & 2010.

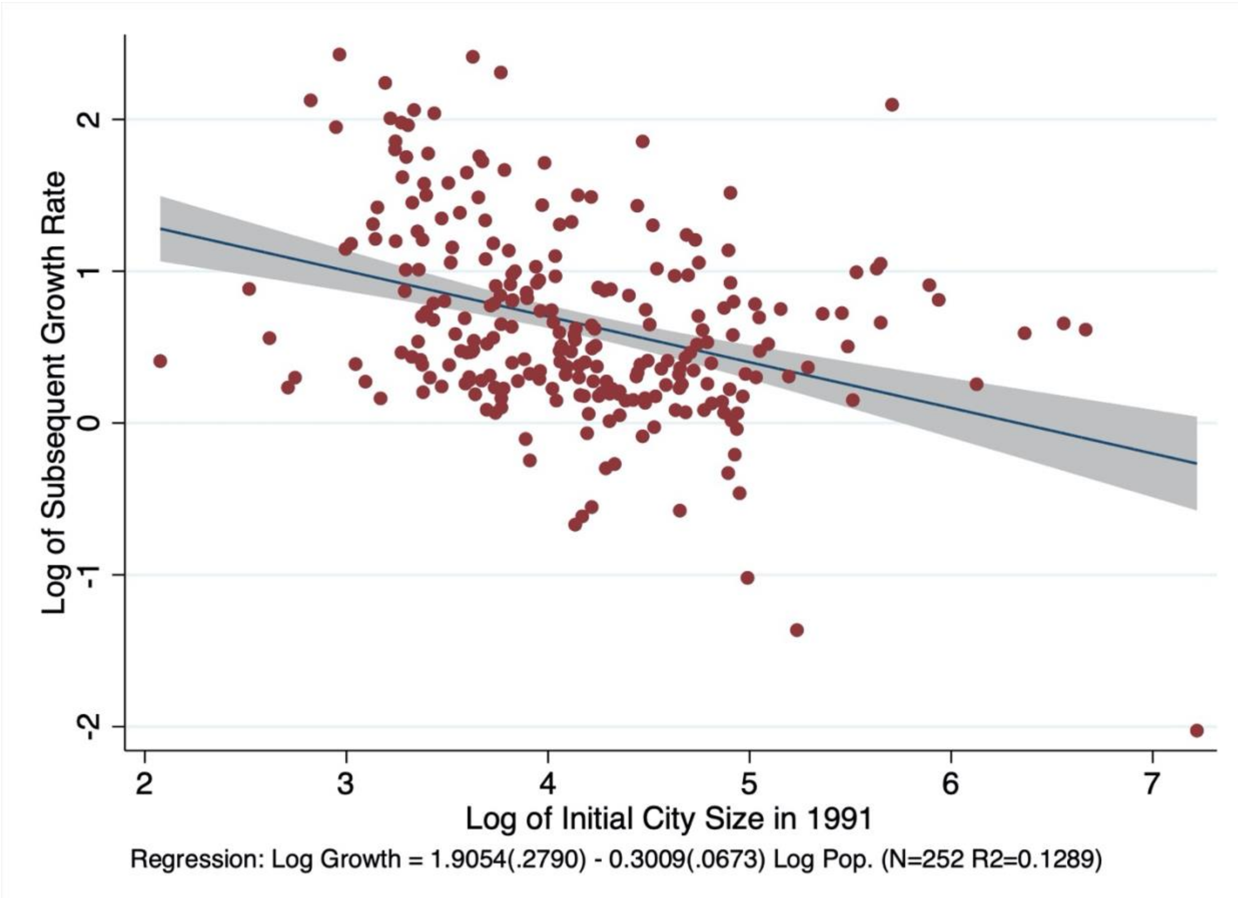


Note: Regression specifications and standard errors are based on Gabaix and Ibragimov (2011). Samples restrict to cities at prefecture level and above. Population data are from NSB (2000 & 2010) measured in 10,000s.

6.2 Correlation between Urban Population Growth and Initial Urban Size

The main results of testing homogeneous growth paths (or Gibrat's law) are presented in Figure 6 and Figure 7. Figure 6 shows the OLS regression of the logarithm of gross growth rates on the logarithm of initial population size, as specified in Equation 27, over the entire period from 1991 to 2017. The scatter plot and the negative slope of the regression line suggest a strong mean reversion of city populations in the past three decades. On average, large cities grow slower than small and medium-size cities. From the estimated value of the coefficient β_1, a one percent increase in initial city size in 1991 predicts a 0.3 percent decrease subsequent population growth over the subsequent 26 years, which is a relatively substantial drop in the context of urban growth. This result implies that urban growth is negatively correlated with initial city size. In the context of Equation 7 in the Zipf's law model, mean growth rate $\mu(S)$ decreases in city size *S*, which leads, ceteris paribus, the absolute value of the local Zipf coefficient, $\zeta(S)$, to increase in *S*. This implication of mean reversion coincides with the concave shape of the rank-size scatter plot. As shown in Figure 3, the first derivative at data points closer to the vertical axis (which represent smaller cities) is lesser in magnitude compared to that at data points closer to the horizontal axis (which represent larger cities). Therefore, China's mean reversion over the past three decades may well account for its deviation from Zipf's law.

Figure 6. Gibrat's law: Urban Population Growth and Initial Urban Population, 1991-2017.



Note: Sample restricts to cities at prefecture level and above. Robust standard errors are in parentheses. Log of subsequent growth rate is calculated by subtracting the logarithm of population size in 1991 from the logarithm of population size in 2017. Population data are from NSB (2017) measured in 10,000s.

Table 8. Gibrat's law: Urban Population Growth and Initial Urban Population.

	(1) 1991-2017	(2) 1991-2000	(3) 2001-2010	(4) 2011-2017
Coef.	-0.3009	-0.2071	-0.0963	-0.0669
Std. Error	(0.0673)***	(0.0456)***	(0.0385)*	(0.0287)*
Adj. R-squared	0.1254	0.1166	0.0411	0.0203
N	252	240	242	246

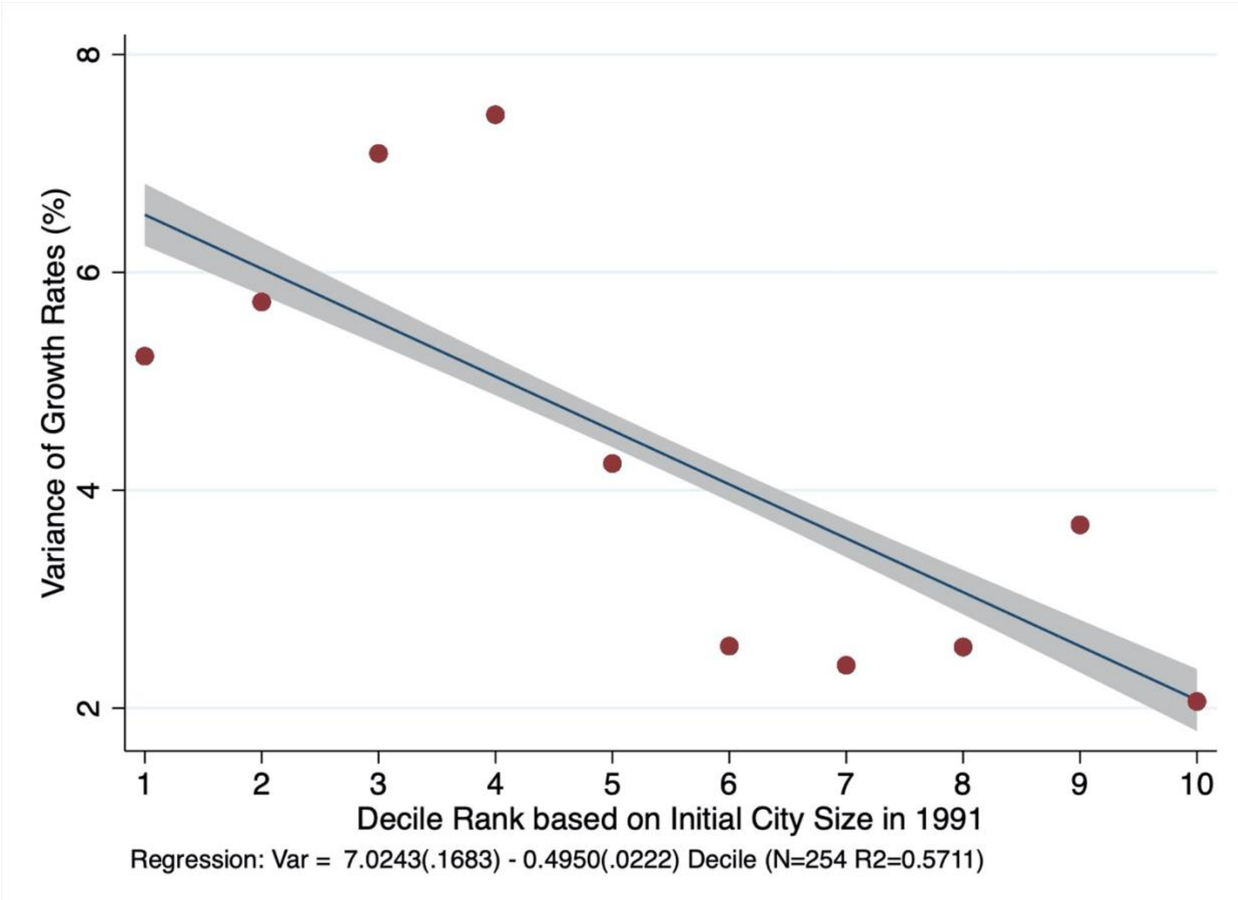
* p<0.05; ** p<0.01; *** p<0.001

Note: All figures reported correspond to city-level regressions of the logarithm change in urban population on the logarithm of initial urban population in the specified periods. Samples restrict to cities at prefecture level and above. Robust standard errors are in parentheses. Population data are from NSB (1991-2017).

Further, we break down the growth over the entire time period into the three individual decades and summarize the estimated values of the coefficient of interest, β_1, in Table 8 Columns 2-4. The strong mean reversion in the 1990s accounts for the majority of mean reversion over the entire period, with the estimated coefficient equal to -0.2071 and R-squared around 0.12. However, this pattern of mean reversion wears off over the past two decades. In the 2000s and the 2010s, the magnitude of coefficients drops to below 0.1 with both 99 percent confidence intervals containing zero. R-squared also decreases over time from 0.12 to 0.02, reinforcing that mean reversion becomes less obvious over time. We thus reject Gibrat's law for Chinese cities in the 1990s; yet, we cannot reject the hypothesis that Gibrat's law hold in the 2000s and the 2010s. Such departure from Gibrat's law in the 1990s may explain the simultaneous deviation from Zipf's law and the rising Zipf coefficient during that period. Moreover, in the most recent two decades, the failure to reject Gibrat's law may help explain the slow conversion of Zipf coefficient to 1 as shown in Figure 4. If Chinese cities continue to shift from mean reversion to an independent growth path, we may expect China to conform to Zipf's law in the future.

In addition to the average growth over the past three decades, results regarding the second aspect of Gibrat's law - whether the variance of growth is size-independent – are shown in Figure 7. Overall, the larger half of all provincial and prefecture cities experience greater variances of urban growth than the smaller half. However, variation in growth rates does not exhibit a linear association with the initial population size. Cities in the third and fourth decile groups, which correspond to the medium-large cities, have the highest variances of growth rates, whereas cities in the sixth, seventh, and eighth decile groups, which correspond to the small-medium cities, have lower variances than those in the ninth decile group. Among the largest 100 cities, the larger the initial population size, the lesser the variance of growth rates. This result is robust even if we divide cities into rank groups using other quantiles, like ventiles. We present the results from decile ranks such that each group has a sufficient number of observations to calculate a sensible variance. Though we may not conclude on a straightforward linear relation, we may still reject the hypothesis that the variance of growth rates is independent of the initial population size, given that the larger half of all prefecture cities have significantly higher variances than the smaller half. Thus, now that we have shown that overall $\frac{\partial \sigma^2(S)}{\partial S} > 0$, the third term in Equation 7, $+\frac{S}{\sigma^2(S)} \cdot \frac{\partial \sigma^2(S)}{\partial S}$, is nontrivial and contributes positively to a higher local Zipf coefficient in absolute value, which accords with the more evenly distributed city sizes. Yet, the result of variance test in China disagrees with the common observation that variance of growth rates depends negatively on city size using U.S. data (González-Val, 2010; González-Val et al., 2014). One possible explanation to reconcile the difference is that the small cities in China are underdeveloped and thus medium-size cities in China behave more like the small cities in the United States with significantly larger variances of growth.

Figure 7. Gibrat's Law: Variance of Urban Population Growth and Initial Population Size, 1991-2017.



Note: Sample restricts to cities at prefecture level and above. Robust standard errors are in parentheses. Cities are divided into ten decile groups based on their size ranks in 1991 in descending order. Variance within each decile group is calculated using annualized rates of population growth from 1991 to 2017. Population data are from NSB (1991-2017).

6.3 City Growth Regressions

Our main results on urban population growth and initial urban conditions are shown in Table 9 below. Columns 1-3 test the framework as laid out in Equation 30 using average wage as a measure of city-level economic productivity, whereas Columns 4-6 replace average wage with per capita gross regional product for an alternative measure of the economic well-being and labor productivity of cities. Columns 1 and 4 show the positive partial effect of economic productivity on subsequent urban growth holding geographical location and initial population fixed; Columns 2 and 5 add local government budget expenditure as a proxy for urban amenity provisions and show that better amenity provisions contribute to future urban growth; Columns 3 and 6 further include initial population density as a proxy for disamenities and show that higher disamenity levels do not appear to be negatively associated with population growth. In addition, the coefficient of the initial population size in 1991 is significantly negative in all six columns even after we control for omitted variables, which checks the robustness of the previous mean reversion results (Table 8). Finally, the coefficients of geographical dummies reveal that (the omitted) Eastern cities grow the fastest, followed by Western and Central cities, and Northeastern cities grow the slowest during the sample period.

Results of whether economic productivity promote population growth are mixed. In Columns 1-3, the coefficient of average wage turns from positive to negative after we add government budget expenditure and population density as explanatory variables, though lacking statistical significance. At first glance, these coefficients appear to suggest that higher productivity discourages subsequent urban growth if we consider average wage a measure of economic productivity. However, this interpretation contradicts the positive coefficients in front of per capita gross regional product in Columns 4-6. In particular, a ten percent increase in per capita gross regional product predicts a 1.5 percent rise in subsequent population growth, holding initial population size, population density, local government budget expenditure, and geographical regions constant. If we interpret per capita gross regional product as the average productivity of labor, then higher productivity seems to promote future urban growth moderately. One possible explanation to reconcile these two contradicting results is that China's average wage in 1991 does not reflect the marginal productivity as derived in the basic spatial equilibrium model. Although the central economic planning waned in the 1980s, the dual-track pricing system still affected a large portion of wages, especially those paid by the state-owned enterprises, and employees enjoy a significant amount of social transfers, including subsidized public housing, in lieu of wage income as employee benefits (Gu, 2002). Thus, per capita gross regional product may serve as a better measure of economic productivity. In general, we expect higher productivity to be positively associated with urban growth, as supported by the regression results of per capita gross regional product. By all means, there may be omitted variables in the long regression that preclude us from a conclusion on whether higher economic productivity causes greater population growth.

Table 9. Urban Population Growth and Initial Urban Conditions, 1991-2017.

	(1)	(2)	(3)	(4)	(5)	(6)
	Log pop growth (1991-2017)	Log pop growth (1991-2017)	Log pop growth (1991-2017)	Log pop growth (1991-2017)	Log pop growth (1991-2017)	Log pop growth (1991-2017)
Population size	-0.0015	-0.0016	-0.0018	-0.0015	-0.0016	-0.0018
	(0.0003)***	(0.0002)***	(0.0002)***	(0.0002)***	(0.0002)***	(0.0002)***
Average wage	0.3040	-0.6838	-0.3368			
	(0.2530)	(0.2678)*	(0.2345)			
Per capita GRP				0.4341	0.1471	0.1520
				(0.0588)***	(0.0856)	(0.0677)*
Local gov budget		0.3956	0.2525		0.2433	0.1271
expenditure		(0.0509)***	(0.0453)***		(0.0627)***	(0.0542)*
Population density			0.2161			0.2280
			(0.0381)***			(0.0386)***
Geographical dummies						
Central	-0.2344	-0.3022	-0.2703	-0.1247	-0.1562	-0.1838
	(0.0962)*	(0.0836)***	(0.0760)***	(0.0883)	(0.0860)	(0.0742)*
Northeast	-0.6514	-0.7168	-0.5463	-0.6274	-0.6311	-0.4910
	(0.0933)***	(0.0899)***	(0.0930)***	(0.0819)***	(0.0832)***	(0.0876)***
West	-0.3800	-0.3293	-0.1230	-0.1744	-0.2358	-0.0412
	(0.0988)***	(0.0850)***	(0.0878)	(0.0936)	(0.0885)**	(0.0854)
Adj. R-squared	0.1928	0.4038	0.4942	0.3428	0.3811	0.4903
N	249	248	248	249	248	248

* $p<0.05$; ** $p<0.01$; *** $p<0.001$

Note: Regressions are at city level restricted to cities at prefecture level or above. Robust standard errors are in parentheses. All regressions include a constant. All explanatory variables are in logarithm form. Gross regional product and local government budget expenditure are in per capita terms. Geographical dummies are based on the four economic regions of China. Units are as presented in Table 5. Data are from NSB (1991-2017).

As for urban amenities and disamenities, both contribute positively to urban growth. Table 10 and Table 11 below test the validity of using local government budget expenditure and population density as proxies for urban amenities and disamenities, respectively. Table 10 examines the relationship between local government budget expenditure and some urban amenity measures. Columns 1 and 2 present the regression results using urban data in 1991, and Columns 3 and 4 use data in 2005. We observe strong positive correlations between government budget expenditure and the three amenity measures, number of hospital beds, area of paved road, and area of public green space. In particular, a one percent increase in the total number of hospital beds is associated with a nearly one percent increase in the total government expenditure; a one percent increase in the area of paved road is associated with a 0.06-0.09% increase in the total government expenditure; a one percent increase in the area of public green space is associated with a 0.03% increase in the total government expenditure. This result holds across different periods. Variations in the three urban amenity provisions account for a moderately large degree of variations in local government expenditure, as justified by values of R-squared above 0.5. There are no striking results on the relationship between urban amenity provisions and local government budget expenditure as we expect a large portion of government spending is devoted to maintaining the current level of amenity provisions. These results suggest that the proposal of using government budget expenditure as a proxy for the level of urban amenity provisions is quite plausible. In addition, it is noteworthy that local government expenditure exhibits significant geographical differences. From the coefficients of the geographical controls, we observe that cities in the Central economic region and the Western economic region consistently enjoy a lesser amount of government spending than the Eastern economic region, which implies lower levels of urban amenities.

Similarly, Table 11 examines the relationship between population density and some urban disamenity measures. We observe positive partial correlations between the population density and the majority of pollution measures, such as the amount of wastewater, waste gas, and dust emitted per square meter. Among them, the emissions of wastewater are significantly correlated with the population density, where a one percent increase in the total amount emitted is associated with a 0.2–0.4% increase in population size, holding all else equal.[5] However, the amount of solid waste emitted in 1991 is an exception that is negatively correlated with population density. One possible excuse may be that different waste emissions are highly correlated among themselves. Variations in the pollution measures explain a decent amount of variations in population density, as justified by values of R-squared around 0.4–0.6. Overall, the positive partial correlations between urban pollution and population density are intuitive as we expect the heavily populated cities to discharge more domestic and industrial waste due to higher production and consumption levels. Thus, our findings suggest that the proposal of using population density as a proxy for the severity of urban problems may be sensible.

Back to the long regressions in Table 9, the coefficients in front of the per capita government budget expenditure and the population density are positive and statistically significant. In particular, from Column 6, a ten percent increase in initial per capita government budget expenditure has a partial effect of a 1.3 percent increase in subsequent urban population growth, and a ten percent increase in initial population density predicts a 2.3 percent increase in urban

5 Note that since the dependent and explanatory variables are both in logarithm of variable and in per square meter terms, the coefficients shown in Table 11 are equal to the coefficients representing partial correlations between the corresponding variables in total terms.

growth. In the case of government expenditure, it seems likely to interpret the positive coefficient as a positive effect of urban amenities on population growth. Cities starting with higher levels of amenities enjoy greater population growth in the following years since they attract more migrants as well as newborns. However, it is counterintuitive that disamenities and urban issues also promote urban growth. There are several possible explanations due to using a proxy for the elusive concept of disamenities. To name but a few, greater emissions and pollution may as well indicate the vibrancy of urban life and urban economy in addition to reflecting the disamenity levels in the city, so cities with higher population density and greater pollution may grow faster if the benefits of these waste-producing economic activities outweigh the costs of pollution. Alternatively, one could argue that population density itself embodies the agglomeration effect, where urban agglomerations give rise to clusters of businesses, accumulation of knowledge and innovations, which, in turn, facilitate urban growth. Thus, it remains unclear whether urban disamenities discourage future population growth. Nevertheless, the results for local government budget expenditure in Table 9 together with Table 10 suggest that a higher level of amenities is positively associated with urban population growth, which accords with comparable studies on the U.S. cities (Glaeser et al., 2001; Glaeser and Gottlieb, 2006).

Table 10. Amenities Regressions.

	(1) Gov bdgt exp 1991	(2) Gov bdgt exp 1991	(3) Gov bdgt exp 2005	(4) Gov bdgt exp 2005
Num. hospital beds	0.9706	1.0946	0.6375	0.6727
	(0.0765)***	(0.0783)***	(0.0612)***	(0.0679)***
Paved road area	0.1122	0.0888	0.0616	0.0563
	(0.0268)***	(0.0256)***	(0.0070)***	(0.0081)***
Public green area			0.0333	0.0261
			(0.0093)***	(0.0089)**
Geographical dummies				
Central		-0.5432		-0.2949
		(0.0921)***		(0.0697)***
Northeastern		-0.4692		0.0390
		(0.1105)***		(0.0890)
Western		-0.3222		-0.3840
		(0.0993)**		(0.0911)***
Adj. R-squared	0.5309	0.5868	0.5645	0.6214
N	249	249	243	243

* $p<0.05$; ** $p<0.01$; *** $p<0.001$

Note: Regressions are at city level restricted to cities at prefecture level or above. Robust standard errors are in parentheses. All regressions include a constant. All dependent and explanatory variables are in logarithm form. Government budget expenditure, number of hospital beds, paved road area, and public green area are in per capita terms. Geographical dummies are based on the four economic regions of China. Units are as presented in Table 5. Data are from NSB (1991 & 2005).

Table 11. Disamenities Regressions.

	(1) Population density 1991	(2) Population density 1991	(3) Population density 2005	(2) Population density 2005
Wastewater	0.4629	0.3997	0.2204	0.1544
	(0.0557)***	(0.0543)***	(0.0499)***	(0.0537)**
Waste gas	0.1310	0.1367	0.0492	0.0640
	(0.0677)	(0.0635)*	(0.0754)	(0.0760)
Dust	0.0378	0.0412	0.1034	0.1199
	(0.0247)	(0.0245)	(0.0925)	(0.0994)
Solid waste	-0.1500	-0.1237		
	(0.0557)**	(0.0535)*		
Geographical dummies				
Central		-0.2288		-0.0963
		(0.0900)*		(0.1204)
Northwest		-0.4915		-0.1861
		(0.1573)**		(0.1664)
West		-0.5768		-0.4575
		(0.1263)***		(0.1385)**
Adj. R-squared	0.5989	0.6353	0.3981	0.4207
N	237	237	238	238

* p<0.05; ** p<0.01; *** p<0.001

Note: Regressions are at city level restricted to cities at prefecture level or above. Robust standard errors are in parentheses. All regressions include a constant. All dependent and explanatory variables are in logarithm form. Wastewater, waste gas, dust, and solid waste are in per square meter terms. Geographical dummies are based on the four economic regions of China. Units are as presented in Table 5. Data are from NSB (1991 & 2005).

Our next main findings on urban characteristics and population sizes are presented in Table 12 and Table 13 below, which show the results of single variable regressions as specified in Equation 31 for 1991 and 2017, respectively. Large cities have better economic performances, as evaluated by per capita gross regional product, per capita industrial output value, and employment rate; on the other hand, large cities suffer more from urban issues like overcrowding, as captured by population density, and pollution, as measured by wastewater, waste gas, dust, and solid waste in per square meter terms. As for amenities, although per capita local government expenditure and public green space increase in population size, per capita number of hospital beds and paved road areas decrease as city size increases. It seems striking that large cities have more spending on infrastructure, healthcare, and education, but the outcome of these amenity provisions is worse compared to smaller cities. However, this may be the case if we believe amenity provisions have decreasing returns to scale: Even though a large city has fewer hospital beds and paved roads per person, its huge population indicates that the total amount is much higher than that of small and medium-size cities, so more investment is needed to maintain the current level of public services and facilities. Another notable finding is that the amount of foreign capital utilized increases in city size, and such relationship grows stronger over time as illustrated by a higher coefficient and a lower p-value in 2017. This result implies that large cities are more affected by the open-door policies, which benefit their economic well-being. Comparing the two tables for 1991 and 2017, we observe that the signs of the coefficients are consistent (except for residential savings), and thus these correlations are robust throughout the time frame. It is noteworthy that the probability values for all but hospital beds and paved roads decrease from 1991 to 2017 whereas the magnitude of coefficients increases, suggesting a greater statistical and economic significance of the correlations between urban characteristics and population sizes over time.

Table 12. Urban Characteristics and Population Size, 1991.

Dependent Var	Coef	Stderr	Pval	N
Population density	0.357	0.115	0.002	252
Employment rate	0.242	0.692	0.727	250
Gross regional product	0.022	0.053	0.680	249
Gross industrial output value	0.042	0.067	0.534	250
Foreign capital utilized	0.126	0.195	0.521	184
Local government budget expenditure	0.053	0.074	0.476	249
Residential savings	-0.048	0.058	0.410	250
Average wage	0.034	0.017	0.049	249
Hospital beds	-0.094	0.041	0.024	250
Paved roads	-0.395	0.169	0.020	250
Wastewater	0.254	0.151	0.094	246
Waste gas	0.435	0.144	0.003	244
Dust	0.546	0.159	0.001	241
Solid waste	0.397	0.140	0.005	244

Note: All figures reported correspond to city-level single variable regressions of the specified dependent variables on a constant and the logarithm of urban population size in 1991. Samples restrict to cities at prefecture level or above. Robust standard errors are recorded. All explanatory variables except employment rate are in logarithm form. Gross regional product, gross industrial output value, foreign capital utilized, local government budget expenditure, residential savings, hospital beds, and paved roads are in per capita terms; wastewater, waste gas, dust, and solid waste are in per square meter terms. Units are as presented in Table 5. Data are from NSB (1991).

Table 13. Urban characteristics and Population Size, 2017.

Dependent Var	Coef	Stderr	Pval	N
Population density	0.556	0.087	0.000	253
Employment rate	6.669	2.848	0.020	249
Gross regional product	0.244	0.049	0.000	252
Gross industrial output value	0.304	0.089	0.001	251
Foreign capital utilized	0.681	0.139	0.000	217
Local government budget expenditure	0.053	0.053	0.315	253
Residential savings	0.144	0.044	0.001	247
Average wage	0.102	0.017	0.000	245
Hospital beds	-0.016	0.044	0.717	247
Paved roads	-0.494	0.571	0.388	237
Wastewater	1.048	0.166	0.000	214
Waste gas	0.546	0.163	0.001	216
Dust	0.509	0.144	0.000	215
Public green space	0.032	0.402	0.937	250
Average house price	0.419	0.046	0.000	253

Note: All figures reported correspond to city-level single variable regressions of the specified dependent variables on a constant and the logarithm of urban population size in 2017. Samples restrict to cities at prefecture level or above. Robust standard errors are recorded. All explanatory variables except employment rate are in logarithm form. Gross regional product, gross industrial output value, foreign capital utilized, local government budget expenditure, residential savings, hospital beds, paved roads, and public green space are in per capita terms; wastewater, waste gas, dust, and house price are in per square meter terms. Units are as presented in Table 6. Data are from NSB (2017) and CREA (2017).

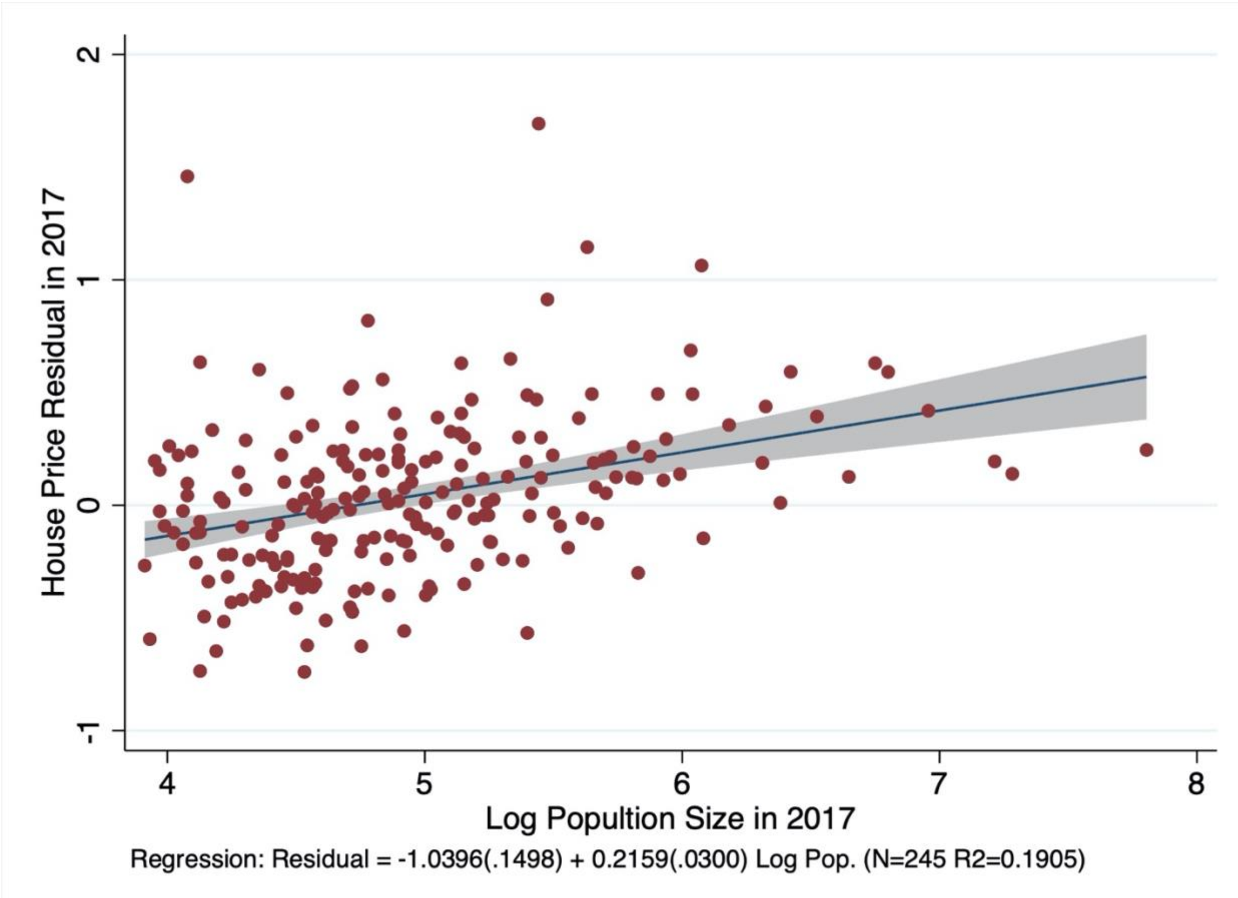
In addition to the correlations presented above, Figure 8 visualizes the connection between house price residuals, a potential "amenity index," and city sizes. Results of the house price wage regression specified in Equation 32 are shown in Figure a. 6 in the Appendix. We observe a positive correlation between house prices and wages in 2017. On average, a one percent increase in wages is associated with a 2% increase in house prices. In line with the spatial equilibrium model prediction, higher nominal income is offset by higher housing costs. On a side note, we observe that large cities tend to have both higher wages and higher house prices, as supported by results in Figure a. 4 and Figure a. 5 in the Appendix. We then estimate amenities using the residuals from regressing average house prices on average wages. Figure 8 shows that the estimated amenities are positively associated with urban population size in 2017. This result agrees with the relationship between amenities and urban population size that we obtain while using local government budget expenditure as a proxy for amenities (see the positive coefficient of regressing government budget expenditure on population size in Table 13). Thus, we conclude that amenities are higher in large cities than in small cities in 2017.

Figure 9 visualizes the coefficients of labor productivity recorded in Table 12 and Table 13. In particular, there does not seem to be an obvious linear relationship between per capita gross regional product and urban population size in 1991 but exists a significantly positive correlation in 2017. Nonetheless, it seems rather suspect that labor productivity declines in population size. Further evidence on labor skills and urban population size is shown in Figure 10, which supports the common view that large cities enjoy higher productivity as they attract more skilled and productive workers. As we can see from the scatter plot and the best linear fit line, labor skills, proxied by the share of university graduates in urban population, increase in population size. Although the relationship may not be perfectly linear, it is clear that more large cities enjoy an abundance of skilled labor than small cities given the shape of the scatter plots. This result is robust over time as we obtain similar trends in both census years 2000 and 2010.

Overall, empirical evidence suggests that large cities experience consistently higher economic productivity (Figure 9 and Figure 10) and higher amenities at least in 2017 (Table 13 and Figure 8). Contrary to the predictions of the spatial equilibrium model where the high productivity of large cities must be offset by worse amenities, residents of large cities in China seem to reap both benefits. This striking result suggests that spatial equilibrium may be violated.

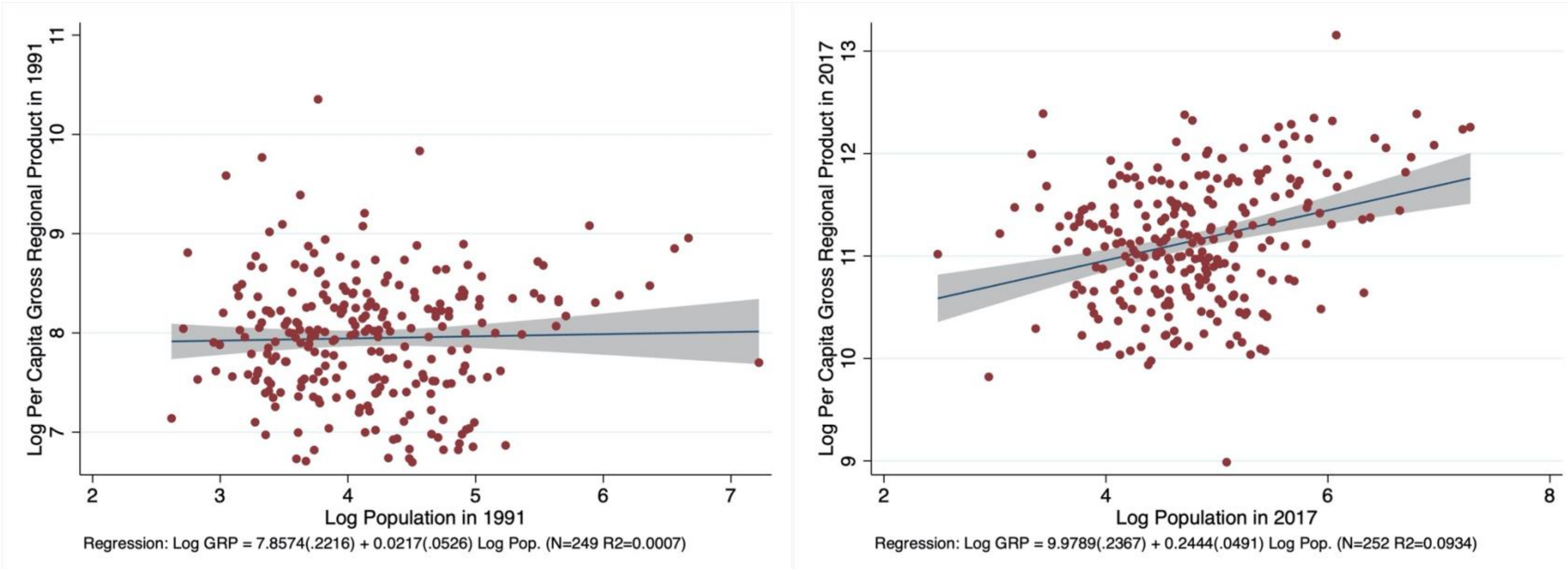
In a nutshell, results from city growth regressions do not explain why Chinese urban population growth experiences significant mean reversion. Although large cities have greater disamenities like pollution, they do not appear to have hindered further population growth. Moreover, while higher amenities help promote urban population growth, large cities enjoy higher levels of amenities proxied by either government spending or house price residuals. As for economic productivity, large cities exhibit significantly better economic performance in 2017, and higher productivity appears to have a slightly positive effect on population growth. Based on these observations, we would expect large cities to grow at least as fast as, if not faster than, small and medium-size cities, which contradicts the mean reversion results in Section 6.2. This leads us to our third hypothesis that the population growth of large cities is bindingly constrained by the public policy rule, such as the hukou system and the one-child policy.

Figure 8. House Price Residual and Urban Population Size, 2017.



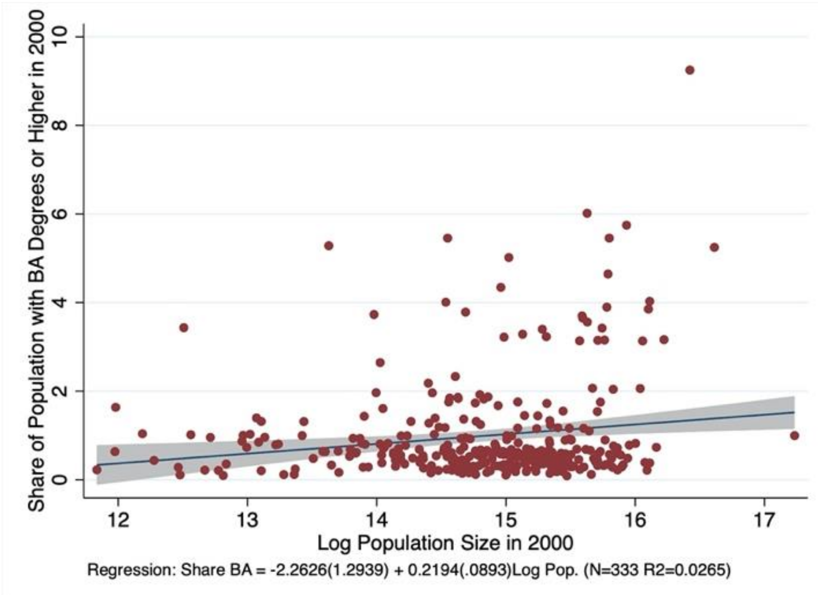
Note: Sample restricts to cities at prefecture level and above. Robust standard errors are in parentheses. Population data are from NSB (2017) measured in 10,000s. House price residuals are estimated from a city-level regression, in which the logarithm of average house price is regressed on the logarithm of average wage. Data on average house prices and wages are from NSB (2017).

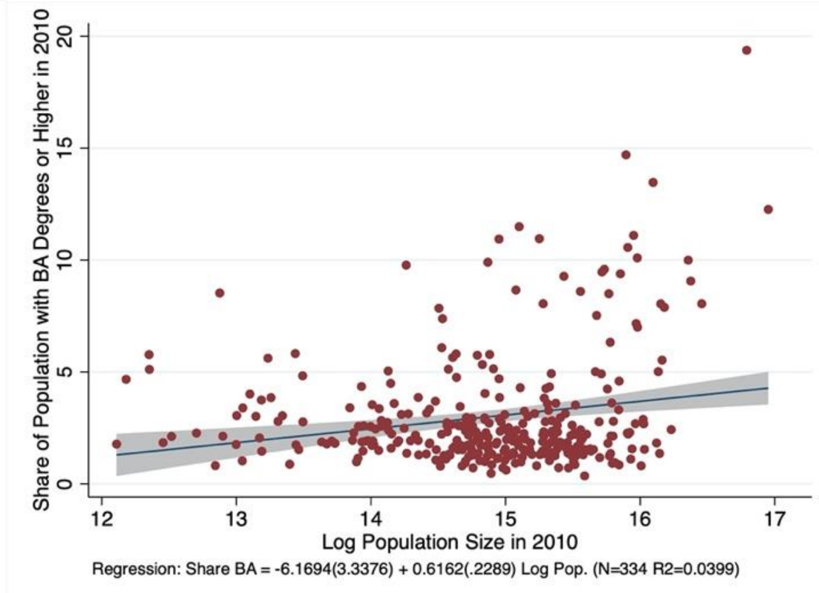
Figure 9. Productivity and Urban Population Size, 1991 & 2017.



Note: Samples restrict to cities at prefecture level and above. Robust standard errors are in parentheses. Population data are measured in 10,000s and per capita gross regional product in Chinese Yuan. All data are from NSB (1991 & 2017).

Figure 10. Labor Skills and Urban Population Size, 2000 & 2010.





Note: Samples restrict to cities at prefecture level and above. Robust standard errors are in parentheses. Population data are residence-based and measured in 10,000s. The share of population with BA degrees or higher is calculated by dividing population with BA degrees or higher by total population and measured in percentage. All data are from *Tabulation on the Population Census of China* (NSB, 2000 & 2010).

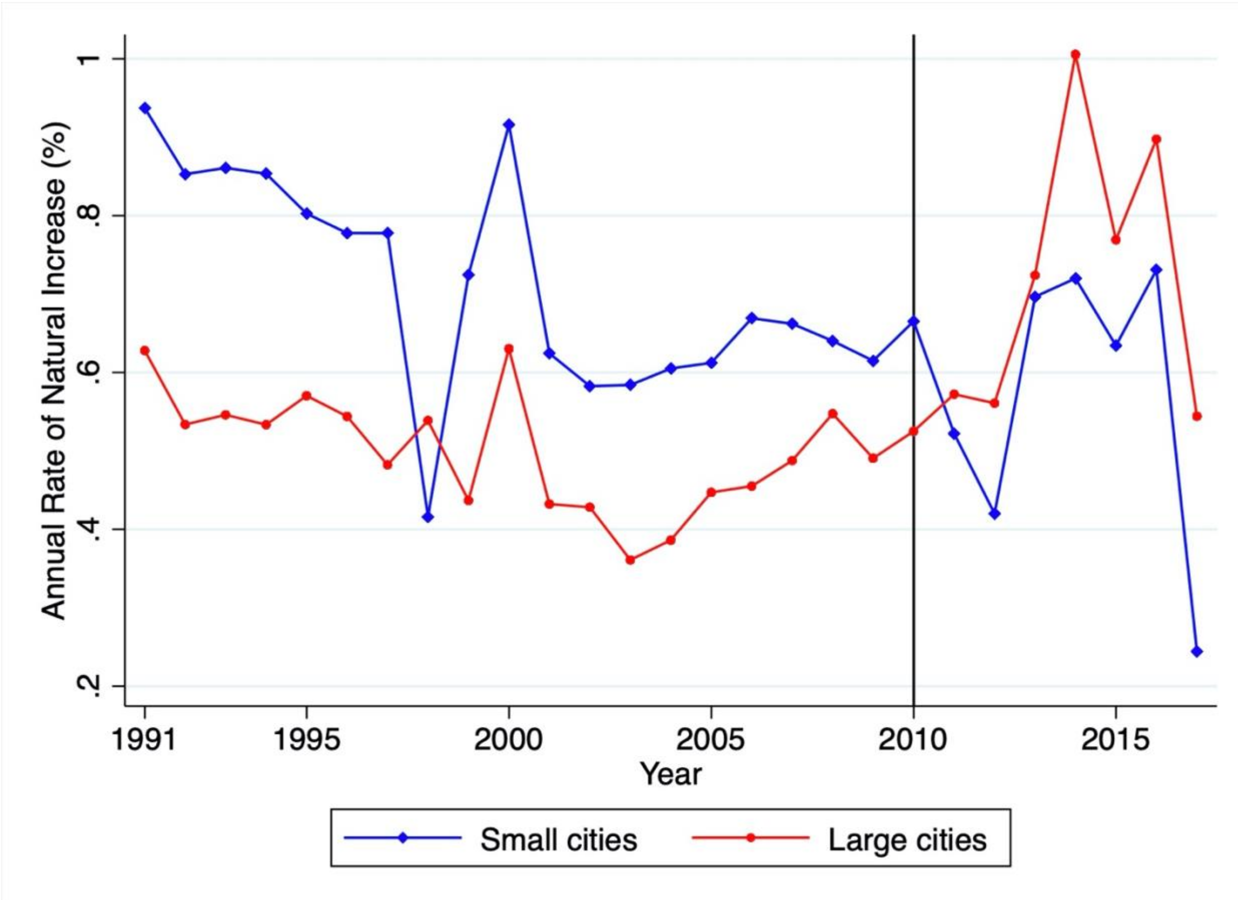
6.4 Public Sector Rule Regressions

Figure 11 and Table 14 focus on the effects of the one-child policy relaxation on urban population growth. Figure 11 below shows that changes in annual rates of natural increase are quite different between large cities (ranked 1 through 50 in 1991) and small cities (ranked 201-250 in 1991). Before 2011, we observe plausible parallel pre-trends of RNI for large cities and small cities (see also Footnote 4), albeit with slight convergence. Small cities experience significantly higher rates of natural increase. One exception is the year 1998, which, we suspect, is caused by issues in data and does not reject the parallel trend assumption or invalidate the specification of our differences-in-differences estimation. After China relaxed the universal one-child policy by allowing a second child for parents who are both the only child of their families, the average RNI for large cities rebounds in 2011 and becomes higher than that of small cities, whereas small cities undergo a drop in RNI for two consecutive years. Post impact, the two trend lines return to being somewhat parallel after 2014.

Table 14 presents the results of the differences-in-differences estimation as specified in Equations 34-35. Both Columns include city fixed effects and year fixed effects; Column 2 adds the number of hospital beds per person as a healthcare control to obtain a more accurate point estimate and check the robustness of the results. In accordance with the time trends shown in Figure 11, large cities experience lower rates of natural increase due to the strict enforcement of the one-child policy before 2011, and the universal relaxation of the policy significantly raises the rates of natural increase for large cities relative to small cities. In particular, the rates of natural increase of large cities, on average, are -0.5 percentage points lower than those of small cities before 2011. The point estimate of the differences-in-differences is that relaxing the one-child policy raises the rate of natural increase by 0.4 percentage points for large cities relative to small cities, which is a considerable effect given that the average annual rate of natural increase for large cities is only about 0.5 percent. It is also worth noting that the coefficient before the healthcare control in Column 2 is positive with 99 percent statistical significance, suggesting that better healthcare facilities have a positive partial effect on the rate of natural increase.

These results imply that the one-child policy was not uniformly enforced and, accordingly, relaxed across China. It was more strictly enforced in cities with larger populations before 2011, as they experienced lower rates of natural increase and thus lower birth rates if we assume death rates to be relatively constant. It follows that, unsurprisingly, the relaxation of the one-child policy affected large cities more positively than small cities in 2011 as the rates of natural increase for large cities significantly increased relative to small cities. Since urban population growth consists of natural increase and migration, we can infer that the enforcement of the one-child policy during the 1990s and the 2000s restricted population expansion and resulted in somewhat slower urban growth for large cities. Therefore, the different degrees to which the one-child policy was enforced among cities provide one possible explanation for the mean reversion of city size obtained in Section 6.2. As China gradually rescinds its one-child policy in the 2010s, we observe weaker mean reversion than the previous two decades, as shown in Table 8 Column 4. Taking a step further, it seems possible for large cities to catch up with small and medium-size cities on the population growth rates in the future, which may be favorable for potential convergence to Zipf's law.

Figure 11. Impact of the One-Child Policy Relaxation on Annual Rate of Natural Increase.



Note: Large cities refer to cities ranked from 1 to 50 and small cities refer to cities ranked from 151 to 200 by population size in the year 1991. Annual Rate of Natural Increase (RNI) for each city are from NSB (1991-2017). We take the simple average of annual RNI over all cites in the group (large or small) in any given year. 2010 represents the last year during which China maintained the universal one-child policy before introducing a nationwide relaxation in 2011.

Table 14. Impact of the One-Child Policy Relaxation on Annual Rate of Natural Increase.

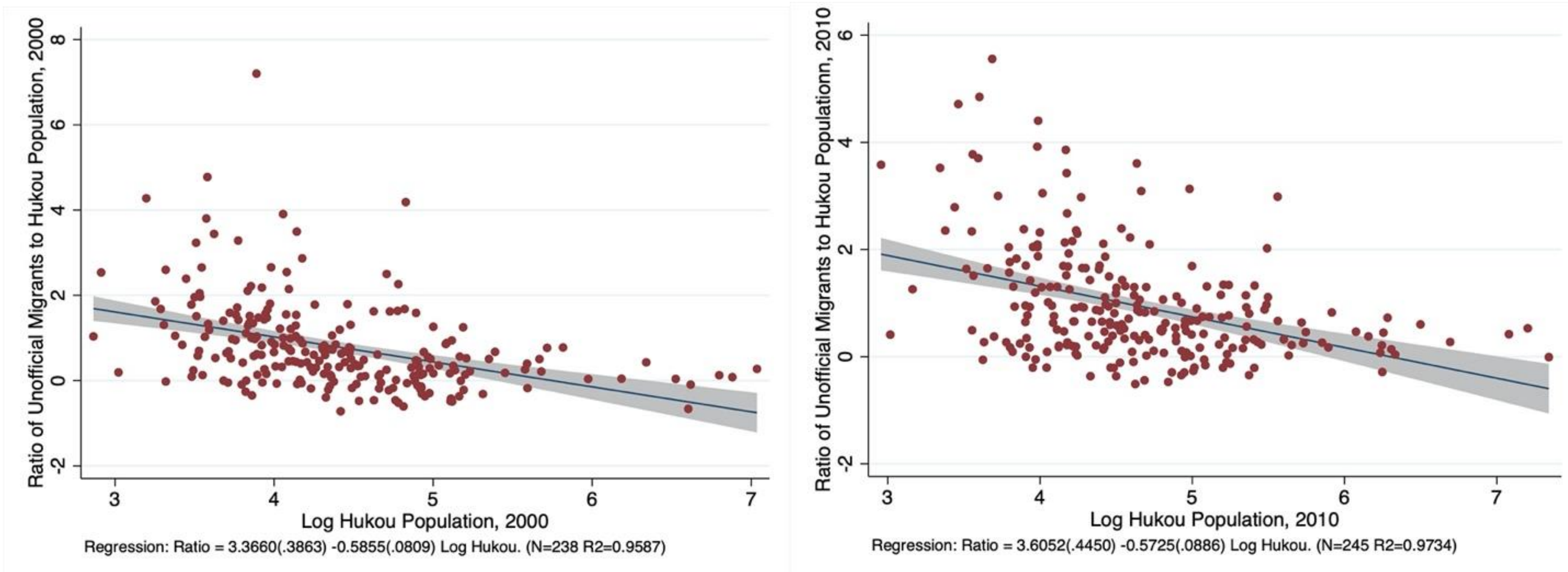
	(1) Rate of natural increase	(2) Rate of natural increase
Post2011	-0.5826	-0.6619
	(0.0767)***	(0.0790)***
Large	-0.4990	-0.4888
	(0.0205)***	(0.0212)***
Post2011#Large	0.3811	0.3993
	(0.0661)***	(0.0651)***
Healthcare control		0.1486
		(0.0488)**
Constant	0.8412	0.2219
	(0.0210)***	(0.1990)
Adjusted R-squared	0.4939	0.4974
N	2,573	2,457

* p<0.05; ** p<0.01; *** p<0.001

Note: Area-level regressions include city dummies and year dummies to control for city fixed effects and year fixed effects. Standard errors, clustered by cities, are in parentheses. *Large* is a dummy variable that takes on value one if the city ranked from 1 to 50 and zero if ranked from 151 to 200 by population size in the year 1991. We exclude cities that do not fall into either category in the regressions. *Post2011* is a dummy variable that takes value one for years from 2011 to 2017 and zero for years from 1991 to 2010. *Post2011#Large* is the interaction term whose coefficient estimates the differences-in-differences. Annual rate of natural increase (RNI) for each city is measured in percentage points. Healthcare control is a proxy for healthcare provision in each city and is measured in the number of hospital beds per 10,000 persons. All data are compiled from NSB (1991-2017).

The main results on the enforcement of the hukou system are presented in Figure 12. The scatter plots and the best linear fit lines suggest an inverse relationship between urban population size and the unofficial migration share of the urban population. In particular, a ten percent increase in the hukou population is associated with a 0.06 unit increase in the ratio of unofficial migrants to hukou population. Although the magnitude of the coefficients is not as major, the negative correlation is statistically significant, and the high values of R-squared suggest a linear relationship is plausible. From the scatter plots, we can see that the top large cities have roughly the same limited degrees of migration shares; small and medium-size cities, in contrast, differ greatly among themselves, and unofficial migrants even constitute the majority of residential population in many cities (as the ratio of unofficial migrants to hukou population is greater than one). Note that data points with negative ratios represent a net outflow of migrants, which is the case for a handful of medium-size cities. These observations are consistent in both census years, 2000 and 2010. Thus, these results suggest that large cities experience greater control over unofficial migrations. As discussed in Section 2.2, measures to achieve such stricter enforcement include limiting unofficial migrants' access to healthcare and education resources, housing supplies, and the local job market, thus making it rather hard for those without hukou status to stay in the city. In addition to minimizing unofficial migration, it is likely that large cities also issue fewer new permits each year compared to small and medium-size cities to contain the hukou population (Zhang and Tao, 2012). Consequently, stricter hukou control in large cities hinders urban growth both in terms of residence-based population and in terms of hukou population, which gives us another important policy factor that sheds light on the mean reversion of Chinese city sizes.

Figure 12. Unofficial Migration and Urban Population Size, 2000 & 2010.



Note: Samples restrict to cities at prefecture level and above. Robust standard errors are in parentheses. We calculate the number unofficial migrants by subtracting the hukou population reported in *Chinese Urban Statistical Yearbooks* from the residence-based population reported in *Tabulation on the Population Census of China* (NSB, 2000 & 2010).

7 Discussion

The goal of this paper is to analyze why China's city size distribution deviates from Zipf's law, and in particular, why the large cities do not grow as large as Zipf's law predicts. First, we find that China's city size distribution deviates consistently from Zipf's law over the past three decades. In particular, the Zipf coefficient has been significantly larger than one, suggesting a more even distribution of the population. There is some evidence suggesting potential convergence to Zipf's law in the most recent years from the time series plot, though this observation is sensitive to the truncating point of our sample. Moreover, we find that the city size distribution based on residence, which includes unofficial migrants, does not provide significantly different results from the distribution based on hukou status in terms of the Zipf coefficient.

To explain such deviation from Zipf's law, we find that the assumption of a homogeneous growth process fails in Chinese cities. Instead, the urban growth rate decreases in the initial population size whereas the variance of growth rates increases in size. These observations can explain why the local Zipf coefficient for large cities are greater than one using Gabaix's (1999) general expression (Equation 7). Intuitively, the top large cities grow slower, allowing other small or medium-size cities with higher growth rates to catch up, leading to a more even city size distribution. This pattern of growth fits well with Chinese government's overall urban development strategy, which focuses on promoting growth for small and medium-size cities and containing any further expansion of metropolitan cities. In comparison with other countries, China's mean reversion of city sizes appears to be much stronger, especially in the 1990s. Concurrently, Gibrat's law has been shown to hold in most countries, such as the United States

(Eeckhout, 2004; Chauvin et al., 2017), France (Eaton and Eckstein, 1997), Japan (Eaton and Eckstein, 1997), and Brazil (Resende, 2004). Even though India also exhibits mean reversion from 1980 to 2010, the degree is much smaller with a corresponding coefficient of -0.05 (Chauvin et al., 2017) compared to the -0.30 for China (Figure 6). Such comparison helps explain why China may be an outlier to Zipf's law. Another interesting finding is that China's mean reversion becomes weaker over time, as the regression coefficient drops to -0.07 in the 2010s and loses statistical significance (Table 8). We may expect this trend of diminishing mean reversion to continue and eventually support convergence to Zipf's law.

Among our three main hypotheses for explaining why large cities experience slower urban growth in China, namely, economic productivity slowdown, amenity deterioration, and direct government interventions, we find that large cities are more appealing in terms of higher productivity and better amenities, but populations grow slower due to stricter enforcement of the one-child policy and the hukou system. Productivity and amenities are universal to other countries in explaining urban growth patterns (Glaeser et al., 2001; Glaeser and Gottlieb, 2006). We study how economic productivity and urban amenities connect to urban growth and population size in China using a linear regression model and find that these two urban characteristics indeed promote urban growth as observed in other countries, and large cities in China enjoy both higher productivity and better amenities. The better economic performance of large cities implies that many of the economic reforms happened during 1991-2017 benefit the large cities more than small and medium-size cities. For instance, China's accession to the WTO brings in more foreign capital for large cities (see the significantly positive coefficient of 0.68 in Table 13) and opens up more opportunities to foreign trade for large companies that mostly reside in the highly populated areas. In addition, large cities see greater government expenditure and house price residuals, which are indicators of urban amenities. As for urban issues, while large cities experience more pollution, such disamenities are byproducts that have been overshadowed by the benefits of economic prosperity. Thus, we do not observe any deterrence of population growth caused by crowding in large cities. Also, China has been making special efforts to ensure better air and water quality by moving heavy industries out of large cities like Beijing and Shanghai. Overall, it is plausible that large cities are not yet close to their carrying capacities and still have the potential to grow further larger given their economic well-being and social amenities. Such potential could lead to convergence to Zipf's law should there be free labor mobility across cities.

Finally, we justify the mean reversion of Chinese cities with government's direct interventions in population growth, a unique factor in China's case. From our differences-in-differences analysis for the one-child policy, we find that compared to small cities, large cities suffered from lower natural increase under the universal one-child policy from 1991 to 2011 and experienced a significant relative rise in the rate of natural increase after China's first nationwide relaxation of the policy in 2011, which allow parents who are both the only child to have a second child. Likewise, our regression result of the hukou system shows that the ratio of unofficial migrants to hukou population decreases in population size. Both results indicate that in the past, China's population regulations were more constraining for large cities than for small and medium-size cities. Yet, in the most recent decade, China continues to loosen its birth planning program by replacing the one-child policy with a universal two-child policy in 2016, whose impact may further reduce the disadvantages in natural increase for large cities; meanwhile, China published new regulations regarding its migration restrictions in 2014, which abolish the official distinction between agriculture and non-agricultural hukou and extend more social welfare to unofficial migrants living in the cities. As China reduces the intensity of its direct population control through these reforms, future convergence of city sizes to Zipf's law may be expected.

One avenue for further research on China's population policies and city sizes is an exploration into the effect of China's migration control on the economic units of cities. Since China's heavy population regulations are imposed within the administrative boundary of each city, many satellite towns emerge around large cities in recent years, and new migrants move to these border towns to take advantage of the positive spillover effects of the neighboring city and circumvent issues with official permits. This new phenomenon leads to the question of how effective migration control really is. One may construct metropolitan areas for China using commuting data or night lights data and examine whether the size dependency of the enforcement intensity of the hukou system that we observe using administrative units is salient in metropolitan areas as well. If that is the case, then China's migration control has a far-reaching effect on migration behaviors and population distribution beyond administrative purposes. Yet, given that China's lights-based metropolitan areas obey Zipf's law (Dingel et al., 2019), we might suspect that the hukou system is effective only in constraining population growth within administrative boundaries but fails to impede the formation of ultra-large urban agglomerations.

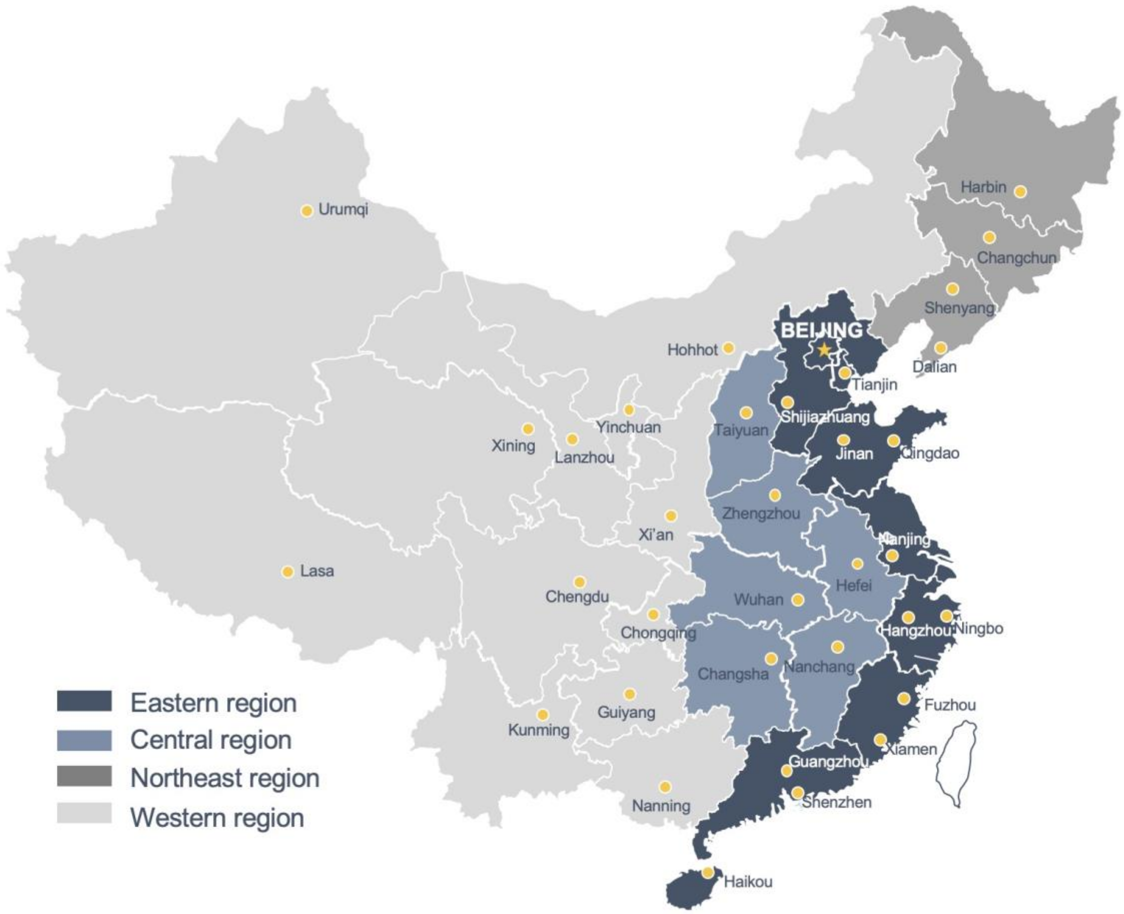
8 Appendix

Table a. 1. Values of Zipf Coefficients, 1949-2017.

Year	Zipf Coefficient	Standard Error	R-Squared
2017	1.061	0.068	0.846
2016	1.075	0.069	0.847
2015	1.179	0.051	0.896
2014	1.211	0.050	0.905
2013	1.232	0.049	0.914
2012	1.236	0.049	0.918
2011	1.242	0.049	0.916
2010	1.259	0.049	0.919
2009	1.248	0.050	0.912
2008	1.246	0.051	0.910
2007	1.259	0.050	0.915
2006	1.256	0.050	0.914
2005	1.244	0.050	0.915
2004	1.259	0.051	0.913
2003	1.258	0.050	0.914
2002	1.282	0.051	0.918
2001	1.321	0.051	0.921
2000	1.358	0.051	0.925
1999	1.322	0.054	0.920
1998	1.309	0.057	0.913
1997	1.295	0.061	0.903
1996	1.274	0.064	0.894
1995	1.286	0.068	0.893
1994	1.275	0.070	0.884
1993	1.274	0.067	0.898
1992	1.284	0.069	0.898
1991	1.249	0.070	0.892
1990	1.243	0.070	0.892
1989	1.231	0.069	0.892
1988	1.214	0.068	0.891
1987	1.180	0.069	0.887
1986	1.198	0.072	0.883
1985	1.228	0.077	0.882
1984	1.194	0.078	0.888
1978	1.066	0.103	0.845
1865	0.959	0.089	0.881
1957	0.977	0.089	0.891
1949	0.944	0.076	0.916

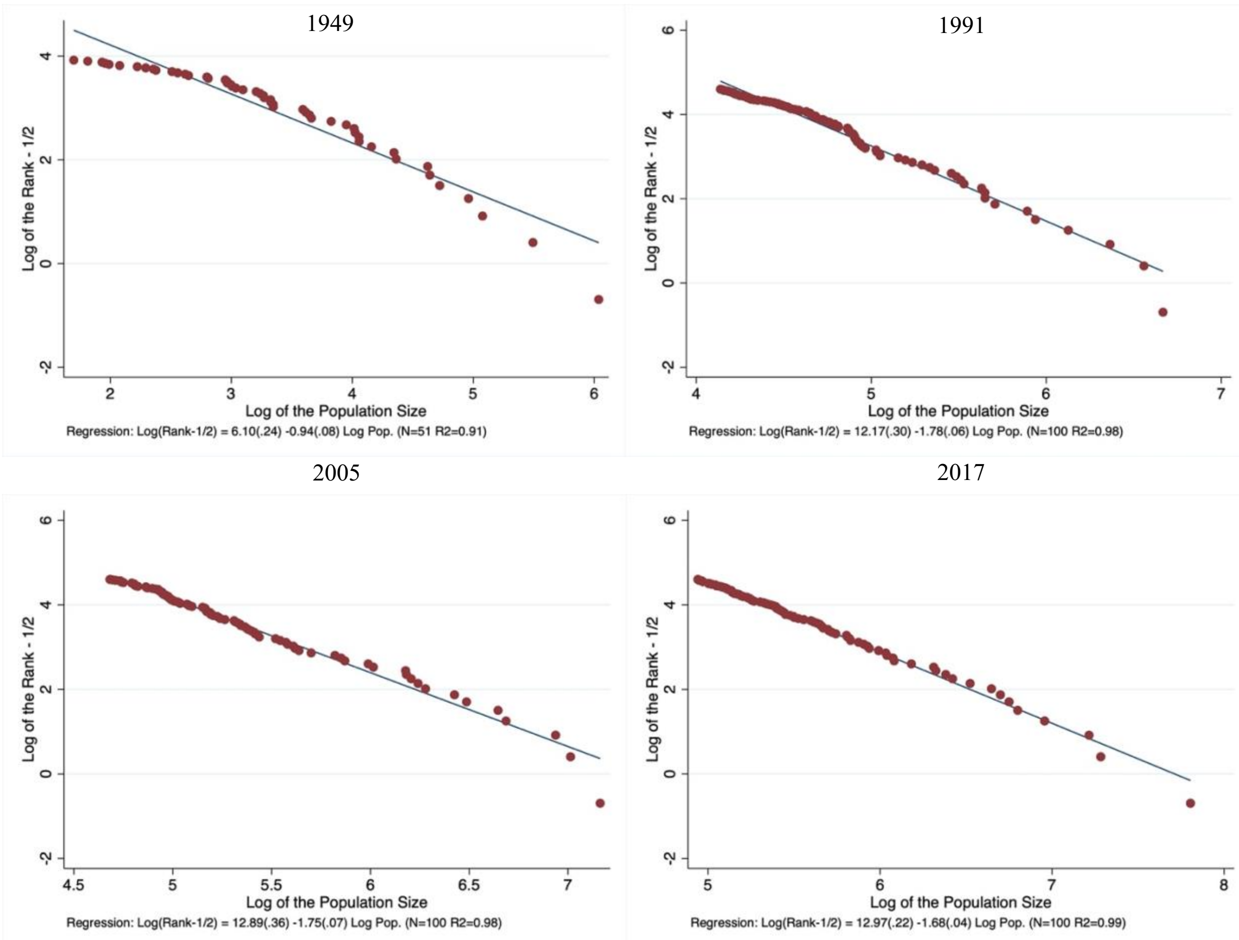
Note: Regression specifications and standard errors are based on Gabaix and Ibragimov (2011). Samples restrict to cities at prefecture level and above in each year.
Source: Author's calculation using population data from NSB (2000, 1984-2017).

Figure a. 1. Illustration of the Four Economic Regions of China.



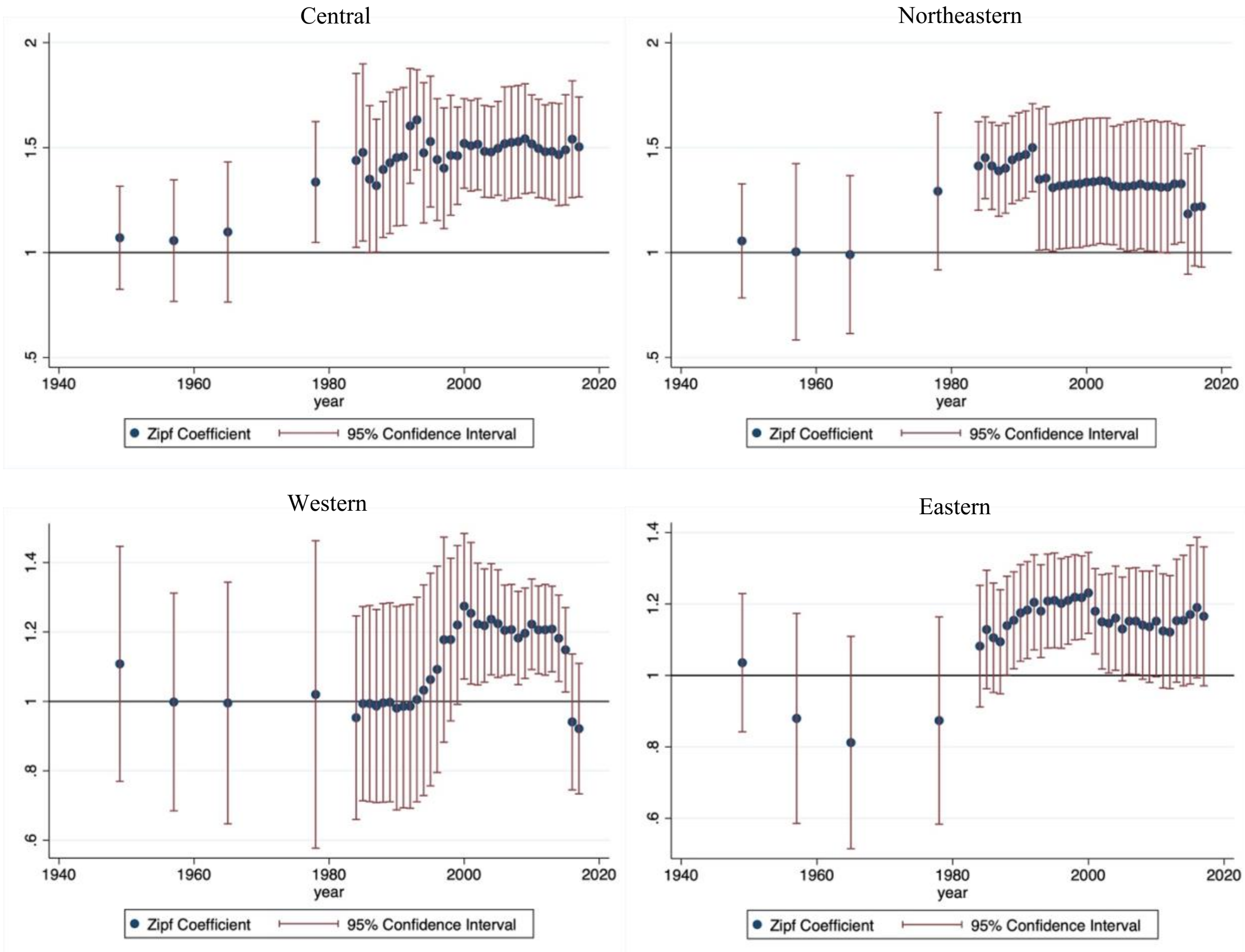
Source: Author’s compilation.

Figure a. 2. Zipf's Law: Urban Population and Urban Population Ranks (Largest 100 Cities).



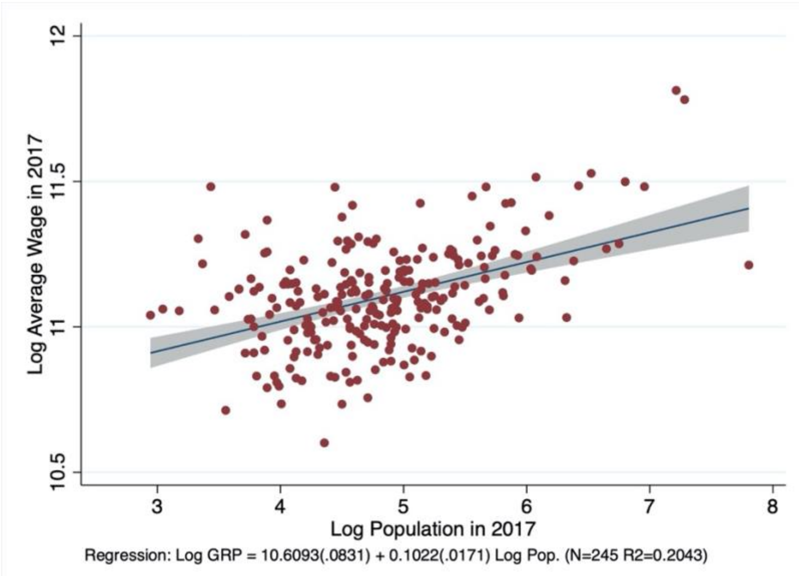
Note: Regression specifications and standard errors are based on Gabaix and Ibragimov (2011). Samples restrict to cities at prefecture level and above. Population data are from NSB (various years) measured in 10,000s.

Figure a. 3. Zipf Coefficients for the Four Economic Regions, 1949-2017.



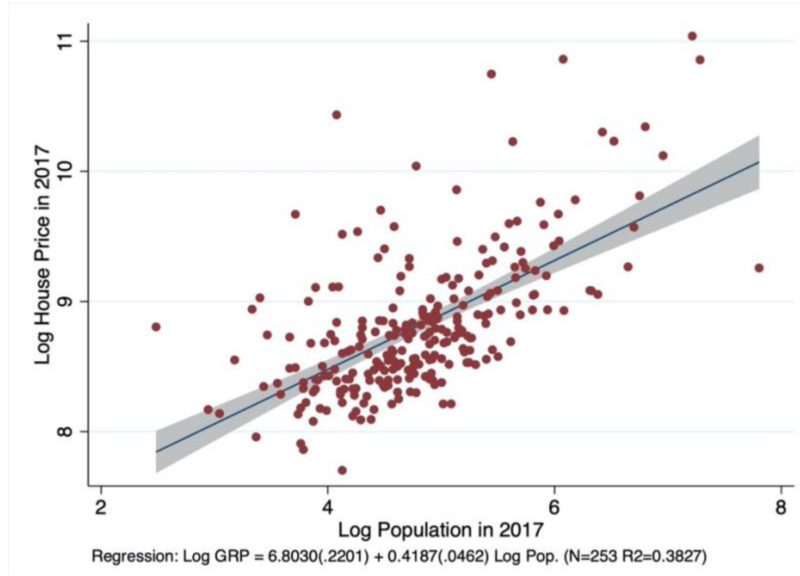
Note: Regression specifications and standard errors are based on Gabaix and Ibragimov (2011). Samples restrict to cities at prefecture level and above in each year. Population data are from NSB (various years) measured in 10,000s. The horizontal line at 1 indicates convergence to Zipf's law.

Figure a. 4. Average Wage and Population Size, 2017.



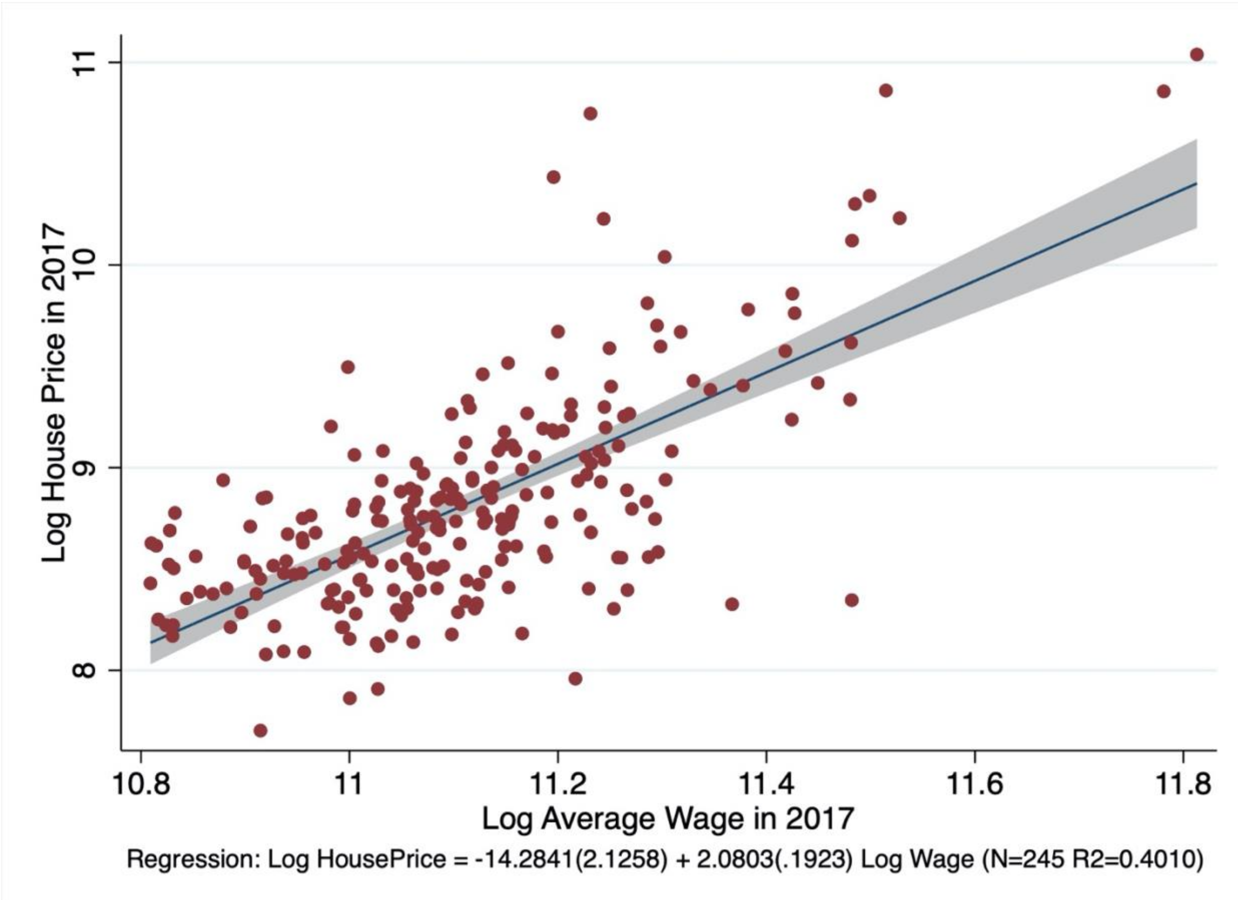
Note: Sample restricts to cities at prefecture level and above. Robust standard errors are in parentheses. Population data are measured in 10,000s and average wage measured in Chinese Yuan. All data are from NSB (2017).

Figure a. 5. Average House Price and Population Size, 2017.



Note: Sample restricts to cities at prefecture level and above. Robust standard errors are in parentheses. Population data are from NSB (2017) and measured in 10,000s. House price data are compiled from CREA (2017) and measured in Chinese Yuan per square meter.

Figure a. 6. House Price and Wage, 2017.



Note: Sample restricts to cities at prefecture level and above. Robust standard errors are in parentheses. Data on average wages are from NSB (2017) and measured in Chinese Yuan. House prices are from CREA (2017) and measured in Chinese Yuan per square meter.

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